The origin of dwarf carbon stars

Lewis James Whitehouse

Thesis submitted in partial fulfilment of the requirements for the Degree of
Doctor of Philosophy of University College London

Supervisors:
Prof. Jay Farihi
Prof. Ian Howarth

Examiners:
Dr. Rob Izzard
Dr. Stephen Fossey

18th of September, 2022
Declaration

I, Lewis James Whitehouse, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis. In particular, I would like to point out the following contributions:

- Work presented in Chapter 4 has been published in Whitehouse et al. (2021).
- Work presented in Chapter 5 has been published in Whitehouse et al. (2018).
- Lightcurve simulations presented in Chapter 4 were developed by Ian Howarth and are discussed in Howarth (1982) and Howarth (2011).
- Synthetic stellar templates used in Chapter 4, and 5 were synthesised by Sara Mancino using the AUTOKUR software.
- I also acknowledge the following people for contributing to this thesis through their hospitality during travels and useful discussions with myself: Henri Boffin, Cecilia Farina, Boris Gänsicke, Paul Green, Hugh Harris, Prashin Jethwa, Dave Jones, Alain Jorissen, Sophia Lilleengen, Fiona Riddick, Jon Subasavage, and the anonymous referee of Whitehouse et al. (2018).

London, UK, 18th of September, 2022

Lewis James Whitehouse
Abstract

Dwarf carbon (dC) stars are low-mass, main-sequence stars that exhibit spectra analogous to carbon-rich giants found along the asymptotic giant branch. An almost half a century old hypothesis postulates that the peculiar atmospheric chemistry of dC stars is owed to mass transfer from an evolved companion. Specifically, while the former primary ascended the asymptotic giant branch, triple-\(\alpha\)-processed material was dredged to the surface before the stellar wind liberated the carbon-enhanced outer layers. The liberated material is then transferred to the main-sequence companion, polluting its atmosphere, and forming a dC star, while the original primary becomes a white dwarf.

In this thesis, I present the results of a decade-long spectroscopic survey of 37 dC stars to test this binary evolution hypothesis. Using MCMC simulations to analyse the radial-velocity variations of all 37 dC stars in this sample, it has been possible to show that the population is consistent with a 100 per cent binary fraction and are thus likely the product of mass transfer.

Furthermore, the orbital parameters of nine dC stars are reported for the first time, increasing the number of known dC binaries by 220 per cent. Interestingly, eight of these newly constrained dC binaries exhibit emission features in their optical spectrum. Moreover, the orbital periods of these emission-line dC stars are all shorter than 12 d, implying that these stars have evolved through a common envelope. Thus, the discovery of eight short-orbital period dC stars indicates that low-mass, metal-weak or metal-poor stars can accrete substantial material before entering the common-envelope phase.

Finally, a kinematical study of 1200 candidate dC stars is presented, with the analysis indicating that as much as 70 per cent of the population may possess Galactic orbits that are inconsistent with thin disc membership. This result, therefore, suggests that dC stars are generally old and likely metal-poor.
Impact Statement

The analyses and conclusions presented in this thesis address an astrophysical problem that has puzzled astronomers for over half a century. The benefits of this work are primarily academic, deepening our understanding of stellar and binary evolution.

In Chapter 4, the results of an observational study into the origin of emission in the optical spectra of a sub-sample of dwarf carbon stars are presented. The subsequent analysis of the data indicates that the origin of this emission can be explained through close binary evolution. The results presented in this chapter double the number of known binary dwarf carbon stars and increased the number of dwarf carbon stars with variable luminosity sixfold. These results are novel and contribute to the broader field of binary star evolution.

In Chapter 5, the results of a decade long radial-velocity survey of dwarf carbon stars are presented. Through careful analysis of the data, the dwarf carbon star population is shown to be consistent with possessing a binary fraction upwards of 94 per cent - a result that was long hypothesised but not proven. These results carry significant implications for the existence of carbon-rich planets, which are unlikely to exist around dwarf carbon stars.

In Chapter 6, state-of-the-art astrometric data has been used to study the kinematics of a sample of over 1000 dwarf carbon stars. The results of this chapter indicate that the dwarf carbon star population is generally consistent with being old and likely metal-poor. If dwarf carbon stars are generally old and metal-poor, they may hold vital clues into the formation and evolution of the Milky Way. The field of Galactic archaeology has grown in recent years and may now have up to 1000 additional stars to study.

Several of the results presented in this thesis have been peer-reviewed and published in leading astrophysical journals, where the wider academic community has cited them. The results presented in Chapter 4 have been published in Whitehouse et al. (2021) and the analysis presented in Chapter 5 are updated results previously published in Whitehouse et al. (2018).

While the benefits of this thesis are primarily academic, the results presented help constrain many important areas of astrophysics and allow us to better understand the Universe around us.
Acknowledgements

Over the past several years, I have interacted with numerous people who have influenced my journey toward submitting this thesis. First and foremost, I would like to thank my supervisors, Jay Farihi and Ian Howarth, for their help and support during my PhD. Jay, thank you for always being available to talk science, whether in the office or over a glass of whisky. Ian, thank you for offering your wisdom; it was invaluable to my work and development.

I would also like to thank Henri Boffin for offering me the opportunity to spend a year working at the European Southern Observatory headquarters in Garching bei München. During this year away from UCL, I gained a deeper understanding of statistics and established confidence in my computational skills. I will always be grateful to Henri, my European Southern Observatory colleagues, and the wonderful people I met while living in München.

Over the past few years, I have been fortunate to embark on several observing trips to the island of La Palma. I would therefore like to thank the whole island of La Palma, not just for its beauty and friendly locals, but for allowing me the opportunity to use its world-class telescopes. Working with the staff at the Roque de los Muchachos observatory has been a pleasure.

Any thesis could not be completed without the support of academic peers. I wish to thank the astrophysics group at UCL for making my PhD much more enjoyable. Particularly, I would like to thank Tom for our office chats about future careers and the activities of our mutual friends. I would also like to thank Martin for always being available for a long lunch or post-work pub trip. Outside of academia, I would like to thank the Boltzmann Wanderers boys for keeping me entertained on the weekends.

A person who has been a pivotal in the completion of this thesis is my partner, Sophia. To you, Sophia, I would like to say: Du erfüllst mich mit Zuversicht. Danke, dass du mich unterstützt, an mich glaubst und mich ermutigst, diese Arbeit zu beenden. Ich weiß deine Geduld und Liebe sehr zu schätzen.

Throughout my PhD and, indeed, my life, I have been fortunate enough to enjoy the strong support of my family. Your encouragement and support have provided the foundations for the
person I have become. I would like to thank my wonderful sister for inspiring me with her strength and resilience, which encourages me to continue, even when times are challenging. To my Dad, I am wholly grateful to you for teaching me that I will never walk alone. I know I will always have your support in anything I pursue. Finally, I will be forever indebted to my mum for your unwavering love and belief in me. I am proud to be part of this family. Therefore, I take great pleasure in dedicating this thesis to my family; without you, completing this thesis would not have been possible.
Contents

List of Figures 17

List of Tables 19

1. Introduction 21
   1.1. Prelude .............................................................. 21
   1.2. Single star evolution ............................................. 23
       1.2.1. Hertzsprung-Russell diagram .............................. 23
       1.2.1.1. The main sequence ....................................... 23
       1.2.1.2. Main sequence turn-off to the asymptotic giant branch ................................. 27
       1.2.1.3. Asymptotic giant branch ................................ 30
       1.2.1.4. Post-AGB evolution ...................................... 33
       1.2.2. Dwarf carbon stars and single-star evolution .................. 33
   1.3. Binary star evolution ............................................. 34
       1.3.1. Keplerian orbits ............................................. 35
       1.3.2. Orbital evolution ............................................ 35
       1.3.3. Mass transfer ................................................. 36
           1.3.3.1. Roche-lobe overflow ................................... 36
           1.3.3.2. Wind capture ........................................... 38
           1.3.3.3. Wind-Roche lobe overflow ............................... 38
       1.3.4. Carbon-enhanced post-mass transfer binaries .................. 39
       1.3.5. Dwarf carbon stars and binary stellar evolution .................. 41
   1.4. Thesis outline .................................................... 43

2. Data acquisition and reduction 45
   2.1. Introduction ...................................................... 45
   2.2. Charge-coupled devices ......................................... 45
2.3. Spectroscopy ........................................... 46
  2.3.1. Overview ........................................ 46
  2.3.2. Bias subtraction ................................ 46
  2.3.3. Flat-fielding ................................... 47
  2.3.4. Spectral extraction ............................. 48
  2.3.5. Wavelength calibration ......................... 48
  2.3.6. Flux calibration and normalisation ............ 50
2.4. Photometry ............................................. 51
  2.4.1. Overview ........................................ 51
  2.4.2. Bias subtraction ................................ 51
  2.4.3. Dark-current subtraction ....................... 51
  2.4.4. Flat-fielding ................................... 51
  2.4.5. Differential photometry ......................... 52
2.5. Cross-correlation ..................................... 52
2.6. Conclusions .......................................... 54

3. Time-series analysis .................................. 55
  3.1. Introduction ......................................... 55
  3.2. Fourier-like analysis ............................... 55
    3.2.1. Classical Fourier analysis ..................... 56
    3.2.2. Lomb-Scargle analysis ........................ 63
    3.2.3. Multi-harmonic analysis of variance ........... 65
  3.3. Monte Carlo sampling methods ..................... 66
    3.3.1. Introduction to Monte Carlo sampling........... 66
    3.3.2. Rejection sampling ............................ 67
    3.3.3. Applying Bayes’ theorem to sampling ......... 68
    3.3.4. The JOKER ..................................... 70
    3.3.5. Markov chain Monte Carlo sampling ............ 72
  3.4. Comparison between Fourier-like analysis and rejection sampling .. 74
    3.4.1. Simulating radial-velocity data ................. 74
    3.4.2. Generalised Lomb-Scargle vs MHAOV .......... 78
    3.4.3. The JOKER vs MHAOV ........................... 79
    3.4.4. Combining the JOKER and MCMC sampling ....... 83
  3.5. Conclusions .......................................... 86
4. The nature of dwarf carbon stars with Hα emission

4.1. Introduction ......................................................... 89
4.2. Observations and data reduction .................................... 91
  4.2.1. Target selection .............................................. 91
  4.2.2. Radial velocities ............................................ 92
  4.2.3. Photometry .................................................. 93
4.3. Methods and period analysis ...................................... 94
  4.3.1. Radial-velocity cross-correlation ............................ 94
  4.3.2. Radial-velocity period determinations ........................ 95
  4.3.3. Periodogram analysis of photometry .......................... 97
  4.3.4. Multi-harmonic analysis of photometry ....................... 102
4.4. Results for individual stars ...................................... 102
  4.4.1. SDSS J084259+225729 ...................................... 102
  4.4.2. SDSS J090128.28+323833.5 .................................. 104
  4.4.3. SDSS J101548.90+094649.7 .................................. 105
  4.4.4. CLS 29 ...................................................... 106
  4.4.5. SDSS J125017.90+252427.6 .................................. 106
  4.4.6. SBSS 1310+561 .............................................. 107
  4.4.7. CBS 311 .................................................... 107
4.5. Discussion ......................................................... 108
  4.5.1. Binary light-curve simulations ............................... 109
  4.5.2. Source of photometric variability ............................ 110
    4.5.2.1. Rotation versus orbital motion .......................... 111
    4.5.2.2. Tidal synchronisation ................................... 112
    4.5.2.3. Differential rotation ................................... 113
  4.5.3. Source of stellar activity ................................... 114
  4.5.4. Period distribution .......................................... 116
4.6. Conclusions ....................................................... 117

5. The binary fraction of dwarf carbon stars .......................... 119

5.1. Introduction ....................................................... 119
5.2. Observations and data reduction .................................. 120
  5.2.1. Target selection .............................................. 120
  5.2.2. Spectroscopic survey ........................................ 121
  5.2.3. Radial-velocity measurements ................................ 124
5.2.4. Measuring the offsets in wavelength solutions at different observing facilities ........................................... 128

5.3. Radial-velocity period determinations ................................................................. 130

5.3.1. Results for individual stars ................................................................. 133

5.3.1.1. SDSS J01215050.42+011301.4 ..................................................... 133

5.3.1.2. G77-61 ................................................................................. 134

5.3.1.3. SDSS J091007.60+521612.5 ..................................................... 135

5.3.1.4. SDSS J154859.72+341821.7 ..................................................... 136

5.3.1.5. LP225−12 ............................................................................ 136

5.3.1.6. LSR J2105+2514 .................................................................. 137

5.4. The binary fraction and orbital period distribution of dC stars ..................... 138

5.4.1. A maximum likelihood analysis of the dC star sample ......................... 140

5.4.2. Exploring the orbital period distribution of dC stars ............................ 143

5.5. Discussion ......................................................................................... 150

5.5.1. Binary fraction and orbital evolution ............................................. 150

5.5.2. Carbon chemistry exoplanets ....................................................... 154

5.6. Conclusions ...................................................................................... 155

6. A dynamical study of the dwarf carbon star population .............................. 157

6.1. Introduction ....................................................................................... 157

6.2. Sample selection and data ...................................................................... 158

6.3. Calculating distances to dC stars ......................................................... 159

6.3.1. Probabilistic inference using Gaia eDR3 data ................................... 159

6.3.2. Constructing a Hertzsprung-Russell diagram prior ......................... 161

6.3.3. Computing the parallax posterior PDFs of all candidate dC stars ....... 163

6.4. Hertzsprung-Russell diagram of dC stars ........................................... 164

6.4.1. Identifying true dC stars .................................................................. 164

6.4.2. The location of the dC population on the HRD .............................. 168

6.5. Orbital actions .................................................................................... 170

6.5.1. Galactic potential ........................................................................... 171

6.5.2. Calculating orbital actions ............................................................... 173

6.5.3. Results ......................................................................................... 173

6.6. Conclusion ......................................................................................... 178
List of Figures

1.1. Example Hertzsprung-Russell diagram constructed using Gaia eDR3 data. . . 24
1.2. A schematic of the zero-age main sequence . . . . . . . . . . . . . . . . . . . . 26

2.1. Example of averaged bias and flat frames . . . . . . . . . . . . . . . . . . . . . 47
2.2. 2D spectral image of G77-61 . . . . . . . . . . . . . . . . . . . . . . . . . . . . 49
2.3. Cross-correlation function of two spectra for the target G77-61 . . . . . . . 53

3.1. An example power spectrum . . . . . . . . . . . . . . . . . . . . . . . . . . . . 58
3.2. Comparison between uniformly and non-uniformly sampled time-series . . . 62
3.3. Rejection sampler example . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 68
3.4. Simulated radial-velocity time-series data . . . . . . . . . . . . . . . . . . . . . 75
3.5. Comparison between the generalised Lomb-Scargle periodogram and MHAOV 76
3.6. Comparison between MHAOV and the JOKER . . . . . . . . . . . . . . . . . . . . 80
3.7. Posterior probability distribution derived by the JOKER for simulated dataset . 84
3.8. Posterior probability distribution derived through MCMC sampling for simu-
   lated dataset . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 85

4.1. Phase-folded radial-velocity curves of six emission-line dC stars . . . . . . . 98
4.2. Lomb-Scargle periodograms computed of seven emission-line dC stars . . . 99
4.3. The optical spectrum spectrum and SED fits of the emission-line dC star, 
   SDSS J084259.79+225729.8 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 104
4.4. Light-curve simulations showing the effects heating on the luminosity of a 
   typical emission-line dC star . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 108
4.5. A comparison between the orbital and photometric periods of seven emission-
   line dC stars . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 112
4.6. Lomb-Scargle periodograms and phase-folded light-curves corresponding the 
   TESS sectors 15, 16, and 22 of the emission-line dC star, SBSS 1310+561 . . . . 114
5.1. Spectrum of LP 225-12 ......................................................... 124
5.2. Summary of synthetic stellar templates ...................................... 127
5.3. Overview of instrumental offsets in radial velocity survey .......... 129
5.4. Phase-folded radial-velocity curves of six dC stars with orbital parameters determined by the joker .............................................. 139
5.5. The distribution of \( p \)-values associated with the probability that any dC star is a binary ................................................................. 141
5.6. The posterior PDF of the binary fraction and period distribution of dC stars ................................................................. 144
5.7. The posterior PDF of a bimodal period distribution of dC stars .......... 147
5.8. The CDF of the orbital period distribution of dC stars .................... 148
5.9. The posterior probability distribution functions derived for the dC stars that exhibit H\( \alpha \) emission, and those that do not, respectively ..................... 151

6.1. The Hertzsprung-Russell diagram of dC stars plotted with errorbars .......... 165
6.2. The Hertzsprung-Russell diagram of dC stars compared to isochrones of stellar populations of various ages ........................................ 166
6.3. A visual display of the cut used to define the main sequence of the Hertzsprung-Russell diagram .............................................. 167
6.4. A cut to define the main sequence of the Hertzsprung-Russell diagram .......... 169
6.5. Comparison of \( M_r \) of the dC population to the background sample over a restricted colour range .............................................. 171
6.6. Orbital actions of the population of dC stars .................................... 174
6.7. Orbital energies of the dC population ........................................... 175
6.8. Action-space map of the dC population ........................................... 177

A.1. List of posterior distribution functions of all H\( \alpha \) emitting dC stars discussed in Chapter 4 ............................................................ 186

C.1. List of posterior distribution functions for all dC stars assuming circular orbits 212
C.2. List of posterior distribution functions for all dC stars assuming eccentric orbits 232

D.1. Probability distribution function of a sine wave ............................ 253
# List of Tables

4.1. Observed dC stars with Hα emission. ........................................... 92
4.2. Wavelength regions and atomic transitions used for spectral cross-correlation
to determine absolute velocities. .................................................... 94
4.3. Summary of period and close binary parameter estimates for dC stars, and
possible sources of photometric variability. ...................................... 101
5.1. Results from the radial-velocity survey ........................................ 131
5.2. A summary of the orbital parameters of dC stars with constrained orbital
parameters with fixed circular orbits. ................................................ 133
5.3. A summary of the orbital parameters of dC stars with constrained orbital
parameters with eccentricity allowed to vary. ................................... 134
B.1. Radial velocities of the target LHS 1075 ....................................... 192
B.2. Radial velocities of the target SDSS J012028.56−083630.9 ............... 192
B.3. Radial velocities of the target SDSS J012150.42+011301.4 ................. 193
B.4. Radial Velocities of the target SDSS J013007.13+002635.3 ............... 193
B.5. Radial Velocities of the target SDSS J022304.43+004501.3 ............... 193
B.6. Radial Velocities of the target G77-61 ....................................... 194
B.7. Radial Velocities of the target SDSS J074257.17+465917.9 ................ 195
B.8. Radial Velocities of the target SDSS J081157.14+143533.0 ............... 195
B.9. Radial Velocities of the target SDSS J081807.45+223427.6 ............... 196
B.10. Radial Velocities of the target PG 0842+288 ............................... 196
B.11. Radial Velocities of the target SDSS J084259.80+225729.0 ............... 197
B.12. Radial Velocities of the target SDSS J090128.28+323833.5 ............... 197
B.13. Radial Velocities of the target SDSS J090302.86+385527.4 ............... 198
B.14. Radial Velocities of the target SDSS J091007.60+521612.5 ............... 198
B.15. Radial Velocities of the target C 0930-00 ................................... 199
<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.16</td>
<td>Radial Velocities of the target SDSS J093334.14+074812.6</td>
<td>199</td>
</tr>
<tr>
<td>B.17</td>
<td>Radial Velocities of the target SDSS J095545.84+443640.4</td>
<td>200</td>
</tr>
<tr>
<td>B.18</td>
<td>Radial Velocities of the target SDSS J101548.90+094649.7</td>
<td>200</td>
</tr>
<tr>
<td>B.19</td>
<td>Radial Velocities of the target CLS 29</td>
<td>201</td>
</tr>
<tr>
<td>B.20</td>
<td>Radial Velocities of the target CLS 31</td>
<td>202</td>
</tr>
<tr>
<td>B.21</td>
<td>Radial Velocities of the target SDSS J110458.97+274311.8</td>
<td>202</td>
</tr>
<tr>
<td>B.22</td>
<td>Radial Velocities of the target KA 2</td>
<td>202</td>
</tr>
<tr>
<td>B.23</td>
<td>Radial Velocities of the target SDSS J112633.94+044137.7</td>
<td>203</td>
</tr>
<tr>
<td>B.24</td>
<td>Radial Velocities of the target SDSS J120024.09+381720.3</td>
<td>203</td>
</tr>
<tr>
<td>B.25</td>
<td>Radial Velocities of the target CLS 50</td>
<td>204</td>
</tr>
<tr>
<td>B.26</td>
<td>Radial Velocities of the target SDSS J122328+353251.9</td>
<td>204</td>
</tr>
<tr>
<td>B.27</td>
<td>Radial Velocities of the target SDSS J130744.53+600903.7</td>
<td>204</td>
</tr>
<tr>
<td>B.28</td>
<td>Radial Velocities of the target SBSS 1310+561</td>
<td>205</td>
</tr>
<tr>
<td>B.29</td>
<td>Radial Velocities of the target SDSS J145725.86+234125.4</td>
<td>206</td>
</tr>
<tr>
<td>B.30</td>
<td>Radial Velocities of the target CBS 311</td>
<td>206</td>
</tr>
<tr>
<td>B.31</td>
<td>Radial Velocities of the target SDSS J154859.72+341821.7</td>
<td>206</td>
</tr>
<tr>
<td>B.32</td>
<td>Radial Velocities of the target CLS 96</td>
<td>207</td>
</tr>
<tr>
<td>B.33</td>
<td>Radial Velocities of the target LP 225-12</td>
<td>207</td>
</tr>
<tr>
<td>B.34</td>
<td>Radial Velocities of the target SDSS J184735.67+405944.1</td>
<td>207</td>
</tr>
<tr>
<td>B.35</td>
<td>Radial Velocities of the target LSR 2105+2514</td>
<td>208</td>
</tr>
<tr>
<td>B.36</td>
<td>Radial Velocities of the target LP 758-43</td>
<td>208</td>
</tr>
<tr>
<td>B.37</td>
<td>Radial Velocities of the target SDSS J235443.13+362907.1</td>
<td>209</td>
</tr>
</tbody>
</table>
Introduction

1.1 Prelude

Historically, carbon stars were thought to exclusively be evolved stars lying along the asymptotic giant branch (AGB). Defined as possessing molecular absorption features of C$_2$, CN, or CH in their optical spectra (Secchi, 1869), carbon stars are the product of intrinsic carbon enhancement via triple-\(\alpha\) burning products (namely carbon) processed in the stellar interior, that have been dredged to the surface of the star (Iben and Renzini, 1983a). Once carbon has been dredged to the upper atmosphere of the star, it readily bonds with oxygen to form the CO molecule (Russell, 1934). Crucially, if the ratio of carbon to oxygen exceeds unity, all of the oxygen becomes locked up in CO molecules, thus leaving the excess carbon free to form molecular bonds with other elements present in the atmosphere (Spinrad and Wing, 1969). This excess of atmospheric carbon is the feature that makes carbon stars distinct from stars like the Sun (that possesses an excess of oxygen relative to carbon), and gives carbon stars their characteristic optical spectra and red colour. Because triple-\(\alpha\) processed material is not dredged to the stellar surface until a star ascends the AGB, single star evolution predicts that all carbon stars are evolved giants.

The first dwarf carbon (dC) star, G77-61, was discovered by the Lowell proper-motion survey (Giclas et al., 1961), and apart from its large proper motion, was initially noted as unremarkable. It was more than a decade later, after the United States Naval Observatory (USNO) measured a trigonometric parallax, that an inconsistency was noted between the absolute magnitude \(M_V \approx 9.5\) Vega mag and colour \((B - V = 1.70)\) of G77-61 (Dahn et al., 1977). Spectroscopic follow-up soon confirmed that G77-61 possesses an optical spectrum typical of a classical carbon star, however, its large proper motion implies that G77-61 lies on the main sequence. Because main sequence stars are not able to ignite the triple-\(\alpha\) process and produce carbon, let alone dredge carbon-rich material to the surface, the existence of a dC star poses a challenge for single star evolution. Hence, the discovery of G77-61 casts doubt on the idea that all carbon stars are evolved giants.

Two leading theories were formed to explain the existence of G77-61 (Dahn et al., 1977). First, G77-61 is a member of an intrinsically carbon-rich halo population, whereby carbon-rich material ejected by massive first generation population II stars had coalesced and collapsed to
form an intrinsically carbon-rich star. While the existence of a handful of carbon-rich subgiants (commonly referred to as CH stars) with similar kinematical and spectroscopic properties to G77-61 do lend marginal evidence to this theory (Bond, 1974; Sneden and Bond, 1976), the dearth of low-luminosity candidates for this hypothetical population would refute this idea.

Second, the atmosphere of G77-61 has been polluted by mass transfer from a close binary companion. This theory provides an enticing solution to the problem because it can be directly tested with observations. Furthermore, mass transfer from an evolved companion is known to pollute the atmospheres of Ba and CH stars (Keenan, 1942; McClure et al., 1980a). However, at the time of its discovery, there was no photometric or spectroscopic evidence of a companion to G77-61, nor any evidence of a ultraviolet (UV) excess that could be attributed to a degenerate companion (Chandrasekhar, 1939; Koester and Chanmugam, 1990).

Confirmation that G77-61 is indeed a member of a binary star system came through radial-velocity monitoring (Dearborn et al., 1986). In this scenario, the binary companion to G77-61 was initially the more massive component of the binary, and hence, evolved at a faster rate. The companion, having evolved onto the AGB and ignited the triple-\( \alpha \) process, then dredged carbon-rich material to its surface before liberating this material via its stellar wind. This liberated carbon-rich material was subsequently accreted onto G77-61, thus polluting its atmosphere. The companion then evolved off the AGB and contracted to become a white dwarf, where it began its cooling process and eventually faded from visibility. The mass function of G77-61 places an approximate mass of the companion at \( \approx 0.6 \, \text{M}_\odot \), thus supporting the hypothesis that the companion is a white dwarf. Furthermore, the lack of UV excess observed for G77-61 implies that the unseen white dwarf companion is cool (\( T_{\text{eff}} < 6000 \, \text{K} \)), and therefore consistent with the kinematical age of G77-61.

It would be 15 years before additional dC stars were discovered, with the earliest of these uncovered through studies of faint high-latitude carbon stars (Green et al., 1991; Green and Margon, 1994; Margon et al., 2002). However, it was with the era of large-scale spectroscopic surveys such as the Sloan Digital Sky Survey (SDSS) and LAMOST (Ahumada et al., 2020; Cui et al., 2012), that the number of known dC stars has exploded (Green, 2013a; Si et al., 2014). The number of candidate dC stars now stands at over 1000, which is expected to further rise with the commissioning of upcoming spectroscopic surveys such as 4MOST (de Jong et al., 2019), WEAVE (Dalton et al., 2012), and DESI (DESI Collaboration et al., 2016). Despite the rising number of candidate dC stars, little is known of the population or its origins. Thus, this Thesis aims to answer a question that has remained unanswered for over half a century: *Are dC stars intrinsically carbon-rich, or extrinsically carbon-enhanced through mass transfer?*
1.2 Single star evolution

A star can be loosely defined as an object that is bound by its own gravity, that is radiating energy from an internal source (i.e. nuclear fusion; Carroll and Ostlie 2007). The evolution of a star begins with its birth from an interstellar, molecular gas cloud, consisting primarily of hydrogen, helium, and trace amounts of heavier elements. From a star’s birth, single star evolution can be thought of as a careful balancing act, where hydrostatic and thermal equilibrium must be maintained by nuclear burning fuel, where only a finite amount of fuel is available (Pols, 2011). As one fuel source depletes, a star will fall out of hydrostatic and thermal equilibrium, and subsequently evolve to the next phase of its evolution where it can burn the next fuel source, restoring hydrostatic and thermal equilibrium. When a star has burned all available fuel, it will either proceed to the white dwarf sequence or experience a core-collapse supernova, where the star’s fate depends on its mass. Stars with initial masses \( \lesssim 10 \, M_\odot \) will evolve onto the white dwarf cooling sequence, where they will radiate their remaining thermal energy. Alternatively, suppose a star’s initial mass is \( \gtrsim 10 \, M_\odot \). In that case, the star will collapse under its gravity resulting in a spectacular release of energy in a process known as a core-collapse supernova. Hence, stars evolve from their birth to their eventual death.

1.2.1 Hertzsprung-Russell diagram

The Hertzsprung-Russell diagram (HRD) is commonly used to display the stellar evolution of either a specific star, or a population of stars (e.g. a star cluster), in terms of its colour\(^1\) (which is a proxy for effective temperature) and luminosity (Hertzsprung, 1911; Russell, 1914). Because each separate stage of stellar evolution occupies a distinct region of the HRD, it is an incredibly useful tool when analysing and understanding stellar evolution. An example HRD consisting of 300,000 stars with high-quality Gaia eDR3 astrometry is shown in Figure 1.1.

1.2.1.1 The main sequence

The main sequence of the HRD consists solely of stars that are undergoing stable core hydrogen-burning while in hydrostatic and thermal equilibrium. Core hydrogen-burning is accomplished via either the proton-proton (pp) chain, or the CNO cycle depending on the initial mass of the star, with the pp-chain favoured in stars with masses in the range \( 0.3 \lesssim M/M_\odot \lesssim 1.3 \), and the CNO cycle favoured in stars with masses \( M \gtrsim 1.3 \, M_\odot \) (Salaris and Cassisi, 2005). The amount of time a star is able to spend on the main sequence mainly depends on the quantity of available hydrogen fuel in the stellar core, and the rate this core

---

\(^1\)If good quality effective temperature measurements exist for a star, or population of stars, it is common to replace colour with effective temperature and thus plot a Hertzsprung-Russell diagram.
Fig. 1.1: Left: An example HRD constructed for 300,000 stars with good quality astrometric and photometric data from Gaia eDR3. The main sequence, white dwarf sequence, and giant branch are all labelled. Right: The Hertzsprung-Russell diagram for the enclosed area in the left panel. This region of the HRD corresponds to the giant branch.
hydrogen fuel is burned. Because stellar mass and luminosity are closely related \(^2\), stars with higher masses burn at a higher luminosities and subsequently consume their hydrogen fuel faster. Therefore, a star’s main-sequence lifetime is anti-correlated with its initial mass, with more massive stars evolving off the main sequence more rapidly than lower mass stars (Iben, 1965; Thomas, 1967; Faulkner and Cannon, 1973). The main-sequence lifetime of a star can therefore vary dramatically from a few million years for stars with \(M \gtrsim 8 \text{M}_\odot\), to over a Hubble time for stars with initial masses \(M \lesssim 0.8 \text{M}_\odot\). However, regardless of initial mass, all single stars will spend approximately 90 per cent of their total lifetime on the main sequence (Kippenhahn et al., 1990).

A star’s initial position on the HRD is primarily dictated by its mass, but is also affected by its composition (Russell, 1914; Hurley et al., 2000). More massive stars exhibit higher effective temperatures and larger luminosities, and hence occupy the upper main sequence, while lower mass stars (that burn at lower effective temperatures and smaller luminosities) occupy the lower main sequence. Furthermore, at fixed mass, metal-deficient stars will tend leftward and upward on the main sequence (see Figure 1.2). The reason for this leftward trend is because metal-poor stars possess a lower opacity than their metal-rich counterparts. This lower opacity allows a larger proportion of the emitted photons to escape the envelope without being absorbed, reducing the heating and subsequent expansion of the envelope. Thus, lower-metallicity stars are smaller and hotter than their metal-rich counterparts at a given mass (Rogers and Iglesias, 1994). The trend to higher luminosities is most apparent for stars with \(M \lesssim 5 \text{M}_\odot\), because at these masses and low metallicities, opacity from bound-free absorption – the absorption of a photon and subsequent ionisation of an atom – is suppressed compared to metal-normal stars of the same mass, hence raising the luminosity of metal-poor stars. However, at higher masses, bound-free absorption becomes less prevalent because electron scatter becomes the main source of opacity (Pols, 2011). Thus, at a fixed mass, only low-mass, metal-poor stars are more luminous than their metal-normal cousins.

The right panel of Figure 1.2 displays a Hertzsprung-Russell diagram (HRD) of the main sequence phase of stellar evolution for a series of stars with masses in the range \(0.8 < M/\text{M}_\odot < 25\). Notably, at all masses, stars become cooler and more luminous as they evolve away from the main sequence. This is because core hydrogen-burning processes hydrogen to helium, thus increasing the molecular weight of the core. Therefore, for the star to remain in hydrostatic equilibrium, the pressure in the core must increase, which is only possible if the pressure in the stellar envelope decreases. Thus, the stellar envelope expands, decreasing the pressure in

\[ \frac{\text{L}}{\text{L}_\odot} = \left( \frac{\text{M}}{\text{M}_\odot} \right)^a ; 1 \leq a \leq 6 \]
Fig. 1.2.: Left: The zero-age main sequence of stars with a series of masses between 0.8 and $25 \, M_\odot$. The blue solid line and orange dashed line corresponds to the zero-age main sequence computed at [Fe/H] = 0.0 and −2.0, respectively. The blue crosses and orange points correspond to the labelled masses. Right: The zero-age main sequence is shown at the same masses discussed in the left panel and [Fe/H] = 0.0, however, the main sequence is also displayed for each mass labelled.

the envelope and the core, maintaining hydrostatic equilibrium. The expansion of the stellar envelope leads to a decrease in the effective temperature of the star, resulting in the main sequence evolutionary tracks shown in Figure 1.2.

Furthermore, as stars evolve along the main sequence, those that possess initial masses $\gtrsim 1.3 \, M_\odot$ display a clear hook in their evolutionary track (see the right panel of Figure 1.2), whereas lower-mass stars do not (Paczyński, 1970). This feature can be understood by considering whether the stellar core is radiative or convective. Because stars with masses $\gtrsim 1.3 \, M_\odot$ possess convective cores (owing to the high temperature dependence of the CNO cycle), hydrogen in the core is well mixed and its abundance can therefore be assumed to be homogeneous throughout the core. Hence, the depletion of hydrogen occurs approximately uniformly throughout convective cores. When hydrogen levels in a convective core fall below what is necessary to maintain stable core hydrogen-burning, the star suddenly falls out of thermal equilibrium, causing the star to contract. This contraction continues until the temperature surrounding the
core is hot enough to ignite shell hydrogen-burning, and restore thermal equilibrium. It is this contraction that causes the hook in the evolutionary track of higher mass stars.

By contrast, stars with masses $0.3 \lesssim M / M_\odot \lesssim 1.3^3$ possess radiative cores. The presence of a radiative core results in the rate of hydrogen-burning being strongly peaked towards the centre of the core. The gradient in hydrogen-burning across the core thereby results in the hydrogen abundance being inhomogeneous throughout the core, with the hydrogen abundance increasing radially away from the centre of the core. Consequently, the transition from core hydrogen-burning to shell hydrogen-burning is smooth and does not produce a hook in the HRD.

The commencement of shell hydrogen-burning marks the end of a star’s lifetime on the main sequence. The subsequent stages of stellar evolution are strongly dependent on the star’s initial mass, and occur on timescales much shorter than the main-sequence lifetime. In the following discussion, low-mass stars are loosely defined as those with initial masses $M \lesssim 1.3 M_\odot$, and intermediate-mass stars defined as stars with initial masses $1.3 \lesssim M / M_\odot \lesssim 7$. These boundaries in initial mass are only intended as a rough guide and are not to be taken as confident limits.

1.2.1.2 Main sequence turn-off to the asymptotic giant branch

After evolving off the main sequence, all stars undergo a phase of shell hydrogen-burning. The duration of this stage of stellar evolution is again dictated by the initial mass of a star. Upon leaving the main sequence, intermediate-mass stars (defined here by masses in the range; $1.3 \lesssim M / M_\odot \lesssim 7$) possess non-degenerate cores, and therefore, the temperature and density gradient between the shell hydrogen-burning layer and the core is small. This small gradient allows the star to remain in thermal equilibrium with a thick region of shell hydrogen-burning around its core, for several Myr.

However, as shell hydrogen-burning progresses and nuclear burning products are deposited onto the core, the core begins to contract. Initially, the contraction is slow, but once the core mass exceeds the Schönberg-Chandrasekhar limit\(^4\), the rate of contraction increases drastically (Schönberg and Chandrasekhar, 1942). Once this point is reached, the temperature and density gradient between the shell hydrogen-burning layer and the core becomes large, which causes

---

\(^1\)Stars with masses on the main sequence below $M \lesssim 0.3 M_\odot$ are fully convective and hence do not possess a convective or radiative core. Furthermore, their expected main-sequence lifetime is longer than a Hubble time and hence, little is known of how these stars evolve off the main sequence.

\(^4\)The Schönberg-Chandrasekhar limit describes the maximum mass of an isothermal, non-fusing core that can support an enclosing envelope. Typically, the limit is taken as between 10 and 15 per cent of the total stellar mass.
the thickness of the hydrogen shell to decrease. While the core contracts, the envelope grows in a process that lasts less than a Myr. During the envelope’s expansion, the effective temperature steadily decreases and deep convection regions begin to form in the envelope. This region of the HRD is called the “Hertzsprung gap” because few stars are observed during this phase of single star evolution, owing to how rapidly stars evolve through the Hertzsprung gap (Mengel et al., 1979; Ayres et al., 1998).

Stars with initial masses $\lesssim 1.3 M_\odot$ are close to degenerate by the time they evolve off the main sequence (Renzini and Fusi Pecci, 1988). Thus, the temperature and pressure gradient between the core and the envelope remains relatively constant throughout shell hydrogen-burning. Because the transition from core to shell hydrogen-burning is gradual in low-mass stars, due to their radiative cores they can maintain hydrostatic and thermal equilibrium throughout shell hydrogen-burning, which can last for a few Gyr. This phase of stellar evolution is known as the “subgiant branch” of the HRD and is relatively well populated owing to the relatively slow process of shell hydrogen-burning. During the subgiant branch phase of stellar evolution, the stellar core will begin to contract due to its increasing mass as material is deposited on it from the shell hydrogen-burning layer above. The contraction of the core drives an expansion in the envelope that results in a decrease in effective temperature. Thus, the subgiant branch is characterised by a general reddening of a star while it emits with roughly constant luminosity.

During the next phase of stellar evolution stars of all masses will ascend the red giant branch (RGB) of the HRD. This region of the HRD is characterised by a rapid increase in luminosity and subtle decrease in effective temperature, owing to the expansion of the stellar envelope. The increase in luminosity requires a higher rate of shell hydrogen-burning, which in turn increases the mass of the core, and increases the rate of core contraction. As the core continues to contract, the radial expansion of the envelope continues, which further increases the stellar luminosity. This produces a cyclical process that causes stars to accelerate their evolution as they ascend the RGB (Eggleton, 1971).

Furthermore, the radially expanding envelope causes the effective temperature to decrease, resulting in an increase in opacity within the stellar envelope. With the steady increase in opacity, convection zones throughout the envelope begin to reach deeper into the stellar interior. Eventually, the convection zones reach deep enough into the stellar interior that nuclear processed material can be dredged to the star’s surface. This process is called “the first dredge-up” and is the first in a series of convective episodes that can alter the surface chemistry of a star (Brown et al., 1989; Gratton et al., 2000).
Due to the inflated radius of stars on the RGB, the role of mass loss becomes important (Reimers, 1975; Iben and Renzini, 1984). Because the stellar envelope may expand to several hundred solar radii (dependent on mass; Hurley et al. 2000), the stellar surface becomes only loosely gravitationally bound to the star. Hence, a large photon flux carrying a high radiation pressure can cause material to be liberated from the stellar surface. The exact mechanism that drives mass loss in RGB stars is not well understood, but studies suggest that around 20 per cent of a star’s mass is lost on the RGB (McDonald and Zijlstra, 2015).

As a star ascends further up the RGB, its helium core continues to contract, and consequently the core temperature begins to rise. Once this temperature approaches $10^8$ K, helium burning commences via the triple-$\alpha$ process, thus synthesising helium into carbon (Salpeter and van Horn, 1969). Unlike hydrogen burning, where the exact nuclear reactions (i.e. pp-chain or CNO cycle) depend on the stellar mass, all stars burn helium via the triple-$\alpha$ process, regardless of mass. However, the onset of helium ignition is vastly different in low-mass stars compared to intermediate and high-mass stars, due to the considerably different pressures in their cores. Because low-mass stars possess degenerate cores, helium ignition is fast and violent, whereas in intermediate and high-mass stars, helium ignition is initially much more stable.

At the tip of the RGB, and hence, the onset of helium burning, all low-mass stars possess a degenerate core of mass $M_{\text{core}} \approx 0.7 M_\odot$ (Pols, 2011). Initially, helium burning in low-mass stars begins in a shell around the core, quickly spreading to consume the entire core. Because the core is degenerate, it is supported by electron degeneracy pressure, and hence, the pressure in the core is independent of its temperature. Thus, as the onset of core helium-burning raises the core temperature, the core is unable to expand. This lack of expansion leads to a dramatic increase in the rate of core helium-burning in a process known as the “helium flash”. Around 3 per cent of all helium in the core is consumed during the helium flash which lasts for no longer than a few seconds (Vassiliadis and Wood, 1993). Furthermore, because all low-mass stars possess a degenerate core of roughly equal mass, the helium flash is relatively uniform in all low-mass stars.

After the helium flash, low-mass stars will settle into thermal equilibrium with stable core helium-burning, and shell hydrogen-burning fuelling the star. Because the core mass dictates the stellar luminosity, and all low-mass stars possess cores of roughly equal mass, these low-mass stars form an overdensity in the HRD. This clustering of low-mass, core helium-burning stars is not well understood, but studies suggest that around 20 per cent of a star’s mass is lost on the RGB (McDonald and Zijlstra, 2015).

\[3\] In degenerate material, fermion particles must obey the Pauli exclusion principle - no two fermions may occupy the same energy state. Thus, when the lowest energy state is filled, other electrons are forced to occupy higher and higher energy states. This process creates a pressure, commonly referred to as the electron degeneracy pressure.
known as the “red clump”. While the luminosities of red clump stars are approximately uniform, their effective temperatures can, and do, vary due to their differing envelope masses (Girardi and Salaris, 2001). Stars with less massive envelopes possess hotter surface temperatures in the red clump as they do not expand to radii as large as more massive stars.

Where the ignition of helium burning in low-mass stars is violent and unstable, the same process in higher-mass stars is much more mundane. In intermediate and high-mass stars, helium burning begins in the centre of the core due to the high temperature and pressure, and proceeds via the triple-$\alpha$ process. Once the carbon abundance begins to rise, it too can fuse with an $\alpha$ particle to form oxygen, with this reaction becoming the dominant nuclear reaction in the core. The final ratio of carbon to oxygen in the core is dependent on the rate of this reaction, which in turn depends on the stellar mass.

The entirety of core helium-burning lasts for several Myr, though this is mass dependent. In low and intermediate-mass stars ($M \lesssim 6 M_\odot$), core helium-burning occurs in a region of the HRD known as the “horizontal branch”. The early phase of the horizontal branch is characterised by an increase in effective temperature, owing to the contraction of the stellar envelope. During these early phases of the horizontal branch, the luminosity is dictated by the mass of the core, and therefore remains almost constant throughout core helium-burning. Eventually, the core will again begin to contract due to its rising molecular weight. This is proceeded shortly after by the exhaustion of helium fuel in the core. The contracting core causes radial expansion in the stellar envelope, thereby reducing the effective temperature and raising the stellar luminosity. The star is now predominately fueled by shell helium-burning (though shell hydrogen-burning does continue in a layer above the helium-burning layer), and the star begins its ascent of the asymptotic giant branch (AGB).

1.2.1.3 Asymptotic giant branch

The early phase of the AGB is similar to the early phase of the RGB. The envelope of the star expands, while the core contracts, leading to a drop in effective temperature and a rise in luminosity. The vast majority of the energy is now provided by shell helium-burning, while shell hydrogen-burning is almost extinguished. A temperature gradient builds up within the envelope due to its expansion, causing the convection zones in the envelope to reach deeper into the stellar interior. For stars of intermediate mass $\gtrsim 4 M_\odot$, the deeper convection leads to the “second dredge-up”, where the convective zones reach deep enough into the stellar envelope that helium and nitrogen (processed from the CNO cycle) are dredged to the stellar surface (Becker and Iben, 1979). Stars with masses below $4 M_\odot$ avoid the second dredge-up because
their hydrogen-burning layer is not entirely extinguished by this stage of evolution and can not be penetrated by the convection zones.

As time progresses, the luminosity of the helium-burning shell begins to fall as the helium fuel is spent. This fall in luminosity causes the layers above the helium shell to begin contracting, consequently increasing the rate of shell hydrogen-burning (in the layer directly above the helium shell). However, the rates of shell hydrogen and helium-burning are not identical. Because the rate of helium-burning is quicker than hydrogen-burning, the helium shell expends its fuel faster than burning in the hydrogen shell can replenish it. Thus, the helium shell begins to burn quasi-periodically in a series of intermittent helium flashes (Marigo et al., 2008; Maraston and Strömbäck, 2011). These unstable episodes of shell helium-burning cause the envelope to expand, subsequently cooling the hydrogen-burning shell, and lowering its reaction rate. After the short helium flash, helium-burning momentarily ceases, and the envelope once again contracts. The rate of shell hydrogen-burning may then again increase, depositing more fuel onto the helium shell. These thermal pulses repeat for the remainder of the ascent along the AGB in a phase of evolution dubbed the “thermally-pulsing AGB”. Furthermore, the thermal pulses drive strong stellar winds that directly lead to efficient mass loss at the surface of the AGB star (Iben and Renzini, 1983a; Bloecker, 1995). In fact, observations of many AGB stars reveal that they are enshrouded in a cloud of dust that has been liberated from the upper atmosphere of the star by the radiation pressure of the stellar wind.

The periodic ignition of the helium-burning shell results in the convective zones within the envelope reaching deeper into the stellar interior. Furthermore, the vast amounts of energy produced by each helium flash causes convection to manifest between the helium and hydrogen-burning shells. Eventually, these two convective regions (one in the envelope, the other between the burning shells) merge and proceed to dredge carbon produced via the triple-$\alpha$ process in the helium shell to the upper atmosphere of the AGB star (Iglesias and Rogers, 1993; Busso et al., 1999; Herwig, 2005). This process is known as the “third dredge-up”. Contrary to its name, the third dredge-up is not a single episode (like the preceding dredge-ups) but instead is a series of deep convection events that probe into the burning layers of the star. With every pulsation, more processed nuclear material is dredged to the stellar surface. In stars of masses $1.3 \lesssim M/M_\odot \lesssim 4$ (depending on metallicity), the abundance of carbon in the upper atmosphere will surpass that of oxygen (Karakas, 2010; Rau et al., 2017). The relatively cool temperatures in the upper atmosphere of AGB stars make it possible for the CO molecule to form, which subsequently consumes all of the available oxygen (Russell, 1934; Spinrad and Wing, 1969), leaving the excess carbon to form molecules with other elements present in
the upper atmosphere (e.g. CN, C, CH; Johnson 1927). Therefore, thermally-pulsating AGB (TP-AGB) stars often appear red in colour, owing to their cool effective temperature and high carbon abundance, and are usually observed enshrouded in dust due to their strong stellar winds driving high rates of mass loss.

