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Edited by: Hans-Georg Weigand, Alison Clark-Wilson, Ana Donevska-Todorova, Eleonora Faggiano, Niels Grønbæk and Jana Trgalova

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Teacher Knowledge for Teaching Geometric Similarity with Technology: A Review of Literature

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Teacher knowledge for the teaching of key topics in secondary mathematics (in particular geometric similarity) is of prime interest to the mathematics education community. This theoretical paper, which forms part of a literature review conducted for a doctoral study, aims to discuss the few existing research studies addressing teacher knowledge of this topic and to highlight how the integration of digital tools requires the nature of this knowledge to be reconsidered. The implications of these findings for the future research agenda are outlined.

Keywords: Teacher Knowledge, Geometric Similarity, Technology.

INTRODUCTION

Teacher knowledge is a key factor for successful teaching of mathematics. A variety of theoretical frameworks provide useful conceptual lenses for analysing teacher mathematical knowledge (e.g., Ball, Thames and Phelps, 2008; Mishra & Koehler, 2006; Shulman, 1987). Shulman's (1987) two content-related categories of knowledge, content knowledge (CK) and pedagogical content knowledge (PCK), have become foundational ideas for many subsequent researchers.

Geometric similarity (GS), an aspect of most school geometry curricula, is a mathematical concept for which researchers have sought to deeply examine teacher knowledge for teaching. GS is considered a fundamental integrative concept in both secondary and higher-level school mathematics (Cox & Lo, 2014; Watson, Jones & Pratt, 2013) as it connects the two core concepts of proportionality and geometric transformations, linking both numeric and geometric reasoning (Cox, 2013). However, as a result, students are known to encounter difficulties in making sense of GS (Chazan, 1988; Edwards & Cox, 2011, Noss & Hoyles, 1996). For example, students tend to fail to apply multiplicative strategies and proportional reasoning in the context of solving problems relating to GS due to an apparent over-reliance on additive strategies. Given both the significance of GS for school mathematics and the difficulties that it presents to students, identifying and describing teacher mathematical knowledge for teaching GS appears to be valuable and important endeavour to elucidate what is necessary for teachers to know in order to teach it efficiently. It is crucial for teachers to have a wide and deep understanding of GS, which involves appreciating the different approaches to its definition and how this relates to the properties of similar shapes.

As part of an ongoing doctoral study, the first author of this paper performed the literature search for articles, conference proceedings, and dissertations published in English using the following data-bases: British Education Index (BEI) (EBSCO), ERIC (EBSCO), Google Scholar, UCL Discovery, and EThOS (the British Library's Electronic Theses Online Service). The key search terms used were "similarity", "geometric similarity", "proportionality", "teacher", "classroom practice", and "technology". This method revealed surprisingly few research studies on teachers' mathematical knowledge for teaching with regard to GS (both with and without reference to any technology), evidence of a clear gap in the literature. This paper aims to synthesise key findings from the identified studies and, based on this review, to provide recommendations for future research.

RESEARCH ON TEACHERS' KNOWLEDGE OF GEOMETRIC SIMILARITY

Identified research studies have tended to examine GS as a sub-concept of proportionality (treated numerically), despite its fundamental role within aspects of geometric reasoning (Cox, 2013). The research has tended to focus on students and be centred on the diagnostic, aiming to identify the kinds of difficulties students encounter (and the strategies they use) to solve particular sets of problems (e.g. Cox, 2013; Cox & Lo, 2014; Friedlander, Lappan & Fitzgerald, 1985).

By comparison, there has been little research undertaken that focuses on teacher knowledge of GS (e.g, Clark-Wilson & Hoyles, 2017; Cunningham & Rappa, 2016; Seago, Jacobs, Heck, Nelson & Malzahn, 2014; Son, 2013). In these studies, the broad aim was either to examine the types of teacher mathematical knowledge of GS (such as CK or PCK) or to explore the growth of teachers' mathematical knowledge for teaching.

For example, Son (2013) examined 57 primary and secondary pre-service teachers' (PSTs') CK and PCK in relation to GS, paying particularly close attention to their additive reasoning in 'missing value' tasks. She argued that these tasks lead students to understand proportionality and GS deeply, from both conceptual and procedural aspects.

