Can GeoGebra's augmented reality tool provide a looking glass into a mathematical wonderland?

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GeoGebra has been well researched within the mathematics education community; however, the majority of this literature does not examine the recent edition to the GeoGebra family, GeoGebra 3D Calculator with Augmented Reality (GeoGebra 3D/AR). This master's study sought to examine how primary school students (age 7 to 12 years old) used 'AR manipulatives' to model familiar household objects. Due to the pandemic, the study was conducted over two 'virtual workshops' which propelled a second technological tool/environment to the fore; teaching, learning and researching within the 'Zoom classroom'. Participants' interactions were analysed qualitatively through Bruner's enactive-iconic-symbolic framework. All young participants identified real-life objects (enactive mode), constructed virtual objects in GeoGebra 3D/AR (iconic and symbolic modes) then 'augmented' these AR manipulatives alongside real-life artefacts (all modes). Furthermore, the virtual workshops revealed how student-centred orchestrations such as 'spot-and-show' and 'sherpa-at-work' were extremely challenging to replicate in a remote setting.

Keywords: augmented reality; AR manipulatives; orchestrations

Introduction

Augmented reality (AR) and 'AR manipulatives'

The embryonic stages of augmented reality (AR) date back over half a century when Ivan Sutherland created the first head-mounted display (HMD), a display which Sutherland believed would ultimately provide "*a looking glass into a mathematical wonderland*" (Sutherland, 1965, p.506). AR emerged as a distinct mixed reality in the early 1990s, initially conceptualised as 'see-through' virtual reality (VR) which superimposed mathematical models onto physical objects. Eventually, VRs transparent younger sibling was defined as follows: an augmented reality system combines real and virtual, is interactive in real time and is registered in 3-D (Azuma, 1997). Most digital environments are all encompassing, containing all mathematical learning within the digital environment itself. Augmented reality is different. The way in which augmented reality straddles both the physical and virtual world (superimposing digital information on real life artefacts), means that AR technology will never replace the real (or physical), but instead supplement it. As tech giants such as Apple and Google continue to invest in augmented reality (Tomaschko & Hohenwarter, 2019), AR environments are now easily accessed through 'AR-enabled' smart devices. Subsequently, there has been a surge in AR studies within education (see extensive literature review in Ibáñez & Delgado-Kloos, 2018). Through the lens of embodied cognition theory, Bujak et al. (2013) predicted that 'AR manipulatives' (manipulatives created using AR technology) would support students in creating personal, embodied representations of specific mathematical concepts. Given the central role manipulatives play in Bruner's (1966) modes of thinking, how would these enactive (concrete), iconic (pictorial) and symbolic (abstract) representations 'play out' in an augmented reality environment? Tomaschko & Hohenwarter (2019) described how 'AR manipulatives' constructed in GeoGebra's 3D/AR application (app) fostered situated, context-aware learning, enabling learners to examine mathematical objects/manipulatives from multiple perspectives. Similar findings were echoed by Lavicza, et al. (2020) who depicted students creating unique, personalised visual-spatial memories when engaging in AR modelling activities using GeoGebra 3D/AR. The 'authoring capability' of a technological tool (or environment) is a very powerful notion, and something which Seymour Papert strongly advocated; *"anything is easy if you can assimilate it to your collection of models"* (Papert, 1980, p.vii).