AGB stars are also responsible for the majority of slow-neutron capture (s-process) elements in the Universe (Gallino et al., 1998; Busso et al., 1999). Through the s-process, Fe nuclei undergo neutron capture to form an isotope with an atomic mass one larger than its pre-neutron capture atomic mass. If the isotope is stable, further neutrons can be captured forming a series of stable isotopes. If any of the isotopes are unstable, then $\beta$-decay occurs, producing an element with the same atomic number as the previous isotope in the series. This new element can then capture additional neutrons and continue the series of isotopes.

The s-process relies on an abundance of free neutrons for nuclei to capture, that are produced in the helium shell surrounding the core – primarily through the synthesis of carbon-13 to oxygen-16. These free neutrons are then captured by iron peak nuclei to create heavier elements. Because the s-process is slow, beta decay may occur for unstable isotopes between successive neutron capture events, leading to the production of barium, strontium, yttrium, etc. Finally, the s-process elements are brought to the upper atmosphere of the AGB star through the episodes of deep convection (the third dredge-up). Mass loss from the surface of the star can then liberate the s-process elements from the AGB star, enriching the interstellar medium with heavy elements.

The number of thermal pulses an AGB star can sustain is a function of the decreasing mass of the stellar envelope and the increasing mass of the core. The exact number of thermal pulses a star will exhibit depends strongly on its mass, metallicity, and the point of onset of the superwind (Vassiliadis and Wood, 1993). However, the TP-AGB typically lasts for $< 2$ Myr.

Observational evidence showing that TP-AGB stars possess strong mass loss is evident through inspection of their spectral energy distributions, that show large excesses in the infrared (Höfner and Olofsson, 2018). While the exact mechanism driving strong mass-loss in TP-AGB stars is not known (Höfner, 2009; Bladh and Höfner, 2012), it is likely that the combination of thermal pulses and radiation pressure are the leading contributors (Bloecher, 1995). In this picture, the thermal pulses cause a series of shock waves throughout the stellar envelope, that in turn, cause the radius of the star to expand periodically. These shock waves can lead to the radius of the stellar envelope doubling. At such large distances from the burning layers (where energy is produced in the star) the temperature can drop as low as a few thousand
Kelvin – cool enough for molecular dust to form. If the opacity of the dust is high enough, then radiation pressure from the star drives the dust to larger distances, thus inducing a wind. This process of rapid mass loss is called the “AGB superwind” and strips the AGB star of the majority of its envelope, thus, ending evolution along the AGB.

1.2.1.4 Post-AGB evolution

Upon leaving the AGB, a star will evolve horizontally (at constant luminosity) across the HRD towards higher effective temperatures (making the star appear bluer). Depending on the mass of the core, the remaining stellar envelope (post-AGB superwind) is typically only a few hundredths of a solar mass, and sustains shell hydrogen-burning. The products from shell hydrogen-burning are deposited onto the core, increasing its mass. Because the core is degenerate, the increase in core mass results in the stellar radius contracting. Furthermore, wind driven mass-loss continues to strip away what is left of the envelope, contributing to the shrinking stellar radius.

Shell hydrogen-burning is finally extinguished once the envelope mass falls to around \( \approx 10^{-5} \, M_{\odot} \) (Pols, 2011). With no more fuel to burn, the star begins to cool. This is the start of the white dwarf cooling track, where the star will spend the rest of its life radiating its remaining thermal energy, unable to produce more energy sustainably (Iben and Renzini, 1983a). Thus, the star will now evolve down the HRD to lower effective temperatures and luminosities, while maintaining a constant radius (owing to its degenerate nature). Because no (stable) nuclear reactions occur in white dwarfs, the star can now be considered dead.

1.2.2 Dwarf carbon stars and single-star evolution

Single-star evolution states that carbon is produced through shell or core helium-burning via the triple-\( \alpha \) process. The conditions necessary for helium to begin fusing in single stars is therefore not achieved until the tip of the RGB. Furthermore, material processed by the triple-\( \alpha \) process remains deep in the stellar interior until it can be brought to the surface by the occurrence of the third dredge-up during the TP-AGB. Therefore, the existence of a population of bona-fide carbon dwarfs is an oxymoron from the point of view of single star evolution. Because dC stars lie solely on the main sequence, they are fuelled by stable hydrogen burning, and hence, can not have intrinsically produced carbon. This does not, however, discount the possibility that dC stars are formed intrinsically carbon-rich, for example, if they are formed in a carbon-rich environment. However, assuming that dC stars, like the majority of other stars in the Galaxy, are not born in a carbon-rich environment, then single star evolution is a poor model for understanding the existence of dC stars.
1.3 Binary star evolution

At least half of all stars in the sky are actually members of multiple systems, that consist of two or more stars that are gravitationally bound to each other, and orbit a common centre of mass (Carroll and Ostlie, 2007). Binary stars are multiple systems that possess exactly two stars that are gravitationally bound to each other. Commonly, the brightest star of a binary system is referred to as the primary, and the fainter star in the binary is referred to as the secondary. Furthermore, binaries can be identified as visual, astrometric, eclipsing, or spectroscopic binaries depending on the physical parameters of the constituents of the binary.

Astrometric, visual, and eclipsing binaries are all detected through photometric time-series observations. Visual binaries are observed as pairs of stars that are gravitationally bound, where each binary component can be individually resolved in photometric observations. The orbits of visual binaries can be traced around the centre of mass of the binary and typically last from a few months to several years (where the exact orbital period is a function of the mass and orbital separation of the binary components).

Astrometric binaries are pairs of gravitationally bound stars where the photocentre of the binary is observed to move relative to the binary’s centre of mass. Unlike visual binaries, astrometric binaries do not require that the secondary is visible. The motion of the binary’s photocentre is typically observed as an oscillation in photometric time-series observations and is caused by the gravitational interactions between the stars within the binary.

Eclipsing binaries are again detected through photometric time-series observations and consist of two gravitationally bound stars where, due to their orientation in space relative to Earth, the light from one of the binary components is periodically hidden behind the other component. Hence, the luminosity of an eclipsing binary is observed to dim periodically as one star passes behind its binary companion.

As two gravitationally bound stars orbit their centre of mass, their spectral lines experience a Doppler shift. These wavelength shifts can be measured with a spectrograph and used to calculate radial velocities. Binaries detected through this method are known as spectroscopic binaries. If an observation of the binary reveals that spectral features of both binary components are present, then the binary is classified as an SB2. Otherwise, if only the spectral features of the primary are visible, the binary is classified as an SB1.

Regardless of how a binary is detected or classified, the physics that dictate the orbital motions of a binary system are well defined in a set of three laws. These laws are commonly known as Kepler’s laws, and were originally derived to characterise the orbits of the planets.
around the Sun. Kepler’s laws can, however, be generalised to any two-body gravitational problem.

### 1.3.1 Keplerian orbits

Kepler’s laws characterise the binary motion of both stars within a binary system (Russell, 1964). Originally, these laws were determined empirically and were later mathematically derived through Newton’s laws of gravitation. Kepler’s laws state the following:

1. Stars orbit each other in elliptical orbits, with their centre of mass as one common focus.
2. A line segment connecting each of the stars with the centre of mass sweeps out over equal areas during equal time intervals.
3. The square of a star’s orbital period is proportional to the cube of its distance from the centre of mass.

A practical application of Kepler’s laws is to determine the orbital parameters of a binary system, such as: the orbital period, velocity semi-amplitude, systemic velocity, eccentricity, and orbital inclination (if possible). A popular method to determine these orbital parameters is through radial-velocity monitoring of a binary system. The centre of mass of each binary system has an intrinsic line-of-sight velocity referred to as the systemic velocity. However, by taking time-resolved radial velocity measurements of each visible component of the binary, it is possible to measure the Doppler shifts in each spectrum and hence, deduce the orbital motion of each visible component around the centre of mass. Furthermore, if it is possible to determine the orbital parameters of each component of the binary (e.g. in an SB2 binary), then the mass ratio of the binary can be derived. Alternatively, if only one component is visible, radial velocity measurements alone can not yield the mass ratio of the binary, but useful constraints can be placed on each of the orbital parameters (e.g. by computing the binary mass function).

### 1.3.2 Orbital evolution

Kepler’s laws are a useful tool for understanding the orbital motion of a binary at any moment in time. However, because orbital motion arises due to gravitational interactions between the components of a binary, the orbital parameters of a binary depend on the mass of each component in the binary. Therefore, any change in the mass of either the primary or secondary will produce a change in the orbital parameters of each component of the binary. Because stars evolve along the HRD and undergo mass loss with varying intensities at numerous phases of stellar evolution, the orbital motion of a binary system will also evolve depending on how the total mass of the system changes.
If the initial orbital separation of a binary is sufficiently long that the individual stellar evolution of each component does not interfere with its companion, then each component of the binary will evolve as if it is a single star. However, if at any stage of a binaries’ lifetime, the separation between the components becomes short, such that the evolution of one (or both) components may interfere with the other, then these interactions between the components can significantly alter the stellar evolution of one or both stars within the binary. The consequences of any such interactions (e.g. mass transfer) can affect the atmospheric chemistry and orbit of the binary.

Observational evidence of binary interactions altering stellar evolution are abundant. For instance, blue stragglers and gamma-ray bursts are thought to be the product of merging binary components (Preston and Sneden, 2000; Fregeau et al., 2004; Woosley and Heger, 2006), while novae and the Algol class of stars are thought to be produced through mass transfer from one binary component to the other (Lynden-Bell and Pringle, 1974; Warner, 2003; Budding et al., 2004). Furthermore, observations of hot subdwarfs suggest that while ascending the RGB, their atmospheres were stripped away by close binary companions (Mengel et al., 1976; Han et al., 2002). Thus, single star evolution alone is not adequate to understand the menagerie of stars spread across the HRD, and numerous different astrophysical events.

1.3.3 Mass transfer

Under certain conditions, the interactions between binary components can result in mass being stably transferred from the primary onto the secondary star. While the efficiency of the mass transfer process and the quantity of mass transferred can vary depending on many factors, mass transfer, in general, will alter the stellar evolution of the mass donor and accretor and the orbital evolution of the binary. Specifically, mass transfer can either truncate or extend the lifetime of a star, in addition to modifying atmospheric abundances (Hurley et al., 2002; Chen and Han, 2008).

1.3.3.1 Roche-lobe overflow

Every star possesses a gravitational potential. If a star is a component of a binary system, its gravitational potential is perturbed by its companion, thus resulting in a shared gravitational potential around the whole binary system, known as the Roche geometry. Close to the surface of each star, the potential is almost spherical, however, at larger distances from each star, the potential is distorted by the gravitational potential of the binary companion. The Roche lobe is defined as the region surrounding a star within a binary system where orbiting material is gravitationally bound to that star, and material outside is lost (Kopal, 1959).
If both stars do not fill their respective Roche lobes, the binary is said to be detached. While both members of the binary remain on the main sequence, they essentially evolve as single stars, though evolved stars in a detached binary may interact through their stellar winds. However, it is still possible for the binary companions to exert tidal forces on each other, thereby affecting their rotation.

While some binaries will remain detached throughout their lifetime, the stellar evolution of the primary, secondary, or both stars within the binary may lead to either one or both stars filling their respective Roche lobes, thus forming a semi-detached or contact binary. In the scenario where one star fills its Roche lobe, the binary is referred to as semi-detached, and a binary with both stars filling their Roche lobes is called a contact binary. Interactions between the binary components in semi-detached and contact binaries can result in the transfer of mass and angular momentum and thus affect the chemical evolution of the binary components and orbital evolution of the binary, respectively.

Consider a binary system containing two stars with unequal masses where the primary possesses a mass \( \lesssim 8 \, M_\odot \). The more massive star will evolve off the main sequence earlier than its less massive companion. As discussed in Section 1.2, when stars depart the main sequence and begin their ascent of the giant branch, their radii begin to expand. If the radii extend far enough to fill the stars Roche lobe (forming a semi-detached binary), then material can transfer from the primary to the secondary through any point on the surface of the Roche lobe where the gravitational force from each star is in equilibrium (notably this occurs at the Lagrangian point L1). Because mass may transfer through the L1 point, there is a decrease in mass within the Roche lobe of the expanding star, and hence, the Roche lobe of the expanding star begins to shrink. If the amount of material transferred through the L1 point is large enough to cause the radius of the expanded star to decrease faster than its Roche lobe shrinks, then the transferred mass can be stably accreted by the secondary star (Hurley et al., 2002; Paxton et al., 2011). This process is known as Roche-lobe overflow, and can increase the mass of the secondary, while also changing its atmospheric chemistry (if the composition of the accreted material is significantly different to the atmospheric chemistry before accretion).

Alternatively, if the radius of the evolved star continues to grow despite mass escaping through the L1 point, then the mass transfer is unstable. Rather than being accreted by the secondary, material begins to build up in the Roche lobe of the secondary until it is filled as well. At this point, a common envelope is developed where the Roche lobes of both stars are filled (Izzard et al., 2012; Ivanova et al., 2013). Because the material filling the Roche lobes is not necessarily rotating with the binary, both stars experience a drag force. Energy from the
orbit of each star within the binary is lost overcoming the drag force, and hence, to conserve angular momentum, the separation between the primary and secondary must shrink. In some cases, the separation becomes small enough that the binary can merge (e.g. in the case of blue straggler stars; Preston and Sneden 2000). Observations of post-common envelope binary systems consist solely of binaries that are separated by a few tenths of an au, and are hence too close to accommodate the giant progenitor of the white dwarf (Kruckow et al., 2021). Hence, the post-common envelope binary systems suggest that large amounts of angular momentum are lost during the common envelope phase (Rebassa-Mansergas et al., 2007; Rebassa-Mansergas et al., 2010).

1.3.3.2 Wind capture

Through the mechanism of wind capture, mass transfer can also occur in binary star systems that remain detached throughout their entire lifetime. Consider again the scenario where a binary system consists of a primary that is more massive than the secondary. The primary therefore evolves off the main sequence sooner than the secondary, and subsequently begins its ascent of the giant branch. During the RGB and AGB phases of evolution, a star will experience large rates of mass loss due to the strong stellar wind (Reimers, 1975; Bloecker, 1995). Once material is no longer gravitationally bound to the primary, it expands radially away from the star. Eventually, the separation of the expelled material becomes comparable to the orbital separation of the primary and the secondary. At this point, the secondary may begin to accrete the expelled material. The mean accretion rate on the secondary can be estimated by the Bondi-Hoyle-Lyttleton accretion rate, that assumes that the velocity of the expelled material is much larger than the orbital velocity of the secondary (Bondi and Hoyle, 1944). Furthermore, because the process of wind capture may entail a change in mass for both the primary and the secondary, the binary may undergo orbital evolution. If the amount of mass (and angular momentum) stably transferred is large, the separation between the primary and secondary can shrink (possibly leading to the binary becoming semi-detached). However, if little or no mass is accreted, then the stellar wind carries away mass and angular momentum from the binary system, thus leading to the primary and secondary drifting to larger separations.

1.3.3.3 Wind-Roche lobe overflow

The need for an additional mass transfer mechanism was born out of the failure of binary population synthesis models to correctly and accurately reproduce observations of various binary populations (Izzard et al. 2009; and references therein). For these problematic binary populations (that are discussed in greater detail in Section 1.3.4), neither Roche-lobe overflow,
nor wind capture are able to transfer sufficient amounts of mass or angular momentum. Wind-Roche lobe overflow is therefore proposed as a theoretical mass transfer mechanism that walks the line between classical Roche-lobe overflow and Bondi-Hoyle-Lyttleton accretion, in an attempt to maximise the efficiency of mass and angular momentum transfer (Abate et al., 2013).

The Bondi-Hoyle-Lyttleton prescription of wind mass transfer is a good estimation when the wind velocity is fast compared to the orbital velocity (Edgar, 2004). This typically occurs in binaries with a wide separation as it affords the stellar wind ample time to accelerate to its maximum velocity. However, for smaller orbital separations where the wind may not accelerate to its full escape velocity, Bondi-Hoyle-Lyttleton accretion becomes a poor mass transfer model (Karovska et al., 2005; Blind et al., 2011).

Wind-Roche lobe overflow is an alternative mass transfer model specifically for the regime where the wind velocity is approximately equal to the orbital velocity of the secondary. In the wind-Roche lobe overflow model, the radius of the primary’s Roche lobe is smaller than the distance required for the wind to reach its escape velocity. Thus, the stellar wind of the primary fills its Roche lobe, and material begins to flow through the L1 point into the Roche lobe of the secondary. This has the effect of focusing the stellar wind into the plane of the secondary, thus increasing the efficiency of mass transfer (Mohamed and Podsiadlowski, 2007; de Val-Borro et al., 2009).

The wind-Roche lobe overflow model has been particularly successful in modelling the mass transfer from AGB stars onto their companions. Typical AGB wind velocities are on the order of a few km s$^{-1}$ and are comparable to the orbital velocities of a binary companion located at several au. As such, Bondi-Hoyle-Lyttleton accretion may be a poor mass transfer model for these systems. Mass transfer simulations comparing wind-Roche lobe overflow to Bondi-Hoyle-Lyttleton accretion and Roche lobe overflow indicate that accretion efficiency may be up to several times higher for the wind-Roche lobe overflow model (de Val-Borro et al., 2009; Abate et al., 2013). This greater accretion efficiency is consistent with observations of post-mass transfer binary systems where the primary has ascended the AGB (Jorissen et al., 2019).

1.3.4 Carbon-enhanced post-mass transfer binaries

Prime examples of how binary evolution can drastically affect the atmospheric chemistry of the secondary star can be found in the various populations of carbon-enhanced post-mass transfer binaries. The existence of these populations was established through large spectroscopic
surveys, that revealed individual stars with peculiar atmospheric abundances given their location on the HRD. These stars were initially discovered on the main sequence turn-off and the RGB (owing to their relatively high luminosities), however, atmospheric abundance measurements displayed a higher than expected abundance of elements associated with nucleosynthesis on the AGB (e.g. barium, carbon, and other s-process elements). The stars were then categorised into separate populations based on their atmospheric abundances.

The first of these populations are the CH stars (McClure, 1984; McClure, 1997) - named due to the presence of the strong CH molecular bands in their optical spectra (amongst other carbon molecules; e.g. CN). These stars typically lie on the RGB and hence, would not be expected to possess such high carbon abundances according to single star evolution. Overall, the CH population exhibits weak metal lines indicating that the population is old.

Barium (Ba) stars account for \(\approx 1\) per cent of all G and K giants found on the RGB, and were classified based on their overall abundance of barium – an s-process element synthesised during the stars evolution along the AGB (McClure and Woodsworth, 1990; Luck and Bond, 1991). The Ba population is effectively the higher metallicity counterpart to the CH star population. However, their carbon abundance is not sufficient to be considered true carbon stars (despite showing carbon enhancements).

The last of these populations are the carbon-enhanced metal-poor-s stars (CEMP-s; Aoki et al. 2007; Yong et al. 2013). As their name suggests, these stars are classified based on their optical spectra that possess weak metal lines, clear carbon absorption features, and a multitude of s-process elements. Similarly to the Ba and CH populations, CEMP-s stars also lie on the RGB, which is unexpected given their optical spectra. CEMP-s stars are more metal-deficient than CH and Ba stars, and typically possess greater s-process enhancements than CH stars.

Binary evolution was hypothesised as a possible explanation for the elemental abundances measured in the optical spectra of the Ba, CH, and CEMP-s populations. The specific scenario proposed begins with a binary with unequal initial masses, where the primary had evolved onto the giant branch more rapidly than the secondary. Carbon and s-process elements were then synthesised during the primary’s ascent of the AGB, before being transferred to the secondary, thus polluting its atmosphere. The primary then evolved onto the white dwarf cooling track, leaving the polluted secondary to be observed (McClure, 1984; McClure and Woodsworth, 1990).

Despite the proposed binary nature of Ba, CH, and CEMP-s stars, none of these populations display any optical evidence for a binary companion. This is perhaps unsurprising, because if
this hypothesis is correct, it is unlikely that a white dwarf could be seen next to an RGB star due to the vast discrepancy in their respective luminosities. Confirmation that these populations were binary systems eventually arrived through dedicated radial-velocity monitoring surveys (McClure et al., 1980a; McClure and Woodsworth, 1990; Lucatello et al., 2005b; Starkenburg et al., 2014). All three populations possess orbital period distributions that range from a few hundred to several thousand days, and their mass functions are consistent with possessing white dwarf companions (Hansen et al., 2016; Jorissen et al., 2016a; Jorissen et al., 2019). Hence, Ba, CH, and CEMP-s stars are consistent with being the product of mass transfer during their companions’ ascent of the AGB.

The Ba, CH, and CEMP-s populations effectively form one continuum of carbon-enhanced post-mass transfer binary stars, that at least depends on initial metallicity, mass transfer process, and initial masses of the binary. While the exact process of mass transfer remains uncertain for all three populations, it is clear that efficient mass transfer can occur from an AGB star onto a lower mass companion (Izzard et al., 2010; Matrozis et al., 2017). More recent research has suggested that wind-Roche lobe overflow is the preferred mechanism for mass transfer, however, no observational data currently exists to reinforce simulations (Abate et al., 2013).

1.3.5 Dwarf carbon stars and binary stellar evolution

At a first glance, dC stars appear to be similar to other populations of carbon-enhanced post-mass transfer binaries. Their optical spectra clearly possess strong molecular carbon absorption features, and their position on the HRD – the main sequence – indicates that they can not intrinsically produce carbon.

Furthermore, kinematical studies of dC stars have suggested that the population is broadly consistent with membership to the thick disc and stellar halo (Farihi et al., 2018), implying that the dC population, in general, is metal-poor. The only dC star with a determined metallicity is the prototype dC star, G77-61 ([Fe/H] = −4.0; Plez and Cohen 2005), and is therefore one of the most metal-poor stars discovered to-date. Binary population synthesis models actually predict that it is easier to alter atmospheric chemistry in metal-deficient stars (de Kool and Green, 1995). For example, consider the mass required to enhance the C/O ratio above unity in the Sun. Spectroscopic measurements of the Sun’s atmosphere indicate that elements heavier than helium account for approximately one per cent of the Sun’s total mass (Lodders, 2003). Moreover, oxygen accounts for 40 per cent of the mass of these elements heavier than helium and C/O ≈ 0.3. Therefore, the Sun would need to accrete ≥ 0.0027 M⊙ of pure carbon to achieve C/O ≥ 1.
Now consider a star with $M_\ast = 1M_\odot$, but with elements heavier than helium accounting for only 0.1 per cent of the star’s total mass. Again, assume this star is similar to the Sun such that oxygen accounts for 40 per cent of the mass of the heavier than helium elements and $C/O \approx 0.3$. This star would only have to accrete $\geq 0.00027M_\odot$ of carbon to attain $C/O \geq 1$. Thus, metal-deficient stars require less mass to be transferred before enhancing $C/O$ beyond unity.

The binary formation mechanism of dC stars is certainly an attractive hypothesis to explain their existence. As with many mass transfer systems, a dC star will begin its life as the secondary star in a binary system. The more massive primary will then evolve off the main sequence and ascend the giant branch. During the primary’s ascent of the AGB, carbon is produced via the triple-$\alpha$ process and is subsequently dredged to the surface through a series of dredge-up episodes that are driven by the numerous thermal pulsations towards the tip of the AGB. Once at the surface, carbon becomes well mixed throughout the envelope and the atmospheric $C/O$ ratio exceeds unity, leading to the formation of various carbon molecules. These molecules subsequently condense to form dust, before radiation pressure drives the carbon-rich material away from the primary.

The next phase of the formation of a dC star is ambiguous at best. While it is obvious that the carbon-rich material liberated from the primary is transferred to the secondary, the exact mechanism of how this happens is unknown. To-date, there is an insufficient amount of data to determine whether mass transfer proceeds via Roche-lobe overflow, wind capture, or wind-Roche lobe overflow. However, assuming dC stars pre-accretion have an initial mass of approximately $0.4M_\odot$, elements heavier than helium account for 0.1 per cent of the stars total mass, carbon accounts for 40 per cent of the heavier than helium mass, and that initially $C/O = 0.3$, then the star would need to accrete $\approx 0.000112M_\odot$ or 0.03 per cent of the star’s initial mass in pure carbon to enhance $C/O$ above unity.

Once the mass transfer process is complete, the dC star will have accreted a sufficient amount of carbon-rich material so that its atmosphere is now carbon-enhanced. The primary, having completed its ascent of the AGB, now joins the white dwarf cooling sequence, where it radiates away its remaining thermal energy. At the end of this process, the binary system consists of a white dwarf and a dC star.

Historically, observational evidence supporting the binary formation mechanism has been sparse (Green and Margon, 1994; Margon et al., 2002; Downes et al., 2004). Despite the theoretical formation mechanism predicting that dC stars possess white dwarf binary companions, only a
dozen out of over a thousand candidate dC stars possess a composite spectrum (Green, 2013a; Si et al., 2014). Furthermore, before the results presented in this thesis, only the prototype dC was known to be a spectroscopic binary (Dearborn et al., 1986).

1.4 Thesis outline

This thesis presents observational evidence supporting the binary nature of the dC star population, through a radial-velocity monitoring survey.

Chapter 2 contains a discussion on the reduction of spectroscopic and photometric data. Chapter 3 discusses the various time-series analysis methods that have been used to analyse photometric and spectroscopic datasets discussed in this thesis, in addition to a comparison between various time-series methods in the regime of sparsely sampled time-series data. The discovery of six Hα, emission-line, short-orbital and photometric period dC stars is reported in Chapter 4, along with a discussion on the nature of the emission and photometric variability. Chapter 5 concludes the results of a decade long radial-velocity survey dedicated to measuring the binary fraction of the dC star population. Based on these results, the period distribution of the whole dC star population is estimated, and all dC stars with constrained orbital parameters are discussed. The HRD and orbital actions for all candidate dC stars are shown in Chapter 6, and are accompanied by a discussion of the implications on the population’s metallicity. Finally, the conclusions of the thesis are presented in Chapter 7.
Data acquisition and reduction

2.1 Introduction

This chapter will discuss the observational and data reduction techniques utilised while processing 2D images of the sky into workable astrophysical datasets and cover some basic observational methods. This chapter is separated into sections discussing methods commonly used while reducing spectroscopic and photometric data. Each of these sections begins with a brief description of the observational techniques before discussing the process of data reduction.

2.2 Charge-coupled devices

A charge-coupled device (CCD) is a detector used to convert incoming photons from an object into an electronic signal that a computer can interpret. CCDs are arranged in large arrays of pixels. Each pixel in a CCD uses a thin layer of silicon to convert incoming photons to electrons through the photoelectric effect. The number of emitted electrons is proportional to the intensity of the incident photons. These liberated electrons are then collected by a positively charged capacitor connected to the thin silicon layer. This process of collecting electrons converted from incoming photons occurs while the CCD is illuminated (the length of an exposure). At the end of each exposure, each column of capacitors in the array passes their charge to the neighbouring column in the array. The final column of the array passes its charge to a charge amplifier where the charge is converted to a voltage. This process repeats until all columns of the CCD array have been read out.

CCD detectors are, however, not perfect. Each CCD possesses a read noise that contributes to the total error in the flux measurement. The read noise can be broken down into two main sources of uncertainty. First, converting an analogue signal to a digital number is not perfect. For every signal the charge amplifier converts, there exists a statistical distribution (which is not necessarily Gaussian) of possible voltages. Typically, the mean of this distribution is taken as the conversion. However, as the distribution may be non-Gaussian, the mean is not necessarily the true conversion. Second, spurious electrons can be released in any pixel of the array and introduce noise to the flux measurement. Neither source of noise is distinguishable from the other, and both sources combine to contribute an additive uncertainty in the final flux measurement.
Despite their read noise, CCDs are incredibly powerful devices for measuring the flux of an object, owing to their high quantum efficiency and the linearity of their outputs. Their use in astronomy has advanced the field dramatically, and they will likely remain the astronomers’ detector of choice in the wavelength region of 3,000–11,000 Å for many years to come.

2.3 Spectroscopy

2.3.1 Overview

The use of spectroscopy as an observational technique in astrophysics dates back to the 19th century when Fraunhofer and Kirchhoff measured the Solar spectrum. Spectroscopy measures the flux of an object per unit of wavelength and thus yields important information on the physical parameters of the object being observed (e.g. atmospheric abundances, velocity, etc.). These observations are performed using a spectrograph. In a spectrograph, light first passes through a slit before eventually being dispersed into different wavelengths by a diffraction grating and finally recorded. The purpose of the slit is to limit the amount of light entering the spectrograph, ensuring that the light behaves as though originating from a point source. Once through the slit, the light is passed through a collimator mirror, ensuring that the light rays are accurately parallel. The collimation process aligns all components of the spectrograph, bringing the light into focus. The light then passes to the diffraction grating, where it is dispersed. The magnitude of the dispersion depends on the number of rulings within the diffraction grating, which dictates the spectrograph’s instrumental resolution. The higher the number of rulings, the greater the resolution. Finally, the dispersed light is collected by a detector. Historically, photometric plates were used as detectors; however, astronomers have moved towards using CCDs due to their high quantum efficiency ($\geq 90\%$), greater dynamic range, and linear response.

Spectroscopy is an essential tool for measuring the orbital parameters of binary stars. However, spectroscopic data must first be reduced before it can be used to answer any astrophysical questions. The steps required to reduce spectroscopic data are outlined and discussed in the following subsections.

2.3.2 Bias subtraction

A bias frame is a snapshot of the pixel-to-pixel variations in electrical current across the CCD. To measure these variations, a zero second exposure under no illumination is taken. It is typical for several bias frames to be averaged together to reduce the statistical variance. The average-combined bias frame is a good measure of the electrical offset across the CCD and
Fig. 2.1: Left panel: The averaged bias frame taken on the William Herschel Telescope using the blue arm of the ISIS spectrograph. The vertical line at just under 800 pixels is an artefact of the detector. Right panel: The average-combined flat-field frame before normalisation also taken on the William Herschel Telescope using the blue arm of the ISIS spectrograph. The saturation limit of the EEV12 detector used is around 65,000 counts. Hence, the peak counts in the flat-field are at around 40,000 counts. The signal is fainter at the top and bottom of the detector due to vignetting.

is subtracted from all science exposures and other calibration frames. An example average combined bias frame is shown in Figure 2.1.

2.3.3 Flat-fielding

A flat-field frame is a measurement of the individual pixel response to a uniform source of illumination and is used to correct for any possible sensitivity changes from one pixel to the next. To accurately measure the response of the pixels to illumination, an exposure resulting in one-half to one-third the saturation limit of the CCD is taken. This interval is used to ensure the response of the CCD is linear. Typically, several flat-field frames are taken at the start or end of a night’s observations. A tungsten lamp inside the spectrograph is usually used as the standard light source while taking flat-field frames. After observing a series of flat-field images, all of these images are median-combined. An example of a median-combined flat-field image is shown in Figure 2.1.

In spectroscopy, it is standard practice to normalise the combined flat-field image because the response across the CCD is not uniform, owing to the wavelength-dependent behaviour of the lamp. To normalise the combined flat-field image, the sum of each row/column (depending on the dispersion axis of the spectrograph) is taken before fitting this 1D array of sums with a polynomial function. The 2D flat-field image is then divided by this function to remove the wavelength-dependent behaviour of the lamp. Finally, all science frames are divided by the normalised flat-field frame.
2.3.4 Spectral extraction

For spectroscopic analysis, the 1D spectrum needs to be extracted from the 2D image. Figure 2.2 provides a visualisation of the extraction process.

Spectral extraction begins by identifying the spectrum in the 2D image, typically achieved by identifying the strongest peak in counts along any line of pixels perpendicular to the dispersion axis. Having identified the location of the spectrum, an aperture is placed around the spatial profile of the spectrum. However, owing to imperfections in the telescope optics, the spectrum is seldom perpendicular to the dispersion axis. Therefore, it is necessary to fit a low-order polynomial to the spectrum along the dispersion axis. This fit is typically referred to as the “trace”.

With an aperture in place and the trace defined, the next problem is that the light within the aperture has two origins - the object of interest and the sky. Thus, to measure the flux of the object, the background noise from the sky must be removed. Therefore, two additional regions are defined in the spatial direction that flank the aperture and follow the trace. These additional regions are placed sufficiently far from the spectrum so that no light from the object is present.

At each step along the dispersion axis, the sum of the flux within the aperture is computed. During this process, a polynomial is fit to the flux enclosed within the regions used to measure the sky background (the polynomial is interpolated over the aperture). Therefore, at each step along the dispersion axis, the estimated background signal is subtracted from the sum of the flux within the aperture, yielding the 1D spectrum.

One caveat to this approach is that the spectral trace can be difficult to define if the signal-to-noise is not sufficiently high. Therefore, for observations of low signal-to-noise spectra, it is possible to use the trace of a high signal-to-noise target (that has been observed on the same night under similar observing conditions) to extract the spectra of low signal-to-noise targets. When using a pre-defined spectral trace for extraction, the aperture is placed as usual, and hence just re-centres the pre-defined spectral trace.

2.3.5 Wavelength calibration

After completing the spectral extraction, the next step is to calibrate the 1D spectrum. Wavelength calibration begins at the telescope with the observation of arc frames directly before and after observing each target. An arc frame is a single observation of an arc lamp that emits at numerous precisely-known wavelengths (e.g. CuNe + CuAr lamps). Because the
Fig. 2.2.: The 2D spectral image for the dC star G77-61 taken using the blue arm of the ISIS spectrograph based at the William Herschel Telescope. The colour-map corresponds to the flux. The 1D spectrum of G77-61 is highlighted between the offset magenta vertical lines, which are shown as a rudimentary aperture. The green offset vertical lines represent the regions defined for calculating and modelling the sky background level.
telescope is partial to flexing – potentially altering the optical path – an arc taken before and after each set of science exposures are median averaged to provide a wavelength solution. An arc spectrum is extracted from each arc frame (with no background subtraction) using the spectral trace defined while extracting the corresponding science exposure. The dispersion solution describes the translation from pixels to wavelength and is determined for each arc spectrum by fitting a low-order polynomial to the correctly identified emission features. The dispersion solution for each arc spectrum is then applied to the corresponding science spectrum completing the wavelength calibration.

2.3.6 Flux calibration and normalisation

Depending on the observing objective, it may be necessary to perform either flux calibration, flux normalisation, or both to the science spectrum.

Flux calibration is used to convert the collected pixel counts into an absolute or relative flux and requires the observation of at least one spectrophotometric standard star observed in similar observing conditions to the science frames. An ideal spectrophotometric standard star is defined as a bright star that is devoid of spectral features in the wavelength region observed in the science frames, though, in practice, all stars possess at least some spectral features. Typically hot subdwarfs and hot white dwarfs make the best spectrophotometric standard stars.

The detector sensitivity function of each spectrophotometric standard star spectrum is computed from either the tabulated flux values of that specific standard star or from a model flux distribution based on stellar type and magnitude. Once the detector sensitivity function is determined, it is applied to the extracted science spectrum to convert counts per pixel to an absolute flux. However, flux calibrating spectra is often difficult and seldom accurate. The most common problem in flux calibration is that tabulated flux measurements are observed using a wide slit, and most science observations use a narrow slit. Thus, slit losses in the observations of spectrophotometric standard stars may lead to large errors in absolute flux calibration.

Continuum normalisation is used to correct the continuum level of a spectrum to unity. It is typical to fit a low-order polynomial or cubic spline to the continuum before dividing the spectrum by this fit. Normalisation can be helpful where flux calibration is erroneous and can be particularly beneficial when measuring radial velocities.
2.4 Photometry

2.4.1 Overview

Photometry is used to measure the brightness of an object by collecting the flux emitted by that object. While spectroscopy uses dispersed light to gather detailed information per resolution element, photometry simply counts the number of photons that fall on the detector. Therefore, exposure times associated with photometric measurements are much shorter than those for spectroscopic measurements for a given signal-to-noise under identical observing conditions.

Photometric observations can be a single observation of a target or form part of a temporal series of observations designed to monitor the flux emitted by an object (i.e. a lightcurve). Photometric filters can be placed in the telescope optics to restrict whether photons are allowed to enter the detector based on their energy levels (or wavelengths). These filters are particularly useful as they can isolate a specific wavelength region or an atmospheric feature of interest. For example, the Sloan Digital Sky Survey (SDSS) uses five colour filters to measure fluxes between 3,000 and 10,000 Å.

2.4.2 Bias subtraction

The process of bias subtraction used in photometry does not differ from the process used in spectroscopy. Bias subtraction is the subtraction of the background resting voltage in the CCD from all science and later calibration frames. Theoretically, a bias frame is a zero-second exposure.

2.4.3 Dark-current subtraction

An issue with using CCD detectors is the presence of dark current. The thermal energy generated in the detector while taking an image can result in a thermal signal and a thermal random noise in the detector when electrons gain enough energy to jump into the conduction band. To reduce the effects of the thermal signal and noise, CCDs are typically cooled to very low temperatures. Additionally, subtraction of a dark frame can be used to correct for the thermal signal in the detector. A dark frame is a long exposure while the detector is not illuminated. Therefore, the dark frame is an accurate reading of the thermal signal in the CCD and can be subtracted from all science and later calibration frames.

2.4.4 Flat-fielding

The flat-fielding process is similar for photometry and spectroscopy. The goal of applying a flat-field correction is to remove the pixel-to-pixel variations within the CCD. It is achieved
by taking an image of a uniformly illuminated light source. Most observatories provide large flat field screens for this purpose. Alternatively, flat-field frames can be taken of the sky during twilight in what are known as ‘sky flats’. Starlight is removed from sky flats by median averaging over the CCD. Regardless of the method used to obtain the flat-field frames, they are median combined and then normalised before all science images are divided by this combined, normalised flat-field.

2.4.5 Differential photometry

The evolution of a target’s luminosity can be monitored using temporal photometric observations. Differential photometry compares the flux from an object in a series of frames relative to the flux measured for other stars in the same series of frames. The underlying assumption in this technique is that all of the comparison stars used to measure the relative flux exhibit non-variable flux emission. To mitigate errors arising from variable comparison stars, it is common practice to measure fluxes relative to several comparison stars, with stars of similar brightness to the target and small angular separation from the target providing the ideal comparisons.

Adopting relative flux to measure flux variations over time has the benefit of removing any effects of variable observing conditions and instrumental effects, as each flux is relative to another star in the same frame. Therefore, a direct comparison of relative flux measurements for the same object over numerous frames is relatively easy.

2.5 Cross-correlation

The radial velocity of an object can be measured using the method of cross-correlation. Any observed spectrum is effectively a mathematical function representing flux as a function of wavelength. If an object exhibits changes in its radial velocity, $\Delta v$, over some time period, the spectrum of the star will be shifted in wavelength, $\Delta \lambda$, due to the following relationship between velocity and wavelength also known as Doppler shift:

$$\Delta \lambda = \frac{\lambda \Delta v}{c}$$  \hspace{1cm} (2.1)

where $c$ and $\lambda$ correspond to the speed of light and wavelength, respectively.

Cross-correlation is a tool that measures the similarity of two functions as a function of their displacement relative to each other. Therefore, cross-correlation can be used to measure the offsets in the wavelength (and hence velocity) of spectra measured at different times relative to
Fig. 2.3.: Top left: An example template spectrum for the dC star G77-61 taken in 2017 August. This spectrum has been normalised by fitting a third order polynomial to the continuum. Top right: Another spectrum of the dC star G77-61 observed in 2013 February. This spectrum was cross-correlated against the template spectrum. Bottom: The cross-correlation function between the template and the comparison spectrum. The cross-correlation function reaches a maximum at 0.72 Å, and hence this value is adopted as the shift in wavelength between the two spectra. The corresponding velocity shift is labelled in the top right of this panel.

a reference spectrum. This process is sometimes referred to as the inner dot product of two functions and for a continuous function is mathematically expressed as:

\[ R_{fg}(\tau) = \int_{-\infty}^{\infty} f(t) g^*(t - \tau) dt \] (2.2)

where \( f(t) \) corresponds to the function acting as the template, and \( g^*(t - \tau) \) is the complex conjugate of the function that possesses a displacement \( \tau \). For functions that are discretely sampled, the integral collapses to the following sum:

\[ R_{fg}(m) = \sum_{n=-\infty}^{\infty} f(n) g^*(n - m) \] (2.3)

where again, the two functions \( f(n) \) and \( g^*(n - m) \) correspond to the template and the displaced function (in this occasion displaced by \( m \) discrete steps). Qualitatively, Equations 2.2 and 2.3 shift the function \( g \), by \( \tau \) or \( m \) in the continuous and discrete examples, respectively, in an iterative process, with the integral (or sum) of the inner product of \( f \) and \( g \) computed at each step. As the displacement reaches a minimum, the integral reaches a maximum (see Figure 2.3).
To determine the offset in wavelength and its associated error, a Gaussian profile is fit to the cross-correlation function. The centre of the fitted Gaussian is then taken as the offset between the two functions that have been cross-correlated. Additionally, the error on this measurement is taken as the full width at half maximum of the fitted Gaussian.

In the case of two stellar spectra, the shift measured by cross-correlation corresponds directly to the wavelength offset between the two spectra. This wavelength offset can then be translated to an offset in radial velocity using Equation 2.1. Typically, one template spectrum is chosen to cross-correlate a series of target spectra against. The resulting radial velocity offsets are therefore relative to the velocity of the template. Thus, if the goal is to measure absolute velocities, the velocity of the template should be known and corrected to the heliocentric reference frame.

2.6 Conclusions

In this chapter, I have outlined the procedures used when reducing spectroscopic and photometric data. These reduction techniques are the important first step needed to analyse any set of astrophysical data, and will feature in the chapters 4 and 5. In the next chapter, I will introduce various time-series analysis methods that can be used to probe for variability in reduced spectroscopic and photometric data.
In this chapter, the time-series analysis tools used to search for, and classify periodic variations in spectroscopic and photometric time-series data in this thesis are discussed. An introduction to each method is presented followed by a brief discussion of their advantages and disadvantages.

3.1 Introduction

Time-series data are a set of discrete observations that have been measured at either regular or semi-regular time intervals. These data are typically expressed with time as the independent variable and the observable of interest as the dependent variable. Time-series data in astronomy fall into three categories; those that are periodic in their variations, those that display transient variations, and those that exhibit stochastic variations. Examples of each of these categories are: orbits of binary stars and exoplanets, supernovae light-curve decay, and accretion onto compact objects (Remillard and McClintock, 2006; Drake et al., 2009; Borucki et al., 2010). The work completed in this thesis considers only periodic variations in the context of constraining the orbital motions of binary stars, and determining photometric variability therein.

3.2 Fourier-like analysis

An important tool when searching for periodic variations in time-series data is the Fourier transform (Bracewell, 1986). Qualitatively, a Fourier transform is a mathematical operator that transforms any waveform from the time domain to the frequency domain. Thus, the Fourier transform of a time-dependent signal results in a frequency dependent function.

The Fourier transform is particularly powerful if the time-dependent signal is periodic. In fact, any periodic signal in the time domain can be written as a discrete sum of sine functions, otherwise known as a Fourier series (Fourier, 1822). Therefore, applying a Fourier transform to any signal that is periodic in the time domain, decomposes the signal into a series of discrete sine functions in the frequency domain.

Typically, a Fourier transform is expressed as a power spectrum in frequency, where each discrete peak present in the periodogram corresponds to a specific element in the Fourier series of the original periodic signal. The power of any particular peak at a specific frequency in
the periodogram is proportional to the strength of that element in the Fourier series. These principles are the underlying basis that all Fourier-like analyses are built.

3.2.1 Classical Fourier analysis

Having qualitatively outlined the principles behind Fourier analysis in Section 3.2, a quantitative description of Fourier analysis now follows. Consider a continuous function $f(t)$. The Fourier transform of this function is given by the following integral, where $i$ is the imaginary number $i = \sqrt{-1}$:

$$
\hat{f}(t) = \int_{-\infty}^{\infty} f(t)e^{-2\pi i\nu t}dt
$$

For convenience, the Fourier operator can be written as:

$$
\mathcal{F}\{f(t)\} = \hat{f}(t) \quad (3.2)
$$

The Fourier transform is a linear operator. Therefore, the Fourier transform of any two linearly combined functions $f(t)$ and $g(t)$, multiplied by constants $a$ and $b$, respectively, can be written as:

$$
\mathcal{F}\{af(t) + bg(t)\} = a\mathcal{F}\{f(t)\} + b\mathcal{F}\{g(t)\} \quad (3.3)
$$

Hence, the amplitude of any periodic signal that undergoes a Fourier transform does not affect the frequency of the transformed function. It does, however, affect the magnitude of the transformed function.

The focus of this thesis is the detection of periodic signals in spectroscopic and photometric time-series data. In the most basic instance, a periodic signal is a pure sinusoid with some amplitude $A$ and frequency $\nu_0$. This sinusoid, when expressed in the time domain, can be expressed as a complex exponential, and its Fourier transform can therefore be written as:

$$
\mathcal{F}\{A\sin(2\pi\nu_0 t)\} = A\int_{-\infty}^{\infty} e^{-2\pi i\nu t} \left( \frac{e^{2\pi i\nu_0 t} - e^{-2\pi i\nu_0 t}}{2i} \right) dt \quad (3.4)
$$
The integral on the right-hand side of Equation 3.4 can be simplified using the definition of the Dirac delta function \(^1\). Thus, the Fourier transform of a sinusoid is mathematically expressed as:

\[
\mathcal{F}\{A \sin(2\pi \nu_0 t)\} = \frac{A}{2i}(\delta(\nu_0 - \nu) - \delta(\nu_0 + \nu))
\]

(3.5)

Equation 3.5 shows that the Fourier transform of a sinusoid with amplitude \(A\), and frequency \(\nu_0\) is the sum of Dirac delta functions at \(\pm \nu_0\). Subsequently, because any periodic function can be expressed as a series of sinusoids, the Fourier transform of any periodic continuous function is a composition of a series of Dirac delta functions located at frequencies corresponding to the frequency of each sinusoid in the series. Hence, the Fourier transform is a powerful tool in decomposing a periodic function into each of its component sinusoids, while helpfully revealing the frequency of each sinusoid.

Fourier transforms take a function in the time domain and transform them to the frequency domain. It can be useful to express the Fourier transform of a function through a power spectrum, which presents the intrinsic power of each frequency present in the original function as a function of frequency. The power spectrum is real and mathematically expressed as:

\[
P = |\mathcal{F}\{f(t)\}|^2
\]

(3.6)

An example power spectrum is shown in Figure 3.1 and corresponds to a periodic function consisting of two sinusoids with frequency 2 and 5 d\(^{-1}\), and amplitude 10 and 20, respectively.

