Distributed Model Predictive Control for Heterogeneous Vehicle Platoon with Unknown Input of Leading Vehicle

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ABSTRACT

In this paper, a distributed model predictive control (DMPC) algorithm for a platoon of heterogeneous vehicles is proposed. The leading vehicle is allowed to be driven by a non-zero and time-varying input, rather than travelling at a constant velocity. Except for individual state and input constraints for each vehicle, all vehicles are coupled via state-coupled inter-vehicular spacing constraints and state-coupled cost functions, which maintain the unidimensional platoon formation with satisfactory transient performance. Each vehicle communicates with its neighboring vehicles, and may not know the leading vehicle's kinetic status information. The control input of each following vehicle is computed by a local optimization problem established by each vehicle's local information and the assumed state information from its neighbors. By designing distributed terminal control laws for following vehicles, dividing each state-coupled set into several specific subsets, and then forcing each following vehicle to optimize its state constrained in the assigned subsets, the coupled constraints and cost functions can be decoupled, and thus a distributed and parallel computing method can be adopted to compute the control inputs of all following vehicles. Based on the tailored terminal equality constraints together with the tailored terminal control laws, the recursive feasibility of local MPC optimization problems is achieved at all time steps and the asymptotic stability of each vehicle is also guaranteed. The effectiveness of the proposed DMPC method is demonstrated in simulation, and the advantage of the proposed DMPC dealing with the leading vehicle's non-zero, inaccessible, and time-varying input is highlighted by a comparison simulation for heterogeneous vehicle platoon with a continuously changing leading vehicle velocity.

1. Introduction

For the benefit of increasing the capacity of highways, enhancing road safety and improving energy efficiency, many efforts have been devoted to the study of Intelligent Transportation Systems (ITS) [1–3]. As one application of ITS, grouping autonomous vehicles in the same lane into a dense platoon provides an economical and effective solution to benefit road transportation [4, 5]. In the platoon, vehicles should drive at the same speed and a pre-determined distance must be maintained between any two consecutive vehicles. Meanwhile, the platoon control strategy must be able to take account of physical constraints and guarantee safety [6–8]. For ease of implementation, the dominant architecture is distributed control, where each following vehicle in the platoon is equipped with its own controller and makes its control decision by utilizing its kinetic status information, such as position, velocity and acceleration, and kinetic status information from other vehicles via Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I) communication [9, 10]. To achieve a safe and efficient platoon, various aspects of vehicle platooning are investigated, such as spacing policy [11, 12], communication topology [13–15], communication and dynamic delays [16, 17], and heterogeneity of vehicle dynamics [18–20]. For better control performance, several cooperative adaptive cruise control (CACC) mechanisms [21, 22], including sliding-mode control (SMC) [23, 24], H_∞ control [25, 26], optimal control [27, 28], and model predictive control (MPC) [3, 29], have been explored.

Considering a more general scenario where the speed of the leading vehicle changes, a consensus control-based platoon control method is proposed in [30], where coupled inter-vehicle safety constraints are considered for collision avoidance guarantees. Moreover, additional physical operating constraints are addressed in [31] with considering also more accurate nonlinear motion dynamics. Both algorithms in [30] and [31] are proven with guaranteed asymptotic

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convergence properties and string stability when the leading vehicle's speed remains constant. Alternatively, [32–34] propose distributed model predictive control (DMPC) methods for following vehicles to track the leading vehicle. However, the feasibility of the DMPC problems and the stability of the platoon control are not analyzed. In [35], the authors consider a scenario with a time-varying leading vehicle speed and provide a complete theoretical analysis of the Lyapunov stability. However, inter-vehicle safety constraints to guarantee collision avoidance are not taken into consideration. Despite the abundance of literature on platoon control schemes, physical operating constraints, coupled inter-vehicle safety constraints and speed variations of the leading vehicle that exist in practice are rarely considered at the same time. The lack of consideration of these aspects can lead to degraded control performance, instability or even catastrophic collisions.

MPC has been widely used in many areas over the last decades due to its ability to explicitly handle various constraints and allow for optimization considerations [36, 37]. To avoid overloaded communication and computation in large-scale systems, DMPC for multi-agent systems has recently been studied in depth [38–40]. Several DMPC algorithms are proposed in [41–45] for vehicle platoon control, and the simulation or experiment results verify the good performance of the designed DMPC controllers. However, the theoretical analyses, including recursive feasibility and closed-loop stability, are not considered in [41–45]. To ensure a rigorous justification of the feasibility and stability properties of the DMPC controllers, a positive definite cost function associated with the underlying DMPC optimization problem is commonly used to penalize the deviation between the state and input point of the actual system and the predefined setpoint or the reference signal containing the position, velocity and acceleration information of the leading vehicle [46–49]. However, it may not be applicable in a platoon control problem where the setpoint or the reference signal may not be known to all following vehicles due to communication restrictions. To overcome the challenge of missing information from the leading vehicle for some following vehicles, terminal equality constraints are introduced into the formulated DMPC optimization problem [49–52]. The terminal state of each following vehicle at all times must be equal to the terminal state average of its neighboring vehicles [50, 53, 54]. Besides, the recursive feasibility in the first few steps can not be proved theoretically, and it leads to an incomplete feasibility proof. Alternatively, the works in [51, 52] design terminal equality constraints that only rely on the local state information and are independent of the state information from the neighboring vehicles for each following vehicle. The recursive feasibility can then be fully proved. The existing literature [47–52] generally assumes that the leading vehicle travels at a constant speed, which implies that the input of the leading vehicle is always zero (or a constant depending on the dynamics). If the input of the leading vehicle is inaccessible to all following vehicles, and can be non-zero and time-varying, the existing DMPC framework will fail due to the lack of real-time trajectory of the leading vehicle. To avoid collisions, coupled state constraints between any two consecutive vehicles are considered in a few cases [46, 49]. However, to the best of our knowledge, under the condition that the leading vehicle's state information can only be transmitted to following vehicles that are directly connected, and the input of the leading vehicle is inaccessible to all followers and can be time-varying, the satisfaction of the state and input constraints, especially the state-coupled inter-vehicular spacing constraints, has not been taken into consideration.

With the aim of addressing the aforementioned challenges, this paper proposes a novel DMPC scheme in a parallel computational protocol for controlling the platoon of heterogeneous vehicles. The contributions of this paper are summarized as follows:

1. It is taken into consideration under the condition that the control targets change over time and are unknown for a subset of the following vehicles, due to the time-varying input of the leading vehicle not being accessible to all following vehicles and the lack of the leading vehicle's kinetic status information. Meanwhile, the recursive feasibility, especially the satisfaction of the state-coupled inter-vehicular spacing constraints, of the DMPC algorithm can be completely guaranteed. The key is to (1) divide each state-coupled set into several specific subsets, and then force each following vehicle to optimize its state constrained in the assigned subsets; (2) design a local terminal feedback control law, inspired by distributed control for multi-agent systems, and resort to centrally derived kinetic status error invariant set for each following vehicle. Such a DMPC framework also ensures the asymptotic stability of the platoon system.

2. The proposed DMPC method considers the unknown input of the leading vehicle, and theoretical properties including recursive feasibility and stability are rigorously proved under such conditions. As such, the proposed algorithm permits more flexible velocity changes of the leading vehicle compared to the existing DMPC-based platoon control approaches. It turns out to be useful in practice to accommodate traffic perturbations.

The rest of this paper is organized as follows. In Section 2, the communication topology, the dynamics of vehicles, and the control problem for the platoon are presented. The DMPC algorithm is proposed in Section 3 including the formulation of the decoupled MPC optimization problem and the design of the terminal control law for each vehicle. Recursive feasibility, constraints satisfaction and closed-loop stability under the proposed algorithm are established in Section 4. Section 5 provides numerical simulations and Section 6 delivers concluding remarks and indicates further research endeavors.

Notation 1.1. *Given $\mathcal{A}, \mathcal{B} \subseteq \mathbb{R}^n$, the Minkowski set addition is defined by $\mathcal{A} \oplus \mathcal{B} = \{a + b \in \mathbb{R}^n \mid a \in \mathcal{A}, b \in \mathcal{B}\}$. Given sets $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$, $\Pi_{i=1}^{N} \mathcal{A}_i = \{a = [a_1^\top, a_2^\top, \dots, a_N^\top]^\top \mid a_i \in \mathcal{A}_i,\ i = 1, 2, \dots, N\}$. Given sets $\mathcal{A}$ and $\mathcal{B}$, $\mathcal{A} \setminus \mathcal{B} = \{c \in \mathcal{A} \mid c \notin \mathcal{B}\}$. Denote $\mathcal{T}_a^b = \{\tau \in \mathbb{R} \mid a \leq \tau \leq b\}$ and $\|x\|_Q^2 = x^\top Q x$. Matrix inequality $A \preceq B$ means that $A - B$ is a non-positive definite matrix. Denote $\mathbf{1}_N$ as $[1, 1, \dots, 1]^\top \in \mathbb{R}^N$, $\mathbf{0}_{1\times N}$ as $[0, 0, \dots, 0]^\top \in \mathbb{R}^N$, $\otimes$ as the Kronecker product, and $\mathrm{sgn}(x)$ as the sign function. Consider a differential discontinuous system $\dot{x} = f(x, t)$, where the map $f(\cdot, \cdot) : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ is measurable and essentially locally bounded. A function $x(t)$ is a Filippov solution of $\dot{x} = f(x, t)$ for $t \in \mathcal{T}_0^T$, if $x(t)$ is absolutely continuous and $\dot{x}(t) \in^{a.e.} \mathcal{K}[f(x, t)]$ for $t \in \mathcal{T}_0^T$, where a.e. stands for 'almost everywhere' and $\mathcal{K}[f(x, t)] = \bigcap_{\Delta > 0} \bigcap_{\mu(\bar{N})=0} \bar{co}(f(\mathcal{B}(z, \Delta) - \bar{N}), t)$, with $\bigcap_{\mu(\bar{N})=0}$ being the intersection over all sets $\bar{N}$ of Lebesgue measure zero, $\bar{co}(L)$ being the convex closure of set L, and $\mathcal{B}(z, \Delta)$ being the open ball of radius Δ centred at z. For more details about $\mathcal{K}[f(x, t)]$, see the non-smooth analysis in [55–57].*

2. Problem Description

In this section, the modelling of a platoon formed by heterogeneous vehicles is considered. The communication topology is first described. Then the longitudinal vehicle dynamics is modelled, after which the control objectives are formulated.

2.1. Communication Topology

Consider a platoon of $N + 1$ heterogeneous vehicles consisting of a leading vehicle and N following vehicles travelling on a flat road, as shown in Figure 1. Without loss of generality, the leading vehicle is indexed by 0 and

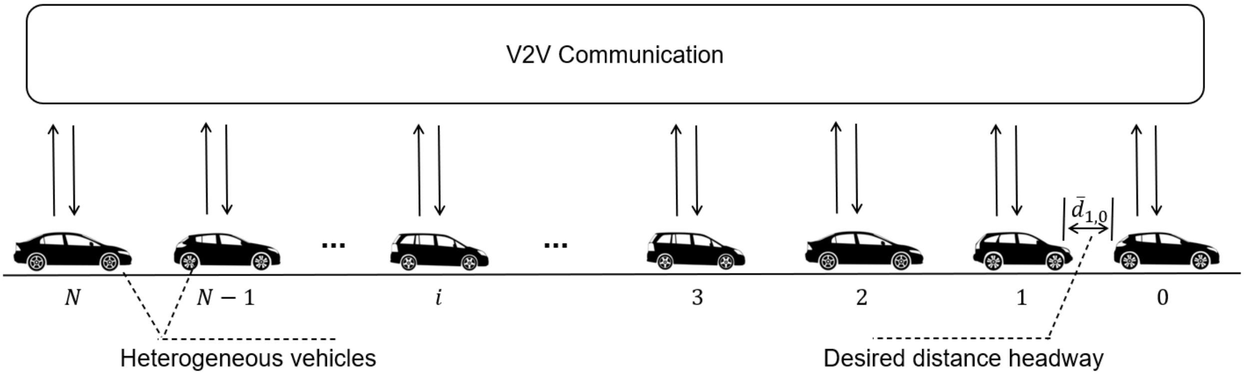

Figure 1: A platoon of heterogeneous vehicles.

the following vehicles are indexed by $1, 2, \dots, N$ in order. To characterize the inter-vehicular communication among the $N + 1$ vehicles, let us first introduce the following definitions. Consider a directed graph $\mathcal{G}^+ = \{\mathcal{V}^+, \mathcal{E}^+\}$, where $\mathcal{V}^+ = \{0, 1, \dots, N\}$ is the set of vehicles and $\mathcal{E}^+ = \mathcal{V}^+ \times \mathcal{V}^+$ is the set of edges between distinct vehicles. An edge of the graph $\mathcal{G}^+$ is described by $\varepsilon_{ij} = (i, j)$, $i \neq j$, $i, j \in \mathcal{V}^+$. The set of vehicles transmitting information to vehicle i in graph $\mathcal{G}^+$ is defined as $\mathcal{N}_i^+ = \{j \in \mathcal{V}^+ \setminus \{i\} \mid \varepsilon_{ji} \in \mathcal{E}^+\}$. Besides, the set of vehicles receiving information from vehicle 0 in graph $\mathcal{G}^+$ is defined as $\mathcal{O}_0 = \{i \in \mathcal{V}^+ \setminus \{0\} \mid 0 \in \mathcal{N}_i^+\}$, and the set of vehicles not communicating with vehicle 0 in graph $\mathcal{G}^+$ is defined as $\bar{\mathcal{O}}_0 = \{i \in \mathcal{V}^+ \setminus \{0\} \mid i \notin \mathcal{O}_0\}$. Denote $\mathcal{A}^+ = [a_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$ as the adjacency matrix associated with the graph $\mathcal{G}^+$, where the weight $a_{ij} > 0$ if and only if $\varepsilon_{ji} \in \mathcal{E}^+$, and $a_{ij} = 0$ otherwise. The in-degree matrix of the graph $\mathcal{G}^+$ is described by $\mathcal{D}^+ = \mathrm{diag}(\sum_{j=0}^{N} a_{0j}, \sum_{j=0}^{N} a_{1j}, \dots, \sum_{j=0}^{N} a_{Nj})$. The Laplacian matrix of the graph $\mathcal{G}^+$ is defined as $\mathcal{L}^+ = \mathcal{D}^+ - \mathcal{A}^+$. Similarly, the sub-graph among the N following vehicles is presented by an undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{1, 2, \dots, N\}$ and $\mathcal{E} = \mathcal{V} \times \mathcal{V}$. The set of vehicles transmitting information to vehicle i in graph $\mathcal{G}$ is defined as $\mathcal{N}_i = \{j \in \mathcal{V} \setminus \{i\} \mid \varepsilon_{ji} \in \mathcal{E}\}$. The following assumptions about the platoon's communication topology are then presented.

