Multipartite State Distribution for Quantum Networks with Probabilistic Entanglement Generation and Swapping

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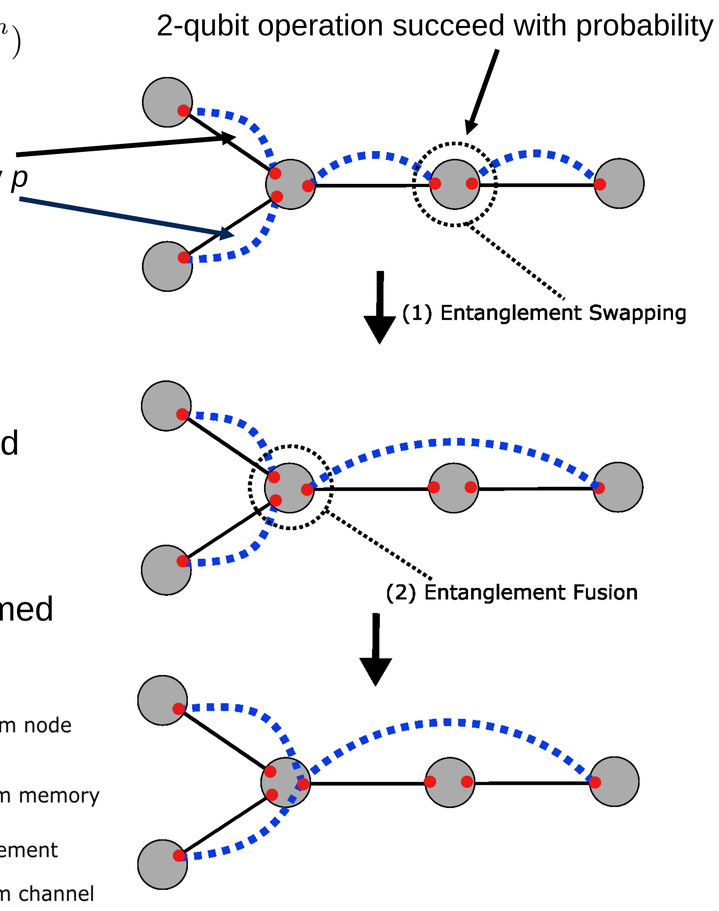
1. INTRODUCTION

- **Aim:** Distribute multipartite states over a quantum network (at a high rate)
- **Applications:** QKD, distributed quantum computation and distributed quantum codes [1].
- **Previous work:** focused on single (shortest) path (SP) routing [2]
- Multipath routing shown to improve entanglement rate for the two-user use cases [3]
- **Multipath routing has not** been applied to multipartite state distribution (multi-users)

2. NETWORK MODEL

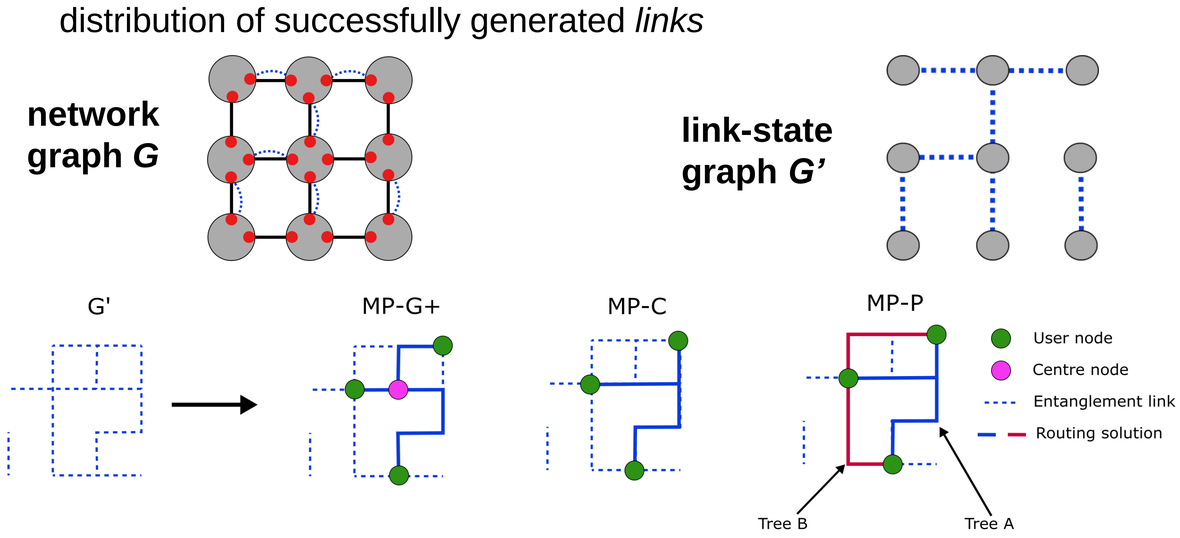
- **Multipartite states,** such as the Greenberger–Horne–Zeilinger (GHZ) state, can be generated from multiple **Bell pairs via LOCC**
- $|\mathrm{GHZ}_n\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes n} + |1\rangle^{\otimes n})$



- A tree of Bell pairs can be used to generate a GHZ between distant users
- Bell pairs (***links***) generated with probability *p* during a timeslot T_{slot}
- 2-qubit operations performed at a device succeed with probability *q*