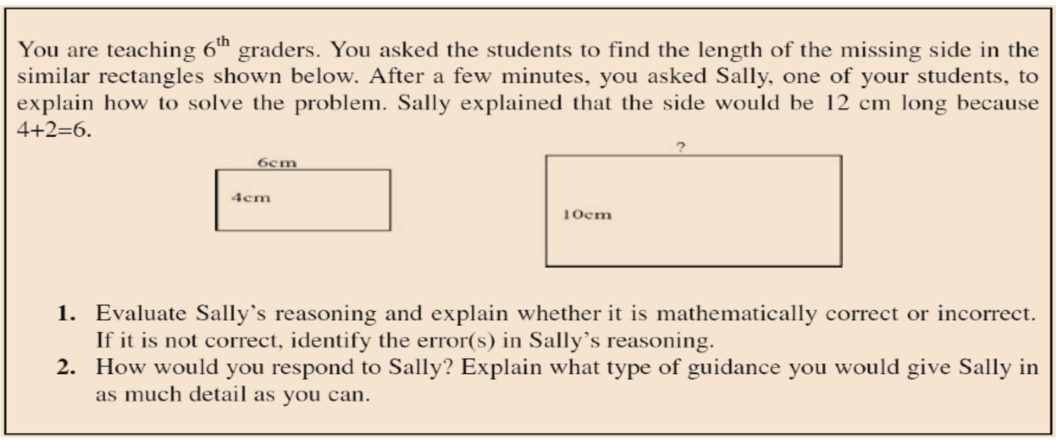

Figure 1: Pedagogical missing value problem: "What is the length of the missing length in similar rectangles?"

In Son's study, the PSTs were first asked to produce an answer to a particular missing value problem (as in Figure 1). They were then invited to interpret and respond to the student's error(s) through a teaching scenario task (Figure 1), which stems from the incorrect use of an additive strategy, and to suggest strategies to help Sally to make sense of here error.

According to Son, the successful steps to solve missing value problems featuring similar figures are: (1) understand the concept of GS; (2) be able to recognise the proportionality embedded in similar shapes by comparing lengths and widths between figures (between ratio) or by comparing the length to width within a rectangle (within ratio) or determining a scale factor; (3) explain the relationship between two similar figures using a ratio, a proportion, or a scale factor; and (4) carry out the calculation correctly. In relation to these four steps, Son noted that while the first two are related to conceptual aspects of GS, the others are associated with procedural aspects.

Son's data analysis categorised three approaches by the PSTs in the ways that they identified and interpreted Sally's misconception in terms of a procedural and conceptual approach.

- *Concept-based approach*: the PSTs paid attention to the meaning of GS in rectangles in that "two figures are similar if (1) the lengths of their corresponding sides increase (or decrease) by the same factor, called the scale factor, while their corresponding angles are equal, and (2) the perimeter from one rectangle to another rectangle also increases by the same scale factor" (p. 59).
- *Procedure-based approach*, concerning finding the value of missing side in similar figures. The PSTs underscored that in the procedure-based approach, one needs to calculate a ratio, a proportion, or a scale factor to find a missing length without necessarily understanding the meaning of GS. i.e. this approach relates to building a numerical expression indicating an equivalence between the two rectangles.
- *Misidentification* of the error(s) in terms of additive reasoning or improper focus.

One of the most notable findings of Son's study is that, despite the fact that Sally's misconception related to her use of an additive strategy might be due to a limited understanding of the concept of similarity, most of the PSTs considered that the error was due to her procedural misunderstanding.

A second study, conducted by Seago et al. (2014), aimed to promote secondary mathematics teachers' MKT in relation to a transformations-based approach to the definition of GS. It is noteworthy to state here that in the literature, GS can be conceptualised in three related but distinctive ways. The first conceptualises GS as "the same shape, but not necessarily the same size". The second, which is named as a static-based approach, conceptualises GS on the basis of a numeric relationship between measures of lengths of figures and their sizes of angles. This implies that if two figures are similar, the measures of their corresponding lengths are proportional, and the sizes of their corresponding angles are equal. The third is a transformations-based approach, whereby GS is conceptualised in terms of translations, reflections, rotations and dilations.

Seago et al. (2014) define GS based on a transformations-based approach as follows: in order for two figures to be similar, it is required that the second figure can be acquired from the first one by applying a sequence of translations, reflections, rotations, and dilations. This pedagogical approach encourages students to solve GS problems by reasoning and applying geometric transformations, rather than by merely applying numeric strategies. The researchers hypothesised that incorporating a transformations-based approach into the process of teaching GS enables students to develop a deeper understanding. For this reason, they aimed to support teachers to "gain a robust conception of similar figures as part of an infinite family that can be formed by applying one or more geometric transformations" (p. 632).

Their study involved a professional development programme (PD) in which a sequence of video cases was used to present the mathematical ideas to teachers so as to address the challenges teachers may face when adopting a transformations-based approach. The research findings indicated that through the PD provided, the teachers improved their understanding of GS for teaching, in particular concerning their mathematical knowledge regarding definitions of GS relating to congruence and dilation.