Assumption 2.1. *Graph $\mathcal{G}^+$ contains a directed spanning tree and the leading vehicle is the root.*

Assumption 2.2. *For each vehicle i, $i \in \mathcal{V}$, $i-1 \in \mathcal{N}_i^+$ and if $i \neq N$, $i+1 \in \mathcal{N}_i^+$.*

According to Assumptions 2.1 and 2.2, the inter-vehicular communication protocol follows 1) the leading vehicle does not receive any information from the following vehicles whereas it is allowed to send its planned state trajectory information to a subset of the following vehicles $j \in \mathcal{O}_0$, 2) the communication topology between the N following vehicles is undirected, and 3) each following vehicle is at least coupled with its adjacent vehicles, i.e., each following vehicle must be able to obtain information from its adjacent vehicles. This communication protocol is commonly used in vehicle platoon, such as the bidirectional topologies shown in Figure 2.

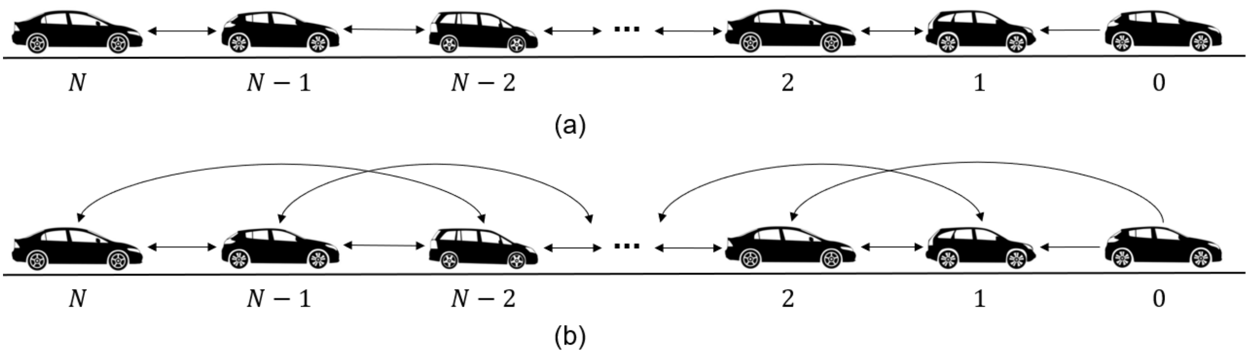

Figure 2: A platoon of heterogeneous vehicles with bidirectional communication topologies.

Since the leading vehicle receives no information from any following vehicle, the Laplacian matrix $\mathcal{L}^+$ can be partitioned as

$$\mathcal{L}^+ = \begin{bmatrix} 0 & \mathbf{0}_{1\times N} \\ \mathcal{L}_0 & \mathcal{L} \end{bmatrix}, \tag{1}$$

with $\mathcal{L}_0 \in \mathbb{R}^{N\times 1}$ and $\mathcal{L} \in \mathbb{R}^{N\times N}$. Matrix $\mathcal{L}$ in (1) is symmetric due to the undirected subgraph $\mathcal{G}$. Thus there exists an unitary matrix $U \in \mathbb{R}^{N\times N}$ such that $U^\top \mathcal{L} U = \Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \dots, \lambda_N)$, where

$$\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_N \tag{2}$$

are the eigenvalues of $\mathcal{L}$, and according to Assumption 2.1, $\lambda_i > 0, \forall i \in \mathcal{V}$.

2.2. Longitudinal Vehicle Dynamics and Constraints

The longitudinal dynamics of each vehicle i, $i \in \mathcal{V}^+$ in the platoon is modelled as

$$\dot{x}_i = A_i x_i + B_i u_i, \tag{3}$$

where $x_i = [p_i, v_i, a_i]^\top \in \mathbb{R}^3$ and $u_i \in \mathbb{R}$ are respectively the state and input of vehicle i. The variables p_i, v_i and a_i are respectively the position, velocity and acceleration of vehicle i. $A_i \in \mathbb{R}^{3\times 3}$ and $B_i \in \mathbb{R}^3$ are constant matrices given by

$$A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_i} \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau_i} \end{bmatrix}, \tag{4}$$

where the heterogeneity of the vehicle model lies in τ_i, which is the inertial time-lag of vehicle longitudinal dynamics. The inertial time-lag τ_0 is assumed to be known to all following vehicles $i \in \mathcal{V}$.

For each vehicle i, $i \in \mathcal{V}^+$, the state x_i and the input u_i are constrained due to the operational limitations given by

$$v_i \in \mathcal{X}_{v,i}, \tag{5}$$

$$a_i \in \mathcal{X}_{a,i}, \tag{6}$$

$$u_i \in \mathcal{U}_i, \tag{7}$$

where $\mathcal{X}_{v,i} = \{v_i \in \mathbb{R} \mid v_i^{\min} \le v_i \le v_i^{\max}\}$, $\mathcal{X}_{a,i} = \{a_i \in \mathbb{R} \mid a_i^{\min} \le a_i \le a_i^{\max}\}$ and $\mathcal{U}_i = \{u_i \in \mathbb{R} \mid u_i^{\min} \le u_i \le u_i^{\max}\}$ are denoted as the admissible sets of velocity, acceleration and control input of vehicle i, respectively. It is obvious that sets $\mathcal{X}_{a,i}$ and $\mathcal{U}_i$ should contain the origin as an interior point such that operations of acceleration and deceleration can be realized. For the sake of narrative convenience, denote $\mathcal{X}_i = \{x_i \mid v_i \in \mathcal{X}_{v,i}, a_i \in \mathcal{X}_{a,i}\}$ as the state constraint set of vehicle i. In this paper, the motion of the leading vehicle is required to be more tightly constrained in comparison to the following vehicles. It implies that $\mathcal{X}_0$ is required to be properly sized such that the state constraint set $\mathcal{X}_i$ can be designed satisfying $\mathcal{X}_0 \subset \mathcal{X}_i$ for all $i \in \mathcal{V}$.

In the previous papers [47–52], the leading vehicle is generally assumed to be driving at a constant velocity, which implies that the input of the leading vehicle, in terms of the model given in (3), is zero, which has limited applicability as it can not effectively deal with the potentially time-varying leading vehicle velocity (to cope with dynamic traffic conditions). This paper bridges the gap by considering a more challenging problem, where the input u_0 of the leading vehicle can be non-zero and time-varying, and is inaccessible to all following vehicles.

Assumption 2.3. *The time-varying input u_0 of the leading vehicle 0 is bounded satisfying $|u_0| \le \bar{c}$.*

From a practical point of view, vehicles in the platoon must follow the spatial formation constraints to maintain a close enough following distance (which can potentially mitigate network latency or packet loss) and also to avoid collisions. However, in the existing works, spatial formation constraints are seldom considered to streamline the theoretical analysis. In this paper, to guarantee the satisfaction of the spatial formation constraints, each vehicle i, $i \in \mathcal{V}^+$ is coupled with the adjacent vehicles by the following constraints

$$p_i - p_{i-1} - \bar{d}_{i,i-1} \in \mathcal{X}_{\Delta p,i,i-1},\ i \neq 0, \tag{8}$$

$$p_{i+1} - p_i - \bar{d}_{i+1,i} \in \mathcal{X}_{\Delta p,i+1,i},\ i \neq N, \tag{9}$$

where $\bar{d}_{i,i-1}$, $i \in \mathcal{V}$ is the desired constant position gap between vehicles i and $i-1$. The admissible set $\mathcal{X}_{\Delta p,i,i-1} = \{\Delta p_i \in \mathbb{R} \mid \Delta p_i^{\min} \le \Delta p_i \le \Delta p_i^{\max}\}$ forces vehicles i and $i-1$, $\forall i \in \mathcal{V}$ to maintain an appropriate following distance. As it can be noticed, sets $\mathcal{X}_{\Delta p,i,i-1}$, $\forall i \in \mathcal{V}$ should contain the origin as an interior point, i.e., $\Delta p_i^{\min} < 0 < \Delta p_i^{\max}$, such that the desired constant position gap $\bar{d}_{i,i-1}$, $\forall i \in \mathcal{V}$ can be achieved at steady state.

2.3. Control Objectives

This paper focuses on cooperative longitudinal platoon control of heterogeneous vehicles with a dynamic leader driven by a time-varying input $u_0(t)$. The control objective is to ensure that all the following vehicles can track the velocity and acceleration of the leading vehicle subject to constraints (5)-(9), while keeping a small constant inter-vehicular distance between any two consecutive vehicles to improve road capacity, i.e., for all $i \in \mathcal{V}$,

$$\lim_{t\to\infty} |p_i(t) - p_{i-1}(t) - \bar{d}_{i,i-1}| = 0, \tag{10a}$$

$$\lim_{t\to\infty} |v_i(t) - v_0(t)| = 0, \tag{10b}$$

$$\lim_{t\to\infty} |a_i(t) - a_0(t)| = 0. \tag{10c}$$

In particular, (10a) for each following vehicle i, $i \in \mathcal{V}$ is equivalent to

$$\lim_{t\to\infty} |p_i(t) - p_0(t) - \bar{d}_{i,0}| = 0, \tag{11}$$

with $\bar{d}_{i,0} = \sum_{j=1}^{i} \bar{d}_{j,j-1}$.

3. DMPC Algorithm for Heterogeneous Vehicles Platooning

In this section, a DMPC scheme is developed for the heterogeneous platoon control problem formulated in Section 2. First, the coupled state constraints (8)-(9) between any two consecutive vehicles are decoupled by introducing assumed trajectories, such that each following vehicle can solve a local optimization problem for platoon formation in parallel. After that, closed-loop stability can be ensured (proof is given in Section 4) by designing a terminal control law for each following vehicle $i \in \mathcal{V}$.

3.1. Local Optimization Problem

To achieve the control goal via DMPC, the local optimization problem for each following vehicle is first formulated. Let t_k, $k \in \mathbb{N}$, be the sampling time and $\delta > 0$ be the sampling interval, i.e., $t_{k+1} = t_k + \delta$. A local cost function at time t_k over the prediction horizon H can be defined for each vehicle $i \in \mathcal{V}$, as specified by

$$J_i(x_i, u_i; \bar{x}_i, \bar{x}_{j\in\mathcal{N}_i^+}, t_k) = \int_0^H l_i(x_i, u_i; \bar{x}_i, \bar{x}_{j\in\mathcal{N}_i^+}, t_k + \tau | t_k) d\tau. \tag{12}$$

