Estimation of Channel Parameters for Port Selection in Millimeter-Wave Fluid Antenna Systems

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***Abstract*—The fluid antenna system (FAS) exploits spatial diversity by adjusting a reconfigurable antenna to a position/port with the highest channel gain, and the FAS has the potential to promote the development of 6G technology [1]. Conventionally, the optimal port is chosen when all the ports are observed and all the channels are estimated, which can be impractical when the port number is large. In this paper, we propose a new method based on the least squares regression to estimate channel parameters in a multi-ray millimeter-wave (mmWave) FAS. In particular, we first estimate the channel gains at a number of ports (shall be greater than or equal to the number of paths) and then use these estimated data to reconstruct the above channel as well as to select the FAS' best port. By using our method, the number of required channel estimates can be massively reduced. More importantly, simulation results show that our method achieves a close performance compared with the conventional method, in terms of the outage probability.**

***Index Terms*—Fluid antenna system, port selection, channel estimation, Vandermonde matrix, outage probability, 6G.**

I. Introduction

Fluid antennas are a class of antennas that are position-reconfigurable and made to exploit the spatial diversity [2], [3]. Inspired by fluid antennas, Wong *et al.* proposed a new concept termed the fluid antenna system (FAS) [4]. A FAS enables a fluid antenna to switch to the optimal reception port among N ports within a given linear space, resulting in the maximum received signal-to-noise ratio (SNR); and it could achieve a better outage probability performance compared with L-antenna maximum ratio combining (MRC) systems if N is large enough [4], [5].

It is crucial for a FAS to have a simple yet efficient port selection scheme for signal reception. The conventional approach is to observe all FAS ports' channels and then choose the optimal port with the highest channel gain. However, this scheme would be too complicated to implement in reality due to the large number of ports required in FASs. Recently, Chai *et al.* [6] proposed several port selection algorithms using machine learning and analysis approximation, which requires 10% of ports to be observed. Skouroumounis *et al.* [7] developed an LMMSE-based channel estimation method and port selection scheme that utilizes strong correlations between ports for channel estimation and port selection. The above work was based on the assumption that the environment has rich scattering so that users have independent Rayleigh channel envelopes, which may not be the case in millimeter-wave (mmWave) communications.

For mmWave FASs, wireless channels have less multipath and are more directional. As a result, a new channel model has been proposed [8]. Port selection schemes in [4], [6], [7] may not be suitable for channels that comprise both line-of-sight (LoS) and non-LoS paths, and moreover, they may underestimate the correlation's influence on the single-user FAS [9]. Therefore, it's important to investigate the FAS port selection approach based on this new channel model.

To ensure good performance, the number of ports N in FAS is usually large, so it may be impractical to select the optimal port by estimating the channels of all ports. Therefore, this paper aims to present a new port selection scheme for the FAS based on channel parameter estimation for mmWave FASs. In particular, this new scheme enables FASs to approach optimal performance using the new mmWave channel model. Since the channel parameters across all FAS ports remain the same, by estimating these parameters based on channel observations of a few ports, we can identify the channels for all ports and perform the port selection. We also investigate channel parameter estimation errors, and the influence of spacing between the observed ports on these errors, and propose an appropriate selection scheme for the ports' observation spacing. Finally, we demonstrate the efficacy of the proposed method through simulations.

The paper is organized as follows. Section II outlines the FAS model, presents the new channel model, and the port selection principle. In Section III, we present the proposed method and an observation interval selection scheme for ports based on error analysis. Simulation results are provided in Section IV. Section V concludes the paper.

II. Millimeter-Wave FAS Model

A. mmWave FAS

Fig. 1 shows point-to-point mmWave communications from a transmitter equipped with a standard antenna to a receiver equipped with a FAS. The FAS is capable of receiving signals from one position/port out of N ports that are uniformly distributed along a linear space of $W\lambda$. Here, W and λ are the normalized size of the fluid antenna and the wavelength, respectively.