How to introduce new technology: Instrumental genesis and TPACK

Individuals rarely master technological tools completely unassisted, or in Touche terms (Touche, 2005, cited in Drijvers et al., 2010), instrumental genesis rarely happens akin to osmosis. If a student is given an instrument (in our case, an AR-enabled mobile device), the student must go through a process of instrumentation before that instrument can be used to perform a specific activity (such as actively constructing an 'AR manipulative'). Fundamentally, instrumentation is inextricably linked to instruction. Laborde (2008) proposed that students also develop knowledge of a specific domain through the process of instrumentation; this would imply that constructing knowledge about 3D geometry may be a biproduct of learning to master the GeoGebra 3D/AR app. When introducing new technological tools to a class, Drijvers et al. (2010) described a series of 'instrumental orchestrations' which teachers can adopt; from 'teacher-centred' modes such as 'technical-demo' and 'explain the screen', through to more 'student-led' approaches described as 'spot-and-show' and 'sherpa-at-work'. Obviously, these instrumental orchestrations were proposed for the 'traditional classroom' and not the 'synchronous classroom' which the post-Covid world were having to become accustomed. All of the studies reported in the AR learning literature were conducted in-person; the researchers, or teacher-researchers were *with* the students, be it a traditional classroom, or an out-of-school setting such as a museum. The 'unknown technological tool' under investigation through the analytic lens of TPCK (Mishra & Koehler, 2006) was the AR tool. Nonetheless, the pandemic brought with it a new technological tool, the virtual learning environment (VLE), but this time the tool was not *for* the classroom, it was in place of it.

Methodology

GeoGebra's 3D/AR app is capable of creating 'AR manipulatives', moreover, the dynamic sliders provide intrinsic, visual feedback which may support students' self-directed modelling journeys. Since the in-person study was scuppered by the pandemic, I needed to consider the impact of introducing the novel, AR tool through the 'Zoom classroom'. As such, the research questions were revised as follows:

1) Can an augmented reality tool, GeoGebra 3D/AR, be introduced effectively through a virtual learning environment?
2) How do primary students construct and augment 3D prisms in GeoGebra 3D/AR?

Since the Government were instructing everyone to stay at home, I designed a 'Zoom workshop' to explore the 3D geometry of household objects using augmented reality: ‘Maths Around the House (MATH)’. Prior to a workshop, participants were asked to download the GeoGebra 3D/AR app and collect an array of household cubes (e.g. die), spheres (e.g. football), cylinders (e.g. tin of tuna) and cuboids (e.g. box of tea). At the start of a workshop participants were asked to order ‘real-life’ household objects by volume. The focus of the workshop was to try and uncover the ‘mathematical wonderland’ of household manipulatives through the lend of the AR app which integrated pictorial (geometric screen) and abstract (algebra screen) elements. The dynamic nature of GeoGebra’s sliders and moveable points enabled users to manipulate some of the variables underpinning the ‘AR manipulatives’. In essence, I was hoping participants would ‘discover’ why household cuboids were the most difficult to order by volume, and likewise, why cuboid ‘AR manipulatives’ were the most taxing to construct. To support participants acquiring the relevant technical skills related to the AR tool, the app was introduced through a ‘synchronous technical demo’. Participants were recruited through a partnership with an Arts space in London and an independent social media campaign. Interested parties ranged from age 7 to age 12, which meant communication (including ethics approval) was via the participants’ parents. If parental approval was secured, the workshops were recorded and data derived using an inductive approach (Erikson, 2006). Further data were elicited through a post-workshop questionnaire. The first workshop (MATH1) consisted of 6 children aged between 7 and 12, however, one parent did not approve filming. Two participants (a 7-year-old and a 10-year-old) and two respective parents attended the second workshop, MATH2. Filming was approved for MATH2 which is why the data in the following section is predominantly from the MATH2 workshop. All participants have been pseudonymized.

Findings and analysis

Figure 1a Participants share household cylinders

Figure 1b Researcher shares household cylinders

The use of familiar household manipulatives helped situate the remote participants in a shared context, and many of the participants were eager to share their objects with the rest of the group (see figures 1a and 1b). These low-level, ice-breaker activities were intended to guide participants to think about prisms in terms of ‘variables’. As the following extract highlights, most participants (along with parents) struggled to articulate why the cylinders and cuboids were more difficult to line up in order of volume:

> Dad (MATH2), in response to seeing the tin of tuna: Wow - that's a cylinder, isn't it? Of course! It’s flat though!
>
> Researcher: If you have a think about it, why are cylinders trickier to line up than spheres?
>
> Adam (10-year-old, MATH2): Because the tops are bigger than the bottom?

