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Disclosure of Bank-Specific Information and the Stability of Financial Systems

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We find that disclosing bank-specific information reallocates systemic risk, but whether it mitigates systemic bank runs depends on the nature of information disclosed. Disclosure reveals banks' resilience to adverse shocks and shifts systemic risk from weak to strong banks. Yet, only disclosure of banks' exposure to systemic risk can mitigate systemic bank runs because it shifts systemic risk from more vulnerable banks to those less vulnerable. Disclosure of banks' idiosyncratic shortfalls of funds does not differentiate such exposure, rendering the resultant reallocation of systemic risk ineffective in mitigating systemic runs. (*JEL* D83, G01, G21, G28)

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How to prevent systemic bank runs has been a central topic for policy makers and researchers since the 2007−2008 financial crisis, with the focus on the role of individual banks[1] in initiating and amplifying systemic risk

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[1] Our label "banks" should be interpreted broadly as financial institutions that are affected by and contribute to the liquidity of the financial system, such as investment banks, mutual funds, and hedge funds. The disclosures

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through interbank linkages. To improve the stability of the whole financial system, public disclosure of bank-specific information has subsequently become a regular occurrence, as exemplified by stress tests. However, the existing literature on regulatory disclosure either focuses on the disclosure of aggregate states or abstracts from systemic risk and the consequent strategic complementarity between investors of different banks. Thus, it does not consider that optimal disclosure depends on the nature of bank-specific information.

This paper fills this gap by studying how the disclosure of *different kinds* of *bank-specific* information affects the stability of a banking system *facing systemic risk*. Systemic risk stems from the interdependence of banks in the system: runs on one bank will adversely affect other banks. We focus on two important kinds of information: a bank's exposure to systemic risk ("systemic vulnerability" hereafter) and its idiosyncratic shortfall of funds ("idiosyncratic shortfall" hereafter). In practice, the former may correspond to the magnitude of interbank lending or to the positions of publicly traded assets, and the latter may correspond to cash and cash equivalents or to nonperforming loans of little systemic consequence. We find that the disclosure of banks' systemic vulnerabilities can mitigate *systemic* bank runs, while the disclosure of banks' idiosyncratic shortfalls cannot. This is due to a novel channel that we identify: the disclosure of banks' systemic vulnerabilities shifts systemic risk from the banks more vulnerable to it to those less vulnerable, and thus reduces the adverse impact of systemic risk on the whole banking system. But the disclosure of banks' idiosyncratic shortfalls does not generate such beneficial negative assortative matching, since it does not differentiate banks by their systemic vulnerabilities. Based on this mechanism, we characterize the properties of optimal disclosures for general policy objectives. Our analysis provides insights into what and how bank-specific information should be disclosed to enhance financial stability.

Theoretically, we explore information design on top of global games. Consider a system of many banks, each with a representative investor.[2] The probability of a bank surviving or failing depends on the macro state,

that we study are the most relevant to regulated banks, and regulated banks nowadays take over the historic role of large investment banks as potential triggers of large positive spillover. In particular, many major investment banks that triggered or were about to trigger large positive spillover during the 2007–2008 financial crisis, such as Morgan Stanley, Goldman Sachs, Merrill Lynch, and Bear Stearns, became or were acquired by regulated banks. Moreover, from a normative perspective, our analysis also sheds light on the regulation for financial institutions other than regulated banks. Indeed, real-life regulators debate whether run-prevention regulations should be extended to nonbank financial institutions. For example, in 2014, the Security and Exchange Commission (SEC) strengthened the regulation for money market funds to address risks of investor runs (SEC 2014). While other forms of regulations for nonbank financial institutions might better mitigate systemic risk, given the scope of our model, we focus on public disclosure of firm-specific information.

2 Since we are focusing on the coordination problem at the level of the whole banking system, we intentionally assume a representative investor to mute the coordination problem within each individual bank. In Section 5.4 we will show that adding the coordination problem at the level of individual banks does not change our main results.

bank-specific factors (i.e., the aforementioned systemic vulnerability and idiosyncratic shortfall), and the total liquidity in the system. Each investor observes a noisy signal about the macro state and then decides whether to run. Runs reduce the liquidity in the system and adversely affect all banks, giving rise to systemic risk. This also generates strategic complementarity between investors: an investor is more willing to run if more of others run. Investors perceive systemic risk differently because of the different information they receive, both from the signal and from the regulator's disclosure. The regulator discloses information about bank-specific factors to mitigate systemic risk.

To understand the overall impact of disclosures on the banking system, we can regard the disclosure of either kind of bank-specific information as effectively reallocating two fixed "budgets" across banks that are otherwise homogeneous to investors, and the stability of the banking system as a result of a negative assortative matching between the two reallocated budgets. The first fixed "budget" is the constant expected systemic vulnerability or the constant expected idiosyncratic shortfall of the whole system, respectively. This stems from an alternative interpretation of the standard Bayesian plausibility constraint: disclosures effectively reallocate this fixed budget across banks by differentiating them for their investors.

The second fixed "budget" is novel in the information design literature: we show that the aggregate systemic risk perceived by all marginal investors (who are indifferent between running and not) is constant regardless of disclosures. However, investors whose banks are disclosed to be stronger ("informationally stronger") believe that other investors are less optimistic about the survival of their banks and are thus more likely to run than themselves. Consequently, when they are marginal, they perceive a greater mass of running investors than others. In other words, disclosures reallocate more of this fixed budget to informationally stronger banks.

Taken together, disclosures of systemic vulnerability differentiate investors with respect to perceived systemic risk as well as vulnerability to systemic risk, generating a beneficial negative assortative matching between them: marginal investors of less-vulnerable banks perceive more systemic risk than those of more-vulnerable banks. This improves the average likelihood to be immune from runs ("robustness" henceforth) of all banks. In contrast, disclosures of idiosyncratic shortfalls differentiate investors with respect to perceived systemic risk but not vulnerability to it, and does not generate the beneficial negative assortative matching. Therefore, disclosures of systemic vulnerabilities can help mitigate systemic bank runs, but disclosures of idiosyncratic shortfalls cannot.

To illustrate how this insight shapes the regulator's information design, suppose her objective is to maximize the possibility that all banks are immune from runs. We find that regardless of which kind of bank-specific information is revealed, an optimal disclosure always equalizes the robustness of all banks. Given this premise, nondisclosure is always an optimal disclosure of idiosyncratic shortfalls, but to take advantage of beneficial negative assortative matching, an optimal disclosure of systemic vulnerabilities maximizes the informational heterogeneity of all banks.[3] If the actual systemic vulnerabilities of the banks do not differ very much from each other, such that they are equally robust even if full disclosure is applied, then full disclosure is the desired optimal disclosure. Otherwise, the optimal disclosure assigns as many scores as possible, and maximizes informational heterogeneity between any two scores, provided that all banks are equally robust. We further show that as the number of scores allowed approaches infinity, the optimal disclosures converge to a limiting disclosure. We characterize this limiting disclosure, and show that the resultant common robustness of all banks is the supremum of that resulting from all optimal disclosures with finite scores. This sets a limit for the mitigation of systemic bank runs through the disclosure of systemic vulnerabilities, and articulates how it is achieved.

To obtain general implications, instead of assuming a particular objective function, we assume only a partial order that the regulator prefers a disclosure A to a disclosure B if A always shields more banks from runs than B. Generically, a disclosure can be viewed as a collection of subdisclosures, each of which is imposed on a separate group of banks with resultant robustness similar within the group but significantly different from that of the other groups.[4] We find that for a disclosure to be optimal, its subdisclosures must all be robust disclosures, defined as those maximizing the robustness of the weakest constituent of the corresponding group, taking as given the mass of banks facing runs outside the group.[5] Moreover, robust disclosures in each dimension have the same qualitative features as the optimal disclosures in the corresponding dimensions in the previous example.

Our results shed light on the public disclosure of stress-test results in the presence of systemic risk. Suppose the regulator's objective is to maximize the mass of banks immune from runs in a hypothetical adverse state of the economy. Then the subdisclosure of her optimal disclosure for the bank group immune from runs must be its corresponding robust disclosure. To maximize the robustness of this group, banks subject to runs consist exclusively of physically weak banks (i.e., those that *actually* have greater systemic vulnerabilities or idiosyncratic shortfalls), with full disclosure applied. Consequently, two novel implications of systemic risk are

[3] Our assumption that each bank has a representative investor informs this result. It also precludes the role of disclosure of bank-specific information in mitigating miscoordination between investors of the same bank, which has been well studied in the literature. Our results concerning how such disclosures affect the stability of the banking system by reallocating systemic risk across banks persist even if this assumption is relaxed.

[4] By "significantly different," we mean that the robustness of the two banks is so different that, as investors are almost certain about the macro state, when the investor of the less (more) robust bank is marginal, he is almost sure that the investor of the more (less) robust bank is staying (running). By "similar," we mean the opposite.

[5] The bank group as a whole is immune from runs only if its weakest constituent is immune. A so-called "robust disclosure" maximizes the robustness of the weakest constituent and thus of the whole group, to adverse fundamental shocks.

underscored. First, unlike that of idiosyncratic shortfalls, optimal disclosure of systemic vulnerabilities entails further differentiation of banks immune from runs, due to the aforementioned beneficial negative assortative matching. Second, when the impact of systemic risk is large, under the optimal disclosure of bank-specific information, as the quality of the banking system deteriorates, a banking crisis unfolds as follows: first, a substantial mass of banks are run simultaneously, and then the remaining banks are run gradually. This is because, while the sacrifice of physically weak banks enhances the informational strength of the rest, it also increases the mass of banks facing runs and thus the systemic risk faced by the investors of unsacrificed banks. When the second effect dominates, an infinitesimal sacrifice of weak banks would worsen the robustness of the others, calling for further sacrifice, until the first effect dominates. Moreover, more information should be disclosed at worse states of the economy, in the sense that more physically weak banks should be fully revealed and "sacrificed", and that in the presence of systemic risk, more banks with low systemic vulnerabilities also should be fully revealed.

Our paper is mainly related to two strands of the literature. The first strand is the discussion of bank transparency and disclosures. A particularly prevalent question is how to design bank stress tests. Goldstein and Sapra (2014) comprehensively review the nature and the cost-benefit analysis of disclosing the results of stress tests. Our paper centers around the two effects of stress tests that they highlight: market discipline and coordination failure. Subsequent work applies the framework of information design to study the design of stress tests. Like us, Bouvard, Chaigneau, and Motta (2015), Williams (2017), Goldstein and Leitner (2018) and Orlov, Zryumov, and Skrzypacz (2023) study the optimal disclosure of bank-specific information, but they do not consider the strategic interaction between investors of different banks. Goldstein and Huang (2016) and Inostroza and Pavan (2022) consider information design concerning the aggregate state of nature in bank-run problems with homogeneous investors. Leitner and Williams (2023) study whether regulators should reveal the models that they use to stress test banks, facing the trade-off between gaming with banks under revelation and banks' underinvestment under secrecy. Parlatore and Philippon (2022) explore how a regulator learns the unknown risk exposures of a set of banks from the estimated losses of these banks under different stress-test scenarios that she picks. Instead, we focus on how the disclosure of bank-specific information creates heterogeneous interests among the investors of different banks who face coordination problems between each other due to endogenous systemic risk, and on how the disclosure of different kinds of bank-specific information affects the stability of banking systems.

The second strand is the literature on global games with heterogeneous players. Frankel, Morris, and Pauzner (2003) prove equilibrium uniqueness for a large class of these games; Corsetti et al. (2004) characterize the impact of a large trader on a population of small ones; and Sákovics and Steiner (2012) provide a criterion that can be used to find the optimal targets for a variety of interventions in regime-change games with heterogeneous agents. Based on Sákovics and Steiner (2012), Drozd and Serrano-Padial (2018) discuss a credit-enforcement cycle driven by the collective default of borrowers, and Leister, Zenou, and Zhou (2022) study strategic interaction in networks, and Serrano-Padial (2020) explore global games with heterogeneous agents based on potential maximization. Invoking Morris and Yang (2021), Dai and Yang (2022) study the role of organizations in coordinating the actions of individuals with heterogeneous interests. Some papers also study systemic bank runs with a global game setup. For instance, Choi (2014) studies whether weak or strong financial institutions should be bolstered to ease financial contagion, and in a model featuring the interaction between within- and cross-bank strategic uncertainty among depositors, Liu (2023) studies the interaction between bank runs and asset prices. In all these models, players' preferences are heterogeneous in only one dimension. But the information design that we study can create two dimensions of heterogeneity in players' preferences. These dimensions differ in whether they interact with the aggregate strategic profile, and our optimal design hinges on their qualitatively different roles in shaping players' equilibrium strategies. Moreover, the incorporation of global games into an information-design problem allows the information designer to endogenously determine the magnitude of strategic uncertainty between any two players, as her means of minimizing the mass of bank runners, given different states of nature.

Contemporaneously with us, Goldstein et al. (2022), using the global-game technique and a Diamond-Dybvig style setup that features the interaction between within- and cross-bank strategic uncertainty among depositors, find that an increase in heterogeneity among banks makes all banks more stable, provided that cross-bank strategic uncertainty remains. Instead, we consider an information design problem, and explore what kind of informational heterogeneity best mitigates systemic bank runs. Our setup accommodates the design of informational heterogeneity in various dimensions to uncover the qualitatively different impact of disclosures in these dimensions. We find that although informational heterogeneity in systemic vulnerabilities enhances the robustness of the banking system, that in idiosyncratic shortfalls does not. This differential impact highlights the novel channel that we identify: disclosures essentially reallocate the constant aggregate systemic risk perceived by marginal investors, and it is its interaction with the reallocation of expected systemic vulnerability and of expected idiosyncratic shortfall that results in the different impact of disclosures. This result manifests an economic mechanism different from that in Goldstein et al. (2022), since the representative-investor setup in our baseline model precludes within-bank strategic uncertainty for a better focus on the role of across-bank strategic uncertainty, which is more important in designing macroprudential policies for modern financial systems.

We show in Section 5.4 that our results are robust to the introduction of within-bank strategic uncertainty. Moreover, we characterize the properties of optimal disclosures for general policy objectives, and point out that a sudden run on a huge mass of banks may be inevitable, even with optimal disclosures of bank-specific information.

1. Model Setup

1.1 Agents

We consider a three-date economy consisting of a regulator ("she"), a continuum of financial institutions ("banks") and a continuum of investors. Only the regulator and the investors are active players, all of whom are risk neutral. Both the gross discount rate and the gross risk-free rate are normalized to one. At date 0, the regulator designs rules for the disclosure of bank-specific information from all banks to investors. The total mass of banks is normalized to 1. Each bank i has a representative investor, henceforth called investor i.[6] At date 1, each investor i ("he") chooses to stay ($l^i = 1$) or to run ($l^i = 0$) based on the information available to him by then. If he runs, he secures the one unit of consumption good invested in bank i's long-term project before date 0, and bank i definitely fails at date 2. If he stays, then at date 2 he receives R units of consumption good from the project if bank i survives, and nothing if it fails.

Remark 1. The term "investors" here refers to wholesale investors and large depositors who are not fully insured through depositor insurance or collateral. "Running" refers to reducing or ceasing liquidity provision, such as not rolling over short-term debt, imposing higher margin requirements, and redeeming shares in the context of mutual or hedge funds (see Brunnermeier 2009).

1.2 Banks' survival probabilities

To focus on how the regulator's information design affects investors' actions, we assume that the probability that bank i survives at date 2, P^i, follows the following reduced form, and abstract from the details of its microfoundation:[7]

$$P^i = \frac{1}{R}\left[\theta - r^i \cdot a(l) + 1 - c^i\right]. \tag{1}$$

The "fundamental" θ is an aggregate state of the economy capturing all exogenous factors that simultaneously affect the survival probability of all

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[6] See footnote 2 for an explanation.

[7] The setup is designed such that the net return to investor i's investment follows the two-factor model in Equation (2) analogous to those in the macro finance and asset pricing literature. The loading on the exogenous factor θ is normalized to one, and that on the endogenous factor $-a(l)$ is r^i. The expected idiosyncratic shortfall $-\mathbb{E}c$ can be viewed as "alpha" and $\mathbb{E}c - c^i$ as the residual. Like a standard factor pricing model, Equation (2) can be viewed as a decomposition of all factors affecting the survival of bank i, given its investor's action. We adopt such a factor model because it is technically convenient and easy to interpret. Our main results hold qualitatively beyond this particular functional form.

banks, such as macroeconomic conditions. We assume that θ is distributed over $[\underline{\theta}, \bar{\theta}] \subset \mathbb{R}$.

The second term in Equation (1) is our key addition to capture the strategic interaction among banks. In particular, $l \triangleq \int l^i di$ is the mass of all investors who stay ("stayers"), which can be interpreted as total liquidity. The loss function a, which is decreasing in l, captures the systemic risk faced by all banks. Hereafter, we directly refer to $a(l)$ as systemic risk. We allow for a generic functional form of a, as long as it is positive and Lipschitz continuous almost everywhere. The coefficient r^i captures the vulnerability of bank i to systemic risk (i.e., systemic vulnerability). This could be due to the heterogeneity in banks' magnitude of interbank lending or positions of publicly traded assets. We assume that $r^i = \underline{r} > 0$ with probability q^r and $r^i = \bar{r} > \underline{r}$ with probability $1 - q^r$. Let $\mathbb{E}r = q^r \underline{r} + (1 - q^r)\bar{r}$.

The idiosyncratic shortfall of funds c^i (i.e., idiosyncratic shortfall) captures all factors exogenous to our model that affect only the survival probability of bank i regardless of the funding condition of the banking system, such as its cash and cash equivalents, or nonperforming loans of little systemic consequence. We assume that $c^i = \underline{c}$ with probability q^c and $c^i = \bar{c} > \underline{c}$ with probability $1 - q^c$. Let $\mathbb{E}c = q^c \underline{c} + (1 - q^c)\bar{c}$.

The following parametric restriction is needed to guarantee that the survival probability P^i is always in $[0, 1]$:

$$-1 \leq \underline{\theta} - \bar{r}a(0) - \bar{c} < \bar{\theta} - \underline{r}a(1) - \underline{c} \leq R - 1.$$

Given the survival probability P^i, the incremental payoff for investor i from staying relative to running is

$$\pi \triangleq P^i R - 1 = \theta - r^i a(l) - c^i. \tag{2}$$

Remark 2. This paper focuses on the mitigation of systemic bank runs and systemic risk; that is, positive spillover across banks due to systemic risk is the major concern (Brunnermeier 2009).[8] Systemic risk amplifies the impact of

[8] Positive spillover across banks has been manifested in several systemic events. On one hand, the failure of financial institutions may significantly affect the functioning of financial markets and asset values. For example, in the years following the fall of Long-Term Capital Management, credit spreads, mortgage spreads, and the 10-year-on-the-run swap spread became too high to be entirely credit-related, so that they must also have included liquidity spreads attributed to the consequent diminishing number of liquidity providers (Scholes 2012). Also, the problems with Bear Stearn's hedge funds and BNP Paribas and the run on the Reserve Primary Fund due to its exposure to the bankrupt Lehman Brothers triggered marketwide runs on asset-backed commercial paper (ABCP) and drastically reduced the total value of ABCP outstanding (Kacperczyk and Schnabl 2010). On the other hand, the malfunction of financial markets and decreases in asset values render banks less robust. In their analysis of the repo market, Goldstein and Metrick (2012) concluded that concerns about the liquidity of markets for the bonds used as collateral led to increases in repo haircuts, which, together with declining asset values, pushed the US banking system to the edge of insolvency. Covitz, Liang, and Suarez (2013) found that uncertainties about the interbank funding market and subprime mortgage values may have been important determinants of runs on ABCP in the last five months of 2007. Concerns about positive spillovers also motivated regulators' interventions. For example, the Fed facilitated the negotiated takeover of Long-Term Capital Management in the 1998 hedge fund crisis (Scholes 2012) and promptly bailed out AIG creditors in the Global Financial Crisis (Brunnermeier 2009).

deteriorating aggregate fundamentals, and our regulator's optimal disclosures are designed to minimize such amplification. As pointed out by Choi (2014), systemic risk may result from fire-sale externalities or information spillovers, among other channels, or the contagion of financial distress via direct or indirect exposures. To obtain general implications of systemic risk that are robust to the fine details of these different channels, we base our analysis only on their commonality that runs on more banks increase the threat to banks not being run; that is, systemic risk is captured by a decreasing function $a(\cdot)$. Admittedly, there are also factors that may generate negative spillover across banks. In particular, as analyzed by Egan, Hortacsu, and Matvos (2017), severe competition for deposits may undermine banks' profitability and thus reduce their robustness. Failure of some banks may weaken such competition for the rest and thus generate negative spillover. But in the context of a financial crisis, usually the aggregate impact of liquidity withdrawals is still a positive spillover across banks.[9]

1.3 The regulator's information design

The focus of this paper is on the regulator's optimal design of disclosure rules ("disclosures" hereafter) at date 0 about relevant bank-specific information (i.e., r^i or c^i) to mitigate systemic bank runs caused by strategic uncertainty. To highlight our main insights, we assume in our baseline model that investors rely completely on the regulator's disclosure to learn about r^i and c^i: without her disclosure, investors know only their expected values, $\mathbb{E}r$ and $\mathbb{E}c$. In Section 5.4 we show that our main insights are robust to the relaxation of this assumption. To better contrast disclosures of bank-specific information in different dimensions, we assume that r^i and c^i are independently distributed, so that disclosures about r^i do not reveal information about c^i, and vice versa. We discuss the case of correlated information in Section 5.4.