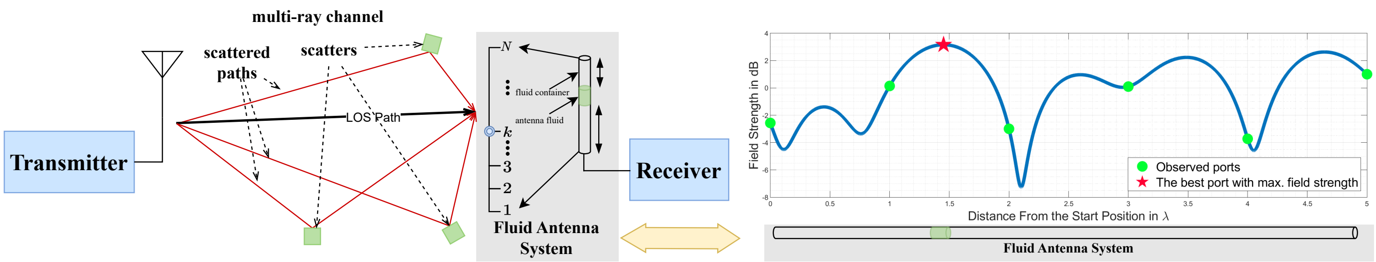

Fig. 1: Optimal port selection in a mmWave FAS system.

The received signal at the k^{th} port, without the time index, can be expressed as

$$\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_N \end{bmatrix} s + \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_N \end{bmatrix}, \qquad (1)$$

where s is the transmitted pilot signal (known at both the transmitter and the receiver), and r_k, g_k and η_k are the received signal, complex channel and complex additive white Gaussian noise (AWGN) at the k^{th} port, respectively. Moreover, η_k has zero-mean and variance of σ_η^2. Let the power gain $\mathrm{E}[|g_k|^2] = \Omega$. The average received SNR at each port Γ

$$\Gamma = \Omega \frac{E[|s|^2]}{\sigma_\eta^2}. \qquad (2)$$

We further assume that the complex channel coefficient $\{g_k\}_{\forall k}$ remains constant over a channel coherence time. During this duration, the average received signal-to-noise ratio (SNR) at the k^{th} port is related to the value of g_k.

B. mmWave Channel Model

As shown in Fig. 1, a mmWave channel model consists of a specular component (LoS) and N_p scattered components (non-LoS). The channel at each port of the fluid antenna system (FAS) is expressed as [8]

$$g_k = x_0 e^{-j\frac{2\pi(k-1)W}{N-1}\cos\theta_0} + \sum_{l=1}^{N_p} x_l e^{-j\frac{2\pi(k-1)W}{N-1}\cos\theta_l}, \qquad (3)$$

where $x_0 = \sqrt{\frac{K\Omega}{K+1}} e^{j\alpha}$, K is the Rice factor, α is the random phase of the LoS, Ω is the average energy of the channel, N_p is the number of scattered paths, x_l is the random complex coefficient of the l^{th} scattered path and satisfies $\sum_{l=1}^{N_p} |x_l|^2 = \frac{\Omega}{K+1}$. Furthermore, θ_0 and θ_l in (3) respectively represent the azimuth angles of arrival (AoAs) of the specular component and the l^{th} scattered path.

C. Optimal Port Selection

The received signal r_k is measured at port k. Since the transmitted signal s is assumed to be known at the receiver side, the estimated channel gain at the k^{th} port is

$$\hat{g}_k = \frac{r_k - \eta_k}{s}. \qquad (4)$$

If $\{\hat{g}_k\}_{\forall k}$ are estimated, then the optimal port is the port with the highest channel gain:

$$\hat{k}^* = \arg\max_k \{|\hat{g}_1|,\ |\hat{g}_2|,\ \dots,\ |\hat{g}_N|\}, \qquad (5)$$

so, if all ports are scanned and the channel state of each port is estimated, then the optimal port can be selected using formula (5). However, in practical situations, having all the estimated $\{\hat{g}_k\}_{\forall k}$ may be impossible because that requires all the $\{r_k\}_{\forall k}$ according to (4).

III. Estimation of Channel Parameters and Suboptimal Port Selection

Compared with the optimal but complicated port selection scheme in Section II-C, we propose a new suboptimal scheme if the following parameters are predefined:

1) the number of scatted paths N_p;
2) azimuth AoAs θ_0 and $\{\theta_l\}_{\forall l}$.

The above two assumptions are valid if certain signal processing techniques are used in advance, such as the multiple signal classification (MUSIC) algorithm for azimuth AoAs. Our scheme requires only a small subset of all N ports based on the mmWave channel model of (3) and is developed in the sections below.