Classical Fourier transforms are particularly useful where data are continuous and evenly sampled in time. However, real-world observations are not continuous, and are seldom evenly sampled. Any set of time-series observations span a finite length of time, and thus, any observations within a time-series are the point-wise product of the continuous periodic function (e.g. the flux of an object) and a window function that describes the series of observations. Therefore, the observed function, \(f_{obs}\), given some continuous underlying periodic function, \(f(t)\), that is observed within some observing window, \(W(t)\), is mathematically expressed as:

\[
f_{obs}(t) = f(t)W(t)
\]

(3.7)

---

\(^1\delta(\nu) = \int_{-\infty}^{\infty} e^{-2\pi i x \nu} dx\)
Fig. 3.1: *Top panel:* The blue solid line represents a continuous periodic function in the time domain that is the linear combination of two sinusoids. Each sinusoid has a frequency $\nu_0$, 2 and 5 $d^{-1}$, and amplitude 10 and 20, and is displayed as the dashed orange and green lines, respectively. *Lower panel:* The normalised power spectrum displaying the Fourier transform of the continuous periodic function (the blue solid line) shown in the upper panel. The peaks in the power spectrum correspond to the frequencies of the individual sinusoids constituent in the periodic function. Each sinusoid yields two Dirac functions at their respective frequencies $\pm \nu_0$, and the power of each peak is proportional to the amplitude of each sinusoid.
By applying the linear behaviour of the Fourier transform stated in Equation 3.3 to the observed function in Equation 3.7, the Fourier transform of a continuous periodic function, \( f(t) \), observed through an observing window \( W(t) \), is given as:

\[
\mathcal{F}\{f_{\text{obs}}(t)\} = \mathcal{F}\{f(t)\} \ast \mathcal{F}\{W(t)\}
\]  

(3.8)

Hence, the Fourier transform of any observed time-series depends on the sampling of that time-series dataset.

It is common to take time-series measurements at regularly spaced intervals where the length of each measurement is assumed to be small compared to the time interval between measurements. Consequently, real-world observations are not continuous and hence any observed time-series will possess a finite number of observations. Applying a Fourier transform to a periodic function that possesses \( N \) observations with even temporal sampling results in the Fourier integral outlined in Equation 3.1 collapsing to form the Fourier sum:

\[
\hat{f}_{\text{obs}}(t) = \sum_{n=1}^{N} f_n e^{-2\pi i \nu n \Delta t}
\]  

(3.9)

where \( \Delta t \) and \( f_n \) correspond to the constant time interval between observations, and the discretely sampled underlying function, respectively. This operation is known as the discrete Fourier transform and is particularly useful when analysing time-series data. By applying the definition of the power spectrum (Equation 3.6) to the discrete Fourier transform (Equation 3.9), the classical or Schuster periodogram is described as:

\[
P_S(f) = \frac{1}{N} \left| \sum_{n=1}^{N} f_n e^{-2\pi i \nu n \Delta t} \right|^2
\]  

(3.10)

where mathematically, the only difference between the power spectrum of a continuous Fourier transform (Equation 3.6) and the classical periodogram is that the latter possesses a normalisation factor proportional to the inverse of the number of observations (Schuster, 1898).

Owing to the finite sampling of the time-series when computing the discrete Fourier transform, the classical periodogram is only an estimation of the true power spectrum of a continuous Fourier transform. In other words, because the sampling rate of the time-series results in a loss of information from the underlying function, computing the classical periodogram (and indeed all periodograms) is not the same as computing the true power spectrum, even when the number of observations approaches infinity (Anderson, 1971). Thus, computing the classical
periodogram from the discrete Fourier transform does not necessarily provide insight into the periodicity of the underlying continuous function. Nevertheless, the discrete Fourier transform and classical periodogram remain strong tools when searching for periodic signals.

Thus far, only evenly sampled discrete time-series have been discussed. However, not all temporal sampling is even. In astronomy in particular, due to the dependence on the weather, the time of day, and the visibility of the target (assuming ground-based observations), most time-series data are taken with an uneven sampling cadence.

The window function of a discretely sampled time-series forms a series of Dirac functions located precisely at the times of each observation. In the limit where data are evenly sampled, these Dirac functions form a Dirac comb, where each tooth of the comb is separated by the time between observations, $\Delta t$. This Dirac comb is advantageous because the Fourier transform of a Dirac comb is simply another Dirac comb (where the teeth of the comb are separated by $1/\Delta t$). Hence, the Fourier transform of an evenly sampled time-series contains a series of predictable aliases. However, while the window function of an unevenly sampled time-series does form a series of Dirac functions, these functions are not evenly spaced and hence do not form a Dirac comb. The window function of an unevenly sampled time-series of $N$ observations can be mathematically expressed as:

$$W_{\{t_n\}}(t) = \sum_{n=1}^{N} \delta(t - t_n)$$ (3.11)

Taking the Fourier transform of a continuous periodic function observed through an unevenly sampled window function described in Equation 3.11 gives:

$$\hat{f}_{\text{obs}}(t) = \sum_{n=1}^{N} f(t_n) \delta(t - t_n)$$ (3.12)

Unfortunately, the Fourier transform of a non-uniformly sampled window function does not possess the same symmetry as the Fourier transform of a uniformly sampled window function. Therefore, rather than an easily predictable Dirac comb, the Fourier transform of a non-uniform window function is noisy due to the uneven sampling, and thus makes any analysis of the power spectrum difficult. Effectively, any random structure that exists in the window function (or indeed the sampling) can produce random peaks in the power spectrum.

Figure 3.2 provides an example comparison between the periodogram for the Fourier transform of a continuous periodic function that has been sampled uniformly and non-uniformly. To compare the different sampling methods, a sinusoid was generated with an amplitude of 10
and frequency of \(4 \text{ d}^{-1}\). The same sinusoid was used in both analyses. The uniformly-sampled time-series is sampled every 0.195 d with a total of 13 observations. The non-uniformly sampled time-series was sampled to ensure that the mean of the sampling rate is the same as for the uniformly sampled dataset (i.e. \(\mu_{\Delta t} = 0.195 \text{ d}\)).

The periodogram corresponding to the uniformly sampled dataset (the lower left panel of Figure 3.2) is clearly a Dirac comb. The teeth of the comb are separated by 5.2 \(\text{d}^{-1}\) which corresponds to the reciprocal of the time interval between observations. The peak furthest left in this periodogram is due to the true signal of the sinusoid and is hence located at 4 \(\text{d}^{-1}\). Therefore, the periodogram of the uniformly sampled time-series dataset is simply the Dirac function corresponding to the true signal, which is then repeated at constant steps separated in frequency by the reciprocal of the interval between observations.

The bottom right panel of Figure 3.2 displays the periodogram for the Fourier transform of the non-uniformly sampled time-series. Unlike the periodogram corresponding to the uniformly sampled time-series, the periodogram is noisy and difficult to interpret. The non-uniform sampling of the time-series has broken the symmetry of the Dirac comb and as a result, the sampling has introduced spurious peaks to the periodogram. In this example, the peak corresponding to the true frequency can be recovered at 4 \(\text{d}^{-1}\), and this peak is indeed the strongest in the periodogram. However, when data become sparse or low-quality, the noise introduced to the periodogram by uneven sampling can greatly increase the difficulty in identifying the true signal.

Further to breaking the symmetry in the periodogram, applying a discrete Fourier transform to unevenly sampled time-series data has problems with the statistical interpretation of the periodogram, in addition to spectral leakage in the periodogram (also known as aliasing).

Where data are unevenly sampled, the periodogram of the discrete Fourier transform can be noisy, even when the data are of good quality. Importantly, the amplitude of the noise in the periodogram does not decrease with increased sample size (Bartlett, 1950; Richards, 1967). While the additional data do increase the signal from the underlying periodic function, the additional measurements also increase the complexity of the window function (see Equation 3.11). Hence, increasing the size of the dataset increases the signal in the power spectrum, but does not decrease the noise.

A further statistical problem of the non-uniformly sampled discrete Fourier transform arises from the distribution of its periodogram. Consider a continuous periodic function that is sampled evenly. The power spectrum of the Fourier transform of this function follows a \(\chi^2\)
Fig. 3.2: Top panels: The blue lines show the underlying periodic function that the time-series data were sampled from, while the black squares correspond to the actual observations used to compute the periodograms. The uniformly sampled data possess temporal sampling of 0.195 d, which is equal to the average of the temporal sampling for the non-uniformly sampled time-series. Bottom panels: The left and right panels present the periodograms corresponding to the Fourier transform for the uniformly and non-uniformly sampled time-series, respectively. These periodograms show how uneven sampling of the time-series breaks all symmetry in the periodogram.
distribution and, hence, the significance of any peak in the power spectrum is easily interpreted. Now consider the same continuous periodic function, however, it is unevenly sampled. It is no longer true that the periodogram of the Fourier transform of this function is \( \chi^2 \) distributed (Scargle, 1982a; VanderPlas, 2018a). Therefore, hypothesis testing becomes increasingly difficult for time-series where the sampling is non-uniform, rendering the analysis uncertain.

Qualitatively, spectral leakage is the term used to describe the situation where power from a peak in a power spectrum has leaked to its surrounding frequencies (Beutler, 1966; Gaster and Roberts, 1977; Kar et al., 1981). It is common for power to leak to frequencies nearby the signal, creating side-lobes, owing to the sampling of the data lying within a finite time period. Alternatively, power can also leak to higher frequencies, typically known as aliasing. All spectral leakage has the effect of diluting the true signal, while at the same time, introducing more power to spurious signals. The exact nature of the specific spectral leakage in a particular power spectrum depends on the sampling rate of the time-series data, and can therefore make the detection of the true period tricky.

The accumulation of these issues with the discrete Fourier transform led to the development of more sophisticated periodograms, designed to mitigate the issues associated with the statistics of the classical periodogram, and spectral leakage.

### 3.2.2 Lomb-Scargle analysis

The Lomb-Scargle analysis builds upon the classical periodogram and Fourier analysis by attempting to better deal with non-uniformly sampled time-series data. To mitigate the problems associated with using discrete Fourier transforms to search for periodicity in non-uniformly sampled time-series data, Lomb (1976a), and Scargle (1982a), both independently developed a technique for period detection known as the Lomb-Scargle periodogram. By generalising the classical periodogram defined in Equation 3.10 with a careful selection of arbitrary functions of the frequency and the observing cadence, the Lomb-Scargle periodogram can effectively deal with unevenly sampled time-series data, while ensuring that the statistics of the periodogram remain analytically computable. The Lomb-Scargle periodogram is mathematically expressed as:

\[
P_{LS}(\nu) = \frac{A^2}{2} \left( \sum_n \nu_n \cos(2\pi \nu [t_n - \tau]) \right)^2 + \frac{B^2}{2} \left( \sum_n \nu_n \sin(2\pi \nu [t_n - \tau]) \right)^2
\] (3.13)
where $A^2$, $B^2$ are arbitrary functions of the frequency $\nu$ and $\tau$ is a function of the observation times. These functions ensure that the periodogram is generalised, and that in the regime where observations are taken with uniform temporal sampling, the Lomb-Scargle periodogram reduces to the classical periodogram. The functions $A^2$ and $B^2$ normalise the periodogram, while $\tau$ ensures that there is a time-shift invariance (e.g. introducing a linear time shift into the data does not affect the computation of the periodogram). Each of the functions are given by:

$$A^2 = \frac{1}{\sum_n \cos^2(2\pi\nu|t_n - \tau|)},$$

$$B^2 = \frac{1}{\sum_n \sin^2(2\pi\nu|t_n - \tau|)}$$

and

$$\tau = \frac{1}{4\pi\nu} \tan^{-1}\left(\frac{\sum_n \sin(4\pi\nu t_n)}{\sum_n \cos(4\pi\nu t_n)}\right).$$

(3.14)

If phase coverage of the data is close to complete, the expectation values of $A^2$ and $B^2$ simplify to $N/2$ as is the case for the classical periodogram in Equation 3.10 (Schuster, 1898). Furthermore, the Lomb-Scargle periodogram is analogous to performing a least-squares fit between a sinusoid and the time-series data at each frequency of the periodogram. Hence, the power of the Lomb-Scargle periodogram can be thought of as the $\chi^2$ goodness of fit at each frequency between the data and a sinusoid. Perhaps unsurprisingly then, given the similarities between the Lomb-Scargle periodogram and the discrete Fourier transform, the Lomb-Scargle periodogram assumes a simple sinusoidal model for period variations.

The Lomb-Scargle periodogram is, however, not perfect, and two important modifications have been made in recent years to improve the method.

The first enhancement to the Lomb-Scargle periodogram was to incorporate measurement errors into the algorithm (Scargle, 1989). In its simplest form, the Lomb-Scargle periodogram could not discriminate between data based on the magnitude of their measurement errors. To correct for noisy measurements in a dataset, it is necessary to weight each observation by the corresponding measurement error before computing the periodogram. Thus, higher quality data become more constraining than poor quality data. This process is equivalent to performing least-squares fitting between a sinusoid and the data at each frequency in the periodogram, reinforcing that the Lomb-Scargle periodogram follows a $\chi^2$ statistic (Zechmeister and Kürster, 2009).
The second improvement was required because the Lomb-Scargle periodogram assumes that the time-series data are centred around zero (e.g., the mean value of the data has been subtracted from the data). In most cases, this assumption is not a problem; however, if the underlying periodic signal is poorly sampled such that only a small region of phase is observed, then the mean of the data will not necessarily match the mean of the underlying periodic function (Cumming et al., 1999). Thus, in the regime where phase coverage is poor, the Lomb-Scargle periodogram can lead to suppression of real peaks in the periodogram. Intuitively, the solution to not explicitly knowing the mean of the underlying periodic function is to introduce an additional free parameter, \( C \), to Equation 3.13 that fits the mean of the dataset (VanderPlas and Ivezić, 2015).

The modified Lomb-Scargle periodogram, accounting for the floating mean and measurement errors, is typically referred to as the generalised Lomb-Scargle periodogram (Zechmeister and Kürster, 2009), and is a powerful tool for probing for periodic variations in time-series data. The generalised Lomb-Scargle periodogram is an improvement over the classical periodogram, as it can be used to analyse unevenly sampled time-series, and even simplifies to the classical periodogram for evenly sampled time-series. However, the generalised Lomb-Scargle periodogram is not a fool-proof method for finding periodic signals. The major issues that affect the Lomb-Scargle periodogram are that spectral leakage and aliasing are not corrected for, and that the analysis assumes all periodic variations are due to sinusoidal signals. The former of these issues can lead to difficult-to-interpret periodograms, while the latter assumption is seldom true in nature.

### 3.2.3 Multi-harmonic analysis of variance

The Multi-Harmonic Analysis of Variance (MHAOV) is a statistical method to identify periodic variations in time-series data, particularly when the temporal sampling is non-uniform (Schwarzenberg-Czerny, 1996a). Breaking from the discussion thus far, MHAOV does not rely on a Fourier transform, but instead relies on phase-folding the time-series data on a series of defined frequencies, before fitting a model to each set of phase-folded data. Hence, MHAOV is similar to the generalised Lomb-Scargle periodogram in the sense that they are both equivalent to fitting a model to the data. However, where the generalised Lomb-Scargle periodogram assumes a sinusoidal function to model the periodic variations, MHAOV models the periodic variations with periodic orthogonal polynomials (e.g., Szegő polynomials). Thus, the MHAOV analysis is far more sensitive to periodic variations that are non-sinusoidal. Furthermore, it
should be noted that the first harmonic of the Szegő polynomials used in MHAOV is identical to a sinusoid.

Perhaps the most fundamental difference between the generalised Lomb-Scargle and MHAOV is in the choice of statistic each method uses to measure the goodness of fit for their respective models. As discussed previously, the generalised Lomb-Scargle uses a $\chi^2$ distribution to assess how well a particular sinusoid fits the data, whereas MHAOV uses the Fisher analysis-of-variance statistic (Fisher, 1992). The statistical disadvantage of the $\chi^2$ statistic is that the variance of the noise and the signal are not independent of each other. In other words, the variance of the noise depends on the periodogram and is not explicitly known from the data. Therefore, applying the generalised Lomb-Scargle periodogram to a dataset with unknown noise variance actually causes the true distribution of the periodogram to deviate away from theoretical distribution (e.g. the periodogram no longer follows a $\chi^2$ distribution). Thus, using a generalised Lomb-Scargle periodogram (or a discrete Fourier transform) to test the hypothesis that a dataset is periodically varying is susceptible to uncertainty (Lachowicz et al., 2006a).

Adopting the Fisher analysis-of-variance statistic ensures that the variance of the signal and the noise remain independent from each other. The probability distribution of the MHAOV periodogram therefore follows a Fisher-Snedecor distribution, rather than a $\chi^2$ distribution. Owing to the intrinsic statistical differences between these two probability distributions, and holding all other variables constant, any detection of a signal using MHAOV is less significant than if the same signal is detected by a generalised Lomb-Scargle periodogram. However, due to the independence of the signal and noise, any detection made by the MHAOV periodogram is more reliable (Schwarzenberg-Czerny, 1989). A direct comparison of the generalised Lomb-Scargle periodogram and the MHAOV periodogram for sparsely sampled time-series data is presented in Figure 3.5, and discussed in Section 3.4.2.

3.3 Monte Carlo sampling methods

3.3.1 Introduction to Monte Carlo sampling

The previous sections of this chapter have outlined various valuable tools that can be used when searching for periodic variations in time-series data. Methods such as the generalised Lomb-Scargle periodogram and MHAOV periodogram are reliable statistical methods for identifying periodic variations in large datasets. However, when datasets become small, or if the data need a probabilistic interpretation (e.g. if a specific physical model predicts the observed
periodic variations better than a simple mathematical function), then other analysis methods must be used.

Monte Carlo sampling is commonly used to evaluate the probability that some event will happen given a certain set of conditions. This is achieved by drawing a typically large number of random samples from a distribution, and using those samples to integrate the probability distribution of an event happening. For example, when using a model to compute a target variable, the parameters of the model may be uncertain (perhaps because of the stochastic nature of the variables or a large number of variables). In this scenario, each parameter of the model may be assigned a probability distribution (where the exact shape of the distribution is motivated by prior knowledge of the variable). Monte Carlo sampling then draws many samples from these probability distributions and for each set of samples, it computes the target variable. Eventually, after many sets of samples are drawn, a probability distribution for the target variable will form. Thus, Monte Carlo sampling evaluates the integral of the probability distribution for the target variable through random sampling.

The technique of Monte Carlo sampling refers to the method of drawing random samples to estimate a probability distribution, and forms the basis for numerous sampling techniques. The following sections will describe several of these sampling techniques and their applications to astrophysical research.

### 3.3.2 Rejection sampling

One application of Monte Carlo sampling is through its use in the statistical method of rejection sampling. As explained in Section 3.3.1, sampling is an important tool used to integrate the probability distribution of some target variable that can be modelled by some combination of parameters. However, Monte Carlo sampling alone can be uninformative if the physical model itself is not constraining. Rejection sampling aims to solve this by introducing additional criteria that the prior samples must pass before they can be used to compute probability distribution of the target variable.

Qualitatively, the process of rejection sampling relies on drawing a number of random samples of each model parameter from a set of defined probability distributions. These samples are then either rejected or accepted depending on whether they pass a certain rejection criterion specified for the specific problem at hand. For example, consider using rejection sampling to estimate the numerical value of \( \pi \). The basic algorithm is to draw samples from a set of known probability distributions – in this example samples are drawn uniformly in \( x \) and \( y \) (see Figure 3.3). The rejection criteria for the drawn samples is whether they lie within a circle of
Fig. 3.3: An example of the rejection sampling algorithm. Samples are drawn from the rectangular region $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$. The samples that fall within the defined boundary, shown as the black circle, are accepted. The samples that do not satisfy this condition are rejected.

unit radius centred at the origin. The samples that lie within this boundary are accepted, while those drawn outside of the boundary are rejected. The ratio of the number of accepted samples compared to the total number of prior samples drawn is equal to the ratio of the area enclosed by the circular boundary ($A_{\text{circ}} = \pi r^2$) to the area from which the original samples were drawn ($A_{\text{rect}} = xy$). This formula can be re-arranged and solved for $\pi$. The accuracy of this algorithm to determine $\pi$ improves with an increased sample size.

Rejection sampling draws many samples from a set of initial probability distributions, and then rejects some of these samples based on problem-specific constraints placed on these samples. This process can lead to improved results over Monte Carlo sampling especially where physical models are not constraining. The drawback of rejection sampling is that many samples may need to be drawn for the algorithm to work effectively and it can therefore be somewhat inefficient and computationally expensive.

3.3.3 Applying Bayes’ theorem to sampling

Thus far in this section, the sampling techniques discussed have drawn samples from a set of probability distributions corresponding to some model parameters and used those samples to evaluate the probability distribution of a target variable. However, neither sampling technique discussed thus far has actually taken observational data into account. For example, consider the
radial-velocity curve of a binary star. The orbital motion of the binary is well defined by Kepler’s laws, and can be modelled as such. Additionally, observational measurements of the radial velocity can be taken with a spectrograph and hence used to constrain the posterior probability distribution function for each set of parameters. The most common technique for evaluating a posterior distribution given a model and some observational data is to combine Monte Carlo sampling techniques with Bayes’ theorem. This process involves randomly sampling the model parameters from a set of prior probability distribution functions (each parameter is drawn from its own unique probability distribution function), before calculating a likelihood that the model, given these randomly sampled parameters, fit the observed data well. Bayes theorem provides a robust method for evaluating the posterior probability distribution through sampling, without having to directly compute the posterior distribution (Joyce, 2019). Mathematically, Bayes’ theorem is expressed as:

$$ P(H|E) = \frac{P(E|H)P(H)}{P(E)} $$

(3.15)

where $P$ corresponds to the probability, and $H$ and $E$ correspond to the model (or hypothesis), and the data (or evidence), respectively.

Qualitatively, Bayes’ theorem evaluates the posterior distribution function (the probability that the hypothesis is correct given some data), $P(H|E)$, by computing the prior belief that the hypothesis is true, $P(H)$, the likelihood of observing the evidence if the hypothesis is true (known as the likelihood function), $P(E|H)$, and the probability that the evidence is drawn from the prior distribution, $P(E)$ (which normalises the right-hand side of Equation 3.15).

Because the posterior probability distribution function is difficult to compute directly, a combination of Monte Carlo sampling and Bayes’ theorem can be used to make an estimation of the posterior probability distribution function. In order to combine Monte Carlo sampling with Bayes’ theorem, a mathematical function must be assigned to the prior probability distribution and the likelihood function. These functions are typically assumed to be Gaussian because many natural processes can be accurately modelled as such. However, any function can be used.

Now, consider again the example of taking radial-velocity measurements of a binary star. The hypothesis is that the binary motion can be modelled as a Keplerian orbit with some set of orbital parameters, and the evidence corresponds to the radial-velocity measurements. In this example, the objective is to determine the optimal orbital parameters for a Keplerian orbit that fit the observational data. In other words, the objective is to determine the posterior probability
distribution of each of the orbital parameters. To achieve this, Monte Carlo sampling is used to draw a large number of samples from the prior probability distribution functions of each of the orbital parameters. These prior samples are then used to calculate a likelihood that a Keplerian orbit, given these prior samples, is able to produce the observed radial velocities. This likelihood is then normalised by the probability that observed radial-velocity measurements are consistent with the prior samples for the orbital parameters.

The algorithm outlined above is repeated many times through Monte Carlo sampling to build up a posterior probability distribution function. As the number of samples becomes large, the probability distribution function is considered well explored and, as such, the expectation of the posterior probability distribution function is expected to tend towards the true value. A more detailed explanation of the above example is presented in the next section.

### 3.3.4 The JOKER

The JOKER (Price-Whelan et al., 2017) is software developed to use rejection sampling to generate posterior samples in Keplerian orbital parameters for single-lined binary systems, given some observational radial-velocity measurements. The JOKER makes the assumptions that: a binary system is described as a two-body system with no gravitational perturbations, the error in the time of each single observation is negligible, and that errors in radial-velocity measurements are Gaussian with zero-mean (Price-Whelan et al., 2020). Under these assumptions, the orbital motion that is characterised by the radial-velocity measurements can be described by six orbital parameters; orbital period, eccentricity, pericentre phase and argument, velocity semi-amplitude, and systemic velocity, expressed as the variables; $P, e, \phi_0, \omega, K, \gamma$, respectively. While these assumptions are not perfect (e.g. it is not guaranteed that any particular binary system is not a triple star system), it does allow for reasonable prior constraints to be made on each of the Keplerian orbital parameters.

Following Price-Whelan et al. (2017), the radial velocity of any two-body binary system measured at some time $t$ with orbital parameters $\theta = (P, e, \phi_0, \omega, K, \gamma)$ can be mathematically expressed through the following set of Equations 3.16–3.19:

$$v(t, \theta) = \gamma + K \left( \cos(g_{\text{anom}} + \omega) + e \cos\omega \right)$$

(3.16)

The function $g_{\text{anom}}$ in Equation 3.16 is the true anomaly, and is given by

$$\cos g_{\text{anom}} = \frac{\cos E - e}{1 - e \cos E}$$

(3.17)
where the eccentric anomaly is given by \( E \). The eccentric anomaly can be expressed using the mean anomaly \( M \), where:

\[
M = 2\pi \frac{t}{P} - \phi_0
\]  
\[ \text{(3.18)} \]

\[
M = E - E \sin e
\]  
\[ \text{(3.19)} \]

Having established the relationship between the observed radial velocity and the six Keplerian orbital parameters that characterise the orbit of a binary star, the JOREK then separates these six parameters into non-linear (\( P, e, \omega, \phi_0 \)) and linear parameters (\( K, \gamma \)). The reason for this separation is to reduce the computational complexity of the algorithm. Rather than sampling across all six Keplerian parameters, the JOREK instead only draws samples from the non-linear parameters. The linear parameters are then determined for each set of non-linear samples through fitting a least-squares minimisation between the model adopting the non-linear parameters of each iteration to the data. Thus, each set of non-linear samples possesses their own unique linear samples.

The linear parameters used in the least-square fitting routine are generated from prior probability distributions. For the systemic velocity of the binary, this can simply be defined by any prior the user may have (e.g. if a binary is a known disc or halo member). However, the velocity semi-amplitude is degenerate with the orbital period and eccentricity. In other words, a binary with a large orbital period and low eccentricity will not possess a large velocity semi-amplitude. But, if the same binary were in a short orbital period, or possessed a high eccentricity, the binary will possess a large velocity semi-amplitude (at inclinations that are not near face-on). Therefore, velocity semi-amplitudes are drawn from a Gaussian prior probability distribution where the variance of the distribution is given as:

\[
\sigma_K^2 = \sigma_{0,K}^2 \left( \frac{P}{P_0} \right)^{-\frac{3}{2}} (1 - e^2)^{-1}
\]  
\[ \text{(3.20)} \]

where \( \sigma_{0,K}^2 \) corresponds to a user defined variance on the velocity semi-amplitude (this can be large if unconstraining priors are required), and \( P_0 \) is another user-defined parameter used to scale the orbital period. Generating prior samples in velocity semi-amplitude through Equation 3.20 ensures that the prior probability distribution, for a fixed primary mass, has a fixed form in companion mass and does not depend on orbital period or eccentricity.
Upon generating each set of prior samples in the non-linear parameters, and subsequently determining the corresponding linear parameters, the marginal log-likelihood can then be calculated. Uncertainties associated with determining the linear parameters through least-squares fitting are accounted for by calculating a covariance matrix of errors. The marginal log-likelihood, $Q_j$, is shown in Equation 3.21, where the prediction, $v_n - v(t_n; \theta_j)$, corresponds to the least-squares fit for a set of prior samples $j$ in the non-linear parameters, and $C_j$ is the covariance matrix containing the uncertainty associated with determining the linear parameters of samples $j$. The log-likelihood is given as:

$$
\ln Q_j = -\frac{1}{2} \sum_{n=1}^{N} \left( \frac{(v_n - v(t_n; \theta_j))^2}{\sigma_n^2} + \ln(2\pi\sigma_n) \right) - \frac{1}{2} \ln ||2\pi C_j||
$$

(3.21)

Because most radial-velocity data are either sparse, of low quality, or both, the JOKER uses rejection sampling over the marginal log-likelihood to determine the posterior probability distribution. In this application, the rejection sampling algorithm is as follows: each sample $j$ has an associated marginal log-likelihood $Q_j$ that lies between 0 and $Q_{\text{max}}$. For each set of prior samples $j$, a random number $r_j$ is generated between the values 0 and $Q_{\text{max}}$. If the rejection condition that $Q_j < r_j$ is met, then this set of samples is rejected. All surviving samples then form the posterior distribution function. It is worth noting that this algorithm ensures that at least one sample will survive the rejection sampling process.

The rejection sampling algorithm used by the JOKER is not fair if the number of surviving samples is small. Thus, this method is only useful if either the number of prior samples drawn is sufficiently large that many survive rejection sampling, or if the data are relatively unconstraining - resulting in many samples surviving rejection, thus yielding a large posterior distribution. Therefore, in the regime of high quality data, rejection sampling will likely not explore the posterior distribution fully, thus other sampling methods must be used (e.g. Markov Chain Monte Carlo sampling).

### 3.3.5 Markov chain Monte Carlo sampling

Similarly to the process of rejection sampling discussed in Section 3.3.2, Markov chain Monte Carlo (MCMC) is a sampling method used to evaluate a posterior probability distribution function for some model given some observed data (see Hogg and Foreman-Mackey 2018, and references therein for a full review). The posterior probability distribution function is typically computed using Equation 3.15, where samples are drawn from some prior probability
distribution, and a likelihood is calculated per sample. The difference between MCMC sampling
and rejection sampling is the process in which samples are drawn.

MCMC sampling relies on constructing a number of Markov chains and sampling from
within these chains. A Markov chain can be summarised as a series of random variables, where
the state at any given time depends on the previous state only, and not on the history of states.
Hence, sampling from a Markov chain is a random stochastic process. In the limit where the
number of samples is large, the Markov chain is stationary, meaning that the mean and the
variance of the Markov chain remain constant as more samples are drawn. Stationarity in the
Markov chain is important for sampling as it ensures that all states in the Markov chain have an
equal probability of being drawn. Furthermore, as the current state of a Markov chain depends
only on the previous state, any state can occur multiple times during the sampling process. The
advantages of using Markov chains to generate samples lies in their stationary behaviour with
large sample sizes.

MCMC sampling is the process of constructing a Markov chain over a probability distribution
and drawing a large number of samples from the chain, to adequately explore the probability
distribution. Because the stationarity of the Markov chain is important to the statistical proper-
ties of the chain, the MCMC is run for a so-called burn-in phase before sampling begins. The
burn-in phase discards a predetermined number of samples ensuring that the Markov chain
is stationary and hence each state equally likely to be drawn. The samples drawn after the
burn-in phase are thus used to explore the desired probability distribution.

The advantages of MCMC sampling over rejection sampling are most obvious with highly
constraining data. The number of samples that survive rejection sampling is proportional to
the number of prior samples drawn, in addition to how constraining the rejection criteria is. In
the case of the JOKER, the rejection criteria are harsh if data are highly constraining. Effectively,
if the maximum log-likelihood is large compared to the log-likelihood of all other samples,
then few samples can survive the rejection sampling process, resulting in a poorly explored
posterior probability distribution. MCMC sampling does not have this problem as after the
burn-in phase is complete, no samples are discarded.

MCMC sampling can be used in unison with rejection sampling for datasets that are of
unknown quality. Rejection sampling can be used in the first instance to explore the posterior
probability distribution. If the data are constraining and hence, few samples survive the rejection
sampling process, the posterior probability distribution returned through rejection sampling
can be used to build a prior probability distribution for MCMC sampling. By combining the
two methods for datasets with uncertain quality, the posterior probability is likely to be fully explored.

3.4 Comparison between Fourier-like analysis and rejection sampling

Throughout this chapter, several methods to identify and constrain period variations in time-series data have been discussed. It is important to understand when it is appropriate to apply a given statistical method to a particular dataset. Where data are plentiful and high-quality, Fourier-like analyses are superior due to their simplicity and computational efficiency. However, it is difficult to identify when a dataset becomes too sparse for Fourier-like analyses to produce reliable results. In this section, Fourier-like analyses will be directly compared to sampling methods by applying each method to a series of simulated radial-velocity datasets. The aim of this exercise is to quantitatively determine at which size a dataset qualifies as sparse.

3.4.1 Simulating radial-velocity data

In order to compare each analysis method, a radial-velocity curve was simulated for a binary system using masses and orbital parameters expected for dC stars. The total mass of the binary system was set at $1M_\odot$, and assumes the masses of the evolved companion and dC star are $0.6M_\odot$ and $0.4M_\odot$, respectively. The orbital period was selected to match the measured orbital period of the prototype dC star, G77-61, at 245 d, and the simulated radial-velocity curve spans six years, or approximately nine full orbits. Furthermore, the inclination of the binary was set to $i = 60^\circ$, and the eccentricity was set to zero. These parameters were used to calculate the expected velocity semi-amplitude of the dC star within the binary system yielding $K \approx 24.5\, \text{km s}^{-1}$. Using these parameters, a simulated radial-velocity curve was generated and then injected with artificial Gaussian noise. A number of radial-velocity measurements were then randomly drawn from this simulated radial-velocity curve. Errors were assigned to each simulated radial-velocity measurement by sampling from a Gaussian distribution centred at $5\, \text{km s}^{-1}$. These assigned uncertainties are representative of real velocity measurement uncertainties taken with a medium resolution spectrograph in the radial-velocity survey of Whitehouse et al. (2018). The assigned uncertainties characterise the artificial Gaussian noise injected to the radial velocity curve well.

The simulated radial-velocity measurements were then analysed using a generalised Lomb-Scargle periodogram, the MHAOV method, and rejection sampling implemented through the joker, to identify the limit where a dataset becomes too sparse for each analysis method to be
Fig. 3.4: The blue line represents the underlying pure sinusoidal signal intended to act as a mock radial-velocity curve given binary orbital parameters specified on the previous page. This signal was injected with Gaussian noise consistent with the radial-velocity campaign of Whitehouse et al. (2018). The orange points represent the time-series observations sampled from the noisy sinusoid, along with the simulated observational errors.

insightful. In order to keep this test relevant to astrophysical datasets, and more specifically, this thesis, the sampling of the of the underlying periodic signal (a sinusoid) was designed to mimic the sampling of the radial-velocity survey of Whitehouse et al. (2018). Thus, mock observations were selected pseudo-randomly given the following criteria. First, as stars are only visible for a portion of the calendar year, and observations can only be taken in a given semester while the star is visible, a mock observation is initially taken once during this window. Following the observing strategy of Whitehouse et al. (2018), a second mock observation was taken at a time corresponding to the following night of the first mock observation, before taking a third observation at a time corresponding to a night between 2–6 weeks after these first two observations. This set of observations is then repeated for the following calendar year, by randomly selecting a time for the first observation within the window that the mock star is visible. Furthermore, all mock observations occur at times that are consistent with night-time. This process was repeated until the simulated time-series dataset contained 20 mock radial-velocity measurements, a sample size that is large enough that each analysis method would be able to identify the true period. The simulated radial-velocity curve, with corresponding mock radial-velocity measurements generated using the method described here, is shown are Figure 3.4.
Fig. 3.5: A comparison of the generalised Lomb-Scargle periodogram to the MHAOV analysis. Each method was used to retrieve the period of a simulated sine wave with a defined period of 245 d, given a decreasing number of data points in the process outlined in Subsection 3.4.1. The true period of the simulated data is marked by the horizontal dashed black line. The distribution of best solutions is shown for each analysis method with the generalised Lomb-Scargle and MHAOV represented by blue and orange, respectively.

Each analysis method was then used to examine the simulated dataset. To identify the regime where data become too sparse to retrieve the true signal, a random observation was removed from the time-series dataset after the analysis was complete. The different analysis methods were then repeated on this smaller dataset. This process was repeated iteratively until the time-series contained only five mock observations. The logic behind randomly removing data from the time-series, rather than simply truncating the time-series, is that random data removal simulates observations lost due to poor weather and, hence, works to form a more accurate window function on the data.

The purpose of sampling the simulated radial-velocity curve with the described methodology is to intentionally introduce window effects into the time-series. Due to the nature of how astrophysical observations are taken (i.e. at night and while targets are visible), it is important to understand which analysis methods succeed under imperfect sampling. While any particular analysis method could perform better with truly randomly sampled data, it is necessary to understand where any particular analysis method succeeds and fails with data that are subject to windowing effects that influence radial-velocity surveys.
Fig. 3.5. continued
3.4.2 Generalised Lomb-Scargle vs MHAOV

This section discusses how the generalised Lomb-Scargle and MHAOV periodograms perform on sparse time-series data. Each method is used to analyse increasing sparse simulated radial velocity datasets to better understand how their respective periodograms react to sparse sampling.

Figure 3.5 shows the direct comparison between the generalised Lomb-Scargle periodogram and MHAOV periodogram for the simulated dataset. Both methods are successful at identifying the correct period in the time-series data where the number of measurements is plentiful (e.g. each dataset contains 20 radial-velocity measurements) and, unsurprisingly, this success diminishes as the datasets becomes smaller. The common pitfall for the generalised Lomb-Scargle periodogram and the MHAOV periodogram is that they find solutions at arbitrarily short periods. While the power in the periodograms at short period solutions is not necessarily strong where data are plentiful, the power in these short period solutions does increase as the dataset becomes smaller. In fact, as the MHAOV periodogram analyses smaller datasets, the power assigned to arbitrarily short periods increases dramatically compared to other signals identified in the periodogram. For instance, the MHAOV periodogram in the bottom panel of Figure 3.5 does contain peaks at other periods (e.g. 245, 365 and 735 d), however, these peaks are not visible due to the large power assigned to the peaks at short periods. The reason that both methods assign power to arbitrarily short periods is simply because as the data become sparse, it becomes increasingly easier to fit higher frequency sinusoids, and therefore, any peak
associated with the true period will not possess a high power in the periodogram compared to those at short periods.

Now, directly comparing the generalised Lomb-Scargle to the MHAOV periodogram, several key features stand out. First, the generalised Lomb-Scargle periodogram possesses many more peaks than the MHAOV periodogram for a fixed number of measurements in the time-series. This is because the $\chi^2$ statistic calculated by the generalised Lomb-Scargle assigns more power to spurious peaks than the analysis-of-variance statistic used in the MHAOV analysis. In fact, the analysis-of-variance statistic damp the spurious peaks, resulting in a less noisy periodogram with more power assigned to the true period.

The second key feature is that the power in the peak of the MHAOV periodogram located at the true period (245 d) decreases as the number of measurements in the time-series becomes small, whereas, the power in the peak of the generalised Lomb-Scargle periodogram located at the true period does not significantly vary relative to the other periodogram peaks as the dataset becomes small. Additionally, as the datasets become sparse, the MHAOV periodogram assigns more power to peaks corresponding to short period solutions, whereas the generalised Lomb-Scargle assigns short period solutions with a large power regardless of the size of the dataset. Thus, Figure 3.5 provides a prime example of how the generalised Lomb-Scargle periodogram is difficult to interpret for astrophysical data, even when the data are plentiful.

Notably, both periodograms assign power to periods of $\approx 1$, 30, and 365 d (albeit to a much lesser extent in the latter), which correspond to the windowing effect of the daily, monthly, and yearly sampling of the underlying signal, respectively. As the datasets become small, these signals corresponding to the nightly and monthly sampling rate begin to dominate the periodograms, despite the true signal remaining in both periodograms even when the dataset consists of only five observations. However, due to the large power in the MHAOV periodogram assigned to these periods that correspond to the sampling, the true signal become invisible in Figure 3.5 at fewer than seven data points. Therefore, while neither method is immune to these aliasing affects, the MHAOV periodogram does appear superior in this example owing to the relatively low-noise periodogram, and the predictability of the strongly aliased peaks.

### 3.4.3 The joker vs MHAOV

Owing to the relative success of the MHAOV periodogram against the generalised Lomb-Scargle periodogram, Figure 3.6 compares how the MHAOV periodogram compares to rejection sampling implemented through the joker for the simulated data described in Section 3.4.1.
To search for periodicity in each of the simulated time-series datasets, the JOKER was run by drawing $2^{26}$ (≈ $7 \times 10^7$) prior samples from the following prior probability distributions. Orbital periods were sampled from a uniform distribution bound between 0.1 and 1000 d (matching the frequency bounds of the frequency grid used for the MHAOV and generalised Lomb-Scargle periodograms). The velocity semi-amplitude was generated for each set of prior samples by drawing a sample from a Gaussian distribution centred at zero with a standard deviation calculated by Equation 3.20, where $\sigma_{0,K}^2 = 30$ km s$^{-1}$, and $P_0 = 1$ yr. The velocity semi-amplitude was taken as the absolute value of the prior sample. The systemic velocity of the system was sampled from a Gaussian distribution centred at zero with a standard deviation of 100 km s$^{-1}$. Finally, because the MHAOV and generalised Lomb-Scargle analyses assume that the underlying periodic function is a perfect sinusoid, the eccentricity of the JOKER was fixed at zero (i.e. only circular orbits were considered).

The panels in Figure 3.6 display the posterior probability distribution for the period found by the JOKER, compared to the MHAOV periodogram for the time-series datasets with a decreasing number of observations described in Section 3.4.1. The first noticeable difference between the posterior probability distribution of the JOKER, and the MHAOV periodogram, is that the JOKER does not find arbitrarily short period solutions until the datasets contain 12 observations or
Fig. 3.6. continued
fewer. The dataset containing 12 observations also corresponds to the MHAOV periodogram where the power in peak corresponding to the true signal is approximately equal to the strongest peak in the MHAOV periodogram corresponding to the ensemble of short periods. In fact, the strongest peak in the MHAOV periodogram for the dataset containing 12 observations corresponds to the 1 \text{ d} nightly observation alias. This is the same periodic signal that is found by the \text{joker} for the same dataset, however, where the MHAOV periodogram assigns a large power to this nightly alias, the \text{joker} finds few posterior samples corresponding to this alias compared to the true signal at 245 \text{ d}.

The performance of the \text{joker} on datasets containing fewer than 12 observations deteriorates in general, and results in a multi-modal posterior distribution for the period. However, the ability of the \text{joker} to find many solutions near the true period does depend on the sampling of the time-series. When the \text{joker} is applied to datasets containing fewer than 12 observations, a peak corresponding to a period of approximately 148 \text{ d} is present. Comparing the number of posterior samples that correspond to the peak at 148 \text{ d} to the true period at 245 \text{ d} shows no particular trend as the time-series datasets become smaller. Therefore, in the regime of sparsely sampled time-series data, the posterior samples returned by the \text{joker} are dependent on the sampling of the time-series.

While there is no silver bullet for identifying the correct period from a non-uniformly, sparsely sampled time-series of a periodic function, rejection sampling implemented through
the **joker** does appear to perform better than methods that fit sinusoids to phase-folded data (e.g. generalised Lomb-Scargle and MHAOV).

### 3.4.4 Combining the **joker** and MCMC sampling

The **joker** excels at identifying the frequency of a periodic function when that function is sampled unevenly, and sparsely. However, because the **joker** is built on a rejection sampling algorithm, the number of posterior samples that survive rejection depends on the largest maximum likelihood of those posterior samples. Therefore, where time-series data are constraining, the **joker** returns few posterior samples. Where the time-series data are well sampled, the posterior probability distribution is typically unimodal, however, due to few surviving posterior samples, the posterior probability distribution is not well explored.

One potential solution to this problem is to draw many more prior samples. By increasing the number of prior samples, the probability of drawing samples with a high likelihood increases, however, the probability does not increase linearly with the number of prior samples. Therefore, generating enough posterior samples to fully explore the posterior probability distribution effectively becomes computationally difficult owing to the large number of prior samples needed.

Because it is difficult to know how constraining a time-series dataset is before actually analysing it, a useful technique to use is rejection sampling (here specifically the **joker**) with a reasonable number of prior samples. If the resulting posterior distribution from the **joker** is unimodal and small, the posterior samples can be used to constrain the prior probability distribution used for MCMC sampling. Thus, where data are constraining, rejection sampling is a good method for reducing the available prior probability distribution function from which prior samples can be drawn from MCMC sampling. Because MCMC sampling does not reject any posterior samples, and instead seeks to find the solution with the highest likelihood, it is a good method for fully exploring the posterior probability distribution.

Consider the radial-velocity curve and sampling technique discussed in Section 3.4.1, along with the **joker** analysis conducted in Section 3.4.3. For the dataset containing 20 observations, the **joker** was run with $2^{26}$ prior samples which resulted in 1158 posterior samples surviving rejection. This analysis took around four hours to complete on 50 CPUs running in parallel ($\approx 200$ CPU hours). The final posterior distribution of this analysis is shown in Figure 3.7 and is unimodal and the parameters space appears well explored. However, the posterior probability distribution functions of the period and the systemic velocity do not appear to be well sampled.
Thus, MCMC sampling could be used to better explore the posterior probability distributions of the orbital parameters.

Figure 3.8 shows the posterior probability distribution for the period, velocity semi-amplitude, and systemic velocity derived through MCMC sampling for the time-series dataset containing 20 observations. The MCMC was run using the NUTS sampler (Hoffman and Gelman, 2011) with four chains. Each chain draws 3 500 samples and has a burn-in of 1 000 samples yielding a total of 10 000 posterior samples. Prior samples were drawn from the same prior probability distribution functions as the JOKER (see Section 3.4.3). The analysis was carried out on four CPUs in parallel and took under two minutes ($\approx 0.13$ CPU hours), and hence, MCMC is much more computationally efficient than rejection sampling.

Comparing the results from the JOKER with the results from MCMC sampling reveals a slight offset in all orbital parameters. While the posterior distributions found by the JOKER are closer
Fig. 3.8: The posterior probability distribution containing 10 000 derived through MCMC sampling for the simulated radial-velocity curve discussed in Section 3.4.1. The orbital period, velocity semi-amplitude, and systemic velocity are shown. The green lines correspond to the true parameters of the simulated data, while the dashed magenta lines correspond to the median of the posterior probability distributions of each orbital parameter.
to the true orbital parameters, MCMC sampling is able to recover a similar posterior distribution with $10^{-3}$ times the computational power. The orbital parameters derived through MCMC sampling are consistent to within four-sigma, one-sigma, and two-sigma for the orbital period, velocity semi-amplitude, and systemic velocity respectively.

MCMC sampling becomes useful when time-series data are well sampled. The example presented here shows the direct comparison between the joker and MCMC sampling for a dataset that is relatively constraining, and the posterior distributions of both analyses methods are broadly consistent with each other. In the regime of high-quality data, rejection sampling, and hence the joker, will only return few posterior samples compared to the number of prior samples drawn. Therefore, to avoid large computation overheads needed to draw increasingly large sets of prior samples, it can be useful to combine rejection and MCMC sampling techniques together. Rejection sampling can be used to measure the quality of a dataset, and MCMC sampling can be used to explore the posterior distribution fully if the data are indeed constraining.

3.5 Conclusions

In this chapter, several tools for time-series analysis have been discussed. From classical Fourier transforms, to various sampling techniques, each tool possesses strengths and weaknesses. In the context of astrophysical time-series data which suffers heavily from nightly, monthly, and yearly aliases, the quality and sparsity of the dataset dictates which time-series analysis tool is appropriate to search for and determine periodic variation.