A disclosure specifies how scores are assigned to banks based on their r^i and c^i, so that investors can distinguish only between banks with different scores, but not between those with the same score. Without loss of generality, any disclosure about r^i and c^i with n scores can be represented by the conditional means of r^i and c^i for each score and the mass of banks receiving that score; that is, with $\{(r_k, c_k, w_k)\}_{k=1}^n$, where $r_k = \mathbb{E}[r^i|\text{score } k]$, $c_k = \mathbb{E}[c^i|\text{score } k]$, and w_k is the mass of banks receiving score k. By construction, $w_k \in [0,1]$ for all k, and $\sum_{k=1}^n w_k = 1$. As a well known result in the literature of information design, a disclosure $\{(r_k, c_k, w_k)\}_{k=1}^n$ is feasible if and only if it satisfies Bayesian

[9] In the recent crisis triggered by the failure of Silicon Valley Bank (Jiang et al. 2023), while strong banks enjoyed incoming deposits from weak banks, the banking system as a whole also lost deposits to money market funds, and even the largest banks experienced a fall in their stock prices (Bhattarai 2023). Indeed, it is for fear of systemic risk that the Treasury, Fed, and FDIC jointly decided to fully protect all depositors of Silicon Valley Bank (U.S. Department of the Treasury 2023).

plausibility; that is, $r_k \in [\underline{r}, \bar{r}]$ and $c_k \in [\underline{c}, \bar{c}]$ for all k, $\sum_{k=1}^{n} w_k r_k = \mathbb{E}r$, and $\sum_{k=1}^{n} w_k c_k = \mathbb{E}c$. For both expositional convenience and practical consideration, we focus on finite disclosures that assign no more than N scores to banks. In Section 3.5, we present our results concerning the limiting case where infinitely many scores are allowed.

Henceforth, superscripts represent exogenous objects, and subscripts represent conditional means given a disclosure and the resultant endogenous objects. For ease of presentation, we refer directly to r_k, c_k, or (r_k, c_k) as a "score." Thus, investors perceive a bank with a lower score to be stronger. We refer to such a bank as "informationally stronger," and to the heterogeneity in scores as "informational heterogeneity." We also refer to r_k as *"perceived systemic vulnerability"* (PSV) and c_k as *"perceived idiosyncratic shortfall"* (PIS) when articulating economic mechanisms. In contrast to "scores," we refer to $r^i \in \{\underline{r}, \bar{r}\}$, $c^i \in \{\underline{c}, \bar{c}\}$, or $(r^i, c^i) \in \{\underline{r}, \bar{r}\} \times \{\underline{c}, \bar{c}\}$ as a "type." We refer to a bank with a lower type as "physically stronger," and to the heterogeneity in types as "physical heterogeneity." In addition, we use "a type-r^i investor," "a score-r_k investor," and "a score-(r_k, c_k) investor" or simply "a score-k investor" to denote the representative investor of a type-r^i bank, that of a score-r_k bank, and that of a score-(r_k, c_k) bank, respectively. Moreover, we refer to disclosures that potentially reveal something about banks' systemic vulnerabilities but nothing about their idiosyncratic shortfalls (i.e., with $c_k = \mathbb{E}c$ for all k) as "disclosures in dimension r," and "disclosures in dimension c" are defined analogously.[10]

Remark 3. Equation (1) essentially assumes that the survival probability of a bank is determined by its *original* type (r^i, c^i) regardless of the regulator's disclosure. Since the regulator's disclosure changes only the investors' beliefs but involves no physical actions from the regulator (e.g., adjusting capital requirements or injecting liquidity), it has no *direct* impact on the bank's type. Disclosure may have an *indirect* impact resulting from a bank's endogenous adjustment of its balance sheet. For example, it may sell assets exposed to systemic risk. However, if a bank has little market power and its asset holdings are fully marked to market, selling assets only converts publicly traded assets approximately one-for-one to cash, with little impact on its net liquidation value. Thus, the bank's survival probability still largely depends on its original type.[11]

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[10] We do not use "disclosures of r^i (c^i)" to avoid the confusion with the treatment of a specific bank.

[11] As a concrete illustration, suppose that a bank's survival probability depends on its net liquidation value (NLV):

$$NLV = r^i(1-a(l)) + (1-r^i) + \theta - c^i - d = \theta - r^i \cdot a(l) + 1 - c^i - d.$$

Here, r^i is its holdings of publicly traded assets with market price $1-a(l)$, $1-r^i$ is the value of its secured lending, $\theta - c^i$ captures bank i's profit from banking service, and d is the bank's liability. Because of its holdings of public traded assets, the bank gets exposed to systemic risk. While the (infinitesimal) bank can adjust the holdings r^i to $r^{i\prime}$, its trading does not move the market price $1-a(l)$, but only converts such assets of value

For simplicity, we assume that systemic risk $a(\cdot)$ is invariant to the regulator's disclosures. This is largely innocuous if disclosures have little impact on the determinants of $a(\cdot)$ outside our model. For example, suppose $a(\cdot)$ is microfounded by the market price of publicly traded assets as in footnote 11. In this case, the function $a(\cdot)$ is determined by the total supply of the asset, which varies little over time, and by its demand from investors other than the banks in the model. If these investors care mainly about long-run asset fundamentals, but little about short-run price fluctuations and thus the survival of the banks,[12] then the regulator's disclosures have little impact on the demand from these investors, and consequently on $a(\cdot)$. For a similar consideration, existing literature, such as Stein (2012), Liu (2023), and Goldstein et al. (2022), also assumes invariant demand functions for investors other than the banks in the model.

1.4 Information about the fundamental

At date 0, the regulator and all investors share a common prior of the fundamental θ, represented by a probability density function (pdf) $h(\cdot)$. At date 1, the investor of each bank i observes a private signal about θ, $x^i = \theta + \sigma \cdot \varepsilon^i$, where ε^i is independent and identically distributed according to a probability density function $\phi(\cdot)$, with corresponding cumulative distribution function (cdf) $\Phi(\cdot)$, and the parameter σ determines the magnitude of the signal noise, which captures the magnitude of fundamental uncertainty (about θ) faced by investors. An investor's signal can be understood as his private information or opinion about the macroeconomy. The pdf's $h(\cdot)$ and $\phi(\cdot)$ are continuous, bounded, and fully supported over $\left[\underline{\theta}, \bar{\theta}\right]$ and $(-\infty, +\infty)$, respectively.[13] As is common in the global games literature, we assume the existence of dominance regions. That is, when θ is sufficiently low (high), it is the dominant strategy of any investor to run (to stay), regardless of the actions of other investors; that is,

$$\underline{\theta} - \underline{r}a(1) - \underline{c} < 0 < \bar{\theta} - \bar{r}a(0) - \bar{c}. \tag{3}$$

Note that a stayer's payoff (2) is strictly increasing in the total mass of stayers. This creates motives for an investor to coordinate his decision with others in the game at date 1. However, the idiosyncrasy of signal noise prevents investors from perfectly knowing others' signal realizations and thereby inferring their actions. As highlighted by the global games literature, strategic uncertainty (about others' actions) as such could persist and thus lead

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$\left(r^i \quad - r^{i\prime}\right)(1-a(l))$ to cash of the same value, and thus does not change its NLV. As such, the bank's survival probability essentially depends on its *original* type (r^i, c^i).

[12] Real-life examples of such investors include insurance companies and pension funds.

[13] Unbounded support of $\phi(\cdot)$ provides convenience of exposition. Our results remain valid for bounded support with minor modification.

to miscoordination among investors, even if fundamental uncertainty vanishes (i.e., $\sigma \to 0$). To explore the impact of the regulator's information design on strategic uncertainty and to sharpen its implication on the stability of the banking system, we follow the convention of the global games literature and focus on the limit of $\sigma \to 0$.[14] This also guarantees equilibrium uniqueness. As is well known in this literature, it is without loss of generality in this limit to focus on symmetric equilibria with switching strategies. That is, an investor stays if and only if he observes x^i above a switching cutoff $\hat{x}(r_k, c_k)$, which depends on the score (r_k, c_k) assigned to his bank by the regulator. Therefore, we say a bank is *immune from runs* given the fundamental θ if $\hat{x}(r_k, c_k) \leq \theta$ and is *subject to runs* if $\hat{x}(r_k, c_k) > \theta$.[15]

1.5 The regulator's objective

Finally, we introduce the regulator's objective. A disclosure $\{(r_k, c_k, w_k)\}_{k=1}^n$ results in a set of limiting switching cutoffs $\left\{\hat{x}_k\right\}_{k=1}^n$, such that all banks with the kth score are subject to runs if $\theta < \hat{x}_k$ and immune from runs if $\theta \geq \hat{x}_k$. Suppose $\left\{\hat{x}_k\right\}_{k=1}^n$, the set of the cutoffs, has T distinct elements ranked as $\theta_1 < \theta_2 < \ldots < \theta_T$. Note that $T \leq n$ by definition. For $i \in \{1, \cdots, T\}$, let $K_i = \sum_{\{k \mid \hat{x}_k \leq \theta_i\}} w_k$ denote the mass of banks whose cutoffs are no greater than θ_i. For ease of notation, define $\theta_{T+1} = \bar{\theta}$. Then, the mass of banks immune from runs given the fundamental θ is essentially

$$K\left(\theta; \{K_j, \theta_j\}_{j=1}^T\right) \triangleq \sum_{i=1}^{T} K_i \cdot 1_{\{\theta_i \leq \theta < \theta_{i+1}\}}.$$

We refer to $K\left(\cdot; \{K_j, \theta_j\}_{j=1}^T\right)$ as a stability scheme.

Stability schemes can be partially ordered according to first-order stochastic dominance (FOSD). A stability scheme that is first-order stochastically dominated by another has a greater mass of banks immune from runs than does the latter under *any* circumstance (i.e., any value of θ). Therefore, the regulator prefers disclosure A to disclosure B if the stability scheme resulting from A (the blue solid line in Figure 1) is first-order stochastically dominated by that resulting from B (the red dashed line in Figure 1).

On the other hand, if neither disclosure A nor B is first-order stochastically dominated by the other, it is difficult to determine which is preferred by the regulator without knowing the fine details of her preferences. Specifically,

14 See Corsetti et al. (2004), Goldstein and Pauzner (2005), and He, Krishnamurthy, and Milbradt (2019) for examples.

15 Formally, $\hat{x}\left(r_k, c_k\right)$ is the limiting switching cutoff at vanishing fundamental uncertainty σ. Thus, given θ, a score-k bank is almost surely immune from runs if $\hat{x}\left(r_k, c_k\right) < \theta$ and subject to runs if $\hat{x}\left(r_k, c_k\right) > \theta$. Since the probability that $\theta = \hat{x}\left(r_k, c_k\right)$ is zero, whether the bank is immune from runs in that case is immaterial in our analysis. For expositional convenience, we assume that in that case the bank is also immune from runs.

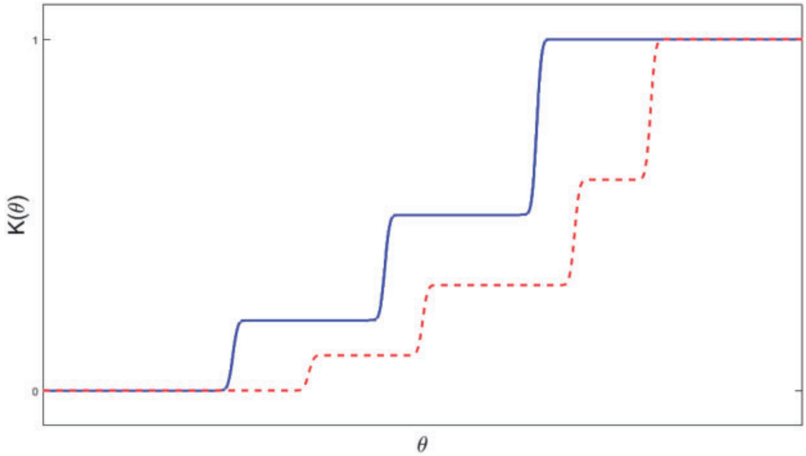

Figure 1
Partial order of stability schemes

since disclosure A leads to a greater mass of banks immune from runs than disclosure B for some fundamentals and a smaller mass for others, the regulator's preference depends on the weight that she attaches to each fundamental. To obtain results robust to such fine details, in our baseline analysis we characterize the general properties of the optimal disclosure under this minimum requirement of FOSD. As an application in stress tests, Section 4 illustrates the central role of the results of our baseline analysis in the construction of the regulator's optimal disclosure given a practical objective function.

2. An Intuitive Illustration

This section illustrates the main idea of the paper using an example of binary-score disclosures, $\{(r_k, c_k, w_k)\}_{k=1}^2$, with fixed masses w_1 and w_2. By construction, $w_1+w_2=1$. In this context, disclosures in dimension r refer to those with $r_1 \le \mathbb{E}r \le r_2$ but $c_1=\mathbb{E}c=c_2$, and disclosures in dimension c refer to those with $r_1=\mathbb{E}r=r_2$ but $c_1 \le \mathbb{E}c \le c_2$. For concreteness of illustration, in this section, the regulator is assumed to maximize the probability that all banks are immune from runs.

2.1 Equilibrium switching cutoffs

We start with an intuitive derivation of equilibrium switching cutoffs (6) given a disclosure.[16] For a given magnitude of fundamental uncertainty σ, let $\hat{x}_i^\sigma$

[16] See the proof of Proposition 1 in the appendix for a rigorous derivation.

denote a score-(r_i, c_i) investor's switching cutoff. Conditional on fundamental θ, the probability that he stays is

$$m_i^\sigma(\theta) \triangleq \Pr(x^i > \hat{x}_i^\sigma | \theta) = 1 - \Phi\left(\frac{\hat{x}_i^\sigma - \theta}{\sigma}\right), \tag{4}$$

and the total mass of stayers is[17]

$$M^\sigma(\theta) \triangleq \sum_i w_i m_i^\sigma(\theta). \tag{5}$$

Call an investor *marginal* if his signal realization equals his switching cutoff. Observe from (4) that when σ is small, $m_i^\sigma(\theta)$ is also a marginal investor i's posterior cdf of θ (up to $O(\sigma)$). A marginal investor i should be indifferent between staying and running:

$$\begin{aligned} 0 &= E_\theta[P^i R - 1 | x_i = \hat{x}_i^\sigma] \\ &= \int_{\underline{\theta}}^{\bar{\theta}} \left[\theta - r_i a(M^\sigma(\theta)) - c_i\right] \left(m_i^\sigma\right)'(\theta) d\theta + O(\sigma); \end{aligned}$$

that is,

$$\hat{x}_i^\sigma - r_i \int_{\theta=\underline{\theta}}^{\bar{\theta}} a(M^\sigma(\theta)) dm_i^\sigma(\theta) - c_i = O(\sigma). \tag{6}$$

The system of equations (6) characterizes the equilibrium cutoffs $\hat{x}_1^\sigma$ and $\hat{x}_2^\sigma$ with small fundamental uncertainty σ. In (6), $\hat{x}_i^\sigma = E[\theta | x_i = \hat{x}_i^\sigma] + O(\sigma)$; that is, $\hat{x}_i^\sigma$ is (up to $O(\sigma)$) investor i's expectation of the fundamental θ conditional on himself being marginal; and $\int_{\theta=\underline{\theta}}^{\bar{\theta}} a(M^\sigma(\theta)) dm_i^\sigma(\theta)$ is (up to $O(\sigma)$) a marginal score-(r_i, c_i) investor's *perception of systemic risk* (PSR). From (6), an investor's switching cutoff $\hat{x}_i^\sigma$ equals the product of his *perception of systemic vulnerability* (PSV; i.e., r_i) and PSR when he is marginal, plus his *perceived idiosyncratic shortfall* (PIS; i.e., c_i). Thus, he is more reluctant to stay if his PSV, PSR when marginal, or PIS is higher.

2.2 Perception of systemic risk

This subsection establishes two properties of the perception of systemic risk (PSR). First, regardless of disclosures, the aggregate PSR of all marginal investors is constant. Second, disclosures in either dimension differentiate banks in that dimension, and reallocate more of the constant aggregate PSR to informationally stronger banks (i.e., banks with better scores), and more so if disclosures entail more such differentiation.

[17] The law of large numbers is not well defined for a continuum of random variables (Sun 2006). Our law of large numbers convention is equivalent to assuming that opponents' play is the limit of play of finite selections from the population.

2.2.1 Constant aggregate PSR Observe that regardless of disclosures, the aggregate PSR of all marginal investors is always given by

$$\sum_i w_i \int_{\theta=\underline{\theta}}^{\bar{\theta}} a(M^\sigma(\theta))dm_i^\sigma(\theta) = \int_{\theta=\underline{\theta}}^{\bar{\theta}} a(M^\sigma(\theta))dM^\sigma(\theta)$$

$$= \int_0^1 a(M^\sigma)dM^\sigma + O(\sigma), \tag{7}$$

where the first equality in (7) is due to (5), and the second is due to assumption (3). While derived from our specific setup, the constant-aggregate-PSR condition (7) echoes the belief constraint in alternative settings in Sákovics and Steiner (2012) and Serrano-Padial (2020); that is, the weighted average strategic belief is the uniform belief on $[0,1]$.

2.2.2 Reallocation of PSR Let

$$\Delta_{i,j}^\sigma \triangleq (\hat{x}_j^\sigma - \hat{x}_i^\sigma)/\sigma \tag{8}$$

be the relative distance between the switching cutoffs of score-j and score-i investors. Disclosures in either dimension r or c increase informational heterogeneity in that dimension, and thus increase $\Delta_{1,2}^\sigma$ from zero.[18] Observe that a marginal score-1 investor's PSR

$$\int_{\theta=\underline{\theta}}^{\bar{\theta}} a\left(M^\sigma(\theta)\right)dm_1^\sigma(\theta)$$

$$=\int_{\theta=\underline{\theta}}^{\bar{\theta}} a\left(w_1 m_1^\sigma(\theta) + w_2 m_2^\sigma(\theta)\right)dm_1^\sigma(\theta)$$

$$=\int_0^1 a\left(w_1 m_1^\sigma + w_2\left[1-\Phi\left(\Phi^{-1}(1-m_1^\sigma)+\Delta_{1,2}^\sigma\right)\right]\right)dm_1^\sigma + O(\sigma) \tag{9}$$

increases with $\Delta_{1,2}^\sigma$. Thus, disclosures reallocate more of the constant aggregate PSR to score-1 investors, who are informationally stronger, and more so with a greater $\Delta_{1,2}^\sigma$.

To understand this, note that a marginal score-1 investor believes that the fundamental θ is around $\hat{x}_1^\sigma$ with high probability. A larger $\Delta_{1,2}^\sigma$ makes him believe that θ is more likely to be below $\hat{x}_2^\sigma$, and thus makes him more pessimistic about the chance that score-2 investors stay. This reduces the mass of score-2 stayers that he perceives. However, since $\Delta_{1,1}^\sigma = 0$ by definition, he perceives roughly the same mass of score-1 stayers regardless of disclosures. Therefore, the total mass of stayers perceived by this marginal score-1 investor

[18] Since score-1 banks are informationally stronger, we must have $\hat{x}_1^\sigma \le \hat{x}_2^\sigma$. We show this more rigorously in Section 3.1.

decreases with $\Delta^{\sigma}_{1,2}$, resulting in a greater PSR. A symmetric argument and an opposite conclusion hold for a marginal score-2 investor.[19]

Such reallocation of PSR happens only before $\Delta^{\sigma}_{1,2}$ reaches infinity, for which the reallocation regions in Figure 2 are named. In this case, the limiting switching cutoffs, $\hat{x}_1$ and $\hat{x}_2$, must coincide. Once $\Delta^{\sigma}_{1,2}$ approaches infinity, a marginal score-1 investor believes that score-2 investors are almost surely running, and thus that stayers come only from score-1 investors. His PSR reaches its maximum, $\int_0^1 a(w_1 m_1^{\sigma})dm_1^{\sigma}$, and stays there as the limiting switching cutoffs diverge (as in the separation regions in Figure 2). In this case, disclosures affect switching cutoffs only through the direct impact of scores.

2.3 Only disclosures in dimension r are beneficial

We now explain why only disclosures in dimension r can improve the stability of all banks, which is captured by a reduction in the average cutoff (11) derived from (6):

$$\sum_i w_i \hat{x}_i^{\sigma} = \sum_i w_i r_i \int_{\theta=\underline{\theta}}^{\bar{\theta}} a(M^{\sigma}(\theta))dm_i^{\sigma}(\theta) + \mathbb{E}c + O(\sigma), \tag{11}$$

whose limit as $\sigma \to 0$ gives rise to the common switching cutoffs in the reallocation regions in Figure 2. Besides the aforementioned constant-aggregate-PSR condition (7), as in a standard information-design problem, Bayesian plausibility constraints

$$\sum_i w_i r_i = \mathbb{E}r, \tag{12}$$

and

$$\sum_i w_i c_i = \mathbb{E}c. \tag{13}$$

also must be satisfied.

Disclosures in dimension r reduce the average cutoff (11), due to negative assortative matching between investors' perception of systemic vulnerability (PSV), r_i, and their perception of systemic risk (PSR), $\int_{\theta=\underline{\theta}}^{\bar{\theta}} a(M^{\sigma}(\theta))dm_i^{\sigma}(\theta)$. Given their respective aggregates (12) and (7), low PSV r_1 is matched with high PSR $\int_{\theta=\underline{\theta}}^{\bar{\theta}} a(M^{\sigma}(\theta))dm_1^{\sigma}(\theta)$, which reduces the first term of Equation (11); that is, marginal score-r_1 investors, whose banks are known to be less vulnerable to systemic risk, bear more of the constant aggregate systemic risk. This improves

19 The argument can be rigorously obtained from a marginal score-i investor's belief about the proportion m_j of stayers out of all score-j investors

$$\Pr\left[m_j \le \tilde{m}_j | x^i = \hat{x}_i^{\sigma}\right] = 1 - \Phi\left(-\Delta^{\sigma}_{i,j} + \Phi^{-1}\left(1 - \tilde{m}_j\right)\right) + O(\sigma). \tag{10}$$

This is induced by his belief (4) about the fundamental θ.

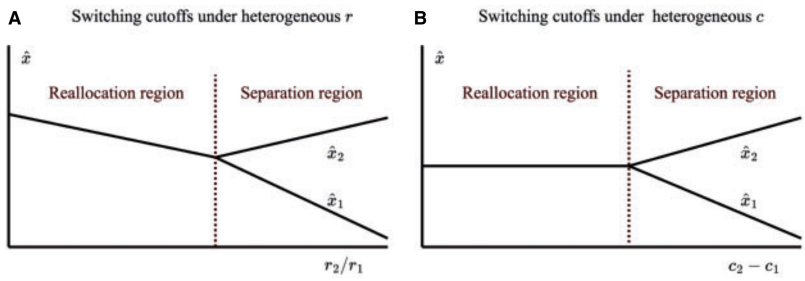

Figure 2
Switching cutoffs resulting from binary-score disclosure rules

the stability of all banks, and more so with a greater r_2/r_1. But disclosures in dimension c fail to achieve this, since they do not reallocate PSV (i.e., $r_1 = \mathbb{E}r = r_2$). Indeed, Equation (11) becomes

$$\sum_i w_i \hat{x}_i^\sigma = \mathbb{E}r \cdot \int_0^1 a(M^\sigma) dM^\sigma + \mathbb{E}c + O(\sigma), \tag{14}$$

which is independent of the distribution of scores in dimension c.