According to the control task in (10), the stage cost function $l_i(x_i, u_i; \bar{x}_i, \bar{x}_{j\in\mathcal{N}_i^+}, t_k + \tau | t_k)$ is given by

$$\begin{aligned} & l_i(x_i, u_i; \bar{x}_i, \bar{x}_{j\in\mathcal{N}_i^+}, t_k + \tau | t_k) \\ &= \|x_i(t_k + \tau|t_k) - \bar{x}_i(t_k + \tau|t_k)\|_{F_i} + \sum_{j\in\mathcal{N}_i^+} \|x_i(t_k + \tau|t_k) - \bar{x}_j(t_k + \tau|t_k) - d_{i,j}\|_{E_i}, \end{aligned} \tag{13}$$

where $d_{i,j} = [\bar{d}_{i,j}, 0, 0]^\top$ with $\bar{d}_{i,j} = \sum_{s=j+1}^{i} \bar{d}_{s,s-1}, j \le i-1$, and $\bar{x}_i(t_k + \tau|t_k) = [\bar{p}_i(t_k + \tau|t_k), \bar{v}_i(t_k + \tau|t_k), \bar{a}_i(t_k + \tau|t_k)]^\top, \tau \in \mathcal{T}_0^H$ is the assumed state trajectory of vehicle i, $i \in \mathcal{V}$ at time t_k and will be constructed in (19). Matrix weights $F_i \in \mathbb{R}^{3\times3}$ and $E_i \in \mathbb{R}^{3\times3}$ are positive definite and are used to penalize respectively the deviation of the optimal state trajectory from its assumed trajectory and the inter-vehicular state deviations. Besides, to guarantee the stability of the platoon control, which will be discussed in Section 4, matrices E_i and F_i, $i \in \mathcal{V}$ must be selected satisfying

$$\sum_{j\in\mathcal{N}_i} E_j - F_i \preceq 0. \tag{14}$$

Now, for a given current state $x_i(t_k)$, the local finite-horizon optimization problem $\mathcal{P}_i(t_k)$ for each following vehicle i, $i \in \mathcal{V}$ at time t_k can be designed as

$$\min_{u_i} J_i(x_i, u_i; \bar{x}_i, \bar{x}_{j\in\mathcal{N}_i^+}, t_k) \tag{15a}$$

s.t. for all $\tau \in \mathcal{T}_0^H$,

$$\dot{x}_i(t_k + \tau|t_k) = A_i x_i(t_k + \tau|t_k) + B_i u_i(t_k + \tau|t_k), \tag{15b}$$
$$x_i(t_k|t_k) = x_i(t_k), \tag{15c}$$
$$x_i(t_k + \tau|t_k) \in \mathcal{X}_i, \tag{15d}$$
$$u_i(t_k + \tau|t_k) \in \mathcal{U}_i, \tag{15e}$$
$$p_i(t_k + \tau|t_k) - \bar{p}_i(t_k + \tau|t_k) \in \bar{\mathcal{X}}_{\Delta p,i,i-1}(t_k + \tau|t_k), \tag{15f}$$
$$\bar{p}_i(t_k + \tau|t_k) - p_i(t_k + \tau|t_k) \in \bar{\mathcal{X}}_{\Delta p,i+1,i}(t_k + \tau|t_k), \text{if } i \neq N, \tag{15g}$$
$$x_i(t_k + H|t_k) = \bar{x}_i(t_k + H|t_k), \tag{15h}$$

where

$$\bar{\mathcal{X}}_{\Delta p,i,i-1}(t_k + \tau|t_k) = \{\Delta p_i \mid 2\Delta p_i + \bar{p}_i(t_k + \tau|t_k) - \bar{p}_{i-1}(t_k + \tau|t_k) - \bar{d}_{i,i-1} \in \mathcal{X}_{\Delta p,i,i-1}\}. \tag{16}$$

Constraint (15b) is the prediction model of vehicle i, $i \in \mathcal{V}$, with the initial predicted state equal to the actual state at time t_k by constraint (15c). Constraint (15d) and (15e) are respectively the state and input constraints of vehicle

i, $i \in \mathcal{V}$ corresponding to constraints (5)-(7). Constraints (15f) and (15g) are imposed on the position trajectory $p_i(t_k + \tau|t_k), \tau \in \mathcal{T}_0^H$ to ensure the spatial formation constraints (8)-(9). In (16), a specific set $\bar{\mathcal{X}}_{\Delta p,i,i-1}(t_k + \tau|t_k)$, $i \in \mathcal{V}$ is designed for vehicles i and $i-1$ to respectively optimize their position states $p_i(t_k + \tau|t_k)$ and $p_{i-1}(t_k + \tau|t_k)$ such that the closed-loop constraint (8)-(9) can be satisfied. Since the assumed position trajectories $\bar{p}_{i-1}(t_k + \tau|t_k)$ and $\bar{p}_{i+1}(t_k + \tau|t_k)$ of adjacent vehicles $i-1$ and $i+1$ are taken as known in $\bar{\mathcal{X}}_{\Delta p,i,i-1}(t_k + \tau|t_k)$ and $\bar{\mathcal{X}}_{\Delta p,i+1,i}(t_k + \tau|t_k)$, constraints (15f) and (15g) are only related to the local optimization variables $p_i(t_k + \tau|t_k), \tau \in \mathcal{T}_0^H$, and thus are decoupled with the decisions of other vehicles. Terminal constraint (15h) is crucial for the guarantee of recursive feasibility and closed-loop stability, which will be elaborated in Section 4.

When problem $\mathcal{P}_i(t_k)$ is solved at time t_k, vehicle i, $i \in \mathcal{V}$ obtains its optimal input trajectory $u_i^*(t_k+\tau|t_k)$, $\tau \in \mathcal{T}_0^H$ and the corresponding optimal state trajectory $x_i^*(t_k + \tau|t_k)$, $\tau \in \mathcal{T}_0^H$. The control actions $u_i^*(t_k + \tau|t_k)$, $\tau \in \mathcal{T}_0^\delta$ are applied to vehicle i during the time interval $[t_k, t_{k+1}]$. At next time step t_{k+1}, each vehicle i, $i \in \mathcal{V}$ updates its assumed trajectories $\bar{u}_i(t_{k+1} + \tau|t_{k+1})$ and $\bar{x}_i(t_{k+1} + \tau|t_{k+1})$, $\tau \in \mathcal{T}_0^H$ for problem $\mathcal{P}_i(t_{k+1})$ by using the optimal solution at time t_k and send them to all vehicles j, $j \in \mathcal{N}_i$. The assumed input trajectory $\bar{u}_i(t_{k+1}+\tau|t_{k+1})$ is constructed, for $\tau \in \mathcal{T}_0^{H-\delta}$, by

$$\bar{u}_i(t_{k+1} + \tau|t_{k+1}) = u_i^*(t_k + \delta + \tau|t_k), \tag{17}$$

and for $\tau \in \mathcal{T}_{H-\delta}^H$

$$\bar{u}_i(t_{k+1} + \tau|t_{k+1}) = \kappa_i(\bar{x}_i(t_{k+1} + \tau|t_{k+1}), \bar{x}_{j\in\mathcal{N}_i^+}(t_{k+1} + \tau|t_{k+1})), \tag{18}$$

where $\kappa_i(\cdot,\cdot)$ is the terminal control law which will be designed in Section 3.2. The corresponding assumed state trajectory $\bar{x}_i(t_{k+1} + \tau|t_{k+1})$, $\tau \in \mathcal{T}_0^H$ is generated by the dynamics of vehicle i, $i \in \mathcal{V}$ in (3) under the control of $\bar{u}_i(t_{k+1} + \tau|t_{k+1})$, $\tau \in \mathcal{T}_0^H$, as specified by

$$\dot{\bar{x}}_i(t_{k+1} + \tau|t_{k+1}) = A_i\bar{x}_i(t_{k+1} + \tau|t_{k+1}) + B_i\bar{u}_i(t_{k+1} + \tau|t_{k+1}). \tag{19}$$

It is immediate to show that, for $\tau \in \mathcal{T}_0^{H-\delta}$,

$$\bar{x}_i(t_{k+1} + \tau|t_{k+1}) = x_i^*(t_k + \delta + \tau|t_k). \tag{20}$$

Remark 3.1. *Note that the local optimization problem $\mathcal{P}_i(t_k)$ in (15) is only designed for the following vehicles $i \in \mathcal{V}$. For the leading vehicle, its driving plan is determined independently without participating in the control decisions, and its planned state trajectory $x_0(t_k + \tau)$, $\tau \in \mathcal{T}_0^H$ is only transmitted to some following vehicles $i \in \mathcal{O}_0$ at each time step t_k in accordance to the communication topology. The information of the leading vehicle $x_0(t_k + \tau)$, $\tau \in \mathcal{T}_0^H$ will be utilized in the cost function (15a) and the coupled constraints (15f) of following vehicles i, $i \in \mathcal{O}_0$. For the sake of narrative convenience, in what follows we define $x_0(t_k + \tau) = x_0^*(t_k + \tau|t_k) = \bar{x}_0(t_k + \tau|t_k) = x_0(t_k + \tau|t_k)$.*

3.2. Guidelines on Terminal Control Law Design

As the foundation of theoretical analysis when using a terminal region approach in MPC [36–40], the key to ensure recursive feasibility and stability is that the terminal region is invariant under a terminal control law. To this end, we introduce an analogous proposition on the terminal control law $\kappa_i(\cdot,\cdot)$ in (18). Denote $d = [d_{1,0}^\top, d_{2,0}^\top, \dots, d_{N,0}^\top]^\top$, $x = [x_1^\top, x_2^\top, \dots, x_N^\top]^\top$, $u = [u_1^\top, u_2^\top, \dots, u_N^\top]^\top$, $\mathcal{X} = \prod_{i=1}^N \mathcal{X}_i$ and $\mathcal{U} = \prod_{i=1}^N \mathcal{U}_i$.

Proposition 3.1. *There exist a set $\mathcal{E}$ satisfying $\mathcal{E} \oplus (\mathbf{1}_N \otimes \mathcal{X}_0) \subseteq \mathcal{X}$ and a terminal control law $\kappa_i(x_i(t), x_{j\in\mathcal{N}_i^+}(t))$ for each following vehicle $i \in \mathcal{V}$ such that for all $x(t_k) - \mathbf{1}_N \otimes x_0(t_k) - d \in \mathcal{E}$, if each following vehicle $i \in \mathcal{V}$ is controlled by $u_i(t) = \kappa_i(x_i(t), x_{j\in\mathcal{N}_i^+}(t))$, then*

(i) $x(t) - \mathbf{1}_N \otimes x_0(t) - d \in \mathcal{E}$, $\forall t \geq t_k$;

(ii) $u(t) \in \mathcal{U}$, $\forall t \geq t_k$;

(iii) $p_i(t) - p_{i-1}(t) - \bar{d}_{i,i-1} \in \mathcal{X}_{\Delta p,i,i-1}$, $\forall i \in \mathcal{V}$, $\forall t \geq t_k$;

(iv) $\lim_{t\to\infty} x_i(t) - x_0(t) - d_{i,0} = 0$.

In Proposition 3.1, (i) implies the invariance of $\mathcal{E}$, (ii) and (iii) ensure the satisfaction of input constraints and spatial formation constraints inside the set $\mathcal{E}$, and (iv) the convergence to the target trajectory when applying the terminal control law $\kappa_i(\cdot,\cdot)$.

To give an exact definition of the terminal control law, the invariant set is commonly defined by a sublevel set of a positive-definite cost function. This cost function is related to the error between the actual state and the predefined set point or the reference signal of the leading vehicle [46–49]. However, this choice of invariant set might be not applicable since the set point or the reference signal is unknown to following vehicles $i \in \bar{\mathcal{O}}_0$ due to the communication restrictions. To handle the lacking of leading vehicle's information for vehicles $i \in \bar{\mathcal{O}}_0$, in the formulated DMPC problem in [51, 52], a terminal equality constraint instead of an invariant set is introduced and then a terminal control law for each following vehicle $i \in \mathcal{V}$ is designed by utilizing only state information from its connecting vehicles $j \in \mathcal{N}_i$. Nevertheless, in [51, 52], the DMPC scheme is designed for the homogeneous vehicle platoon with consideration of only input constraints, and the leading vehicle is assumed driving at a constant velocity which implies $u_0 = 0$. For the more practical case considered in this paper, the design method of terminal control law in [42]–[43] can not ensure the satisfaction of Proposition 3.1 if we consider the heterogeneity of vehicles in the platoon, state constraints (especially coupled state constraints), and the non-zero, time-varying and inaccessible input of the leading vehicle.

Inspired by [55] for multi-agent systems, a method to construct the invariant set $\mathcal{E}$ and the terminal control law $\kappa_i(x_i(t), x_{j\in\mathcal{N}_i^+}(t))$, $i \in \mathcal{V}$ satisfying Proposition 3.1 is proposed below. For each following vehicle $i \in \mathcal{V}$, choose the feedback control law $\kappa_i(x_i(t), x_{j\in\mathcal{N}_i^+}(t))$ as

$$\kappa_i(x_i(t), x_{j\in\mathcal{N}_i^+}(t)) = G_i(x_i(t) - d_{i,0}) + g_i r_i(t), \tag{21}$$

$$r_i(t) = c_1 K \sum_{j\in\mathcal{N}^+} a_{ij}(x_i(t) - x_j(t) - d_{i,j}) + c_2 \operatorname{sgn}\left(K \sum_{j\in\mathcal{N}^+} a_{ij}(x_i(t) - x_j(t) - d_{i,j})\right), \tag{22}$$

where $g_i = \frac{\tau_i}{\tau_0}$, $G_i = [0, 0, 1 - g_i]$, and parameters $c_1, c_2 \in \mathbb{R}$ and $K \in \mathbb{R}^{1\times 3}$ are chosen such that

$$c_1 \geq \frac{\rho}{2\lambda_1}, \tag{23}$$

$$c_2 \geq \bar{c}, \tag{24}$$

$$K = -R^{-1} B_0^\top P, \tag{25}$$

with $0 < \rho < 1$. In (25), matrix P is the solution of the modified algebraic Riccati equation:

$$A_0^\top P + P A_0 + Q - \rho P B_0 R^{-1} B_0^\top P = 0, \tag{26}$$

where positive-definite matrices $Q \in \mathbb{R}^{3\times 3}$ and $R \in \mathbb{R}$ are pre-specified.

The candidate invariant set $\mathcal{E}$ is selected as

$$\mathcal{E} = \{e \in \mathbb{R}^{3N} \mid e^\top(\mathcal{L} \otimes P)e \leq \epsilon\}, \tag{27}$$

with $\epsilon \geq 0$. Find the largest possible ϵ such that the inclusion $\mathcal{E} \oplus (\mathbf{1}_N \otimes \mathcal{X}_0) \subseteq \mathcal{X}$ is satisfied.

Below we will show that the designed feedback control law $\kappa_i(x_i(t), x_{j\in\mathcal{N}_i^+}(t))$ in (21) and the constructed invariant set $\mathcal{E}$ in (27) satisfy conditions (i)-(iv) in Proposition 3.1.