Despite Zoom confining visual communication to the internal cameras of two or more devices, this particular 'show and tell' activity exemplifies Zoom replicating face-to-face communication relatively well. Participants were encouraged to 'come along for the ride' and construct their own AR models in real-time with me as I mirrored GeoGebra 3D/AR app through Zoom. To sidestep the technical mathematical language associated with 3D space (such as 3D co-ordinates), I instructed participants to *"click on the blue 3"* when constructing a sphere (see fig. 2a), or *"click the red 2"* when constructing the cube (see fig 2c).

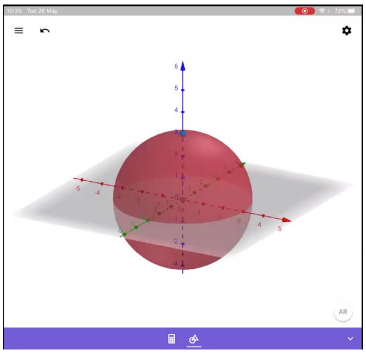
Figure 2a Constructing sphere

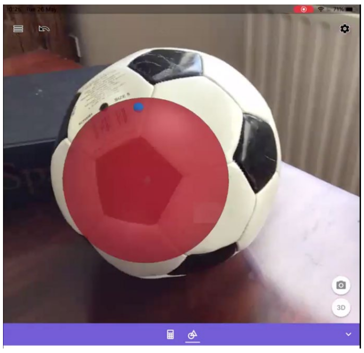
Figure 2b Augmenting a sphere

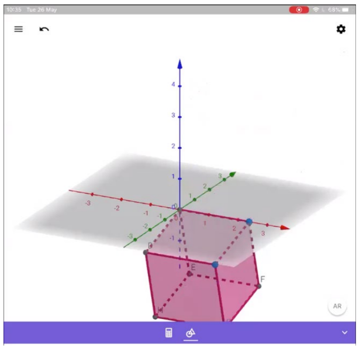
Figure 2c Constructing a cube

One 9-year-old participant in MATH1 asked why only one point was needed to construct the cube and sphere. Using the Zoom whiteboard, I attempted to explain how the volumes of spheres and cubes were solely dependent on the distance between two dynamic points. All the MATH1 participants were very quiet at the point, so I am unsure if the algebra was too abstract for the young group.

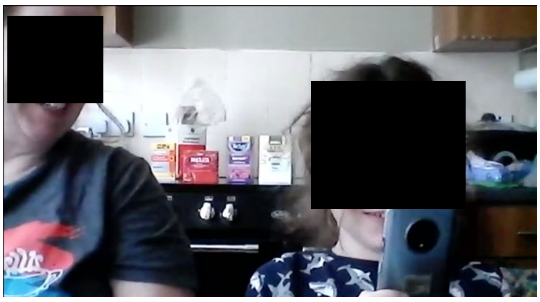
Figure 3a Participants discovering the 'net'

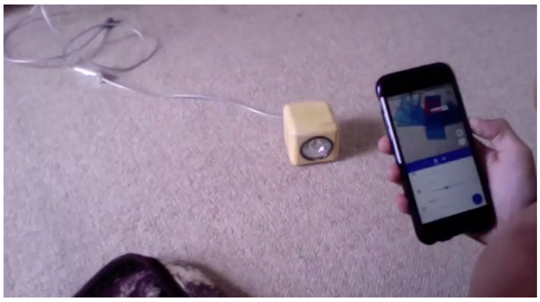
Figure 3b Participant constructing three nets

Eliciting 'over the shoulder' feedback was extremely challenging to replicate in Zoom (see figures 3a and 3b). Subsequently, the variety of orchestrations that teachers are able to perform in the Zoom classroom do not mirror the 'traditional classroom' orchestrations: didactic, teacher-centred orchestrations are relatively easy to imitate in Zoom, whereas student-centred ones are not. Even if students had the technological skills to share their screens and fulfill the 'sherpa-at-work' orchestration, this scenario is likely to favour more confident students since teachers are unable to 'spot-and-show' shy Sherpa-students in virtual settings. Given the lack of intrinsic, visual feedback (usually afforded by the natural world), virtual environments rely more on extrinsic, evaluative feedback which can present reliability issues. I suspect some students may respond with false positives and be reluctant to ask for help, particularly in a (virtual) room full of strangers. When conducting research remotely, there is no way for a researcher to evaluate the 'unshared' or the unconscious.

