Implementation through adaptation: Relations between mathematics, programming and computational thinking

Morten Misfeldt,[1] Thomas Brahe,[1] Uffe Thomas Jankvist,[2] Raimundo Elicer,[1] Eirini Geraniou,[3] Kajsa Bråting[4] and Andreas Lindenskov Tamborg[1]

[1]University of Copenhagen, Centre for Digital Education, Denmark; misfeldt@ind.ku.dk, tbrahe@ind.ku.dk, reli@ind.ku.dk, andreas_tamborg@ind.ku.dk

[2]Aarhus University, Danish School of Education, Denmark; utj@edu.au.dk

[3]University College London, Institute of Education, United Kingdom; e.geraniou@ucl.ac.uk

[4]Uppsala University, Department of Education, Sweden; kajsa.brating@edu.uu.se

This paper examines the experience of three teachers in developing teaching resources for programming and computational thinking (PCT) as part of a project that integrates mathematics and PCT. Using interviews, we explore the teachers' understanding of this integration and their approach to transmitting it through teaching sequences. The findings reveal that the teachers face challenges in exerting agency over the innovation and adapting it for implementation, affecting their ability to integrate PCT and mathematics effectively.

Keywords: Co-creation, computational thinking, implementation, mathematical competencies.

Introduction

There is growing interest in integrating programming and computational thinking (PCT) into primary and secondary education. In Denmark, a pilot project was launched in 2018 to teach "technology comprehension" to K-9 students in 46 schools, using a curriculum focused on four areas: digital empowerment, digital design, computational thinking, and technological agency. The project tested two approaches: technology comprehension as an independent subject and as part of traditional subjects, including mathematics. The evaluation showed that teachers largely followed the materials developed, thus highlighting the importance of teaching resources for successful implementations (Rambøll, 2021). Analysis of existing materials shows varying levels of integration between programming and mathematics (Elicer & Tamborg, 2022; Elicer et al., 2023). The authors of this paper have conducted comparative explorations of PCT integration in Denmark, England, and Sweden (Elicer et al. 2023), accompanied by a number of design experiments testing preliminary findings in Danish classrooms (Elicer et al., 2022). The findings from these experiments were used to initiate a co-creation process with five teachers, who were to develop resources intended to take advantage of the explored synergies between mathematics and PCT. The development process of these resources is the outset of this paper, in which we focus on the experience of three of the five teachers engaged in the work. We aim at answering the following question: *How do the three teachers* *adapt the idea of integrating PCT and mathematics in their design of teaching resources, and what considerations did they have in this respect?*

In order to study conceptual integration, we have based ourselves in the emerging literature of implementation research in mathematics education (Koichu et al., 2021). The idea is that challenges with integrating PCT and mathematics can be viewed as a resource for implementation and change. In the following, we first describe in detail the interplay between PCT and mathematics as well as the adopted framework of co-creation implementation. Next, we describe the empirical bases and results. Finally, we discuss potential challenges with PCT in the Danish mathematics classrooms.

Programming, computational thinking and mathematics

A number of characterizations of computational thinking already exists. Yet, they generally refer to a collection of knowledge and skills to solve problems modeled in such a way that problems can be unambiguously communicated to and solved by a computer (Shute et al., 2017; Wing, 2006). Wing (2006) describes computational thinking as decomposition, data representation, pattern recognition, abstractions, and algorithms. Here, we refer to programming and computational thinking (PCT) in order to stress the inclusion of actual programming in our focus. This is related to mathematics, of course, e.g., through problem solving, data handling, and modeling.

Elicer and Tamborg (2022) developed six categories or "levels" describing the integration between mathematics and PCT. These are: (1) no mathematics involved; (2) no PCT involved; (3) mathematics as a context; (4) PCT as a context; (5) conceptual integration involving concepts from both mathematics and PCT in the scene-setting and wrap-up, but not necessarily in the specific solution methods involved; and (6) operational integration where mathematics and PCT competencies are interdependent. Building on a competence approach to mathematics (Niss & Højgaard, 2019), these categories are defined by distinguishing actions and concepts in both mathematics and PCT as depicted in available teaching materials. In turn, we aim at using them for the production of new ones.

