Exploratory analysis of a survey item designed to capture teachers' mathematical digital competency in geometry

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The use of dynamic geometry environments to support the teaching and learning of geometry has been relatively well-researched. However, by comparison, teachers' competencies in using digital technology in their teaching of geometry have not been widely investigated. We carried out a small-scale research study investigating mathematics teachers' competencies in teaching with technology. We designed and trialled a prototype survey for assessing teachers' mathematical digital competency (MDC) with 114 preservice and early career teachers, supported by follow-up interviews with six survey respondents. In this paper, we focus on analysing teachers' responses to one survey item designed to capture teachers' competencies in using dynamic geometry software in the teaching of circle theorems. We present initial findings and discuss what this reveals about teachers' competencies in using digital technology in their teaching of geometry.

Keywords: Mathematical digital competency, geometry, teacher survey design, digital technology, circle theorems.

Introduction

The introduction to the TWG04 papers from CERME12 called for further research on the geometrical knowledge needed by teachers (Brunheira et al., 2022). This paper responds to the call by investigating teachers' competencies in using digital technology in their teaching of geometry. Papers from TWG04 highlight a plethora of tools, including digital technologies, for the teaching and learning of geometry (Bimova et al., 2023; Palatnik, 2023; Papadaki, 2023). However, relatively little is known about the competencies that mathematics teachers need to integrate such tools, and digital technologies in particular, into their teaching. We have thus been interested in investigating how to conceptualise, assess and support mathematics teachers' Mathematical Digital Competency (MDC; Geraniou & Jankvist, 2019) for teaching with the use of digital technologies. The use of dynamic geometry environments in teaching circle theorems has been relatively well-researched (Bozkurt & Ruthven, 2017; Bretscher, 2022; Komatsu & Jones, 2020; Ruthven et al., 2008), showing that this is a potentially fertile context for investigating teachers' mathematical digital competency in the subject-specific context of teaching geometry. Hence our research question is: *How does a survey item designed around using dynamic geometry software to teach circle theorems capture teachers' mathematical digital competency in geometry?*

In the next section, we explain how we conceptualise teachers' mathematical digital competency (MDC) and contextualise elements of MDC within the teaching of geometry. We then describe the design and implementation of a prototype survey for assessing teachers' MDC with 114 preservice and early career teachers, focusing on one item situated in the context of using dynamic geometry software to teach circle theorems. Finally, we analyse teachers' survey responses to this item, supported by interview data, and present the initial findings in relation to our research question.

Theoretical background

We chose to define mathematical digital competency for teaching (MDCT) in terms of the set of elements proposed by Geraniou et al. (2022) as follows:

- MDCT1: Being able to engage in a techno-mathematical discourse at a meta-pedagogic level.
- MDCT2: Being aware of which digital tools to apply within different mathematical situations and context, and being aware of the different tools' capabilities and limitations, so as to think, and act, pedagogically with these tools, while considering the benefits and limitations of these.
- MDCT3: Being able to use digital technology reflectively in problem solving and when doing (learning or teaching) mathematics. (Geraniou et al., 2022)

These elements were derived from Geraniou and Jankvist's (2019) characterisation of mathematical digital competency for students and informed by Niss and Højgaard's (2019) definition of mathematical competence as "someone's insightful readiness to act appropriately in response to all kinds of mathematical challenges pertaining to given situations" (p. 12).