2.4 Summary

Figure 2 summarizes the impact of disclosures on investors' switching cutoffs under vanishing fundamental uncertainty σ. First, all investors have the same limiting cutoff if and only if r_2/r_1 or $c_2 - c_1$ is not large. Second, disclosures in dimension r can reduce the switching cutoffs of all investors. Thus, the regulator should disclose as much information in that dimension as possible, provided that all investors have the same limiting cutoff. Third, disclosures in dimension c cannot reduce the average cutoff of all investors. Thus, it is optimal for the regulator not to disclose information in that dimension.

For expositional convenience, this section has focused on binary disclosures with fixed w_1 and w_2. In Section 3, where such restrictions are lifted, nondisclosure is still optimal in dimension c, but optimal disclosures in dimension r exploit the insight of our intuitive illustration to the extreme: the regulator assigns as many scores as possible, such that for any two scores r_i and r_j, we have $\hat{x}_i = \hat{x}_j$ while $\Delta_{i,j} \triangleq \lim_{\sigma \to 0} \Delta_{i,j}^\sigma = +\infty$, unless such practice is restricted by physical heterogeneity $\bar{r}/\underline{r}$, where full disclosure is optimal.

3. Optimal Disclosures beyond Binary Scores

Based on the key messages from Section 2, this section goes beyond binary-score disclosures and studies disclosures that meet the minimum optimality requirement introduced in Section 1.5: they yield stability schemes not first-order stochastically dominating those resulting from any other feasible disclosures. Section 3.1 solves the equilibrium given a finite disclosure, and introduces the concepts of entanglement, separation, and adjacency that characterize the strategic relationship between the investors of banks that receive different scores. Section 3.2 introduces the building blocks of optimal disclosures: robust disclosures of bank groups. Section 3.3 shows that nondisclosure is a robust disclosure in dimension c for any bank group. Section 3.4 shows that if the physical heterogeneity of a bank group is weak, its robust disclosure in dimension r is full disclosure. Otherwise, its robust disclosure assigns as many adjacent scores as possible, and we construct a limiting disclosure with infinitely many scores in Section 3.5, showing that this outperforms all finite disclosures, and that the sequence of robust disclosures converges to the limiting disclosure as the number of scores allowed approaches infinity. Lastly, we confirm in Section 3.6 that essentially, any optimal disclosure must be a combination of robust disclosures.

3.1 Equilibrium given a disclosure

Consider a disclosure $\{(r_i, c_i; w_i)\}_{i=1}^n$ with n different scores and associated mass $w_1, w_2, ..., w_n$, respectively. Again, without loss of generality, we can focus on symmetric equilibria in which all investors are playing switching strategies. Let $\hat{x}_i^\sigma$ be the switching cutoff of score-(r_i, c_i) investors for $\sigma > 0$, and $\Delta_{i,j}^\sigma$ be given by Equation (8). Then, the probability that a score-(r_i, c_i) investor chooses to stay if the fundamental is θ, $m_i^\sigma(\theta)$, is still given by Equation (4); the total mass of stayers, $M^\sigma(\theta)$, is still given by Equation (5); and $\hat{x}_i^\sigma$ still satisfies Equation (6). Proposition 1 characterizes the equilibrium given any finite disclosure in the limit $\sigma \to 0$.

Proposition 1. As $\sigma \to 0, \forall i, j \in \{1, 2, \ldots, n\}, \hat{x}_i^\sigma \to \hat{x}_i$ and $\Delta_{i,j}^\sigma \to \Delta_{i,j}$, where $\{\hat{x}_i, \Delta_{i,j}\}_{i,j=1}^n$ satisfies the system of equations

$$\hat{x}_i = c_i + r_i \int_0^1 a\left(\sum_{j=1}^n w_j \left[1 - \Phi\left(\Phi^{-1}(1-m_i) + \Delta_{i,j}\right)\right]\right) dm_i, \tag{15}$$

with

$$\Delta_{i,j} \begin{cases} = +\infty, & \text{if } \hat{x}_j > \hat{x}_i \\ = -\infty, & \text{if } \hat{x}_j < \hat{x}_i \\ \in [-\infty, +\infty], & \text{if } \hat{x}_j = \hat{x}_i \end{cases}, \tag{16}$$

and

$$-\Delta_{i,j} = \Delta_{j,i} = \sum_{k=j+1}^{i} \Delta_{k-1,k}. \tag{17}$$

Conversely, if $\left\{\hat{x}_i, \Delta_{i,j}\right\}_{i,j=1}^n$ satisfies this system of equations, then investors' switching cutoffs converge to $\left\{\hat{x}_i\right\}_{i=1}^n$ under the disclosure $\{(r_i, c_i; w_i)\}_{i=1}^n$ as $\sigma \to 0$.

Equations (16) and (17) follow the definition of $\Delta^\sigma_{i,j}$. As a generalization of Equations (6) and (9), Equation (15) characterizes investors' limiting cutoffs. Notably, these conditions are not only necessary but also sufficient for $\{\hat{x}_i\}_{i=1}^n$ to be the limiting cutoffs. This guarantees that the disclosure derived from these conditions can surely induce the desired equilibrium.

Why does the equilibrium operate in this manner? We exemplify the rationale with binary-score disclosures in dimension c (i.e., $r_1=\mathbb{E}r=r_2$ but $c_1<\mathbb{E}c<c_2$) for expositional convenience. A similar argument applies to disclosures in dimension r. By Equation (9) and its counterpart for score-c_2 investors, the indifference condition (6) becomes

$$\hat{x}_1^\sigma-\mathbb{E}r\cdot\int_0^1 a\left(w_1m_1^\sigma+w_2\left[1-\Phi\left(\Phi^{-1}(1-m_1^\sigma)+\Delta^\sigma_{1,2}\right)\right]\right)dm_1^\sigma-c_1=O(\sigma) \tag{18}$$

and

$$\hat{x}_2^\sigma-\mathbb{E}r\cdot\int_0^1 a\left(w_1\left[1-\Phi\left(\Phi^{-1}(1-m_2^\sigma)-\Delta^\sigma_{1,2}\right)\right]+w_2m_2^\sigma\right)dm_2^\sigma-c_2=O(\sigma). \tag{19}$$

Recall that the difference between the first terms in Equations (18) and (19), $\hat{x}_2^\sigma-\hat{x}_1^\sigma=E[\theta|x_i=\hat{x}_2^\sigma]-E[\theta|x_i=\hat{x}_1^\sigma]+O(\sigma)$, and that the difference between the second terms in Equations (18) and (19) is (up to $O(\sigma)$) that between marginal investors' PSRs, which is of magnitude $\Delta^\sigma_{1,2}\triangleq(\hat{x}_2^\sigma-\hat{x}_1^\sigma)/\sigma$, multiplied by their common PSV, $\mathbb{E}r$. To make marginal investors of banks with both scores indifferent, we must have $\hat{x}_1^\sigma<\hat{x}_2^\sigma$: marginal score-$c_1$ investors, whose banks are informationally stronger, must be less optimistic about the fundamental θ and perceive more systemic risk.

When σ is small, $\Delta^\sigma_{1,2}$ and thus the difference in PSRs can be substantial even if $x_2^\sigma-\hat{x}_1^\sigma$ is small. If c_2-c_1 is not too large (as in the reallocation region in Figure 2), it can be made up with $\hat{x}_2^\sigma-\hat{x}_1^\sigma=O(\sigma)$ but $\Delta^\sigma_{1,2}>0$, so that the limiting switching cutoff $\hat{x}_2=\hat{x}_1$ and $\Delta_{1,2}$ is finite. But the difference in PSRs is at most $\int_0^1 a\left(w_1m_1^\sigma\right)dm_1^\sigma-\int_0^1 a\left(w_1+w_2m_2^\sigma\right)dm_2^\sigma$, so if c_2-c_1 is large (as in the separation region in Figure 2), investors must have substantially different cutoffs: $\hat{x}_2^\sigma-\hat{x}_1^\sigma>0$ and $\Delta^\sigma_{1,2}\to\infty$ at vanishing σ, so that $\hat{x}_2>\hat{x}_1$ and $\Delta_{1,2}=+\infty$.

Hereafter, without loss of generality, we reorder the scores such that $\Delta_{i-1,i}\geq 0$ for all i. Based on Proposition 1, we define the concepts of entanglement, separation and adjacency. Figure 3 illustrates these concepts with a disclosure only in dimension r; that is, with $c_i=\mathbb{E}c$ for all i.

Definition 1. For a pair of scores (r_i,c_i) and $\left(r_j,c_j\right)$ with $i<j$,

- if $\Delta_{i,j}<+\infty$, then we must have $\hat{x}_i=\hat{x}_j$, and we say scores (r_i,c_i) and $\left(r_j,c_j\right)$ are *entangled*;
- if $\hat{x}_j>\hat{x}_i$, then we say scores (r_i,c_i) and $\left(r_j,c_j\right)$ are *separate*;
- if $\Delta_{i,j}=+\infty$ and $\hat{x}_i=\hat{x}_j$, then we say scores (r_i,c_i) and $\left(r_j,c_j\right)$ are *adjacent*.

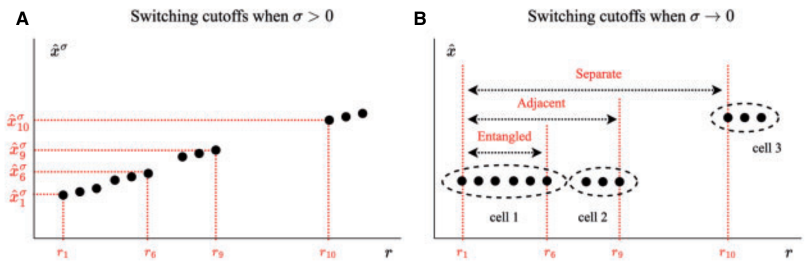

Figure 3
Switching cutoffs given a finite disclosure rule

Entanglement refers to the situation in which the investors of two banks that receive different scores face *strategic uncertainty* from each other, as for r_1 and r_6 in Figure 3, panel B: a marginal score-r_1 (-r_6) investor is uncertain whether a score-r_6 (-r_1) investor is running (staying).[20] Separation refers to the situation in which the investors of the two banks with different scores have distinct cutoffs. Necessarily, there is no strategic uncertainty between them, as for r_1 and r_{10} in Figure 3, panel B. Adjacency refers to the knife-edge situation in which the investors of the two banks with different scores have the same limiting cutoff but no strategic uncertainty, as for r_1 and r_9 in Figure 3, panel B. In the binary-score example in Figure 2, entanglement, separation and adjacency correspond to the interior of the reallocation regions, that of the separation regions, and the vertical boundaries between them, respectively.

Moreover, entanglement defines an equivalence relation on scores, and divides all investors into several partition cells: there is strategic uncertainty between the investors of banks with scores in the same cell, but not between those of banks with scores in different cells. As illustrated in Figure 3, panel B, investors in different partition cells share the same switching cutoff if they are adjacent (as cell 1 and cell 2), and have different switching cutoffs if they are separate (as cell 1 and cell 3). Proposition 2 shows that given the partition defined by entanglement, we can obtain explicit expressions for the switching cutoffs in the limiting case.

Proposition 2. Given a finite disclosure, the limiting Bayes Nash equilibrium is characterized by a consecutive Z−partition of $\{1,\ldots,n\}$, $\{\{i : p_z \le i < p_{z+1}\}|z = 1,2,\ldots,Z\}$, with $1 = p_1 < \cdots < p_z < \cdots < p_{Z+1} = n+1$, such that

- Scores in the same partition cell are all entangled with each other;
- Scores in different partition cells are adjacent or separate; and

[20] This can be seen from (10) in footnote 19.

- If $i \in \{p_z, p_z+1, \cdots, p_{z+1}-1\}$, then

$$\hat{x}_i = \left(\sum_{j=p_z}^{p_{z+1}-1} \frac{w_j}{r_j}\right)^{-1} \left[\sum_{j=p_z}^{p_{z+1}-1} \frac{c_j}{r_j} w_j + A\left(\sum_{j=1}^{p_{z+1}-1} w_j\right) - A\left(\sum_{j=1}^{p_z-1} w_j\right)\right], \quad (20)$$

where

$$A(l) \triangleq \int_0^l a(\tilde{l})\, d\tilde{l}. \quad (21)$$

Moreover, $\{\hat{x}_i\}_{i=1}^n$ is a weakly increasing sequence.

Hereafter, we focus on limiting switching cutoffs (20) and suppress the adjective "limiting" unless otherwise specified.

3.2 Robust disclosures of bank groups

In the next three subsections, we focus on robust disclosures of an arbitrary bank group, formally defined as follows:

Definition 2. A *bank group* (W, Q^r, Q^c) refers to a mass W of banks, mass Q^r of which are type-$\underline{r}$ banks and mass Q^c of which are type-$\underline{c}$ banks.

Definition 3. A robust disclosure in dimension r or c of a bank group is a disclosure in that dimension for this group that minimizes the maximum switching cutoffs of its investors, given that all banks outside this group are almost surely subject to runs.

Several remarks are in order. First, the bank group as a whole is immune from runs only if its weakest constituent is immune; that is, the bank whose investor's switching cutoff is the maximum among the whole group. By minimizing this maximum cutoff, a robust disclosure maximizes the robustness of the weakest constituent, and thus that of the whole group, to adverse fundamental shocks, for which it is so named. Second, if the regulator wants to maximize the probability that all banks are immune from runs, as in Section 2, then her optimal disclosure(s) is the robust disclosure(s) for the whole banking system; that is, for bank group $(1, q^r, q^c)$. Lastly, recall from Section 1.5 that our minimum requirement for an optimal disclosure is that it yields a stability scheme that does not first-order stochastically dominate those resulting from any other feasible disclosures. We establish in Section 3.6 that such an optimal disclosure must be a combination of generalized robust disclosures, where the generalization allows the mass of stayers outside the corresponding bank group to be constants other than zero.

3.3 Robust disclosures in dimension c

First, we consider disclosures in dimension c, where $r_i = \mathbb{E}r$ for all i. Since they do not reallocate PSV, as discussed in Section 2.3 and implied by Equation (20), such disclosures are not conducive to mitigating systemic bank runs.

Proposition 3. Nondisclosure is a robust disclosure in dimension c for any bank group (W, Q^r, Q^c), in which all investors share the switching cutoff

$$\hat{x}_c\left(W, Q^c, A(\cdot)\right) = \frac{\left[Q^c \cdot \underline{c} + (W - Q^c) \cdot \bar{c}\right]}{W} + \mathbb{E}r \cdot \frac{A(W)}{W}. \tag{22}$$

Equation (22) generalizes Equation (14), where $\frac{\left[Q^c \cdot \underline{c} + (W - Q^c) \cdot \bar{c}\right]}{W}$ is the expected idiosyncratic shortfall of the bank group (W, Q^r, Q^c).

3.4 Robust disclosures in dimension r

Now we consider disclosures in dimension r, where $c_i = \mathbb{E}c$ for all i. As discussed in Section 3.1, if the heterogeneity between two bank scores is sufficiently large, their investors will have different switching cutoffs. Then, as indicated in Figure 2, reducing the heterogeneity can always lower the maximum of the switching cutoffs until they become equal:

Proposition 4. In a robust disclosure in dimension r, all scores must be either entangled or adjacent to each other.

By Equation (20), the common switching cutoff of all investors is

$$\hat{x} = \mathbb{E}c + \left(\sum_j \frac{w_j}{r_j}\right)^{-1} \cdot A\left(\sum_j w_j\right).$$

As an integral over $a(\cdot) > 0$, $A\left(\sum_j w_j\right) > 0$ and thus $\hat{x}$ is strictly increasing in $\left(\sum_j \frac{w_j}{r_j}\right)^{-1}$. Lemma 1 establishes that when two scores are entangled, marginally increasing informational heterogeneity in dimension r with a mean-preserving spread of scores reduces $\left(\sum_j \frac{w_j}{r_j}\right)^{-1}$ and thus $\hat{x}$. This captures the essence of the beneficial negative assortative matching discussed in Section 2.3.

Lemma 1. Suppose $r_i' \le r_i \le r_j \le r_j'$, $w_i r_i + w_j r_j = w_i' r_i' + w_j' r_j'$, and $w_i + w_j = w_i' + w_j'$. Then we have

$$\frac{w_i}{r_i} + \frac{w_j}{r_j} \le \frac{w_i'}{r_i'} + \frac{w_j'}{r_j'},$$

and the equality holds if and only if $r_i' = r_i$ and $r_j' = r_j$.

Proof. The proof is straightforward from the convexity of $f(r)=1/r$ in $(0,+\infty)$. ■

Hence, a robust disclosure should maximize the heterogeneity in dimension r, provided that all investors have the same switching cutoff. But the extent of such maximization is restricted by the original physical heterogeneity among banks, as captured by $\bar{r}/\underline{r}$.

Proposition 5. Consider a bank group (W, Q^r, Q^c).

- If $\bar{r}/\underline{r} \leq \frac{A(Q^r)}{Q^r}\frac{W-Q^r}{A(W)-A(Q^r)}$, then its robust disclosure in dimension r is full disclosure.
- If $\bar{r}/\underline{r} > \frac{A(Q^r)}{Q^r}\frac{W-Q^r}{A(W)-A(Q^r)}$, under its robust disclosure in dimension r, all scores must be adjacent to each other, and the resultant common switching cutoff is strictly decreasing in the number of scores allowed.

That is, if $\bar{r}/\underline{r} \leq \frac{A(Q^r)}{Q^r}\frac{W-Q^r}{A(W)-A(Q^r)}$, then the physical heterogeneity of the bank group is so weak that all its investors share the same switching cutoff even under full disclosure, so full disclosure is its robust disclosure. Otherwise, robust disclosures assign adjacent scores, and allowing for more scores always reduces the common switching cutoff induced by a robust disclosure. To see this, note that for any such disclosure, we can regard one of its scores as two scores with the same mass but forced to coincide by the constraint on the total number of scores. If one more score is allowed, the regulator can at least make these two scores also adjacent, which further reduces the common switching cutoff.

3.5 Limiting robust disclosures of bank groups with strong physical heterogeneity

By Proposition 5, for a bank group with strong physical heterogeneity, the regulator would prefer to implement a disclosure with as many scores as possible to lower the common cutoff. What if she is allowed to assign as many scores as she likes? In this subsection, we construct the limiting robust disclosure and study its properties. We write this candidate as the function

$$\Omega(\cdot; W, Q^r, A(\cdot)) : \left[\underline{r}, \bar{r}\right] \to [0, W],$$

where $\Omega(r; W, Q^r, A(\cdot))$ represents the mass of banks whose scores are less than or equal to r. Figure 4 illustrates its structure.

The essence of adjacency is the maximization of informational heterogeneity under the constraint of a common switching cutoff. First, consider the situation in which $\underline{r}$ is so low and $\bar{r}$ is so high that they do not restrain the regulator through Bayesian plausibility from pushing such maximization to the extreme, as illustrated in Figure 4, panel A. Such maximization reduces the mass of banks sharing the same score to zero, so that $\Omega(\cdot; W, Q^r, A(\cdot))$ is continuous. Otherwise, further reduction of the common switching cutoff is feasible by

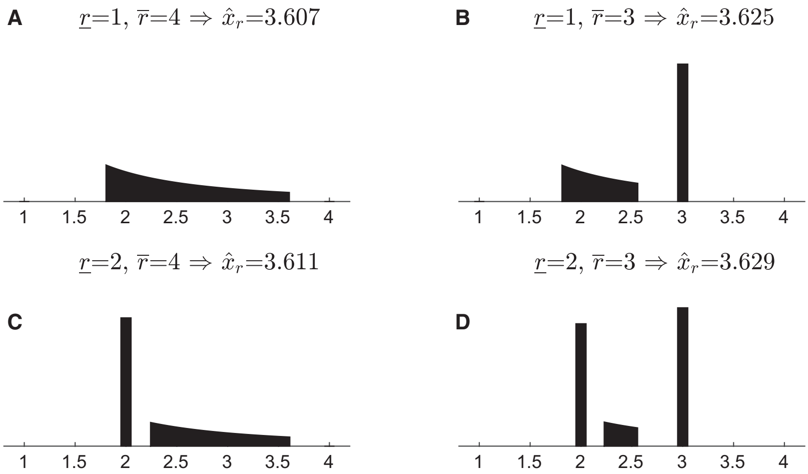

Figure 4
Limiting robust disclosures in dimension *r*
This figure illustrates the structure of limiting robust disclosures in dimension r when $\bar{r}/\underline{r} > \frac{A(Q^r)}{Q^r}\frac{W-Q^r}{A(W)-A(Q^r)}$. For all panels, $W=1$, $\mathbb{E}r=2.5$, $\mathbb{E}c=0$, and $a(l)=2-l$. In panel A, neither $\bar{r}$ nor $\underline{r}$ restricts the design of limiting robust disclosures; in panel B, only $\bar{r}$ is restrictive; in panel C, only $\underline{r}$ is restrictive; in panel D, both $\bar{r}$ and $\underline{r}$ are restrictive.

replacing a score of positive mass with its mean-preserving spread. Such maximization also eliminates the strategic uncertainty faced by all investors: a marginal score-r investor believes that investors with scores $r'<r$, whose mass is $\Omega(r;W,Q^r,A(\cdot))$, almost surely stays, and the others almost surely run, so that his switching cutoff, which he shares with all investors because of adjacency of scores, is

$$\hat{x}_r\left(W,Q^r,A(\cdot)\right)\equiv\mathbb{E}c+r\cdot a\left(\Omega(r;W,Q^r,A(\cdot))\right). \tag{23}$$

In this situation, the right-hand side of Equation (23) must be constant over all r in the support of $\Omega(\cdot;W,Q^r,A(\cdot))$,[21] and we refer to Equation (23) as the common-switching-cutoff constraint. Lastly, the total mass of banks and the mass of type-$\underline{r}$ banks given by $\Omega(\cdot;W,Q^r,A(\cdot))$ must be consistent with the group; that is, $\Omega(\bar{r};W,Q^r,A(\cdot))=W$ and $\int_{r=\underline{r}}^{\bar{r}} r\cdot d\Omega(r;W,Q^r,A(\cdot))=\underline{r}\cdot Q^r+\bar{r}\cdot[W-Q^r]$. A unique $\Omega(\cdot;W,Q^r,A(\cdot))$ satisfies all these properties. To avoid distracting readers with technicalities, we relegate its detailed construction to Section A of the appendix.