Implementation as collaborative creation

In education, there is always a continuity between implementation and creation. As stated by Bruckheimer (1979), "Generally, curricula are not implemented" (p. 43). He continues to argue that implementation should be regarded as a dynamic process, and that the process of adoption is also a process of manipulation, adaptation and additions. Jankvist et al. (2021) argue that "the actual implementations should themselves become creative" (p. 4). In order to understand implementation in this way, we build on a definition of implementation developed by Koichu et al. (2021). They conceptualize implementation as an *ecological disruption* to state of affairs that happens through a combination of: "(a) endorsements of innovations; (b) an action plan; and (c) a perceived problem or challenge that can be addressed through (a) and (b)" (p. 986). Koichu and colleagues (2021) describe the different stakeholders as *proponents* of an innovation and innovation plan and *adapters* of the innovation, respectively. Innovation adapters will go through a process in four parts:

> (1) constructing agency over the innovation, (2) gradually changing within-community communication or across-community communication, (3) gradually changing practice so that it accommodates the innovation, (4) adapting the innovation to their needs and aspirations. (p. 986)

This does, however, also mean that there is a constant interplay between proponents and adapters. Hence, neither plan nor innovation is left unaffected by the implementation process. Building on the four sub-processes from Koichu et al. (2021) and the six categories from Elicer and Tamborg (2022), we aim to understand how our three teachers combine elements of PCT and mathematics in the various processes of developing teaching resources.

Method

The empirical foundation for this paper consists of interviews conducted with three teachers to develop resources and was led by the second author of this paper. The teachers were given: 1) relative freedom on deciding and developing the themes of the resources; 2) support for developing their ideas and discuss the integration of PCT and mathematics with the researchers from the team; and 3) a clear objective to create tasks and teaching scenarios that require combination of PCT and mathematics. In the time period dedicated for the teachers to develop their resources, three workshops were held where members from the research team jointly discussed what integrating PCT and mathematics means.

Despite the large degree of "freedom" and open-endedness in how to scope the resources, we asked the teachers specifically to address the interplay between PCT and mathematics. We were interested in how this setup led practitioners to reflect, develop and find solutions and attach meaning to this integration. Looking at the first draft of the resources, it became clear that the teachers used rather different approaches and levels of integration between PCT and mathematics.

The interview study

In order to understand the teachers' difficulties, the second author conducted interviews with the three teachers, henceforth referred as Michael, John and Lucas. Michael was a mathematics teacher in his 40s with around 20 years of experience. He had no previous experience as an author of teaching resources, but broad experience in teaching technology, albeit not programming. John, a younger mathematics teacher in his 30s, had experience with teaching programming and as author of teaching resources about block programming. Lucas was in his 50s and had a mixed background. He graduated as a teacher in the mid 1990s and always had an interest in integrating programming and computational thinking into school subjects. After ten years as a teacher he began as a developer of educational technologies for children. The interviews were audio recorded and later transcribed verbatim for analysis. A semi-structured approach was developed in order to explore the participants' experiences and conception. The interview guide was focused on the interplay between mathematics and PCT. We analyzed the interviews in a thematic manner (Braun & Clarke, 2006), in which we began by identifying their perception of the synergy between PCT and mathematics. Subsequently, we identified the teachers' conception of the implementation, and what they believed to be needed to support the implementation of the teaching resources they had produced.

The synergy between PCT and mathematics

The synergy between PCT and mathematics was difficult for the teachers to use and adapt. Michael pointed to a specific presentation slide describing the computational thinker and said: "It was only really clear to me when you showed me the slide" because "it was very practical. The name 'construct' for example. Exactly that word helped me." This indicates both difficulties in the operationalization of the synergy, i.e., levels 5–6 in Elicer and Tamborg's (2022) integration model—but it also says something about the value of tangible manifestations. According to Michael, what PCT can offer is actually not unique to mathematics:

> Michael: I would rather say that PCT provides something special in ALL subjects. I understand why we are doing it in mathematics due to the official curricula, but the exploring approach to create something physical, I think is a fit for all subjects.