Denote $z_i(t) = x_i(t) - d_{i,0}$, $\forall i \in \mathcal{V}$. The dynamics of $z_i(t)$, $i \in \mathcal{V}$ by implementing $u_i(t) = \kappa_i(x_i(t), x_{j\in\mathcal{N}_i^+}(t))$ is given by

$$\dot{z}_i(t) = A_i x_i(t) + B_i u_i(t) = A_i(z_i(t) + d_{i,0}) + B_i(G_i z_i(t) + g_i r_i(t)) = A_0 z_i(t) + B_0 r_i(t). \tag{28}$$

By letting $z_0(t) = x_0(t)$ and $r_0(t) = u_0(t)$, it can be obtained that $z_0(t)$ also evolves with $\dot{z}_0(t) = A_0 z_0(t) + B_0 r_0(t)$. Furthermore, by defining $e_i(t) = z_i(t) - z_0(t)$, $i \in \mathcal{V}$, the dynamics of $e_i(t)$, $i \in \mathcal{V}$ is then

$$\dot{e}_i(t) = A_0 e_i(t) + c_1 B_0 K \left(\sum_{j=1}^{N} a_{ij}(e_i(t) - e_j(t)) + a_{i0} e_i(t)\right)$$

$$+ c_2 B_0 \operatorname{sgn}\left(K\left(\sum_{j=1}^{N} a_{ij}(e_i(t) - e_j(t)) + a_{i0}e_i(t)\right)\right) - B_0 r_0(t), \tag{29}$$

and these variables $e_i(t)$, $i \in \mathcal{V}$ can be written in a compact form $e(t) = [e_1^\top(t), e_2^\top(t), \dots, e_N^\top(t)]^\top$, whose dynamics are written as

$$\dot{e}(t) = (I_N \otimes A_0 + c_1 \mathcal{L} \otimes B_0 K)e(t) + c_2(I_N \otimes B_0) \operatorname{sgn}\left((\mathcal{L} \otimes K)e(t)\right) - (\mathbf{1}_N \otimes B_0)r_0(t). \tag{30}$$

Condition (iv) in Proposition 3.1 is then transformed into the asymptotic stability of dynamic system (30). Note that the system equation (30) is discontinuous. To analyze the stability of (30), we utilize differential inclusions and non-smooth analysis [55–57], and rewrite (30) in terms of differential inclusions as

$$\dot{e}(t) \in^{a.e.} \mathcal{K}[(I_N \otimes A_0 + c_1 \mathcal{L} \otimes B_0 K)e(t) + c_2(I_N \otimes B_0) \operatorname{sgn}\left((\mathcal{L} \otimes K)e(t)\right) - (\mathbf{1}_N \otimes B_0)r_0(t)]. \tag{31}$$

A Lyapunov function candidate for system (31) is constructed as

$$V(t) = e^\top(t)(\mathcal{L} \otimes P)e(t). \tag{32}$$

It follows from Assumption 2.1 that $\mathcal{L} \geq 0$, and therefore $V(t) \geq 0$. The set-valued Lie derivative of $V(t)$ [55–57] along the trajectory of (31) satisfies

$$\begin{aligned}\dot{\tilde{V}}(t) = \mathcal{K}[&e^\top(t)(\mathcal{L} \otimes (PA_0 + A_0^\top P) + 2c_1 \mathcal{L}^2 \otimes PB_0 K)e(t) + 2c_2 e^\top(t)(\mathcal{L} \otimes PB_0) \operatorname{sgn}\left((\mathcal{L} \otimes K)e(t)\right) \\ &- 2e^\top(t)(\mathcal{L}\mathbf{1}_N \otimes P^{-1} B_0)r_0(t)].\end{aligned} \tag{33}$$

By using (25) and the facts that $x^\top \operatorname{sgn}(x) = ||x||_1$ and $\mathcal{K}[f(x,t)] = \{f(x,t)\}$ if $f(x,t)$ is continuous [55–57], we can obtain that

$$\begin{aligned}\dot{\tilde{V}}(t) = &e^\top(t)(\mathcal{L} \otimes (PA_0 + A_0^\top P) - 2c_1 \mathcal{L}^2 \otimes PB_0 R^{-1} B_0^\top P)e(t) \\ &- 2c_2 R \|(\mathcal{L} \otimes R^{-1} B_0^\top P)e(t)\|_1 - 2Re^\top(t)(\mathcal{L}\mathbf{1}_N \otimes PB_0 R^{-1})r_0(t).\end{aligned} \tag{34}$$

By the Hölder's inequality and the definition of $\bar{c}$, it follows from (34) that

$$\begin{aligned}\dot{\tilde{V}}(t) \leq &e^\top(t)(\mathcal{L} \otimes (PA_0 + A_0^\top P) - 2c_1 \mathcal{L}^2 \otimes PB_0 R^{-1} B_0^\top P)e(t) \\ &- 2c_2 R \|(\mathcal{L} \otimes R^{-1} B_0^\top P)e(t)\|_1 + 2R\bar{c}\|(\mathcal{L} \otimes R^{-1} B_0^\top P)e(t)\|_1 \\ \leq &e^\top(t)(\mathcal{L} \otimes (PA_0 + A_0^\top P) - 2c_1 \mathcal{L}^2 \otimes PB_0 R^{-1} B_0^\top P)e(t),\end{aligned} \tag{35}$$

where (24) is utilized to obtain the second inequality. Denote $\bar{e}(t) = [\bar{e}_1^\top(t), \bar{e}_2^\top(t), \dots, \bar{e}_N^\top(t)]^\top = (U^\dagger \otimes I_N)e(t)$, and rewrite the inequality (35) in terms of $\bar{e}(t)$ as

$$\begin{aligned}\dot{\tilde{V}}(t) &\leq \bar{e}^\top(t)(\Lambda \otimes (PA_0 + A_0^\top P) - 2c_1 \Lambda^2 \otimes PB_0 R^{-1} B_0^\top P)\bar{e}(t) \\ &= \sum_{i=1}^{N} \lambda_i \bar{e}_i^\top(t)(PA_0 + A_0^\top P - 2c_1 \lambda_i PB_0 R^{-1} B_0^\top P)\bar{e}_i(t) \\ &\leq \sum_{i=1}^{N} \lambda_i \bar{e}_i^\top(t)(PA_0 + A_0^\top P - \rho PB_0 R^{-1} B_0^\top P)\bar{e}_i(t) \\ &= -\sum_{i=1}^{N} \lambda_i \bar{e}_i^\top(t) Q \bar{e}_i(t) \\ &\leq 0\end{aligned} \tag{36}$$

where the second inequality holds due to (2) and (23), and the last inequality holds due to the facts that $\lambda_i > 0$, $i \in \mathcal{V}$ and matrix Q is positive definite. This proves $e(t) \to 0$ as $t \to \infty$, and therefore Proposition 3.1 (iv) is satisfied.

Since the set $\mathcal{E}$ is defined as a sublevel set of the Lyapunov function $V(t)$ in (27) and $V(t)$ is proved to be a monotonically nonincreasing function in (36), it holds that for all $x(t_k) - \mathbf{1}_N \otimes x_0(t_k) - d \in \mathcal{E}$, we have $x(t) - \mathbf{1}_N \otimes x_0(t) - d \in \mathcal{E}$, $t \geq t_k$, i.e., Proposition 3.1 (i) holds. Proposition 3.1 (i) together with the fact that $p_i(t) - p_{i-1}(t) - \bar{d}_{i,i-1} = [1,0,0](e_i(t) - e_{i-1}(t))$ ensures the satisfaction of Proposition 3.1 (iii), if the parameter ϵ is tuned such that for all $[e_1^\top, e_2^\top, \ldots, e_N^\top]^\top \in \mathcal{E}$,

$$[1,0,0](e_i - e_{i-1}) \in \mathcal{X}_{\Delta p,i,i-1}. \tag{37}$$

Besides, by defining $\mathcal{E}_i = \{e_i \in \mathbb{R}^3 \mid [e_1^\top, e_2^\top, \ldots, e_N^\top]^\top \in \mathcal{E}\}$ and adjusting parameters c_1, c_2, K, ρ and ϵ such that

$$(1-g_i)\mathcal{X}_{a,i} \oplus g_i \left(c_1 K \left(\oplus_{j=0}^{N} a_{ij} \left(\mathcal{E}_i \oplus (-\mathcal{E}_j) \right) \right) \oplus \Omega_{\text{sgn}} \right) \subseteq \mathcal{U}_i, \tag{38}$$

with $\Omega_{\text{sgn}} = \{\phi \in \mathbb{R} \mid |\phi| \leq c_2\}$, it can be ensured that the feedback control laws $\kappa_i(x_i(t), x_{j\in\mathcal{N}_i^+}(t))$ in (21) satisfies Proposition 3.1 (ii).

To sum up, by following the guidelines, if the parameters c_1, c_2, K, ρ and ϵ are chosen satisfying (23)-(25), (37) and (38), then the designed terminal control law $\kappa_i(x_i(t), x_{j\in\mathcal{N}_i^+}(t))$ in (21) for each vehicle $i \in \mathcal{V}$ and the set $\mathcal{E}$ in (27) satisfy Proposition 3.1.

Remark 3.2. *The parameter λ_1 is usually a small positive number depending on the communication topology. By taking the bidirectional topology adopted in Section 5 as an example, the parameter $\lambda_1 = 0.0581$. Thus, in [55], the parameter c_1 has to be chosen large enough such that the required condition $c_1 \geq \frac{1}{\lambda_1}$ holds, thereby leading to a relatively large input in (21). As a result, the control law design method in [55] is only applicable for the case of a large input constraint set $\mathcal{U}_i$ for each following vehicle i, $i \in \mathcal{V}$, which might be extremely restrictive in practical applications. In this paper, the design of matrices K in (25) and P in (26) is modified compared to [55] (where $K = -B_0^\top P^{-1}$ and $A_0 P + P A_0^\top - 2B_0 B_0^\top < 0$), and then the selection of parameter c_1 in (23) can be relaxed by introducing a tuneable parameter ρ. As such, a suitable c_1 can be found (such that condition (23) and further Proposition 3.1 hold) even in the presence of a tight input constraint set $\mathcal{U}_i$ for each following vehicle $i \in \mathcal{V}$.*

Remark 3.3. *Note that the terminal control law (21) for vehicle i only depends on the states of vehicles $i \in \mathcal{N}_i^+$, whereas a "centralized" invariant set $\mathcal{E}$ is designed in (27) for the whole platooning system. This is because in typical cooperative control tasks for example the formation control problems, only the decreasing of the total error $e(t)$ (see (36)) can be ensured rather than the local errors $e_i(t)$. As a result, only a centralized set can be guaranteed to be invariant. To achieve a fully distributed control, we observe that the required centralized invariant set $\mathcal{E}$ can be ensured by local terminal equality as used in (15h). For each vehicle $i \in \mathcal{V}$ optimizing its local problem $\mathcal{P}_i(t_k)$ at time t_k, (15h) will force the terminal state $x_i(t_{k+H}|t_k)$ to be identical to the terminal state $\bar{x}_i(t_{k+H}|t_k)$ of the assumed trajectory. Combining this and Proposition 3.1 (i), if $x(t_{k+H}|t_k) - \mathbf{1}_N \otimes x_0(t_{k+H}|t_k) - d \in \mathcal{E}$ is valid, then $x(t_{k+1+H}|t_{k+1}) - \mathbf{1}_N \otimes x_0(t_{k+1+H}|t_{k+1}) - d \in \mathcal{E}$ for time t_{k+1} can be ensured. This property is important to prove the recursive feasibility of problem $\mathcal{P}_i(\cdot)$ for each vehicle $i \in \mathcal{V}$, which will be discussed further in Section 4.*

3.3. DMPC Algorithm Design

After designing the local MPC optimization problem (15) and the terminal control law (21) used to construct the assumed input trajectory in (18), the DMPC algorithm for a platoon of heterogeneous vehicles with unknown input u_0 of the leading vehicle and inter-vehicular spacing constraints (8)-(9) is now summarized in Algorithm 1.

At the beginning of the actual control process, all vehicles are tasked with their initialization. The initialization step is performed by solving an optimization problem (15a) once offline for a collection of admissible control process $\bar{x}_i(t_0+\tau|t_0)$ and $\bar{u}_i(t_0+\tau|t_0)$, $\tau \in \mathcal{T}_0^H$ satisfying constraints (3), (5)-(9) and $\bar{x}(t_0+H|t_0) - \mathbf{1}_N \otimes x_0(t_0+H|t_0) - d \in \mathcal{E}$. The initialization term is another research topic about distributed optimization. In previous works on DMPC, the initialization was either implemented by a centralized way or by a trial-and-error method [38] when it was performed in the distributed fashion.

4. Closed-loop Theoretic Properties

In this section, the closed-loop theoretic properties under the proposed distributed control framework in Algorithm 1 are analyzed. We start with the recursive feasibility of the MPC optimization problem before discussing closed-loop constraints satisfaction and stability of the closed-loop platoon control system. In particular, note that only the asymptotic stability of the platoon is considered in this paper and the string stability is left for future discussion.