A. Estimation of Channel Parameters

Let us first simplify (3):

$$g_k = \sum_{l=0}^{N_p} x_l e^{-j\frac{2\pi(k-1)W}{N-1}\cos\theta_l}. \qquad (6)$$

By substituting g_k of (1) with (3), we have

$$r_k = \sum_{l=0}^{N_p} x_l e^{-j\frac{2\pi(k-1)W}{N-1}\cos\theta_l} s + \eta_k. \qquad (7)$$

We then substitute r_k of (4) with (7) and have

$$\hat{g}_k = \sum_{l=0}^{N_p} x_l e^{-j\frac{2\pi(k-1)W}{N-1}\cos\theta_l} + n_k, \qquad (8)$$

where $n_k = \eta_k / s$. Eq. (8) has $N_p + 1$ unknown variables, i.e., $x_0, x_1, \cdots, x_{N_p}$. Therefore, it is required to measure at least

N_p+1 ports. If ports are observed every Δ ports from port 1, then the following equations can be established based on (8):

$$\begin{cases} \hat{g}_{i_0} = \sum_{l=0}^{N_p} \hat{x}_l + n_{i_0} \\ \hat{g}_{i_1} = \sum_{l=0}^{N_p} \hat{x}_l e^{-j\frac{2\pi\Delta W}{N-1}\cos\theta_l} + n_{i_1} \\ . \\ . \\ . \\ \hat{g}_{i_{N_{\text{obs}}}} = \sum_{l=0}^{N_p} \hat{x}_l e_l^{-j\frac{2\pi N_p \Delta W}{N-1}\cos\theta_l} + n_{i_{N_{\text{obs}}}} \end{cases}, \quad (9)$$

where $\hat{g}_{i_k}$ is the estimated channel gain of the observed k^{th} port. The method of solving equations is often used for channel estimation [10]–[12]. More importantly, if we let $\omega_l = e^{-j\frac{2\pi\Delta W}{N-1}\cos\theta_l}$, then the matrix form of (9) can be expressed as

$$\hat{\mathbf{g}} = \mathbf{W}\mathbf{x} + \mathbf{N}, \quad (10)$$

where

$$\mathbf{W} = \begin{pmatrix} 1 & 1 & \dots & 1 \\ \omega_0 & \omega_1 & \dots & \omega_{N_p} \\ \dots & \dots & \dots & \dots \\ \omega_0^{N_{\text{obs}}} & \omega_1^{N_{\text{obs}}} & \dots & \omega_{N_p}^{N_{\text{obs}}} \end{pmatrix}, \quad (11)$$

$$\hat{\mathbf{g}} = (\hat{g}_{i_1}, \hat{g}_{i_2}, \cdots, \hat{g}_{i_{N_{\text{obs}}}})^T, \quad (12)$$

$$\mathbf{x} = (x_0, x_1, \cdots, x_{N_p})^T, \quad (13)$$

$$\mathbf{N} = (n_{i_1}, n_{i_2}, \cdots, n_{\text{obs}})^T. \quad (14)$$

Note that $\mathbf{W}$ of (11) is the Vandermonde matrix and is assumed to be known in advance because of the above assumptions.

The objective is to minimize the squared errors:

$$\min_{\mathbf{x}} \|\hat{\mathbf{g}} - \mathbf{W}\mathbf{x}\|_2^2. \quad (15)$$

One common way to determine the value of $\mathbf{x}$ that achieves the minimum in (15) is to use the least square regression so $\mathbf{x}$ can be estimated by

$$\hat{\mathbf{x}} = \mathbf{W}^+\mathbf{g}, \quad (16)$$

where $\mathbf{W}^+$ is the pseudoinverse of $\mathbf{W}$. Specifically, $\mathbf{W}^+ = (\mathbf{W}^*\mathbf{W})^{-1}\mathbf{W}^*$, where $(\cdot)^*$ is the conjugate transpose operator.