Michael believed that the creation of tangible solutions to problems should be a guiding principle in all education. He traced this back to his childhood, building things with his father, where they learned from mistakes and adjusted their work until they succeeded. This hands-on approach is evident in his teaching resource, where he leads students on a fictional journey through a dystopian universe that requires them to solve tasks and survive through hands-on work.

For John, the connection between PCT and mathematics was more artificial.

> John: In what I ended up working with, I actually found it difficult to integrate mathematics and programming. It has been more like a project, where the students are working on an innovative project, and I was careful not to tell them "You MUST use a specific mathematical concept, or you MUST use a specific way to program." So, I left it open for the students to think along those lines themselves.

In his first iterations, John presented a specific model for project-oriented work without any relation to PCT and mathematics. This may be due to his general educational philosophy with a high degree of open-endedness for the students to work within. Still, it could also signal that he found it difficult to integrate PCT into mathematics. He touches upon this, later in the interview, when talking about the transferability to other teachers, who may have different backgrounds and different prerequisites:

> John: (...) We talked about the teacher guide and whether it makes sense to other teachers, or only to me. Does it make sense? And the first time that we talked, you were talking about

mathematics and analog to digital coding… And now, I've ended up in a completely different place without the concrete coding part and without the concrete mathematics part. It is a more diffuse way of working.

For John, the interplay between PCT and mathematics was not easy to plan. It takes reflection, work and re-thinking in order to arrive at a higher level of integration with regards to Elicer and Tamborg's (2022) integration model and to get past level 3, where for example mathematics is used as a context to a programming task without adding anything from a mathematical learning perspective.

Lucas, on the other hand, immediately seemed to adopt the idea of integrating PCT into mathematics. He had a background as a mathematics teacher in primary school, and he had worked for more than 15 years developing educational technology in the industry. At first, the interview touched on how to teach programming and technology in general:

Lucas: In my opinion, a lot of it [PCT] comes by itself if you provide the students with a concrete problem or task that they have to solve. I think that the best way to learn programming and computational thinking is to actually do it and try it out.

His approach was centered round small, simple scenarios in which there is a problem for students to solve. For example, in one scenario, a girl wants to build a treat-dispensing robot for her dog. The student use a Lego Spike set and a Scratch-like programming environment to program the robot to dispense a treat when a sensor detected a certain distance. This task required both PCT and mathematical skills, and therefore demonstrated level 6 integration (Elicer & Tamborg, 2022), where PCT and mathematics are tightly intertwined and a solution requires both.

The resources as implementation

An important aspect of the integration between PCT and mathematics is the approach to making its use as clear as possible to practitioners. As seen above, each teacher had his own personal background, making up an important bag of knowledge, experience, values and worldviews. To varying degrees, this may have helped shape the resources and the underlying and implicit demands for uses to possess. Hence, the question is how the teachers reflected on this aspect with regard to making the resources accessible. For Michael, there is a certain degree of self-contradiction in that a resource calls for innovation and at the same time has a detailed set of instructions or guidelines for the users—but he also acknowledges the need for such guidelines:

Michael: (...) Of course, the teacher guide has huge implications. If it [the resource] seems too unmanageable, some teachers may give it a pass.

The view on teacher guiding was one of the central themes in the interview guide as part of our assessment of the path from initial idea to eventual implementation. This in particular with regard to the PCT area that is highly demanding from a technological point of view. As for John, he sees a potential implementation problem that may stem from the basic structures that mathematics teachers are working within; a mathematics teacher is required to complete or touch upon certain competency areas during a school year. Hence, adding PCT, which the teacher may not be familiar with, can prove to be an obstacle that some want to avoid:

John: Certainly, there are teachers who, from the outset of the school year, have made specific plans as to what to accomplish and when: "We are going to slavishly follow the textbook." In math, that is pretty easy, since all schools have a textbook system for the individual age groups and classes. It is easy to go: "Well then, we'll go with the mathematics textbook and don't look back."