Algorithm 1 DMPC for Heterogeneous Vehicle Platoon with Unknown Input of Leading Vehicle

Offline:

a) Choose prediction horizon H;

b) Determine matrices E_i, F_i, $i \in \mathcal{V}$ satisfying (14);

c) Construct invariant set $\mathcal{E}$ in (21) and terminal control law $\kappa_i(\cdot,\cdot), i \in \mathcal{V}$ in (27) according to the guidelines in Section 3.2;

d) Set $k = 0$ and the maximum operation time $\bar{k}$;

e) At the beginning time t_0, each vehicle $i \in \mathcal{V}^+$ measures its state $x_i(t_0)$, and finds an initial state trajectory $\bar{x}_i(\tau + t_0|t_0)$, and an initial input trajectory $\bar{u}_i(\tau + t_0|t_0), \tau \in \mathcal{T}_0^H$ satisfying dynamics (3), constraints (5)-(9) and $\bar{x}(t_0 + H|t_0) - \mathbf{1}_N \otimes x_0(t_0 + H|t_0) - d \in \mathcal{E}$, where $\bar{x} = [\bar{x}_1^\top, \bar{x}_2^\top, \ldots, \bar{x}_N^\top]^\top$.

Online: For each vehicle $i \in \mathcal{V}$:

while $k < \bar{k}$ **do**
    Measure the state $x_i(t_k)$
    Send $\bar{x}_i(\tau + t_k|t_k), \tau \in \mathcal{T}_0^H$ to vehicles $j \in \mathcal{N}_i$
    Receive $\bar{x}_j(\tau + t_k|t_k), \tau \in \mathcal{T}_0^H$ from vehicles $j \in \mathcal{N}_i^+$
    Solve $\mathcal{P}_i(t_k)$ for $x_i^*(t_k + \tau|t_k)$ and $u_i^*(t_k + \tau|t_k)$
    Apply input $u_i(t_k + \tau) = u_i^*(t_k + \tau|t_k), \tau \in \mathcal{T}_0^\delta$
    Construct $\bar{x}_i(t_{k+1} + \tau|t_{k+1})$ and $\bar{u}_i(t_{k+1} + \tau|t_{k+1}), \tau \in \mathcal{T}_0^H$, via (17)-(19)
    $k \leftarrow k + 1$
end while

Theorem 4.1. *(Recursive Feasibility) Suppose Assumptions 2.1 and 2.2 hold. If the initial input trajectory $\bar{u}_i(t_0+\tau|t_0)$, $\tau \in \mathcal{T}_0^H$ and the corresponding initial state trajectory $\bar{x}_i(t_0+\tau|t_0)$, $\tau \in \mathcal{T}_0^H$ at the beginning time t_0 satisfies dynamics (3) and constraints (5)-(9), and the terminal states of the initial state trajectory for all following vehicles satisfy*

$$\bar{x}(t_0 + H|t_0) - \mathbf{1}_N \otimes x_0(t_0 + H|t_0) - d \in \mathcal{E}, \tag{39}$$

then problem $\mathcal{P}_i(t_k)$ in Algorithm 1 is recursively feasible at all subsequent time steps t_k for all following vehicles i, $i \in \mathcal{V}$.

Proof: The proof of this theorem is broken down mainly into two parts:

1) prove that the initial input trajectory $\bar{u}_i(t_0 + \tau|t_0)$, $\tau \in \mathcal{T}_0^H$ and the corresponding initial state trajectory $\bar{x}_i(t_0 + \tau|t_0)$, $\tau \in \mathcal{T}_0^H$ satisfying dynamics (3) and constraints (5)-(9) is a feasible solution for problem $\mathcal{P}_i(t_0)$, $i \in \mathcal{V}$ at the beginning time t_0;

2) prove that the assumed input trajectory $\bar{u}_i(t_{k+1} + \tau|t_{k+1})$, $\tau \in \mathcal{T}_0^H$ together with the assumed state trajectory $\bar{x}_i(t_{k+1} + \tau|t_{k+1})$, $\tau \in \mathcal{T}_0^H$ constructed by (17)-(19) is a feasible solution for problem $\mathcal{P}_i(t_{k+1})$, $i \in \mathcal{V}$ at any time t_{k+1}.

Step 1): It is straightforward to see that constraints (15b)-(15e) are satisfied with the initial input trajectory $\bar{u}_i(t_0 + \tau|t_0)$, $\tau \in \mathcal{T}_0^H$ and the initial state trajectory $\bar{x}_i(t_0 + \tau|t_0)$, $\tau \in \mathcal{T}_0^H$ due to the satisfaction of dynamics (3) and constraints (5)-(7). Since the initial state trajectory $\bar{x}_i(t_0 + \tau|t_0)$, $\tau \in \mathcal{T}_0^H$, $i \in \mathcal{V}$ satisfies constraints (8)-(9), we have, for any $i \in \mathcal{V}, \tau \in \mathcal{T}_0^H$,

$$\bar{p}_i(t_0 + \tau|t_0) - \bar{p}_{i-1}(t_0 + \tau|t_0) - \bar{d}_{i,i-1} \in \mathcal{X}_{\Delta p,i,i-1}, \tag{40}$$

which implies

$$0 \in \bar{\mathcal{X}}_{\Delta p,i,i-1}(t_0 + \tau|t_0),\ \tau \in \mathcal{T}_0^H, \tag{41}$$

and further constraint (15f) is satisfied if $p_i(t_0 + \tau|t_0)$ is replaced by $\bar{p}_i(t_0 + \tau|t_0)$, $\forall \tau \in \mathcal{T}_0^H$. Similarly, constraints (15g) and (15h) are satisfied. Hence, $\bar{u}_i(t_0 + \tau|t_0)$, $\tau \in \mathcal{T}_0^H$ is a feasible solution for problem $\mathcal{P}_i(t_0)$, $i \in \mathcal{V}$ at time t_0.

Step 2): Firstly, suppose problem $\mathcal{P}_i(t_k)$ in Algorithm 1 is feasible at some time t_k for all following vehicles i, $i \in \mathcal{V}$ and

$$0 \in \bar{\mathcal{X}}_{\Delta p,i,i-1}(t_k + \tau|t_k),\ \tau \in \mathcal{T}_0^H. \tag{42}$$

This implies that there exists the optimal solution $x_i^*(t_k+\tau|t_k)$, $\tau \in \mathcal{T}_0^H$ and $u_i^*(t_k+\tau|t_k)$, $\tau \in \mathcal{T}_0^H$ for problem $\mathcal{P}_i(t_k)$, $i \in \mathcal{V}$ at time t_k, and based on the definition of $\bar{\mathcal{X}}_{\Delta p,i,i-1}(t_k+\tau|t_k)$ in (16), it holds that

$$\bar{p}_i(t_k+\tau|t_k) - \bar{p}_{i-1}(t_k+\tau|t_k) - d_{i,i-1} \in \mathcal{X}_{\Delta p,i,i-1}. \tag{43}$$

Using the definitions of the assumed trajectories (17)-(19) and the fact that $\bar{x}_i(t_{k+1}|t_{k+1}) = x_i^*(t_k+\delta|t_k) = x_i(t_{k+1})$ under the control of Algorithm 1, we obtain $\bar{x}_i(t_{k+1}+\tau|t_{k+1}) = x_i^*(t_k+\tau+\delta|t_k)$, $\tau \in \mathcal{T}_0^{H-\delta}$ and $\bar{u}_i(t_{k+1}+\tau|t_{k+1}) = u_i^*(t_k+\tau+\delta|t_k)$, $\tau \in \mathcal{T}_0^{H-\delta}$ for all $i \in \mathcal{V}$. Since $x_i^*(t_k+\tau|t_k)$, $\tau \in \mathcal{T}_0^H$ and $u_i^*(t_k+\tau|t_k)$, $\tau \in \mathcal{T}_0^H$ for each following vehicle i, $i \in \mathcal{V}$ are the solution of problem $\mathcal{P}_i(t_k)$ satisfying constraints (15b)-(15e) at time t_k, it can be inferred that the first part of the constructed assumed state trajectory $\bar{x}_i(t_{k+1}+\tau|t_{k+1})$, $\tau \in \mathcal{T}_0^{H-\delta}$ and input trajectory $\bar{u}_i(t_{k+1}+\tau|t_{k+1})$, $\tau \in \mathcal{T}_0^{H-\delta}$ satisfy constraints (15b)-(15e) at time t_{k+1}.

Besides, since $x_i^*(t_k+\tau|t_k)$, $\tau \in \mathcal{T}_0^H$ and $u_i^*(t_k+\tau|t_k)$, $\tau \in \mathcal{T}_0^H$ for each following vehicle $i \in \mathcal{V}$ satisfy constraints (15f)-(15g) at time t_k, it holds, for $i \in \mathcal{V}, i \neq 1$, that

$$p_i^*(t_k+\tau|t_k) - \bar{p}_i(t_k+\tau|t_k) \in \bar{\mathcal{X}}_{\Delta p,i,i-1}(t_k+\tau|t_k), \tag{44}$$
$$\bar{p}_{i-1}(t_k+\tau|t_k) - p_{i-1}^*(t_k+\tau|t_k) \in \bar{\mathcal{X}}_{\Delta p,i,i-1}(t_k+\tau|t_k). \tag{45}$$

Combining (43) with (44)-(45) yields, for all $i \in \mathcal{V}, i \neq 1$, $\tau \in \mathcal{T}_0^H$

$$\begin{aligned}
&p_i^*(t_k+\tau|t_k) - p_{i-1}^*(t_k+\tau|t_k) - d_{i,i-1} \\
&= (p_i^*(t_k+\tau|t_k) - \bar{p}_i(t_k+\tau|t_k)) + (\bar{p}_{i-1}(t_k+\tau|t_k) - p_{i-1}^*(t_k+\tau|t_k)) \\
&\quad + (\bar{p}_i(t_k+\tau|t_k) - \bar{p}_{i-1}(t_k+\tau|t_k) - d_{i,i-1}) \\
&\in 2\bar{\mathcal{X}}_{\Delta p,i,i-1}(t_k+\tau|t_k) \oplus \{\bar{p}_i(t_k+\tau|t_k) - \bar{p}_{i-1}(t_k+\tau|t_k) - d_{i,i-1}\} \\
&\subseteq \mathcal{X}_{\Delta p,i,i-1}.
\end{aligned} \tag{46}$$

Similarly, the same conclusion can be concluded for $i = 1$. Thus, for all $i \in \mathcal{V}$, $\tau \in \mathcal{T}_0^{H-\delta}$,

$$\bar{p}_i(t_{k+1}+\tau|t_{k+1}) - p_{i-1}^*(t_{k+1}+\tau|t_{k+1}) - d_{i,i-1} = p_i^*(t_k+\delta+\tau|t_k) - p_{i-1}^*(t_k+\delta+\tau|t_k) - d_{i,i-1} \in \mathcal{X}_{\Delta p,i,i-1}, \tag{47}$$

which implies for $\tau \in \mathcal{T}_0^{H-\delta}$,

$$0 \in \bar{\mathcal{X}}_{\Delta p,i,i-1}(t_{k+1}+\tau|t_{k+1}), \tag{48}$$

and further the satisfaction of constraints (15f)-(15g).

Moreover, suppose the terminal states of the assumed state trajectory for all following vehicles at time t_k satisfy

$$\bar{x}(t_k+H|t_k) - \mathbf{1}_N \otimes x_0(t_k+H|t_k) - d \in \mathcal{E}. \tag{49}$$

By (15h), we obtain $x_i^*(t_k+H|t_k) = \bar{x}_i(t_k+H|t_k)$ and therefore

$$\bar{x}(t_{k+1}+H-\delta|t_{k+1}) = x^*(t_k+H|t_k) = \bar{x}(t_k+H|t_k). \tag{50}$$

where $x^* = [x_1^{*\top}, x_2^{*\top}, \dots, x_N^{*\top}]^\top$. Utilizing (15h), the construction method of the assumed trajectory (17)-(19) and Proposition 3.1 (i), it can be obtained that constraint (15b) is fulfilled and the rest part of the constructed assumed state trajectory $\bar{x}_i(t_{k+1}+\tau|t_{k+1})$ satisfies constraints (15d) for $\tau \in \mathcal{T}_{H-\delta}^H$, and

$$\bar{x}(t_{k+1}+H|t_{k+1}) - \mathbf{1}_N \otimes x_0(t_{k+1}+H|t_{k+1}) - d \in \mathcal{E}. \tag{51}$$

Similarly, it can be obtained that the rest part of the corresponding input trajectory $\bar{u}_i(t_{k+1}+\tau|t_{k+1})$ satisfies constraints (15e) for $\tau \in \mathcal{T}_{H-\delta}^{H}$ via Proposition 3.1 (ii).

Based on Proposition 3.1 (iii), the position trajectories in the assumed state trajectories $\bar{p}_i(t_{k+1}+\tau|t_{k+1})$, $\tau \in \mathcal{T}_{H-\delta}^{H}$ for all following vehicles satisfy, for any $i \in \mathcal{V}$, $\tau \in \mathcal{T}_{H-\delta}^{H}$

$$\bar{p}_i(t_{k+1}+\tau|t_{k+1}) - \bar{p}_{i-1}(t_{k+1}+\tau|t_{k+1}) - \bar{d}_{i,i-1} \in \mathcal{X}_{\Delta p,i,i-1}, \tag{52}$$

which implies for $\tau \in \mathcal{T}_{H-\delta}^{H}$, (48) holds, and further constraint (15f) is valid if $p_i(t_{k+1}+\tau|t_{k+1})$ is replaced by $\bar{p}_i(t_{k+1}+\tau|t_{k+1})$ for $\tau \in \mathcal{T}_{H-\delta}^{H}$. Constraint (15g) and (15h) are satisfied in the similar way.