B. Suboptimal Port Selection

Now we aim to find an optimal observation difference Δ in (9) that minimize the deviation between $\hat{\mathbf{x}}$ and $\mathbf{x}$:

$$\begin{aligned} &\min_{\Delta} \mathbb{E}[\|\hat{\mathbf{x}} - \mathbf{x}\|_2^2] \quad (17) \\ &\text{subject to} : \Delta \in \mathbb{N} \\ &\Delta \le \lfloor \frac{N-1}{N_{\text{obs}}} \rfloor \end{aligned}$$

$\mathbb{E}[\cdot]$ is an expectation operator. The value of Δ has to be less than or equal to $\lfloor \frac{N-1}{N_{\text{obs}}} \rfloor$. Otherwise, N_{obs} would be greater than the number of ports N. $\|\hat{\mathbf{x}} - \mathbf{x}\|_2^2$ in (17) can be further expanded:

$$\begin{aligned} \|\hat{\mathbf{x}} - \mathbf{x}\|_2^2 &= \left\|\mathbf{W}^+\hat{\mathbf{g}} - \mathbf{W}^+\mathbf{g}\right\|_2^2 = \left\|\mathbf{W}^+\mathbf{N}\right\|_2^2 \\ &= \left\|\mathbf{N}^*(\mathbf{W}^+)^*\mathbf{W}^+\mathbf{N}\right\|_2^2. \quad (18) \end{aligned}$$

By using the singular value decomposition (SVD), $\mathbf{W}$ can be written as a multiplication of three matrices $\mathbf{U}$, $\mathbf{\Sigma}$ and $\mathbf{V}$:

$$\mathbf{W} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^*, \quad (19)$$

while the inverse of $\mathbf{W}$ is

$$\mathbf{W}^+ = \mathbf{V}\mathbf{\Sigma}^+\mathbf{U}^*. \quad (20)$$

Substituting $\mathbf{W}$ of (18) with (20) gives us

$$\begin{aligned} \|\hat{\mathbf{x}} - \mathbf{x}\|_2^2 &= \left\|\mathbf{N}^*(\mathbf{W}^+)^*\mathbf{W}^+\mathbf{N}\right\| \quad (21) \\ &= \left\|\mathbf{N}^*(\mathbf{V}\mathbf{\Sigma}^+\mathbf{U}^*)^*\mathbf{V}\mathbf{\Sigma}^+\mathbf{U}^*\mathbf{N}\right\| \quad (22) \\ &= \left\|(\mathbf{U}^*\mathbf{N})^*(\mathbf{\Sigma}^+)^2\mathbf{U}^*\mathbf{N}\right\|. \end{aligned}$$

Because $\mathbf{U}$ is a rotation matrix, $\mathbb{E}[(\mathbf{U}^*\mathbf{N})^*\mathbf{U}^*\mathbf{N}] = \mathbb{E}[\mathbf{N}^*\mathbf{N}]$. Therefore, (17) is equivalent to the following problem:

$$\begin{aligned} &\min_{\Delta} \operatorname{Tr}((\mathbf{\Sigma}^+)^2) \quad (23) \\ &\text{subject to} : \Delta \in \mathbb{N} \\ &\Delta \le \lfloor \frac{N-1}{N_{\text{obs}}} \rfloor \end{aligned}$$

The above problem is a nonlinear integer programming problem, which is known to be NP-complete. In our work, we use the exhaustive search to find the optimal Δ_{opt}. Although the computational complexity of the exhaustive search is $O(N)$, we only need N_{obs} ports ($\frac{N_{\text{obs}}}{N}100\%$ of total ports) to be measured.

TABLE I: Simulation Parameters

Parameter	Value
Average SNR: Γ	30 dB
Rician factor: K	$\{1, 2, \cdots, 10\}$
Number of paths: N_p	$\{1, 2, \cdots, 5\}$
Number of observed ports: N_{obs}	$N_p + \{1, 2, \cdots, 5\}$
Phase of LoS path: α_0	2.6202
AoA of LoS path: θ_0	4.5259
Gains of scattered paths relative to the first scatter path: $\{a_l\}_{\forall l}$	$\{1, 0.9041, 2.1925$ $0.7649, 0.3823\}$
Phases of scattered paths: $\{\alpha_l\}_{\forall l}$	$\{-2.1626, 2.1802, -1.4532$ $1.7919, -0.3925\}$
AoA of scatted paths: $\{\theta_l\}_{\forall l}$	$\{0.0007, 1.8996, 0.9221$ $0.5802, 1.1703\}$