Now consider the situation in which $\bar{r}$ is so low that it restrains the regulator from further spreading scores beyond it, as illustrated in Figure 4, panel B. In this situation, a positive mass has to be "piled" at $\bar{r}$. Note that there is a gap between $\bar{r}$ and the supremum of the continuous component

[21] In general, Equation (23) holds for all scores of mass zero in limiting robust disclosures.

of the distribution, r^+. Strategic complementarity between all investors and the strategic uncertainty among investors of these score-$\bar{r}$ banks leads to the isolation of these banks from the rest. To see this, while both a marginal score-r^+ investor and a marginal score-$\bar{r}$ investor believe that investors with scores other than $\bar{r}$ are almost surely staying, the marginal score-r^+ investor believes that all score-$\bar{r}$ investors are almost surely running, while the marginal score-$\bar{r}$ investor believes that only some of them are running.[22] Thus, as long as the mass piled at $\bar{r}$ is positive, there is a noninfinitesimal difference in the systemic risk that they expect, and the common-switching-cutoff constraint (23) requires a gap between r^+ and $\bar{r}$. Similar phenomena occur when only $\underline{r}$ is restrictive, as in Figure 4, panel C, and when both $\underline{r}$ and $\bar{r}$ are restrictive, as in Figure 4, panel D. When $\underline{r}$ and $\bar{r}$ become so restrictive that the bank group has weak physical heterogeneity (i.e., $\overline{r}/\underline{r} \leq \frac{A(Q^r)}{Q^r}\frac{W-Q^r}{A(W)-A(Q^r)}$), the continuous component of the distribution vanishes, consistent with Proposition 5 that full disclosure is its robust disclosure in dimension r.

Proposition 6. Consider robust disclosures with at most $t \geq 1$ scores in dimension r for any bank group (W, Q^r, Q^c) with $\overline{r}/\underline{r} > \frac{A(Q^r)}{Q^r}\frac{W-Q^r}{A(W)-A(Q^r)}$.

- $\hat{x}_r(W, Q^r, A(\cdot))$ is the infimum of the switching cutoffs of all such disclosures.
- Such disclosures converge to $\Omega(\cdot; W, Q^r, A(\cdot))$ as $t \to \infty$, in the sense that the distance between their quantile functions converges to 0 in the L^1-norm.

Proposition 6 provides two pieces of important information. First, as the number of scores t goes to infinity, the common cutoff of robust disclosures converges downward to $\hat{x}_r(W, Q^r, A(\cdot))$. Hence, $\hat{x}_r(W, Q^r, A(\cdot))$ can be considered as the (asymptotically) lowest cutoff that the regulator can achieve with sufficient scores. Second, $\Omega(\cdot; W, Q^r, A(\cdot))$ is indeed the limit of robust disclosures as the number of scores t goes to infinity.

Although $\Omega(\cdot; W, Q^r, A(\cdot))$ involves infinitely many scores and thus is not feasible in a practical design problem, it still provides a meaningful benchmark. First, robust disclosures with sufficient scores are arbitrarily close to $\Omega(\cdot; W, Q^r, A(\cdot))$. Second, $\Omega(\cdot; W, Q^r, A(\cdot))$ can be explicitly characterized, and thus serves as a tractable tool for studying the nature of optimal disclosures with many scores. In addition, the notion of $\Omega(\cdot; W, Q^r, A(\cdot))$ also can be extended to disclosures in dimension c (which is nondisclosure, by Proposition 3), and to those in dimension r of bank groups with $\overline{r}/\underline{r} \leq \frac{A(Q^r)}{Q^r}\frac{W-Q^r}{A(W)-A(Q^r)}$ (which is full disclosure, by Proposition 5). Later in the application in Section 4, we characterize optimal disclosures based on robust disclosures in this notion in all these cases.

22 More precisely, they believe that the proportion of score-$\bar{r}$ investors who stay is uniformly distributed in $[0,1]$. This can be seen from Equation (10) in footnote (19) with $\Delta^{\sigma}_{i,j} = 0$, since they all share the same score.

3.6 Robust disclosures given general objective functions

Recall from Section 1.5 that our ultimate goal is to characterize the general properties of disclosures that meet the minimum optimality requirement of yielding stability schemes that do not first-order stochastically dominate those resulting from any other feasible disclosures. In this subsection, we show that any optimal disclosure in this sense must be a combination of a generalized version of the robust disclosures just studied, which maintain the same qualitative properties.

Formally, consider disclosures with no more than $n \geq 2$ scores.[23] Since the set of n-score disclosures is closed and bounded, optimal disclosures exist. Now suppose an optimal disclosure induces $T \leq n$ distinct switching cutoffs in equilibrium, ranked as $\theta_1 < \theta_2 < \ldots < \theta_T$. We can then regard a disclosure as a collection of subdisclosures, each of which is imposed on a group of banks whose investors share the same switching cutoff. We define (κ, t)-robust disclosures of a bank group as follows, and show in Proposition 7 that any subdisclosure of an optimal disclosure must be a certain (κ, t)-robust disclosure of the bank group.

Definition 4. A (κ, t)-*robust disclosure* in dimension r or c of a bank group (W, Q^r, Q^c) is a disclosure with no more than t scores in that dimension for this group and one that minimizes the maximum switching cutoffs of its investors, given that among all banks (of mass $1-W$) outside this group, mass κ are almost surely immune from runs and the rest are almost surely subject to runs.

Proposition 7. Suppose $K\left(\cdot;\{K_i, \theta_i\}_{i=1}^{T}\right)$ is a stability scheme resulting from an n-score optimal disclosure. Let t_i denote the number of scores whose corresponding investors share the switching cutoff θ_i. Then for any i, the subdisclosure of the bank group consisting of all the banks whose investors share the switching cutoff θ_i must be the (K_{i-1}, t_i)-robust disclosure of the group.

To see the intuition of Proposition 7, consider an optimal disclosure and the resultant stability scheme $K\left(\cdot;\{K_i, \theta_i\}_{i=1}^{T}\right)$. Consider the bank group consisting of all the banks whose investors share the switching cutoff θ_i and whose mass is $K_i - K_{i-1}$. As long as their signal realizations are in $(\theta_{i-1}, \theta_{i+1})$, which includes θ_i, these investors think that among investors outside this bank group, whose mass is $1-(K_i - K_{i-1})$, those with cutoffs no greater than θ_{i-1}, whose mass is K_{i-1}, almost surely stay, and the rest almost surely run. This is precisely the condition on the outsiders of this bank group for its (K_{i-1}, t_i)-robust disclosure. We write the maximum switching cutoffs under the (K_{i-1}, t_i)-robust disclosure as θ_i'. By definition, we have $\theta_i' \leq \theta_i$. But the optimality of the original disclosure

[23] Note that $n=1$ means nondisclosure.

implies that $\theta_i'=\theta_i$. Otherwise, we can replace the original subdisclosure for this bank group with its (K_{i-1},t_i)-robust disclosure, without changing the original subdisclosures for its outsiders. It can be shown that under this alternative disclosure, investors of this bank group would have switching cutoffs no more than θ_i', while other investors' switching cutoffs do not increase relative to their original levels. This results in a stability scheme that is first-order stochastically dominated by $K\left(\cdot;\{K_j,\theta_j\}_{j=1}^{T}\right)$, violating the optimality of the original disclosure.

Note that the robust disclosure defined by Definition 3 with at most t scores is a $(0,t)$-robust disclosure. We now show the converse: its qualitative properties are preserved by (κ,t)-robust disclosures. Since the mass of stayers outside the bank group is fixed at κ, we can equivalently consider the investors of the given bank group (W,Q^r,Q^c) as playing a coordination game only among themselves, where the systemic risk they face when the mass of stayers among them is l is

$$a_\kappa(l)\triangleq a(l+\kappa)$$

instead of $a(l)$. We then define

$$A_\kappa(l)\triangleq\int_0^l a_\kappa(w)dw=A(l+\kappa)-A(\kappa).$$

By definition, $A_\kappa(0)=0$. Thus, with $a(l)$ and $A(l)$ replaced by $a_\kappa(l)$ and $A_\kappa(l)$, respectively, the equilibrium of the game is still characterized by Propositions 1 and 2, and Propositions 3∼5 still characterize (κ,t)-robust disclosures. Moreover, for a bank group (W,Q^r,Q^c), its limiting (κ,t)-robust disclosure as $t\to+\infty$ is $\Omega(\cdot;W,Q^r,A_\kappa(\cdot))$, and the resultant common switching cutoff is

$$\hat{x}_r\left(W,Q^r,A_\kappa(\cdot)\right)\equiv\mathbb{E}c+r\cdot a_\kappa\left(\Omega(r;W,Q^r,A_\kappa(\cdot))\right). \tag{24}$$

Proposition 8 summarizes these results.

Proposition 8. Propositions 3∼6 with $A(\cdot)$ replaced by $A_\kappa(\cdot)$ hold for (κ,t)-robust disclosures in the corresponding dimension for bank group (W,Q^r,Q^c).

4. An Application: Public Disclosure of Stress-Test Results

Since the 2007−2008 financial crisis, a vigorous debate has arisen concerning whether and how the regulator should disclose the results of the stress tests of individual financial institutions. The essence of this debate is the design of optimal public disclosure of bank-specific information that mitigates systemic bank runs.[24] This section complements existing discussions with two

[24] For example, Goldstein and Sapra (2014) point out that while public disclosure of bank-specific information may enhance market and supervisory discipline on banks' risk-taking behaviors, it may engender the Hirshleifer effect and undermine risk sharing, incentivize banks to boost short-term cash flows at the expense of long-term profitability, trigger ex post coordination failure among market participants, and weaken regulators' ability to learn from the market.

novel implications due to the presence of systemic risk. Theoretically, this section also demonstrates the critical role of the robust disclosures developed in Section 3 in the construction of optimal disclosures, given a complete preference of the regulator that respects the partial order in Section 1.5.

4.1 Optimal disclosures

In practice, the regulator is concerned about whether banks are able to withstand negative economic shocks. As manifested by the design of stress tests, the regulator often focuses on hypothetical adverse scenarios and makes policies accordingly to improve financial stability in these scenarios. Motivated by this observation, we assume that the regulator's objective is to maximize the mass of banks immune from runs in a hypothetical adverse scenario, where the fundamental θ equals an exogenous $\hat{\theta}$. Again, we adopt the law of large numbers convention[25] so that q^r (q^c) is also the mass of type-$\underline{r}$ (type-$\underline{c}$) banks in the system.

Consider disclosures in dimension r. Recall from Section 3.2 that the robust disclosure for the whole system (i.e., for bank group $(1, q^r, q^c)$) maximizes the robustness of the system and prevents all banks from runs when $\hat{\theta}$ is above the resultant common switching cutoff $\hat{x}_r(1, q^r, A(\cdot))$. A bifurcation occurs when $\hat{\theta} < \hat{x}_r(1, q^r, A(\cdot))$: if $\overline{r}/\underline{r} \leq \frac{A(q^r)}{q^r} \frac{1-q^r}{A(1)-A(q^r)}$, physical heterogeneity in systemic vulnerabilities is so weak that all investors always share the same switching cutoff regardless of disclosures. Thus, the whole system is subject to runs if its robust disclosure (which is full disclosure, by Proposition 5) cannot save it; if $\overline{r}/\underline{r} > \frac{A(q^r)}{q^r} \frac{1-q^r}{A(1)-A(q^r)}$, physical heterogeneity in systemic vulnerabilities is strong enough to allow physically strong banks to be separate from weak ones through disclosures. Since the regulator cares only about the mass of banks immune from runs at $\hat{\theta}$, all "sacrificed" banks must be physically weak and fully revealed, and the corresponding robust disclosure is made to "preserved" banks to maximize their joint robustness and consequently their mass. It is possible to preserve some banks through disclosures only if $\hat{\theta} \geq \hat{x}_r(q^r, q^r, A(\cdot))$, where $\hat{x}_r(q^r, q^r, A(\cdot))$ is the common switching cutoff resulting from the robust disclosure for bank group (q^r, q^r, q^c), which consists exclusively of all physically strong banks in the system.

Proposition 9. Consider optimal disclosures in dimension r. Let $\hat{x}_r$ be given by equation (23). Suppose $\overline{r}/\underline{r} > \frac{A(q^r)}{q^r} \frac{1-q^r}{A(1)-A(q^r)}$.

- If $\hat{\theta} \geq \hat{x}_r(q^r, q^r, A(\cdot))$, a mass $1 - W_r\left(\hat{\theta}, q^r\right)$ of type-$\overline{r}$ banks are fully revealed and subject to runs at $\hat{\theta}$ while the remaining banks are revealed as specified by their robust disclosure and are immune from runs at $\hat{\theta}$.

[25] See footnote 17.

Here, $W_r\left(\hat{\theta}, q^r\right)$ is the maximum W in $[q^r, 1]$ such that $\hat{x}_r(W, q^r, A(\cdot)) \leq \hat{\theta}$.

- If $\hat{\theta} < \hat{x}_r(q^r, q^r, A(\cdot))$, no bank is immune from runs at $\hat{\theta}$ regardless of disclosures.

Suppose $\overline{r}/\underline{r} \leq \frac{A(q^r)}{q^r}\frac{1-q^r}{A(1)-A(q^r)}$. If $\hat{\theta} \geq \hat{x}_r(1, q^r, A(\cdot))$, all banks are fully revealed and immune from runs at $\hat{\theta}$; otherwise, they are all subject to runs at $\hat{\theta}$ regardless of disclosures.

Optimal disclosures in dimension c are analogous. Recall from Proposition 3 that nondisclosure is always a robust disclosure in dimension c for any bank group. Thus, Proposition 10 indicates that it is optimal to reveal nothing about any bank in dimension c regardless of $\hat{\theta}$ when $\overline{c}-\underline{c} \leq \left[\frac{A(q^c)}{q^c} - \frac{A(1)-A(q^c)}{1-q^c}\right]\mathbb{E}r$, and to fully reveal "sacrificed" type-$\overline{c}$ banks but to reveal nothing about the rest when $\overline{c}-\underline{c} > \left[\frac{A(q^c)}{q^c} - \frac{A(1)-A(q^c)}{1-q^c}\right]\mathbb{E}r$.

Proposition 10. Consider optimal disclosures in dimension c. Let $\hat{x}_c$ be given by Equation (22). Suppose $\overline{c}-\underline{c} > \left[\frac{A(q^c)}{q^c} - \frac{A(1)-A(q^c)}{1-q^c}\right]\mathbb{E}r$.

- If $\hat{\theta} \geq \hat{x}_c(q^c, q^c, A(\cdot))$, a mass $1-W_c\left(\hat{\theta}, q^c\right)$ of type-$\overline{c}$ banks are fully revealed and subject to runs at $\hat{\theta}$ while the remaining banks are revealed as specified by their robust disclosure and immune from runs at $\hat{\theta}$. Here, $W_c\left(\hat{\theta}, q^c\right)$ is the maximum W in $[q^c, 1]$ such that $\hat{x}_c(W, q^c, A(\cdot)) \leq \hat{\theta}$.
- If $\hat{\theta} < \hat{x}_c(q^c, q^c, A(\cdot))$, no bank is immune from runs at $\hat{\theta}$ regardless of disclosures.

Suppose $\overline{c}-\underline{c} \leq \left[\frac{A(q^c)}{q^c} - \frac{A(1)-A(q^c)}{1-q^c}\right]\mathbb{E}r$. If $\hat{\theta} \geq \hat{x}_c(1, q^c, A(\cdot))$, all banks are revealed as specified by their robust disclosure and immune from runs at $\hat{\theta}$; otherwise, they are all subject to runs at $\hat{\theta}$ regardless of disclosures.

Our key addition to a standard Bayesian persuasion model is systemic risk, which engenders strategic complementarity between the investors of different banks. This is the key concern of the designers of macro-prudential policies and Basel III. In the absence of systemic risk (i.e., when $a(\cdot)=0$), disclosures in dimension r do not matter, and an optimal disclosure in dimension c is nondisclosure for all banks if $\hat{\theta} \geq \mathbb{E}c$, and full disclosure for "sacrificed" type-$\bar{c}$ banks but nondisclosure for "preserved" banks, such that the latter barely survive $\hat{\theta}$ if $\hat{\theta} < \mathbb{E}c$.[26] Systemic risk renders disclosures of systemic

[26] To see this, note that by Equation (15), a score-i investor's switching cutoff is simply $\hat{x}_i = c_i$.

vulnerabilities r meaningful. In practice, such disclosures may comprise the magnitude of interbank lending or positions of publicly traded assets. By Propositions 9 and 10, optimal disclosures in either dimension apply the respective robust disclosures to banks immune from runs. But recall from Section 3 that because of beneficial negative assortative matching, robust disclosures in dimension r assign adjacent scores that maximally differentiates these banks provided that they are equally robust, while disclosures in dimension c (such as those of banks' cash and cash equivalents or nonperforming loans of little systemic consequence) do not entail such differentiation. And the distinction becomes extreme with weak physical heterogeneity; that is, when the impact of systemic risk dominates physical heterogeneity. Then, optimal disclosure in dimension r is full disclosure, while nondisclosure is an optimal disclosure in dimension c. This is in sharp contrast to Bouvard, Chaigneau, and Motta (2015), where systemic risk is not considered.

Implication 1: To mitigate systemic bank runs in the presence of systemic risk, optimal disclosures of systemic vulnerabilities entail significant differentiation among banks not in trouble, while optimal disclosures of idiosyncratic shortfalls need not.

Moreover, as shown in Bouvard, Chaigneau, and Motta (2015), when systemic risk is absent, as the adverse scenario deteriorates, more physically weak banks are fully revealed and "sacrificed" under the optimal disclosure. Proposition 11 confirms that this conclusion continues to holds in the presence of systemic risk. Moreover, Proposition 11 stipulates that this notion of "more information should be disclosed under worse scenario" is strengthened by systemic risk in dimension r. That is, in the presence of systemic risk, optimal disclosures of systemic vulnerabilities also differentiate banks immune from runs by more under worse scenario, in the sense that more physically strong banks are also fully revealed.

Proposition 11. As $\hat{\theta}$ decreases, the mass of type-$\overline{r}$ (type-$\overline{c}$) banks that are fully revealed under optimal disclosures of systemic vulnerabilities (idiosyncratic shortfalls) weakly increases. In addition, the mass of type-$\underline{r}$ banks that are fully revealed under optimal disclosures of systemic vulnerabilities also weakly increases.

We now discuss the practical implications for public disclosure of bank-specific information as the summary of this subsection. Regardless of the nature of bank-specific information, banks that cannot be protected from runs by optimal policies under a given adverse scenario should be fully revealed. But a bifurcation emerges concerning banks that can be immune from runs: Optimal disclosures of information related to their systemic vulnerabilities, such as the magnitude of their interbank lending or positions of publicly traded assets, should entail significant differentiation. But disclosures of

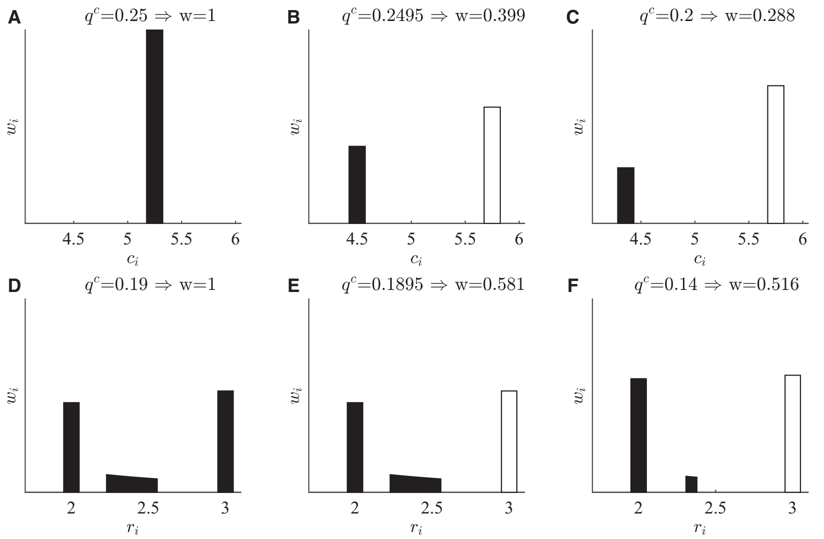

Figure 5
Optimal disclosures in dimensions *c* and *r*, respectively
This figure illustrates how optimal disclosures in either dimension c or dimension r evolve with the deterioration of the average quality of the banking system, represented by a reduction in q^c. Panels A to C illustrate optimal disclosures in dimension c with the corresponding distributions of scores, and panels D to F illustrate those in dimension r. In each panel, the horizontal axis represents disclosed scores, and the vertical axis represents the mass of banks; solid areas represent "preserved" banks when $\theta = \hat{\theta}$, with their total mass represented by the w in the title, and hollow areas represent "sacrificed" banks. For all panels, $(\underline{r}, \overline{r}) = (2,3)$, $q^r = 0.5$, $(\underline{c}, \overline{c}) = (3.75, 5.75)$, $a(l) = 2 - l$, $\hat{\theta} = 9$, and both $\overline{c} - \underline{c} > \left[\frac{A(q^c)}{q^c} - \frac{A(1) - A(q^c)}{1 - q^c}\right] \mathbb{E}r$ and $\overline{r}/\underline{r} > \frac{A(q^r)}{q^r} \frac{1 - q^r}{A(1) - A(q^r)}$ are satisfied.

information of little relation to systemic vulnerabilities, such as cash and cash equivalents or nonperforming loans of little systemic consequence, need not. Moreover, more information should be disclosed given a worse state of the economy.