Thus, the constructed assumed input trajectory $\bar{u}_i(t_{k+1}+\tau|t_{k+1})$, $\tau \in \mathcal{T}_0^H$ and assumed state trajectory $\bar{x}_i(t_{k+1}+\tau|t_{k+1})$, $\tau \in \mathcal{T}_0^H$ satisfy constraints (15b)-(15h) at time t_{k+1}, which means it is a feasible solution for problem $\mathcal{P}_i(t_{k+1})$, and also satisfy (48) for $\tau \in \mathcal{T}_0^H$ and (51).

Hence, it can be concluded by induction that problem $\mathcal{P}_i(t_k)$ in Algorithm 1 is recursively feasible and (42) is satisfied at all subsequent time t_k for all following vehicles $i \in \mathcal{V}$. ■

Then the closed-loop constraints satisfaction in which all constraints (5)-(9) are satisfied under the closed-loop operation with Algorithm 1 is shown in the following theorem.

Theorem 4.2. *(Closed-loop Constraint Satisfaction) Provided that the conditions in Theorem 4.1 hold, then the closed-loop trajectories $x_i(t)$ and $u_i(t)$ for each following vehicle $i \in \mathcal{V}$ under Algorithm 1 satisfy the constraints (5)-(9) over all time.*

Proof: From the online constraints (15d)-(15e) and the recursive feasibility established in Theorem 4.1, the closed-loop constraints (5)-(7) hold.

In the proof of Theorem 4.1, (42) is proved valid at all time steps t_k for all following vehicles $i \in \mathcal{V}$. Similar to the derivation process of (46), it can be obtained that for all $i \in \mathcal{V}$, $\tau \in \mathcal{T}_0^{\delta}$,

$$p_i(t_k+\tau) - p_{i-1}(t_k+\tau) - d_{i,i-1} = p_i^*(t_k+\tau|t_k) - p_{i-1}^*(t_k+\tau|t_k) - d_{i,i-1} \in \mathcal{X}_{\Delta p,i,i-1}, \tag{53}$$

which proves the satisfaction of spatial formation constraints (8)-(9) in the closed-loop operation. ■

Finally, the closed-loop stability is analyzed in the rest of this section by using the Lyapunov method showing that the sum over the optimal value functions J_i will be decreasing as time increases.

Theorem 4.3. *(Asymptotic Stability) Provided that the conditions in Theorem 4.1 hold, if $\Pi_i = \sum_{j\in\mathcal{N}_i} E_j - F_i \preceq 0$, $i \in \mathcal{V}$, then the control objective (10) will be achieved for all follower vehicles $i \in \mathcal{V}$ under Algorithm 1.*

Proof: Firstly, define $\Delta J_i(t_k, t_{k+1})$ as the difference between the optimal value functions of the following vehicle $i \in \mathcal{V}$ at time t_{k+1} and at time t_k:

$$\Delta J_i(t_k, t_{k+1}) = J_i(x_i^*, u_i^*; \bar{x}_i, \bar{x}_{j\in\mathcal{N}_i^+}, t_{k+1}) - J_i(x_i^*, u_i^*; \bar{x}_i, \bar{x}_{j\in\mathcal{N}_i^+}, t_k). \tag{54}$$

Due to the optimality of $x_i^*(t_{k+1}+\tau|t_{k+1})$, $\tau \in \mathcal{T}_0^H$ and $u_i^*(t_{k+1}+\tau|t_{k+1})$, $\tau \in \mathcal{T}_0^H$, we obtain

$$\Delta J_i(t_k, t_{k+1}) \le J_i(\bar{x}_i, \bar{u}_i; \bar{x}_i, \bar{x}_{j\in\mathcal{N}_i^+}, t_{k+1}) - J_i(x_i^*, u_i^*; \bar{x}_i, \bar{x}_{j\in\mathcal{N}_i^+}, t_k). \tag{55}$$

From the definition of cost function $J_i(x_i, u_i; \bar{x}_i, \bar{x}_{j\in\mathcal{N}_i^+}, t)$ in (12), it can be derived that

$$\begin{aligned}
&J_i(\bar{x}_i, \bar{u}_i; \bar{x}_i, \bar{x}_{j\in\mathcal{N}_i^+}, t_{k+1}) - J_i(x_i^*, u_i^*; \bar{x}_i, \bar{x}_{j\in\mathcal{N}_i^+}, t_k) \\
&= \int_0^H \Big(\sum_{j\in\mathcal{N}_i^+} \|\bar{x}_i(t_{k+1}+\tau|t_{k+1}) - \bar{x}_j(t_{k+1}+\tau|t_{k+1}) - d_{i,j}\|_{E_i} + \|\bar{x}_i(t_{k+1}+\tau|t_{k+1}) - \bar{x}_i(t_{k+1}+\tau|t_{k+1})\|_{F_i} \Big) d\tau \\
&\quad - \int_0^H \Big(\sum_{j\in\mathcal{N}_i^+} \|x_i^*(t_k+\tau|t_k) - \bar{x}_j(t_k+\tau|t_k) - d_{i,j}\|_{E_i} + \|x_i^*(t_k+\tau|t_k) - \bar{x}_i(t_k+\tau|t_k)\|_{F_i} \Big) d\tau.
\end{aligned} \tag{56}$$

By defining

$$\eta_i(t_k) = \int_0^{\delta} \Big(\sum_{j\in\mathcal{N}_i^+} \|x_i^*(t_k+\tau|t_k) - \bar{x}_j(t_k+\tau|t_k) - d_{i,j}\|_{E_i} + \|x_i^*(t_k+\tau|t_k) - \bar{x}_i(t_k+\tau|t_k)\|_{F_i} \Big) d\tau, \tag{57}$$

$$\zeta_i(t_k) = \int_H^{\delta+H} \sum_{j\in\mathcal{N}_i^+} \|\bar{x}_i(t_k+\tau|t_{k+1}) - \bar{x}_j(t_k+\tau|t_{k+1}) - d_{i,j}\|_{E_i} d\tau, \tag{58}$$

and based on (56), we obtain

$$\begin{aligned} &\Delta J_i(t_k, t_{k+1}) \\ \le& -\eta_i(t_k) + \zeta_i(t_k) + \int_\delta^H \sum_{j\in\mathcal{N}_i^+} \|\bar{x}_i(t_k+\tau|t_{k+1}) - \bar{x}_j(t_k+\tau|t_{k+1}) - d_{i,j}\|_{E_i} d\tau \\ &- \int_\delta^H \sum_{j\in\mathcal{N}_i^+} \|x_i^*(t_k+\tau|t_k) - \bar{x}_j(t_k+\tau|t_k) - d_{i,j}\|_{E_i} d\tau - \int_\delta^H \|x_i^*(t_k+\tau|t_k) - \bar{x}_i(t_k+\tau|t_k)\|_{F_i} d\tau. \end{aligned} \tag{59}$$

In virtue of the triangle inequality and (20), it yields that

$$\begin{aligned} &\Delta J_i(t_k, t_{k+1}) \\ \le& -\eta_i(t_k) + \zeta_i(t_k) + \int_\delta^H \Big(\sum_{j\in\mathcal{N}_i} \|x_j^*(t_k+\tau|t_k) - \bar{x}_j(t_k+\tau|t_k)\|_{E_i} - \|x_i^*(t_k+\tau|t_k) - \bar{x}_i(t_k+\tau|t_k)\|_{F_i} \Big) d\tau. \end{aligned} \tag{60}$$

Define the sum of all local optimal value functions $J_i(x_i^*, u_i^*; \bar{x}_i, \bar{x}_{j\in\mathcal{N}_i^+}, t_k)$ at time t_k over $i \in \mathcal{V}$ as a Lyapunov function candidate

$$J(x^*, u^*; \bar{x}, t_k) = \sum_{i=1}^{N} J_i(x_i^*, u_i^*; \bar{x}_i, \bar{x}_{j\in\mathcal{N}_i^+}, t_k). \tag{61}$$

It is obvious from the definition of the cost function $J_i(x_i^*, u_i^*; \bar{x}_i, \bar{x}_{j\in\mathcal{N}_i^+}, t_k)$ and the positive definiteness of matrices E_i, F_i, $i \in \mathcal{V}$ that

$$J(x^*, u^*; \bar{x}, t_k) \ge 0. \tag{62}$$

Define $\Delta J(t_k, t_{k+1})$ as the difference between the Lyapunov function candidates at two consecutive time steps t_k and t_{k+1}

$$\Delta J(t_k, t_{k+1}) = J(x^*, u^*; \bar{x}, t_{k+1}) - J(x^*, u^*; \bar{x}, t_k). \tag{63}$$

By summing (60) from $i = 0$ to $i = N$, it can be derived based on (55) and (60) that

$$\begin{aligned} \Delta J(t_k, t_{k+1}) \le& -\sum_{i=1}^{N} \eta_i(t_k) + \sum_{i=1}^{N} \zeta_i(t_k) \\ &+ \sum_{i=1}^{N} \int_\delta^H \Big(\sum_{j\in\mathcal{N}_i} \|x_j^*(t_k+\tau|t_k) - \bar{x}_j(t_k+\tau|t_k)\|_{E_i} - \|x_i^*(t_k+\tau|t_k) - \bar{x}_i(t_k+\tau|t_k)\|_{F_i} \Big) d\tau \\ =& -\sum_{i=1}^{N} \eta_i(t) + \sum_{i=1}^{N} \zeta_i(t) \\ &+ \int_\delta^H \sum_{i=1}^{N} \Big(\sum_{j\in\mathcal{N}_i} \|x_i^*(t_k+\tau|t_k) - \bar{x}_i(t_k+\tau|t_k)\|_{E_j} - \|x_i^*(t_k+\tau|t_k) - \bar{x}_i(t_k+\tau|t_k)\|_{F_i} \Big) d\tau \end{aligned}$$

$$= \alpha(t_k) + \beta(t_k) \tag{64}$$

where

$$\alpha(t_k) = -\sum_{i=1}^{N} \eta_i(t_k) + \int_{\delta}^{H} \sum_{i=1}^{N} \|x_i^*(t_k+\tau|t_k) - \bar{x}_i(t_k+\tau|t_k)\|_{\Pi_i} \tag{65}$$

$$\beta(t_k) = \sum_{i=1}^{N} \zeta_i(t_k). \tag{66}$$

From the positive definiteness of matrices E_i, F_i, and the semi-negative definiteness of matrix Π_i, for all $i \in \mathcal{V}$, it yields that $\alpha(t_k) \leq 0$ and $\beta(t_k) \geq 0$. If one have $\alpha(t_k) = 0$, then $x_i^*(t_k+\tau|t_k) = \bar{x}_i(t_k+\tau|t_k)$ and $x_i^*(t_k+\tau|t_k) = \bar{x}_j(t_k+\tau|t_k) + d_{i,j}$, $j \in \mathcal{N}_i^+$, which implies each following vehicle $i \in \mathcal{V}$ tracks the velocity and acceleration of the leading vehicle, and keeps a constant inter-vehicular distance $\bar{d}_{i,j}$ between other vehicles $j \in \mathcal{N}_i^+$. In other words, the control target (10) of the platoon is achieved. Besides, based on Proposition 3.1 (iv), we have for $\tau \in \mathcal{T}_H^{\delta+H}$,

$$\begin{aligned}
&\lim_{t_k\to\infty} \bar{x}_i(t_k+\tau|t_{k+1}) - \bar{x}_j(t_k+\tau|t_{k+1}) - d_{i,j} \\
&= \lim_{t_k\to\infty} \left(\bar{x}_i(t_k+\tau|t_{k+1}) - x_0(t_k+\tau|t_{k+1}) - d_{i,0}\right) - \left(\bar{x}_j(t_k+\tau|t_{k+1}) - x_0(t_k+\tau|t_{k+1}) - d_{j,0}\right) \\
&= 0,
\end{aligned} \tag{67}$$

and therefore $\beta(t_k) \to 0$ as $t_k \to \infty$.