Owing to the computational simplicity and easily interpretable periodograms, Fourier-like analyses are useful in situations where there is an abundance of data. Because few time-series in astrophysics are evenly sampled, it is necessary to use methods compatible with unevenly sampled time-series such as the generalised Lomb-Scargle and MHAOV. Both of these techniques are useful where data are plentiful and hence, would be ideal tools for probing for photometric variations, as photometric datasets typically consist of many tens or hundreds of observations. However, the ability of these techniques to determine the period of an underlying function diminishes as datasets become smaller and more sparsely sampled (see Figure 3.5).

Radial-velocity surveys typically contain few observations and frequently suffer from sparse sampling. However, under the assumption that radial-velocity variations are the product of two bodies gravitationally interacting, the radial-velocity curve can be modelled using Kepler’s equations. In this regime of small datasets, and in the presence of a model that describes the
periodicity, sampling techniques are more successful than Fourier methods at determining the period of the underlying periodic function that is being measured.

The time-series analysis tools discussed in this chapter will form the basis of the various analyses in the following chapters and are applied to photometric and radial-velocity time-series datasets.
The nature of dwarf carbon stars with Hα emission

Many characteristics of dwarf carbon stars are broadly consistent with a binary origin, including mass transfer from an evolved companion. While the population overall appears to have old-disc or halo kinematics, roughly 2 per cent of these stars exhibit Hα emission, which in low-mass main-sequence stars is generally associated with rotation and relative youth. Its presence in an older population therefore suggests either irradiation or spin-up. This study presents time-series analyses of photometric and radial-velocity data of seven dwarf carbon stars with Hα emission. All are shown to have photometric periods in the range 0.2–5.2 d, and orbital periods of similar length, consistent with tidal synchronisation. It is hypothesised that dwarf carbon stars with emission lines are the result of close-binary evolution, indicating that low-mass, metal-weak or metal-poor stars may accrete substantial material prior to entering a common-envelope phase.

Work shown in this chapter is accepted for publication and presented in Whitehouse et al. 2021, MNRAS.

4.1 Introduction

Binary-star evolution is responsible for a wide array of astrophysical phenomena, including blue stragglers, Type-Ia supernovae, and gravitational waves (Maeder, 1987; Webbink, 1984; Abbott et al., 2016). Interactions between binary components may result in deviations from the course of typical single-star evolution, through mass transfer or mergers, leading to changes in stellar structure and atmospheric chemistry (Hurley et al., 2002; Temmink et al., 2020). A prime example of how binary interactions can influence stellar evolution is given by dwarf carbon (dC) stars. These low-mass, main-sequence stars exhibit spectra with molecular carbon features typically associated with spectra of evolved carbon stars found at the tip of the asymptotic giant branch (AGB; Dahn et al. 1977), but in advance of dredge-up processes being able to modify surface chemistry.

The peculiar atmospheric chemistry (C/O > 1) in the prototype dC star, G77-61, is thought to originate through mass transfer from an evolved, higher-mass companion (Dearborn et al., 1986). In this picture, as the former primary star ascends the AGB, tripleα-processed material
is dredged to its surface before stellar winds liberate the consequently carbon-enhanced outer layers (Iben and Renzini, 1983b). Enriched material is then transferred to the main-sequence companion, polluting its atmosphere and creating a dC star, while the original primary becomes a white dwarf. The exact physical means of mass transfer in these systems remain unknown, but possibilities include classical processes of stellar-wind capture and Roche-lobe overflow, and more-recent models such as wind–Roche-lobe overflow (Abate et al., 2013). Regardless of the detailed mechanisms, metal-poor stars are significantly more susceptible to atmospheric pollution through external carbon enrichment processes (de Kool and Green, 1995).

There is little optical evidence for white-dwarf companions to dC stars, where only \( \lesssim 1 \) per cent of systems exhibit flux consistent with a white dwarf at blue wavelengths (e.g. Heber et al., 1993; Liebert et al. 1994a; Green 2013b). Nonetheless, recent observational evidence supports a binary formation channel for dC stars. A radial-velocity study of 28 dC stars, each with typically 3–5 measurements obtained over several years, suggests a dC binary fraction of at least 75 per cent (Whitehouse et al., 2018). A larger survey of 240 dC stars, at a lower resolution, also found a high binary fraction, possibly as large as 95 per cent, with a mean orbital period on the order of 1 yr (Roulston et al., 2019).

The first four, well-characterised dC binaries all have orbital periods on the order of years, as established by radial-velocity or astrometric monitoring (Dearborn et al., 1986; Harris et al., 2018): G77-61 (0.67 yr), LSPM 0742+4659 (1.23 yr), LP 255-12 (3.21 yr), and LP 758-43 (11.4 yr). These systems are all single-lined spectroscopic binaries or their astrometric equivalent, with neither optical nor ultraviolet indications of flux from their suspected evolved companions.

The existence of long orbital periods (\( \gtrsim 100 \) d) among dC stars is commensurate with better-studied classes of carbon-enhanced binary systems, such as CH, Ba, and carbon-enhanced metal-poor (CEMP) stars, which are all thought to follow analogous evolutionary pathways and which have orbital periods of hundreds to thousands of days (McClure et al., 1980b; Lucatello et al., 2005c; Jorissen et al., 2016b). In contrast, short-period binaries are not necessarily expected, as Roche-lobe overflow tends to be unstable in the case of mass transfer from a higher- to a lower-mass star, and a population-synthesis study for dC stars verifies that short-period systems fail to result if accretion is inefficient during a common-envelope phase (de Kool and Green, 1995).

However, in the same year that three of the long-period orbits were published, the dC star SDSS J125017.90+252427.6 (hereafter J1250) was reported to have a photometric period of 2.9 d, and radial-velocity measurements folded on this period appear to provide a good orbital solution.
This system has two notable features: photometric variability on a period potentially identical to the orbital period, and prominent emission in the Balmer lines and in Ca II H & K, all of which are photospheric (Margon et al., 2018). Such emission features are rare among dC stars, with only a handful known among the ~1200 confirmed and suspected candidates in the Sloan Digital Sky Survey (Green, 2013b). Furthermore, a Chandra study of six emission-line dC stars detected each of them in X-rays – a clear indication of stellar activity (Green et al., 2019). Such activity was not anticipated for dC stars, as they are expected to be drawn from populations of older, metal-weak disc stars as well as truly metal-poor halo stars (including the prototype G77-61 with [Fe/H] = −4; Plez and Cohen 2005) on the bases of their kinematics (Harris et al., 1998; Plant et al., 2016; Farihi et al., 2018) and population-synthesis modelling (de Kool and Green, 1995).

Motivated by these findings, this study investigates the possibility that emission lines observed in some dC stars arise because of unexpectedly rapid rotation, associated with tidal synchronisation in short-period binaries, or with spin-up due to mass transfer. This paper presents the results of radial-velocity and photometric time-series analyses conducted for seven dC stars with Hα emission, and finds that six stars in the sample are binaries with orbital periods in the range 0.2–4.4 d. All seven sources show photometric variability on periods that are either orbital, or are similarly short. The radial-velocity and photometric observations are summarised in Section 4.2, the methods and period analyses are discussed in Section 4.3, the results of each individual target are presented in Section 4.4, and a discussion follows in Section 4.5.

### 4.2 Observations and data reduction

#### 4.2.1 Target selection

The literature was searched to identify dC stars known to exhibit Hα emission (e.g. Lowrance et al. 2003; Green 2013b), with six selected for further study based on brightness, visibility, as well as proper motion consistent with the main sequence, and are listed in Table 4.1. Four have been monitored previously for radial-velocity variations over a several-year baseline, with a median of five observing epochs per target (SDSS J101548.90+094649.7, CLS 29, SBSS 1310+561, CBS 311; Whitehouse et al. 2018). Furthermore, the emission-line dC star J1250 is included in this analysis, though no new data have been obtained for this target.
Tab. 4.1.: Observed dC stars with Hα emission.

<table>
<thead>
<tr>
<th>Target</th>
<th>J2000 Coordinates (h m s)</th>
<th>SDSS r (AB mag)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>SDSS J084259</td>
<td>08 42 59.8 +22 57 29</td>
<td>17.1</td>
<td>1</td>
</tr>
<tr>
<td>SDSS J090128</td>
<td>09 01 28.3 +32 38 33</td>
<td>17.1</td>
<td>1</td>
</tr>
<tr>
<td>SDSS J101548</td>
<td>10 15 48.9 +09 46 49</td>
<td>16.9</td>
<td>2</td>
</tr>
<tr>
<td>CLS 29</td>
<td>10 40 06.4 +35 48 02</td>
<td>14.9</td>
<td>3</td>
</tr>
<tr>
<td>SBSS 1310+561</td>
<td>13 12 42.4 +55 55 54</td>
<td>14.3</td>
<td>4</td>
</tr>
<tr>
<td>CBS 311</td>
<td>15 19 06.0 +50 07 02</td>
<td>17.4</td>
<td>5</td>
</tr>
</tbody>
</table>

Notes. The final column lists either the discovery or the first reference to illuminate the nature of the source: (1) Green 2013b; (2) Whitehouse et al. 2018; (3) Mould et al. 1985; (4) Rossi et al. 2011a; (5) Liebert et al. 1994a.

4.2.2 Radial velocities

Radial-velocity monitoring was conducted using the ISIS spectrograph on the 4.2-m William Herschel Telescope at the Roque de Los Muchachos Observatory. Some of the observations are reported in a previous paper (Whitehouse et al., 2018), and further observing runs have since been carried out in 2018 Aug, 2019 Feb and Apr, and 2020 Feb. All observations prior to 2020 were carried out using only the blue arm of the spectrograph, without any dichroic, in combination with the R1200B grating. An EEV12 detector was employed without binning, providing wavelength coverage of approximately 5000–6000 Å and a resolving power $R \approx 6700$ at the central wavelength (1 arcsec slit). This wavelength range encompasses many strong atomic and molecular lines, including the Mg i b triplet, Na i D doublet, and two prominent C2 Swan bands (see Table 4.2).

In 2020 Feb, both the blue and red arms of the ISIS spectrograph were used, with the GG495 dichroic. The blue arm was equipped with the R600B grating to provide wavelength coverage of 3510–5340 Å, achieving a resolving power $R \approx 2700$ at the central wavelength. These data were obtained using the EEV12 detector without binning. The red arm of the spectrograph was employed with the R1200R grating, covering a wavelength range of 5770–6770 Å, where a resolving power of $R \approx 7400$ was achieved. These data were obtained using the RED+ detector with no binning. Together, these more recent data cover Hα and the higher Balmer lines.

Beginning in 2019, each star was observed a number of times (typically two to four) during each night to probe for short-term changes in radial velocity. Each observation consisted of at least three separate exposures of the science target, with wavelength-calibration frames taken immediately before and after. Individual exposure times for the stars varied between 180 and 600 s, typically, for the brighter and fainter sources, respectively. Standard stars and other calibration data were taken at least once per night.
All science exposures were bias subtracted, flat fielded, and extracted using standard methods in IRAF. For extraction, the 2-dimensional spectral trace was established using the standard star observation with the highest signal-to-noise (S/N) of the night and was re-centred for the extraction of each target spectrum. Individual exposures were wavelength calibrated using the appropriate arc frames for each target. Finally, all consecutive target exposures were combined using inverse variance weighting, to create a single spectrum with S/N $\gtrsim 10$ for analysis.

### 4.2.3 Photometry

Time-series photometry used in this study was sourced primarily from the public data releases of the Palomar Transient Factory (PTF), intermediate-Palomar Transient Factory, and Zwicky Transient Facility (ZTF) surveys (Law et al., 2009; Kulkarni, 2013; Bellm, 2014). The primary science goals of these projects are to monitor the sky for transient events, using the Palomar 48-in Oschin Schmidt Telescope with wide-field imagers, surveying the entire available northern sky roughly every three days. The detection limit of the survey is $R \approx 20.6$ mag (Rau et al., 2009), and all data are reduced and calibrated by the dedicated facility pipelines (Ofek et al., 2012; Masci et al., 2019).

Data were retrieved for J0901, J1015, and CBS 311 by querying the PTF DR3 and ZTF DR5 databases using their Gaia eDR3 astrometry (Gaia Collaboration et al., 2020). For each target, a cone search was made using a 15-arcsec radius centred on the 2015.5 Gaia positions. Only matches within 0.5 arcsec of the expected position were selected for analysis; this is well within the typical diameter of the point spread function of the surveys. All photometric data with poor quality flags were rejected.

Photometric monitoring of the dC star J0842 was conducted by Kepler during K2 campaign 18, with continuous measurements for two months at a cadence of 30 min (Howell et al., 2014). Other sources of photometry were not used for this target, as the Kepler data are of superior quality.

Finally, CLS 29 and SBSS 1310 were observed with the Transiting Exoplanet Survey Satellite (TESS; Ricker et al. 2015), and are sufficiently bright and isolated that useful data were obtained. CLS 29 was observed in Sector 22, and SBSS 1310 in Sectors 15, 16, and 22 (with an interval of 139 days between Sectors 16 and 22). The duration of each TESS sector is approximately 25 d, with photometric observations taken every 30 min.
Tab. 4.2: Wavelength regions and atomic transitions used for spectral cross-correlation to determine absolute velocities.

<table>
<thead>
<tr>
<th>Wavelengths (Å)</th>
<th>Important lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>4092 – 4112</td>
<td>Hδ</td>
</tr>
<tr>
<td>4330 – 4350</td>
<td>Hγ</td>
</tr>
<tr>
<td>4851 – 4871</td>
<td>Hβ</td>
</tr>
<tr>
<td>5163 – 5187</td>
<td>Mg i b</td>
</tr>
<tr>
<td>5197 – 5215</td>
<td>Fe i / Cr i blends</td>
</tr>
<tr>
<td>5258 – 5273</td>
<td>Fe ii</td>
</tr>
<tr>
<td>5325 – 5331</td>
<td>Fe i</td>
</tr>
<tr>
<td>5368 – 5376</td>
<td>Fe i</td>
</tr>
<tr>
<td>5863 – 5924</td>
<td>Na i D</td>
</tr>
<tr>
<td>6553 – 6573</td>
<td>Hα</td>
</tr>
</tbody>
</table>

4.3 Methods and period analysis

4.3.1 Radial-velocity cross-correlation

The most precise measurements of radial-velocity changes are achieved by cross-correlating individual spectra of a given target against its highest S/N counterpart. Such measurements benefit from the strong molecular carbon absorption features (e.g. C<sub>2</sub> Swan bands) in dC stars. To achieve the best cross-correlation possible, all spectra were first continuum-normalised by fitting a cubic spline. The cross-correlation was performed using the IRAF FXCOR package (Tonry and Davis, 1979a). This cross-correlation method reduces the velocity error by around 50 per cent compared to a determination of absolute velocity using the few available (weak) atomic lines. These more precise but relative velocity measurements are then used to determine the orbital parameters of each target.

To establish an absolute velocity scale for these data, the highest S/N spectrum of each target was cross-correlated against a continuum-normalised synthetic spectrum template. These spectra were synthesised using Autokur, which is a wrapper for the Kurucz spectral synthesis codes (Kurucz, 2005), with the underlying atmospheric structures based on MARCS models (Gustafsson et al., 2008). Stellar parameters were chosen to be representative of low-mass main-sequence stars (i.e. appropriate for late-K to early-M dwarfs), and to reflect the older kinematical ages of the dC population as a whole. Templates were generated for $T_{\text{eff}} = 3500$–4500 K in steps of 250 K, and metallicities [Fe/H] from −2 to 0 in integer steps, with the surface gravity and microturbulence kept fixed at $\log_{10}[g \text{ (cm s}^{-2})] = 5.0$ and $v_t = 2.0 \text{ km s}^{-1}$, respectively. The synthetic spectra were generated at resolving powers of $R = 6700$ and 7900 to match the instrumental resolution achieved under standard and best observing conditions at the telescope, respectively.
Because stellar-atmosphere models are not currently available for cool, carbon-rich dwarf stars, the synthetic spectra were generated for stars with solar carbon abundances, at gravities and effective temperatures similar to those expected for a typical dC star. The C₂ Swan bands, which contain myriad individual transitions and are useful for relative velocity measurements, are not present in the solar-abundance templates, and would lead to increased measurement errors in velocity if used as the primary source of cross-correlation. Therefore, to minimise the measurement errors, restricted wavelength regions of the synthetic templates – those containing the strongest atomic lines – were selected for cross-correlation. Using these restricted regions significantly improved the quality of each cross-correlation (albeit while still not as precise as the relative velocity measurements), by mitigating spectral mismatches. The wavelength regions used for the synthetic templates are listed in Table 4.2.

To determine the best synthetic template to measure absolute velocities, each synthetic template was cross-correlated against the highest S/N spectrum of each observed dC star. The template that achieved the strongest cross-correlation function was therefore deemed as the most suitable. In general, templates at cooler temperatures achieved a stronger cross-correlation function than templates at higher temperatures; those with $T_{\text{eff}} \leq 4000$ K consistently outperformed their warmer counterparts across the whole metallicity range. Furthermore, templates with metallicity $[\text{Fe/H}] = 0$ typically performed worse when compared to lower metallicity templates. Overall, the resolving power of the templates exhibited no performance trend in cross-correlations. The stellar parameters of the best performing synthetic spectral template were $[\text{Fe/H}] = -1$, $T_{\text{eff}} = 3750$ K, and $R = 6700$. This template was therefore adopted to measure absolute velocities, as it consistently possessed strong cross-correlation metrics with high S/N observed spectra.

### 4.3.2 Radial-velocity period determinations

The [joker](./sc/k.sc/e.sc/r.sc) is software designed to determine orbital periods from radial-velocity data, under the assumption that all variability is the product of two-body gravitational interactions (Price-Whelan et al., 2017; Price-Whelan et al., 2020). For a given number of samples drawn from a prior probability distribution function (PDF), it uses the method of rejection sampling to eliminate low-likelihood solutions, generating a random value for each posterior sample, $r_j$, between 0 and the maximum likelihood in the posterior distribution, $L_{\text{max}}$. If the likelihood of a specific sample, $L_j$, does not meet the condition that $r_j < L_j$, then the sample is rejected. This approach guarantees at least one surviving sample and is particularly useful where radial-velocity data are sparse.
Prior PDFs were selected for orbital eccentricity, velocity semi-amplitude, and systemic velocity as follows. Only circular orbits were considered, both for simplicity and because the available radial-velocity sampling is insufficient to robustly characterise eccentric orbits. Should a short-period solution result for any given system, then the circular orbit assumption is retrospectively well justified for post-common-envelope binaries (Paczynski, 1976; Podsiadlowski, 2001). Velocity semi-amplitudes were sampled from a Gaussian PDF: for each target, the mean was set to zero, with the standard deviation set to 110 km s\(^{-1}\) (taking the absolute value). The adopted standard deviation is based on the expected velocity semi-amplitude for a binary with a total mass of 1.1 M\(_\odot\), a 5 d period, and 60° inclination. The width of the prior PDF on velocity semi-amplitude is intended to avoid biasing the posterior distributions. Finally, systemic velocities were sampled from a Gaussian distribution also centred at zero, with a standard deviation of 100 km s\(^{-1}\). Again, this prior PDF is uninformative and should not bias against any actual velocities.

Because the JOKER analysis is performed on the more precise, relative radial-velocity data, the systemic velocity returned from the analysis is offset from the true value. This was corrected by applying the absolute velocity offsets as determined by the synthetic spectral templates, while propagating the errors in the offset measurement (which are assumed to be Gaussian) with the systemic velocity posterior distribution.

To ensure that an adequate number of posterior samples survive the JOKER analysis, 10\(^9\) prior samples are generated for each target, with the maximum posterior sample size set to 8192. However, the number of posterior samples removed during the rejection sampling algorithm in the JOKER is a function of the maximum likelihood of those posterior samples. Therefore, for targets possessing more constraining data, fewer posterior samples survive.

In the scenario where few samples survive rejection sampling, the posterior distribution returned from the JOKER is typically unimodal with fewer than 100 samples. Owing to the small number of surviving samples, in order to explore the posterior distribution fully, these surviving samples are used as prior samples to initialise a Markov Chain Monte Carlo (MCMC) simulation. The PYMC3 NO-U-TURN sampler was used with four chains and a tuning length of 1000, yielding a total of 2500 posterior samples per chain (Salvatier et al., 2016; Hoffman and Gelman, 2011). This further analysis with MCMC was necessary for the dC stars CLS 29 and SBSS 1310, as only 10 and 74 samples survived rejection sampling, respectively. All other targets possess sufficiently large posterior distributions from the JOKER such that this additional analysis was obviated.
The orbital parameters of each target are quoted as the median of the final posterior distribution (whether through the JOKER or MCMC) and presented in Table 4.3. The median was selected due to the non-Gaussian nature of some of the distributions. The uncertainties in the orbital parameters of each target are quoted as the 16 per cent and 84 per cent confidence limit in the posterior distribution. This analysis was completed for the six targets discussed in this paper, as well as J1250. The radial-velocity curves corresponding to the median orbital parameters derived through this analysis are shown in Figure 4.1, with the posterior PDFs of each target displayed in Appendix A.

4.3.3 Periodogram analysis of photometry

All photometric time-series were normalised to the median value for each target, then analysed using the ASTROPy implementation of the Lomb-Scargle (LS; Lomb 1976b; Scargle 1982b) periodogram, and the P4J (Huijse et al., 2018) implementation of Multi-Harmonic Analysis of Variance (MHAOV; Schwarzenberg-Czerny 1996b). The results of the LS analysis are presented in Figure 4.2.

The LS periodogram is a method to derive the period of oscillation, for a recurrent signal, in unevenly sampled time-series data through the construction of a Fourier-like power spectrum (VanderPlas, 2018b); it is almost equivalent to performing a least-square fit of a sine wave (cf. Zechmeister and Kürster 2009). Such a periodogram can be relatively difficult to interpret, as the Fourier transform of any continuous signal in the data is convolved with the window function. It can therefore be challenging to identify which periodogram peaks represent real signals in the time-series data. To break the degeneracy between the true and aliased peaks, an LS periodogram was computed for the window function for each target, and the window-function periodogram compared to the periodogram computed from the actual fluxes.

For an LS periodogram peak to be considered real, it must be sufficiently separate in frequency from a significant peak in the periodogram of the window function. Quantitatively, a significant peak in the window function was defined as any peak with power above 20 per cent of the strongest peak in the window function. The minimum separation in frequency space between a window function peak and an LS peak was chosen to be 0.001 d$^{-1}$. Finally, a false-alarm probability was calculated by performing a bootstrap on the photometric data for the top five peaks that satisfied these conditions. All peaks surviving these quality checks were then considered to be real periodic signals in the data.

To calculate the error in the photometric period corresponding to the best LS periodogram signal for a particular target, a random number of data points were dropped from the dataset.
Fig. 4.1.: Radial-velocity curves corresponding to the median, posterior parameter values detailed in Section 4.3.2. For each target, the radial-velocity data and corresponding measurement errors are shown in orange, phase-folded and fitted in blue with the determined orbital parameters. The dashed black line corresponds to the median of the fitted curve, representing the systemic velocity. The orbital period and reduced $\chi^2$ are given in Table 4.3.
Fig. 4.2: Left: Lomb-Scargle periodograms for the six dC stars studied here, in addition to J1250. The most significant peak in the periodogram is highlighted with an orange vertical bar. Right: The phase-folded light-curves corresponding to the strongest periodogram peak for each target.
and replaced by randomly sampling the original dataset. This new dataset was then analysed as outlined above, and the process was repeated $10^3$ times to generate a distribution of most likely photometric periods, where the final error is quoted as the standard deviation of this distribution. The LS periodograms for the seven targets and their most significant frequency peaks are shown in Figure 4.2.
<table>
<thead>
<tr>
<th>Star</th>
<th>$P_{rv}$ (d)</th>
<th>$K_1$ (km s$^{-1}$)</th>
<th>$i_{mod}$ (°)</th>
<th>$\chi^2_{red}$</th>
<th>$P_{phot}$ (d)</th>
<th>$\Delta m$ (mag)</th>
<th>Distance (pc)</th>
<th>$M_r$ (AB mag)</th>
<th>$m_{NUV}$ (AB mag)</th>
<th>Variability?</th>
</tr>
</thead>
<tbody>
<tr>
<td>J0842</td>
<td>...</td>
<td>&lt; 13.8</td>
<td>&lt; 6.1</td>
<td>...</td>
<td>0.945$_{-0.004}^{+0.001}$</td>
<td>±0.014</td>
<td>680</td>
<td>8.0</td>
<td>22.0</td>
<td>− − +</td>
</tr>
<tr>
<td>J0901</td>
<td>2.1133$_{-0.0025}^{+0.0002}$</td>
<td>24.5$_{-1.6}^{+0.7}$</td>
<td>14.3</td>
<td>2.74</td>
<td>2.12$_{-0.01}^{+0.01}$</td>
<td>±0.023</td>
<td>590</td>
<td>8.2</td>
<td>...</td>
<td>− − +</td>
</tr>
<tr>
<td>J1015</td>
<td>0.203$_{-0.002}^{+0.001}$</td>
<td>18.5$_{-2.7}^{+2.5}$</td>
<td>4.9</td>
<td>2.19</td>
<td>0.196$_{-0.003}^{+0.003}$</td>
<td>±0.022</td>
<td>480</td>
<td>8.5</td>
<td>18.0</td>
<td>− + +</td>
</tr>
<tr>
<td>CLS 29</td>
<td>3.14707$_{-0.0010}^{+0.00001}$</td>
<td>78.9$_{-7.0}^{+0.8}$</td>
<td>65.3</td>
<td>2.94</td>
<td>3.62$_{-0.01}^{+0.01}$</td>
<td>±0.014</td>
<td>235</td>
<td>8.0</td>
<td>22.9</td>
<td>− − +</td>
</tr>
<tr>
<td>J1250</td>
<td>2.56$_{-0.13}^{+0.13}$</td>
<td>86.1$_{-3.4}^{+4.2}$</td>
<td>67.2</td>
<td>0.47</td>
<td>2.92$_{-0.01}^{+0.01}$</td>
<td>±0.023</td>
<td>280</td>
<td>9.2</td>
<td>&gt; 22.0</td>
<td>− − +</td>
</tr>
<tr>
<td>SBSS 1310</td>
<td>4.33454$_{-0.00006}^{+0.00006}$</td>
<td>46.5$_{-0.8}^{+0.8}$</td>
<td>36.7</td>
<td>0.53</td>
<td>5.26$_{-0.01}^{+0.01}$</td>
<td>±0.015</td>
<td>100</td>
<td>9.2</td>
<td>20.3</td>
<td>− − +</td>
</tr>
<tr>
<td>CBS 311</td>
<td>0.1866$_{-0.0004}^{+0.0029}$</td>
<td>29.2$_{-1.7}^{+2.0}$</td>
<td>7.6</td>
<td>4.83</td>
<td>0.30$_{-0.01}^{+0.01}$</td>
<td>±0.030</td>
<td>440</td>
<td>9.2</td>
<td>...</td>
<td>− + +</td>
</tr>
</tbody>
</table>

Notes. Errors are shown for the orbital period, velocity semi-amplitude and the photometric period, with posterior distributions shown in the Appendix A. The third column lists a model binary inclination based on the adopted or suspected radial-velocity period and velocity semi-amplitude (or upper limit), under the assumptions of 0.4 $M_\odot$ for the dC star, and 0.6 $M_\odot$ for a white-dwarf companion. The fourth column lists the reduced $\chi^2$ for each of the orbital solutions fitted in Figure 4.1. The final column gives a positive (+) or negative (−) indication as to whether the observed photometric variability amplitude could plausibly result from modulation due to tidal distortion (T), a heating effect from irradiation (I), or rotation (R).
4.3.4 Multi-harmonic analysis of photometry

The MHAOV method was then used as an independent check on the LS periodograms. In principle, this method differs from the LS analysis in terms of the model fitted to the data, and the statistic calculated from that fit. Whereas the LS method assumes the data are well modelled by a sine wave, the MHAOV method instead uses orthogonal Szegö polynomials. It should be noted that the first harmonics of the Szegö polynomials simplify to a sine wave, and thus in this work, both the LS and MHAOV method are fitting sine waves to the photometric data.

The advantage of using MHAOV, therefore, comes from how the goodness of fit statistic is calculated (Lachowicz et al., 2006b). The statistic used in the LS method is simply $\chi^2$, and depends on the variance of the data. However, because the true data variance is not actually known, but rather estimated from the data, the goodness-of-fit statistic for the LS periodogram and variance estimations are not independent. The MHAOV method circumvents this problem by using the Fisher analysis-of-variance statistic instead of a $\chi^2$, thus breaking the dependency between the statistic and the variance. This has the benefit of suppressing the significance of periodogram peaks that are due to aliasing in the data; therefore, allowing for the determination of real peaks to be significantly easier than in the LS analysis. Furthermore, the MHAOV analysis is capable of detecting periodic signals at lower S/N than the LS analysis, while also assigning real signals with a higher significance than the LS analysis (Schwarzenberg-Czerny, 1996b). Based on these statistical advantages, the MHAOV method was used in parallel to the LS method, and in particular to verify the five strongest peaks identified from the LS analysis for each target. The MHAOV analysis agreed with the strongest peak identified through the LS analysis for all targets in this study and hence, only the LS periodograms are shown in Figure 4.2. All photometric periods quoted in this study are the most significant peak in both analyses.

4.4 Results for individual stars

The results of both the photometric and radial-velocity analyses are presented here in detail for each target, with a brief summary provided in Table 4.3.

4.4.1 SDSS J084259+225729

The dC star SDSS J084259+225729 (hereafter J0842) has an unusual spectrum compared to its siblings, as is shown in Figure 4.3. While the spectrum exhibits clear CN features in the red, below roughly 7000 Å the flux is broadly consistent with a late-K dwarf. However, there are subtle but convincing signs of C$_2$ Swan bands in the blue, especially when compared
side by side with an appropriate stellar template of solar abundance (e.g. the late sdK stars SDSS J110140.90+385230.2 or SDSS J142617.53+073749.2). Also noteworthy is the CaH band—
a hallmark of metal-poor, cool subdwarfs (Reid and Gizis, 2005; Lépine et al., 2007). In addition to 
H\alpha, close inspection also reveals emission in Hβ and Hγ. Without the clear CN features, the
spectrum might easily be mistaken for that of a K dwarf, although the emission lines would 
remain remarkable.

Interestingly, the source is detected with Galex in both the far- and near-ultraviolet bandpasses.
There are three and four separate detections respectively, with weighted averages of
m_{FUV} = 22.0 AB mag, m_{NUV} = 21.4 AB mag. It is possible that these ultraviolet detections are a result
of strong cool-star line emission at these wavelengths, but there are few transitions that could be
responsible. In the near-ultraviolet Galex band, the Mg ii 2800 Å doublet is a candidate, but this
is highly unlikely. It is near the red cutoff of the bandpass, and furthermore, the emission would
have to be three orders of magnitude brighter than the stellar continuum. The corresponding
far-ultraviolet detection cannot be due to any common stellar emission features, as the bandpass
cuts off before Ly\alpha. Therefore, the observed ultraviolet flux is almost certainly not intrinsic to
the dC star.

The ultraviolet fluxes appear to be consistent with a white dwarf, and in Figure 4.3 the ultra-
violet and optical photometry for J0842 is approximated using models. Solar-abundance models
(Castelli and Kurucz, 2003) were used for the dC star, as there are currently no atmospheric
models for carbon-enhanced cool dwarfs, while white-dwarf atmospheric models were taken
from Koester (2010). A reasonable fit to the ultraviolet-optical photometry is achieved using
a T_{eff} = 12 000 K, hydrogen-atmosphere white-dwarf model, together with a T_{eff} = 4000 K
main-sequence stellar template, where a modestly metal-poor model atmosphere provides a
somewhat better fit than one with solar metallicity.

If the white dwarf is at the Gaia parallax distance of d = 680 ± 50 pc, then, for the observed
temperature and luminosity, evolutionary models\(^\dagger\) suggest a surface gravity close to a canonical
value of log [g (cm s\(^{-2}\))] = 8.0, with a corresponding mass and radius of 0.61 M_⊙ and 0.013 R_⊙,
and cooling age of around 40 Myr (Fontaine et al., 2001). It is noteworthy that the SDSS u-band
photometry appears to be consistent with the composite stellar fluxes, and the Gaia distance
indicates clearly that this is a main-sequence star with M_r = 8.0 AB mag.

\(^\dagger\)http://www.astro.umontreal.ca/~bergeron/CoolingModels
This target was observed during K2 campaign 18. The LS periodogram analysis yields a clearly-defined photometric period of 0.945 ± 0.001 d with a mean semi-amplitude of ±0.014 mag (Figure 4.2).

Nine radial-velocity observations were obtained over three nights in 2019, with two to four measurements per night. Additionally, five further radial-velocity measurements were taken in 2020 Feb. Perhaps surprisingly, these data do not reveal any radial-velocity variations exceeding the error bars (typically ±5 km s\(^{-1}\)). Thus, if J0842 is binary with the suspected white-dwarf companion described above, and its orbital period is similar to the observed photometric period of 0.95 d, then the inclination must be ≤ 6.1° (3σ) for masses of \(M_{\text{dC}} = 0.4 M_{\odot}\) and \(M_{\text{WD}} = 0.6 M_{\odot}\).

### 4.4.2 SDSS J090128.28+323833.5

This source exhibits a dC optical spectrum with no evidence for a luminous companion, and is, therefore, a more typical example of its class, albeit with the exception of the H\(\alpha\) and likely H\(\beta\) emission features. It is detected in X-rays by Chandra (Green et al., 2019), but Galex observations do not exist for this region of the sky. This study is the first to address its potential binarity, and all radial-velocity observations reported are new.

The joker radial-velocity analysis indicates that the orbital period distribution is approximately bimodal, with one larger peak at 2.11 d and a second, smaller peak at around 2.09 d. The median of the posterior distribution yields a period of 2.12\(^{+0.01}_{-0.03}\) d and a velocity semi-amplitude
of $24.5^{+1.5}_{-1.6}$ km s$^{-1}$. A modest number of additional radial-velocity measurements at around a phase of 0.7 would likely precisely constrain the orbital period.

The photometric analysis for SDSS J090128+323833 (hereafter J0901) was conducted on both ZTF and TESS data. Because of the target faintness, the TESS photometry are not useful, but the ZTF data indicate a possible photometric period of 2.12 d. However, this photometric period remains relatively uncertain.

### 4.4.3 SDSS J101548.90+094649.7

This source does not appear in any compendium of dC stars published to date but is flagged as having carbon lines by the SDSS spectroscopic reduction pipeline in Data Release 9. The SDSS spectrum appears to be a composite of a dC star and a DA-type white dwarf that contributes continuum flux and broad absorption at Hγ, Hδ, and He. It is a binary suspect reported by Whitehouse et al. (2018), based on only two radial-velocity measurements which differ by nearly 7σ.

The white dwarf is detected clearly in Galex ultraviolet photometry, where the weighted average of two observations yield $m_{FUV} = 17.6$ AB mag, $m_{NUV} = 18.0$ AB mag. The ultraviolet colour alone suggests an effective temperature between 20 000 and 25 000 K for a typical white-dwarf surface gravity. The inclusion of the SDSS u-band photometry and the $d \approx 480$ pc Gaia parallax distance then favours $T_{\text{eff}}$ closer to, but not quite, 25 000 K to best match the expected absolute ultraviolet magnitudes for a carbon–oxygen core white dwarf. While J1015 is one of the six dC stars detected in X-rays by Chandra (Green et al., 2019), the white dwarf cannot be the source of hard X-rays.

Owing to the low S/N in the spectra of J1015, it was not immediately possible to cross-correlate each spectrum with the relative velocity template. Therefore, each observed spectrum of J1015 was re-binned to lower resolution, with each bin consisting of three pixels. These re-binned spectra were then cross-correlated against the relative velocity template to yield precise velocities for determining the orbital period, before being cross-correlated against the synthetic template, to determine absolute velocities.

The joker radial-velocity analysis finds that the orbital period is strongly peaked at 0.20 d. There are a modest number of posterior samples discretely scattered at either side of this peak, though these samples make up $\lesssim 5$ per cent compared to the peak. The velocity semi-amplitude of J1015 is measured to be $18.5^{+2.7}_{-2.5}$ km s$^{-1}$, and thus the system inclination is likely to be low.

The photometric analysis reveals a strong frequency peak corresponding to a period of $0.196 \pm 0.003$ d, where two smaller flanking peaks in the power spectrum are the result of
windowing. Relatively large photometric errors (averaging 0.07 mag) for this source give rise to the broadened structures in the periodogram. This photometric period is strongest in both the LS and MHAOV analyses, and the phase-folded light-curve at this period shows an amplitude near 2 per cent.

4.4.4 CLS 29

The optical spectrum of this dC star is relatively unremarkable, except for the presence of Hα and Hβ emission. The source is possibly marginally detected in the near-ultraviolet with Galex at \( m_{\text{NUV}} = 22.9 \pm 0.6 \text{ AB mag} \), but too uncertain to warrant further analysis. The TESS data yield a clear peak in the periodogram, corresponding to a period of 3.62 d. The amplitude of the photometric variation is around 1.4 per cent.

This study is the first to report radial-velocity variations in CLS 29. The radial-velocity analysis is highly constraining, with only 1 in \( 10^8 \) posterior samples surviving. The MCMC analysis yields a bimodal distribution in the orbital parameters, where both peaks in the orbital period are within 1 per cent of 3.14 d. Owing to incomplete phase coverage during the radial-velocity monitoring, the period cannot be uniquely determined. Additional velocity measurements for phases 0.4 – 0.5 should break the degeneracy between the two solutions.

4.4.5 SDSSJ125017.90+252427.6

This dC star is a known single-lined binary and the first shown to have a short-period orbit (Margon et al., 2018), where both photometric and radial-velocity variations are observed. It is included here for context and discussed later, as it belongs to the subgroup of stars exhibiting Hα emission lines, and is detected in X-rays (Green et al., 2019). There are Galex observations for this region of the sky, but no detection near the location of J1250 in either ultraviolet band, placing a rough upper limit at either wavelength of \( m > 22.0 \text{ AB mag} \). For a white-dwarf companion at the Gaia parallax distance, this would imply a star no brighter than around \( m = 15 \text{ AB mag} \) in the ultraviolet, and for a typical mass and radius this would translate to \( T_{\text{eff}} \lesssim 8000 \text{ K} \).

The publicly available photometric data from ZTF for J1250 were analysed using the LS and MHAOV analyses. The photometric period of 2.92 d identified by Margon et al., 2018 was successfully recovered.

While the radial-velocity measurements of J1250 taken by Margon et al., 2018 do appear consistent with the photometric period, no dedicated radial-velocity analysis was performed in that study. Here, the reported radial-velocity data are analysed using the JOKER, where the
median posterior distribution for the orbital period is found to be \(2.56^{+0.13}_{-0.11}\) d, and is notably 12 per cent shorter than the reported photometric period.

4.4.6 SBSS 1310+561

This is one of the nearest and brightest dC stars known, and among only three or four objects within or just outside 100 pc (Harris et al., 2018). The optical spectrum is spectacular and shows emission from higher Balmer lines as well as Ca II H and K (Rossi et al., 2011a). There is a detection of this source in Galex in the near-ultraviolet only, with \(m_{\text{NUV}} = 20.3\) AB mag. Similar to the more detailed argument above for J0842, it is unlikely that emission from the dC star is responsible for near-ultraviolet flux detection, and in this case, stellar emission lines would need to be roughly 400 times brighter than the expected continuum. Based solely on the absolute magnitude of this ultraviolet detection, and assuming it was due to a typical surface gravity white dwarf at the Gaia parallax distance to this target, it would imply \(T_{\text{eff}} \approx 7500\) K.

Whitehouse et al. (2018) reported several radial-velocity observations of this star that strongly indicate binarity at a significance just above 7\(\sigma\). Analysis of the previous and newer radial-velocity data with the joker yields only 70 surviving samples. The subsequent MCMC posterior distribution for SBSS 1310 is unimodal for all parameters, with the orbital period tightly constrained at 4.43 d with a velocity semi-amplitude of \(46.5 \pm 0.84\) km s\(^{-1}\).

The TESS data reveal a strong periodic signal corresponding to a photometric period of \(5.26 \pm 0.01\) d. Unexpectedly, the photometric analysis on the individual, cleaned light-curves from each TESS sector reveals a clear change in the photometric period; Sectors 15, 16, and 22, individually, yield periods of \(5.17 \pm 0.01\), \(5.24 \pm 0.01\), and \(5.30 \pm 0.01\) d, respectively. Also serendipitously, the amplitude of the photometric variation changes significantly between Sector 16 and 22, by around a factor of five. These data are discussed in more detail in Section 4.5.2.3.

4.4.7 CBS 311

This system is a previously known binary with a composite spectrum (Liebert et al., 1994a), consisting of the dC star and a \(T_{\text{eff}} \approx 31000\) K DA (\(\equiv\) strongest spectral lines are hydrogen) white dwarf. The region of the sky occupied by this target lacks any Galex observations that could further constrain the luminosity of the companion. CBS 311 is one of the dC stars detected in X-rays by Chandra (Green et al., 2019), but it should be noted again that the white dwarf cannot be the source of the high-energy emission. This target was also imaged at high spatial resolution using the Hubble Space Telescope, where the binary remained unresolved to at least
Fig. 4.4: Representative \( R \)-band light-curve simulations (detailed in Section 4.5.1) for orbital periods representative of the shortest yet measured or suggested for dC stars. The grey bar in each panel represents the range of peak-to-peak variations observed in the dC-star sample (0.028–0.060 mag). The top-left panel demonstrates the influence of tidal distortion alone, while the models represented at top right have identical parameters but include heating (irradiation). The lower-left panel explores the effect of reduced heating (i.e., from cooler white-dwarf companions), and the bottom-right panel shows the effects of decreasing inclination.

25 mas (Farihi et al., 2010), corresponding to a projected separation on the sky of roughly 10 AU at the 440 pc \( \text{Gaia} \) DR2 parallax distance.

The dC star in this binary is fainter than suggested by the \( r \)-band brightness due to some flux contribution from the white dwarf. This diminishes the effective S/N of the absorption lines used for cross-correlation when measuring relative and absolute velocities. Therefore, each spectrum of CBS 311 was re-binned to a lower resolution by taking the mean of every 3 pixels. The radial-velocity analysis identifies one prominent peak in the posterior distribution of the orbital parameters, at 0.19 d with small peaks to either side, where the velocity semi-amplitude is constrained to be \( 29.2^{+2.0}_{-1.7} \) km s\(^{-1}\).

The photometric analysis was conducted on the available ZTF data for this target. A clear periodic signal corresponding to the strongest periodogram peak is located at 0.302 ± 0.001 d, with a semi-amplitude near ±3 per cent.

4.5 Discussion

Spectroscopic and photometric periods are closely similar for all six systems for which both have been established, and are indistinguishable for only two (Table 4.3). For those two, orbital variability is, ostensibly, a plausible mechanism for the photometric changes. For the remaining
stars (at least), the working hypothesis is that the photometric variability must arise through rotational modulation, with the close similarities between orbital and rotational periods arising through tidal synchronisation.

In this section, binary light-curve simulations are presented, in order to provide context for the observed photometric variability. Potential sources of the periodic flux changes, and their implications for the observed emission-line activity, are then explored. Finally, the origin and evolution of these binaries are discussed in a wider context of carbon-enhanced stars.

4.5.1 Binary light-curve simulations

As an aid to interpreting the observed photometric variability, a range of orbital light-curves are simulated in order to gauge the possible effects of close-binary interaction. The goal is to distinguish purely orbital photometric effects from the rotation of the dC star itself. In particular, the aim is to characterise tidal-distortion and heating (irradiation, or day–night) effects arising from a white-dwarf companion, and thereby to characterise the parameter space where these effects may be significant.

A standard Roche model is adopted, with simple 'deep' heating, using the current version of a code originally described by Howarth (1982). These illustrative calculations are carried out for the Cousins $R$ band, as a rough match to the observed photometry, with the dC component modelled as a star with $T_{dC} = 4000$ K, $M_{dC} = 0.44 M_\odot$, and $R_{dC} = 0.42 R_\odot$ (if the lobe size allows; lobe-filling otherwise), which are approximately the values expected for a star near the bottom of the K-dwarf sequence for a metallicity $[\text{Fe/H}] = -1.0$ and an age of 1 Gyr or older (Dotter et al., 2008).

The choice of metallicity has a strong influence on the inferred mass and radius for a fixed effective temperature, where going from solar metallicity to $[\text{Fe/H}] = -2.0$ changes the implied stellar mass from roughly 0.6 to 0.3 $M_\odot$ at 4000 K. The metallicity of dC stars remains poorly determined, except in a single case of extreme metal poverty (G77-61 with $[\text{Fe/H}] = -4.0$; Plez and Cohen 2005), yet the kinematics of dC stars clearly indicates that the population includes a large fraction of metal-poor members. The adopted metallicity of $[\text{Fe/H}] = -1.0$ yields $M_r = 8.6$ AB mag, and this compares favourably with the stars studied here, whose absolute magnitudes span $M_r = 8.0$–9.2 AB mag (see Table 4.3).

The adopted default white-dwarf parameters are $T_{\text{WD}} = 30000$ K, $M_{\text{WD}} = 0.60 M_\odot$, $R_{\text{WD}} = 0.014 R_\odot$, based on parameters for the white-dwarf companion to CBS 311 (Liebert et al., 1994a), one of only two known hot, bright companions to any dC star. The implied mass–radius ratio is fairly typical of single white dwarfs, as well as those found in binaries.
(although in the models the adopted radius is of consequence only for its role in determining the luminosity, \(L/L_\odot \simeq 0.14\), which determines the magnitude of the irradiation). A white dwarf of this nature gives rise to a significant heating (or day–night) effect on its companion. In order to explore the effect of fainter white dwarfs, each simulation at a given orbital period was run for a range of heating efficiencies from 100 per cent down to 20 per cent, where the latter roughly corresponds to \(T_{WD} = 20\,000\,\text{K}\) for the same white-dwarf mass and radius.

The models incorporate a number of simplifications that are inconsequential for the purposes of this study. The dC surface intensities are calculated using a constant, linear limb-darkening coefficient of 0.6 (roughly appropriate for the range of temperatures found), a gravity-darkening exponent of 0.08, and emergent fluxes (from Howarth 2011) interpolated at fixed \(\log [g\,(\text{cm s}^{-2})] = 5.0\). While the white dwarf is bolometrically luminous (giving rise to heating of the dC-star atmosphere), it contributes no flux directly to the simulated light-curves. Orbital periods are taken from the range \(P_{\text{orb}} = 0.2–1.0\,\text{d}\) – the shortest measured or suggested periods known – in steps of 0.1 d, with inclinations \(i = 10–90^\circ\) in steps of 10°.

Figure 4.4 shows a representative sample of the simulated light-curves, each panel holding some parameters fixed while varying others. Notably for these parameters, the dC star fills its Roche lobe just below the simulation range at a period of 0.1 d \((R \simeq 0.33R_\odot)\). At 0.2 d, the tidal distortion of the star results in a photometric semi-amplitude near ±0.15 mag, and with the characteristic double-peaked, non-sinusoidal pattern. Adding heating at the shortest simulated period results in a single peak and semi-amplitude just over ±0.30 mag. At longer periods (i.e., wider separations), both of these orbital-motion-induced effects diminish rapidly; their full combined effect at 0.4 d is less than half that at 0.2 d. For periods approaching and exceeding 1.0 d, both tidal distortion and irradiation have a negligible impact on the emergent flux.