MCDT1 relates to how teachers support students to communicate their geometrical thinking in the context of dynamic geometry environments. For example, Ruthven et al. (2008) highlight how teachers *incorporate dynamic manipulation into mathematical discourse* by developing vocabulary to talk about dragging and, more rarely, *privilege a mathematical register for framing figural properties*. Healy et al (1994) found that the idea of "messing-up" through dragging was powerful in providing a mutual language for teacher and students to distinguish between a drawing and a figure in dynamic geometry environments. MDCT2 relates to teachers' decision-making about whether and how to employ dynamic geometry environments in lessons and their awareness of the constraints and affordances of such software for teaching geometry. For example, Ruthven et al. (2008) describe how teachers made decisions to either conceal or capitalise on the appearance of rounding errors and issues with angle measurement when using dynamic geometry environments. Komatsu and Jones (2020) also identify ways in which teachers capitalise on affordances of both dynamic geometry environments and physical tools to support students in proving circle theorems. Similarly, Bretscher (2022) analyses teachers' decision-making about how to manage transitions within and beyond dynamic geometry environments for the purposes of teaching circle theorems. MDCT3 relates to how teachers support their students to reason about and solve geometrical problems using dynamic geometry environments. For example, Bozkurt and Ruthven (2017) describe the predict-and-test routine developed by a teacher to encourage his students in confirming or refining their geometrical conjectures using a dynamic geometry environment.

Methods

The prototype survey consisted of three items designed to capture teachers' MDCT, with each item mapped to at least one of the MDCT elements. In addition, Thurm and Barzel's (2022) items were included to survey teachers' beliefs about the use of technology in mathematics teaching and learning, however these are not discussed in this paper. The survey was piloted initially with 11 mathematics educators, colleagues of the authors also based at UCL. Based on the feedback we received from mathematics educator colleagues, we refined the MDCT items and trialled them with 114 pre-service and early-career teachers, studying on three different Initial Teacher Education (ITE) programmes at a University in England. Of our sample, 49 pre-service teachers were studying on the one-year Post-graduate Certificate of Education (PGCE) Mathematics programme. These pre-service teachers typically have an undergraduate degree in mathematics or a mathematics-related subject. Ten pre-service teachers were studying on the PGCE Physics with Mathematics programme. These pre-service teachers primarily focus on teaching physics, but also receive input on teaching mathematics, and their undergraduate degrees are typically mathematics-related, either in physics or engineering. Finally, 55 early-career teachers were in the second year of the two-year Post-graduate Diploma in Education (PGDE) programme and were teaching mathematics nearly full-time in school. These early-career teachers therefore had at least an extra year of teaching experience compared to the pre-service teachers. Early-career teachers on the PGDE programme have a more varied undergraduate background: the majority (30/55) had non-mathematics-related degrees; 14 had mathematics-related degrees; and 11 had mathematics degrees. We invited survey respondents to volunteer to be interviewed using financial incentives. Altogether we interviewed six survey respondents: three from PGDE, two from PGCE Mathematics and one from PGCE Physics with Mathematics (PWM). Survey data was analysed using descriptive statistics. The six teachers who participated in qualitative interviews, helped us gain insight into their survey responses and how the tool supported their reflection on practice and critical research engagement. The sample information is summarised in Table 1.

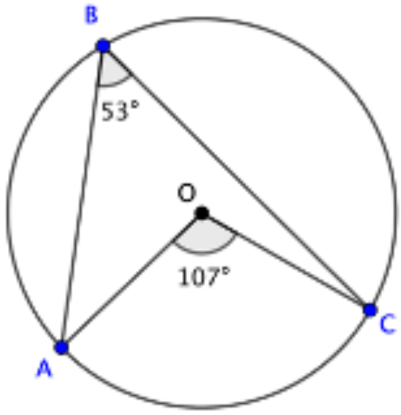
Cohort	**Data Collection**	**Data**
PGDE Mathematics	April 2022	Survey (55)
PGCE Mathematics	June 2022	Survey (49)
PGCE Physics with Mathematics	June 2022	Survey (10)
PGCE Mathematics – Oliver & Wanda	June 2022	Interview
PGCE Physics with Mathematics - Daniel	July 2022	Interview
PGDE programme – Rita, Tim & Richard	July 2022	Interview