IV. Simulation Results and Discussion

In this section, simulation results are provided to evaluate the proposed mmWave FAS suboptimal port selection scheme. Common parameters are first set: $\Omega = 1$, $W = 5$, $N = 1000$. Let N_{obs} be the number of observed ports. The optimal port selection scheme is used as a benchmark or the best-case scenario. For comparison purpose, we consider four port selection schemes:

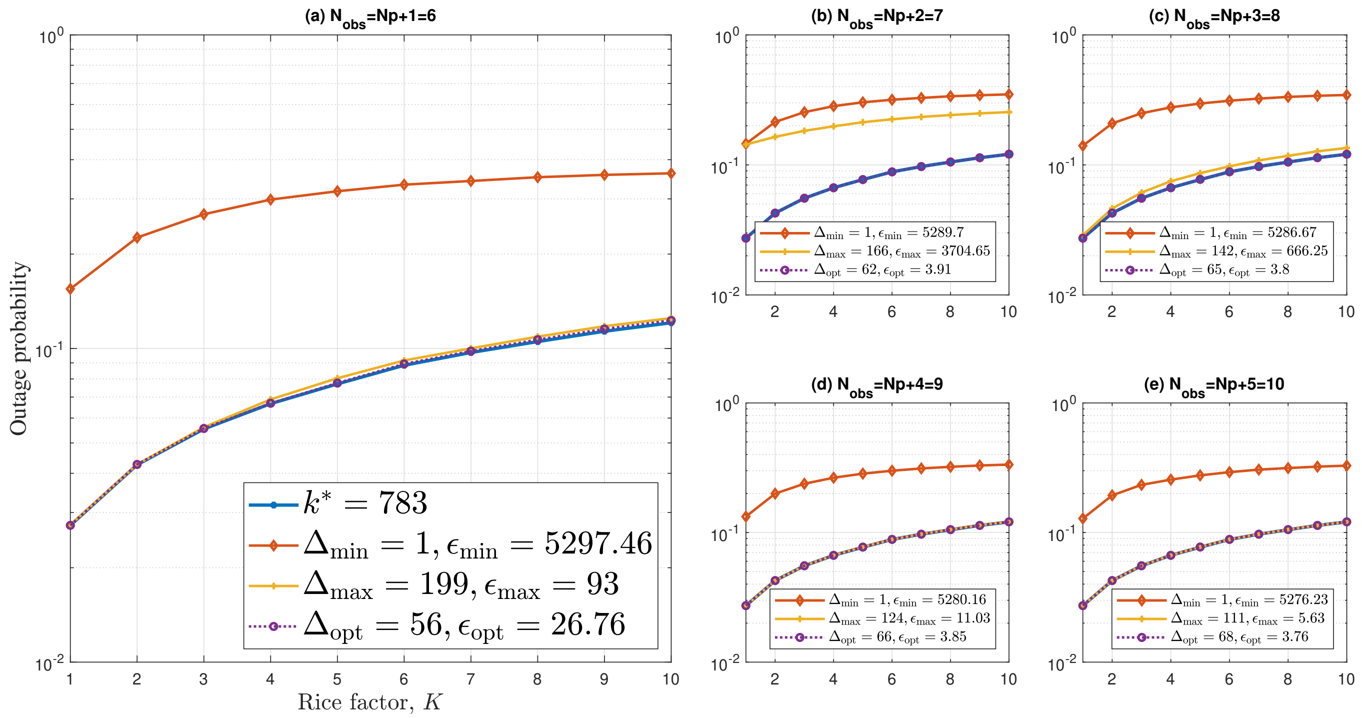

Fig. 2: Outage probability for a mmWave FAS against the rice factor.