The magnitude of these calculated effects are taken here only as a representative guide, and they are accurate only to a degree. For the tidal distortion, a classical Roche lobe is used for the equipotential surface, without the influence of asynchronous rotation. The atmosphere of the irradiated star is only changed insofar as a local effective temperature modification, and without any calculation of the actual atmospheric structure. Despite these potential shortcomings, the simulations should be sufficient to determine whether an observed photometric variation can plausibly be attributable to orbital motion, as opposed to stellar rotation.

4.5.2 Source of photometric variability

Here, the discussion pursues the question of whether the observed photometric variability in any given system can be attributed to orbital motion or to rotation of the dC star.
4.5.2.1 Rotation versus orbital motion

The seven dC stars listed in Table 4.3 exhibit photometric variability amplitudes falling between $\pm 0.014$ and $\pm 0.030$ mag, with no apparent correlations with orbital period, or with (crudely estimated) inclination. Each system is compared with the light-curve simulations (Fig. 4.4) for the appropriate orbital period (or range), and none are consistent with ellipsoidal variations induced by tidal distortion. There are two elements to this inference; first, the amplitudes of the predicted changes are significantly smaller than those observed, and secondly, the dominant photometric period of ellipsoidal variations should be exactly half the orbital period. While ellipsoidal variations would occur more readily in systems with shorter orbital periods, neither J1015 nor CBS 311 (each with $P_{\text{orb}} \approx 0.2$ d) exhibit such light-curve characteristics. As a result of having well-constrained radial-velocity periods for six of the targets, it can be securely stated that none of their light-curves are consistent with an origin in tidal-distortions.

Next, the observed photometric changes are compared with those that might be induced due to irradiation from a relatively hot and luminous white-dwarf component – the day–night, or heating, effect. The predicted amplitudes can match or exceed the observed ranges, and so a more detailed examination is needed. The simulations suggest CBS 311 might experience a $\pm 0.15$ mag modulation in its light-curve due to irradiation, and the observed value is around five times smaller than this, and thus is unlikely. For J1015, the companion appears to be roughly three times less luminous (see below), and because the irradiated flux scales with luminosity, the heating effect will commensurately decrease. In this case, the simulations indicate that a mildly double-peaked structure is expected, with a semi-amplitude near $\pm 0.07$ mag, and thus a possibility.

There are two additional effects that would mitigate the magnitude of any orbital-induced photometric changes. First, in the simulations, the white dwarf contributes no flux to the light-curve, and thus in cases where the white dwarf is visible in the optical spectrum, there will be a corresponding dilution factor from the contribution of its flux. In CBS 311, the two stars contribute nearly equal flux in the $r$ band (Liebert et al., 1994a), and thus the predicted amplitude of the heating effect on the dC star will be diluted by a factor $\sim 2$. For J1015, the ultraviolet through optical spectra and photometry are decently fit by the addition of a $T_{\text{eff}} = 23,500$ K DA-type white dwarf, whose flux in the $r$ band is approximately 25 per cent of the total, and the amplitude of any heating effect will be reduced accordingly. Secondly, for an $i = 0^\circ$ (face-on) binary, there is no expected flux modulation from either tidal distortion or irradiation, as both binary components are in full view for the entire orbit. The amplitude of both these orbital
effects on the light-curve should increase to first order as $\sin i$ (Figure 4.4), and thus in real observations, where present, these effects will typically be modestly decreased.

Taking into account all of these factors as the data permit, the light-curves of both CBS 311 and J1015 become inconsistent with irradiation at relatively low inclinations. Specifically, for $i < 42^\circ$ the observed photometric variation of CBS 311 is more consistent with rotation, and similarly for J1015 for $i < 25^\circ$. For the stars J0842, J0901, CLS 29, J1250, and SBSS 1310, the only realistic source of photometric variability is rotation. This is next considered as the primary or exclusive cause of the observed light-curve variations in the sample of dC stars is axial rotation.

4.5.2.2 Tidal synchronisation

Figure 4.5 shows the relationship between the photometric and orbital periods for the six targets with well-constrained radial-velocity periods (including J1250). Two systems have essentially identical photometric and radial-velocity periods, while the remaining four have photometric periods that are modestly longer than the radial-velocity periods (further undermining the possibility that the photometric variations are orbital).

Those systems with matching photometric and radial-velocity periods – J0901 and J1015 – are neither the shortest period systems nor those with the most luminous companions, as might be expected if tidal distortion or irradiation were responsible for the photometric variability.
The stars J1250, CLS 29, SBSS 1310, and CBS 311 possess photometric periods on average around 15 per cent longer than their orbital periods, which is inconsistent with orbital motion.

The collective data on these emission-line dC stars therefore broadly support a rotational origin for the photometric changes. However, magnetic braking causes spin-down with age in isolated convective stars. Assuming a crude, and probably conservative, age estimate for dC stars of a few to several Gyr (Farihi et al., 2018), the empirical age–rotation relationship in single M dwarfs predicts spin periods on the order of 20–100 d (Engle and Guinan, 2018) – significantly longer than any of the photometric periods measured in this study, implying an additional source of spin angular momentum.

Tidally induced rotational synchronisation is an appealing mechanism for the implied spin-up and relatively rapid rotation, particularly since the radial-velocity data are consistent with circular orbits (and rotational synchronisation is expected to take place on shorter timescales than orbital circularisation). Theoretical timescales for synchronisation depend mainly on the ratio of the radius of the star experiencing tidal forces (here the dC star) to the orbital separation between the binary components. Adopting canonical values for M-dwarf radii, along with the orbital separations determined from this study, it is found that synchronisation timescales are a few Myr up to no more than a Gyr (Zahn, 1989; Zahn, 2008).

Assuming that all of the binary companions are now white dwarfs, their cooling timescales can be estimated and provide a window over which tidal synchronisation could have occurred. Attributing all Galex ultraviolet flux in these systems to white dwarfs – and upper limits for non-detections – at the Gaia distances to the dC stars, and assuming a fixed white-dwarf surface gravity of $\log \left[ g \ (\text{cm s}^{-2}) \right] = 8.0$, cooling-age estimates and lower limits can be derived from models (Bédard et al., 2020). The resulting cooling ages range from approximately 10 Myr (CBS 311) to over 1 Gyr, but in all cases are broadly consistent with the predicted synchronisation timescale.

4.5.2.3 Differential rotation

Of course, for fully synchronised rotation the photometric signal should exactly match the radial-velocity period (as assumed in the analysis of J1250 by Margon et al. 2018). However, there are modest offsets between the orbital and photometric periods for much of the sample, with the photometric period never shorter than the radial-velocity period. To reconcile this result with fully synchronised equatorial rotation, the dC star must rotate differentially, with starspots (or other surface-brightness inhomogeneities) at relatively high latitudes; for example,
The nature of dwarf carbon stars with Hα emission

4.5.3 Source of stellar activity

There are two basic possibilities for the source of stellar activity in dC stars, as manifested by Hα and similar emission features; an intrinsic source via relatively rapid stellar rotation, and an extrinsic source via irradiation.

In order to heat a low-mass star to sufficiently high local surface temperatures that emission lines are excited, a white-dwarf companion must be close to the irradiated star and fairly
luminous. Based on a comprehensive study of stellar activity as traced by H α emission in M dwarf companions to white dwarfs (Rebassa-Mansergas et al., 2013), it was found that activity is nearly always intrinsic for both partly and fully convective stars. Furthermore, while the influence of magnetic braking is seen for partly convective stars in wide binaries, those M dwarfs in short-period orbits resulting from a common envelope all remain active. This fact alone argues that irradiation is not the cause of stellar activity in low-mass stars closely orbiting white dwarfs, and the study furthermore demonstrates that only white dwarfs with an irradiating flux that is at least 3 × higher than the intrinsic M dwarf flux can result in emission (Rebassa-Mansergas et al., 2013). None of the seven short-period dC stars in this study approaches this level of irradiation; e.g. the 30 000 K white dwarf in CBS 311 is only 1.5 × more luminous than its companion, and thus falls drastically short of this criterion at the surface of the dC star. Therefore, irradiation cannot be responsible for the observed activity in dC stars.

The remaining possibility requires relatively rapid rotation to generate a solar-type dynamo (Parker, 1955), and an active chromosphere (Boro Saikia et al., 2018). The fact that dC stars with emission lines are relatively rapid rotators is clearly evidenced by X-ray detections in six out of six observed cases (Green et al., 2019), including four of the stars analysed here. There are three potential angular momentum sources for the rapid rotation observed in dC stars: intrinsic spin of formation, spin-up owing to mass transfer, and tidal synchronisation in a binary.

Three lines of evidence discount the idea that dC stars are relatively young and hence, still rotating rapidly due to their youth. One is their overall kinematical properties that suggest a mixture of old disc and halo stars (Farihi et al., 2018), second is the indication from population synthesis that metal-poor stars are more readily polluted to C/O > 1 than solar abundance stars (de Kool and Green, 1995), and third is through estimating the cooling ages of their white-dwarf companions. The actual masses and temperatures of dC stars are certainly not well constrained, and not established yet at any level of confidence, but based on their positions on the H-R Diagram they should be partly convective stars with parameters comparable to late-type dwarfs, and because of inevitable magnetic braking (Skumanich, 1972; McQuillan et al., 2013), they require an extrinsic origin to their current spins, assuming their total ages are at least a few Gyr.

Thus, only mass transfer or tidal synchronisation remain as possibilities for the observed, spin-induced stellar activity. And it should be noted that both can occur, that is they are not mutually exclusive (however, it is noteworthy that the latter can erase any evidence of the former, e.g. in the case the dC was once spinning faster than the orbital period). If some fraction of the emission-line dC stars are the product of spin-up through mass transfer, and not the
result of tidal synchronisation in short-period binaries, the observed rotation rate would be an
indication of how much mass the system has accreted. Therefore, measurements of the rotation
rates in any systems that are not short-period binaries (such as for the Ba giant HD 165141; Theuns et al. 1996) would provide constraints on the amount of mass, and angular momentum accreted, and thus inform mass transfer models (Izzard et al., 2010; Matrozis et al., 2017).

4.5.4  Period distribution

The overall results indicate clearly that Hα emission in dC stars is linked to short-period
binarity. In fact, of the ten dC stars with constrained orbital parameters, there are two distinct families: those with Hα emission all have orbital periods of a few days or less, those without
detectable Hα emission have orbital periods of several months or longer (Dearborn et al., 1986; Harris et al., 2018; Margon et al., 2018).

The data at hand suggest that the orbital period distribution of the dC star population may be
bimodal, and thus consistent in some key aspects with the (vastly) more commonly occurring
M-type dwarf companions to white dwarfs. In general, such bimodal populations are consistent
with the post-main sequence removal of intermediate-period orbits, via a combination of
common-envelope evolution for shorter periods, and the reduction of gravitational binding energy due to mass loss for longer periods. However, there is a notable difference in the dC star binary population, and that is the orbital periods on the order of years, which are absent in the carbon-normal, M-dwarf companion population orbiting white dwarfs (Farihi et al., 2010; Nebot Gómez-Morán et al., 2011; Ashley et al., 2019). It is tempting to speculate that the dC companion orbits may differ from dM companion orbits owing to mass transfer, but that is beyond the scope of the current study. Nevertheless it is clear that the short-period dC binaries have experienced a common envelope (Izzard et al., 2012; Ivanova et al., 2013). If the emission-line dC stars have evolved through a common-envelope phase, and assuming their carbon enhancement is extrinsic, then this suggests relatively stable mass transfer prior to the common envelope.

Extrinsically carbon-enhanced binaries possessing short-orbital periods are rare. Amongst the Ba, CH, and CEMP-s populations, which typically possess orbital periods of many months or
years (Lucatello et al., 2005c; Jorissen et al., 2016b; Jorissen et al., 2019), only two stars are known
to have orbital periods shorter than ten days. One of these is the central binary in the Necklace
nebula, with a 1.2 d orbital period, and whose exact link to other carbon-enhanced populations
remains to be determined (Miszalski et al., 2013). The other is HE 0024-2523, a 0.9 M⊙ main-
sequence star with a 3.4 d orbital period measured through radial-velocity monitoring (Lucatello
et al., 2003), and a CH spectral class. Similarly to the short-period binaries found here, HE 0024-2523 is suspected of receiving mass transfer from an evolved companion before evolving through a common envelope. However, there is no evidence of short-period photometric variability or rotation in HE 0024-2523, despite decent quality TESS data, and the star lacks emission lines.

It is tempting to speculate why, on the one hand, Ba, CH, and CEMP-s stars appear to result from relatively wide and essentially detached binary evolution; and on the other hand, why the vast majority of post-common envelope companions to white dwarfs appear to be carbon-normal M dwarfs. Unfortunately, there are currently no robust constraints on the fundamental stellar parameters of dC stars (i.e. mass, effective temperature, metallicity), and a distinct lack of appropriate atmospheric models. It is therefore beyond the scope of the current study to address the different pathways in stellar and binary evolution that might give rise to the observed orbital period differences in these populations. However, it is hoped that the empirical results presented here will encourage novel evolutionary modeling. Because dC stars should far outnumber their evolved counterparts, those with short orbital periods provide valuable constraints for models of extrinsic carbon-enrichment.

4.6 Conclusions

This study analyses time-series radial-velocity and photometric data for seven dC stars with Hα emission lines. Six of the dC stars analysed here are found to possess orbital periods in the range 0.2–4.4 d, along with photometric periods that are either identical (in the case of J0901 and J1015), or within a few tens of per cent of the orbital period. The seventh dC star, J0842, lacks significant radial-velocity variations but possesses a 0.945 d photometric period, and if these periods are similar it would be difficult to detect through ground-based radial-velocity measurements.

Binary light-curve simulations suggest that the origin of the photometric variations observed in these emission-line dC stars is consistent with rotation. None of the light-curves show any evidence for a day- and night-side effect or tidal distortion, and while irradiation is physically plausible in two of the seven systems, it is unlikely based on binary inclination and companion brightness estimates. Thus, rotation is the most likely cause for the observed photometric variations. The mostly sinusoidal behavior of the (best) observed light curves is not necessarily expected for a random assortment of starspot latitudes and stellar spin inclinations, but both the light-curve data quality and the number of short-spin period dC stars are insufficient to determine further constraints on the geometry and number of starspots.
The expectations of magnetic braking are such that the dC stars should be spinning an order of magnitude more slowly than observed, and their ages are consistent with sufficient time to have become tidally synchronised to their visible or unseen (and presumed to be) white-dwarf companions. Tidal synchronisation provides the most consistent picture for these short-period dC star binaries, and provides the additional angular momentum needed for spin-up, which in turn powers the emission lines through a solar-type dynamo. However, mass transfer cannot be ruled out, but in the sample studied here, mass transfer is only necessary for the extrinsic source of pollution, and not for spin-up. Lastly, at least a few of the dC stars appear to be differentially rotating, as evidenced by photometric periods that are similar to, but somewhat longer than their orbital periods, as well as at least one system with changing photometric period and variability amplitude.

This study more than doubles the number of dC stars with constrained orbital periods, and increases those known to be in short-period orbits from one to six. It appears the dC binary period distribution may be bimodal, with a post-common-envelope population as studied here, and a wider population with periods on the order of months to years. Interestingly, the other well-studied, carbon-enhanced stellar populations lack a short-period, post-common-envelope population, and it is likely the dC stars will provide key insight into mass transfer models on the basis of these, and future orbital period determinations.
Dwarf carbon stars make up the largest fraction of carbon stars in the Galaxy with ≈ 1200 candidates known to date primarily from the Sloan Digital Sky Survey. They either possess primordial carbon-enhancements, or are polluted by mass transfer from an evolved companion such that C/O is enhanced beyond unity. To directly test the binary hypothesis, a radial-velocity monitoring survey has been carried out on 37 dwarf carbon stars, resulting in the detection of variations in at least 20 targets. Using an MCMC analysis, this detection fraction is found to be consistent with a 94 per cent binary population and orbital periods on the order of hundreds of days. This result supports the post-mass-transfer nature of dwarf carbon stars, and implies they are not likely hosts to carbon planets.


5.1 Introduction

Carbon stars were historically thought to lie on the asymptotic giant branch (AGB), having dredged up triple-$\alpha$ burning products to their surface. This gives rise to distinct atmospheric chemistry when the C/O ratio exceeds unity, revealing strong molecular absorption bands of $\text{C}_2$, CH, and CN. Intriguingly, it is now known that dwarf stars can exhibit the same distinct absorption features, thus indicating that there exists a carbon-rich, main-sequence stellar population.

The first dwarf carbon (dC) star discovered was G77-61, a $T_{\text{eff}} \approx 4100$ K, high proper-motion star that was at first assumed to be an M dwarf. A discrepancy was noticed when the $M_V = +10.08$ mag derived from parallax measurements was compared to the observed colour, with the star appearing far redder than expected. Spectroscopy revealed strong molecular carbon features, typical of classical carbon giants, were responsible for the red colour (Dahn et al., 1977) and established the first known main-sequence star with distinct $\text{C}_2$ absorption bands.

Stellar evolution does not predict the synthesis of carbon in single stars until the AGB, resulting in two possible explanations for the atmospheric chemistry of G77-61. The first
hypothesis is that the star was formed in a carbon-rich environment, and the second is that mass was transferred from an evolved companion (now unseen; Dahn et al. 1977). Radial-velocity monitoring of G77-61 over a baseline of three years revealed variations consistent with a circular orbit and 245 d period (Dearborn et al., 1986). The mass function indicated the unseen component must have a mass of at least $0.55 \, M_\odot$, consistent with a white dwarf.

Carbon is produced via the triple-$\alpha$ process through helium shell burning on the AGB. This material is then mixed into the envelope raising the carbon abundance over time via a series of convection and pulsation episodes, with the largest being the third dredge-up. This process can produce a C/O ratio well above unity for stars of intermediate mass (Iben and Renzini, 1983a). If the star is part of a binary, then this carbon-rich material can be transferred to the companion via Roche-lobe overflow or efficient wind capture. However, the mass transfer mechanism is currently unconstrained for dC stars owing to the lack of information on orbital separations.

Mass transfer of carbon-rich material from an AGB star is widespread amongst binary systems, with other well-known examples being carbon-enhanced, metal-poor s-type (CEMP-s), Ba, and CH stars. CEMP-s stars are defined by their relatively low metallicity, high carbon abundance, and high abundance of Ba ($[\text{Fe/H}] < -2.0$, $[\text{C/Fe}] > +1.0$, $[\text{Ba/Fe}] > +1.0$; Aoki et al. 2007), whereas Ba and CH stars are more loosely defined as containing strong absorption features of Ba and CH respectively. These stars are typically giants with Ba and CH stars found in the red clump and on the main-sequence turn-off respectively (Escorza et al., 2017), while CEMP-s stars populate the first ascent giant branch (RGB). All three populations exhibit radial velocity variations consistent with high binary fractions and orbital periods typically on the order of hundreds to thousands of days (McClure and Woodsworth, 1990; Jorissen et al., 1998; Hansen et al., 2016).

In this chapter, the first results of a radial-velocity monitoring survey are presented for 37 dC stars. The results to date are consistent with a binary fraction possibly as high as 100%, supporting a post-mass transfer origin. In Section 5.2, the target selection and observations are described, with the results given in Section 5.3 and 5.4. The results are discussed in Section 5.5, with the preliminary conclusions presented in Section 5.6.

5.2 Observations and data reduction

5.2.1 Target selection

Potential targets were compiled from the literature based on brightness, and were selected as having a high likelihood of being a main-sequence star. The bulk of potential targets were found
within the Sloan Digital Sky Survey (SDSS), with the first few hundred candidates identified via colour cuts (Downes et al., 2004). The largest sample of dC candidates to date was later discovered via cross-correlation to template spectra in DR7 and DR8 (Green, 2013a). Additional dC stars identified via colour and proper motion were also included among potential targets (Liebert et al., 1994b; Totten et al., 2000; Lowrance et al., 2003; Rossi et al., 2011b), including the prototype G77-61.

Initially, the goal of the radial-velocity survey was to test for binarity, and thus, targets were originally selected solely on their brightness and visibility. However, as the survey developed, a preference was given to dC stars that exhibit Hα emission owing to their high probability of being members of short-orbital-period binaries (see Chapter 4). An initial pool of 73 potential targets brighter than $g = 19.0$ AB mag was formed, where ultimately, nine dC stars possessing Hα emission and 28 dC stars lacking emission were observed at least twice in the survey. The complete list of the 37 targets is presented in Table 5.1.

### 5.2.2 Spectroscopic survey

The radial-velocity monitoring survey began in 2011 September and concluded in 2020 February, and consists of 45 nights of observations spread across three telescopes.

The first observations of the radial velocity survey were taken at the Gemini-North Telescope using the GMOS spectrograph. These observations were obtained over five nights spaced between 2011 September and 2012 February and monitored the radial velocity of G77-61 only. As such, the radial velocities measured at the Gemini-North Telescope form the smallest radial-velocity dataset in this sample. The adopted instrumental setup uses a 1′′ slit, in combination with the B1200+G5321 grating in 1st order. The spectral coverage is 4750–6250 Å and the resolving power at the central wavelength corresponds to $R \approx 4000$. The EEV detector was used to read out the data with $1 \times 1$ binning. Each target exposure is 300 s, where one observation consists of three consecutive exposures average combined to increase S/N and mitigate the effects of cosmic rays. Bias and flat-field frames were observed either at the start or end of each night. Arc frames were observed directly before each set of target exposures to ensure an accurate wavelength calibration was possible. Furthermore, only one observation of G77-61 was taken per night. Hence, this dataset yields five radial-velocity measurements spread over five nights.
These data were reduced using IRAF\textsuperscript{1} following the GMOS reduction cookbook\textsuperscript{2}. Spectra were extracted using the IRAF package APALL.

Further radial-velocity monitoring was conducted over nine nights on the Mayall 4-m Telescope, located at Kitt Peak national observatory. Initially, two nights of observations were taken in 2013 April, utilising the 4-m Ritchey-Chretien Focus (RC) Spectrograph, combined with the T2KA detector and the data were read out with no binning. The instrumental setup employed a 1\textquoteright slit, in unison with the BL 380, 120 line/mm grating in 2\textsuperscript{nd} order, yielding spectral coverage of approximately 5100–5700 Å, and a resolving power $R \approx 6000$ at the central wavelength.

Additional radial-velocity data were taken on the Mayall 4-m Telescope in 2014 April and December over two and three nights, respectively. These observations were conducted again using the RC spectrograph, with the T2KA detector and no binning. However, the instrumental setup for these observations utilised the KPC 18C, 790 line/mm grating in the 2\textsuperscript{nd} order. The instrumental setup covers the wavelength region 5000-6000 Å and achieves a resolving power $R \approx 2000$ at the central wavelength.

The final observations taken on the Mayall 4-m Telescope occurred over two nights in 2015 December and were conducted using the KOSMOS spectrograph, equipped with the Blue VPH grism and a 0.6\textquoteright slit. The spectral coverage of these data spans 4300–7000 Å, and the resolving power of the instrumental setup corresponds to $R \approx 3500$ at the central wavelength.

All observations obtained on the Mayall 4-m Telescope were taken at an airmass below 2.0, with individual exposure times in the range of 300–900 s, yielding S/N $> 10$. Each observation per target typically consists of three consecutive exposures that were combined with a mean weighting. Combining individual exposures has the effect of raising S/N while also mitigating any effects from cosmic rays.

Bias and flat-field frames were taken at the start of each night, and arc frames were taken directly before and after each set of science exposures. Suitable arc lamps were selected for wavelength calibration by ensuring that a high number of emission features were present in the spectrum of the arc lamp over the observed spectral range. The wavelength solution of each set of science exposures was found by taking the average wavelength solution determined

\textsuperscript{1}IRAF is distributed by the National Optical Astronomy Observatory, which is operated by the Association of Universities for Research in Astronomy (AURA) under a cooperative agreement with the National Science Foundation.

\textsuperscript{2}http://ast.noao.edu/sites/default/files/GMOS_Cookbook/Processing/IrafProclS.html
for the arc frames taken directly before and after each set of science exposures, thus correcting for any telescope flexure during the observation.

All spectral images were trimmed, bias subtracted, and flat-fielded using standard routines in IRAF. Spectral extraction was completed using the APALL package, where the 2-dimensional spectral trace for each night of observations was established using a bright standard star observed on a corresponding night. The trace was then re-centred before extracting each the science spectrum.

The vast majority of the radial-velocity observations in this survey were carried out using the ISIS spectrograph on the William Herschel Telescope (WHT) at Roque de los Muchachos. These data represent 31 nights of observations and were obtained between 2013 February and 2020 February. All observations taken before 2020 February utilise the ISIS blue arm only, fitted with the EEV12 detector with no dichroic. These data were read out with no binning. For these observations, the ISIS blue arm was equipped with the R1200B grating and a 1″ slit was used to achieve a resolving power $R \approx 6400$ over the range 5000–6000 Å. Observations were taken at airmasses below 2.0 and used individual exposure times between 300 and 1200 s, with a goal signal-to-noise (S/N) ratio $>10$. In a similar method used at the other facilities, an observation consists of three or more individual exposures that were combined with mean-weighting for each target.

The observations taken in February 2020 utilised both the blue and red arms of the ISIS spectrograph, that was fitted with the GG495 dichroic. The blue arm of the ISIS spectrograph was fitted with the R600B grating, and these data were recorded with the EEV12 detector with no binning. This instrumental setup provides wavelength coverage of 3510–5340 Å, and a resolving power $R \approx 2700$ at the central wavelength. The red arm of the ISIS spectrograph was equipped with the R1200R grating providing spectral coverage of 5770–6770 Å, and achieving a resolving power $R \approx 7400$ at the central wavelength. Data recorded by the red arm were read out using the RED+ detector, again with no binning.

The observations taken at the WHT were reduced and calibrated in precisely the same process as those taken on the Mayall 4-m Telescope. The only notable difference between the observations taken at the WHT compared to the observations obtained at the other facilities is that the sampling rate was increased for the last nine nights of observations taken on the WHT (February and April 2019, and February 2020). Rather than observing each target once per night, several targets were observed several times per night in an attempt to probe for short orbital period binaries. A more thorough discussion of these observations is presented in Chapter 4.
All spectra observed during this survey – except those taken with the WHT in 2020 February – cover a similar spectral range of $\approx 5000-6000\,\text{Å}$. The choice to observe this spectral region was motivated by the presence of several strong absorption features necessary for robust cross-correlation, with the region including the C$_2$ Swan bands at 5165 and 5636 Å, the Mg I triplet at 5183 Å, and the Na I doublet at 5889 Å. An example spectrum is shown in Figure 5.1 which also displays the critical atomic and molecular features used for cross-correlation.

The observations taken with the WHT in 2020 February predominantly searched for short-period variability in dC stars exhibiting H\textalpha emission. Therefore, this instrumental setup was designed to cover the Balmer series, as these emission features could be used for cross-correlation. Thus, the wavelength regions 3510–5340 Å and 5770–6770 Å were observed in the blue and red arms, respectively. Additionally to the Balmer series, these wavelength regions allow for the inclusion of the Mg I triplet at 5183 Å, the Na I doublet at 5889 Å, and a few Fe I and Fe II lines (see Table 4.2).

### 5.2.3 Radial-velocity measurements

Radial-velocity variations were measured using the package \texttt{FXCOR} that performs a Fourier cross-correlation between a series of input spectra and a designated template spectrum (Tonry and Davis, 1979b). Initially, only relative radial velocities were measured for each target owing to the lack of high-quality carbon-enhanced synthetic spectra necessary to establish accurate absolute velocity measurements. Therefore, for each target, the highest S/N spectrum was chosen as the template against which all other observations were cross-correlated (Marsh et al.,...
For dC stars, this method gives the benefit of utilising the strong C$_2$ Swan bands in the cross-correlation process, resulting in a reduction in velocity error of around 50 per cent compared to using carbon-normal synthetic stellar templates. Thus, the relative radial-velocity measurements were used to probe for binarity and constrain orbital parameters for each dC due to superior precision over the measured absolute velocities. The velocity residuals (in km s$^{-1}$) of the cross-correlation were added in quadrature to the velocity uncertainty from the wavelength calibration, yielding a total error for each pair of spectra. This total error depends on the S/N of the target, with higher S/N targets possessing errors < 1 km s$^{-1}$ and lower S/N targets exhibiting errors up to 6 km s$^{-1}$, but typically less than a few km s$^{-1}$.

Absolute radial velocities were subsequently measured by cross-correlating the highest S/N spectrum of each target against a series of continuum normalised synthetic stellar templates. This process yields the offset between the highest S/N spectrum of each target and the laboratory reference frame, which can then be applied to all relative radial velocities measured from the high S/N spectrum. All synthetic stellar templates were synthesised using the AUTOKUR software that acts as a wrapper for the Kurucz spectral synthesis codes (Castelli and Kurucz, 2003), which in turn are based on MARCS atmospheric structure models (Gustafsson et al., 2008). Because little is known about the stellar parameters of dC stars, a total of 30 synthetic models were synthesised, spanning a range of metallicities, effective temperatures, and instrumental resolutions. The stellar parameters of each of these 30 synthetic templates were chosen to represent low-mass main-sequence stars with spectral types of late-K and early-M dwarfs – broadly consistent with the stellar parameters expected for dC stars in this survey. The templates were generated at $T_{\text{eff}} = 3500$–$4500$ K in steps of 250 K, metallicities [Fe/H] = −2 to 0 dex in integer steps, and instrumental resolving powers of 6700 and 7900, corresponding to average and best observing conditions achieved at the telescope, respectively. All templates were synthesised with constant surface gravity and microturbulence of log $[g$ (cm s$^{-2}$)] = 5.0 and $v_t = 2.0$ km s$^{-1}$, respectively.

Because stellar-atmosphere models do not currently exist for cool, carbon-rich dwarf stars, the series of synthetic templates was computed for cool, carbon-normal dwarf stars. Due to the paucity of carbon features present in the synthetic templates, the quality of any cross-correlation between the synthetic templates and the observed dC spectra is reduced. Notably, the absence of the C$_2$ swan bands drastically impairs the cross-correlation process. Furthermore, the multitude of vibrational transitions associated with each molecular carbon feature in the observed dC spectra can lead to spectral mismatches during cross-correlation with the synthetic...
templates. Therefore, the cross-correlation between the dC spectra and synthetic templates yields higher errors than relative velocity measurements.

In an attempt to curtail spectral mismatches between the dC spectra and the synthetic templates, cross-correlation was only performed on selected regions of the spectrum. These finite regions were centred on each of the strongest atomic lines present in the observed dC spectra, with a width sufficient to ensure that the atomic lines were well sampled regardless of their width and any Doppler shift. A complete list of the wavelength regions used is given in Table 4.2.

To ensure the most accurate absolute velocity is measured, each observed dC star must first be matched with its closest matching synthetic stellar template. Therefore, each synthetic template was cross-correlated against the highest S/N spectrum of each dC star using only the pre-defined wavelength regions listed in Table 4.2. The quality of each cross-correlation was measured using the Tonry-Davis ratio (TDR; Tonry and Davis 1979a), that measures the strength and the FWHM of the cross-correlation. The TDR is a real number such that the following is true: $TDR \in \mathbb{R} : TDR \in [0, \infty]$, where the TDR is minimised by a poor cross-correlation, and maximised by a strong cross-correlation. The TDR values of each set of cross-correlations between the highest S/N dC spectra and the synthetic templates are in the range 5–30 and the normalised TDR values are displayed in Figure 5.2. The TDR can take a broad range of values, and therefore, to easily compare the quality of each cross-correlation, the TDR values were normalised\(^3\), where 0 and 1 correspond to the worst and best cross-correlations of each dC star, respectively. Thus, through this normalisation, it is possible to compare the success of each template across all observed dC stars.

Inspecting the TDR values corresponding to the cross-correlation between each pair of observed dC spectra and synthetic templates reveals no optimal template for this sample of dC stars. Instead, each dC star finds its unique template that yields the highest TDR. Several dC stars, such as LHS 1075 and SDSS J012028 (amongst others), achieve their highest TDR with synthetic templates possessing lower metallicities and higher effective temperatures. Whereas dC stars such as SBSS 1310 + 561 and SDSS J145725 achieve their highest TDR with synthetic templates at a specific effective temperature and little dependence on metallicity. Generally, templates corresponding to lower resolving power obtain a higher TDR than templates corresponding to higher resolving power, though this relationship is weak. Due to the lack of a clear optimal synthetic template for the dC sample, absolute velocities were measured by cross-correlating

---

\(^3\)The normalisation was computed using the following equation: $z_i = \frac{x_i - \min(x)}{\max(x) - \min(x)}$
Fig. 5.2.: The results of the cross-correlation between the highest S/N spectrum of each dC star against all synthetic stellar templates, normalised so that the best and worst cross-correlation scores correspond to 0 and 1, respectively. The colour bar indicates the strength (or quality) of each cross-correlation, where darker colours correspond to higher cross-correlation scores. The names of each template correspond to the metallicity, effective temperature, and resolution of each template (e.g. m0t3500R6700 corresponds to $[\text{Fe/H}]=0.0$, $T_{\text{eff}}=3500\,\text{K}$, and $R=6700$).
the highest S/N spectrum of each dC star with the synthetic stellar template possessing the highest TDR for that target.

Because the templates are synthesised for carbon-normal stars, it is unclear whether these results can be used to place any constraints on the true metallicity and effective temperatures of the dC stars in this sample. Furthermore, the highest TDR ratios obtained by cross-correlating the dC spectra to these synthetic templates are typically around 15 per cent of the average TDR ratio obtained by cross-correlating the individual spectra of a given star, with that star’s highest S/N spectrum. Hence, the velocities produced through this method are uncertain compared to the relative radial velocities reported in this study and should therefore be treated with caution.

5.2.4 Measuring the offsets in wavelength solutions at different observing facilities

Because the radial-velocity survey was conducted on three separate telescopes and uses six different instrumental setups, it is essential to identify and correct any systematic offsets in the wavelength solutions measured at each of the different facilities and instrumental setups used. Systematic offsets were identified by comparing the central wavelength of the O i skyline (with vacuum wavelength 5577.338Å) in all science spectra observed throughout the survey. The O i skyline was selected because it is present in all science spectra observed before 2020 February and is the only strong skyline in the spectral region covered in all observations. The O i skyline originates from emission in the Earth’s atmosphere, and therefore, its wavelength is independent of where the observation is taken and the instrumental setup used. Thus, any observed shift in the measured wavelength of the O i skyline is a good approximation of the systematic instrumental offset.

To measure the wavelength of the O i skyline in each dC star observation, all science spectra were re-extracted without any background subtraction, thus ensuring the presence of the O i skyline in these newly extracted spectra. The non-background extracted spectra were subsequently clipped in wavelength to cover the region of 5560–5590Å – the region surrounding the O i skyline. The continuum and the profile of the O i skyline were then simultaneously modelled in each clipped spectrum by performing a least-χ² fit between a linear model and Voigt profile to the continuum and the skyline, respectively. The wavelength of the O i skyline was recorded, along with its uncertainty, as the centre of the Voigt profile for each clipped spectrum of each science spectrum. Figure 5.3 shows a direct comparison between the central wavelength of the O i skyline of each spectrum observed in the radial-velocity survey, colour-coded by
Fig. 5.3.: Left panel: The measured wavelength of the O i skyline of each observation in the radial-velocity survey, colour-coded for each of the instrumental setups used. The laboratory wavelength of O i is shown as a grey horizontal line. Right panel: A histogram binned in wavelength corresponding to the measured O i wavelength in each observation taken in the radial-velocity survey. The colours are identical to those used in the left panel and highlighted in the legend. Notably, the only instrumental setup with a clear offset from the laboratory wavelength of O i is the KOSMOS instrument used at the Kitt Peak National Observatory.
the instrumental setup used to make the observation. All instrumental setups yield broadly consistent measurements of the central wavelength of the O I skyline, except for the KOSMOS spectrograph used on the 4-m Mayall Telescope. The median wavelength offset between the KOSMOS instrument and the blue arm of the ISIS spectrograph fitted with the R1200B grating is 0.42 Å, corresponding to a velocity offset of $\approx -22.9 \text{ km s}^{-1}$. Hence, a wavelength correction was applied only to data observed using the KOSMOS spectrograph by shifting each spectrum by 0.42 Å, bringing the dataset in line with the ISIS blue arm and the other instrumental setups. No other dataset was corrected.

The right panel of Figure 5.3 shows the dispersion in the measured wavelength of the O I skyline of each instrumental setup. Notably, each different instrumental setup possesses a unique dispersion in the measured wavelength of the O I skyline, where the resolving power of the instrument is anti-correlated with the magnitude of the dispersion. The dispersion in the measured wavelength of the O I skyline correlates well with the average uncertainty obtained from the wavelength solution of each instrumental setup. This result is unsurprising, and the dispersion in each instrument is well within the radial-velocity errors of each instrument.

5.3 Radial-velocity period determinations

Having corrected all science spectra of any systematic offsets in the wavelength calibrations, relative radial velocities were determined for each dC star. These corrected radial-velocity measurements were then used to constrain the orbital parameters of each dC star observed in this survey using the JOKER rejection sampler (Price-Whelan et al., 2017; Price-Whelan et al., 2020). A detailed description of the JOKER algorithm is given in Section 3.3.4. Briefly, the JOKER uses rejection sampling to fit radial-velocity curves to radial-velocity data under the assumption that any radial-velocity variations are the result of a two-body gravitational interaction.

The JOKER begins by generating sets of prior samples from specified prior PDFs for each of the orbital parameters. Each set of prior samples describes a radial-velocity curve, which is then fitted to the data. A likelihood is then calculated using this radial-velocity curve and the observed data. Once a likelihood has been computed for all sets of prior samples, rejection sampling is used to remove poor-quality solutions, yielding final posterior distributions of each orbital parameter.

The JOKER was used to constrain the orbit of each dC star listed in Table 5.1, regardless of whether a published orbit exists in the literature (or this thesis). Prior PDFs were defined for the orbital period, orbital eccentricity, velocity semi-amplitude, and systemic velocity as
### Tab. 5.1: Dwarf carbon stars observed during the radial-velocity monitoring survey with at least two observations. Column six shows the $\rho$ of each target expressed logarithmically with an arbitrary lower limit placed at $\log_{10}(\rho) = -0.6$, corresponding to a $1:10^6$ chance of a target being non-binary. The seventh column gives the maximum radial velocity difference between any two sets of observations, and in the eighth column this is divided by the largest error in relative radial velocity, and is thus a measure of the statistical significance. Finally, the ninth column gives the estimated orbital period of each dC star.

<table>
<thead>
<tr>
<th>Name</th>
<th>RA (ICRS (2000))</th>
<th>Dec (ICRS (2000))</th>
<th>Epochs</th>
<th>$\log_{10}(\rho)$</th>
<th>$\Delta v_{\text{rad,max}}$ (km s$^{-1}$)</th>
<th>Significance (cr)</th>
<th>$P_{\text{orb}}$ (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHS 1075</td>
<td>00 26 00.48</td>
<td>$-19.18 52.0$</td>
<td>15</td>
<td>1828</td>
<td>$-3.2$</td>
<td>13.7</td>
<td>1.2</td>
</tr>
<tr>
<td>SDSS J102028</td>
<td>01 20 28.56</td>
<td>$-08.36 30.9$</td>
<td>12</td>
<td>1827</td>
<td>$&lt; -6.0$</td>
<td>25.9</td>
<td>1.1</td>
</tr>
<tr>
<td>SDSS J102150</td>
<td>01 21 50.42</td>
<td>$+01.13 01.4$</td>
<td>11</td>
<td>1827</td>
<td>$&lt; -6.0$</td>
<td>176.3</td>
<td>22.3</td>
</tr>
<tr>
<td>SDSS J13007</td>
<td>01 30 07.13</td>
<td>$+00.26 35.3$</td>
<td>11</td>
<td>1827</td>
<td>$&lt; -6.0$</td>
<td>21.0</td>
<td>1.2</td>
</tr>
<tr>
<td>SDSS J222904</td>
<td>02 23 04.43</td>
<td>$+00.45 10.3$</td>
<td>7</td>
<td>1677</td>
<td>$-2.4$</td>
<td>12.5</td>
<td>1.6</td>
</tr>
<tr>
<td>G77-61</td>
<td>03 32 38.08</td>
<td>$+01.58 00.0$</td>
<td>24</td>
<td>3061</td>
<td>$&lt; -6.0$</td>
<td>42.3</td>
<td>5.6</td>
</tr>
<tr>
<td>SDSS J074257</td>
<td>07 42 57.17</td>
<td>$+46.59 17.9$</td>
<td>17</td>
<td>2488</td>
<td>$-4.1$</td>
<td>28.0</td>
<td>2.6</td>
</tr>
<tr>
<td>SDSS J081157</td>
<td>08 11 57.14</td>
<td>$+14.35 33.0$</td>
<td>12</td>
<td>2210</td>
<td>$&lt; -6.0$</td>
<td>47.6</td>
<td>2.4</td>
</tr>
<tr>
<td>SDSS J081807</td>
<td>08 18 07.45</td>
<td>$+22.34 27.6$</td>
<td>4</td>
<td>622</td>
<td>$-0.7$</td>
<td>23.0</td>
<td>2.6</td>
</tr>
<tr>
<td>PG 0824+288</td>
<td>08 27 05.09</td>
<td>$+28.44 02.4$</td>
<td>10</td>
<td>1059</td>
<td>$&lt; -6.0$</td>
<td>24.8</td>
<td>1.4</td>
</tr>
<tr>
<td>SDSS J084259</td>
<td>08 42 59.80</td>
<td>$+22.57 29.0$</td>
<td>15</td>
<td>345</td>
<td>$-1.5$</td>
<td>10.8</td>
<td>0.6</td>
</tr>
<tr>
<td>SDSS J090128</td>
<td>09 01 28.28</td>
<td>$+32.38 33.5$</td>
<td>25</td>
<td>1514</td>
<td>$&lt; -6.0$</td>
<td>45.2</td>
<td>6.7</td>
</tr>
<tr>
<td>SDSS J090302</td>
<td>09 03 02.86</td>
<td>$+38.55 27.4$</td>
<td>5</td>
<td>1788</td>
<td>$&lt; 0.2$</td>
<td>36.6</td>
<td>0.3</td>
</tr>
<tr>
<td>SDSS J091007</td>
<td>09 10 07.60</td>
<td>$+52.16 12.5$</td>
<td>14</td>
<td>3</td>
<td>$&lt; -6.0$</td>
<td>39.7</td>
<td>6.6</td>
</tr>
<tr>
<td>C0930-00</td>
<td>09 33 24.64</td>
<td>$-00.31 44.5$</td>
<td>17</td>
<td>2558</td>
<td>$-2.9$</td>
<td>8.5</td>
<td>0.9</td>
</tr>
<tr>
<td>SDSS J095334</td>
<td>09 33 34.14</td>
<td>$+06.48 12.6$</td>
<td>11</td>
<td>1044</td>
<td>$-2.3$</td>
<td>10.8</td>
<td>0.7</td>
</tr>
<tr>
<td>SDSS J095545</td>
<td>09 55 45.84</td>
<td>$+44.36 40.4$</td>
<td>8</td>
<td>1060</td>
<td>$-3.1$</td>
<td>38.4</td>
<td>2.2</td>
</tr>
<tr>
<td>SDSS J101548</td>
<td>10 15 48.90</td>
<td>$+09.46 49.7$</td>
<td>23</td>
<td>2550</td>
<td>$-4.1$</td>
<td>99.0</td>
<td>7.8</td>
</tr>
<tr>
<td>CLS 29</td>
<td>10 40 06.36</td>
<td>$+35.48 02.4$</td>
<td>37</td>
<td>2556</td>
<td>$&lt; -6.0$</td>
<td>151.2</td>
<td>11.8</td>
</tr>
<tr>
<td>CLS 31</td>
<td>10 54 29.42</td>
<td>$+34.02 26.0$</td>
<td>5</td>
<td>974</td>
<td>$&lt; -6.0$</td>
<td>103.4</td>
<td>9.4</td>
</tr>
<tr>
<td>SDSS J110458</td>
<td>11 04 58.97</td>
<td>$+27.43 11.8$</td>
<td>4</td>
<td>1059</td>
<td>$&lt; -6.0$</td>
<td>12.6</td>
<td>3.2</td>
</tr>
<tr>
<td>KA 2</td>
<td>11 19 03.90</td>
<td>$-16.44 49.3$</td>
<td>16</td>
<td>2555</td>
<td>$-5.6$</td>
<td>27.0</td>
<td>1.9</td>
</tr>
<tr>
<td>SDSS J112633</td>
<td>11 26 33.94</td>
<td>$+04.41 37.7$</td>
<td>5</td>
<td>1043</td>
<td>$-1.9$</td>
<td>23.8</td>
<td>2.5</td>
</tr>
<tr>
<td>SDSS J120024</td>
<td>12 00 24.09</td>
<td>$+38.17 20.3$</td>
<td>6</td>
<td>2212</td>
<td>$&lt; -6.0$</td>
<td>24.9</td>
<td>2.2</td>
</tr>
<tr>
<td>CLS 50</td>
<td>12 20 00.77</td>
<td>$+36.48 01.7$</td>
<td>23</td>
<td>2558</td>
<td>$&lt; -6.0$</td>
<td>13.8</td>
<td>1.6</td>
</tr>
<tr>
<td>SDSS J122328</td>
<td>12 23 28.13</td>
<td>$+35.32 51.9$</td>
<td>3</td>
<td>677</td>
<td>$-3.6$</td>
<td>8.7</td>
<td>4.3</td>
</tr>
<tr>
<td>SDSS J130744</td>
<td>13 07 44.53</td>
<td>$+60.09 03.7$</td>
<td>4</td>
<td>1653</td>
<td>$-0.3$</td>
<td>5.6</td>
<td>0.5</td>
</tr>
<tr>
<td>SBSS 1310+561</td>
<td>13 12 42.51</td>
<td>$+55.55 54.6$</td>
<td>34</td>
<td>2556</td>
<td>$&lt; -6.0$</td>
<td>75.2</td>
<td>9.8</td>
</tr>
<tr>
<td>SDSS J145725</td>
<td>14 57 25.86</td>
<td>$+23.41 54.5$</td>
<td>7</td>
<td>2072</td>
<td>$-1.7$</td>
<td>5.7</td>
<td>1.6</td>
</tr>
<tr>
<td>CBS 311</td>
<td>15 19 05.99</td>
<td>$+50.07 02.8$</td>
<td>18</td>
<td>2374</td>
<td>$&lt; -6.0$</td>
<td>66.2</td>
<td>1.2</td>
</tr>
<tr>
<td>SDSS J154859</td>
<td>15 48 59.27</td>
<td>$+34.18 21.7$</td>
<td>8</td>
<td>3</td>
<td>$&lt; -6.0$</td>
<td>151.5</td>
<td>42.0</td>
</tr>
<tr>
<td>CLS 96</td>
<td>15 52 37.35</td>
<td>$+29.27 59.1$</td>
<td>10</td>
<td>2484</td>
<td>$&lt; -6.0$</td>
<td>11.0</td>
<td>4.1</td>
</tr>
<tr>
<td>LP 225-12</td>
<td>16 22 32.86</td>
<td>$+42.37 54.2$</td>
<td>10</td>
<td>2142</td>
<td>$&lt; -6.0$</td>
<td>16.6</td>
<td>3.0</td>
</tr>
<tr>
<td>SDSS J184735</td>
<td>18 47 35.67</td>
<td>$+40.59 44.1$</td>
<td>12</td>
<td>2072</td>
<td>$&lt; -6.0$</td>
<td>26.0</td>
<td>2.6</td>
</tr>
<tr>
<td>LSR J2105+2514</td>
<td>21 05 16.54</td>
<td>$+25.14 48.6$</td>
<td>16</td>
<td>2072</td>
<td>$&lt; -6.0$</td>
<td>278.3</td>
<td>23.5</td>
</tr>
<tr>
<td>LP 758-43</td>
<td>21 49 37.84</td>
<td>$-11.38 28.5$</td>
<td>11</td>
<td>1628</td>
<td>$&lt; -6.0$</td>
<td>10.1</td>
<td>0.8</td>
</tr>
<tr>
<td>SDSS J233443</td>
<td>23 34 43.13</td>
<td>$+36.29 07.1$</td>
<td>9</td>
<td>1334</td>
<td>$&lt; 1.0$</td>
<td>11.9</td>
<td>1.8</td>
</tr>
</tbody>
</table>

**Notes.**

$a$ Astrometric period identified by Harris et al. (2018).