Table 1. Sample information

The overall design for the three MDCT items comprised a vignette approach (Skilling & Stylianides, 2020) involving classroom scenarios (or *vignettes*). For each scenario, teachers were presented with several potential teaching approaches in response to the scenario and asked to (1) rate how likely they were to use each approach on a 5-point scale from ‘very unlikely’ to ‘very likely’; (2) select their preferred approach; (3) select the approach an expert teacher would use and (4) explain their selection of preferred and expert teacher approach in an open-response format. The MDCT1 item was based on a scenario inviting an immediate teacher response to two pupils’ coding solutions using a programming language, Scratch, to construct regular polygons. The item mapped to MDCT3 was based on a scenario involving order of operations, specifically how teachers would address pupil errors in entering calculations into a calculator. This paper focuses on the design and analysis of teacher responses to the MDCT2 item (see Figure 1), which was based on a scenario inviting a teacher response to the appearance of rounding errors when using a dynamic geometry app in teaching circle theorems.

Q.1. Dynamic Geometry

<u>**Context**</u>
Year: 10 (GCSE)
Attainment: High
Mathematical Topic: Circle Theorems
Learning Goal: To understand the Circle Theorem **"**the angle subtended at the circumference is twice the angle subtended at the centre of the circle".
Tool/Software: Dynamic Geometry App
Scenario: You drag point A so that the angles AOC and ABC change to demonstrate the doubling relationship. As you do so, rounding errors appear, as shown in the diagram.



1. How likely are you to use any of the following approaches?
(a) Tell your pupils to ignore the rounding error: it is just a glitch in the software.
(b) Show pupils that increasing the number of decimal places corrects the rounding error.
(c) Use the rounding error to justify the need for a mathematical proof of the circle theorem.
(d) Use the rounding error to make a connection with the topic of upper and lower bounds, for example to estimate what the angle could have been.

[very unlikely - unlikely - neither - likely - very likely]

2. Which of the above approaches are you most likely to use and why?
3. Which of the above approaches would an expert teacher most likely use and why?
(Optional) If your preferred approach is different from (a) to (d), what would that be and why?

Figure 1: MDCT2 survey item based on using a dynamic geometry app in teaching circle theorems

The scenario design for the MDCT2 item was based upon research highlighting the appearance of rounding errors as a challenge that teachers regularly encounter when using dynamic geometry environments (e.g., Bozkurt & Ruthven, 2017; Ruthven et al., 2008). Circle theorems was selected since it is a topic which is commonly identified with the use of dynamic geometry environments in England (Ruthven et al., 2008). The design of potential teaching approaches was informed by Ruthven et al.'s (2008) identification of two distinct approaches: either concealing or capitalising on rounding errors (and other issues) to support geometrical learning. Potential teaching approach (a) 'tell your pupils to ignore the rounding error' was intended to convey a concealing approach suggestive of a low level of MDCT2. Informed by Bretscher's (2022) analysis of teacher interview-responses to a similar scenario, potential teaching approaches (b)-(d) were intended to capture more or less well-developed capitalising approaches indicating higher or lower degrees of MDCT2, see Figure 1. For example, approach (b) was intended as a less well-developed approach since selecting 'show pupils that increasing the number of decimal places corrects the rounding error' indicates a teachers' awareness of the limitations of the tool and the value of making pupils aware of such limitations but does not explicitly connect with the geometrical topic of circle theorems. Approach (d) 'use the rounding error to make a connection with the topic of upper and lower bounds' does make a mathematical connection with the issue of rounding errors but draws attention to issues of measurement rather than prompting a geometrical interpretation. Finally, approach (c) 'Use the rounding error to justify the need for a mathematical proof of the circle theorem' was intended to be the most well-developed approach, capitalising on rounding errors to promote *mathematically disciplined interaction through the software* (Ruthven et al., 2008).