Further to the work of Son (2013) and Seago et al. (2014), Cunningham and Rappa (2016) also investigated mathematics teachers' ability to solve GS problems. The researchers surmise that, like Seago et al., when teachers introduce a transformations-based approach together with a static-based approach when teaching GS, students are likely to understand the underlying ideas of GS more deeply. Therefore, they asserted that it is important to investigate teachers' mathematical knowledge of GS from both perspectives because the teachers' mathematical knowledge could play a key role in the development of students' understanding.

In their small-scale study, Cunningham and Rappa asked 15 secondary mathematics teachers to solve seven problems related to GS, in which either a static-based approach or a transformations-based approach was the stipulated method.

The concluded that, while the problems requiring a static perspective were successfully solved by all of the teachers, only eight teachers were able to successfully solve the problems requiring a transformational perspective. Cunningham and Rapp conclude that the latter group of teachers perceived GS more procedurally, which led them to rely only on the numerical relationship embedded in the similar figures. This result resonates with Son's (2013) finding that teachers may favour using procedure-based method to solve problems related to GS.

The aforementioned studies were not designed to research teachers' specific knowledge and practice to use dynamic technology in the teaching of GS, although the study by Seago et al. (2014) did employ technology within the PD.

This aspect has been partially addressed in research conducted by Clark-Wilson and Hoyles (2017), which explored the impact of 40 secondary mathematics teachers' engagement with PD and classroom teaching on their mathematical knowledge for teaching GS. Their study explored the teachers' starting points using data collected through survey-items, PD tasks and lesson plans. Key to the design of the PD were a number of tasks for teachers that required them to closely analyse hypothesised student responses whilst engaging with a particular dynamic mathematical technology (DMT), 'Cornerstone Maths' (CM) (For an example, see Figure 2).

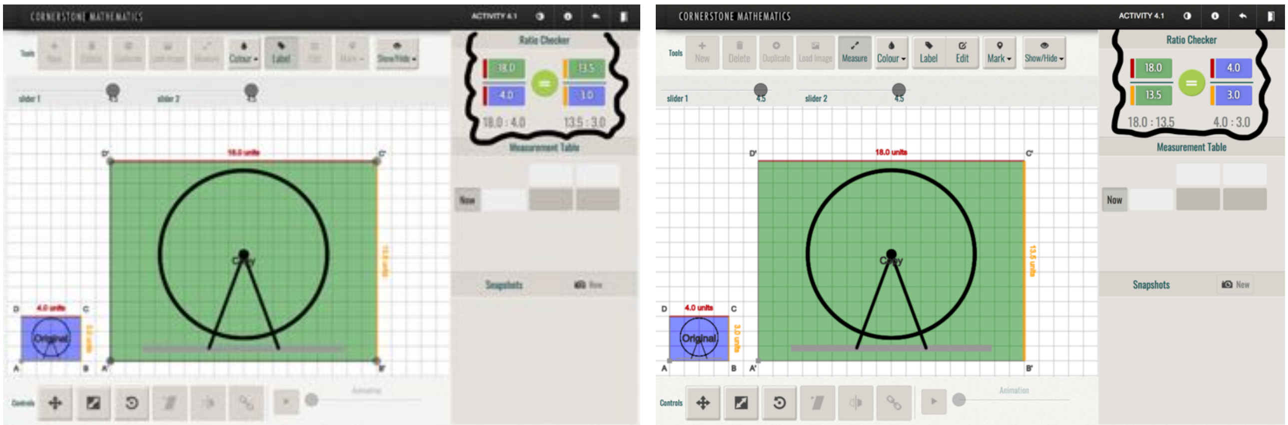

Figure 2: Students' work created in the DMT used as a context for teachers to identify the meaning of invariant ratios revealed by the two statements in the ratio checker.

Clark-Wilson and Hoyles gained further insight into the teachers' enacted knowledge by observing a common lesson from the CM teaching sequence. 7 of the 40 participating teachers were observed and subsequently interviewed about their lesson.

Clark-Wilson and Hoyles found that the combination of PD activity focused on GS and classroom teaching involving DMT led to notable improvements in teachers MKT in relation to GS that "concerned more robust definitions of [GS] for a broader range of polygons and the appreciation of the invariant ratio property for pairs of corresponding sides within similar polygons" (p. 18). According to the researchers, the use of students' work created in the DMT environment (as in Figure 2) encouraged the teachers to think deeply about the "within ratio" invariant property. Having engaged with the task in the DMT environment, they were able to successfully articulate the underlying mathematical ideas related to the property that, for similar shapes, the ratios of the side lengths for any pair of corresponding sides within the shape is invariant.