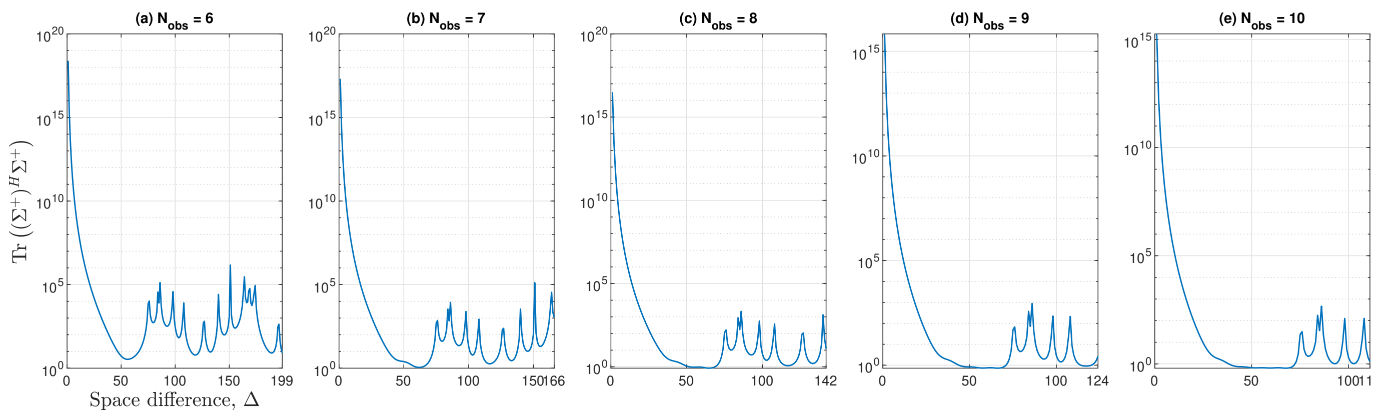

Fig. 3: When K changes,$\text{Tr}\left((\mathbf{\Sigma}(\Delta)^+)^2\right)$ changes with the index spacing of the observation port.

1) k^*: optimal port selection;
2) $\Delta_{\min}$: observed ports are adjacent;
3) $\Delta_{\max}$: observed ports are uniformly distributed from port 1 to port N;
4) Δ_{opt}: suboptimal port selection scheme.

Furthermore, we use the average difference ϵ between the estimated port index and the optimal index in each case as a performance indicator.

Fig. 2 shows the outage probability results using the above four schemes based on 1 set of random AoAs and path gains and $N_p = 5$. Other parameters are shown in Table I. The performance is the worst when $\Delta_{\min}$ is used while the performance is inconsistent when $\Delta_{\max}$ is used. On the other hand, the performance of using Δ_{opt} is consistently close the optimal port selection scheme. The performance difference among different $\Delta_{(\cdot)}$-based schemes can be explained if we investigate the $\text{Tr}\left((\mathbf{\Sigma}^+)^2\right)$ in Fig. 3. Fig. 4 shows the outage probability results based on 10 sets of random AoAs and path gains and $N_p = 5$. Similar conclusion can be made.

As the number of observation ports increases, the performance of the $\Delta_{\max}$-based port selection method might improve, and sometimes even make the performance of the FAS close to optimal. The reason can be seen in Fig. 3. As the number of observation ports increases, the value of $\text{Tr}\left((\mathbf{\Sigma}^+)^2\right)$ is likely to be very small when the observed index spacing is $\Delta_{\max}$.

Fig. 5 illustrates the outage probability based on $K = 10$ and different N_p values. Other parameters can be found in Table I as well. The $\Delta_{\min}$-based port selection scheme offers no advantage to the FAS using the port selection method. Sim-