Discussion

Our first finding is that the classroom scenario of managing rounding errors whilst using a dynamic geometry app in teaching circle theorems in the MDCT2 item was appropriate, enabling insight into pre-service and early career teachers' competencies in using digital technology in their teaching of geometry. Feedback from mathematics educator colleagues indicated the classroom scenario had good face validity. Interviews with six survey-respondents showed that the scenario was realistic and contained sufficient information to elicit teachers' decision-making about how to employ a dynamic geometry app in lessons based on their awareness of the constraints and affordances of such software for teaching geometry. However, due to their limited classroom experience, some pre-service and early career teachers (Wanda, Rita) stated they had not yet had an opportunity to teach circle theorems, which they felt limited their ability to reason about the scenario. In particular, in the English context, learning to prove the circle theorems is usually limited to the highest-attaining pupils and, in some schools, teaching such pupils is reserved for more experienced teachers. Hence whilst this scenario was challenging for pre-service and early career teachers, they were able to articulate their pedagogic reasoning, and this suggests the MDCT2 item might also be appropriate for eliciting more experienced teachers' decision-making.

Our second finding is that the multiple-choice options for the MDCT2 item need further development and, in particular, better 'distractors' need to be identified. For example, only 8.8% chose approach (a) 'tell your pupils to ignore the rounding error' as their most likely approach. This result coupled with interview data suggests that this option may be too easy to dismiss as the 'wrong' answer. For example, one interviewee explained:

> Daniel: It's a terrible approach, you should never tell a student to ignore anything. Every question they have is a worthwhile question and telling them 'don't worry about that' discourages them from asking questions in future. It doesn't actually address the fact that they're right: it is 53, 107 so it's not double. You shouldn't tell them to just ignore that when previously you've told them this circle theorem is true and suddenly they've found a counterexample.

Approach (a) was intended to convey a concealing approach (Ruthven et al., 2008) to the rounding error, suggesting a low-level of MDCT2. However, Ruthven et al. (2008, p.312) reported that the teacher in their study took "great care" to avoid such anomalies through "vigilant dragging" to avoid confusion with pupils of below-average attainment. Instead, such a painstaking approach to conceal rounding errors could perhaps be interpreted as a well-developed concealing strategy, showing a high-level of MDCT2. For example, such an approach could be justified in terms of the teacher's knowledge of their pupils intertwined with knowledge of using digital technology to teach geometry in their specific curricular context. Other interviewees (Richard, Tim) chose (b) as their most likely approach based on a *cost-benefit analysis* (Ruthven et al., 2008) that there wasn't sufficient benefit for their pupils in spending time on circle theorem proofs. If survey items are to be effective in assessing teachers' competencies in using digital technology in their teaching of geometry, then multiple-choice options may need to capture the pedagogical reasoning that underpins teachers' choice of teaching approach.

Relatedly, our third finding is that pedagogical reasoning underpinning the selection of teaching approach may be more important than the actual teaching approach chosen in terms of assessing teachers' MDCT for geometry. For the MDCT2 item, around half the survey respondents indicated approach (b) 'show pupils that increasing the number of decimal places corrects the rounding error' as their most likely approach. This was not surprising given that about 80% reported that they were likely or very likely to adopt approach (b). However, 46.5% responded they were likely or very likely to use approach (c) 'use the rounding error to justify the need for a mathematical proof of the circle theorem' and 38% responded they were likely or very likely to use approach (d) 'use the rounding error to make a connection with the topic of upper and lower bounds'. This means that some survey respondents were saying that they were likely or very likely to use several of these approaches, perhaps in combination. In addition, approaches (c, 43.9%) and (d, 39.5%) were the most popular response for the approach an expert teacher would most likely use, with approach (b) only (12.3%). This suggests that some survey respondents selected (b) as their most likely approach due to a lack of confidence or experience, although believing that (c) and (d) were 'better' approaches. This interpretation was supported by evidence from interview data:

> Wanda: I would just say it's a rounding error because I wouldn't know how else to explain it to students. I don't know enough about GeoGebra, like how do they do their rounding, to justify to students as to why that isn't 106 degrees. So that kind of links in with the [approach (b)], showing people that increasing number of decimal places corrects the error, I've just never done that myself before or that's never been shown to me, so it just wasn't a thought that I had before I had seen this example. Using a rounding error to justify the need for a mathematical proof, again if I was more confident, it would be a really nice way to introduce the theorem because you could say okay well we've said this, this is an example where it doesn't work let's think as to why. So I think it would be a nice activity kind of to unpack all those concepts, but just I don't necessarily feel confident in it and then using a rounding error to make the link about the upper and lower bounds I had never even thought of that as a connection.

Approach (d) was relatively popular as an expert teacher response, which we found surprising because it was intended as a less well-developed capitalising approach compared to (c). Its popularity could be due to the novelty of the approach or due to its position as the last option. However, interview data suggests there may be valid pedagogical reasons for teachers selecting this response:

> Daniel: The circle theorem is true in maths. So is everything else, like trigonometry etc, but whenever it comes to actually taking real life measurements that are real life application of maths, nothing's quite perfect. And so there will always be some sort of rounding error and that's what you're doing in real life and so [the app] really allows you to show that the circle theorem can be true, and there can be a rounding error, at the same time. And that way when the students eventually leave school, not that many of them will go on to become mathematicians, but all of them will have an appreciation of the fact that in real life when you try to apply maths which, hopefully, all of them will do at some point in their lives, it will not always match up to theory perfectly.

Daniel was a pre-service teacher on the PGCE PWM programme. His response may explain wider differences between the cohorts. For example, 50.9% of PGDE respondents selected (c) as the most likely expert approach with 38.2% selecting approach (d). Similarly, 42.9% of PGCE Mathematics respondents selected (c) as the most likely expert approach with 32.7% selecting (d). However, 80%

of PGCE PWM respondents selected (d) as the most likely expert approach. Acknowledging there were only 10 PWM teachers, these differences suggest teachers' backgrounds influence their epistemological beliefs about what is the value of using digital technology in teaching geometry. Teachers on the PGCE PWM appear to privilege applied mathematics, commensurate with their background in physics and engineering, whereas PGDE and PGCE Mathematics lean towards a more geometrically-oriented interaction through the software. Taken together, these results indicate that the quality of pedagogical reasoning behind their selection of the most likely approach might be more important in relation to assessing teachers' MDCT than the selection of a specific teaching approach.

Conclusion

Tool use can enable students to view geometrical figures in new and significant ways (Palatnik, 2023; Papadaki, 2023). However, teachers mediate tool use and the findings from this study suggest that opportunities for viewing geometrical figures depend on the quality of teachers' pedagogic reasoning. Eliciting teachers' reasoning is therefore important to better understand and develop their competencies. The vignette design (Skilling & Stylianides, 2020) of the MDCT2 item was successful in eliciting pre-service and early career teachers' competencies in using digital technology in their teaching of geometry. The classroom scenario of managing rounding errors whilst using a dynamic geometry app in teaching circle theorems in was appropriate, enabling teachers to articulate their pedagogic reasoning, revealing elements of MDCT. Comparing teacher's selection of most likely approach with their choice of expert teacher's most likely approach may reveal teachers' confidence in using dynamic geometry app in teaching circle theorems. In addition, selecting approaches (c) or (d) may indicate teachers' epistemological beliefs about the value of using digital technology in teaching geometry. However, if survey items are to be effective in assessing teachers' competencies in using digital technology in their teaching of geometry, then multiple-choice options may need to capture more nuanced pedagogical reasoning that underpins teachers' choice of teaching approach. In particular, better 'distractors' are needed that capture more and less-developed strategies for concealing anomalies such as rounding errors. Our future work, therefore, entails capturing teachers' reasoning behind the teaching approach chosen in the vignettes to confidently identify their MDCT for geometry, in a study at scale. Furthermore, the MDCT2 item was necessarily limited to one aspect of using dynamic geometry apps in teaching, namely managing (anomalies) in numerical measurement. A key affordance of dynamic geometry software is that geometrical relationships are embedded in the structure of the software. Future development should focus on designing items that capture teachers MDCT in relation to other aspects of geometrical software, including this key affordance.