3. PROTOCOLS

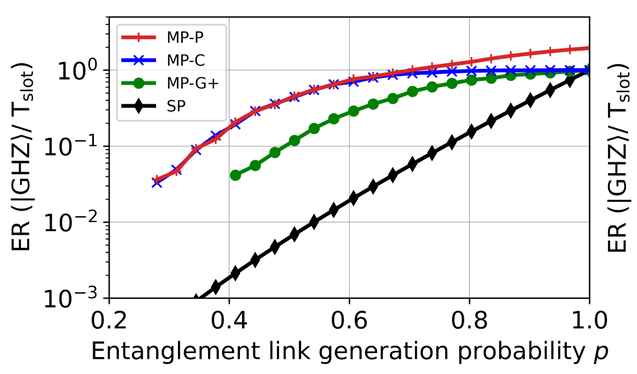
- Three protocol, MP-G+, MP-C & MP-P, developed for distributing *n*-qubit GHZ states between *n* users.
- Multipath protocols use the graph **G'** for routing, which describes the distribution of successfully generated *links*



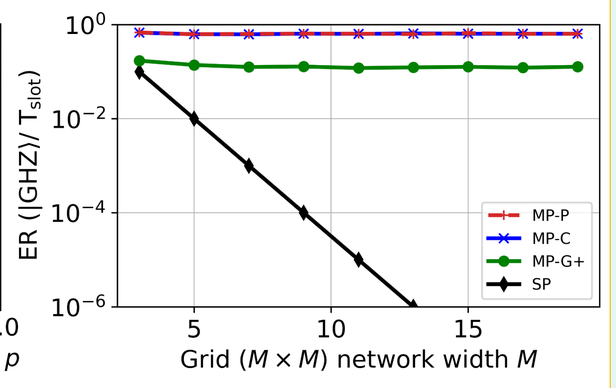
- Protocols compared to **SP protocol** in which links are generated along the minimum (Steiner) tree in the network topology **G** connecting users
- **MP-G+:** Each user is connected to a centre node by a separate edge-disjoint path in ***G'***. These may not be the shortest paths in **G**
- **MP-C:** Protocol connects user by minimum Steiner tree in **G'**
- **MP-P:** Operates similarly to MP-C. However, protocol can generate multiple GHZ states if multiple edge-disjoint trees exist in **G'**.

4. RESULTS

- Protocols assessed using entanglement rate (ER = GHZ/T_{slot})
- Multipath routing outperforms SP routing, with a maximum speedup at intermediate link generation probabilities *p*
- Distance-independent ER achieved by protocols → **exponential speedup** in ER over SP routing with distance
- Observed 6000× speedup
- Distance-independent ER achieved above percolation threshold of link-state graph. Significant speedup achievable at lower *p*

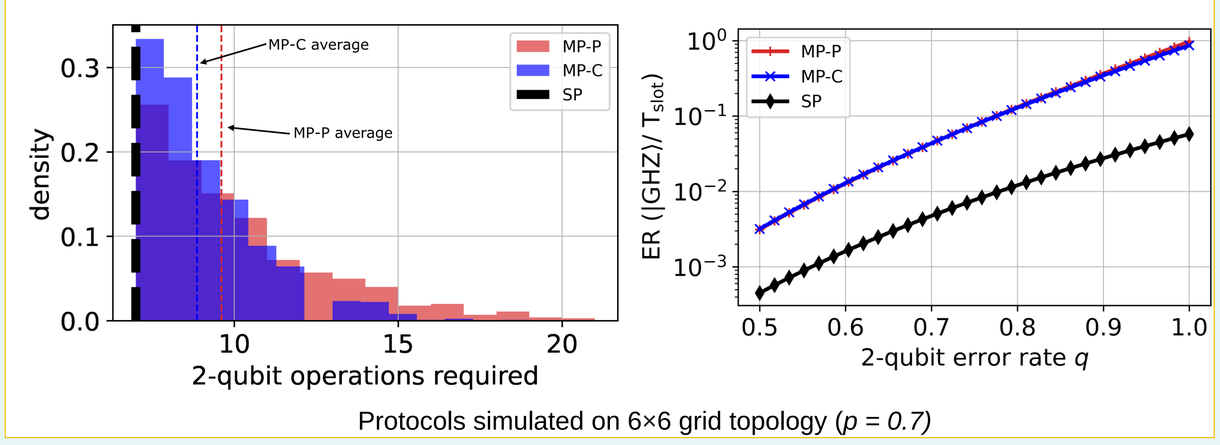


Protocols simulated on 6×6 grid topology with four randomly located users (*q*=1)



Protocols simulated on a grid topologies with users at corner nodes (*p* = 0.75,*q*=1)

- Multipath routing can use routes other than shortest path, proposed protocols utilised routes up to 3× longer than shortest path (on a 6×6 grid topology)
- Longer routes require more 2-qubit operates to generate GHZ from Bell pairs
- Penalty to entanglement rate when error prone (probabilistic) 2-qubit operations are considered (such as entanglement swapping)
- For high error rates (low *q)*, the proposed protocols still achieve entanglement rates a magnitude higher than SP protocol.



Protocols simulated on 6×6 grid topology (*p = 0.7)*

5. CONCLUSIONS

Multipath routing for multipartite state distribution brings clear benefits:

1. Significant speedup above 6000x compared to shortest path routing
2. Multipartite entanglement rate independent of distance (above a threshold link generation probability) → giving exponentially speedup
3. speedup not significantly affected by imperfect gate operations such as entanglement swapping

Future work will consider GHZ state fidelity

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[1] S. de Bone *et al*. *IEEE Transactions on Quantum Engineering* 1 (2020): 1-10.
[2] L. Bugalho, et al. *quantum* 7 (2023): 920..
[3] A. Patil, et al. npj Quantum Information. 2022 May 6;8(1):1-9.