4.2 A sudden run on a huge mass of banks may be inevitable even with optimal disclosures

Figure 5 illustrates how optimal disclosures in dimensions r and c evolve respectively with the deterioration of the average quality of the banking system, represented by a reduction in q^c, the percentage of type-$\underline{c}$ banks in the system, under strong physical heterogeneity. Observe that they preserve two properties of optimal disclosures in the respective dimensions in the absence of systemic risk. First, "sacrificed" banks (marked by hollow areas) are all physically weak and fully disclosed, and their scores are separate from those of "preserved" banks. Second, more information is disclosed as the banking system deteriorates. However, systemic risk makes a key difference in this process:

Implication 2: When the impact of systemic risk is large, under the optimal disclosure of bank-specific information, as the quality of the banking

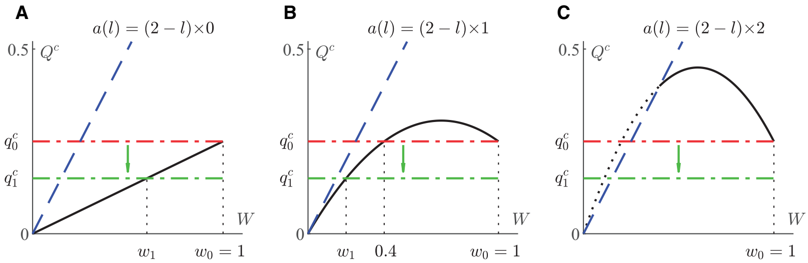

Figure 6
The minimum mass of type-$\underline{c}$ banks
The black solid curves in this figure illustrate the minimum mass Q^C of type-$\underline{c}$ banks among banks of a given mass W that can keep them from runs under optimal disclosures in dimension c when $\theta=\hat{\theta}$, given by $\hat{x}_c\left(W, Q^c, A(\cdot)\right)=\hat{\theta}$. The horizontal axis of each panel represents the mass W of all banks, and the vertical axis represents the mass Q^C of type-$\underline{c}$ banks. The blue dashed lines indicate the natural constraint $Q^C \leq W$. For all panels, $\left(\underline{r}, \overline{r}\right)=(2,3)$, $\mathbb{E}r=2.5$, and $\hat{\theta}=9$. From panels A to C, we scale up $a(\cdot)$ to achieve higher systemic risks. To ensure the feasibility of the same $\hat{\theta}=9$ for disclosures under a uniformly higher systemic risk $a(\cdot)$, we lower $(\underline{c}, \overline{c})$ accordingly: we pick $\overline{c}=9.5-1.5\times\mathbb{E}r\times\frac{a(l)}{2-l}$ and $\underline{c}=\overline{c}-2$.

system deteriorates, a banking crisis unfolds as follows: first, a substantial mass of banks are run simultaneously, and then the remaining banks are run gradually.

To see this, note that the value of q^c in Figure 5, panel A, is the critical level that could prevent all banks from runs with disclosures in dimension c: if it falls a tiny bit to that in panel B, the mass w of "preserved" banks experiences a negative discontinuous jump from 1 down to 0.399. A similar pattern also applies to disclosures in dimension r, as shown in the transition from panel D to panel E. This is due to a new economic force: the "sacrifice" of physically weak banks reduces the total liquidity and increases the systemic risk faced by all the "preserved" banks. This also differentiates our paper from Bouvard, Chaigneau, and Motta (2015), which abstracts from the strategic interaction among investors of different banks.

Figure 6 illustrates the role of systemic risk in shaping this implication with disclosures in dimension c. In each panel, the black solid curve illustrates the minimum mass Q^c of type-$\underline{c}$ banks among banks of a given mass W that can keep them from runs under optimal disclosures in dimension c when $\theta=\hat{\theta}$, given by $\hat{x}_c(W, Q^c, A(\cdot))=\hat{\theta}$. Consequently, the maximum mass w of banks in the system that can be immune from runs at $\hat{\theta}$ given q^c is determined by the (the furthest right, if there are several) intersection of the black curve and the horizontal line $Q^c=q^c$.

Figure 6, panel A, depicts the benchmark without systemic risk. There, a constant average quality of the bank group, and thus a constant Q^c/W, are required for the group to be preserved given the same $\hat{\theta}$. Thus, the black solid curve is a straight line passing through the origin. To keep the whole system from runs, Q^c must reach the critical level q_0^c indicated by the red dot-dashed line. The total mass q^c of physically strong banks in the system determines the maximum mass w of immune banks through the constraint $Q^c \le q^c$. If $q^c > q_0^c$, the constraint slacks and $w = 1$. Now consider a fall in q^c from q_0^c to q_1^c, as shown by the downward shift of the horizontal dot-dashed line indicated by the green arrow. The constraint $Q^c \le q^c$ binds in the process, and the mass w of immune banks adjusts *linearly* from $w_0 = 1$ down to w_1, the level corresponding to $Q^c = q_1^c$ on the black solid line. Thus, without systemic risk, there is no discontinuous jump in w; that is, bank runs under optimal disclosures in dimension c are gradual.

High systemic risk breaks the monotonic relation between Q^c and W, as illustrated in Figure 6, panels B and C: since banks outside the group are almost surely run at $\hat{\theta}$, a reduction in W (e.g., from 1 to 0.9 in both figures) raises the systemic risk expected by all investors in the group at $\hat{\theta}$, and requires a higher Q^c to compensate. This results in a discontinuous jump in w following a fall in q^c from the critical level q_0^c analogous to that in Figure 6, panel A. In this process in Figure 6, panel B, where systemic risk $a(\cdot)$ is identical to that in Figure 5 and $q_0^c = 0.25$ as in Figure 5, panel A, even if the magnitude of the fall in q^c is infinitesimal, the resultant reduction in the mass w of "preserved" banks, $w_0 - w_1$, is greater than $1 - 0.4 = 0.6$! This interprets the large reduction in w from 1 to 0.399 in Figure 5, from panel A to panel B. But there is no further jump in w as q^c decreases further, as shown by the transition in Figure 5, from panel B to panel C, until all the type-$\bar{c}$ banks are sacrificed, when further reduction in q^c instantaneously kills all the type-$\underline{c}$ banks.

When systemic risk is sufficiently high, another constraint by construction, $Q^c \le W$, further worsens the situation. In all panels, this constraint means that the black solid curve cannot go beyond the blue dashed 45-degree line: $Q^c = W$. In Figure 6, panel C, this is violated for low values of W corresponding to the black dotted curve segment, since the impact of systemic risk is so large that $\bar{c} - \underline{c} < \left[\frac{A(q^c)}{q^c} - \frac{A(1) - A(q^c)}{1 - q^c}\right]\mathbb{E}r$. For these values, even if $Q^c = W$, the runs on banks (of mass $1 - W$) outside this group raises the systemic risk perceived by investors in this group by so much, that their switching cutoff cannot be reduced to $\hat{\theta}$. This implies a downward jump in w of magnitude 1: once q^c falls below the critical level q_0^c, no bank is immune from runs regardless of disclosures.

5. Extension and Discussion

5.1 Investors' heterogenous impact on total liquidity

In the baseline model, the action l^i of each investor i affects the total liquidity l equally. What if such impact is heterogeneous? This subsection studies the impact of such heterogeneity.[27] Specifically, suppose $l = \int b^i l^i di$, where

[27] We thank an anonymous referee for suggesting the discussion in this subsection.

systemic impact $b^i = \underline{b} > 0$ with probability q^b and $b^i = \bar{b} \geqslant \underline{b}$ with probability $1-q^b$, such that $\mathbb{E}b = q^b \underline{b} + (1-q^b)\bar{b} = 1$, so that the baseline model is its special case. We assume first that b^i is independent of r^i and c^i, and study the nature of disclosures in this new dimension b: $\{(r_k, c_k, b_k, w_k)\}_{k=1}^n$ with $r_k = \mathbb{E}r$, $c_k = \mathbb{E}c$, $w_k > 0$ and $\underline{b} \le b_k \le \bar{b}$ for all k, $\sum_k w_k = 1$ and $\sum_k w_k b_k = \mathbb{E}b = 1$. We have

Proposition 12. For any disclosure in dimension b, all investors share the switching cutoff $\hat{x} = \mathbb{E}r \cdot A(1) + \mathbb{E}c$.

Note that this cutoff is identical to that given by (22). This is because, while $b_i \neq 1$, since each investor i is still infinitesimal, his choice of l^i is invariant given the original equilibrium aggregate liquidity $l(\theta)$ obtained when $b_i = 1$ for all i.

Usually, banks more vulnerable to systemic risk also have a greater impact on the total liquidity of the banking system and thus on systemic risk, as exemplified by too-big-to-fail financial institutions versus small local banks; that is, b^i is positively correlated with r^i, so that disclosures of the latter reveal the former. To explore the interaction between them, we highlight such correlation by assuming that banks with $r^i = \overline{r}$ ($r^i = \underline{r}$) have $b^i = \overline{b}$ ($b^i = \underline{b}$). Proposition 13 shows that disclosures in dimension r are still beneficial as long as physical heterogeneity in systemic impact is smaller than that in systemic vulnerability, so that the adverse impact of larger banks on systemic risk is dominated by their larger vulnerability to it.

Proposition 13. If $\overline{b}/\underline{b} < \overline{r}/\underline{r}$, then there must exist a disclosure in dimension r under which all banks are more robust than under nondisclosure.

5.2 Implications for the 2023 banking crisis

March 2023 witnessed the second-, third-, and fourth-largest bank failures in the history of the United States. According to Jiang et al. (2023), because of the rise of federal funds rate and quantitative tightening, marked-to-market bank assets have declined by an average of 10% across all the banks from Q1 2022 to Q1 2023. As a victim, Silicon Valley Bank failed following a run by its uninsured depositors, and its contagion effect[28] resulted in runs on Signature Bank and First Republic Bank. The runs, exacerbated by poor management, lead to the failure of these two banks.[29] With fears of further contagion, besides extraordinary liquidity provision (such as Bank Term Funding Program), public

[28] For example, using comprehensive Twitter data, Cookson et al. (2023) quantify the role of social media as a bank run catalyst.

[29] See, for example, https://www.fdic.gov/news/speeches/2023/spmay1723.html, retrieved on July 23, 2023.

disclosure of bank-specific information is also debated in policy proposals.[30] Our model captures the situation, and thus our discussion in Section 4 also contributes to this debate.[31]

Specifically, in Equation (1), the decline in banks' asset values due to the rising interest rate and quantitative tightening is captured by a reduction in θ, the contagion effect by $r^i a(l)$, and the poor management by an increase in c^i. Implication 1 indicates a bifurcation of bank-specific information: optimal disclosures of banks' systemic vulnerabilities (such as those of magnitude of interbank lending or positions of publicly traded assets) entail significant differentiation among banks not in trouble, while optimal disclosures of idiosyncratic shortfalls (such as those of banks' cash and cash equivalents or management skills) need not, since only the former leads to the beneficial negative assortative matching between PSR and PSV. Implication 2 gives a caveat: when the impact of systemic risk is large, simultaneous runs on a substantial mass of banks are inevitable even with the optimal disclosure of bank-specific information.

5.3 Beyond the banking industry

So far, we have been using "banks" to describe the entities facing systemic risk, and public disclosure of stress test results as the leading example. Potentially, our theory can be applied to other industries and settings with positive spillover. There, positive spillover could arise because of complementarities in production networks (Cohen and Lou 2012) or customer-supplier relations among firms (Cohen and Frazzini 2008). However, we think that the banking industry is the most relevant application of our theory. In practice, regulations concerning disclosures of firm-specific information are far more prevalent in the banking industry than in other industries. Besides political and cultural reasons, such prevalence can be understood from an economic perspective.

Public disclosure of firm-specific information is costly, since it requires the collection of confidential data from relevant firms and the mechanism enforcing truthful reporting. Several factors render the benefit of such disclosure potentially larger for the banking industry than for other industries. First, the banking industry is larger than most industries, and has a crucial impact on the whole economy. Second, positive spillover in the banking industry is much more impactful than that in other industries. Lastly, as suggested by a few empirical papers, banks are likely more opaque than firms

[30] See, for example, Maurer (2023). Also, Dursun-de_Neef, Ongena, and Schandlbauer (2023) and Granja (2023) discuss the problem of held-to-maturity accounting.

[31] We thank the editor for suggesting the discussion in this subsection.

in many other industries,[32] leaving larger room for the regulator's disclosure of bank-specific information.[33]

5.4 Robustness check

This subsection discusses the robustness of our results to the relaxation of three assumptions. Mathematical details have been relegated to the Internet Appendix because of length requirements.

In our baseline model, banks' systemic vulnerabilities and idiosyncratic shortfalls are assumed to be independent, allowing the regulator to disclose information in only one dimension. This facilitates our analysis of the different impact of disclosures in different dimensions. In reality, different bank-specific information could be correlated, so that a disclosure in one dimension automatically reveals information in the other. In our first robustness check, we allow for correlation between bank types in different dimensions. We find that if types in different dimensions are not too negatively correlated, or if allocation of systemic risk is sufficiently important,[34] disclosures in dimension r still can be beneficial due to the same mechanism of negative assortative matching as in the baseline model, and disclosures in dimension c affect investors' strategies only through the information they reveal in dimension r.

In our baseline model, investors' priors are assumed to be uninformative about bank-specific information in both dimensions r and c. In reality, investors may have information sources about their banks other than the regulator's disclosures. In our second robustness check, we consider the possibility of informative common priors in dimensions r and c, and show that disclosures in dimension c can only hurt the stability of a bank group (i.e., increase the maximum of its investors' switching cutoffs), while there is always a disclosure in dimension r that improves its stability. In this sense, our main results are robust to this possibility.

In our baseline model, we assume that each bank has a representative investor, so that the model naturally abstracts from coordination problems within a bank. In our third robustness check, we take this layer of coordination into consideration.[35] We make three changes to the baseline model. First,

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32 For example, Bessler and Nohel (1996) identify significantly stronger announcement effect of dividend cuts for banks than for nonbanks; Morgan (2002) and Iannotta (2006) observe that bank bonds are more likely to be rated differently by different bond-rating agencies; Hirtle (2006) finds that, following the SEC mandate that CEOs certify the accuracy of their financial statements, bank holding companies experienced positive and significant abnormal returns, but nonfinancial firms did not, and the abnormal returns are related to measures of opacity; while Flannery, Kwan, and Nimalendran (2004) detect no statistically significant differences between banks' and nonfinancial firms' equity market microstructure properties, they subsequently discover in Flannery, Kwan, and Nimalendran (2013) that this conclusion holds only for normal times, not for crisis periods.

33 We thank the editor for suggesting the discussion in this subsection.

34 A precise if-and-only-if condition is given in the Internet Appendix.

35 We thank an anonymous referee for suggesting this discussion.

each bank has infinitesimal investors with total mass one, and now $l^i \in [0,1]$ represents the mass of stayers of bank i, which can be interpreted as inside liquidity. Second, the probability that bank i survives at date 2 is

$$P^i = \frac{1}{R}[\theta^i - r^i \cdot e(l, l^i) + 1 - c^i],$$

where $e(l, l^i)$ is decreasing in both l and l^i. This allows investors to have a more significant impact on the survival of their own banks, so that an investor cares more about other investors of his own bank. Third, at date 1, each investor j of each bank i observes a private signal about θ, $x^{ij} = \theta + \sigma\varepsilon^i + \sigma_\eta \eta^{ij}$, where $\theta^i = \theta + \sigma\varepsilon^i$ represents the information shared by all investors of bank i, while $\sigma_\eta \eta^{ij}$ represents each investor's idiosyncratic noise. ε^i and η^{ij} are independent, and follow probability density functions $\phi(\cdot)$ and $\psi(\cdot)$, respectively, which are continuous, bounded, and fully supported over $(-\infty, +\infty)$. Parameters σ and σ_η determine the magnitude of noises respectively. To derive the main properties of optimal disclosures, we need to characterize the equilibrium of the coordination game following any feasible disclosure analytically. As pointed out by Liu (2023), this exercise is challenging because one signal x^{ij} is used to infer both total liquidity and inside liquidity. For tractability, we follow the approach of Liu (2023) and assume that the heterogeneity in information across investors within a bank is much smaller than that across banks. Specifically, we first take $\sigma_\eta \to 0$ with σ fixed, and then take $\sigma \to 0$ as in the baseline model. We show that our main results hold with the function $a(\cdot)$ defined as

$$a(\cdot) \equiv \int_0^1 e(\cdot, x)\,dx.$$

6. Conclusion

This paper studies how the disclosure of bank-specific information can mitigate systemic bank runs through a novel channel: the reallocation of systemic risk across banks. We find that regardless of disclosure, the aggregate systemic risk perceived by all marginal investors is constant, and that the disclosure of bank-specific information differentiates banks by their resilience to adverse shocks, and results in a negative assortative matching: it reallocates systemic risk from weak banks to strong ones. However, the disclosure of different kinds of bank-specific information has qualitatively different impacts. The disclosure of information concerning the vulnerability of banks to systemic risk could improve the stability of all banks, because it reallocates more of the constant aggregate systemic risk to banks that are believed to be less vulnerable to such risk. However, the resultant negative assortative matching from the disclosure of banks' idiosyncratic shortfalls of funds is not conducive to mitigating systemic bank runs.

Throughout the paper, we have focused on disclosures in either dimension, but not in both. This enables us to highlight the dependence of optimal disclosure on the nature of the information, which is ignored in the literature. Once the joint design of disclosures in both dimensions with more than two scores is allowed, in addition to the values of scores in each dimension, the regulator can flexibly design the correlation structure between them. While interesting, this would significantly complicate the analysis, as exemplified by the binary-score setup in the Internet Appendix. We leave this promising but technically challenging work for future research.

Appendix A. The Construction of Limiting Robust Disclosures

We take two steps to construct the limiting robust disclosure $\Omega(r;W,Q^r,A(\cdot))$. In step 1, for any bank group with a total mass W of banks, we construct an auxiliary disclosure $\tilde{\Omega}(r;W,\hat{X},A(\cdot))$ that takes the following form: there is a continuous component with support $[r_-,r_+]\subset[\underline{r},\bar{r}]$, a mass $\underline{m}\geq 0$ piled at $\underline{r}$, and a mass $\bar{m}\geq 0$ at $\bar{r}$, such that all scores are adjacent to each other and their common cutoff is $\hat{X}$. In step 2, we show that there exists a unique value of $\hat{X}$, denoted by $\hat{x}_r(W,Q^r,A(\cdot))$, such that the total mass $\tilde{Q}$ of type-$\underline{r}$ banks implied by this auxiliary disclosure is exactly Q^r. We then define $\Omega(r;W,Q^r,A(\cdot))$ as $\tilde{\Omega}(r;W,\hat{x}_r(W,Q^r,A(\cdot)),A(\cdot))$.

A.1 Constructing Auxiliary Disclosures

Given the total mass W of banks, if $\hat{X}\in\left[\underline{r}\frac{A(W)}{W}+\mathbb{E}c,\bar{r}\frac{A(W)}{W}+\mathbb{E}c\right]$, we say $\hat{X}$ is a feasible switching cutoff.[36] Define a distribution of scores $r\in[\underline{r},\bar{r}]$, whose cumulative distribution function is $\tilde{\Omega}(\cdot;W,\hat{X},A(\cdot))$: $[\underline{r},\bar{r}]\to[0,W]$ such that

$$\tilde{\Omega}(r;W,\hat{X},A(\cdot))=\begin{cases}\underline{m}, & \text{if } r\in(\underline{r},r_-)\\ \underline{m}+\int_{r_-}^{r}w(\tau)d\tau, & \text{if } r\in(r_-,r_+]\\ \underline{m}+\int_{r_-}^{r_+}w(\tau)d\tau, & \text{if } r\in(r_+,\bar{r})\\ \underline{m}+\int_{r_-}^{r_+}w(\tau)d\tau+\bar{m}, & \text{if } r=\bar{r}\end{cases}, \tag{A1}$$

where

$$\underline{m}\begin{cases}=0, & \text{if } \hat{X}\geq\mathbb{E}c+\underline{r}a(0)\\ \text{satisfies } \hat{X}=\mathbb{E}c+\underline{r}\frac{A(\underline{m})}{\underline{m}} & \text{if } \hat{X}<\mathbb{E}c+\underline{r}a(0)\end{cases}, \tag{A2}$$

$$\bar{m}\begin{cases}=0, & \text{if } \hat{X}\leq\mathbb{E}c+\bar{r}a(W)\\ \text{satisfies } \hat{X}=\mathbb{E}c+\bar{r}\frac{A(W)-A(W-\bar{m})}{\bar{m}} & \text{if } \hat{X}>\mathbb{E}c+\bar{r}a(W)\end{cases}, \tag{A3}$$

and $w(\cdot)$ is such that for any $r\in[r^-,r^+]$,

$$\hat{X}=\mathbb{E}c+r\cdot a\left(\underline{m}+\int_{r_-}^{r}w(\tau)d\tau\right), \tag{A4}$$

and that the total mass

$$\tilde{\Omega}(\bar{r};W,\hat{X},A(\cdot))=\underline{m}+\int_{r_-}^{r_+}w(\tau)d\tau+\bar{m}=W.$$

The construction of $\tilde{\Omega}(\cdot;W,\hat{X},A(\cdot))$ ensures that all different scores are adjacent. To see this, first observe that by construction, all investors share the same switching cutoff $\hat{X}$. In addition,

[36] Only switching cutoffs in this range are feasible. Given the total mass W of the bank group, if all banks are of type-$\underline{r}$, then the common switching cutoff of their investors is $\underline{r}\frac{A(W)}{W}+\mathbb{E}c$, which is the lowest feasible target switching cutoff. Similarly, $\bar{r}\frac{A(W)}{W}+\mathbb{E}c$ is the highest feasible target switching cutoff.

Equation (A4) indicates that any investor whose score is in the "continuous component" $\left[r^{-}, r^{+}\right]$ of the distribution $\tilde{\Omega}(\cdot; W, \hat{X}, A(\cdot))$ faces no strategic uncertainty. That is, when he is indifferent between running and staying, he believes that stayers exactly consist of investors with scores lower than his own, whose mass is $\underline{m} + \int_{r_-}^{r} w(\tau) d\tau$.