Consequently, we are unable to determine whether the difference $\Delta J(t_k, t_{k+1})$ is positive or negative in a period of time when applying Algorithm 1. However, the closed-loop constraints (5)-(9) for each following vehicle $i, i \in \mathcal{V}$ can be ensured well during this time period. Moreover, note that $\beta(t_k)$ will gradually approach 0 as time goes on, and $\alpha(t_k)$ will keep negative until the control objective (10) is achieved, which in turn implies that when t_k is large enough we must have

$$\Delta J(t_k, t_{k+1}) \leq 0, \tag{68}$$

and further

$$J(x^*, u^*; \bar{x}, t_{k+1}) \leq J(x^*, u^*; \bar{x}, t_k). \tag{69}$$

Due to the positive-definiteness of the Lyapunov function candidate $J(x^*, u^*; \bar{x}, t_k)$ (see (62)), we have $J(x^*, u^*; \bar{x}, t_k) \to 0$ as $t_k \to \infty$. By the definition of stage cost function in (13) and $x_i^*(t_k|t_k) = x_i(t_k)$, it holds, for any $i \in \mathcal{V}$, that

$$\lim_{t_k\to\infty} ||x_i(t_k) - \bar{x}_i(t_k|t_k)|| = 0. \tag{70}$$

$$\lim_{t_k\to\infty} ||x_i(t_k) - \bar{x}_j(t_k|t_k) - d_{i,j}|| = 0, \ \forall j \in \mathcal{N}_i^+. \tag{71}$$

Considering that $i-1 \in \mathcal{N}_i^+$ in Assumption 2.2 leads to

$$\lim_{t_k\to\infty} ||x_i(t_k) - x_{i-1}(t_k) - d_{i,i-1}|| \overset{(70)}{=} \lim_{t_k\to\infty} ||x_i(t_k) - \bar{x}_{i-1}(t_k|t_k) - d_{i,i-1}|| \overset{(71)}{=} 0. \tag{72}$$

Therefore the relation (72) suffices to conclude the control objective (10) will be achieved, i.e., each following vehicle $i \in \mathcal{V}$ will track the velocity and acceleration of the leading vehicle, while keeping a small constant inter-vehicular distance between any two consecutive vehicles. ■

5. Numerical Examples

To illustrate the efficacy of Algorithm 1, numerical simulations are implemented on a heterogeneous vehicle platoon including a leading vehicle with non-zero and time-varying input and 6 following vehicles. The heterogeneity of the

Table 1
Heterogeneous engine time constants in the platoon

Index	0	1	2	3	4	5	6
τ_i	0.51	0.75	0.78	0.70	0.73	0.72	0.62

platooning vehicles is characterized by heterogeneous engine time constants $\tau_i, i \in \mathcal{V}^+$, which are listed in TABLE I [50]. The simplest and most basic bidirectional topology is considered as shown in Figure 2 (a), where each following vehicle only communicates with the adjacent vehicles, i.e.,

$$\mathcal{N}_i^+ = \begin{cases} \{i-1, i+1\}, \text{ if } i \neq N, \\ \{i-1\}, \text{ if } i = N, \end{cases} \tag{73}$$

and the leading vehicle only transmits its planned state trajectory $x_0(t_k + \tau|t_k)$, $\tau \in \mathcal{T}_0^H$ to following vehicle 1, i.e.,

$$\mathcal{O}_0 = \{1\}. \tag{74}$$

The discretization step of dynamic system (3) and the sampling interval δ of the DMPC are respectively set as 0.01s and 0.1s. The operational constraints (5)-(7) are defined by

$$\begin{aligned}
&\mathcal{X}_{v,i} = \{v_i \mid 0 \leq v_i \leq 32\}, \ i \in \mathcal{V}, \\
&\mathcal{X}_{v,0} = \{v_0 \mid 2 \leq v_0 \leq 30\}, \\
&\mathcal{X}_{a,i} = \{a_i \mid -6 \leq a_i \leq 6\}, \ i \in \mathcal{V}, \\
&\mathcal{X}_{a,0} = \{a_0 \mid -3 \leq a_0 \leq 3\}, \\
&\mathcal{U}_i = \{u_i \mid -5 \leq u_i \leq 5\}, \ i \in \mathcal{V}, \\
&\mathcal{U}_0 = \{u_0 \mid -2 \leq u_0 \leq 2\},
\end{aligned}$$

and the spatial formation constraints (8)-(9) are considered as

$$\mathcal{X}_{\Delta p,i,i-1} = \{\Delta p_i \mid -4 \leq \Delta p_i \leq 4\}, \ i \in \mathcal{V}.$$

The desired distance headway is given by $\bar{d}_{i,i-1} = 5$m for all $i \in \mathcal{V}$.

For this example, the weights E_i and F_i in (14) are chosen, for $i \in \mathcal{V}$, as

$$E_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, F_i = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}. \tag{75}$$

According to the guidelines in Section 3.2, by setting $Q = \text{diag}\{2, 2, 2\}$ and $R = 10$, the parameters c_1, c_2, K, ρ and ϵ satisfying (23)-(25),(37) and (38) are chosen as $c_1 = 1.3765$, $c_2 = 2$, $\rho = 0.16$, $\epsilon = 0.9$,

$$\begin{aligned}
P &= \begin{bmatrix} 7.9555 & 14.8226 & 5.7010 \\ 14.8226 & 53.2600 & 22.6781 \\ 5.7010 & 22.6781 & 10.3801 \end{bmatrix}, \\
K &= \begin{bmatrix} -1.1178 & -4.4467 & -2.0353 \end{bmatrix}.
\end{aligned}$$

The initial time t_0 is set to 0s, the prediction horizon H is set to 100, i.e., $t_H = 1$s, and the total running time step $\bar{k}$ is set to 8000, i.e., $t_{\bar{k}} = 80$s. The initial state of the leading vehicle is $x_0(t_0) = [0, 22, 0]^\top$, which means the leading vehicle drives at the origin $p_0(t_0) = 0$m with velocity $v_0(t_0) = 20$m/s and zero acceleration $a_0(t_0) = 0$m/s^2. The vehicle platoon is assumed to be at the steady state when $t = t_0$, that is

$$x_i(t_0) - x_{i-1}(t_0) + d_{i,i-1} = 0, \ \forall i \in \mathcal{V}.$$

All simulations are performed in Matlab environment using YALMIP toolbox [58].

5.1. Platoon Control Performance

To demonstrate the control performance under Algorithm 1, a simulation scenario of a vehicle platoon with a leading vehicle subject to a time-varying input is considered. The input and state trajectories of the leading vehicle are generated by (3) with consideration of constraints (5)-(7). The input profile and the resulting velocity profile of the leading vehicle are shown in Figure 3.

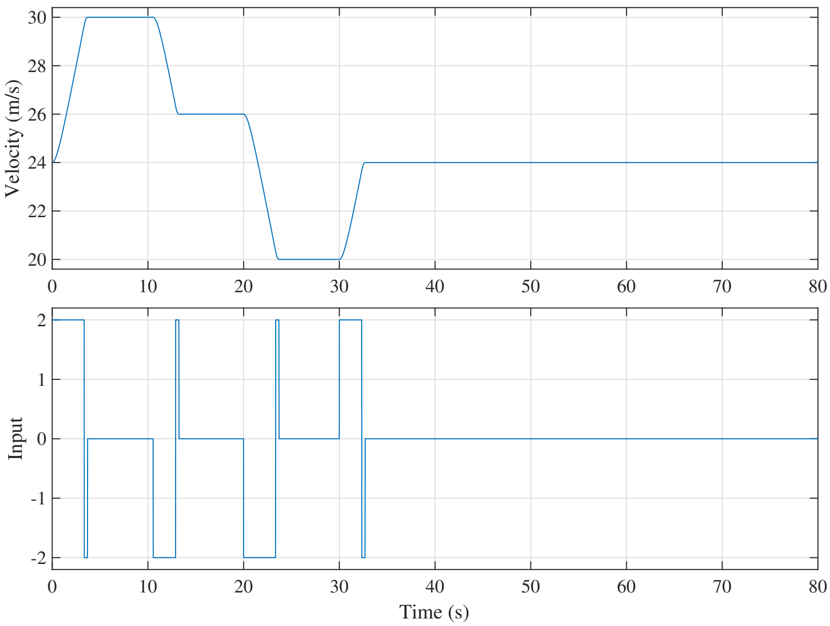

Figure 3: Velocity and input profiles of the leading vehicle. Top: Velocity; Bottom: Input.

Figure 4 depicts the state trajectories of each vehicle $i \in \mathcal{V}^+$ under the control of Algorithm 1. The velocity $v_i(t_k)$ and the acceleration $a_i(t_k)$ of each vehicle $i \in \mathcal{V}$ (plotted in 2 sub-figures from the top to the bottom) converge respectively to $v_0(t_k)$, and $a_0(t_k)$. For clearer demonstration, Figure 5 shows the tracking errors of each following vehicle $i \in \mathcal{V}$ compared with the leading vehicle 0 under Algorithm 1. The position tracking error $p_i(t_k) - p_0(t_k) - \bar{d}_{i,0}$, the velocity tracking error $v_i(t_k) - v_0(t_k)$, and the acceleration tracking error $a_i(t_k) - a_0(t_k)$ for each following vehicle $i \in \mathcal{V}$ (plotted in 3 sub-figures from the top to the bottom) all converge to zero, which demonstrates that the control objective (10) is achieved. The trajectories in Figure 6 show the inter-vehicle distance, denoted as $\Delta_i(t_k) = p_{i-1}(t_k) - p_i(t_k)$, between adjacent vehicles i and $i-1$, $i \in \mathcal{V}$. It can be observed from Figure 4 - Figure 6 that all local constraints and coupled formation constraints are satisfied at all time steps, and the inter-vehicle gaps between all adjacent vehicles converge to the desired distance 5m.

5.2. Performance Comparation

In the simulations of existing works, the velocity of the leading vehicle is piecewise constant, similar to Figure 3. To illustrate the advantages of Algorithm 1 in which the leading vehicle's input can be non-zero and time-varying, we consider a continuously changing leading vehicle velocity depicted in Figure 7. For benchmarking purposes, the control solution of the proposed method is compared with the algorithm in [51]. As [51] only deals with homogeneous vehicles [51] and the state constraints (5)-(6) as well as the coupled constraints (8)-(9) are not taken into consideration, the engine time constants τ_i of all vehicles $i \in \mathcal{V}^+$ are all set to 0.75 and only the input constraints (7) are imposed.

Under the control of the algorithm in [51], it is obvious from the velocity and acceleration sub-graphs in Figure 8 that there exist noticeable phase lags between the state profiles of the leading vehicle and each following vehicle. As a result, the tracking errors in terms of the following distance, velocity and acceleration can not converge to 0, which implies the control objective (10) can not be achieved. The argument is verified in Figure 9, which shows the non-convergence behavior of the tracking error of all following vehicles. Consider a tracking performance index for

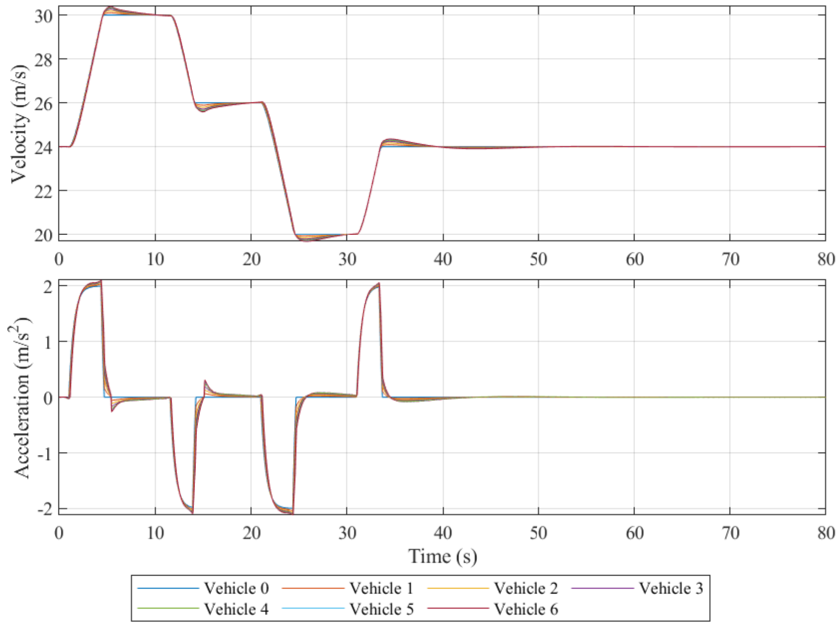

Figure 4: State trajectories of all vehicles under Algorithm 1 subject to a time-varying velocity profile of the leading vehicle according to Figure 3. Top: Velocity; Bottom: Acceleration.

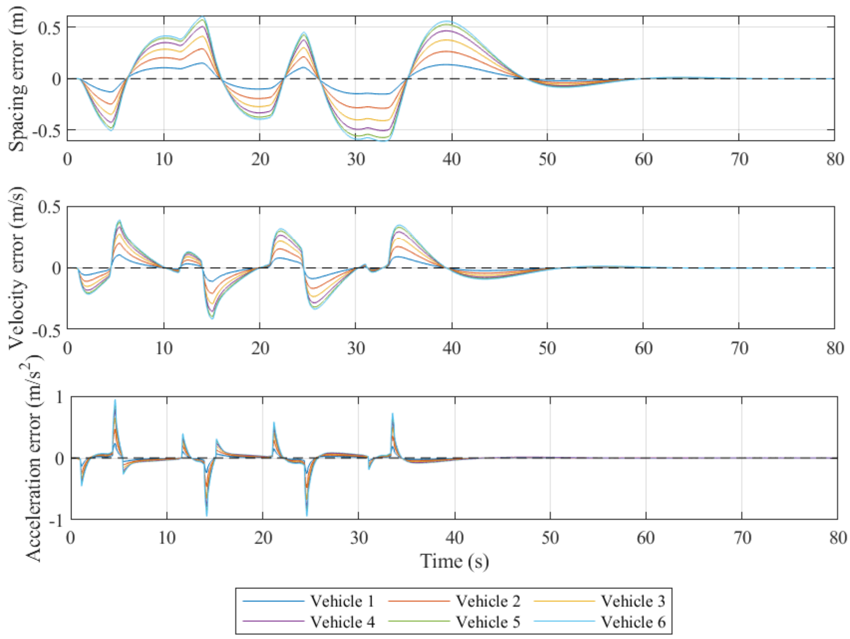

Figure 5: State tracking error trajectories of all following vehicles under Algorithm 1 subject to a time-varying velocity profile of the leading vehicle according to Figure 3. Top: Spacing error; Middle: Velocity error; Bottom: Acceleration error.

the platoon as $\sigma = \sum_{i=1}^{N} \sigma_i$, where $\sigma_i = \frac{\sum_{k=1}^{\bar{k}} ||x_i(t_k)-x_0(t_k)-d_{i,0}||_2^2}{\bar{k}}$ is the mean squared tracking error of following vehicle $i \in \mathcal{V}$ over $\bar{k}$ time steps. Under the control of the algorithm in [51], the tracking performance index is $\sigma = 294.7686$, with $\sigma_1 = 6.4890$, $\sigma_2 = 22.6167$, $\sigma_3 = 43.1071$, $\sigma_4 = 62.7086$, $\sigma_5 = 77.3476$ and $\sigma_6 = 82.4996$.