$b$ Orbital period identified by Dearborn et al. (1986).

$c$ Orbital period identified by Whitehouse et al. (2021)

$d$ Orbital period identified in this analysis.
follows. The orbital period was sampled from a uniform distribution spanning 0.1–10 000 d. This PDF’s upper and lower limits correspond to rough limits on the sensitivity of the radial-velocity survey to the longest and shortest orbital periods. An orbital period of over 10 000 d would produce radial-velocity variations \(< 1 \text{ km s}^{-1}\), and hence would be undetectable at the instrumental resolution used in this survey. Furthermore, due to the sampling of the radial-velocity survey, it is not possible to probe for orbital periods of shorter than 0.1 d owing to the Nyquist limit. Velocity semi-amplitudes were sampled by taking the absolute value from a Gaussian distribution centred at zero, with a standard deviation calculated via Equation 3.20. Systemic velocities were sampled from a Gaussian distribution centred at zero, with a standard deviation of 100 km s\(^{-1}\). The broad PDF is chosen in an attempt to avoid biasing against any systemic velocities. The JOKER analysis was initially performed by only considering circular orbits due to computational simplicity. While the assumption that all orbits are circular is reasonable for binary systems that have undergone orbital circularisation through tidal forces (typically observed in short-orbital period systems Paczynski 1976; Podsiadlowski 2001), it is not a valid assumption for longer-period binaries where tidal circularisation does not occur. Because several long-orbital period dC binaries are known to exist (Harris et al., 2018), the JOKER analysis was re-run allowing orbits with non-zero eccentricity. During this second run of the JOKER, orbital eccentricity sampled from a Beta distribution such that \(e = \text{Beta}(0.867, 3.03)\) (Kipping, 2013).

The relative radial-velocity measurements were used for the JOKER analysis due to their higher precision compared to the absolute radial-velocity measurements obtained via cross-correlation with synthetic templates. Therefore, the systemic velocities returned by the JOKER are offset from their true values by the absolute velocity of the template used to measure the relative radial velocities of each target. To correct this offset, all spectra of each target were shifted by the absolute velocity of the corresponding radial-velocity template. Furthermore, the measurement errors in the offset were propagated to all relative radial velocities during this correction. Tables of the relative and absolute velocities are presented in B.

Because rejection sampling removes low-likelihood solutions, and hence results in a posterior distribution that is smaller than the number of prior samples drawn, a total of \(2^{28} \approx 3 \times 10^8\) prior samples were drawn for each dC star in each analysis. To reduce the overall computation cost of the analysis, a maximum limit was placed on the size of the posterior distribution corresponding to \(2^{13} (8192)\) samples. Above this limit, it was assumed that the posterior distribution was well explored. Because the number of posterior samples that survive rejection sampling depends on the maximum likelihood of those posterior samples, few samples survive
the rejection sampling process in the regime of high-quality data. Hence, few samples survive
the rejection sampling process for stars that possess highly constraining radial-velocity data,
resulting in a poorly explored posterior PDF, despite a large number of prior samples being
drawn.

For the stars where the JOKER analysis was unable to provide well-explored posterior PDFs
(assumed to be fewer than 100 posterior samples), the analysis was repeated using an MCMC
simulation. The PYMC3 NO-U-TURNS sampler was used with four chains and a tuning (or burn-in)
of 1000 samples, yielding 2500 samples per chain, or a total of 10 000 posterior samples (Hoffman
and Gelman, 2011; Salvatier et al., 2016). This MCMC analysis was needed for a total of seven
dC stars.

5.3.1 Results for individual stars

The dC stars that possess well constrained orbital parameters based on the radial-velocity
analysis are discussed below and their orbital parameters are presented in Tables 5.2 and 5.3.
Furthermore, their phase-folded radial-velocity curves are presented in Figure 5.4. The dC
stars that were discussed in Chapter 4 are omitted from this section for brevity, but a detailed
discussion on each of those stars is presented in Section 4.4.

5.3.1.1 SDSS J01215050.42+011301.4

SDSS J01215050.42+011301.4 (hereafter J0121) was first identified as a dC star in Sloan DR7
(Green, 2013a). Since its initial identification as a dC star, J0121 was reported as a likely binary
star with relative radial-velocity variations measured at the 15σ confidence level (Whitehouse
Tab. 5.3: A summary of the orbital parameters of dC stars with constrained orbital parameters with eccentricity allowed to vary.

<table>
<thead>
<tr>
<th>Target</th>
<th>$P_{\text{orb}}$ (d)</th>
<th>$K$ (km s$^{-1}$)</th>
<th>$e$</th>
<th>$\gamma$ (km s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SDSS J012150</td>
<td>7.767$^{+0.0004}_{-0.0004}$</td>
<td>81.4$^{+5.1}_{-5.1}$</td>
<td>0.18$^{+0.02}_{-0.02}$</td>
<td>$-98.1^{+1.2}_{-1.1}$</td>
</tr>
<tr>
<td>G77-61</td>
<td>246.9$^{+0.3}_{-0.4}$</td>
<td>31.3$^{+6.0}_{-6.0}$</td>
<td>0.26$^{+0.06}_{-0.06}$</td>
<td>$-57.3^{+1.8}_{-1.7}$</td>
</tr>
<tr>
<td>SDSS J090128</td>
<td>2.01202$^{+0.00007}_{-0.00007}$</td>
<td>26.2$^{+3.1}_{-3.1}$</td>
<td>0.13$^{+0.07}_{-0.07}$</td>
<td>$56.5^{+1.9}_{-1.9}$</td>
</tr>
<tr>
<td>SDSS J091007</td>
<td>10.1$^{+4.5}_{-3.7}$</td>
<td>66.6$^{+29.8}_{-29.8}$</td>
<td>0.16$^{+0.12}_{-0.12}$</td>
<td>$52.8^{+2.7}_{-2.7}$</td>
</tr>
<tr>
<td>SDSS J101548</td>
<td>0.201202$^{+0.000003}_{-0.000003}$</td>
<td>25.8$^{+5.4}_{-5.4}$</td>
<td>0.46$^{+0.10}_{-0.10}$</td>
<td>$-3.1^{+2.7}_{-2.7}$</td>
</tr>
<tr>
<td>CLS 29</td>
<td>3.147074$^{+0.00001}_{-0.00001}$</td>
<td>78.8$^{+0.6}_{-0.6}$</td>
<td>0.01$^{+0.01}_{-0.01}$</td>
<td>$-19.7^{+0.7}_{-0.7}$</td>
</tr>
<tr>
<td>SBSS 1310+561</td>
<td>4.43457$^{+0.000004}_{-0.000004}$</td>
<td>46.8$^{+0.6}_{-0.6}$</td>
<td>0.01$^{+0.01}_{-0.01}$</td>
<td>$-25.3^{+0.7}_{-0.7}$</td>
</tr>
<tr>
<td>CBS 311</td>
<td>0.250766$^{+0.000001}_{-0.000001}$</td>
<td>55.9$^{+3.6}_{-3.6}$</td>
<td>0.74$^{+0.03}_{-0.03}$</td>
<td>$-80.4^{+6.7}_{-6.7}$</td>
</tr>
<tr>
<td>SDSS J154859</td>
<td>2.77$^{+0.08}_{-0.08}$</td>
<td>78.1$^{+5.1}_{-5.1}$</td>
<td>0.13$^{+0.09}_{-0.09}$</td>
<td>$10.9^{+2.1}_{-2.1}$</td>
</tr>
<tr>
<td>LP 225-12</td>
<td>1145$^{+628.0}_{-628.0}$</td>
<td>8.9$^{+2.7}_{-2.7}$</td>
<td>0.15$^{+0.23}_{-0.23}$</td>
<td>$-56.5^{+1.7}_{-1.7}$</td>
</tr>
<tr>
<td>LSR J2105+2514</td>
<td>3.3173$^{+0.00007}_{-0.00006}$</td>
<td>119.0$^{+15.1}_{-15.1}$</td>
<td>0.66$^{+0.04}_{-0.04}$</td>
<td>$-218.2^{+1.4}_{-1.4}$</td>
</tr>
</tbody>
</table>

Notes. Orbital parameters are quoted as the median of the posterior PDF returned from the JOKER.

et al., 2018). The optical spectrum is typical of a nominal dC star, albeit with the addition of subtle Hα emission. The parallax of J0121 measured by both Gaia eDR3, and the USNO, agree to the 2σ level, and yield a distance of $d \approx 260$ pc, thus indicating the absolute magnitude of J0121 is $M_r \approx 9.9$ AB mag.

Radial-velocity monitoring of this J0121 began in 2013 August and concluded in 2018 August, consisting solely of observations taken with the WHT. The radial-velocity dataset consists of 12 measurements in total. The observing cadence of these observations is relatively sparse, typically consisting of groups of two consecutive observations with each group spaced many months apart.

Analysis by the JOKER reveals that the posterior PDF of the orbital parameters converge to a unimodal distributions that are approximately Gaussian. The posterior PDF of the orbital period indicates that J0121 is likely a short-orbital period binary, with an orbital period of 7.8 d. The orbital period remains constant in the analysis completed with a circular orbit and in the analysis where eccentricity was allowed to vary. Interestingly, the analysis allowing non-circular orbits yields an eccentricity of $e \approx 0.18^{+0.04}_{-0.03}$. Furthermore, the posterior PDFs of the velocity semi-amplitude and the systemic velocity show some co-variance.

5.3.1.2 G77-61

G77-61 is the first dC star ever discovered. It was originally flagged as a peculiar star due to its large proper motion and red $B - V$ colour, with the former suggesting residency on the main-sequence, and the latter suggesting G77-61 is a first ascent red giant star. The optical
spectrum of G77-61 reveals strong molecular carbon features such as the C$_2$ swan band, and numerous CN bands, and hence confirmed G77-61 as the first known dC star (Dahn et al., 1977).

Almost a decade later, G77-61 was confirmed as a spectroscopic binary with a (spectroscopically and photometrically) unseen companion, where the orbital period was measured to be 245 d. The mass function of the binary suggests the unseen companion possesses a mass consistent with those expected for a white dwarf (Dearborn et al., 1986). However, astrometric monitoring conducted over seven years by the USNO failed to identify G77-61 as an astrometric binary (Harris et al., 2018). Furthermore, the parallax measured by the USNO is consistent at the 3σ level with Gaia eDR3, and these measurements place G77-61 as the nearest dC star identified to-date ($d = 82$ pc). The absolute magnitude of G77-61 is $M_R \approx 9.9$ Vega mag, and at the distance reported by Gaia, makes G77-61 the brightest dC star in the sky.

Observations of G77-61 began in 2011 September at the Gemini-North observatory and concluded on 2020 February at the WHT. A total of 24 radial-velocity measurements were taken during this time. Typically, observations are spaced by at least a few months, however, three sets of two observations were taken on consecutive nights.

The posterior PDFs of the orbital parameters of G77-61 are unimodal and well sampled. The medians of the orbital period posterior PDF corresponding to the analysis of fixed and varying eccentricity agree within 3σ ($245.5^{+0.4}_{-0.3}$ d and $246.9^{+0.3}_{-0.4}$ d, respectively). The orbital period found by the analysis with fixed eccentricity is more closely aligned to the results of Dearborn et al. (1986), with these results lying well within 1σ of each other. Alternatively, allowing the eccentricity to vary yields posterior PDFs that indicated a slightly longer orbital period, and an orbit with eccentricity of $0.26^{+0.07}_{-0.06}$. This result is discrepant with the results of Dearborn et al. (1986), where a circular orbit was favoured by their radial-velocity data.

5.3.1.3 SDSS J091007.60+521612.5

SDSS J091007.60+521612.5 (hereafter J0910) is an $M_r \approx 8.6$ AB mag dC star with weak Hα and Hβ emission. It was first identified as a likely binary due to its large radial-velocity variations measured during the Sloan Time-Domain Spectroscopic Survey ($\Delta_{rv} = 59\pm3$ km s$^{-1}$; Roulston et al., 2019).

Radial-velocity monitoring of J0910 was conducted in 2020 Feb using both the red and blue arms of the ISIS spectrograph. A total of 14 observations were taken over three nights, where observations are spaced by around 2.5 hours on average. The short cadence of observations was selected for this target due to the large $\Delta_{rv}$ reported by Roulston et al. (2019).
The analysis of the radial-velocity data yields unimodal, approximately Gaussian posterior PDFs in all of the orbital parameters. The median orbital period corresponding to the analysis with fixed and variable eccentricity agree within 1σ and are \(11.9^{+3.3}_{-2.8}\) d and \(10.1^{+4.5}_{-3.5}\) d, respectively. The posterior PDFs corresponding to the analysis assuming a circular orbit show a large degree of covariance that likely contributes to the relatively large errors on the orbital period. To break the covariances between the posterior PDFs of the orbital parameters, greater phase coverage is needed and hence, future observations should be taken.

The posterior PDF of the eccentricity for J0910 \((e = 0.16^{+0.20}_{-0.12})\) is poorly constrained and additional observations are needed to place meaningful constraints on this orbital parameter.

### 5.3.1.4 SDSSJ154859.72+341821.7

The optical spectrum of SDSS J154859.72+341821.7 (hereafter J1548) possesses strong Balmer emission features from Hα down to Hϵ. Furthermore, Chandra observations of this star detect an X-ray flux, thus suggesting this star possesses an active chromosphere and corona (Green et al., 2019). Interestingly, the optical spectrum also exhibits the CaH band that is typical of metal-poor, cool subdwarfs (Reid and Gizis, 2005; Lépine et al., 2007).

Eight radial-velocity measurements were taken over the course of three nights in 2020 February with the blue and red arms of the ISIS spectrograph. Analysis by the joker assuming a circular orbit yields well sampled, unimodal, posterior PDFs in all of the orbital parameters. The orbital period corresponding to this analysis is \(2.8\) d and sits in the range of orbital periods found for other emission-line dC stars (see Section 4). Unfortunately, no lightcurve for J1548 exists, and hence, no comment on the source of variability can be made.

Allowing eccentricity to vary yields much more irregular posterior PDFs in each of the orbital parameters. For instance, the posterior PDF corresponding to the orbital period is bimodal with one peak centred at 1.7 d and another at 2.8 d. Furthermore, the posterior PDFs show a high degree of covariance. Therefore, the radial-velocity data obtained for J1548 are likely not of sufficient quality to support an eccentric orbit.

### 5.3.1.5 LP225–12

The earliest reference in the literature that LP 225–12 is a dC star came in a study of nearby ultra-cool dwarf stars identified in the 2MASS infrared survey (Cruz et al., 2003). LP 225–12 was originally noted as being of interest due to its high probability of being a nearby, cool dwarf, but was later confirmed as a dC star due to its high proper motion and clear molecular carbon absorption features (Lowrance et al., 2003). Eventually, after almost a decade and a half
of astrometric monitoring by the USNO, LP 225-12 was confirmed as an astrometric binary with an orbital period of 3.2 yr (Harris et al., 2018). Whitehouse et al. (2018) later identified LP 225-12 as a possible spectroscopic binary through radial-velocity monitoring. The optical spectrum of LP 225-12 is typical of a dC star, with the only notable feature being strong Na i absorption. The parallax measured by the USNO is consistent with that measured by Gaia and places LP 225-12 at a distance of $\approx 110$ pc, corresponding to a $M_R \approx 9.9$ AB mag.

A total of ten observations of LP 225-12 were taken commencing in 2013 August and concluding in 2019 February. Nine of these radial velocities were measured using the WHT with an additional measurement taken at Kitt peak. Analysis by the Joker assuming circular orbits reveals that these data produce a multimodal posterior PDF of the orbital period. The first and smallest of these peaks is centred at short orbital periods of a few days or less, and likely corresponds to arbitrarily short periods that can be fit to the data. Thus, these orbital periods are likely not real and simply arise due to the incomplete sampling of the true period.

The largest of the peaks in the orbital period PDF corresponds to a period of 1160 d (approximately 3.2 yr). This solution matches the astrometric period found by Harris et al. (2018) well, and is likely the true orbital period. However, analysis by the Joker identifies an additional peak in the posterior PDF at an orbital period that is centred at approximately 540 d. This shorter orbital period solution probably arises due to the sparse sampling of the radial velocity curve of LP 225-12. Additional radial velocity measurements would provide a more well sampled radial velocity curve and better constrain the posterior PDFs of LP 225-12.

The analysis allowing the eccentricity to vary produces largely the same result as the analysis assuming circular orbits. Perhaps unsurprisingly the dataset is not rich enough to constrain the orbital eccentricity of this target.

5.3.1.6 LSR J2105+2514

LSR J2105+2514 (hereafter LSR 2105) was first identified as a high-proper motion star (Lépine et al., 2002), before being confirmed as a dC star owing to its numerous strong C$_2$ and CN bands present in the optical spectrum (Lowrance et al., 2003). Interestingly, the $J - K_S = 1.2$ is typical of very late-M dwarfs or early-L dwarfs, suggesting LSR 2105 possesses a low mass ($M < 0.2 M_\odot$). This is supported by its absolute magnitude $M_R \approx 11.1$ Vega mag. Furthermore, astrometric monitoring of LSR 2105 suggests it is a part of the stellar halo, and is hence likely old and metal-poor.

Between 2013 August and 2019 April, a total of 16 radial-velocity measurements were taken of LSR 2105 to probe for variations consistent with binary motion. Three of these radial-velocity
measurements were taken using the facilities at Kitt peak, while the remaining data were observed at the WHT. One observation from 2018 August was omitted from this analysis due to poor signal. The radial-velocity variations measured by cross-correlation are typically at least several tens of km s$^{-1}$ and sometimes over 100 km s$^{-1}$, providing strong evidence that LSR 2105 is a short period binary. Analysis by the Joker assuming circular orbits yields only one surviving posterior sample from an initial pool of $2^{28}$ prior samples, therefore, the radial-velocity data were further analysed using MCMC methods. This additional analysis produces a unimodal posterior PDF of the orbital period, centred at 0.15 d. Thus, LSR 2105 is the shortest period dC star found in this radial velocity survey and known to-date.

Analysing the same data but allowing the eccentricity to vary results in quite different PDFs compared to the analysis assuming circular orbits. Allowing the eccentricity to vary favours a longer orbital period, where the median of the orbital period PDF is located at 3.3 d. However, this result may not be reliable due to the high degree of covariance between the PDFs of the different orbital parameters.

Interestingly, a close companion (within 5$''$) to LSR 2105 exists within the Gaia database, however, due to the faintness of the visual companion, the astrometric measurements and uncertainties provided by Gaia are unreliable. Though, its proper motion suggests this is a background source. Finally, no photometric data are currently available for this target and hence, lightcurve variations can not be probed for.

5.4 The binary fraction and orbital period distribution of dC stars

The dC population has been hypothesised to be the product of mass transfer from an evolved companion, and is hence thought to be a binary population. The radial-velocity survey presented here has successfully constrained the orbits of eleven dC star binaries and thus, offers support to this hypothesis. However, the remaining 26 dC stars do not possess adequate radial-velocity data for confident constraints to be placed on any possible binary orbit. Therefore, to test the binary hypothesis for dC stars, it is necessary to understand for each dC star in this sample, how likely is it a member of a binary.

To test whether any particular dC star is consistent with exhibiting a varying radial velocity, and hence binarity, a weighted $\chi^2$ test was used to determine whether the null hypothesis that the radial velocity is constant could be rejected (Lucatello et al., 2005b; Starkenburg et al., 2014). The results of the weighted $\chi^2$ test are given in Table 5.1, where $p(\chi^2 | \nu)$ is the probability that
Fig. 5.4.: For each target, the radial-velocity data have been phase-folded onto the median of the posterior PDF corresponding to the orbital period obtained from the JOKER. The model corresponding to the median sample in the posterior PDF is plotted in grey. The radial-velocity measurements of each target are plotted in colours corresponding to either the instrumental set-up used to obtain the data, or the location from where the data were obtained.
the observed radial-velocity variations are real given the magnitude of errors, and \( \nu \) corresponds to the number of degrees of freedom (which here is the number of observations). A small \( p \)-value thus indicates a low probability that the null hypothesis of constant radial velocity can be accepted, implying the system is likely binary.

Figure 5.5 displays the cumulative distribution of \( p \)-values derived from the results of the weighted \( \chi^2 \) test. The \( p \)-values are expresses as \( \log_{10}(p) \) and a lower limit was placed at \( \log_{10}(p) = -6.0 \). Despite prior knowledge that G77-61, PG 0824+288, LP225-12, J0742, and LP758-43 are members of binary systems (Dearborn et al., 1986; Farihi et al., 2010; Harris et al., 2018), all these stars were included in the statistics as the sample was primarily selected based on brightness and visibility. No significant radial-velocity variations were detected in the dC stars: PG 0824+288 or LP758-43. However, this is unsurprising given the fact PG 0824+288 is a member of a spatially resolved binary with separation of \( \approx 17 \) au (corresponding to an orbital period of \( \gtrsim 60 \) yr; Farihi et al. 2010). Furthermore, LP758-43 is a member of an astrometric binary with a period of \( \approx 11.35 \) yr (Harris et al., 2018), and hence, any radial velocity variations would fall well below the average radial velocity error of this star (\( \approx 3 \) km s\(^{-1} \)). However, many of the dC stars in this sample possess radial-velocity variations that are not consistent with the null hypothesis of exhibiting a constant radial velocity.

Deriving the true number of binaries amongst this sample of dC stars is tricky as it depends on placing an arbitrary limit at some \( p \)-value. Because the \( p \)-value is derived from a \( \chi^2 \) statistic, it is dependent on the magnitude of the velocity errors. Therefore, if the errors are under or overestimated, then the effectiveness of the \( p \)-value breaks down as a statistical test (Lucatello et al., 2005b; Starkenburg et al., 2014; Preston and Sneden, 2001). For this reason, while careful consideration was taken during error calculation and propagation, the \( p \)-values listed in Table 5.1 may not be reliable. Hence, the fraction of dC stars with \( p \)-values below a certain threshold may be a poor indication of the fraction of dC stars that exhibit varying radial velocity, and subsequently, binarity. Therefore, to better understand the binary fraction of dC stars, MCMC simulations were used to find the most likely binary fraction and orbital period distribution of the population simultaneously.

### 5.4.1 A maximum likelihood analysis of the dC star sample

Markov chain Monte Carlo simulations were used to determine the binary fraction and the most likely orbital period distribution of the dC population. The method uses the measured distribution of maximum changes in radial velocity from this survey of 37 dC stars, and compares this distribution to those of simulated stellar populations where the binary fraction...
Fig. 5.5: The cumulative histogram of $p$-values expressed logarithmically for the sample of 37 dC stars that possess radial-velocity measurements. The dashed lines correspond to arbitrary cuts in the $p$-value that could be used to determine the number of binary stars amongst the sample of dC stars.

and orbital period distribution are treated as free parameters in the simulations. For simplicity, and consistency with studies of other binary populations, the orbital period distribution was assumed to be a log-normal distribution (Duquennoy and Mayor, 1991; Starkenburg et al., 2014). Therefore, the mean and standard deviation of this distribution were allowed to vary. Thus, at each step of the MCMC, the binary fraction along with the mean and standard deviation of the orbital period distribution are varied. The MCMC method awards higher likelihoods to simulated stellar populations that possess a distribution of maximum changes in radial velocity that are most similar to those observed for the dC stars.

At each step of each Markov chain, a population of 10 000 stars were generated where a random fraction of these stars were designated as binary stars. Each of these designated binary stars was assumed to be a dC star with a white dwarf companion. Masses were assigned to each binary component based on expected values of dC stars and white dwarfs, respectively. Furthermore, orbital parameters were randomly assigned to each simulated binary such that a simulated radial-velocity curve could be computed. These simulated radial-velocity curves were then sampled with cadence reflecting the true radial-velocity survey. The maximum measured change in these simulated radial-velocity curves were recorded and compared to the observed distribution of maximum changes in radial velocity in the sample of 37 dC stars.
For every simulated population of stars, the binary fraction was sampled from a uniform distribution between 0 and 1. The mean of the orbital period distribution was sampled from a uniform prior PDF where $-1 \leq \log_{10} \mu(d) \leq 6.6$. The upper limit on the mean of the orbital period distribution corresponds to $\approx 10,000 \text{ yr}$, and acts as a conservative cut in the orbital periods to which this radial-velocity survey is sensitive to. The standard deviation of the orbital period distribution was again sampled from a uniform prior PDF, bound in the range $0 < \log_{10} \sigma(d) \leq 2.5$. Masses of each of the stars designated as binaries were sampled from a Salpeter initial mass distribution (Salpeter, 1955) with upper and lower bounds set at $0.8 \text{M}_\odot$ and $0.2 \text{M}_\odot$, respectively, where these masses are typical of late-K and early-M dwarfs. Masses of the white dwarf companions were sampled from a normal distribution with $\langle M_* \rangle = 0.63 \text{M}_\odot \pm 0.14$ (Tremblay et al., 2016). The orbital parameters of each designated binary were assigned as follows. Orbital inclinations were drawn from a uniform distribution in $\cos(i)$, and the argument of pericenter $\omega$ and pericenter phase $\phi$ were assigned from uniform distributions between 0 and $2\pi$. Orbital eccentricity was assigned depending on the orbital period, with $e = 0$ for $P < 1000 \text{ d}$, and longer periods drawn from a normal distribution $\langle e \rangle = 0.4 \pm 0.2$. All stars that were not designated as binaries were assumed to possess a constant radial velocity.

Upon computing the radial-velocity curves of each designated binary in each of the simulated stellar populations, it is essential that each of these radial-velocity curves are sampled with a cadence that is representative of the actual observational cadence of the radial-velocity survey. Thus, by sampling each simulated radial-velocity curve with a cadence representative of the observational campaign, any comparison made between the distributions of maximum radial-velocity changes for a simulated stellar population and that for the observed dC population is fair. For each simulated radial-velocity curve, between four and 33 mock observations were made, where the exact number of observations was drawn from a prior PDF identical to the distribution of the true number of observations taken in the radial-velocity survey.

Furthermore, the actual radial-velocity survey utilised two distinct observing strategies to probe for binarity in the dC population. The first of these strategies was to take two observations of a target on consecutive nights, followed by a third observation around a month after these two initial nights. The second strategy (that was employed primarily for suspected short period binaries) was to take several (typically between two and five) observations per target over the course of one night. Both of these strategies could be used for a particular dC star on a yearly basis, owing to restricted telescope time and the right ascension of the target. Therefore, to reflect the observational strategy of the survey, mock observations of the simulated radial-velocity curves were sampled according to one of these observational strategies, while ensuring
that all mock observation only occurred at times that correspond to night-time. Finally, because
the observational survey spanned ten years in total, the baseline of the simulated observations
was restricted to ten years.

The distributions of maximum changes in radial velocity for the simulated stellar populations
and the measured distribution of maximum changes in radial velocity for the sample of 37 dC
stars were compared using a Kolmogorov–Smirnov (KS) test. The null hypothesis of the KS
test is that both distributions are consistent with being drawn from the same distribution, i.e.
that the simulated distribution of maximum radial-velocity variations is drawn from the same
distribution as the observed distribution of maximum radial-velocity variations. Thus, identical
distributions for the simulated and true maximum velocity changes would correspond to a
maximum in the KS test. Therefore, the MCMC was used to explore the parameter space while
varying the binary fraction and orbital period distribution of the simulated stellar populations,
while ultimately trying find the parameters that maximise the result of the KS test.

The MCMC was initialised with 50 walkers, each taking 5 000 steps. Subsequently, the first
2 500 steps were discarded as burn-in. The final posterior PDF is shown in Figure 5.6. The MCMC
analysis supports a high binary fraction for the dC star population, with the median of the binary
fraction posterior PDF being located at $94^{+4}_{-6}$ per cent. Furthermore, the simulations suggest that
the dC population as a whole is best modelled with a log-normal orbital period distribution with
mean and standard deviation $\log_{10}\mu (d) = 2.17^{+0.30}_{-0.32}$ and $\log_{10}\sigma (d) = 0.66^{+0.59}_{-0.42}$, respectively.

Comparing the posterior PDFs of this analysis to the binary fraction and orbital period
distributions of Ba, CH, and CEMP-s stars, reveals that, while the dC population exhibits a
similarly high binary fraction ($\geq 90$ per cent), the orbital period distribution of dC stars appears
modestly shorter than its carbon-enhanced cousins ($\log_{10}\mu (d) \approx 2.5$ and $\log_{10}\mu (d) \approx 2.8$ for
CEMP-s and CH populations, respectively; Starkenburg et al., 2014).

5.4.2 Exploring the orbital period distribution of dC stars

Across the literature and this thesis, a total of 15 dC stars now possess constrained orbital
parameters (Margon et al., 2018; Harris et al., 2018). However, the orbital periods of these
dC stars with constrained orbital parameters appear to either be short ($P_{\text{orb}} < 12$ d) or long
($P_{\text{orb}} > 245$ d), with a dearth of orbits lasting tens of days to a few months.

Interestingly, white dwarf–M dwarf binaries, that are the likely metal-normal cousins to
dC stars, exhibit a similar bimodal orbital period distribution (Farihi et al., 2010). The white
dwarf–M dwarf binaries that have evolved through a common envelope possess orbital periods
that peak at typically a few days or fewer, and those that avoid the common envelope possess
Fig. 5.6.: The posterior probability distribution functions corresponding to the mean and standard deviation of the period distribution, and the binary fraction of the dC star population.
orbital periods of many years (Willems and Kolb, 2004; Rebassa-Mansergas et al., 2013; Ashley et al., 2019). Currently, it is unclear if dC stars undergo the same orbital evolution as white dwarf–M dwarf binaries. Still, the handful of dC stars with determined orbital periods to-date seem to support a bimodal orbital period distribution, with no dC stars yet found with an orbital period of several tens of days. Furthermore, binary population synthesis models do predict a bimodal orbital period distribution for dC stars (de Kool and Green, 1995).

To investigate the possibility that the dC population possesses a bimodal orbital period distribution, the MCMC analysis was repeated using an orbital period distribution that was constructed from the sum of two log-normal distributions. Each log-normal distribution was used to model the orbital period distribution of the short and long period dC stars, respectively, and the mean and standard deviation of each log-normal distribution were treated as free parameters to be fit. Furthermore, because the ratio of short-orbital period to long-orbital period dC stars is unknown, the fraction of short-orbital period dC stars was added as an additional free parameter to the MCMC analysis. Therefore, this MCMC analysis uses a total of five free parameters to model the orbital period distribution of the dC population.

The mean of the short-orbital period distribution was sampled from a uniform prior PDF $-1 \leq \log_{10} \mu_1 (d) \leq 1.5$, with the corresponding standard deviation sampled from a uniform prior PDF $0 < \log_{10} \sigma_1 (d) \leq 2.0$. The mean and standard deviation of the long-orbital period distribution were sampled from a uniform prior PDF $1.6 \leq \log_{10} \mu_2 (d) \leq 6.6$ and $0 < \log_{10} \sigma_2 (d) \leq 2.5$, respectively. For simplicity, the binary fraction in this analysis was assumed to be 100 per cent.

The process of the MCMC analysis remains unchanged from the analysis used to constrain the binary fraction and unimodal orbital period distribution of the dC star population, and again, the MCMC was run with 50 walkers each taking 5 000 steps.

Figure 5.7 shows the posterior PDFs derived for the bimodal orbital period distribution of the dC population where the first 2 500 steps were discarded as burn-in. The posterior PDFs show that a bimodal period distribution is a poor model to fit the data at hand. The unconstrained posterior PDFs arise due to a lack of a set of high-likelihood parameters that could be used to model a bimodal distribution in the orbital period for this data. For example, the posterior PDF corresponding to the mean of the short-orbital period distribution tends towards its upper limit, and if less restrictive priors were used, this distribution would likely tend towards the median of the long-orbital period distribution. However, it is worth noting that while the short-orbital period distribution is poorly defined, the posterior PDF corresponding to the
long-orbital period distribution is much better defined. The long-orbital period distribution is not dissimilar to the solution for a binary population modelled with a unimodal log-normal orbital period distribution.

The lack of high-quality solutions to the bimodal period distribution model could be due to one of two reasons. First, the dC population possesses a unimodal distribution in orbital period, and hence, a bimodal distribution model fits the observational data poorly. Second, the dataset at hand is insufficiently large to probe for a bimodal orbital period distribution.

The results presented in Chapter 4 indicate that emission-line dC stars possess orbital periods of a few days or less – much shorter than any orbital period expected for carbon-enhanced post-mass transfer binary systems. Comparing the orbital periods of dC stars exhibiting Hα emission with the orbital periods of dC stars that show no emission, in addition to the cumulative distribution function of the derived orbital period distribution of the dC population (shown in Figure 5.8), reveals that the emission-line dC stars are quite distinct from the general dC population. Figure 5.8 clearly indicates that all emission-line dC stars that possess a constrained orbital period lie at the shorter end of the orbital period distribution, and with the exception of LSR 2105, are orders of magnitude shorter than the orbital periods of dC stars that do not exhibit emission. The cumulative distribution function derived from the orbital period distribution of the dC population suggests that, in the most favourable circumstances, the probability of finding ten short orbital period dC binaries is around one in $10^8$. Hence, it may be possible that the dC star population possesses a bimodal orbital period distribution, where emission-line dC stars populate the shorter period peak of the distribution.

In this picture, the peak at shorter orbital periods would be commensurate with the periods found for the emission-line dC stars (Margon et al., 2018; Whitehouse et al., 2021), while the second peak at longer orbital periods would be consistent with the periods for dC stars lacking emission (Dearborn et al., 1986; Harris et al., 2018). Therefore, it is of interest to test whether emission-line dC stars possess an orbital period distribution that is distinct from the rest of the dC population.

Using the assumption that emission-line dC stars are solely short-orbital period binaries, and therefore their orbital periods are drawn from a separate orbital period distribution to non-emitting dC stars, the orbital period distributions of the dC stars exhibiting and lacking Hα emission, may be fit by two separate log-normal distributions, respectively. This logic would therefore yield a bimodal orbital period distribution for the population as a whole.
Fig. 5.7: The posterior probability distribution functions found when simulating the dC population as a bimodal binary population. $\mu_1$ and $\sigma_1$ correspond to the mean and standard deviation of the shorter orbital period Gaussian distribution, respectively, while $\mu_2$ and $\sigma_2$ correspond to the mean and standard deviation of the longer orbital period Gaussian distribution, respectively. $f_1$ corresponds to the fraction of dC stars that populate the shorter orbital period distribution.
Fig. 5.8: The cumulative distribution functions are shown for the median, 16\textsuperscript{th}, and 84\textsuperscript{th} percentile values of the orbital period distribution for dC stars, as solid blue, dashed orange, and dash-dotted green lines, respectively. The region of orbital periods spanned by dC stars exhibiting H\textalpha emission is shaded in gray, while, the constrained orbital periods of dC stars that do not exhibit H\textalpha emission are presented as purple dotted lines.
While this approach of separating the dC population based on the presence of H\(\alpha\) emission is biased because it forces a bimodal orbital period distribution on the population, it does have a physical motivation. Binary population synthesis models of the dC population predict a bimodal period distribution (de Kool and Green, 1995) where the dC stars that populate the shorter period distribution have evolved through a common envelope and subsequently spiralled into short orbital separations. Conversely, the dC stars that populate the longer period distribution have avoided the common envelope phase of evolution and not undergone such dramatic orbital evolution. Moreover, this bimodal distribution in orbital period is observed in carbon-normal white dwarf–M dwarf binaries (Farihi et al., 2010).

Grouping the radial-velocity dataset based on the presence of H\(\alpha\) emission yields a total of nine dC stars with H\(\alpha\) emission and 28 dC stars without H\(\alpha\) emission. The analysis outlined in Section 5.4.1 was then repeated to deduce the binary fraction and orbital period distribution of the dC stars with and without H\(\alpha\) emission, separately. Thus, this analysis returns a mean and standard deviation for the orbital period distribution, in addition to the binary fraction, for each group of dC stars.

Figure 5.9 shows the posterior PDFs of the dC stars exhibiting and lacking H\(\alpha\) emission features, respectively. While both populations maintain a high binary fraction (\(\geq 86\) per cent), the mean of the orbital period distribution corresponding to the emission-line dC stars is two orders of magnitude smaller than the mean of the orbital period distribution corresponding to the dC stars with no emission.

The analysis of the radial-velocity data for the emission-line dC stars shows that this population is consistent with a log-normal orbital period distribution with a mean and standard deviation corresponding to \(\mu_{H\alpha} = 0.77^{+0.51}_{-0.50}\), and \(\sigma_{H\alpha} = 0.91^{+0.94}_{-0.61}\), respectively. Furthermore, the binary fraction of the dC stars exhibiting H\(\alpha\) emission corresponds to \(f_{b,H\alpha} = 0.86^{+0.10}_{-0.13}\). It is important to note the degeneracy between the standard deviation of the orbital period distribution and the binary fraction. For a limited number of observations, any target that possesses small variations in radial velocity could either belong to long-period binary, possess a high-inclination angle, or be a true single star. For example, the emission-line dC star J0842 possesses radial-velocity variations less than 8 km s\(^{-1}\) (with typical velocity errors of 5 km s\(^{-1}\)), and thus, it is unclear based on radial-velocities alone whether this target is single star, high-inclination binary, or long-period binary. While the data at hand can not break the degeneracy between a binary fraction and the standard deviation of the orbital period distribution, it is important to understand how the data may affect the outputs of the analysis, especially when analysing a small dataset of only 37 stars.
The posterior PDFs obtained for the dC stars lacking emission appear more consistent with the posterior PDFs found for the dC population as a whole. Notably, excluding the predominantly short orbital period emission-line dC stars results in the mean of the period distribution becoming larger by a factor of approximately 3. These results for the dC stars without emission are in line with the orbital period distributions and binary fractions of other post-mass transfer, carbon-enhanced binary populations.

While it remains unclear whether the emission-line dC stars are a distinct population of dC stars, the analysis presented here suggests that their orbital period distribution can be modelled as a distinct distribution from the dC stars that exhibit no emission. Further radial-velocity monitoring of a larger sample of dC stars in needed to understand the orbital period distribution of the dC population as a whole.

5.5 Discussion

5.5.1 Binary fraction and orbital evolution

The distribution of relative radial-velocity variations observed in the dC stars in this radial-velocity survey are consistent with a binary population and hence this evidence supports the hypothesis that dC stars are formed through the transfer of carbon-enhanced material from an evolved companion. The dC stars targeted in the survey are well modelled by a population with a binary fraction of $94^{+4}_{-6}$ per cent and an orbital period distribution corresponding to a mean and standard deviation of $\log_{10} \mu(d) = 2.17^{+0.30}_{-0.32}$ and $\log_{10} \sigma(d) = 0.66^{+0.50}_{-0.42}$, respectively. These results are broadly consistent with the binary fraction and orbital period distributions of other carbon-enhanced, post-mass transfer binaries, albeit possessing an orbital period distribution that is peaked at slightly shorter periods (Starkenburg et al., 2014; Jorissen et al., 2016a; Jorissen et al., 2019). Because the orbital period distribution measured for dC stars spans several orders of magnitude (from a few hours to many years), it is unlikely that any single mass transfer process is responsible for the observed pollution in the atmospheres of dC stars. Instead, it is likely that multiple avenues of binary star evolution, and hence mass transfer, are capable of producing dC stars. For instance, over this orbital period distribution for dC stars, both Roche-lobe overflow, efficient wind capture, and Wind Roche-lobe overflow could be capable of transferring sufficient mass to pollute the dC star.

The dC stars with constrained orbital periods appear to fall within two distinct groups, those with orbital periods of a few days or less, and those with orbital periods of a few months of longer. Therefore, these results may imply that the orbital period distribution of dC stars may
Fig. 5.9: Upper panel: The posterior PDFs corresponding to the mean and standard deviation of a log-normal distribution for the orbital period, in addition to the binary fraction of the population of emission-line dC stars. Lower panel: The same as the upper panel but for the population of dC stars lacking emission-lines.
be bimodal. Prior to the results presented in this thesis, the orbits of only five bona fide dC stars were constrained. Four of these dC stars possess orbital periods of several months or longer (Dearborn et al., 1986; Harris et al., 2018), while the final dC star possesses an orbital period of only 3 d (Margon et al., 2018). Interestingly, the optical spectrum of the dC star with a 3 d orbital period exhibits clear H\(\alpha\) emission whereas the longer period dC stars do not. The orbital periods were later constrained for a further six emission-line dC stars with all of these targets possessing orbital periods in the range 0.2–5.2 d (Whitehouse et al., 2021). Therefore, it appears likely that dC stars with H\(\alpha\) emission are exclusively short-period binaries.

These results are generally supported by the findings in this chapter, with the discovery of three new short period emission-line dC stars, all with periods shorter than 12 d. However, the target LSR 2105 appears to possess an orbital period of 0.15 d and yet does not exhibit any H\(\alpha\) emission. Therefore, LSR 2105 is the first short-orbital period dC star to be discovered, that does not exhibit H\(\alpha\) emission in its optical spectrum. The number of short-orbital period dC stars currently stands at ten, with nine of these possessing H\(\alpha\) emission, and one that does not.

Assuming that the chance of observing H\(\alpha\) emission in the optical spectrum of any short-orbital period dC star is even, then the probability of achieving the actual observed rate of emission-line dC stars to dC stars lacking emission-lines amongst the short-orbital period dC population is just under 1 per cent. This result is significant at the 2.6\(\sigma\) level and therefore implies that the frequency of short-orbital period binarity is larger amongst the dC stars that exhibit H\(\alpha\) emission compared to the dC stars that exhibit no emission-lines. Furthermore, the analysis presented in Section 5.4.2 supports a bimodal orbital period distribution of the dC stars where the shorter-period peak is populated by emission-line dC stars, and the longer-period peak is populated by the dC stars lacking any emission-lines. A bimodal orbital period distribution could also explain the lack of dC stars with constrained orbits in the range of 12–245 d.

Interestingly, the carbon-normal cousins to the dC stars – white dwarf–M dwarf binaries – exhibit a bimodal orbital period distribution. In these binaries, mass can overfill the AGB star’s Roche lobe and create a common envelope that causes an initially close companion to inspiral due to friction (Ivanova et al., 2013). In contrast, if the initial binary separation is sufficiently large, then as mass is lost the orbital separation will increase.

These theoretical predictions are strongly confirmed among commonly-occurring white dwarf–M dwarf binary systems, where there is a clear dearth of pairs with orbital separations in the region \(\approx 1–10\) au as established via space-based imaging in the optical (Farihi et al.,
In contrast, there are myriad short-orbital period ($\lesssim 10$ d; Rebassa-Mansergas et al. 2008), post-common envelope systems, and long-period ($\gtrsim 50$ yr) widely separated white dwarf–M dwarf systems detected by common-proper motion (Farihi et al., 2005). Comparing the spectral types of M dwarfs in post-common envelope systems to those in widely separated binaries reveals no obvious differences (Schreiber et al., 2010), therefore suggesting that neither process is capable of efficient mass transfer. This is further supported by the detection of just one dC star among a sample of 1600 white dwarf–red dwarf binaries identified from the SDSS via template matching and identifying excess red fluxes via optical and near-infrared survey data (Rebassa-Mansergas et al., 2010). Though dC stars would not be found via template matching to a white dwarf–red dwarf composite spectrum, it would be expected that binaries identified via a red excess to the white dwarf spectrum could include dC stars. Their rare nature as companions to known white dwarfs is consistent with the fact that only nine of 1211 SDSS dC stars possesses composite spectra.

The indication that the dC population may possess a bimodal orbital period distribution is commensurate with the white dwarf–M dwarf population, and suggests that perhaps these populations undergo similar channels of binary evolution. However, the longer period peak of the dC population does appear to be within the “no man’s land” of orbital periods for white dwarf–M dwarf binaries. Nonetheless, orbits lasting a few hundred days (consistent with this “no man’s land”) are observed in other post-mass transfer binaries such as the Ba, CH, and CEMP-s populations (Pols et al., 2003; Izzard et al., 2010). The notable difference between these other carbon-enhanced, post-mass transfer systems and dC stars is their lack of any binaries possessing orbital periods of a few days or shorter, and their advanced evolutionary stage (lying on the first-ascent giant branch).

The binary evolution channels that are able to produce dC stars are likely similar to those that produce both white dwarf–M dwarf binaries and carbon-enhanced, post-mass transfer binaries. These evolutionary channels must ensure that a sufficient amount of carbon-enhanced material can be transferred to a dC star prior to, or during, the common envelope phase, thus producing the short-orbital period dC stars observed. However, the dC stars that avoid a common envelope phase must undergo an evolutionary channel that ensures sufficient carbon-enhanced material and angular momentum is transferred while polluting the dC binary such that it can maintain its orbit in the “no man’s land” of orbits for white dwarf–M dwarf binaries. These dC stars with longer orbital periods (of several months or more) therefore likely evolve through similar binary evolution channels as the Ba, CH, and CEMP-s populations.
Another potential similarity between the Ba, CH, and CEMP-s populations and dC stars is the metallicity of these systems, with Ba, CH, and CEMP-s stars metal-deficient with respect to solar (most notably the CEMP-s stars). High-resolution spectroscopy has revealed G77-61 is one of the most metal-poor stars known ([Fe/H] = −4.0; Plez and Cohen 2005), and preliminary kinematical results based on Gaia DR1 suggest that the dC population as a whole is old and metal-poor, with roughly 30–60 per cent halo members (Farihi et al., 2018). Furthermore, binary synthesis models suggest that metal-deficiency is a key factor in forming a population of dC stars (de Kool and Green, 1995). Thus, it appears that metal-poverty is important for C/O enhancement.