Moreover, physical heterogeneity may restrict informational heterogeneity; that is, scores cannot go beyond $\left[\underline{r}, \bar{r}\right]$. If $\underline{r}$ is so high that $\hat{X} < \mathbb{E}c + \underline{r}a(0)$, some mass out of W has to be piled at $r = \underline{r}$. These are type-$\underline{r}$ banks whose type is fully revealed and whose mass $\underline{m}$ is such that their investors face strategic uncertainty only among themselves. The high value of $\underline{r}$ impedes the elimination of such uncertainty.

Similarly, consider the situation in which $\bar{r}$ is too low for a given W (recall that $a(\cdot)$ is decreasing), such that $\hat{X} > \mathbb{E}c + \bar{r}a(W)$. While assigning higher scores to banks hurts their investors' confidence, this adverse effect is dominated by the beneficial negative assortative matching, given the large mass of banks to be dealt with. So the regulator would "spread" the score beyond $\bar{r}$ if feasible, which is impeded by the low value of $\bar{r}$. Again, the mass $\bar{m}$ piled at $\bar{r}$ (consisting of type-$\bar{r}$ banks whose type is fully revealed) is such that their investors face strategic uncertainty only among themselves.

A.2 Determining the Common Cutoff

For any $W \in [0,1]$ and any feasible switching cutoff $\hat{X}$, the distribution $\tilde{\Omega}(r; W, \hat{X}, A(\cdot))$ implies a mass of type-$\underline{r}$ banks $\tilde{Q}\left(\hat{X}; W, A(\cdot)\right) \triangleq \frac{\bar{r}W - \int_{r=\underline{r}}^{\bar{r}} r \cdot d\tilde{\Omega}(r; W, \hat{X}, A(\cdot))}{\bar{r} - \underline{r}}$. The auxiliary disclosure constructed in step 1 is continuous in nature. To enhance the stability of the bank group with given total mass W; that is, to induce a lower $\hat{X}$, the auxiliary disclosure has to assign more scores with low r, which requires a larger mass of type-$\underline{r}$ banks out of the fixed total mass W. This monotonicity further implies that there is a unique value of $\hat{X}$, denoted by $\hat{x}_r(W, Q^r, A(\cdot))$, that is consistent with the mass Q^r of type-$\underline{r}$ banks in the bank group (W, Q^r).

Lemma 2. $\tilde{Q}\left(\hat{X}; W, A(\cdot)\right)$ is continuous and strictly decreasing in $\hat{X}$. Thus, there exists a unique $\hat{x}_r(W, Q^r, A(\cdot))$ such that $\tilde{Q}\left(\hat{x}_r(W, Q^r, A(\cdot)); W, A(\cdot)\right) = Q^r$. In addition, $\hat{x}_r(W, Q^r, A(\cdot))$ is continuous and decreasing in Q^r.

Appendix B. Proofs

Proofs of all the lemmas introduced in the appendix have been relegated to the Internet Appendix.

Proof of Proposition 1

We take three steps to prove the proposition. The first two steps are summarized by the following two lemmas.

Lemma 3. For any infinite sequence $\{\sigma_m\}_{m=1}^{+\infty}$ of σ that converges to 0, there exists an infinite subsequence $\{\sigma_m^4\}_{m=1}^{+\infty}$ such that all $\hat{x}_i^{\sigma_m^4}$ and $\Delta_{i-1,i}^{\sigma_m^4}$ either converge to finite numbers or go to infinity. Moreover, their limits $\{(\hat{x}_i^0, \Delta_{j,i}^0)\}_{j,i \in \{1,2,\ldots,n\}}$ satisfy the equation system consisting of (15), (16), and (17).

Lemma 4. The equation system consisting of (15), (16), and (17) has a unique solution.

Based on the two lemmas, we prove that the equation system is the necessary and sufficient condition for $\{\hat{x}_i\}_{i=1}^{n}$ to be the limits of the cutoffs as $\sigma \to 0$.

Suppose as $\sigma \to 0$, $\{\hat{x}_i^{\sigma}\}_{i=1}^{n}$ do not converge to $\{\hat{x}_i^0\}_{i=1}^{n}$. That means, there exists ϵ and an infinite sequence $\{\sigma_m\}_{m=1}^{+\infty}$ such that $\max_i \left|\hat{x}_i^{\sigma_m} - \hat{x}_i^0\right| > \epsilon$. However, according to Part I and Part II, there exists an infinite subsequence $\{\sigma_m^4\}_{m=1}^{+\infty}$ of $\{\sigma_m\}_{m=1}^{+\infty}$ such that $\{\hat{x}_i^{\sigma_m^4}\}_{i=1}^n$ converges to $\{\hat{x}_i^0\}_{i=1}^n$. Contradiction! Therefore, as $\sigma\to 0$, $\{\hat{x}_i^\sigma\}_{i=1}^n$ converge to $\{\hat{x}_i^0\}_{i=1}^n$.

On the other hand, if $\{\hat{x}_i\}_{i=1}^n$ satisfies the equation system, when the disclosure is implemented, as $\sigma\to 0$, $\{\hat{x}_i^\sigma\}_{i=1}^n$ must converge to the solution of the equation system, which is uniquely $\{\hat{x}_i\}_{i=1}^n$.

Proof of Proposition 2

For any z, the scores in the partition cell $[p_z, p_{z+1})$ all have the same cutoff in the limiting case, which is denoted by $\hat{x}(z)$. Then for any $i\in[p_z, p_{z+1})$,

$$\begin{aligned}
&\hat{x}(z)\\
&=\mathbb{E}c+r_i\int_0^1 a\left(\sum_{\left\{j:\Delta_{j,i}=+\infty\right\}} w_j+\sum_{\left\{j:\left|\Delta_{i,j}\right|<\infty\right\}} w_j\left[1-\Phi\left(\Phi^{-1}(1-m_i)-\Delta_{j,i}\right)\right]\right)dm_i\\
&=\mathbb{E}c+r_i\int_0^1 a\left(\sum_{j=1}^{p_z-1} w_j+\sum_{j=p_z}^{p_{z+1}-1} w_j\left[1-\Phi\left(\Phi^{-1}(1-m_i)-\Delta_{j,i}\right)\right]\right)dm_i,
\end{aligned}$$

so

$$\frac{\hat{x}(z)-\mathbb{E}c}{r_i}=\int_0^1 a\left(\sum_{j=1}^{p_z-1} w_j+\sum_{j=p_z}^{p_{z+1}-1} w_j\left[1-\Phi\left(\Phi^{-1}(1-m_i)-\Delta_{j,i}\right)\right]\right)dm_i.$$

For any real number μ_i, we can replace m_i with $1-\Phi(\mu_i-y)$ and write the right-hand side as an integral with respect to y over $(-\infty,+\infty)$, that is,

$$\begin{aligned}
&\frac{\hat{x}(z)-\mathbb{E}c}{r_i}\\
&=\int_{y=-\infty}^{+\infty} a\left(\sum_{j=1}^{p_z-1} w_j+\sum_{j=p_z}^{p_{z+1}-1} w_j\left[1-\Phi\left(\Phi^{-1}(\Phi(\mu_i-y))-\Delta_{j,i}\right)\right]\right)d[1-\Phi(\mu_i-y)]\\
&=\int_{y=-\infty}^{+\infty} a\left(\sum_{j=1}^{p_z-1} w_j+\sum_{j=p_z}^{p_{z+1}-1} w_j\left[1-\Phi\left(\mu_i-y-\Delta_{j,i}\right)\right]\right)d[1-\Phi(\mu_i-y)] \qquad \text{(B1)}
\end{aligned}$$

Specifically, because of Equation (17), we can pick $\{\mu_i\}_{i=1}^n$ such that $\mu_j-\mu_i=\Delta_{i,j}$.

Multiplying Equation (B1) by w_j and sum over $[p_z, p_{z+1})$, we obtain

$$\begin{aligned}
&\sum_{i=p_z}^{p_{z+1}-1}\frac{\hat{x}(z)-\mathbb{E}c}{r_i}w_i\\
&=\sum_{i=p_z}^{p_{z+1}-1}\int_{y=-\infty}^{+\infty} a\left(\sum_{j=1}^{p_z-1} w_j+\sum_{j=p_z}^{p_{z+1}-1} w_j\left[1-\Phi\left(\mu_i-y-\Delta_{j,i}\right)\right]\right)d\left[w_i-w_i\Phi\left(\mu_i-y\right)\right]\\
&=\int_{y=-\infty}^{+\infty} a\left(\sum_{j=1}^{p_z-1} w_j+\sum_{j=p_z}^{p_{z+1}-1} w_j\left[1-\Phi\left(\mu_j-y\right)\right]\right)d\left\{\sum_{i=p_z}^{p_{z+1}-1}\left[w_i-w_i\Phi\left(\mu_i-y\right)\right]\right\}\\
&=\int_{\omega=0}^{\sum_{i=p_z}^{p_{z+1}-1} w_i} a\left(\sum_{j=1}^{p_z-1} w_j+\omega\right)d\omega=A\left(\sum_{j=1}^{p_{z+1}-1} w_j\right)-A\left(\sum_{j=1}^{p_z-1} w_j\right).
\end{aligned}$$

so

$$\hat{x}(z)=\mathbb{E}c+\left(\sum_{j=p_z}^{p_{z+1}-1}\frac{w_j}{r_j}\right)^{-1}\left[A\left(\sum_{j=1}^{p_{z+1}-1}w_j\right)-A\left(\sum_{j=1}^{p_z-1}w_j\right)\right].$$

Proof of Proposition 3
According to (20) and $r_i=\mathbb{E}r$ for all i, if $i\in\{p_k,p_k+1,...,p_{k+1}-1\}$,

$$\sum_{j=p_k}^{p_{k+1}-1}w_j\hat{x}_j=\sum_{j=p_k}^{p_{k+1}-1}w_j\hat{x}_i=\sum_{j=p_k}^{p_{k+1}-1}w_jc_j+\mathbb{E}r\left[A\left(\sum_{j=1}^{p_{k+1}-1}w_j\right)-A\left(\sum_{j=1}^{p_k-1}w_j\right)\right].$$

Then the average cutoff is

$$\frac{\sum_{j=1}^{t}w_j\hat{x}_j}{W}=\frac{\sum_{j=1}^{t}w_jc_j}{W}+\mathbb{E}r\frac{A(W)}{W}=\frac{\left[Q^c\cdot\underline{c}+(W-Q^c)\cdot\bar{c}\right]}{W}+\mathbb{E}r\cdot\frac{A(W)}{W},$$

which is also a lower bound of the maximum cutoff. Under nondisclosure, all investors have the same cutoff, so the maximum cutoff achieves this lower bound.

Proof of Proposition 4
For any k, the scores in the partition cell $[p_k,p_{k+1})$ all have the same cutoff in the limiting case, which is denoted by $\hat{x}(k)$. If two scores have different cutoffs, then there must exist k such that $\hat{x}(k)<\hat{x}(k+1)$. Let k_{max} be the maximum among them. Then $\hat{x}(k_{max}+1)=\hat{x}(k_{max}+2)=\ldots=\hat{x}(K)$.

Let $\tilde{x}$ be the root of

$$\sum_{i=p_{k_{max}+1}}^{p_{K+1}-1}\frac{(\tilde{x}-\mathbb{E}c)w_ir_i}{\hat{x}(k_{max}+1)-\mathbb{E}c}+\sum_{i=p_{k_{max}}}^{p_{k_{max}+1}-1}\frac{(\tilde{x}-\mathbb{E}c)w_ir_i}{\hat{x}(k_{max})-\mathbb{E}c}=\sum_{i=p_{k_{max}}}^{p_{K+1}-1}w_ir_i.$$

Then $\hat{x}(k_{max})<\tilde{x}<\hat{x}(k_{max}+1)$.

Consider an alternative disclosure with

$$r_i'=\begin{cases} r_i, & \forall i<p_{k_{max}} \\ \frac{(\tilde{x}-\mathbb{E}c)r_i}{\hat{x}(k_{max})-\mathbb{E}c} & \forall p_{k_{max}}\le i<p_{k_{max}+1} \\ \frac{(\tilde{x}-\mathbb{E}c)r_i}{\hat{x}(k_{max}+1)-\mathbb{E}c} & \forall i\ge p_{k_{max}+1}\end{cases}$$

and the same mass w_i for each score as the original disclosure. We guess the equilibrium is $(\hat{x}_1',\ldots,\hat{x}_n')$, where $\hat{x}_i'=\hat{x}_i$ if $i<p_{k_{max}}$, $\hat{x}_i'=\tilde{x}$ if $\forall i\ge p_{k_{max}}$, and its $\{\Delta_{i,j}'\}_{j,i=1}^{t}$ is the same as the original one $\{\Delta_{i,j}\}_{j,i=1}^{t}$. To verify that this is indeed the equilibrium, we need to show that $\forall i,j$,

$$\hat{x}_i'=\mathbb{E}c+r_i'\int_0^1 a\left(\sum_{j=1}^{n}w_j\left[1-\Phi\left(\Phi^{-1}(1-m_i)-\Delta_{j,i}'\right)\right]\right)dm_i,$$

and $\hat{x}_i'=\hat{x}_j'$ if Δ_{ij}' is finite. It is easy to see

$$\frac{\hat{x}_i'-\mathbb{E}c}{r_i'}=\frac{\hat{x}_i-\mathbb{E}c}{r_i}=\int_0^1 a\left(\sum_{j=1}^{n}w_j\left[1-\Phi\left(\Phi^{-1}(1-m_i)-\Delta_{j,i}\right)\right]\right)dm_i$$

$$=\int_0^1 a\left(\sum_{j=1}^{n}w_j\left[1-\Phi\left(\Phi^{-1}(1-m_i)-\Delta_{j,i}'\right)\right]\right)dm_i.$$

In addition, for $i,j<p_{k_{max}}$, if Δ_{ij}' is finite, Δ_{ij} is finite, then $\hat{x}_i'=\hat{x}_i=\hat{x}_j=\hat{x}_j'$; for $i<p_{k_{max}}$ and $j\ge p_{k_{max}}$, Δ_{ij}' is infinite; for $i,j\ge p_{k_{max}}$, $\hat{x}_i'=\hat{x}_j'$. So, this is indeed the equilibrium, and it has the same partition structure as the original one. This alternative disclosure has a strictly lower maximum cutoff; that is, $\max_i\left\{\hat{x}_i'\right\}=\tilde{x}<\hat{x}(k_{max}+1)=\max_i\left\{\hat{x}_i\right\}$. To prevent such decrease in the maximum cutoff, the robust disclosure must result in all scores being either entangled or adjacent to each other.

Proof of Proposition 5

The case of $\overline{r}/\underline{r} \le \frac{A(Q^r)}{Q^r}\frac{W-Q^r}{A(W)-A(Q^r)}$ According to Proposition 2, the switching cutoff of investors in the k-th partition cell is

$$\hat{x}(k) = \mathbb{E}c + \left(\sum_{j=p_k}^{p_{k+1}-1} \frac{w_j}{r_j}\right)^{-1} \left[A\left(\sum_{j=1}^{p_{k+1}-1} w_j\right) - A\left(\sum_{j=1}^{p_k-1} w_j\right)\right].$$

So, the maximum cutoff

$$\max_i \{\hat{x}_i\} \ge \mathbb{E}c + \left(\sum_{j=p_k}^{p_{k+1}-1} \frac{w_j}{r_j}\right)^{-1} \left[A\left(\sum_{j=1}^{p_{k+1}-1} w_j\right) - A\left(\sum_{j=1}^{p_k-1} w_j\right)\right]$$

$$\Leftrightarrow \left(\sum_{j=p_k}^{p_{k+1}-1} \frac{w_j}{r_j}\right)\left[\max_i \{\hat{x}_i\} - \mathbb{E}c\right] \ge A\left(\sum_{j=1}^{p_{k+1}-1} w_j\right) - A\left(\sum_{j=1}^{p_k-1} w_j\right).$$

Summing over k, we obtain

$$\left(\sum_{i=1}^{t} \frac{w_i}{r_i}\right)\left[\max_i \{\hat{x}_i\} - \mathbb{E}c\right] \ge A(W) \Rightarrow \max_i \{\hat{x}_i\} \ge \mathbb{E}c + \frac{A(W)}{\sum_{i=1}^{t} \frac{w_i}{r_i}}.$$

By Lemma 1, we obtain

$$\frac{w_i}{r_i} \le \frac{w_i\overline{r} - w_i r_i}{\overline{r} - \underline{r}}\frac{1}{\underline{r}} + \frac{w_i r_i - w_i \underline{r}}{\overline{r} - \underline{r}}\frac{1}{\overline{r}} \Rightarrow \sum_{i=1}^{t} \frac{w_i}{r_i} \le \frac{Q^r}{\underline{r}} + \frac{W - Q^r}{\overline{r}}.$$

The last inequality holds if any r_i is not equal to $\underline{r}$ or $\overline{r}$. So,

$$\max_i \{\hat{x}_i\} \ge \mathbb{E}c + \frac{A(W)}{\frac{Q}{\underline{r}} + \frac{W-Q}{\overline{r}}}.$$

Consider the disclosure $r_1 = \underline{r}$, $r_2 = \overline{r}$, $w_1 = Q^r$ and $w_2 = W - Q^r$. By $\underline{r}\frac{A(Q^r)}{Q^r} \ge \overline{r}\frac{A(W)-A(Q^r)}{W-Q^r}$, they must have the same cutoff, $\mathbb{E}c + \frac{A(W)}{\frac{Q}{\underline{r}} + \frac{W-Q}{\overline{r}}}$. Therefore, the robust disclosure is a uniquely full disclosure.

The case of $\overline{r}/\underline{r} > \frac{A(Q^r)}{Q^r}\frac{W-Q^r}{A(W)-A(Q^r)}$ Suppose k_{max} is the greatest score that is entangled with another score. We prove the statement for any $k_{max} \ge 2$ by mathematical induction.

First, consider $k_{max} = 2$ and $t = 2$. The two scores are entangled and have a common cutoff $\mathbb{E}c + \frac{A(W)}{\frac{w_1}{r_1} + \frac{w_2}{r_2}}$. Consider an alternative disclosure r_1' and r_2' that satisfy $w_1 r_1 + w_2 r_2 = w_1' r_1' + w_2' r_2'$, $w_1 + w_2 = w_1' + w_2'$, and $r_1' = r_1 - \delta(r_1 - \underline{r})$, $r_2' = r_2 + \delta(\overline{r} - r_2)$. Since r_1' and r_2' are entangled when $\delta = 0$ and are separate when $\delta = 1$, there must exist a $\delta' > 0$ such that they are adjacent; that is, $r_1'\frac{A(w_1')}{w_1'} = r_2'\frac{A(w_1+w_2)-A(w_1')}{w_1+w_2-w_1'}$. Note for any $\delta > 0$, $\frac{w_1}{r_1} + \frac{w_2}{r_2} < \frac{w_1'}{r_1'} + \frac{w_2'}{r_2'}$. So when r_1' and r_2' are adjacent, their maximum cutoff is strictly lower than the original one.

Second, consider $k_{max} = 2$ and $t \ge 3$. Only r_1 and r_2 are entangled. The scores $2 \sim t$ are adjacent. All scores have the common cutoff $\hat{x}_3 = \mathbb{E}c + r_3\frac{A(w_1+w_2+w_3)-A(w_1+w_2)}{w_3}$. Consider the mean-preserving spread of $\{(r_1, w_1), (r_2, w_2)\}$, $\{(r_1', w_1'), (r_2', w_2')\}$ where $r_1' = r_1$, $r_2' = r_2 + \delta$, $w_1' r_1' + w_2' r_2' = w_1 r_1 + w_2 r_2$, and $w_1' + w_2' = w_1 + w_2$. We show that there exists δ' such that $r_1'\frac{A(w_1')}{w_1'} = r_2'\frac{A(w_1+w_2)-A(w_1')}{w_1+w_2-w_1'}$. Note that $\delta' \in [0, +\infty)$. To see this, on one hand, since only r_1 and r_2 are entangled under the original disclosure, when $\delta = 0$, $r_1' \frac{A(w_1')}{w_1'} > r_2' \frac{A(w_1+w_2)-A(w_1')}{w_1+w_2-w_1'}$. On the other hand, $\frac{A(w_1+w_2)-A(w_1')}{w_1+w_2-w_1'} \geq a(w_1+w_2)$ and $r_1' \frac{A(w_1')}{w_1'} < r_1 a(0)$, when δ is sufficiently large, $r_1' \frac{A(w_1')}{w_1'} < r_2' \frac{A(w_1+w_2)-A(w_1')}{w_1+w_2-w_1'}$. Therefore, there exists $\delta' > 0$ such that the equation holds. Consider an alternative disclosure $(r_1', r_2', r_3, \ldots r_t)$ that replaces $\{(r_1, w_1), (r_2, w_2)\}$ with the $\{(r_1', w_1'), (r_2', w_2')\}$ associated with δ' while keeping $\{(r_i, w_i)\}_{i=3}^n$ unchanged. Since

$$r_1' \frac{A(w_1')}{w_1'} = r_2' \frac{A(w_1+w_2)-A(w_1')}{w_1+w_2-w_1'} = \left(\frac{w_1'}{r_1'} + \frac{w_1+w_2-w_1'}{r_2'}\right)^{-1} \cdot A(w_1+w_2)$$

$$< \left(\frac{w_1}{r_1} + \frac{w_2}{r_2}\right)^{-1} \cdot A(w_1+w_2) = r_3 \frac{A(w_1+w_2+w_3)-A(w_1+w_2)}{w_3},$$

it is straightforward to see that under the alternative disclosure, the scores $3 \sim t$ are still adjacent to each other and their common switching cutoff is still $\hat{x}_3$; the scores 1 and 2 are adjacent and have a common cutoff strictly lower than $\hat{x}_3$. That means, each partition cell has only one score. Following the proof of Lemma 4, we can find a disclosure under which all scores are adjacent in this case and the maximum cutoff is strictly lower.