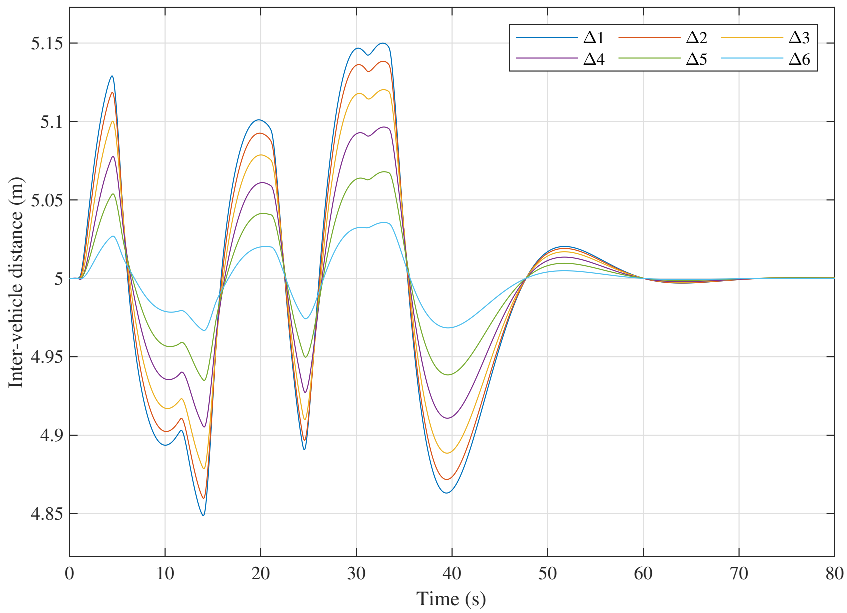

Figure 6: Inter-vehicle distance profiles between adjacent vehicles under Algorithm 1 subject to a time-varying velocity profile of the leading vehicle according to Figure 3.

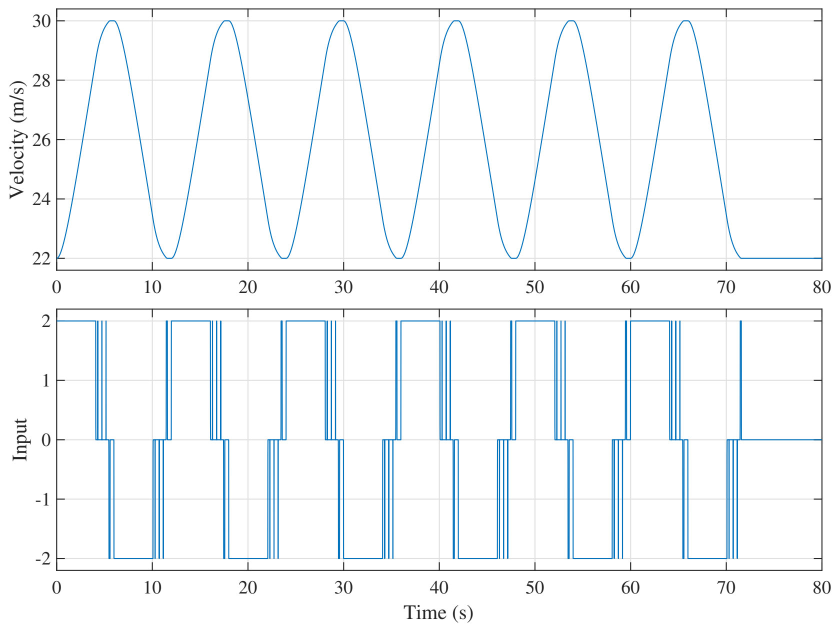

Figure 7: Velocity and input profiles of the leading vehicle. Top: Velocity; Bottom: Input.

The control performance of Algorithm 1 is shown in Figure 10 and Figure 11. The tracking performance index is $\sigma = 4.3299$ for Algorithm 1 with $\sigma_1 = 0.0958$, $\sigma_2 = 0.2600$, $\sigma_3 = 0.4293$, $\sigma_4 = 0.7285$, $\sigma_5 = 1.3131$ and $\sigma_6 = 1.5030$. Compared to the results obtained by [51], the tracking accuracy is noticeably improved by the proposed control solution.

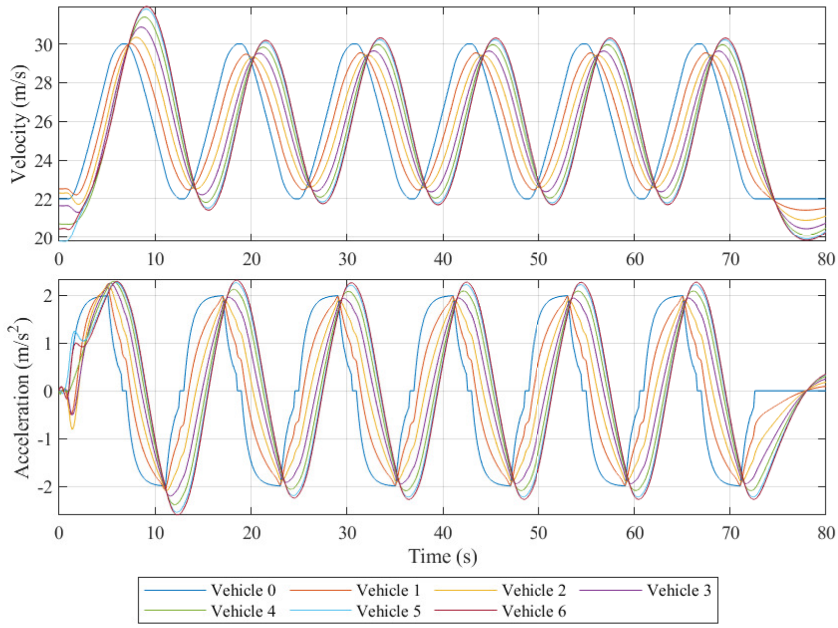

Figure 8: State trajectories of all vehicles under algorithm in [51] subject to a time-varying velocity profile of the leading vehicle according to Figure 7. Top: Velocity; Bottom: Acceleration.

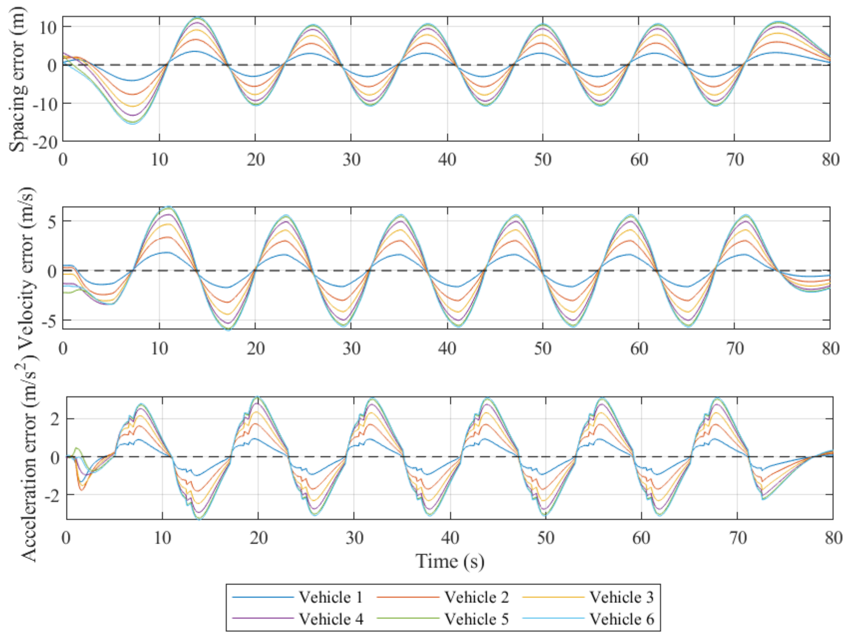

Figure 9: State tracking error trajectories of all following vehicles under algorithm in [51] subject to a time-varying velocity profile of the leading vehicle according to Figure 7. Top: Spacing error; Middle: Velocity error; Bottom: Acceleration error.

6. Conclusions

This paper has proposed a DMPC approach for heterogeneous platoon control problem subject to both local state and input constraints and state-coupled inter-vehicular spacing constraints. The approach is applicable to the situation

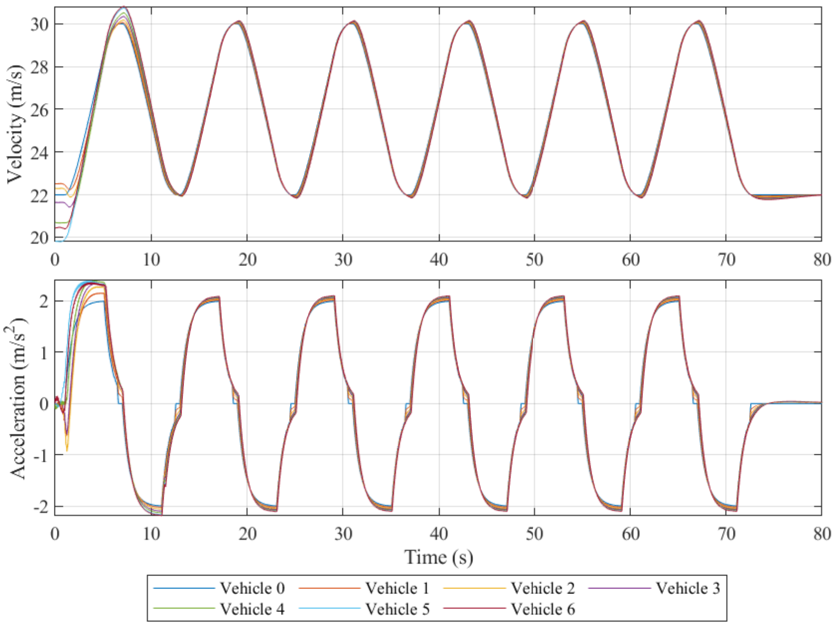

Figure 10: State trajectories of all vehicles under Algorithm 1 subject to a time-varying velocity profile of the leading vehicle according to Figure 7. Top: Velocity; Bottom: Acceleration.

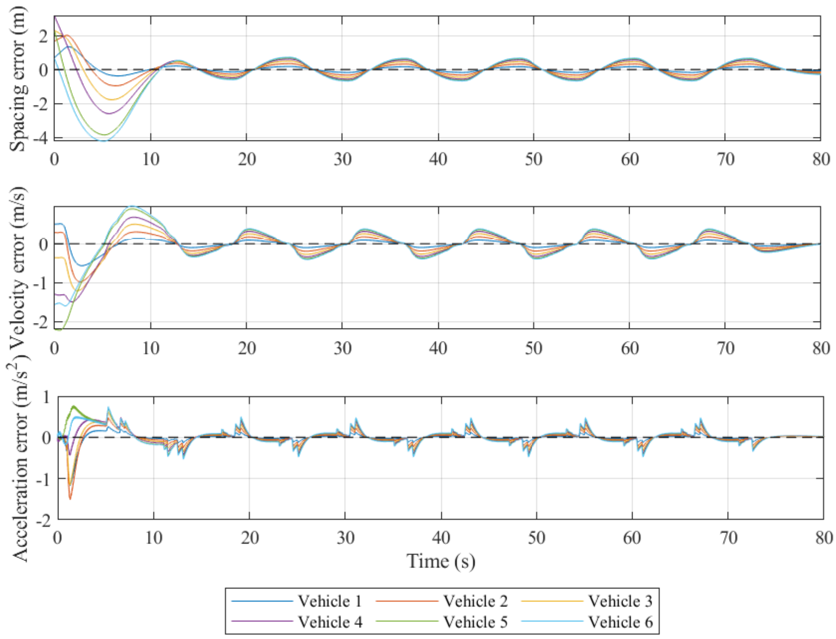

Figure 11: State tracking error trajectories of all following vehicles under Algorithm 1 subject to a time-varying velocity profile of the leading vehicle according to Figure 7. Top: Spacing error; Middle: Velocity error; Bottom: Acceleration error.

that the leading vehicle's state information is unknown to some following vehicles and the non-zero and time-varying input of the leading vehicle is inaccessible to all following vehicles. To achieve distributed implementation, we divide each state-coupled constraint set into several specific local subsets by utilizing the assumed state trajectories of the coupled vehicles, and then allow each following vehicle to optimize its state in the corresponding subsets. The unique feature of the developed algorithm is the design of terminal control laws and an invariant set to deal with the effect of continuously changing leading vehicle trajectory. The feature enables the recursive feasibility of local optimization problems to be ensured at all time steps despite the leading vehicle's velocity changes. The asymptotic stability of the proposed DMPC is also proved to ensure that all following vehicles can track the velocity and acceleration of the leading vehicle, and keep a small constant inter-vehicular distance between any two consecutive vehicles. Comparison simulation results demonstrate the efficacy of the proposed distributed control scheme and the improvement in terms of the flexibility of the leading vehicle's velocity changes caused by the variable input of the leading vehicle.

Future research efforts will be devoted to extending the analysis to address the string stability and the delay in communication for enhanced practicability.

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