5.5.2 Carbon chemistry exoplanets

There has been considerable interest in the existence of exoplanets that exhibit carbon dominated chemistry (Madhusudhan et al., 2011). The existence of such planets requires that the protoplanetary material be intrinsically enriched in carbon such that C/O > 0.8 (Bond et al., 2010). In this scenario, major planet-building materials could be predominantly carbide minerals, allowing for a SiC, TiC, graphite mantle with an Fe–Si–Ni core. Such planets would be chemically distinct from the rocky bodies found within the solar system. Although unrelated to the present study, it is noteworthy that the minor bodies that pollute the surfaces of white dwarf stars exhibit Earth-like or chondritic C/O, with no evidence for carbon-dominated materials (Wilson et al., 2016).

The potential frequency of carbon-rich exoplanets depends on the space density of viable host stars (Fortney, 2012). While dC stars are the most numerous carbon stars in the Galaxy, they are still far less abundant than their oxygen-rich counterparts, with approximately 1:650 000 dC stars relative to low-mass K and M dwarfs (0.1M⊙ < M < 0.8 M⊙; de Kool and Green 1995; Bochanski et al. 2010). With drastically fewer potential hosts with C/O > 1, the expected relative abundance of carbon-rich planets could be vanishing.

Assuming carbon-rich planets can and do form around host stars with C/O > 0.8, the results presented here, that all low-mass, main-sequence stars in the phase space above C/O = 1.0 are consistent with 100 per cent duplicity, therefore diminishes the possibility of single stars with C/O ≥ 1.0, and thus their ability to host planets. Hence, the available real estate for carbon planets may be dismal. However, one subgroup of CEMP stars appears to commonly possess both binary and single members (the CEMP-no stars; Starkenburg et al., 2014) and therefore may contain primordial carbon-enhancement. If these stars are formed from carbon-enhanced nebulae, then presumably they are possible sites for carbon-rich planets (Mashian and Loeb, 2015).
2016), notwithstanding the potentially unfavourable planet hosting frequency of metal-poor stars (Fischer and Valenti, 2005).

5.6 Conclusions

This radial-velocity monitoring survey indicates that the binary fraction of the dC star population is \(94^{+4}_{-6}\) per cent and hence is consistent with duplicity, and the hypothesis that dC stars are the product mass transfer. Using MCMC simulations, the orbital period distribution of the whole dC population has been successfully modelled as a Gaussian distribution in log-space that is centred at \(\log_{10}\mu(d) = 2.17^{+0.30}_{-0.32}\) and possesses a standard deviation \(\log_{10}\sigma(d) = 0.66^{+0.59}_{-0.42}\). Repeating this analysis separately for the dC stars that exhibit H\(\alpha\) emission from those that do not exhibit H\(\alpha\) emission, allows for the orbital period distribution of both of these dC populations to be modelled as two separate log-normal distributions. Comparing the means of these distributions indicated that the emission-line dC stars possess an orbital period distribution centred at much shorter periods compared to the dC stars that lack emission-lines. The peaks of the respective distributions are separated by over an order of magnitude. When compared to white dwarf–M dwarf binaries, which exhibit a bimodal period distribution, the dC stars that lack emission-lines appear to lie between the peaks indicating that efficient mass transfer circumvents migration to short or long periods. The emission-line dC stars appear broadly consistent with post-common envelope binaries, implying mass can be transferred prior to the common-envelope.

A further six dC stars now possess constrained orbital periods, of which three exhibit H\(\alpha\) emission, and three do not. This raises the number of dC stars with determined orbital periods to 16. All of the emission-line dC stars possess orbital periods < 12 d, with LSR 2105 possessing the shortest orbital period found to date in the dC population of 0.15 d (albeit without any clear signs of emission in its optical spectrum). This radial velocity analysis was only able to constrain two dC stars with orbital periods longer than 20 d (G77-61 and LP225-12), where both are previously known binaries. Due to the relatively low number of observations for dC stars lacking H\(\alpha\) emission compared to dC stars that exhibit H\(\alpha\) emission, phase coverage for long period dC stars may be poor, and thus, this result is perhaps unsurprising.

The high binary fraction of dC stars constrains the potential real estate for carbon-rich exoplanets, owing to the extrinsic nature of their high carbon abundance. As dC stars are the product of efficient mass transfer, the chemistry of the system during the planet formation phase would not reflect the chemistry of the dC star observed today. This may also be true
for all main-sequence stars that exhibit C/O significantly above solar; if they exist (which is uncertain; Fortney 2012; Teske et al. 2014) such stars could be the result of binary mass transfer. It is clear from the dC stars that carbon enhancement in a main-sequence star is possible via binary evolution, and thus more subtle C/O enhancements may be more common (e.g. in FGK stars).

Continued radial velocity measurements for the stars in this study are necessary to determine actual orbits. Binary population synthesis models and physical models of mass transfer – for example Roche Lobe overflow or wind capture (Paczyński, 1965; Abate et al., 2015) – can only be tested with tightly constrained binary periods. State-of-the-art mass transfer models currently face challenges in producing carbon-enhanced stars in general (Izzard et al., 2010; Matrozis et al., 2017), and the newly uncovered dC binary population can provide an additional and distinct set of empirical constraints.
6.1 Introduction

Binary population synthesis models predict that metal-deficient stars are more susceptible to atmospheric pollution via mass transfer (de Kool and Green, 1995). As the metallicity of the accretor decreases, the total mass of material needed to be accreted (assuming a constant abundance) to alter the accretor’s atmospheric chemistry also falls. Thus, it is possible that only a modest amount of mass transfer could be necessary to pollute the most metal-poor stars. This hypothesis is supported by observations of carbon-enhanced post-mass transfer binary systems such as Ba, CH, and CEMP’s stars, where atmospheric abundance measurements have confirmed these populations possess modest [Fe/H] compared to solar (Lucatello et al., 2005a; Campbell and Lattanzio, 2008; de Castro et al., 2016).

Dwarf carbon (dC) stars are a population of low-mass main-sequence stars that exhibit optical spectra consistent with carbon enhancement hypothesised to be the result of atmospheric pollution via mass transfer from an evolved companion. Furthermore, the prototype dC star, G77-61, is one of the most metal-poor stars known ([Fe/H] = −4.0; Plez and Cohen 2005). However, little is known regarding the metallicity distribution of the dC population. Due to the faintness of the population, any abundance study using current large-scale spectroscopic surveys is unfeasible.

This chapter aims to broadly constrain the age and thus metallicity of the dC star population by examining its location on the Hertzsprung-Russell diagram (HRD). Moreover, this initial analysis is supplemented by a dynamical study of the dC population that aims to understand how the Galactic orbits of the population are distributed amongst the thin disc, thick disc, and halo. Galactic orbits compatible with membership to the thick disc and halo indicate that these stars are likely to be metal-poor.

The data used in this study are discussed in Section 6.2. The methodology used to calculate distances to the dC stars is discussed in Section 6.3. All prior assumptions used while computing the Hertzsprung-Russell diagrams are discussed in Section 6.3.1, while the Hertzsprung-Russell diagrams are shown and discussed in Section 6.4. High-confidence dC stars are identified, and
their orbital actions are calculated and discussed in Section 6.5. Finally, Section 6.6 presents a brief discussion of the results and their possible implications.

## 6.2 Sample selection and data

Candidate dC stars were sourced from the Green 2013a published catalogue of carbon stars. These dC candidates were identified by performing a cross-correlation between all spectroscopic targets observed during SDSS DR7 and DR8, and a series of carbon star spectral templates, yielding a total of 1204 unique candidate dC stars (Green, 2013a). While this method of identifying candidate dC stars is biased towards identifying stars with spectral types similar to those of the stellar spectral templates, it does provide a kinematically unbiased sample of candidate dC stars to analyse. Because no kinematical cuts were made while assembling the catalogue of candidate dC stars, the catalogue almost certainly contains stars that do not lie on the main sequence and are hence not true dC stars. Any contaminants to the catalogue were subsequently removed later during the analysis (see Section 6.4.1).

For each candidate dC star, photometry and radial velocities were sourced from SDSS DR16 (Ahumada et al., 2020), while 6D astrometry was sourced from Gaia eDR3 (Gaia Collaboration et al., 2020). Sloan photometry was retrieved through casjobs by cross-matching the catalogue of candidate dC stars with the SDSS DR16 database on the J2000 coordinates of each target. This cross-match provides the bestObjID and specObjID, which were used in turn to recover the apparent magnitudes and associated errors in the $g$, $r$, and $i$ bands of each target, and the radial velocities measured by the SEGUE survey, respectively.

SDSS photometric measurements are taken using the 2.5m Sloan Foundation Telescope located at Apache Point Observatory, which is a dedicated survey telescope. The SDSS imaging survey measures the apparent magnitudes in five bands ($u, g, r, i, z$) of all stars within the Sloan footprint down to $\approx 23$ AB mag. All SDSS photometric data are pre-processed and reduced by the dedicated SDSS imaging pipeline, which also provides photometric error estimates (Stoughton et al., 2002; Padmanabhan et al., 2008).

The SEGUE spectroscopic survey is a campaign run by SDSS to measure optical spectra of candidate thick disc and halo stars, intending to understand their metallicities and kinematics (Yanny et al., 2009). All spectra obtained during the SEGUE survey are reduced by the dedicated SEGUE pipeline (Lee et al., 2008a). In this chapter, all radial velocities of the candidate dC stars were sourced from the SEGUE survey and were retrieved by cross-matching the specObjID of each candidate to the SDSS DR16 database in casjobs. The SEGUE pipeline measures radial
velocities of each target spectrum via cross-correlation with either the SDSS commissioning templates or ELODIE spectroscopic templates, degraded to the resolution of the SEGUE spectra (Lee et al., 2008b; Allende Prieto et al., 2008). The template spectrum that best matches the target spectrum is used to measure the radial velocity, with the ELODIE templates frequently providing the best match. Because any target can be observed on multiple occasions during the SEGUE survey, the radial velocity and corresponding error returned for each candidate dC is the weighted average of all radial velocity and error measurements of that target. Despite the binary nature of dC stars – and their potentially variable radial velocities – the averaged radial velocities obtained from SEGUE are approximately equal to the systemic velocity of the binary (For a more detailed discussion, see Appendix D.2).

Astrometry data were sourced from Gaia eDR3 by performing a cone search with a radius of 15″ around the coordinates of each target. The SDSS positions of each dC candidate were corrected from the J2000 to J2015.5 epoch to bring them into line with the positions reported by Gaia. The cone search was then completed for all dC candidates around their estimated J2015.5 epoch coordinates. From the initial catalogue of 1204 candidate dC stars, a total of 1375 matches were returned from the cone search. Duplicated targets were filtered out by converting the Gaia G, B_P, and R_P band photometric magnitudes to Sloan g, r, and i band photometric magnitudes. The Gaia source that provided the closest match between the estimated and measured Sloan magnitudes in each photometric band was selected as the true match. This method produced clear matches and returned no ambiguous results.

6.3 Calculating distances to dC stars

A necessary initial step in computing the absolute magnitude or orbital actions of any star (allowing for analysis into its location on the HRD or its Galactic orbit) is to compute the star’s distance. In this section, stellar distances are computed using probabilistic inference through MCMC methods. This approach helps to reduce any uncertainties that may exist within the astrometric data. The distances computed in this section form the foundations of the analyses that follow later in this chapter.

6.3.1 Probabilistic inference using Gaia eDR3 data

Unfortunately, calculating the distance to a star is not as easy as simply inverting its parallax measurement (Luri et al., 2018). Because the Gaia data reduction pipelines are not immune to introducing bias into the parallax measurements, the true and measured parallaxes are not necessarily equal. The obvious example of bias within the Gaia pipeline is the measurement
of negative parallaxes for faint stars in crowded fields. These negative parallaxes, if inverted, would result in distances that are unphysical.

Although calculating distances and their errors from Gaia parallax measurements is not trivial, it is still possible. The goal is to determine the true parallax of a star – which can then be inverted to yield the distance – given its Gaia parallax measurement. This goal can be achieved by combining Bayes’ theorem (see Equation 3.15) with a probabilistic approach (Bailer-Jones, 2015; Astraatmadja and Bailer-Jones, 2016b). For any particular star, this approach relies on defining a distribution for the likelihood, corresponding to the star’s parallax, along with a set of prior beliefs. Using MCMC methods, it is possible to draw samples from within the likelihood distribution that satisfy the defined prior beliefs and compute a posterior probability distribution function (PDF), where the posterior PDF corresponds to the distribution of possible true parallaxes given the observational data and prior beliefs for any star. Theoretical and empirical evidence suggests that when computing the true parallax from the measured parallax, the likelihood can be assumed to be Gaussian around the observed parallax measurement, with a standard deviation equal to the magnitude of the measurement error (Lindegren et al., 2018).

While computing the posterior PDF corresponding to the true parallax and, hence, the distance of each of the candidate dC stars, a total of three astrometric priors were used. The first prior was placed on the distance and states that Gaia observes stars in an exponentially decreasing space density (Bailer-Jones, 2015). The physical interpretation of this prior is that as the distance from the Sun increases radially, the number density of stars observed within that distance increases up to a maximum number density. Beyond this maximum, the number density begins to decay as the distance increases. The distance that corresponds to the maximum number density is the scale length of the distribution and has been determined from simulations of the Milky Way’s evolution (Bailer-Jones, 2015; Astraatmadja and Bailer-Jones, 2016b). Furthermore, due to the limits of the exponentially decreasing space density, this prior ensures that all distances remain positive, regardless of the magnitude of the measured parallax.

The exact shape of the exponentially decreasing space density profile depends on the line-of-sight direction it is applied. For example, observations into the plane of the thin and thick discs show a different space density profile compared to observations out of the plane of the thin and thick discs. The solution is to allow the scale length to vary as a function of latitude and longitude. Therefore, the exponentially decreasing space density for any particular target is unique and depends on the location of the target on the sky.
The second prior was placed on transverse velocity and requires that a star be bound to the Galaxy. This prior removes the possibility that solutions with arbitrarily large, unphysical velocities could be found. Only transverse velocities in the range $0 \leq v_{\text{trans}} \leq 750\, \text{km s}^{-1}$ in the local standard of rest were considered. The third prior requires that the position angle of a target is in the range $0 \leq P.A \leq 2\pi$, and excludes unphysical angles.

### 6.3.2 Constructing a Hertzsprung-Russell diagram prior

While the prior constraints based on astrometry alone are useful while determining the true parallax, photometric data provide an additional wealth of information. Combining distance and photometry makes it possible to construct an HRD of a sample of stars. The HRD is a tool to visualise stellar evolution in terms of effective temperature (or colour in the absence of reliable temperature measurements) and absolute magnitude. Because a star’s effective temperature and luminosity evolve with the star, the location of a star on the HRD can be used to help discriminate at which evolutionary stage that star is at. Therefore, the structure of the HRD implies that for any given colour, there exists a non-uniform probability distribution of absolute magnitude. Furthermore, through the distance modulus, the absolute magnitude contains information on the true parallax of any given star. Thus, prior constraints can be placed on the true parallax of a target based on its colour (Bailer-Jones, 2011).

The first step towards constructing a HRD prior to constraining the true parallax of a target is to construct a good quality HRD. Because the catalogue of candidate dC stars is sourced from the SDSS and hence possesses SDSS photometry, the HRD prior was constructed from stars with good quality astrometry from Gaia eDR3, and photometry from SDSS DR16. An initial pool of 300,000 stars was created by randomly querying the Gaia eDR3 cross-match with the SDSS DR12 best neighbour catalogue\(^1\). All stars returned by this query were required to meet several quality checks, namely the recommended astrometric quality cuts from the Gaia team (Gaia Collaboration et al., 2018), in addition to RUWE\(^2\) $< 1.2$ (Belokurov et al., 2020), and a measured parallax signal-to-noise greater than 50. Furthermore, this query also provides the Sloan BESTObjID of each of the 300,000 targets, which is necessary to perform a cross-match with the SDSS DR16 database. Distances to each of the 300,000 stars were provided by the Gaia geometric distance catalogue (Astraatmadja and Bailer-Jones, 2016a).

Photometric measurements in the Sloan $g$, $r$, and $i$ bands were retrieved for each of the stars with high quality Gaia astrometry by querying the SDSS DR16 database using the BESTObjID

---

\(^1\)SDSS DR12 represents the most recent SDSS data release to possess a cross-match with Gaia provided by the Gaia team.

\(^2\)RUWE is an acronym for the re-normalised unit weight error in the Gaia astrometry.
(returned by the Gaia query for each star) in casjobs. The photometric quality flags were inspected for each star in the pool to ensure that all targets possess good quality photometric measurements.

Sloan $r$ band absolute magnitude, in addition to $g - r$, $g - i$, and $r - i$ colours, were computed for the 300,000 stars before assembling HRDs in each of these three colours. The Sloan $u$ band photometry was omitted because dC stars are extremely faint in this band due to carbon molecules absorbing blue light. Additionally, due to difficulties with ground-based photometry in the red (e.g. a less efficient detector, and telluric features), the $z$ band photometry was also omitted (Doi et al., 2010).

Each of the three HRDs were converted to a probability distribution function by defining a grid over absolute magnitude and colour in each HRD, with a fixed number of bins in both absolute magnitude and colour. The grid in absolute magnitude spans $-2 \leq M_r \leq 16$ AB mag in all three HRDs, and the grid in colour spans $0 \leq g - r \leq 2.2$, $0 \leq g - i \leq 4.0$, and $0 \leq r - i \leq 2.1$, for each colour, respectively. Each axis was separated into 200 evenly spaced bins, meaning that each HRD was partitioned into 40,000 bins. These bins were then smoothed with a Gaussian kernel density estimate (KDE) with a smoothing parameter of 0.1. The defined grids are intended to encapsulate the whole HRD in each colour, so as not to bias against any stage of evolution, while ensuring sufficient resolution to form a smooth probability distribution function, without incurring large computational overheads.

The probability distribution functions derived from each of the HRDs were used to defined a set of three priors while computing the true parallax of each star. Thus, a star possessing a certain combination of colours has a distinct probability distribution for its absolute magnitude.

Constructing a prior based on the HRD may introduce biases to the determination of the distance. Qualitatively, this prior has the effect of pulling all posterior samples towards areas of the HRD that are heavily populated (e.g. the main sequence). For example, if a dC candidate were to possess a composite spectrum with a white dwarf (at least ten of these are known), the HRD prior will artificially pull the posterior of this candidate towards the main sequence, and hence bias the distance inference. While there is no silver bullet to solve this bias, three HRD priors are computed in the colours $g - r$, $g - i$, and $r - i$, in an attempt not to bias against any particular region of the HRD. In general, the candidate dC stars are faint ($\pi \approx 18.5$ AB mag) and therefore possess poor signal to noise parallax measurements, thus, the addition of the HRD priors are beneficial to the inference of the distance. However, all potential biases have been noted.
6.3.3 Computing the parallax posterior PDFs of all candidate dC stars

By modelling the likelihood of each dC star as a Gaussian distribution centred on its Gaia eDR3 parallax measurement and with standard deviation equal to its corresponding measurement error, and taking the six priors (three astrometric and three photometric), it is possible to infer the posterior PDF corresponding to the true parallax of each candidate dC star. These true parallaxes can then be inverted to yield the distances to each of the dC star candidates.

The posterior PDF corresponding to the true parallax of each candidate dC was computed through MCMC methods using the emcee package (Foreman-Mackey et al., 2013). For each dC candidate, the MCMC draws prior samples from a set of prior probability distribution functions corresponding to the right ascension, declination, proper motions in right ascension and declination, parallax, and $g$, $r$, and $i$ band photometry. These prior probability distribution functions were assumed to be Gaussian distributions centred at their quoted measurement value and with a standard deviation equal to the measurement uncertainty. If the given parallax measurement from Gaia eDR3 were negative, then rather than sampling the parallax directly, the distance would instead be sampled from a Gaussian distribution centred at 5000 pc with a standard deviation of 2000 pc. These distance samples were then inverted to yield prior samples in parallax.

Each set of prior samples was then used to compute the transverse velocity, the absolute magnitude in Sloan $r$ band and $g - r$, $g - i$, $i - r$ colours. Finally, these computed values were combined to calculate the true parallax and hence distance given the prior beliefs outlined in Sections 6.3.1 and 6.3.2. The MCMC process repeats this recipe numerous times to construct a posterior PDF of the distance of each dC candidate. The median of the posterior PDFs were taken as the inferred value with the final error quoted as the 16th and 84th percentile confidence interval (owing to the non-Gaussian nature of some posterior probability distribution functions).

For each dC candidate, an MCMC was initialised with 50 walkers, each taking 5000 steps. The initial 1500 steps taken by each walker were discarded as burn-in. The covariances of all the astrometric parameters are provided by Gaia and are hence accounted for. Furthermore, it was assumed that the Gaia astrometry and the SDSS photometry are independent, and as such, no covariance exists between the astrometric and photometric measurements. However, while the apparent magnitude measurements in each SDSS photometric band are independent, the colours computed from these measurements are not. Therefore, covariances between the
different photometric bands used to calculate the colours were accounted for in the MCMC process (see Appendix D.1 for a more detailed discussion).

6.4 Hertzsprung-Russell diagram of dC stars

6.4.1 Identifying true dC stars

Out of the 1204 candidate dC stars, the MCMC analysis converged for 1168 dC candidates. From the ensemble of candidate dC stars with a converged solution in the true parallax (and hence, distance), 1132 candidates possess reasonable uncertainties in their colours and absolute magnitudes, defined as \( \sigma_{g-r}, \sigma_{g-i}, \sigma_{r-i} \leq 0.5 \), and \( \sigma_{M_r} \leq 5 \). Therefore, it was not possible to infer true parallaxes, and hence, distances for only 64 of the candidate dC stars. These candidate dC stars that do not possess a converged solution for the true parallax have astrometric or photometric data that are poor quality. The cuts used here, particularly in absolute magnitude, are conservative and aim only to remove candidate dC stars for which the MCMC analysis converged poorly. Figures 6.1 and 6.2 present the HRDs of the candidate dC stars for all three colours, with and without errors, respectively.

Inspection of Figures 6.1 and 6.2 reveals that several of the candidate dC stars do not lie on the main sequence and are hence, not true dC stars. Therefore, before continuing with this analysis, a list of candidate dC stars with a high probability of lying on the main sequence was compiled.

Foremost to classifying a candidate dC star as a member of the main sequence, the position of the main sequence on the HRD must be defined. The location of the main sequence was established in each of the three HRDs using the sample of randomly selected stars, mostly of solar metallicity, with high-quality astrometric and photometric data initially used to construct the HRD priors. Upon constructing each of the three HRDs using these stars with high-quality data, each HRD was smoothed by a Gaussian KDE with a smoothing parameter of 0.1, in the exact same process used to construct the initial HRD priors. Because the main sequence possesses the largest number density of stars within the HRD, the main sequence was therefore defined as the region of the HRD having a number density above a certain threshold. A cut to define the main sequence was made by creating a contour in the number density of the Gaussian KDE at 8 per cent of the maximum density of each HRD, where candidate dC stars lying within these boundaries were flagged as likely true dwarf stars. While this cut is motivated

---

The uncertainties quoted here are designed to only remove candidates from the study where no convergence occurred during the MCMC analysis and are hence, unconstraining.
Fig. 6.1: The Hertzsprung-Russell diagrams of the 1132 candidate dC stars that possess reliable absolute magnitude errors, in all three colours, for which the MCMC analysis converged. The shaded region and colour bar in each subplot corresponds to the original HRD prior in that colour, and the colour bar corresponds to the normalised number density of stars. The dC candidates are shown as blue points with the 16th and 84th percentile confidence interval as errors.
Fig. 6.2: Each panel represents the HRD for the candidate dC stars expressed in the colours $g - r$, $g - i$, and $r - i$, from top to bottom, respectively. The shaded region and colour bars in each panel correspond to the HRD prior in that colour. Finally, three isochrones are plotted in orange over each HRD representing a young, intermediate, and old stellar population.
by visual inspection, it is conservative and only excludes stars that clearly do not lie on the main sequence.

However, this cut alone does not include any main-sequence stars lying significantly below or to the left of the main sequence. Therefore, metal-poor stars (that lie to the left of the main sequence due to their lower opacity and thus higher effective temperature; Kaler 1989) and white dwarf-main sequence binaries (Rebassa-Mansergas et al., 2007) would not be classified as main-sequence stars using the cut in number density alone. Thus, an additional region below and to the left of the original cut was defined to capture any potentially metal-poor or white dwarf-dC star binaries. An example of the selection boundary is shown for the \( r - i \) HRD in Figure 6.3, in addition to whether the candidate dC star was classified as a bona fide dwarf star.

Candidate dC stars were then defined as possessing a high probability of lying on the main sequence if 68 per cent (consistent with one sigma) of their error lay within the selection
boundary for every colour. Thus, if a candidate passed the selection criteria in the HRDs corresponding to two colours, but failed the third, it would not be classified as a dC star. This exercise was repeated by requiring 95 and 99.7 per cent of the error (representing two and three sigma cuts, respectively) to be bound by the contour; however, this made minimal difference to the final number of dC stars. A total of 1014 dC candidates were identified as possessing a high probability of lying on the main sequence, meaning that 118 candidates did not pass these selection criteria.

6.4.2 The location of the dC population on the HRD

Initially considering the unfiltered catalogue of dC stars, Figures 6.1 and 6.3 reveals that the list of dC candidates possesses a large number of true dC stars, with the vast majority of candidates clustering on the main sequence. Furthermore, the dC candidates spread along the whole main sequence, indicating that the dC star population spans a range of colours, effective temperatures, and masses, with some candidates possessing colours more consistent with G-type stars. Additionally, these dC candidates also appear to contain a few main sequence turn-off stars, giants, and even white dwarfs (confirmed by visually inspecting the SDSS spectra).

Not all of the candidate dC stars have been pulled onto the main sequence by the HRD priors, suggesting that while these priors are informative, they do not dominate the likelihood. This result is encouraging and provides confidence in the distance inferences made through this analysis method. These locations on the HRD are not expected to be correct for any particular candidate dC star due to the inherent bias in the methodology and uncertainty in the astrometric and photometric measurements. However, this exercise is informative when considering the population of dC stars as a whole. Hence, general trends that span the whole population are likely authentic.

The HRDs constructed using photometry from the Sloan $g$ band do not show as much detail on the main sequence as the HRD in the $r - i$ colour. Furthermore, those HRDs containing $g$ band photometry show multiple stars above the main sequence that are much redder than expected for main-sequence turn-off or giant stars. The likely cause of these peculiar characteristics is the strong carbon molecular absorption features present in the $g$ band. Stars possessing a C/O ratio greater than unity form carbon molecules in their atmospheres. Depending on the effective temperature of the star, these molecular absorption features can be strong. Notably, at low effective temperatures, the $C_2$ swan bands suppress the $g$ band luminosity, while the CH band can suppress the luminosity in the $g$ band at higher effective temperatures. This suppression of the $g$ band magnitude has the effect of making the dC stars appear redder than
The HRDs displaying the high confidence main-sequence stars amongst the list of candidate dC stars. The shaded region and colour bar represents the HRD prior used to compute distances to the dC stars, while the orange lines correspond to metal-normal stellar isochrones at various ages and metallicities. (Dotter et al., 2008)
their metal-normal cousins and is likely the reason for many candidate dC stars lying above the main sequence in the HRDs calculated with \( g \) band photometry.

Figure 6.4 provides the best schematic view of how the dC candidates with a high probability of lying on the main sequence occupy the HRD. Several isochrones that correspond to young, intermediate, and old metal-normal stellar populations are overplotted. Isochrones were computed for metal-normal stars due to a lack of theoretical stellar evolutionary models of carbon-enhanced stars and are therefore likely inadequate to deduce the age and metallicity of the dC population. The choice of ages and metallicities of the isochrones are arbitrary and only intended as a general guide towards how metallicity and age affect a population’s position on the HRD. In each of the HRDs, particularly in the \( r - i \) colour HRD, a significant proportion of the dC stars on the main sequence tend towards the left of the sequence and thus indicate an older, more metal-poor population. This behaviour is clearest in the \( r - i \) colour HRD, and is seen in the region \( 0.5 \lesssim r - i \lesssim 0.8 \) and \( 8.5 \lesssim M_r \lesssim 11 \) (see Figure 6.5). If dC stars are genuinely an old, metal-poor population, this result has important implications regarding the mass transfer process responsible for polluting dC stars’ atmospheres. Binary population synthesis models predict that metal-poor stars are more susceptible to atmospheric pollution via mass transfer because the amount of mass required to increase the atmospheric C/O ratio beyond unity is anti-correlated with the metallicity of the accreting star. Hence, if the accreting star possesses less oxygen, it would require less carbon-rich material (relative to an oxygen-normal star) to be accreted to enhance the C/O ratio. Thus, in terms of accreting sufficient mass to change the atmospheric chemistry of a star, metal-deficient stars may offer a more attractive prospect than their metal-normal cousins due to only needing to accrete a modest mass relative to metal-normal stars.

The results reported in this section indicate that a large proportion of the dC population may lie to the left of the main sequence – a region typically occupied by metal-poor dwarfs. However, due to the absence of any high-resolution spectroscopic data necessary to measure abundances and metallicities of dC stars, it is currently not possible to confirm this result. To further explore the possibility that the dC population is old and metal-poor, the orbital actions of each dC candidate are computed, and the population’s distribution across the thin and thick discs and the stellar halo is analysed.

### 6.5 Orbital actions

Since there are insufficient observational data to characterise the age and metallicity of the dC population, these properties can be inferred – to first order – through the population’s
association to the Galactic, thin disc, thick disc, and halo. Each dC star can be classified accordingly by analysing its dynamical characteristics, such as its orbital actions, that describe how the star moves in relation to the Galaxy (Binney and Tremaine, 2008).

In an axisymmetric gravitational potential, as the Milky Way is often modelled, the orbital actions are the integrals of motion and quantify the oscillation of an orbit in the radial, vertical, and azimuthal directions (Trick et al., 2019). Therefore, the orbital actions in an axisymmetric potential can be interpreted as the following physical properties: the radial action, $J_R$, quantifies a star’s orbital excursion away from a circular orbit, and can be therefore interpreted as a measure of the Galactic orbital eccentricity. The azimuthal action, $J_\phi$, expresses the magnitude of rotation around the Galactic centre, and thus, the azimuthal action is exactly equal to the $z$-component of the angular momentum. Finally, the vertical action, $J_z$, quantifies the Galactic orbital excursion directly out of the plane.

### 6.5.1 Galactic potential

In order to calculate orbital actions, it is necessary to assume a Galactic potential (Binney and Tremaine, 2008). In recent years, several theoretical and empirical Galactic potential models have been proposed (e.g. Irrgang et al., 2013; Piffl et al., 2014; Bovy, 2015). Unfortunately, the Galaxy’s gravitational field can not be measured directly because stars undergo only a small
acceleration that can not be detected by Gaia. Therefore, modelling the Galactic potential of the Milky Way requires detailed mass distribution models of each component of the Galaxy. McMillan (2017) presented an important update to these mass distribution models, with several stellar, dark matter, and gas components of the Milky Way fitted to a larger and extremely precise set of data. The orbital actions computed for the dC stars in the section are calculated within the gravitational potential that this model provides.

The mass distribution models of McMillan (2017) decompose the Milky Way into density profiles of six key components: the bulge, thin and thick discs, the dark matter halo, and H I and molecular gas clouds. Briefly, each of the components, excluding the dark matter halo, are modelled as an exponentially decreasing density profile and are calibrated using the latest observational constraints on each component (barring the dark matter halo due to a lack of direct observations). The dark matter halo is modelled as a Navarro-Frenk-White density profile (Navarro et al., 1996).

The Poisson equation was then used to derive a Galactic gravitational potential from the set of density profiles used to describe the Milky Way, displayed in Equation 6.1. The desired potential is denoted as \( \phi \), \( G \) represents the gravitational constant, and \( \rho \) corresponds to the density profile describing the mass distribution.

\[
\nabla^2 \phi = -4\pi G \rho
\]

Finally, the gravitational potentials from each component of the Milky Way are summed to construct the Galactic potential.

With a Galactic potential now in place, the final step before calculating the orbital actions of the dC stars is to perform a coordinate transformation from a heliocentric to a Galactocentric system. For this transform, the Sun’s distance from the Galactic centre is taken as \( R_0 = 8.3 \) kpc and is derived from the latest observations of Sagittarius A* (Gravity Collaboration et al., 2021). Furthermore, the Sun’s velocity is taken relative to the Galactic standard of rest as \( 11.1, 245, \) and \( 7.3 \) km s\(^{-1}\) in the \( x, y, \) and \( z \)-directions, respectively. The \( y \) component of the Sun’s velocity is measured from the proper motion of Sagittarius A* (Reid and Brunthaler, 2004), while the \( x \) and \( z \) components are derived from a comparison between local stellar kinematics and chemodynamical models (Schönrich et al., 2010).
6.5.2 Calculating orbital actions

Orbital actions were calculated for the catalogue of 1014 high-confidence dC stars derived in Section 6.4.1, using the AGAMA software package (Vasiliev, 2019a) in the Galactic gravitational potential of McMillan (2017). For each dC star, orbital actions were calculated by Monte Carlo sampling in right ascension, declination, proper motions in right ascension and declination, radial velocity, and distance. All of these quantities except for the distance were sampled from Gaussian distributions centred at the corresponding measurement value with a standard deviation equal to the corresponding measurement error. Distances for each dC star were sampled directly from the distance posterior probability distribution computed in Section 6.3. For each dC star, 1000 samples were drawn from these distributions, and each set of samples was used to calculate the orbital actions. Therefore, every dC star that orbital actions were calculated for possesses a distribution of size 1000 in each orbital action, where the median of these distributions is taken as the quoted value, and errors are given as the 16th and 84th percentile confidence limits. Due to poor quality radial velocity data, or lack thereof, the orbital actions of 15 dC stars could not be calculated. Therefore, it was only possible to calculate the orbital actions of 999 dC stars.

6.5.3 Results

The orbital actions \( (J_R, L_z, J_z) \) of the dC population are displayed in Figure 6.6, along with the orbital actions of the Sun (as a rough guide to where thin disc stars exist in these figures). It is clear from the broad distribution of orbital actions that the dC population does not exist within just one component of the Milky Way and is likely spread throughout the thin and thick discs and the halo.

The distribution of radial actions displayed by the dC population is continuous and spans the range \( 0 \leq J_R \lesssim 5000 \text{ kpc km s}^{-1} \). Physically, a small radial action corresponds to a Galactic orbit that is near-circular, where a radial action of \( J_R = 0 \) corresponds to a perfectly circular Galactic orbit. On the other hand, a large radial action indicates that the Galactic orbit is highly eccentric. Thus, the broad distribution of radial actions exhibited by the dC population implies that the dC population possess Galactic orbits with a large range of eccentricities.

The distribution of angular momentum in the \( z \)-direction of the dC star population spans \( -2500 \lesssim L_z \lesssim 2500 \text{ kpc km s}^{-1} \). As a general guide to the location of the thin disc, the \( z \)-component of the angular momentum of the Sun is shown in Figure 6.6 and is \( L_z \approx 2000 \text{ kpc km s}^{-1} \). Because \( L_z = 0 \text{ kpc km s}^{-1} \) corresponds to no rotation around the Galactic centre, stars with a positive \( L_z \) possess prograde Galactic orbits, and stars with negative \( L_z \)
Fig. 6.6: The orbital actions of the high confidence dC stars. The top row shows orbital actions with their corresponding errors calculated through a Monte Carlo process. The lower panel omits the errors for clarity. The Sun is marked in each of the lower panels by an orange star as a rough guide to the location of thin disc.
have retrograde Galactic orbits. Thus, stars with $L_z < 0$ are consistent with halo membership. It is difficult to differentiate between the thin and thick discs because they overlap in action space. However, due to the large number of dC stars possessing retrograde orbits, it is likely that a significant fraction of the population are old and hence, metal-poor.

The vertical action measures an orbit’s oscillation out of the plane, and therefore, stars that orbit within the plane of the thin disc will possess a small vertical action. The majority of the dC population possess vertical actions $< 500 \text{kpc km s}^{-1}$ and, as such, few dC stars possess orbits that venture far out of the plane. This result is perhaps unsurprising considering the faintness of the dC population on average. Because dC stars are predominantly low-luminosity dwarfs, they are only detectable to relatively short distances and, hence, must currently be in the Solar neighbourhood. Therefore, it is more likely to find dC stars with orbits that remain close to the plane of the disc. The dC stars that deviate far from the plane are likely only visible in a small fraction of their Galactic orbit. Hence, they are probably underrepresented in this sample due to the low luminosities of dC stars.

Figure 6.7 displays the relationship between the orbital energy and $z$-component of the angular momentum of the population of dC stars. The right panel of Figure 6.7 clearly shows a cluster of stars with angular momenta and orbital energies comparable to the Sun and hence consistent with membership to the disc. However, around half of the dC population displays a significant deviation away from this cluster and are likely members of the halo.
Generally, it is difficult to distinguish between thin and thick disc stars based solely on their kinematics or dynamics. It is typical in the kinematics literature to differentiate the thin and thick disk by placing cuts in offset velocity from the local standard of rest. However, these cuts are arbitrary and do not define a definitive boundary between the thin and thick disc. Furthermore, the thin and thick disc occupy a continuous region in action space and, therefore, no clear dynamical boundary exists between these Galactic components. The most accurate boundary between thin and thick disc stars comes from abundance measurements, with the boundary usually defined in the \([\alpha/Fe]\)-metallicity plane. Without data describing stellar abundances, it is not possible to confidently make an accurate cut between the thin and thick disc. Thus, with the data at hand, it is difficult to confidently classify what proportion of the dC population falls into each Galactic component, and in turn, the proportion of dC stars that may be old and metal-poor.

Nevertheless, it is clear from the angular momenta and orbital energies that a significant fraction of the dC population does not possess Galactic orbits commensurate with thin disc membership. The average metallicities of stars found in the thick disc and the halo are \([Fe/H]= -0.8\), and \(= -1.5\), respectively (Beers and Christlieb, 2005). Therefore, if the dC population is genuinely dominated by members of the thick disc and the halo, this would imply that the dC population, in general, is metal-poor.

Figure 6.8 decomposes the Galactic orbits of the dC population into axes that express the direction of rotation, the eccentricity, and excursion out of the plane of the disc. Stars that occupy the right-hand side of the figure correspond to prograde orbits and hence orbit the Galactic centre in the same direction as the disc. Conversely, the left-hand side of the plot corresponds to retrograde orbits. The vertical axis of Figure 6.8 expresses the orientation of the Galactic orbits, with stars near the top of the figure possessing polar orbits (at right angles to the disc). In contrast, stars at the bottom of the figure possess radial orbits (within the disc). The Sun is again plotted as an orange star to identify the approximate location of the thin disc. Unsurprisingly the Sun, as an indicator of thin disc stars, is located at the extreme right-hand corner of the plot. The stars in this corner are on circular, prograde orbits that lie within the plane of the disc. Thus, thin disc stars are expected to cluster tightly to the right-hand corner. Many of the dC stars tend towards this corner of the figure, implying that they possess prograde orbits near the plane and with low eccentricity. However, the cluster of dC stars in this corner of the figure is quite broad and suggests that a reasonable proportion are therefore members of the thick disc. All other dC stars that do not cluster in this corner of the figure possess orbits
Fig. 6.8:
The action-space map of the 1014 high confidence dC stars classified in Section 6.4.1, where each dC star is represented as a blue point. This visualisation was introduced by Vasiliev (2019b). The top, bottom, left, and right corners of the plot correspond to polar, radial, retrograde, and prograde orbits, respectively. The upper vertices correspond to circular orbits, while the lower vertices correspond to orbits that lie in the plane. The thin disc occupies a distinct region in action space, consisting mainly of stars with approximately circular prograde orbits that lie somewhat within the plane. Thus, the disc is concentrated toward the right corner of this plot (as indicated by the location of the Sun; the plotted orange star). However, the orbits of disc stars may be non-circular and may also stray out of the plane. The dashed black line represents an approximate boundary between the halo to the left, and the thin and thick disc, to the right of the line. The position of the line is motivated by prior knowledge of disc dynamics (e.g. Binney and Tremaine, 2008). The ‘action diamond’ has since been used in observational and theoretical studies where the disc is identified in this region (e.g. Lane et al., 2022). This line should not be considered a hard cut between the disc and the halo but as an approximate guide.
that are any combination of eccentric, retrograde, and out of the plane of the disc and are, hence, consistent with membership of the halo.

At this stage, it is difficult to estimate precisely what proportion of the dC population belongs to the thin disc, thick disc, and halo. However, by inspecting the orbital actions of the dC population, it is clear many dC stars possess orbital actions that are inconsistent with membership to the thin disc. Thus, it is likely that a significant proportion of the dC population is old and metal-poor.

6.6 Conclusion

Distances to 1044 candidate dC stars taken from Green (2013a) were inferred through MCMC methods using astrometry from Gaia eDR3 and photometry from SDSS DR16. These distances were subsequently used to compute HRDs in $g - r$, $g - i$, and $r - i$ colours for the list of candidate dC stars, revealing a total of 1014 high confidence dC stars. To-date, this is the only catalogue of bona-fide dC stars, as no other catalogue has been published.

Analysing the locations of the dC stars on the HRDs reveals that the population likely possess a continuum of masses and effective temperatures. Furthermore, a large proportion of the population lies to the left of the main sequence indicating that the population as a whole is likely to be metal-poor. Comparing the HRDs to isochrones of metal-normal stars indicates that the populations may be most consistent with $[\text{Fe/H}] < -1$, however, it is unknown whether metal-normal isochrones may be applied to carbon-enhanced populations. The only dC with a constrained metallicity is G77-61 with $[\text{Fe/H}] = -4.0$, placing it as one of the most metal-deficient stars known. The low metallicity of G77-61 is consistent with the results of this analysis.

The orbital actions of the dC population indicate that the population is spread throughout the thin and thick disc, and the halo, with the majority of the dC stars analysed in this chapter possessing orbital actions expected for thick disc and halo stars. These results are consistent with the kinematical study of Farihi et al. (2018), that found between 30 and 60 per cent of dC stars were likely to be members of the halo. If dC stars are predominantly formed in the thick disc and halo, this would again imply that the population is in general metal-poor.

The results presented in this chapter indicate that the dC population is likely to be metal-poor based on their location on the HRD and the orbital actions of the population. Binary population synthesis models state metal-poor stars are likely more susceptible to atmospheric pollution via mass transfer that could increase the C/O ratio to above unity. The possible metal-poor nature
of the dC population supports this theoretical prediction. Based on these results, a spectroscopic follow-up campaign is encouraged to measure the metallicities of dC stars, and would provide conclusive evidence on the metallicity of the population as a whole, possibly opening up new avenues to discover very metal-poor stars.
Conclusions

7.1 Summary

The chapters in this thesis have presented the results of a decade-long radial-velocity survey dedicated to measuring the binary fraction and properties of dC stars. This survey has utilised three telescopes and observed a total of 37 dC stars with varying temporal resolutions. The results of this survey indicate that the dC population is consistent with duplicity, with simulations suggesting the population possesses a binary fraction of \(94^{+4}_{-6}\) per cent. Furthermore, under the assumption that the orbital period distribution of the dC population is well modelled as a Gaussian distribution in log-space, MCMC simulations find that the distribution is centred at \(\log_{10} \mu (d) = 2.17^{+0.30}_{-0.32}\), with standard deviation \(\log_{10} \sigma (d) = 0.66^{+0.39}_{-0.42}\).

Through radial-velocity monitoring, the orbital parameters of a total of 11 dC stars have been constrained, nine of which are reported for the first time. Interestingly, all of these dC stars with newly constrained orbits possess orbital periods shorter than 12 d. Furthermore, all but one of these short-orbital period dC stars exhibit clear H\(\alpha\) emission in their optical spectra.

The analysis presented in Chapter 4 finds that H\(\alpha\) emission in dC stars is accompanied by short-period photometric and radial-velocity variations, where the latter is the result of binary motion consistent with a white dwarf companion. Comparing the photometric variations to binary light-curve simulations reveals that rotation is the most likely cause of these variations. Furthermore, age estimates of the white dwarf companions suggest that a sufficient amount of time has passed for magnetic braking in the dC stars to slow their rotation rates. Instead, the rotation periods of these dC stars with H\(\alpha\) emission are typically within several per cent of their orbital periods, implying that these systems have become tidally synchronised. Hence, tidal synchronisation provides the necessary energy and angular momentum to maintain the short rotation period of these stars and counteract the effects of magnetic braking.

Intriguingly, one short-orbital period dC star (LSR 2105) does not exhibit H\(\alpha\) emission in its optical spectrum. Notably, this dC star is one of the faintest dC stars known with an absolute magnitude of \(M_R \approx 11.1\) Vega mag. Furthermore, the \(J - K_S = 1.2\) colour is typical of very late-M or early-L dwarfs, thus suggesting that its mass be as low as \(M < 0.2 M_\odot\). At these low masses, stars are expected to be fully convective and consequently possess a strong internal dynamo driving high levels of chromospheric activity (resulting in H\(\alpha\) emission). The
absence of emission in the spectrum of LSR 2105 is puzzling and should be the topic of further investigation.

The only other dC stars where it was possible to constrain an orbital period are G77-61 and LP225−12, both of which have been reported as long-period binaries in the literature (Dearborn et al., 1986; Harris et al., 2018). Comparing the orbital periods of the dC stars possessing constrained orbits that show Hα emission in their spectra against those that do not exhibit Hα emission reveals what appear to be distinct orbital period distributions. The dC stars possessing Hα emission appear consistent with solely short-orbital period binaries ($P_{\text{orb}} < 12$ d). In contrast, the dC stars without Hα emission tend to possess orbital periods of many months or longer.

MCMC simulations presented in Chapter 5 indicate that the orbital period distribution of dC stars that exhibit Hα emission is centred at orbital periods of almost two orders of magnitude shorter than the orbital period distribution of dC stars that lack Hα emission. These results support a bimodal orbital period distribution of the dC star population. However, repeating the same simulation while considering all of the dC stars observed in the radial-velocity survey suggests that a single Gaussian distribution is able to model the orbital period distribution of the population well. One drawback of these simulations is the relatively small sample size of only 37 dC stars. As the number of dC stars with constrained orbital periods grows, the accuracy of these simulations will improve.

The broad range of orbital periods found for dC stars suggests that atmospheric pollution required to create a dC star may be possible via several mass transfer mechanisms. The dC stars that possess orbital periods of a few days have likely evolved through a common-envelope phase. Thus, mass transfer may have occurred via Roche lobe overflow or wind capture before entering the common envelope. Moreover, the dC stars observed today with orbital periods of several months or longer may have received carbon-enhanced material via a wind mass transfer mechanism. There is considerable work to be completed in the theoretical understanding of the fine details of mass transfer, and the dC population can provide a novel testing site for simulations.