Next, suppose the statement holds for $k_{max} < k (\geq 3)$ and any t. Consider $k_{max} = k$ and any t. Consider an alternative disclosure that replaces (r_{k-1}, r_k) with one of its mean-preserving spread (r_{k-1}', r_k') while keeping other scores unchanged. Specifically, $r_{k-1}' = r_{k-1} - \delta$ and $r_k' = r_k$. Consider the process that δ increases until $r_{k-1}' = r_{k-2}$ or r_{k-1}' and r_k' are adjacent, whichever is first. We conjecture that the cutoff of r_k' is always decreasing and $r_{k+1}, \ldots r_t$ have the same cutoff as under the original disclosure. Suppose the partition cell that contains r_{k-1}' and r_k' consists of the scores $\{n(\delta), \ldots, k-1, k\}$. We write this as $P(\delta)$. In this process, $P(\delta)$ may experience three kinds of changes.

1. $P(\delta)$ does not change. Then the cutoff of r_k'

$$\hat{x}_k' = \mathbb{E}c + \left(\sum_{j=n(\delta)}^{k-2} \frac{w_j}{r_j} + \frac{w_{k-1}}{r_{k-1}'} + \frac{w_k}{r_k'}\right)^{-1} \left[A\left(\sum_{j=1}^{k} w_j\right) - A\left(\sum_{j=1}^{n(\delta)-1} w_j\right)\right]$$

is strictly decreasing in δ.

2. $P(\delta)$ absorbs some scores below $n(\delta)$. We write the $P(\delta)$ before and after the change as $P(\delta_-)$ and $P(\delta_+)$, respectively. In the instant when the change happens, the score $n(\delta_-)-1$ must be adjacent to $P(\delta_-)$ and the scores $\{n(\delta_+), \ldots, n(\delta_-)-1\}$ are either entangled or adjacent. So, in this instant,

$$\hat{x}_k' = \mathbb{E}c + \left(\sum_{j=n(\delta_-)}^{k-2} \frac{w_j}{r_j} + \frac{w_{k-1}}{r_{k-1}'} + \frac{w_k}{r_k'}\right)^{-1} \left[A\left(\sum_{j=1}^{k} w_j\right) - A\left(\sum_{j=1}^{n(\Delta_-)-1} w_j\right)\right],$$

and also

$$\hat{x}_k' = \mathbb{E}c + \left(\sum_{j=n(\delta_+)}^{k-2} \frac{w_j}{r_j} + \frac{w_{k-1}}{r_{k-1}'} + \frac{w_k}{r_k'}\right)^{-1} \left[A\left(\sum_{j=1}^{k} w_j\right) - A\left(\sum_{j=1}^{n(\delta_+)-1} w_j\right)\right].$$

This implies that $\hat{x}_k'$ has no jump when the change happens.

3. $P(\delta)$ drop some scores in $\{n(\delta), \ldots k-2\}$. Following the same analysis of the second case, $\hat{x}_k'$ has no jump when the change happens.

We have verified the conjecture. Then, no matter for what reason the process stops, we end up with a new disclosure with $k_{max} \leq k-1$. If $k=t$, its maximum cutoff is strictly lower than the original one. If $k<t$, when the process stops, $\hat{x}_k' < \hat{x}_k \leq \hat{x}_{k+1}$. Following the proof of Lemma 4, we can find another disclosure whose maximum cutoff is strictly lower than $\hat{x}_{k+1}$ and $k_{max} \leq k-1$.

We can iterate this procedure finite times and end up with a disclosure with all scores adjacent and its maximum cutoff strictly lower than that of the original one.

Proof of Proposition 6

By Proposition 5, we focus on disclosures with all scores adjacent. For any disclosure like this, suppose their common cutoff is $\hat{x}$. Then $\forall k$, $\hat{x} = \mathbb{E}c + r_k \frac{A(\sum_{i=1}^{k} w_i) - A(\sum_{i=1}^{k-1} w_i)}{w_k}$, where $\sum_{i=1}^{t} w_i = W$ and $\sum_{i=1}^{t} w_i r_i = (W - Q^r)\underline{r} + Q^r \overline{r}$.

Part I: $\hat{x}_r(W, Q^r, A(\cdot))$ is smaller than the maximum cutoff under any finite disclosure. Consider $\tilde{\Omega}(r; W, \hat{x}, A(\cdot))$ that is defined in Section A.1. Since $r_1 \geq \underline{r}$ and $r_1 \frac{A(w_1)}{w_1} = \hat{x} - \mathbb{E}c = \underline{r}\frac{A(\underline{m})}{\underline{m}}$, it is easy to see that $\underline{m} \leq w_1$. Similarly, $\overline{m} \leq w_t$. Then for any k, there exists $\tilde{r}_k$ such that $\tilde{\Omega}(\tilde{r}_k; W, \hat{x}, A(\cdot)) = \sum_{i=1}^{k} w_i$. For the part of $\tilde{\Omega}(r; W, \hat{x}, A(\cdot))$ over $[\tilde{r}_{k-1}, \tilde{r}_k]$, we have

$$(\hat{x} - \mathbb{E}c) \int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k} \frac{1}{r} d\tilde{\Omega}(r; W, \hat{x}, A(\cdot)) = \int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k} a\left(\tilde{\Omega}(r; W, \hat{x}, A(\cdot))\right) d\tilde{\Omega}(r; W, \hat{x}, A(\cdot))$$

$$= A\left(\sum_{i=1}^{k} w_i\right) - A\left(\sum_{i=1}^{k-1} w_i\right) = \frac{\hat{x} - \mathbb{E}c}{r_k} w_k. \tag{B2}$$

Since by Cauchy-Schwarz inequality,

$$\int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k} \frac{1}{r} d\tilde{\Omega}(r; W, \hat{x}, A(\cdot)) \cdot \int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k} r d\tilde{\Omega}(r; W, \hat{x}, A(\cdot)) \geq w_k^2 = \frac{w_k}{r_k} w_k r_k,$$

we have $\int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k} r d\tilde{\Omega}(r; W, \hat{x}, A(\cdot)) \geq w_k r_k$. Summing over k, we have

$$\underline{r} \cdot \tilde{Q}\left(\hat{X}; W, A(\cdot)\right) + \bar{r} \cdot \left[W - \tilde{Q}\left(\hat{X}; W, A(\cdot)\right)\right] = \int_{r=\underline{r}}^{\overline{r}} r d\tilde{\Omega}(r; W, \hat{x}, A(\cdot))$$

$$\geq \sum_{i=1}^{t} w_i r_i = \underline{r} \cdot Q^r + \bar{r} \cdot \left(W - Q^r\right),$$

so $\tilde{Q}\left(\hat{x}; W, A(\cdot)\right) \leq Q^r$. Since $\tilde{Q}\left(\hat{X}; W, A(\cdot)\right)$ is decreasing in $\hat{X}$, $\hat{x} \geq \hat{x}_r(W, Q^r, A(\cdot))$. Therefore, $\hat{x}_r(W, Q^r, A(\cdot))$ is the lower bound of the maximum cutoff.

Part II: there exists a sequence of t-score disclosures $\left\{\Omega_{(t)}\right\}$ such that their maximum cutoff converges to $\hat{x}_r(W, Q^r, A(\cdot))$ as $t \to +\infty$. For any feasible $\hat{X}$, consider a t-score disclosure $(r_1, \ldots r_t)$ as follow: $(r_1, w_1) = (\underline{r}, \underline{m})$, $(r_t, w_t) = (\overline{r}, \overline{m})$; for $2 \leq k \leq t-1$, $w_k = \delta = \frac{W - w_1 - w_t}{t-2}$, and r_k satisfies $\hat{X} = \mathbb{E}c + r_k \frac{A(w_1 + (k-1)\delta) - A(w_1 + (k-2)\delta)}{\delta}$.

Let $S(\hat{X}; t) \triangleq w_1 \underline{r} + \sum_{i=2}^{t-1} \delta r_i + w_t \overline{r}$ be the sum of r. $S(\hat{X}; t)$ is continuous in $\hat{X}$. There exists an $\hat{x}$ such that $S(\hat{x}; t) = \underline{r} \cdot Q^r + \bar{r} \cdot (W - Q^r)$. In this case, the disclosure is feasible and all scores are adjacent and have the common cutoff $\hat{x}$. We write this disclosure as $\Omega_{(t)}$. Let $S(\hat{x}) \triangleq \int_{r=\underline{r}}^{\overline{r}} r d\tilde{\Omega}(r; W, \hat{x}, A(\cdot))$.

Next, we compare $S(\hat{x};t)$ with $S(\hat{x})$. Similar to the above, suppose $\tilde{r}_k$ satisfies $\tilde{\Omega}(\tilde{r}_k;W,\hat{x},A(\cdot))=\sum_{i=1}^{k} w_i = w_1+(k-1)\delta$. Then We have

$$(\hat{x}-\mathbb{E}c)\int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k}\frac{1}{r}d\tilde{\Omega}(\tilde{r}_k;W,\hat{x},A(\cdot))=A(w_1+(k-1)\delta)-A(w_1+(k-2)\delta)=\frac{\hat{x}-\mathbb{E}c}{r_k}\delta$$

$$S(\hat{x})-S(\hat{x};t)=\int_{r=r^-}^{r^+} r\,d\tilde{\Omega}(r;W,\hat{x},A(\cdot))-\sum_{k=2}^{t-1}\delta r_k=\sum_{k=2}^{t-1}\left[\int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k} r\,d\tilde{\Omega}(r;W,\hat{x},A(\cdot))-\delta r_k\right].$$

Note that

$$\begin{aligned}
&\left[\int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k} r\,d\tilde{\Omega}(r;W,\hat{x},A(\cdot))-\delta r_k\right]\frac{\delta}{r_k}\\
&=\int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k} r\,d\tilde{\Omega}(r;W,\hat{x},A(\cdot))\cdot\int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k}\frac{1}{r}d\tilde{\Omega}(r;W,\hat{x},A(\cdot))-\delta^2\\
&=\int_{z=\tilde{r}_{k-1}}^{\tilde{r}_k}\int_{y=\tilde{r}_{k-1}}^{\tilde{r}_k} y\cdot\frac{1}{z}d\tilde{\Omega}(y;W,\hat{x},A(\cdot))d\tilde{\Omega}(z;W,\hat{x},A(\cdot))-\delta^2\\
&\leq\int_{z=\tilde{r}_{k-1}}^{\tilde{r}_k}\int_{y=\tilde{r}_{k-1}}^{\tilde{r}_k}\frac{\tilde{r}_k}{\tilde{r}_{k-1}}d\tilde{\Omega}(y;W,\hat{x},A(\cdot))d\tilde{\Omega}(z;W,\hat{x},A(\cdot))-\delta^2\\
&=\frac{\tilde{r}_k-\tilde{r}_{k-1}}{\tilde{r}_{k-1}}\delta^2
\end{aligned}$$

Since

$$w(r)=\frac{da^{-1}\left(\frac{\hat{x}-\mathbb{E}c}{r}\right)}{dr}=\frac{1}{-a'\left[a^{-1}\left(\frac{\hat{x}-\mathbb{E}c}{r}\right)\right]}\frac{\hat{x}-\mathbb{E}c}{r^2}\geq\frac{1}{\sup\{-a'\}}\underline{r}\frac{A(W)}{W}\frac{1}{\overline{r}^2}>0$$

is bounded from below by a positive number, $\inf\{w(r)\}$ exists and is positive. Then

$$\delta=\int_{\tilde{r}_{k-1}}^{\tilde{r}_k} w(r)dr\geq(\tilde{r}_k-\tilde{r}_{k-1})\inf\{w(r)\}\Rightarrow\frac{\tilde{r}_k-\tilde{r}_{k-1}}{\tilde{r}_{k-1}}\leq\frac{\delta}{\tilde{r}_{k-1}\inf\{w(r)\}}\leq\frac{\delta}{\underline{r}\inf\{w(r)\}},$$

so

$$\int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k} r\,d\tilde{\Omega}(r;W,\hat{x},A(\cdot))-\delta r_k\leq\frac{r_k}{\underline{r}\inf\{w(r)\}}\delta^2\leq\frac{\overline{r}}{\underline{r}\inf\{w(r)\}}\delta^2.$$

Summing over k, we obtain

$$S(\hat{x})-S(\hat{x};t)\leq\frac{(t-2)\overline{r}}{\underline{r}\inf\{w(r)\}}\delta^2\leq\frac{W\overline{r}}{\underline{r}\inf\{w(r)\}}\delta.$$

As $t\to+\infty$, we have $\delta\to 0$, so

$$S(\hat{x})\to\underline{r}\cdot Q^r+\overline{r}\cdot\left(W-Q^r\right)\Rightarrow\tilde{Q}\left(\hat{x};W,A(\cdot)\right)\to Q^r.$$

Since $\hat{x}_r(W,Q^r,A(\cdot))$ is continuous in Q^r, as $t\to+\infty$, $\hat{x}_r\left(W,\tilde{Q}\left(\hat{x};W,A(\cdot)\right),A(\cdot)\right)\to\hat{x}_r(W,Q^r,A(\cdot))$, that is, $\hat{x}\to\hat{x}_r(W,Q^r,A(\cdot))$. So, by increasing the number of scores, we can make the common cutoff of $\Omega_{(t)}$, which is also its maximum cutoff, arbitrarily close to $\hat{x}_r(W,Q^r,A(\cdot))$.

Part III: The quantile functions of robust disclosures with t scores converge to that of $\Omega(\cdot;W,Q^r,A(\cdot))$ in L^1-norm as $t\to\infty$. Write the robust disclosure as $\{(r_i;w_i)\}_{i=1}^t$ and its corresponding common cutoff as $\hat{x}$. Suppose $\tilde{r}_k$ satisfies $\tilde{\Omega}(\tilde{r}_k;W,\hat{x},A(\cdot))=\sum_{i=1}^k w_i$. As $t\to+\infty$, $\hat{x}\to\hat{x}_r(W,Q^r,A(\cdot))$. By the continuity of $\tilde{\Omega}(\cdot;W,\hat{x},A(\cdot))$ in $\hat{x}$, it is easy to see that the quantile functions of $\tilde{\Omega}(\cdot;W,\hat{x},A(\cdot))$ converge to that of $\Omega(\cdot;W,Q^r,A(\cdot))$ in L^1-norm as $t\to\infty$. Then our goal is to prove that as $t\to+\infty$, $\sum_{k=1}^t\int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k}|r-r_k|d\tilde{\Omega}(r;W,\hat{x},A(\cdot))\to 0$. Since

$$\sum_{k=1}^t\int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k}\left|\frac{1}{r}-\frac{1}{r_k}\right|d\tilde{\Omega}(r;W,\hat{x},A(\cdot))\geq\frac{1}{\overline{r}^2}\sum_{k=1}^t\int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k}|r-r_k|d\tilde{\Omega}(r;W,\hat{x},A(\cdot)),$$

it suffices to prove that as $t\to+\infty$, $\sum_{k=1}^t\int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k}\left|\frac{1}{r}-\frac{1}{r_k}\right|d\tilde{\Omega}(r;W,\hat{x},A(\cdot))\to 0$.

Consider any $1\leq k\leq t$. Let $\Lambda_k\equiv\int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k}\left|\frac{1}{r}-\frac{1}{r_k}\right|d\tilde{\Omega}(r;W,\hat{x},A(\cdot))$. Following Equation (B2), $\int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k}\frac{1}{r}d\tilde{\Omega}(r;W,\hat{x},A(\cdot))=\frac{w_k}{r_k}$. Then it is easy to see $\tilde{r}_{k-1}\leq r_k\leq\tilde{r}_k$. Suppose y and z satisfy $\int_{r=\tilde{r}_{k-1}}^{r_k}\frac{1}{r}d\tilde{\Omega}(r;W,\hat{x},A(\cdot))=\frac{1}{y}\int_{r=\tilde{r}_{k-1}}^{r_k}d\tilde{\Omega}(r;W,\hat{x},A(\cdot))$ and $\int_{r=r_k}^{\tilde{r}_k}\frac{1}{r}d\tilde{\Omega}(r;W,\hat{x},A(\cdot))=\frac{1}{z}\int_{r=r_k}^{\tilde{r}_k}d\tilde{\Omega}(r;W,\hat{x},A(\cdot))$. Then $y\leq r_k\leq z$. We write $\int_{r=\tilde{r}_{k-1}}^{r_k}d\tilde{\Omega}(r;W,\hat{x},A(\cdot))$ as u, so we have

$$\frac{1}{y}u+\frac{1}{z}(w_k-u)=\frac{w_k}{r_k},\tag{B3}$$

$$\left(\frac{1}{y}-\frac{1}{r_k}\right)u+\left(\frac{1}{r_k}-\frac{1}{z}\right)(w_k-u)=\Lambda_k.\tag{B4}$$

Moreover, by Cauchy-Schwarz inequality

$$\begin{aligned}\int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k}rd\tilde{\Omega}(r;W,\hat{x},A(\cdot))&=\int_{r=\tilde{r}_{k-1}}^{r_k}rd\tilde{\Omega}(r;W,\hat{x},A(\cdot))+\int_{r=r_k}^{\tilde{r}_k}rd\tilde{\Omega}(r;W,\hat{x},A(\cdot))\\&\geq\frac{u^2}{\int_{r=\tilde{r}_{k-1}}^{r_k}\frac{1}{r}d\tilde{\Omega}(r;W,\hat{x},A(\cdot))}+\frac{(w_k-u)^2}{\int_{r=r_k}^{\tilde{r}_k}\frac{1}{r}d\tilde{\Omega}(r;W,\hat{x},A(\cdot))}\\&=yu+z(w_k-u).\end{aligned}$$

Next, we derive a lower bound of $yu+z(w_k-u)-w_kr_k$. From Equations (B3) and (B4), we can obtain $\frac{1}{y}=\frac{1}{r_k}+\frac{\Lambda_k}{2u}$ and $\frac{1}{z}=\frac{1}{r_k}-\frac{\Lambda_k}{2(w_k-u)}$. Then

$$\begin{aligned}yu+z(w_k-u)-w_kr_k&=\frac{u}{\frac{1}{r_k}+\frac{\Lambda_k}{2u}}+\frac{w_k-u}{\frac{1}{r_k}-\frac{\Lambda_k}{2(w_k-u)}}-w_kr_k\\&=r_k^2\Lambda_k\left[-\frac{u}{2u+r_k\Lambda_k}+\frac{(w_k-u)}{2(w_k-u)-r_k\Lambda_k}\right].\end{aligned}$$

Its derivative with respect to u is $r_k^2\Lambda_k\left[-\frac{r_k\Lambda_k}{(2u+r_k\Lambda_k)^2}+\frac{r_k\Lambda_k}{(2w_k-2u-r_k\Lambda_k)^2}\right]$, which is increasing in u. The minimum is attained at $\frac{r_k\Lambda_k}{(2u+r_k\Lambda_k)^2}=\frac{r_k\Lambda_k}{(2w_k-2u-r_k\Lambda_k)^2}\Leftrightarrow u=\frac{w_k-r_k\Lambda_k}{2}$. Hence, $yu+z(w_k-u)-w_kr_k\geq\frac{r_k^3\Lambda_k^2}{w_k}$.

Further, $\sum_{k=1}^t\int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k}rd\tilde{\Omega}(r;W,\hat{x},A(\cdot))\geq\sum_{k=1}^t\left(w_kr_k+\frac{r_k^3\Lambda_k^2}{w_k}\right)=\sum_{k=1}^t w_kr_k+\sum_{k=1}^t\frac{r_k^3\Lambda_k^2}{w_k}$. By Cauchy-Schwarz inequality, $\sum_{k=1}^t\frac{r_k^3\Lambda_k^2}{w_k}\cdot\sum_{k=1}^t\frac{w_k}{r_k^3}\geq\left(\sum_{k=1}^t\Lambda_k\right)^2$, so $\sum_{k=1}^t\int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k}rd\tilde{\Omega}(r;W,\hat{x},A(\cdot))-\sum_{k=1}^t w_kr_k\geq\left(\sum_{k=1}^t\frac{w_k}{r_k^3}\right)^{-1}\left(\sum_{k=1}^t\Lambda_k\right)^2\geq\frac{\underline{r}^3}{W}\left(\sum_{k=1}^t\Lambda_k\right)^2$.

As $t \to +\infty$, $\hat{x} \to \hat{x}_r(W, Q^r, A(\cdot))$, so $\sum_{k=1}^{t} \int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k} r d\tilde{\Omega}(r; W, \hat{x}, A(\cdot)) \to \underline{r} \cdot Q^r + \bar{r} \cdot (W - Q^r) = \sum_{k=1}^{t} w_k r_k$. This implies $\sum_{k=1}^{t} \int_{r=\tilde{r}_{k-1}}^{\tilde{r}_k} \left| \frac{1}{r} - \frac{1}{r_k} \right| d\tilde{\Omega}(r; W, \hat{x}, A(\cdot)) = \sum_{k=1}^{t} \Lambda_k \to 0$.

Proof of Proposition 7

Suppose the proposition does not hold. Specifically, the subdisclosure of the bank group whose investors have the switching cutoff θ_k is not the (K_{k-1}, t_k)-robust disclosure of the group. Notice that the maximum of the switching cutoffs is also θ_k. Write the maximum of the switching cutoffs under the (K_{k-1}, t_k)-robust disclosure as $\hat{x}_{(K_{k-1}, t_k)}$. Then by the definition, $\hat{x}_{(K_{k-1}, t_k)} < \theta_k$. We show that if this subdisclosure is replaced with the (K_{k-1}, t_k)-robust disclosure of the group, all investors have weakly lower cutoffs, and a positive mass of them have strictly lower cutoffs.

Part I: investors with cutoffs smaller than θ_k under the original disclosure. Suppose these investors correspond to the first m scores. The investors' cutoffs are $\{\hat{x}_i, \Delta_{i,j}\}_{j,i=1}^{m}$ under the original disclosure and $\{\hat{x}'_i, \Delta'_{i,j}\}_{j,i=1}^{m}$ under the new disclosure respectively. Since $\Delta_{i,j} = +\infty$ for $i \le m$ and $j > m$, $\{\hat{x}_i\}_{i=1}^{m}$ satisfies

$$\hat{x}_i = c_i + r_i \int_0^1 a \left(\sum_{j=1}^{m} w_j \left[1 - \Phi\left(\Phi^{-1}(1-m_i) - \Delta_{j,i} \right) \right] \right) dm_i.$$

Suppose there exists $i \le m$ such that $\hat{x}'_i > \hat{x}_i$ and such i constitutes the set $\mathcal{T} = \{\tau_1, \tau_2, \ldots, \tau_L\}$ where $\tau_1 < \tau_2 \ldots < \tau_L$. Consider $i \in \mathcal{T}$. There must exist $j \le m$ such that $\Delta'_{j,i} < \Delta_{j,i}$; otherwise, $\hat{x}'_i \le \hat{x}_i$. Let $\xi(i)$ be the smallest j such that $\Delta'_{j,i} < \Delta_{j,i}$.