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References

Bímová, D., Břehovský, J., Pirklová, P., Eichler, K-P., Seibold, M., & Mushtaq, A. (2023). Developing students' visuospatial abilities in geometry using various tangible and virtual 3D geometric models. Paper submitted to TWG04, CERME13. Hungary: Budapest.

Bozkurt, G., & Ruthven, K. (2017). Classroom-based professional expertise: a mathematics teacher's practice with technology. *Educational Studies in Mathematics*, *94*(3), 309–328. https://doi.org/10.1007/s10649-016-9732-5

Bretscher, N. (2022). Conceptualising TPACK Within Mathematics Education: Teachers' Strategies for Capitalising on Transitions Within and Beyond Dynamic Geometry Software. *Digital Experiences in Mathematics Education*, https://doi.org/10.1007/s40751-022-00115-0

Brunheira, L., Maschietto, M., Palatnik, A., & Papadaki, C. (2022). Introduction to the papers of TWG04: geometry teaching and learning. Twelfth Congress of the European Society for Research in Mathematics Education (CERME12), (pp. 688–695), Bozen-Bolzano: Italy. https://hal.science/hal-03808387v1/document

Geraniou, E., & Jankvist, U. T. (2019). Towards a definition of "mathematical digital competency." *Educational Studies in Mathematics*, *102*(1), 29–45. https://doi.org/10.1007/s10649-019-09893-8

Geraniou, E., Jankvist, U. T., Elicer, R., Tamborg, A. L., & Misfeldt, M. (2022). On mathematical digital competency for teaching: The cae of an expert teacher. In J. Medova, H-G. Weigand, & J. Trgalová (Eds.), *Proceedings of the 13th ERME Topic Conference (ETC13) – Mathematics Education in the Digital Age 3 (MEDA3)* (pp. 368–375), Nitra: Slovakia.

Healy, L., Hölzl, R., Hoyles, C. & Noss, R. (1994). "Messing up", *Micromath, 10*(1), 14–16.

Komatsu, K., & Jones, K. (2020). Interplay between paper-and-pencil activity and dynamic-geometry-environment use during generalisation and proving. *Digital Experiences in Mathematics Education*, *6*(2), 123–143. https://doi.org/10.1007/s40751-020-00067-3

Niss, M., & Højgaard, T. (2019). Mathematical competencies revisited. *Educational Studies in Mathematics*, *102*(1), 9–28. https://doi.org/10.1007/s10649-019-09903-9

Palatnik, A. (2023). Interaction of spatial and theoretical aspects in spatial geometry problem solving through construction: the case of 3D sketching. Paper submitted to TWG04, CERME13. Hungary: Budapest.

Papadaki, C. (2023). Visualisation and spatial manipulation in argumentation: creating a hypothesis. Paper submitted to TWG04, CERME13. Hungary: Budapest.

Ruthven, K., Hennessy, S., & Deaney, R. (2008). Constructions of dynamic geometry: A study of the interpretative flexibility of educational software in classroom practice. *Computers and Education*, *51*(1), 297–317. https://doi.org/10.1016/j.compedu.2007.05.013

Skilling, K., & Stylianides, G. J. (2020). Using vignettes in educational research: a framework for vignette construction. *International Journal of Research & Method in Education, 43*(5), 541–556. https://doi.org/10.1080/1743727X.2019.1704243

Thurm, D., & Barzel, B. (2022). Teaching mathematics with technology: a multidimensional analysis of teacher beliefs. *Educational Studies in Mathematics, 109,* 41–63. https://doi.org/10.1007/s10649-021-10072-x