This is potentially important to remember when developing resources that are trying to "shake things up" by implementing new ways of working and combining subject areas, i.e., a lot hinges on the individual teacher's personal preferences. Hence, having a very stringent focus on teacher guiding is crucial. It needs to be easy and accessible. Along those lines, Lucas provided a similar perspective and mentioned the lack of programming in the Danish teacher education (as he recalled it):

Lucas: One could argue that it is a problem that is amplified by the fact that Danish teachers are not trained in programming. During the teacher training, you are not provided with a coherent and elaborate course in how to program—and that creates uncertainty. They don't believe that they can. Hence, they don't believe that their students can.

Now, Lucas utterances are of course based on his personal beliefs and experiences. Yet, he does touch upon a theme that may be a real-life barrier in some cases where individual teachers choose not to use PCT resources because of lack of experience with (basic) programming.

Discussion: Implementing the interplay between PCT and mathematics

Even if teachers agree on the potential value of integration of PCT into mathematics education, other factors (including prior experiences, expertise, beliefs and motivations) come into play, when we aim for implementing the interplay between PCT and mathematics in teaching practices and resources.

Applying Koichu et al.'s (2021) implementation framework, we recognized the challenges that the three *innovation adapters*, John, Michael and Lucas, were faced with, as well as their views and strengths. In terms of the first stage in the implementation process, i.e., **(1) constructing agency over the innovation,** both John and Michael had difficulties with constructing agency over the strong interrelation between PCT and mathematics. For both of them, the interplay between mathematics and PCT was considered rather abstract. On the contrary, Lucas found this first stage easier, since he had much more experience in developing resources as compared to John and Michael. In addition, he seemed to have already internalized the interplay between PCT and mathematics. Moving on to the second stage, **(2) gradually changing within-community communication or across-community**

communication, the co-designing and co-editing work, and the ongoing debate with the second author, did gradually change John's and Michael's resources in that the sequences and tasks developed a closer and more interdependent relationship between PCT and mathematics. This, quite crucially, also changed their conception of the interplay between PCT and mathematics in their own practice and the teaching practice in general. Over the duration of the project, the communication and interactions between the researchers and John and Michael progressed from "telling" John and Michael about how PCT and mathematics integration may be achieved to offering feedback on their ideas and prototypes of teaching materials. This process was not required for Lucas. Lucas was able to accommodate the PCT and mathematics integration to his existing ways of thinking about teaching resources. Regarding the third stage in the implementation process, **(3) gradually changing practice so that it accommodates the innovation,** we find that the main problem for John and Michael was the lack of time for achieving a change in their teaching practice in order to accommodate the integration of PCT into mathematics. This of course was not an issue for Lucas, as the innovation in question was already embedded (to a certain level) in his practice. Finally, the fourth stage in the implementation process involves **(4) adapting the innovation to their needs and aspirations.** This happened in all three cases. John relied on students finding the meeting points between PCT and mathematics on their own, while he wanted to have a relatively "hands-off" approach. He adapted the integration of PCT to mathematics, as his aspirations were to prepare the ground for project-oriented pedagogy. Michael adapted the integration between PCT and mathematics in the direction of narrative teaching. Finally, for Lucas the whole idea of PCT-mathematics integration was already part of his way of thinking. He adapted the integration in question in a direction of problem solving scenarios, closely aligned with how he had worked with the development of resources before.

The question that navigated the work presented in this paper was: "*How do the three teachers adapt the idea of integrating PCT and mathematics in their design of teaching resources, and what considerations did they have in this respect?*" Co-designing resources with researchers was a valuable, yet complex exercise for the participant teachers, which also revealed the different needs each teacher had. An awareness of these challenges and difficulties, as well as the contributions and expertise offered by researchers, places us in as position towards integrating PCT into mathematics education on a basis informed and enlightened by implementation research.

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