Note that for $j \notin \mathcal{T}$, since $\hat{x}'_j \le \hat{x}_j$ and $\hat{x}'_i > \hat{x}_i$, $\Delta'_{j,i} \ge \Delta_{j,i}$. Hence, $\xi(i) \in \mathcal{T}$. Since $\xi(\tau_1) \in \mathcal{T}$, $\xi(\tau_1) > \tau_1$. Consider $\xi^{(2)}(\tau_1) = \xi(\xi(\tau_1))$. It must be in $\mathcal{T}$. By the definition of $\xi(\tau_1)$, for any $j \in \mathcal{T}$ and $j < \xi(\tau_1)$, $\Delta'_{j,\tau_1} \ge \Delta_{j,\tau_1}$, and $\Delta'_{\xi(\tau_1),\tau_1} < \Delta_{\xi(\tau_1),\tau_1}$. So, for these j,

$$\Delta'_{j,\xi(\tau_1)} = \Delta'_{j,\tau_1} - \Delta'_{\xi(\tau_1),\tau_1} > \Delta_{j,\tau_1} - \Delta_{\xi(\tau_1),\tau_1} = \Delta_{j,\xi(\tau_1)},$$

which implies $\xi(\xi(\tau_1)) > \xi(\tau_1)$. Iterating the procedure, we end up with an infinite sequence $\{\xi^{(j)}(\tau_1)\}_{j=1}^{+\infty}$ in $\mathcal{T}$. This is impossible because $\mathcal{T}$ is a finite set. Therefore, for $i \le m$, $\hat{x}'_i \le \hat{x}_i$.

Part II: investors with cutoffs equal to θ_k under the original disclosure. Write the (K_{k-1}, t_k)-robust disclosure of the group as $\{(r'_i, c'_i, w'_i)\}_{i=m+1}^{m+t_k}$. By the definition of the (K_{k-1}, t_k)-robust disclosure and $K_{k-1} = \sum_{j=1}^{m} w_j$, there exists $\{\hat{x}''_i, \Delta''_{i,j}\}_{i,j=m+1}^{m+t_k}$ such that $\hat{x}''_i \le \hat{x}_{(K_{k-1}, t_k)}$, where

$$\hat{x}''_i = c'_i + r'_i \int_0^1 a \left(\sum_{j=1}^{m} w_j + \sum_{j=m+1}^{m+t_k} w'_j \left[1 - \Phi\left(\Phi^{-1}(1-m_i) - \Delta''_{j,i} \right) \right] \right) dm_i,$$

$$\Delta''_{i-1,i} \begin{cases} = +\infty, & \text{if } \hat{x}''_i > \hat{x}''_{i-1} \\ = -\infty, & \text{if } \hat{x}''_i < \hat{x}''_{i-1} \\ \in [-\infty, +\infty], & \text{if } \hat{x}''_i = \hat{x}_{i-1}" \end{cases},$$

and $-\Delta''_{i,j} = \Delta''_{j,i} = \sum_{u=j+1}^{i} \Delta''_{u-1,u}$. Write the cutoffs of the t_k scores specified by the (K_{k-1}, t_k)-robust disclosure under the new disclosure as $\{\hat{x}'_i, \Delta'_{i,j}\}_{j,i=m+1}^{m+t_k}$. Suppose there exists $m+1 \le i \le m+t_k$ such that $\hat{x}'_i > \max\{\hat{x}_{(K_{k-1}, t_k)}, \theta_{k-1}\}$ and such i's constitute a set $\mathcal{T} = \{\tau_1, \tau_2, \ldots, \tau_L\}$ where $\tau_1 < \tau_2 \ldots < \tau_L$. Consider $i \in \mathcal{T}$. There must exist $m+1 \le j \le m+t_k$ such that $\Delta'_{j,i} < \Delta''_{j,i}$; otherwise,

$$\hat{x}'_i \le c'_i + r'_i \int_0^1 a \left(\sum_{j=1}^{m} w_j + \sum_{j=m+1}^{m+t_k} w'_j \left[1 - \Phi\left(\Phi^{-1}(1-m_i) - \Delta''_{j,i} \right) \right] \right) dm_i$$

$$= \hat{x}''_i \le \hat{x}_{(K_{k-1}, t_k)}.$$

Let $\xi(i)$ be the smallest j such that $\Delta'_{j,i} < \Delta''_{j,i}$. Similar to Part I, we will encounter contradiction. Hence, for $m+1 \le i \le m+t_k$, $\hat{x}'_i \le \max\{\hat{x}_{(K_{k-1}, t_k)}, \theta_{k-1}\} < \theta_k$.

Part III: investors with cutoffs greater than θ_k under the original disclosure. Note that all other investors have cutoffs smaller than θ_k under the new disclosure. Since the equation system in Proposition 1 has a unique solution, the cutoffs of these investors must be the same under the new disclosure as they are under the original disclosure.

Proof of Proposition 8

It suffices to replace the $a(\cdot)$ and $A(\cdot)$ in the proofs of Propositions 3-6 with $a_\kappa(\cdot)$ and $A_\kappa(\cdot)$, respectively, since the only difference is that the mass of stayers outside the corresponding bank group is now fixed at κ instead of zero.

Proof of Proposition 9

Throughout this proof, the last argument of $\hat{x}_r$ is always $A(\cdot)$, so we suppress it for notational convenience.

Part I: $\overline{r}/\underline{r} \le \frac{A(q^r)}{q^r}\frac{1-q^r}{A(1)-A(q^r)}$. If $\hat{\theta} \ge \hat{x}_r(1,q^r) = \mathbb{E}c + \left(\frac{q^r}{\underline{r}} + \frac{1-q^r}{\overline{r}}\right)^{-1} \cdot A(1)$, by full revelation, all investors have a common switching cutoff $\hat{x}_r(1,q^r)$, so all banks are immune from runs. Moreover, full revelation minimizes the common switching cutoff of all banks.

Next, consider the case $\hat{\theta} < \hat{x}_r(1,q^r)$. Suppose the proposition does not hold. That means, under a disclosure, a group of banks (W,Q) can be immune from runs. It is easy to see that $\underline{r}\frac{A(q^r)}{q^r} \ge \frac{A(1)}{\frac{q^r}{\underline{r}}+\frac{1-q^r}{\overline{r}}} \ge \overline{r}\frac{A(1)-A(q^r)}{1-q^r}$.

If $\overline{r}/\underline{r} > \frac{A(Q)}{Q}\frac{W-Q}{A(W)-A(Q)}$, then according to the construction of its 0-robust disclosure, $\hat{x}_r(W,Q) \ge \mathbb{E}c + \underline{r}\lim_{x\to \underline{m}}\frac{A(x)}{x} \ge \mathbb{E}c + \underline{r}\frac{A(q^r)}{q^r} > \hat{\theta}$. Contradiction!

If $\overline{r}/\underline{r} \le \frac{A(Q)}{Q}\frac{W-Q}{A(W)-A(Q)}$, then its maximum cutoff cannot be smaller than $Ec + \frac{A(W)}{\frac{Q}{\underline{r}}+\frac{W-Q}{\overline{r}}}$. Since

$$\frac{A(1)-A(W)}{\frac{1-W}{\overline{r}}} < \frac{A(W)-A(Q)}{\frac{W-Q}{\overline{r}}} \le \frac{A(W)}{\frac{Q}{\underline{r}}+\frac{W-Q}{\overline{r}}} \le \frac{A(Q)}{\frac{Q}{\underline{r}}},$$

$\frac{A(1)}{\frac{q^r}{\underline{r}}+\frac{1-q^r}{\overline{r}}} \le \frac{A(1)}{\frac{Q}{\underline{r}}+\frac{1-Q}{\overline{r}}} = \frac{1}{\frac{Q}{\underline{r}}+\frac{W-Q}{\overline{r}}+\frac{1-W}{\overline{r}}}[A(W)+A(1)-A(W)] < \frac{A(W)}{\frac{Q}{\underline{r}}+\frac{W-Q}{\overline{r}}}$. So $\mathbb{E}c + \frac{A(W)}{\frac{Q}{\underline{r}}+\frac{W-Q}{\overline{r}}} > \hat{\theta}$. Contradiction!

Part II: $\overline{r}/\underline{r} > \frac{A(q^r)}{q^r}\frac{1-q^r}{A(1)-A(q^r)}$. We prove Proposition 9 for this case based on Lemma 5.

Lemma 5. Suppose $\overline{r}/\underline{r} > \frac{A(q^r)}{q^r}\frac{1-q^r}{A(1)-A(q^r)}$.

- For any $Q \in [0,q^r]$ and $\hat{\theta} \in \left[\hat{x}_r(Q,Q), \hat{x}_r(1,q^r)\right)$, $\hat{x}_r(W,Q) = \hat{\theta}$ has a unique solution in $[Q,1)$, $W_r(\hat{\theta},Q)$.
- Moreover, $\lim_{r\to\overline{r}-}\Omega\left(r; W_r(\hat{\theta},Q), Q, A(\cdot)\right) = W_r(\hat{\theta},Q)$, and $W_r(\hat{\theta},Q)$ is continuous and increasing in Q and $\hat{\theta}$.

If $\hat{\theta} \ge \hat{x}_r(1,q^r)$, the 0-robust disclosure of the whole banking system, $\Omega(\cdot;1,q^r,A(\cdot))$, can ensure that all banks survive.

If $\hat{\theta} < \hat{x}_r(q^r,q^r)$, then $\hat{\theta} < \mathbb{E}c + \underline{r}\frac{A(q^r)}{q^r}$. This implies that for any bank group with no more than a mass q^r of $\underline{r}$-type banks, there does not exist a 0-robust disclosure such that the common cutoff is not higher than $\hat{\theta}$. So, no bank can be immune from runs.

Next, we consider $\hat{\theta}\in\left[\hat{x}_r(q^r,q^r),\hat{x}_r(1,q^r)\right)$. First, any group of banks (W,Q) that has a cutoff smaller than $\hat{x}_r(1,q^r)$ must have strong 0-heterogeneity. Suppose not, that is, $\overline{r}/\underline{r}\leq\frac{A(Q)}{Q}\frac{W-Q}{A(W)-A(Q)}$. Then its lowest cutoff is

$$\mathbb{E}c+\frac{A(W)}{\frac{Q}{\underline{r}}+\frac{W-Q}{\overline{r}}}\geq\mathbb{E}c+\overline{r}\frac{A(W)-A(Q)}{W-Q}\geq\mathbb{E}c+\overline{r}\frac{A(1)-A(q^r)}{1-q^r}>\hat{x}_r\left(1,q^r\right).$$

Second, consider any bank group (W,Q) that can be immune from runs. Suppose its maximum cutoff under its 0-robust disclosure is $\theta'\leq\hat{\theta}$. Then W solves $\hat{x}_r(W,Q)=\theta'$ and $W\leq 1$. By the proof of Lemma 5, we know that it must be $W=W_r(\theta',Q)$. Since $W_r(\theta,Q)$ is strictly increasing in θ and Q, $W\leq W_r(\hat{\theta},q^r)$. And $W_r(\hat{\theta},q^r)$ can be attained uniquely by the 0-robust disclosure of bank group $(W_r(\hat{\theta},q^r),q^r)$, which consists of measure q^r of $\underline{r}$-type banks and measure $W_r(\hat{\theta},q^r)-q^r$ of $\overline{r}$-type banks.

Proof of Proposition 10

Throughout this proof, the last argument of $\hat{x}_c$ is always $A(\cdot)$, so we suppress it for notational convenience.

Part I: $\overline{c}-\underline{c}\leq\left[\frac{A(q^c)}{q^c}-\frac{A(1)-A(q^c)}{1-q^c}\right]\mathbb{E}r$. Suppose there exists a disclosure such that banks have different switching cutoffs. Because the average switching cutoff of all investors is always $\hat{x}_c(1,q^c)$, there exists a group of banks (W,Q) such that $(W,Q)\leq(1,q^c)$ and $\hat{x}_c(W,Q)<\hat{x}_c(1,q^c)$, that is,

$$\frac{Q}{W}\left[\underline{c}+\mathbb{E}r\frac{A(Q)}{Q}\right]+\frac{W-Q}{W}\left[\overline{c}+\mathbb{E}r\frac{A(W)-A(Q)}{W-Q}\right]$$
$$<q^c\left[\underline{c}+\mathbb{E}r\frac{A(q^c)}{q^c}\right]+(1-q^c)\left[\overline{c}+\mathbb{E}r\frac{A(1)-A(q^c)}{1-q^c}\right].$$

If $\underline{c}+\mathbb{E}r\frac{A(Q)}{Q}<\overline{c}+\mathbb{E}r\frac{A(W)-A(Q)}{W-Q}$, $\hat{x}_c(W,Q)>\underline{c}+\mathbb{E}r\frac{A(Q)}{Q}\geq\underline{c}+\mathbb{E}r\frac{A(q^c)}{q^c}\geq\hat{x}_c(1,q^c)$. Contradiction!

If $\underline{c}+\mathbb{E}r\frac{A(Q)}{Q}\geq\overline{c}+\mathbb{E}r\frac{A(W)-A(Q)}{W-Q}$, then $\overline{c}+\mathbb{E}r\frac{A(W)-A(Q)}{W-Q}\leq\hat{x}_c(W,Q)$. Since

$$\begin{aligned}&\hat{x}_c(1,Q)\\&=Q\left[\underline{c}+\mathbb{E}r\frac{A(Q)}{Q}\right]+(W-Q)\left[\overline{c}+\mathbb{E}r\frac{A(W)-A(Q)}{W-Q}\right]+(1-W)\left[\overline{c}+\mathbb{E}r\frac{A(1)-A(W)}{1-W}\right]\\&=W\hat{x}_c(W,Q)+(1-W)\left[\overline{c}+\mathbb{E}r\frac{A(1)-A(W)}{1-W}\right]\leq\hat{x}_c(W,Q),\end{aligned}$$

$\hat{x}_c(1,q^c)\leq\hat{x}_c(1,Q)\leq\hat{x}_c(W,Q)$. Contradiction!

So, no matter what the disclosure is, all banks have the same cutoff, $\hat{x}_c(1,q^c)$.

Part II: $\overline{c}-\underline{c}>\left[\frac{A(q^c)}{q^c}-\frac{A(1)-A(q^c)}{1-q^c}\right]\mathbb{E}r$. If $\hat{\theta}\geq\hat{x}_c(1,q^c)$, nondisclosure can ensure all banks immune from runs.

If $\hat{\theta}<\hat{x}_c(1,q^c)$, only part of the banks can be immune from runs. Suppose a group of banks (W,Q) are. Then their average cutoff must be weakly smaller than $\hat{\theta}$, that is,

$$\hat{x}_c(W,Q)=\frac{Q\cdot\underline{c}+(W-Q)\cdot\overline{c}}{W}+\mathbb{E}r\cdot\frac{A(W)}{W}\leq\hat{\theta}.$$

We want to find the maximum W subject to this constraint. Notice that $\hat{x}_c(W,Q)$ is decreasing in Q. It is easy to see that the maximum W must be a solution to $\hat{x}_c(W,q^c)=\hat{\theta}$. Last, we prove the following lemma in the Internet Appendix.

Lemma 6. For $\hat{\theta}<\hat{x}_c(1,q^c)$, $\hat{x}_c(W,q^c)=\hat{\theta}$ has a unique solution $W_c(\hat{\theta},q^c)$.

Also, notice that $\hat{x}_c(W,Q)\geq\hat{x}_c(W,q^c)\geq\hat{x}_c(q^c,q^c)$. So, if $\hat{\theta}<\hat{x}_c(q^c,q^c)$, no bank can be immune from runs under any disclosure.

Proof of Proposition 11

The claim is trivial for $\hat{\theta}$ in the range high enough for all banks to be immune from runs, and for $\hat{\theta}$ in the range too low for any bank to be immune from runs. For the complementary intermediate range of $\hat{\theta}$, according to Proposition 9, under optimal disclosures of systemic vulnerabilities, type-$\bar{r}$ banks that are fully revealed are exactly those subject to runs at $\hat{\theta}$. By the definition of optimal disclosures with respect to $\hat{\theta}$, the mass of banks subject to runs at $\hat{\theta}$ must be weakly lower for a higher $\hat{\theta}$. That means, the mass of type-$\bar{r}$ banks that are fully revealed weakly increases as $\hat{\theta}$ decreases. The similar logic applies to optimal disclosures of idiosyncratic shortfalls.Under optimal disclosures of systemic vulnerabilities, the mass of banks with $r^i = \underline{r}$ that are fully revealed is $\underline{m}$ characterized by Equation (A2) as

$$\underline{m}\begin{cases} =0, & \text{if } \hat{\theta} \geq \mathbb{E}c + \underline{r}a(0) \\ \text{satisfies } \hat{\theta} = \mathbb{E}c + \underline{r}\frac{A(\underline{m})}{\underline{m}} & \text{if } \hat{\theta} < \mathbb{E}c + \underline{r}a(0) \end{cases}.$$

Since $a(l)$ is decreasing, $\frac{A(\underline{m})}{\underline{m}}$ is decreasing in $\underline{m}$, so $\underline{m}$ weakly increases as $\hat{\theta}$ decreases.

Proof of Proposition 12

Let $\hat{x}_i$ denote the switching cutoff of score-i investors. WLOG, assume $\hat{x}_1 \leq \hat{x}_2 \leq \ldots \leq \hat{x}_n$. Analogous to , we have

$$\hat{x}_i = \mathbb{E}r \cdot \lim_{\sigma \to 0} \int_{\theta=\underline{\theta}}^{\bar{\theta}} a(\sum_j w_j b_j m_j^{\sigma}(\theta)) dm_i^{\sigma}(\theta) + \mathbb{E}c. \tag{B5}$$

Suppose there exists k such that $\hat{x}_k < \hat{x}_{k+1}$. Then we have the contradiction that

$$\begin{aligned}
\hat{x}_k &= \mathbb{E}r \cdot \lim_{\sigma \to 0} \int_{\theta=\underline{\theta}}^{\bar{\theta}} a(\sum_{j=1}^{k} w_j b_j m_j^{\sigma}(\theta)) dm_k^{\sigma}(\theta) + \mathbb{E}c \\
&> \mathbb{E}r \cdot \lim_{\sigma \to 0} \int_{\theta=\underline{\theta}}^{\bar{\theta}} a(\sum_{j=1}^{k} w_j b_j) dm_k^{\sigma}(\theta) + \mathbb{E}c \\
&> \mathbb{E}r \cdot \lim_{\sigma \to 0} \int_{\theta=\underline{\theta}}^{\bar{\theta}} a(\sum_{j=1}^{k} w_j b_j + \sum_{j=k+1}^{n} w_j b_j m_j^{\sigma}(\theta)) m_k^{\sigma}(\theta) + \mathbb{E}c \\
&= \hat{x}_{k+1},
\end{aligned}$$

where the first equality results from the separation between score k and score $i = k+1, \ldots, n$, the last equality results from the separation between score $(k+1)$ and score $i = 1, 2, \ldots, k$, both because $\hat{x}_k < \hat{x}_{k+1}$. Then, by (B5),

$$\begin{aligned}
\hat{x} &= \sum_k w_k b_k \hat{x}_k \\
&= \mathbb{E}r \cdot \lim_{\sigma \to 0} \int_{\theta=\underline{\theta}}^{\bar{\theta}} a(\sum_k w_k b_k m_k^{\sigma}(\theta)) d \sum_k w_k b_k m_i^{\sigma}(\theta) + \sum_k w_k b_k \mathbb{E}c \\
&= \mathbb{E}r \cdot A(1) + \mathbb{E}c.
\end{aligned}$$

Proof of Proposition 13

Consider any finite-score disclosure in dimension r, $\{(r_i, w_i)\}_{i=1}^{n}$. The expected systemic impact of score-i banks is

$$b_i = \frac{r_i - \underline{r}}{\overline{r} - \underline{r}}\overline{b} + \frac{\overline{r} - r_i}{\overline{r} - \underline{r}}\underline{b}.$$

Following the derivation of Proposition 1 and Proposition 2, we obtain

$$\sum_{j=1}^{n} \frac{w_j b_j}{r_j}\hat{x}_j = \sum_{j=1}^{n} \frac{w_j b_j}{r_j}\mathbb{E}c + A\left(\sum_{j=1}^{n} w_j b_j\right) - A(0) = \sum_{j=1}^{n} \frac{w_j b_j}{r_j}\mathbb{E}c + A(\mathbb{E}b) - A(0).$$

In the case of nondisclosure, all banks have the common cutoff

$$\hat{x} = \mathbb{E}c + \frac{\mathbb{E}r}{\mathbb{E}b}\left[A(\mathbb{E}b) - A(0)\right].$$

Notice that

$$\sum_{j=1}^{n} \frac{w_j b_j}{r_j} = \sum_{j=1}^{n} \frac{w_j}{r_j}\left(\frac{r_j - \underline{r}}{\overline{r} - \underline{r}}\overline{b} + \frac{\overline{r} - r_j}{\overline{r} - \underline{r}}\underline{b}\right) = \sum_{j=1}^{n} \frac{w_j}{r_j} \cdot \frac{\underline{b}\overline{r} - \overline{b}\underline{r}}{\overline{r} - \underline{r}} + \frac{\overline{b} - \underline{b}}{\overline{r} - \underline{r}}.$$

Compared to nondisclosure, any disclosure in dimension r increases $\sum_{j=1}^{n} \frac{w_j}{r_j}$. If $\underline{b}\overline{r} - \overline{b}\underline{r} > 0$ or $\overline{b}/\overline{r} < \underline{b}/\underline{r}$, then there must exist a disclosure in dimension r such that all banks have a common cutoff smaller than $\hat{x}$.

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