CONCLUDING COMMENTS AND FUTURE DIRECTIONS

The aforementioned studies sought to elicidate and/or improve teacher knowledge in relation to GS and they provide multiple insights into both what teachers understand concerning GS and how they develop related MKT. For example, Son's (2013) revealed a lack of teachers GS-related PCK as evidenced by the misidentification and/or misinterpretation of Sally's error. The studies by Seago et al. (2014) and Cunningham and Rappa (2016), reveal that definitions of GS from the perspective of geometric transformations is novel to teachers and the key role that PD plays in the development of teachers' understanding of the underlying curriculum links. Likewise, Clark-Wilson and Hoyles' (2017) study suggests that although gaps in teachers' knowledge for teaching GS are apparent, PD programmes in which teachers use dynamic technology to engage with mathematical activities away from and in the classroom do stimulate notable improvements in their knowledge.

Furthermore, research has consistently highlighted that GS is a mathematical topic with which both students and teachers encounter difficulties. Simultaneously, some studies do suggest that carefully designed DMTs, might help students (and teachers) to overcome difficulties and misconceptions with regard to GS as the dynamic and visual nature of digital technology offers opportunities (e.g., dragging, visualisation, measurement) to explore the underlying concepts and discover the embedded variant and invariant relationships (Chazan, 1988; Denton, 2017; Edwards & Cox, 2011). Such opportunities might enable teachers and students to experience and examine the dynamic nature of GS in more tangible ways. For example, teachers can exploit the affordances of digital technology to help students build connections between geometric transformations and GS so that students understand how to use translations, reflections, rotations and dilations to determine if two figures are similar. Additionally, making use of technology in a dynamic environment where students can formulate, test, and verify mathematical conjectures, teachers can support students to surmount their misconceptions about the ideas of GS, particularly those who make the incorrect use of an additive strategy as the student in Son's (2013) study. How digital technology can support students in addressing their misconceptions related to non-multiplicative strategies has been illustrated by Edwards and Cox (2011).

We conclude that carefully designed DMT can be a useful didactical tool that can provide teachers with both a context and opportunities to develop their students' understanding of the ideas of GS. A focused and longitudinal investigation into teacher knowledge for teaching GS using digital technology could identify and articulate teachers' relevant mathematical knowledge, an aspect that none of the aforementioned research studies have *specifically* explored. Consequently, such a study would focus the research lens on characteristics of teacher knowledge (both espoused and enacted in the classroom) in relation to using digital technology to teach GS.

Moreover, researchers underline that one of the key factors for the success of the integration of digital technology into classroom practice is the teacher, and the interactive and dynamic nature of their knowledge plays a central role in underpinning the practice (Ruthven, 2014). Nonetheless, it is widely acknowledged that that the integration of new digital technologies into ordinary classroom practices poses a significant challenge for mathematics teachers.

In terms of the concept of GS, Clark-Wilson and Hoyles' (2017) research project is the only study in the literature that focuses on selected teachers' classroom practices with DMT in relation to GS. However, as their research probed only one lesson of each teacher, their data provides useful but limited insights into the development of their mathematical knowledge and associated classroom practices on their teaching of GS with dynamic technology. Hence, very little is currently known as to *how and why* teachers exploit the opportunities that dynamic digital technology offers when teaching GS in the mainstream classroom and how their associated knowledge shapes and is shaped by their thoughts and actions, and thereby their practices. There is a need to identify the important aspects of teacher knowledge and classroom practices that promote students' robust understanding of GS within technology enhanced classroom environments through conducting more systematic investigation. This necessity, therefore, calls for more research aiming to ascertain what mathematical knowledge for teaching and mathematical pedagogic practices are required for teachers to productively make use of dynamic digital technologies in their classroom teaching of GS.

To address the identified gap, the first author's doctoral study is researching the actual classroom practices of three English secondary mathematics teachers using a particular DMT to teach GS explored through classroom observation, teacher interview, and the scrutiny of lesson plans and resources. The combination of the Structuring Features of Classroom Practices (Ruthven, 2014) and Instrumental Orchestration (Drijvers et al., 2010) frameworks guide both the data collection and analysis. The research aims, in particular, to develop a more comprehensive understanding of the development of teachers' classroom practices when teaching GS with dynamic technology along with the nature and content of their associated MKT and, in general, to add to the growing body of knowledge on teachers' integration of DMT in mathematics classrooms.

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