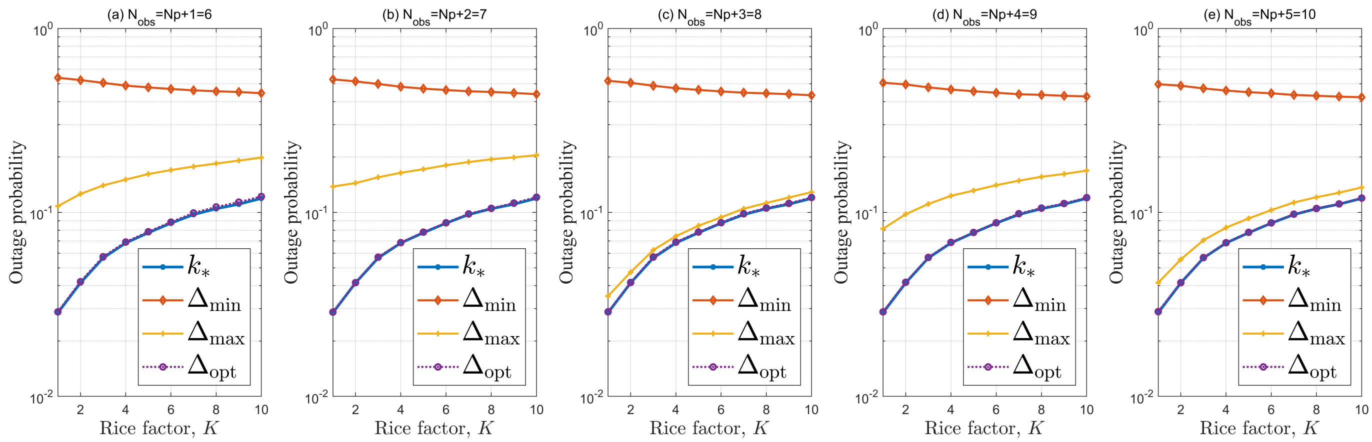

Fig. 4: Outage probability for a mmWave FAS against the rice factor when angles change.

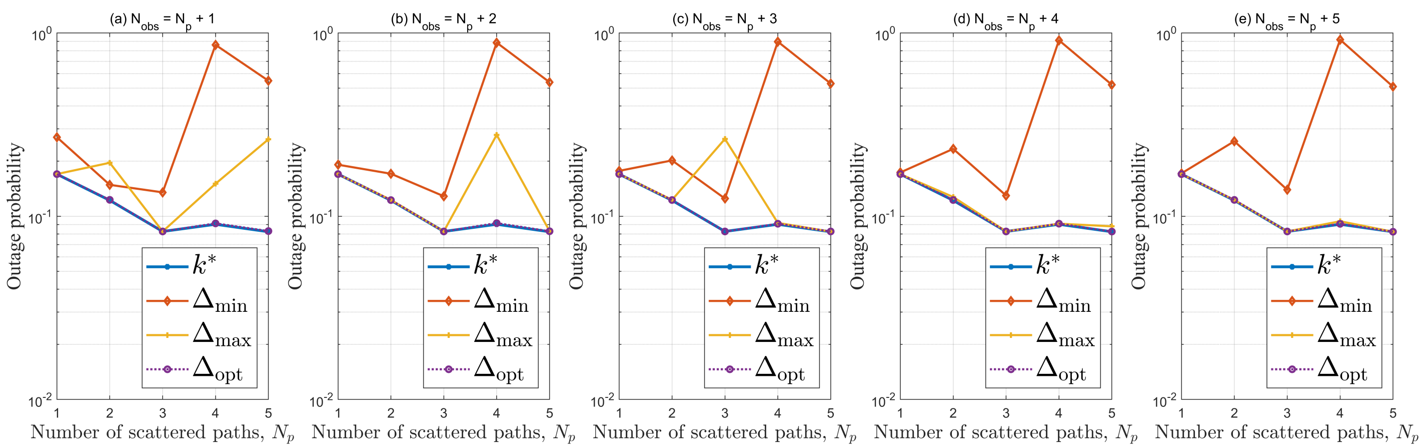

Fig. 5: Outage probability for a mmWave FAS against the number of scattered paths.

TABLE II: Selected port using optimal and sub-optimal port selection schemes with different Δ values

Number of scattered paths, N_p	k^*	Δ_{min}	Δ_{max}	Δ_{opt}
1	540	1	999	572
2	722	1	499	261
3	722	1	333	75
4	421	1	249	220
5	119	1	199	166

ilarly, The Δ_{max}-based port selection scheme cannot achieve a good result, and the gap between FAS performance and optimal performance may be significant when N_p changes. Choosing observed ports according to Δ_{opt} optimizes matrix $\mathbf{W}$, leading to the FAS port selection method offering optimal performance closely. In addition, when the number of observed ports is $N_p + 1$, the corresponding k^*, Δ_{min}, Δ_{max} and Δ_{opt} for different N_p is shown in TABLE II for reference purpose.

V. Conclusion

This paper introduced a new port selection method based on the estimation of channel parameters, which enables the millimeter-wave fluid Antenna System (FAS) to select ports by observing a number of ports as paths in the channel. Simulation results indicate that selecting the ports for observation based on an appropriate Δ yields a high degree of similarity between the FAS performance and its optimal counterpart.

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