All the participants successfully constructed AR manipulatives using GeoGebra 3D/AR within the first half hour of the workshops. Although the constructions needed a lot of guidance and were often met with moments of despair, the 'synchronous, guided instruction' section certainly assisted participants' process of instrumental genesis. Each participant shaped their respective mobile device (artefact) into an AR-assisted mathematical tool (process of instrumentalisation). The (instrumentalised) AR tool then positioned AR manipulatives within the real environment, thus realising mathematical ideas in a unique setting as a direct consequence of action with the artefact (process of instrumentation). Participants were certainly intrigued by the AR representations and the AR environment in general. Throughout both workshops I witnessed expressions of amazement and joy, along with comments such as *"That's so cool"* or *"Oh, my God, I made a cube"*. Overall, the most captivating features were 'the net' (particularly opening and closing this) and the volume calculation button. Yet these moments of captivation were often juxtaposed by confusion; some participants struggled to find an appropriate surface on which to render an AR manipulative, others did not understand why we used an algebraic value ('r') for the radius of the cylinder'. These frustrations could be attributed to the complexity of the app, vague instructions on my part, or perhaps elements of both. Towards the end of the MATH2 workshop Adam (age 10) described how he *"went inside a Rubik's cube it was really weird"* and Mark (age 7) impressed his mother when he 'discovered' the animated slider for 'the net'. Similarly, Benjamin (age 9, MATH1) reported "*I've played with it (the app) more and drawn a solar system with different colour planets!*". These moments of student-led 'hands-on' explorations felt like the vignettes of Brunerian (1966) discovery learning. Although these extracts cannot tell us much about the epistemological value of the autonomous explorations, they do exemplify how participants successfully 'instrumented' their apps to useful, augmented, mathematical tools.

Conclusion

Despite bumps along the AR road, all young participants managed to construct AR manipulatives using GeoGebra's 3D/AR app. These AR journeys could be described in Bruner's non-sequential terms; first, participants identified 'real-life' objects (enactive mode), next, participants constructed virtual objects in GeoGebra 3D (iconic and symbolic modes), and finally, participants 'augmented' 3D objects alongside real-life counterparts, hence realising 'AR manipulatives'. In principle, this final stage (superimposing an AR manipulative onto a corresponding household object) describes an environment which combines all of Bruner's representation modes and more. Augmented learning does not a replace Bruner's enactive-iconic-symbolic modes of thinking, but instead situates the framework within a modern-day digital tool set. Moreover, the way in which GeoGebra 3D/AR reveals the mathematics underpinning dynamic 3D prisms merges some of Papert's (1980) constructionist principals. If Papet's Turtles are objects-to-think-with, could dynamic, mathematical AR environments be described as environments-to-think-within? Nonetheless, accessing these environments (or 'mathematical wonderlands') is far from straightforward. The process of instrumental genesis needs careful guidance, and the leaning environment (in-person or virtual) influences this guidance. Through the lens of TCK (how a tool can best communicate content and afford a greater variety of representations), we can appreciate the range of representational modes (video, screenshare, whiteboard) that Zoom offers. These representational modes enable didactic demonstrations to be delivered highly effectively through Zoom. In contrast, the pedagogic capabilities of Zoom (or TPK) are limited. When working remotely, gauging the mood of the (Zoom) room is extremely challenging, as are the student-focussed orchestrations such as 'spot-and-show' and 'sherpa-at-work'. If virtual and blended learning are here to stay, then instrumental orchestrations may need to be elaborated for the virtual classroom setting.

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