The dynamical analysis of the dC population presented in Chapter 6 indicates that a large proportion of the population possesses Galactic orbits that are not consistent with the thin disc. The orbital actions of the dC population are generally elliptical and exhibit excursions out of the Galactic plane. Thus, a substantial proportion of the dC population likely belongs to the thick disc or the halo. Because the average metallicity of stars found in the thick disc and halo is
low relative to solar metallicity (Beers and Christlieb, 2005), these results may imply that the dC population is old and metal-poor. Interestingly, binary population synthesis models predict a larger number of dC stars at lower metallicities (de Kool and Green, 1995), and observationally, many carbon-enhanced populations are also metal-deficient compared to solar (e.g. Ba, CH, and CEMP stars). Therefore, metallicity appears to be an important factor in whether the accretor becomes carbon-enhanced during the mass transfer process.

The analysis presented in this thesis suggests that dC stars possess modest metallicities relative to solar metallicity, and are formed through mass transfer from an evolved companion. Because the orbital period distribution of the dC star population spans from a few hours to several years, it is likely that dC stars are formed through at least two different channels, Roche lobe overflow and wind capture. Thus, these data suggest that metallicity plays no role in the mechanism of mass transfer. Moreover, because dC stars are formed through multiple channels of mass transfer, it is unlikely that metallicity is an important factor in the efficiency of mass transfer. The role of metallicity in mass transfer becomes important when considering the composition of the material being transferred and how that can pollute the atmosphere of the accretor. Thus, these data suggest the role of metallicity in mass transfer is confined to the chemical evolution of the accretor.

### 7.2 Final remarks

Returning now to the question posed at the beginning of this thesis: *Are dC stars intrinsically carbon-rich or extrinsically carbon-enhanced through mass transfer?*

The evidence presented in this thesis suggests that dC stars are members of binary star systems with a typically unseen, evolved companion (consistent with a white dwarf). Therefore, dC stars likely owe their peculiar atmospheric chemistry to mass transfer from their evolved companions. Thus, dC stars are formed by accreting a sufficient amount of carbon-enhanced material to raise their C/O ratio above unity, rather than intrinsically possessing carbon to exhibit a C/O ratio greater than unity at the onset of hydrogen burning.

The origin of a dC star is, therefore, as follows. Initially, a binary star system consisting of two stars of unequal masses is formed. In this binary system, the secondary star is a late-K/early-M dwarf and spends all of its life from the onset of core hydrogen-burning to today on the main sequence. Moreover, the primary star in this binary possesses a larger mass than the secondary. Thus, the primary evolves off the main sequence before the secondary. After evolving away from the main sequence, the primary eventually begins its ascent of the AGB and consequently
fuses helium into carbon in its core through the triple-$\alpha$ process. Carbon-rich material is then dredged to the surface of the primary during the third dredge-up, causing its C/O ratio in its atmosphere to rise beyond unity. After the primary’s atmosphere becomes carbon-rich, mass is transferred from the primary onto the secondary, thus minting a fresh dC star. The primary then evolves onto the white dwarf cooling sequence where it radiates its remaining energy and fades from visibility, leaving behind these faint carbon-enhanced main-sequence stars for astronomers to puzzle over for half a century.

Over the next few decades, large-scale surveys will provide an unprecedented volume of spectroscopic and photometric data for targets across the sky. Through spectroscopic surveys such as WEAVE, 4MOST, and DESI (Dalton et al., 2012; DESI Collaboration et al., 2016; de Jong et al., 2019), the number of candidate dC stars is almost certain to rise. The spectra observed during these surveys will be important in the discovery of new dC binaries and may help to constrain the orbits of a larger number of dC stars, thus creating a larger observational dataset to calibrate binary synthesis and mass transfer models. Furthermore, the medium resolution spectra obtained from these surveys may allow abundance measurements to be made for the dC population and hence help astronomers better understand their chemical evolution.
Appendix: Posterior probability distribution functions of the orbital parameters of seven emission-line dC stars

Presented in this Appendix are the posterior probability distribution functions for the orbital period, velocity semi-amplitude, and systemic velocity (relative to the dC star for each target) for seven emission-line dC stars, discussed in Chapter 4. The analysis was conducted using the JOKER.
Fig. A.1.: The posterior probability distribution functions are shown for orbital period, velocity semi-amplitude, and systemic velocity for all H\(\alpha\) emitting dC stars discussed in Chapter 4.

Fig. A.1. continued
Fig. A.1. continued
Fig. A.1. continued

$$P \ [d] = 2.56^{+0.13}_{-0.11}$$

$$J1250$$

$$K \ [\text{km s}^{-1}] = 86.17^{+4.21}_{-3.42}$$

$$\gamma \ [\text{km s}^{-1}] = -39.06^{+4.79}_{-3.68}$$

Fig. A.1. continued

$$P \ [d] = 4.4345^{+0.0006}_{-0.0006}$$

$$SBSS \ 1310$$

$$K \ [\text{km s}^{-1}] = 46.52^{+0.83}_{-0.84}$$

$$\gamma \ [\text{km s}^{-1}] = -46.92^{+0.83}_{-0.82}$$

Fig. A.1. continued
Fig. A.1. continued
Appendix: Tabulated absolute radial velocity data

In this Appendix, the tabulated radial velocity data are presented for all 37 dC stars that were studied in this thesis. Both the relative and absolute velocities are given, along with their corresponding uncertainties. Furthermore, the template used to measure the absolute velocities for each star is given in the header to each table. Finally, the instrumental set-up is provided for all radial-velocity measurements, along with the Julian date of each observation relative to the Julian date of the start of the radial velocity campaign.

Because SDSS J125017.90+252427.6 was not observed as part of this radial-velocity survey, neither the absolute nor relative radial velocity measurements are presented here. These measurements can, however, be found in Margon et al. 2018.
Tab. B.1.: Radial velocities of the target LHS 1075. Absolute radial velocities were measured relative to the m2t4500R7900 synthetic stellar template. The Julian date of each observation is given in first column. The barycentric corrected relative radial velocity measurements and errors are given in the second and third columns. The absolute velocities and corresponding errors are reported in the fourth and fifth columns. Finally, the instrument and the grating used to take the measurement are presented in the sixth and seventh columns.

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2455827</td>
<td>687.63</td>
<td>16.73</td>
<td>2.58</td>
<td>−24.02</td>
<td>9.89 ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td></td>
<td>688.57</td>
<td>17.46</td>
<td>1.51</td>
<td>−23.29</td>
<td>9.66 ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td></td>
<td>1180.33</td>
<td>11.93</td>
<td>2.54</td>
<td>−28.82</td>
<td>9.88 ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td></td>
<td>1184.60</td>
<td>13.58</td>
<td>9.63</td>
<td>−27.17</td>
<td>13.56 RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td></td>
<td>1185.64</td>
<td>6.30</td>
<td>3.09</td>
<td>−34.45</td>
<td>10.03 RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td></td>
<td>1186.58</td>
<td>6.33</td>
<td>2.94</td>
<td>−34.42</td>
<td>9.99 RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td></td>
<td>1449.60</td>
<td>14.35</td>
<td>3.24</td>
<td>−26.40</td>
<td>10.08 ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td></td>
<td>1546.88</td>
<td>−12.65</td>
<td>6.01</td>
<td>−53.40</td>
<td>11.28 KOSMOS Blue VPH</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1547.37</td>
<td>17.22</td>
<td>2.23</td>
<td>−23.53</td>
<td>9.80 ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td></td>
<td>1547.86</td>
<td>−2.72</td>
<td>4.87</td>
<td>−43.47</td>
<td>10.71 KOSMOS Blue VPH</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2155.68</td>
<td>16.32</td>
<td>1.75</td>
<td>−24.42</td>
<td>9.70 ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td></td>
<td>2156.68</td>
<td>15.24</td>
<td>1.47</td>
<td>−25.50</td>
<td>9.66 ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td></td>
<td>2181.55</td>
<td>16.43</td>
<td>2.18</td>
<td>−24.31</td>
<td>9.79 ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td></td>
<td>2514.72</td>
<td>16.88</td>
<td>2.00</td>
<td>−23.87</td>
<td>9.75 ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td></td>
<td>2515.68</td>
<td>13.99</td>
<td>2.78</td>
<td>−26.76</td>
<td>9.94 ISIS blue arm</td>
<td>R1200B</td>
</tr>
</tbody>
</table>

$^a$ Ritchey-Chretien Focus Spectrograph  
$^c$ KPC 18C, 790 line/mm grating, used in 2nd order

Tab. B.2.: Radial velocities of the target SDSS J012028.56−083630.9. Absolute radial velocities were measured relative to the m2t4500R7900 synthetic stellar template. These data are formatted in the same way as Table B.1

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2455827</td>
<td>687.64</td>
<td>16.11</td>
<td>1.92</td>
<td>59.21</td>
<td>9.22 ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td></td>
<td>688.63</td>
<td>23.65</td>
<td>1.48</td>
<td>66.75</td>
<td>9.14 ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td></td>
<td>1180.35</td>
<td>18.70</td>
<td>23.5</td>
<td>61.80</td>
<td>25.17 ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td></td>
<td>1449.67</td>
<td>11.32</td>
<td>2.44</td>
<td>54.42</td>
<td>9.34 ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td></td>
<td>1546.90</td>
<td>6.56</td>
<td>10.89</td>
<td>49.66</td>
<td>14.14 KOSMOS Blue VPH</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1547.41</td>
<td>23.88</td>
<td>2.07</td>
<td>66.97</td>
<td>9.25 ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td></td>
<td>1547.89</td>
<td>10.35</td>
<td>8.27</td>
<td>53.44</td>
<td>12.24 KOSMOS Blue VPH</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2155.65</td>
<td>11.89</td>
<td>1.69</td>
<td>54.98</td>
<td>9.17 ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td></td>
<td>2156.65</td>
<td>17.05</td>
<td>1.58</td>
<td>60.14</td>
<td>9.15 ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td></td>
<td>2180.66</td>
<td>16.10</td>
<td>3.14</td>
<td>59.20</td>
<td>9.55 ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td></td>
<td>2513.71</td>
<td>7.13</td>
<td>1.83</td>
<td>50.22</td>
<td>9.20 ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td></td>
<td>2514.69</td>
<td>7.23</td>
<td>1.97</td>
<td>50.32</td>
<td>9.23 ISIS blue arm</td>
<td>R1200B</td>
</tr>
</tbody>
</table>
Tab. B.3.: Radial velocities of the target SDSS J012150.42+011301.4. Absolute radial velocities were measured relative to the m2t3500R7900 synthetic stellar template. These data are formatted in the same way as Table B.1.

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$</th>
<th>$v_{\text{rad}}$</th>
<th>$\sigma_{v_{\text{rad}}}$</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2455827$</td>
<td>-36.13</td>
<td>1.56</td>
<td>95.47</td>
<td>5.44</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>687.68</td>
<td>25.29</td>
<td>1.50</td>
<td>156.9</td>
<td>5.42</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1449.58</td>
<td>-5.77</td>
<td>5.32</td>
<td>125.83</td>
<td>7.45</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1546.94</td>
<td>8.47</td>
<td>2.04</td>
<td>140.08</td>
<td>5.59</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1547.92</td>
<td>-48.72</td>
<td>5.81</td>
<td>82.89</td>
<td>7.80</td>
<td>KOSMOS Blue VPH</td>
<td>Blue VPH</td>
</tr>
<tr>
<td>2155.6</td>
<td>-44.52</td>
<td>1.75</td>
<td>87.09</td>
<td>5.50</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2156.59</td>
<td>4.94</td>
<td>3.33</td>
<td>136.54</td>
<td>6.18</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2180.70</td>
<td>86.36</td>
<td>2.27</td>
<td>217.97</td>
<td>5.68</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2513.65</td>
<td>60.99</td>
<td>5.72</td>
<td>192.59</td>
<td>7.74</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2514.63</td>
<td>131.81</td>
<td>2.13</td>
<td>263.41</td>
<td>5.63</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
</tbody>
</table>

Tab. B.4.: Radial Velocities of the target SDSS J013007.13+002635.3. Absolute radial velocities were measured relative to the m0t3750R6700 synthetic stellar template. These data are formatted in the same way as Table B.1.

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$</th>
<th>$v_{\text{rad}}$</th>
<th>$\sigma_{v_{\text{rad}}}$</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2455827$</td>
<td>25.35</td>
<td>3.15</td>
<td>46.26</td>
<td>5.34</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>687.70</td>
<td>25.28</td>
<td>1.50</td>
<td>46.19</td>
<td>4.57</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1449.62</td>
<td>31.32</td>
<td>7.16</td>
<td>52.23</td>
<td>8.36</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1546.96</td>
<td>10.47</td>
<td>16.79</td>
<td>31.38</td>
<td>17.33</td>
<td>KOSMOS Blue VPH</td>
<td>Blue VPH</td>
</tr>
<tr>
<td>1547.95</td>
<td>-5.00</td>
<td>8.55</td>
<td>15.91</td>
<td>9.57</td>
<td>KOSMOS Blue VPH</td>
<td>Blue VPH</td>
</tr>
<tr>
<td>2155.63</td>
<td>31.55</td>
<td>3.13</td>
<td>52.46</td>
<td>5.33</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2156.62</td>
<td>36.11</td>
<td>2.81</td>
<td>57.02</td>
<td>5.15</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2181.56</td>
<td>36.00</td>
<td>3.85</td>
<td>59.51</td>
<td>5.78</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2513.68</td>
<td>30.32</td>
<td>3.74</td>
<td>51.23</td>
<td>5.71</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2514.66</td>
<td>30.41</td>
<td>3.67</td>
<td>51.32</td>
<td>5.66</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
</tbody>
</table>

Tab. B.5.: Radial Velocities of the target SDSS J022304.43+004501.3. Absolute radial velocities were measured relative to the m2t4500R7900 synthetic stellar template. These data are formatted in the same way as Table B.1.

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$</th>
<th>$v_{\text{rad}}$</th>
<th>$\sigma_{v_{\text{rad}}}$</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2455827$</td>
<td>25.35</td>
<td>3.15</td>
<td>46.26</td>
<td>5.34</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>687.70</td>
<td>25.28</td>
<td>1.50</td>
<td>46.19</td>
<td>4.57</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1449.62</td>
<td>31.32</td>
<td>7.16</td>
<td>52.23</td>
<td>8.36</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1546.96</td>
<td>10.47</td>
<td>16.79</td>
<td>31.38</td>
<td>17.33</td>
<td>KOSMOS Blue VPH</td>
<td>Blue VPH</td>
</tr>
<tr>
<td>1547.95</td>
<td>-5.00</td>
<td>8.55</td>
<td>15.91</td>
<td>9.57</td>
<td>KOSMOS Blue VPH</td>
<td>Blue VPH</td>
</tr>
<tr>
<td>2155.63</td>
<td>31.55</td>
<td>3.13</td>
<td>52.46</td>
<td>5.33</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2156.62</td>
<td>36.11</td>
<td>2.81</td>
<td>57.02</td>
<td>5.15</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2181.56</td>
<td>36.00</td>
<td>3.85</td>
<td>59.51</td>
<td>5.78</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2513.68</td>
<td>30.32</td>
<td>3.74</td>
<td>51.23</td>
<td>5.71</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2514.66</td>
<td>30.41</td>
<td>3.67</td>
<td>51.32</td>
<td>5.66</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
</tbody>
</table>

193
Tab. B.6.: Radial Velocities of the target G77-61. Absolute radial velocities were measured relative to the m2t4500R7900 synthetic stellar template. These data are formatted in the same way as Table B.1

<table>
<thead>
<tr>
<th>JD</th>
<th>( \Delta v_{\text{rad}} ) (km s(^{-1}))</th>
<th>( \sigma_{\Delta v_{\text{rad}}} ) (km s(^{-1}))</th>
<th>( v_{\text{rad}} ) (km s(^{-1}))</th>
<th>( \sigma_{v_{\text{rad}}} ) (km s(^{-1}))</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2455827</td>
<td>0.89</td>
<td>9.08</td>
<td>2.30</td>
<td>65.32</td>
<td>7.37</td>
<td>GMOS</td>
</tr>
<tr>
<td></td>
<td>9.88</td>
<td>4.59</td>
<td>2.10</td>
<td>60.83</td>
<td>7.31</td>
<td>GMOS</td>
</tr>
<tr>
<td></td>
<td>26.84</td>
<td>−2.34</td>
<td>2.64</td>
<td>53.90</td>
<td>7.49</td>
<td>GMOS</td>
</tr>
<tr>
<td></td>
<td>126.54</td>
<td>−13.95</td>
<td>1.86</td>
<td>42.30</td>
<td>7.25</td>
<td>GMOS</td>
</tr>
<tr>
<td></td>
<td>135.53</td>
<td>−3.37</td>
<td>1.84</td>
<td>52.87</td>
<td>7.24</td>
<td>GMOS</td>
</tr>
<tr>
<td></td>
<td>505.42</td>
<td>9.20</td>
<td>1.17</td>
<td>65.44</td>
<td>7.10</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>510.39</td>
<td>7.13</td>
<td>1.24</td>
<td>63.37</td>
<td>7.11</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>687.73</td>
<td>28.33</td>
<td>1.36</td>
<td>84.58</td>
<td>7.14</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>898.35</td>
<td>12.05</td>
<td>1.13</td>
<td>68.29</td>
<td>7.10</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>1126.52</td>
<td>5.34</td>
<td>1.23</td>
<td>61.59</td>
<td>7.11</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>1180.43</td>
<td>24.46</td>
<td>1.20</td>
<td>80.70</td>
<td>7.11</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>1180.53</td>
<td>22.40</td>
<td>1.20</td>
<td>78.64</td>
<td>7.11</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>1184.63</td>
<td>18.55</td>
<td>2.39</td>
<td>74.79</td>
<td>7.40</td>
<td>RC(^a)</td>
</tr>
<tr>
<td></td>
<td>1185.69</td>
<td>24.35</td>
<td>2.24</td>
<td>80.60</td>
<td>7.36</td>
<td>RC(^a)</td>
</tr>
<tr>
<td></td>
<td>1186.66</td>
<td>17.56</td>
<td>2.48</td>
<td>73.81</td>
<td>7.43</td>
<td>RC(^a)</td>
</tr>
<tr>
<td></td>
<td>1449.71</td>
<td>25.53</td>
<td>1.18</td>
<td>81.77</td>
<td>7.10</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>1547.96</td>
<td>−7.11</td>
<td>5.40</td>
<td>49.14</td>
<td>8.84</td>
<td>KOSMOS</td>
</tr>
<tr>
<td></td>
<td>2155.72</td>
<td>26.62</td>
<td>1.15</td>
<td>82.87</td>
<td>7.10</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>2156.73</td>
<td>25.96</td>
<td>1.12</td>
<td>82.20</td>
<td>7.09</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>2194.55</td>
<td>16.56</td>
<td>2.62</td>
<td>72.80</td>
<td>7.48</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>2194.62</td>
<td>24.46</td>
<td>1.24</td>
<td>80.70</td>
<td>7.11</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>2515.72</td>
<td>−8.26</td>
<td>1.19</td>
<td>47.98</td>
<td>7.11</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>3056.36</td>
<td>−11.05</td>
<td>1.66</td>
<td>45.20</td>
<td>7.20</td>
<td>ISIS red arm</td>
</tr>
<tr>
<td></td>
<td>3061.40</td>
<td>−21.72</td>
<td>1.09</td>
<td>44.53</td>
<td>7.09</td>
<td>ISIS red arm</td>
</tr>
</tbody>
</table>

\(^a\) Ritchey-Chretien Focus Spectrograph  
\(^c\) KPC 18C, 790 line/mm grating, used in 2nd order
Tab. B.7.: Radial Velocities of the target SDSS J074257.17+465917.9. Absolute radial velocities were measured relative to the m2t3500R7900 synthetic stellar template. These data are formatted in the same way as Table B.1.

<table>
<thead>
<tr>
<th>JD</th>
<th>Δ$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>σΔ$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>σ$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>573.63</td>
<td>11.09</td>
<td>2.11</td>
<td>24.2</td>
<td>6.04</td>
<td>RC$^a$</td>
<td>BL 380$^b$</td>
</tr>
<tr>
<td>898.40</td>
<td>14.9</td>
<td>1.35</td>
<td>28.02</td>
<td>5.82</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1180.50</td>
<td>11.9</td>
<td>2.05</td>
<td>25.01</td>
<td>6.02</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1184.72</td>
<td>−3.45</td>
<td>6.81</td>
<td>9.66</td>
<td>8.85</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>1185.87</td>
<td>8.70</td>
<td>5.51</td>
<td>21.81</td>
<td>7.89</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>1449.73</td>
<td>13.58</td>
<td>1.62</td>
<td>26.69</td>
<td>5.88</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1546.99</td>
<td>−14.81</td>
<td>8.90</td>
<td>−1.70</td>
<td>10.55</td>
<td>KOSMOS Blue VPH</td>
<td></td>
</tr>
<tr>
<td>1547.49</td>
<td>11.05</td>
<td>1.09</td>
<td>24.16</td>
<td>5.76</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1548.00</td>
<td>1.86</td>
<td>9.47</td>
<td>14.97</td>
<td>11.03</td>
<td>KOSMOS Blue VPH</td>
<td></td>
</tr>
<tr>
<td>2715.42</td>
<td>−11.47</td>
<td>2.42</td>
<td>1.64</td>
<td>6.15</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3056.39</td>
<td>−10.21</td>
<td>2.87</td>
<td>2.90</td>
<td>6.35</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3057.37</td>
<td>−11.04</td>
<td>1.90</td>
<td>2.08</td>
<td>5.97</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3059.36</td>
<td>−10.61</td>
<td>1.53</td>
<td>2.51</td>
<td>5.86</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3059.36</td>
<td>−12.63</td>
<td>1.09</td>
<td>0.48</td>
<td>5.76</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
</tbody>
</table>

$^a$ Ritchey-Chretien Focus Spectrograph
$^b$ BL 380, 120 line/mm grating, used in 2nd order
$^c$ KPC 18C, 790 line/mm grating, used in 2nd order

Tab. B.8.: Radial Velocities of the target SDSS J081157.14+143533.0. Absolute radial velocities were measured relative to the m1t4250R7900 synthetic stellar template. These data are formatted in the same way as Table B.1.

<table>
<thead>
<tr>
<th>JD</th>
<th>Δ$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>σΔ$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>σ$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>505.44</td>
<td>−9.45</td>
<td>1.51</td>
<td>−27.52</td>
<td>2.58</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>510.42</td>
<td>−20.59</td>
<td>9.94</td>
<td>−38.66</td>
<td>10.16</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>573.67</td>
<td>−43.97</td>
<td>5.30</td>
<td>−62.04</td>
<td>5.70</td>
<td>RC$^a$</td>
<td>BL 380$^b$</td>
</tr>
<tr>
<td>898.37</td>
<td>−4.58</td>
<td>6.60</td>
<td>−22.66</td>
<td>6.92</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>925.66</td>
<td>−28.45</td>
<td>19.26</td>
<td>−46.52</td>
<td>19.37</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>1180.55</td>
<td>−2.52</td>
<td>5.29</td>
<td>−20.59</td>
<td>5.69</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1184.77</td>
<td>−15.59</td>
<td>6.57</td>
<td>−33.67</td>
<td>6.89</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>1186.85</td>
<td>−8.85</td>
<td>6.24</td>
<td>−26.92</td>
<td>6.58</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>1547.01</td>
<td>−29.18</td>
<td>7.46</td>
<td>−47.25</td>
<td>7.74</td>
<td>KOSMOS Blue VPH</td>
<td></td>
</tr>
<tr>
<td>1548.01</td>
<td>−21.04</td>
<td>7.04</td>
<td>−39.11</td>
<td>7.35</td>
<td>KOSMOS Blue VPH</td>
<td></td>
</tr>
<tr>
<td>1564.46</td>
<td>−7.24</td>
<td>3.56</td>
<td>−25.31</td>
<td>4.13</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2715.42</td>
<td>3.60</td>
<td>4.29</td>
<td>−14.47</td>
<td>4.78</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
</tbody>
</table>

$^a$ Ritchey-Chretien Focus Spectrograph
$^b$ BL 380, 120 line/mm grating, used in 2nd order
$^c$ KPC 18C, 790 line/mm grating, used in 2nd order
Tab. B.9.: Radial Velocities of the target SDSS J081807.45+223427.6. Absolute radial velocities were measured relative to the m0t4000R6700 synthetic stellar template. These data are formatted in the same way as Table B.1.

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2455827$</td>
<td>925.68</td>
<td>36.91</td>
<td>3.07</td>
<td>88.88</td>
<td>4.90</td>
<td>RC$^a$ KPC 18C$^c$</td>
</tr>
<tr>
<td></td>
<td>1185.91</td>
<td>16.17</td>
<td>10.9</td>
<td>68.15</td>
<td>3.97</td>
<td>RC$^a$ KPC 18C$^c$</td>
</tr>
<tr>
<td></td>
<td>1547.03</td>
<td>2.19</td>
<td>6.60</td>
<td>54.17</td>
<td>7.62</td>
<td>KOSMOS Blue VPH</td>
</tr>
<tr>
<td></td>
<td>1548.03</td>
<td>16.53</td>
<td>7.25</td>
<td>68.51</td>
<td>8.19</td>
<td>KOSMOS Blue VPH</td>
</tr>
</tbody>
</table>

$^a$ Ritchey-Chretien Focus Spectrograph
$^c$ KPC 18C, 790 line/mm grating, used in 2$^{nd}$ order

Tab. B.10.: Radial Velocities of the target PG 0842+288. Absolute radial velocities were measured relative to the m0t4000R6700 synthetic stellar template. These data are formatted in the same way as Table B.1.

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2455827$</td>
<td>505.49</td>
<td>-8.92</td>
<td>1.56</td>
<td>11.25</td>
<td>3.28</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>510.45</td>
<td>-6.95</td>
<td>5.62</td>
<td>13.22</td>
<td>6.32</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>573.71</td>
<td>-15.46</td>
<td>2.88</td>
<td>4.70</td>
<td>4.08</td>
<td>BL 380$^b$</td>
</tr>
<tr>
<td></td>
<td>898.42</td>
<td>-13.16</td>
<td>4.75</td>
<td>7.00</td>
<td>5.56</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>925.65</td>
<td>3.79</td>
<td>16.59</td>
<td>23.96</td>
<td>16.84</td>
<td>RC$^a$ KPC 18C$^c$</td>
</tr>
<tr>
<td></td>
<td>1180.57</td>
<td>-10.09</td>
<td>4.12</td>
<td>10.07</td>
<td>5.03</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>1180.72</td>
<td>-7.02</td>
<td>4.52</td>
<td>13.14</td>
<td>5.37</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>1184.81</td>
<td>-20.69</td>
<td>5.94</td>
<td>-0.52</td>
<td>6.61</td>
<td>RC$^a$ KPC 18C$^c$</td>
</tr>
<tr>
<td></td>
<td>1186.82</td>
<td>-18.13</td>
<td>5.81</td>
<td>2.04</td>
<td>6.48</td>
<td>RC$^a$ KPC 18C$^c$</td>
</tr>
<tr>
<td></td>
<td>1564.44</td>
<td>-14.11</td>
<td>3.71</td>
<td>6.06</td>
<td>4.70</td>
<td>ISIS blue arm R1200B</td>
</tr>
</tbody>
</table>

$^a$ Ritchey-Chretien Focus Spectrograph
$^b$ BL 380, 120 line/mm grating, used in 2$^{nd}$ order
$^c$ KPC 18C, 790 line/mm grating, used in 2$^{nd}$ order
Tab. B.11.: Radial Velocities of the target SDSS J084259.80+225729.0. Absolute radial velocities were measured relative to the m0t4000R6700 synthetic stellar template. These data are formatted in the same way as Table B.1.

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>2714.37</td>
<td>$-14.29$</td>
<td>18.4</td>
<td>$-39.38$</td>
<td>18.57</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2714.37</td>
<td>$-17.65$</td>
<td>4.27</td>
<td>$-42.73$</td>
<td>4.95</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2714.46</td>
<td>$-17.88$</td>
<td>4.27</td>
<td>$-42.97$</td>
<td>4.95</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2714.56</td>
<td>$-15.17$</td>
<td>1.48</td>
<td>$-40.25$</td>
<td>2.91</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2714.65</td>
<td>$-19.43$</td>
<td>4.81</td>
<td>$-44.51$</td>
<td>5.42</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2715.47</td>
<td>$-13.36$</td>
<td>4.91</td>
<td>$-38.45$</td>
<td>5.51</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2715.64</td>
<td>$-17.24$</td>
<td>4.00</td>
<td>$-42.32$</td>
<td>4.72</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2715.60</td>
<td>$-14.17$</td>
<td>3.78</td>
<td>$-39.26$</td>
<td>4.54</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2715.53</td>
<td>$-8.58$</td>
<td>4.16</td>
<td>$-33.66$</td>
<td>4.86</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>3058.37</td>
<td>$-18.99$</td>
<td>10.14</td>
<td>$-44.07$</td>
<td>10.45</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3058.42</td>
<td>$-4.30$</td>
<td>1.48</td>
<td>$-29.38$</td>
<td>2.91</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3058.58</td>
<td>$-15.23$</td>
<td>4.98</td>
<td>$-40.31$</td>
<td>5.57</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3059.38</td>
<td>$-11.22$</td>
<td>9.01</td>
<td>$-36.30$</td>
<td>9.36</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3059.48</td>
<td>$-15.09$</td>
<td>7.05</td>
<td>$-40.17$</td>
<td>7.48</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
</tbody>
</table>

Tab. B.12.: Radial Velocities of the target SDSS J090128.28+323833.5. Absolute radial velocities were measured relative to the m0t4000R6700 synthetic stellar template. These data are formatted in the same way as Table B.1.

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>1547.07</td>
<td>$-35.59$</td>
<td>4.02</td>
<td>$-102.47$</td>
<td>5.70</td>
<td>KOSMOS Blue VPH</td>
<td>Blue VPH</td>
</tr>
<tr>
<td>1548.08</td>
<td>$-24.50$</td>
<td>4.19</td>
<td>$-91.37$</td>
<td>5.82</td>
<td>KOSMOS Blue VPH</td>
<td>Blue VPH</td>
</tr>
<tr>
<td>2714.39</td>
<td>$-29.75$</td>
<td>2.82</td>
<td>$-96.62$</td>
<td>4.92</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2714.49</td>
<td>$-23.59$</td>
<td>2.55</td>
<td>$-90.47$</td>
<td>4.77</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2714.59</td>
<td>$-14.19$</td>
<td>1.61</td>
<td>$-81.06$</td>
<td>4.35</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2714.68</td>
<td>$-8.65$</td>
<td>2.24</td>
<td>$-75.52$</td>
<td>4.61</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2715.35</td>
<td>15.43</td>
<td>2.95</td>
<td>$-51.45$</td>
<td>5.00</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2715.45</td>
<td>10.08</td>
<td>2.58</td>
<td>$-56.80$</td>
<td>4.79</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2715.62</td>
<td>1.35</td>
<td>3.13</td>
<td>$-65.52$</td>
<td>5.11</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2759.39</td>
<td>7.80</td>
<td>2.48</td>
<td>$-59.07$</td>
<td>4.74</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2759.47</td>
<td>12.30</td>
<td>3.88</td>
<td>$-54.58$</td>
<td>5.60</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>3057.40</td>
<td>$-2.16$</td>
<td>2.36</td>
<td>$-69.03$</td>
<td>4.67</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3057.47</td>
<td>$-9.71$</td>
<td>1.53</td>
<td>$-76.59$</td>
<td>4.32</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3058.40</td>
<td>$-6.60$</td>
<td>1.55</td>
<td>$-73.47$</td>
<td>4.32</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3058.49</td>
<td>$-20.97$</td>
<td>4.99</td>
<td>$-87.85$</td>
<td>6.41</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3058.68</td>
<td>$-23.13$</td>
<td>1.47</td>
<td>$-90.91$</td>
<td>4.30</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3059.42</td>
<td>$-18.28$</td>
<td>1.52</td>
<td>$-85.18$</td>
<td>4.31</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3059.55</td>
<td>4.59</td>
<td>1.49</td>
<td>$-62.28$</td>
<td>4.30</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3059.64</td>
<td>6.43</td>
<td>1.23</td>
<td>$-60.44$</td>
<td>4.22</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3060.38</td>
<td>6.35</td>
<td>4.62</td>
<td>$-60.53$</td>
<td>6.14</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3060.42</td>
<td>3.02</td>
<td>1.54</td>
<td>$-63.86$</td>
<td>4.32</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3060.43</td>
<td>$-6.70$</td>
<td>1.37</td>
<td>$-73.58$</td>
<td>4.26</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3060.52</td>
<td>0.05</td>
<td>1.23</td>
<td>$-66.82$</td>
<td>4.22</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3060.69</td>
<td>$-16.36$</td>
<td>1.53</td>
<td>$-83.24$</td>
<td>4.32</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
</tbody>
</table>
Tab. B.13.: Radial Velocities of the target SDSS J090302.86+385527.4. Absolute radial velocities were measured relative to the m0t3750R6700 synthetic stellar template. These data are formatted in the same way as Table B.1

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2455827</td>
<td>926.76</td>
<td>53.45</td>
<td>29.81</td>
<td>54.40</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td></td>
<td>1185.96</td>
<td>16.85</td>
<td>1.09</td>
<td>17.80</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td></td>
<td>1547.10</td>
<td>26.24</td>
<td>9.82</td>
<td>27.19</td>
<td>KOSMOS</td>
<td>Blue VPH</td>
</tr>
<tr>
<td></td>
<td>1548.11</td>
<td>17.42</td>
<td>25.03</td>
<td>18.37</td>
<td>KOSMOS</td>
<td>Blue VPH</td>
</tr>
<tr>
<td></td>
<td>2714.42</td>
<td>19.76</td>
<td>116.4</td>
<td>20.71</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
</tbody>
</table>

$^a$ Ritchey-Chretien Focus Spectrograph  
$^c$ KPC 18C, 790 line/mm grating, used in 2nd order

Tab. B.14.: Radial Velocities of the target SDSS J091007.60+521612.5. Absolute radial velocities were measured relative to the m2t3500R7900 synthetic stellar template. These data are formatted in the same way as Table B.1

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2455827</td>
<td>3058.34</td>
<td>-5.22</td>
<td>1.09</td>
<td>-106.85</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td></td>
<td>3058.55</td>
<td>2.58</td>
<td>1.96</td>
<td>-99.05</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td></td>
<td>3059.45</td>
<td>8.62</td>
<td>1.79</td>
<td>-93.01</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td></td>
<td>3059.46</td>
<td>8.10</td>
<td>1.65</td>
<td>-93.53</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td></td>
<td>3060.36</td>
<td>4.52</td>
<td>2.31</td>
<td>-97.11</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td></td>
<td>3060.36</td>
<td>4.19</td>
<td>1.86</td>
<td>-97.44</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td></td>
<td>3060.47</td>
<td>1.28</td>
<td>2.71</td>
<td>-100.35</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td></td>
<td>3060.59</td>
<td>1.14</td>
<td>2.06</td>
<td>-100.49</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td></td>
<td>3061.34</td>
<td>-20.15</td>
<td>5.99</td>
<td>-121.78</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td></td>
<td>3061.43</td>
<td>-19.48</td>
<td>2.38</td>
<td>-121.11</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td></td>
<td>3061.44</td>
<td>-17.09</td>
<td>2.15</td>
<td>-118.72</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td></td>
<td>3061.55</td>
<td>-25.69</td>
<td>4.12</td>
<td>-127.32</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td></td>
<td>3061.56</td>
<td>-28.47</td>
<td>2.62</td>
<td>-130.10</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td></td>
<td>3061.65</td>
<td>-31.08</td>
<td>2.59</td>
<td>-132.71</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
</tbody>
</table>
Tab. B.15: Radial Velocities of the target C0930-00. Absolute radial velocities were measured relative to the m14000R7900 synthetic stellar template. These data are formatted in the same way as Table B.1.

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$</th>
<th>$v_{\text{rad}}$</th>
<th>$\sigma_{v_{\text{rad}}}$</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2455827$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>503.64</td>
<td>17.58</td>
<td>1.27</td>
<td>1.41</td>
<td>2.44</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>504.63</td>
<td>16.26</td>
<td>1.37</td>
<td>0.10</td>
<td>2.50</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>573.75</td>
<td>16.55</td>
<td>2.99</td>
<td>0.38</td>
<td>3.65</td>
<td>RC$^a$</td>
<td>BL 380$^b$</td>
</tr>
<tr>
<td>898.49</td>
<td>14.70</td>
<td>1.80</td>
<td>$-1.47$</td>
<td>2.76</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>925.71</td>
<td>18.02</td>
<td>7.51</td>
<td>1.85</td>
<td>7.79</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>1180.58</td>
<td>15.56</td>
<td>2.23</td>
<td>$-0.61$</td>
<td>3.05</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1184.87</td>
<td>13.00</td>
<td>5.74</td>
<td>$-3.16$</td>
<td>6.11</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>1186.93</td>
<td>19.65</td>
<td>5.06</td>
<td>3.48</td>
<td>5.47</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>1547.14</td>
<td>$-10.81$</td>
<td>4.73</td>
<td>$-26.97$</td>
<td>5.17</td>
<td>KOSMOS</td>
<td>Blue VPH</td>
</tr>
<tr>
<td>1548.13</td>
<td>$-3.94$</td>
<td>4.41</td>
<td>$-20.10$</td>
<td>4.88</td>
<td>KOSMOS</td>
<td>Blue VPH</td>
</tr>
<tr>
<td>1564.52</td>
<td>20.36</td>
<td>1.48</td>
<td>4.19</td>
<td>2.56</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2714.45</td>
<td>18.77</td>
<td>2.35</td>
<td>2.61</td>
<td>3.15</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2715.44</td>
<td>15.46</td>
<td>1.49</td>
<td>$-0.70$</td>
<td>2.56</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>3056.48</td>
<td>5.71</td>
<td>2.21</td>
<td>$-10.45$</td>
<td>3.04</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3056.59</td>
<td>8.08</td>
<td>1.88</td>
<td>$-8.08$</td>
<td>2.81</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3056.60</td>
<td>5.58</td>
<td>1.16</td>
<td>$-10.59$</td>
<td>2.39</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3061.51</td>
<td>1.40</td>
<td>1.60</td>
<td>$-14.76$</td>
<td>2.63</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
</tbody>
</table>

$^a$ Ritchey-Chretien Focus Spectrograph
$^b$ BL 380, 120 line/mm grating, used in 2nd order
$^c$ KPC 18C, 790 line/mm grating, used in 2nd order

Tab. B.16: Radial Velocities of the target SDSS J093334.14+074812.6. Absolute radial velocities were measured relative to the m2t4500R7900 synthetic stellar template. These data are formatted in the same way as Table B.1.

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$</th>
<th>$v_{\text{rad}}$</th>
<th>$\sigma_{v_{\text{rad}}}$</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2455827$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>503.66</td>
<td>2.74</td>
<td>1.48</td>
<td>$-10.95$</td>
<td>3.45</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>504.64</td>
<td>2.48</td>
<td>1.75</td>
<td>$-11.22$</td>
<td>3.57</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>504.66</td>
<td>4.38</td>
<td>2.64</td>
<td>$-9.31$</td>
<td>4.09</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>504.66</td>
<td>6.52</td>
<td>1.94</td>
<td>$-7.18$</td>
<td>3.67</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>573.74</td>
<td>$-2.01$</td>
<td>3.48</td>
<td>$-15.71$</td>
<td>4.67</td>
<td>RC$^a$</td>
<td>BL 380$^b$</td>
</tr>
<tr>
<td>925.7</td>
<td>$-0.58$</td>
<td>4.88</td>
<td>$-14.28$</td>
<td>5.79</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>1180.62</td>
<td>4.37</td>
<td>3.26</td>
<td>$-9.33$</td>
<td>4.51</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1184.90</td>
<td>2.05</td>
<td>15.00</td>
<td>$-11.65$</td>
<td>15.33</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>1186.89</td>
<td>$-4.24$</td>
<td>13.10</td>
<td>$-17.94$</td>
<td>13.47</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>1547.12</td>
<td>$-26.14$</td>
<td>5.03</td>
<td>$-39.83$</td>
<td>5.92</td>
<td>KOSMOS</td>
<td>Blue VPH</td>
</tr>
<tr>
<td>1548.12</td>
<td>$-22.76$</td>
<td>5.31</td>
<td>$-36.45$</td>
<td>6.16</td>
<td>KOSMOS</td>
<td>Blue VPH</td>
</tr>
</tbody>
</table>

$^a$ Ritchey-Chretien Focus Spectrograph
$^b$ BL 380, 120 line/mm grating, used in 2nd order
$^c$ KPC 18C, 790 line/mm grating, used in 2nd order
**Tab. B.17.** Radial Velocities of the target SDSS J095545.84+443640.4. Absolute radial velocities were measured relative to the m2t4500R7900 synthetic stellar template. These data are formatted in the same way as Table B.1

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2455827</td>
<td>503.68</td>
<td>−6.00</td>
<td>3.59</td>
<td>0.67</td>
<td>7.70</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>504.69</td>
<td>−2.05</td>
<td>1.50</td>
<td>4.61</td>
<td>6.97</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>504.69</td>
<td>−1.46</td>
<td>5.59</td>
<td>5.2</td>
<td>8.81</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>504.70</td>
<td>−6.61</td>
<td>4.17</td>
<td>0.05</td>
<td>7.98</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>1180.6</td>
<td>−27.47</td>
<td>9.45</td>
<td>−20.8</td>
<td>11.64</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>1547.16</td>
<td>−11.77</td>
<td>16.66</td>
<td>−5.11</td>
<td>18.00</td>
<td>KOSMOS Blue VPH</td>
</tr>
<tr>
<td></td>
<td>1548.14</td>
<td>−33.44</td>
<td>9.08</td>
<td>−26.77</td>
<td>11.35</td>
<td>KOSMOS Blue VPH</td>
</tr>
<tr>
<td></td>
<td>1564.50</td>
<td>−2.02</td>
<td>5.63</td>
<td>4.64</td>
<td>8.83</td>
<td>ISIS blue arm</td>
</tr>
</tbody>
</table>

**Tab. B.18.** Radial Velocities of the target SDSS J101548.90+094649.7. Absolute radial velocities were measured relative to the m0t3750R6700 synthetic stellar template. These data are formatted in the same way as Table B.1

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2455827</td>
<td>510.46</td>
<td>−0.47</td>
<td>44.96</td>
<td>5.99</td>
<td>45.19</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>898.53</td>
<td>12.37</td>
<td>6.71</td>
<td>18.83</td>
<td>8.13</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>2714.43</td>
<td>4.16</td>
<td>7.02</td>
<td>10.62</td>
<td>8.39</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>2714.52</td>
<td>6.11</td>
<td>4.85</td>
<td>12.58</td>
<td>6.67</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>2714.70</td>
<td>−2.78</td>
<td>1.69</td>
<td>3.68</td>
<td>4.89</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>2715.37</td>
<td>4.62</td>
<td>8.33</td>
<td>11.08</td>
<td>9.51</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>2715.50</td>
<td>−6.67</td>
<td>7.03</td>
<td>−0.21</td>
<td>8.39</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>2715.67</td>
<td>−13.67</td>
<td>7.41</td>
<td>−7.20</td>
<td>8.72</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>2759.42</td>
<td>9.65</td>
<td>14.86</td>
<td>16.11</td>
<td>15.55</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>2759.55</td>
<td>−22.11</td>
<td>11.61</td>
<td>−15.65</td>
<td>12.48</td>
<td>ISIS blue arm</td>
</tr>
<tr>
<td></td>
<td>3058.47</td>
<td>8.40</td>
<td>6.43</td>
<td>14.86</td>
<td>7.90</td>
<td>ISIS red arm</td>
</tr>
<tr>
<td></td>
<td>3058.61</td>
<td>35.56</td>
<td>7.08</td>
<td>39.02</td>
<td>8.44</td>
<td>ISIS red arm</td>
</tr>
<tr>
<td></td>
<td>3058.62</td>
<td>54.66</td>
<td>13.18</td>
<td>61.12</td>
<td>13.95</td>
<td>ISIS red arm</td>
</tr>
<tr>
<td></td>
<td>3059.51</td>
<td>−4.35</td>
<td>1.48</td>
<td>2.11</td>
<td>4.82</td>
<td>ISIS red arm</td>
</tr>
<tr>
<td></td>
<td>3059.52</td>
<td>−26.12</td>
<td>6.77</td>
<td>−19.66</td>
<td>8.18</td>
<td>ISIS red arm</td>
</tr>
<tr>
<td></td>
<td>3060.45</td>
<td>10.42</td>
<td>10.95</td>
<td>29.51</td>
<td>−0.75</td>
<td>ISIS red arm</td>
</tr>
<tr>
<td></td>
<td>3060.54</td>
<td>−45.32</td>
<td>8.79</td>
<td>−38.86</td>
<td>9.92</td>
<td>ISIS red arm</td>
</tr>
<tr>
<td></td>
<td>3060.67</td>
<td>23.00</td>
<td>12.71</td>
<td>29.47</td>
<td>13.51</td>
<td>ISIS red arm</td>
</tr>
<tr>
<td></td>
<td>3060.68</td>
<td>21.80</td>
<td>6.33</td>
<td>28.26</td>
<td>7.82</td>
<td>ISIS red arm</td>
</tr>
</tbody>
</table>
Tab. B.19.: Radial Velocities of the target CLS 29. Absolute radial velocities were measured relative to the m04000R6700 synthetic stellar template. These data are formatted in the same way as Table B.1

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2455827$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>505.51</td>
<td>82.73</td>
<td>1.56</td>
<td>155.74</td>
<td>3.75</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>510.49</td>
<td>-10.61</td>
<td>1.18</td>
<td>62.40</td>
<td>3.61</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>573.78</td>
<td>-23.21</td>
<td>2.41</td>
<td>49.80</td>
<td>4.18</td>
<td>RC$^a$</td>
<td>BL 380</td>
</tr>
<tr>
<td>898.50</td>
<td>26.75</td>
<td>1.50</td>
<td>99.76</td>
<td>3.73</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>925.73</td>
<td>7.98</td>
<td>6.81</td>
<td>81.00</td>
<td>7.62</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>1180.67</td>
<td>6.35</td>
<td>1.34</td>
<td>79.36</td>
<td>3.66</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1184.92</td>
<td>25.19</td>
<td>4.84</td>
<td>98.20</td>
<td>5.92</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>1186.96</td>
<td>4.99</td>
<td>4.97</td>
<td>78.01</td>
<td>6.03</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>1547.21</td>
<td>68.29</td>
<td>11.71</td>
<td>141.31</td>
<td>12.2</td>
<td>KOSMOS Blue VPH</td>
<td></td>
</tr>
<tr>
<td>1548.18</td>
<td>104.2</td>
<td>4.61</td>
<td>177.22</td>
<td>5.73</td>
<td>KOSMOS Blue VPH</td>
<td></td>
</tr>
<tr>
<td>1564.54</td>
<td>19.51</td>
<td>1.09</td>
<td>92.53</td>
<td>3.58</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2714.72</td>
<td>81.41</td>
<td>1.91</td>
<td>154.43</td>
<td>3.91</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2715.57</td>
<td>124.61</td>
<td>2.64</td>
<td>197.63</td>
<td>4.31</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2759.50</td>
<td>127.98</td>
<td>2.28</td>
<td>201.00</td>
<td>4.10</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2759.58</td>
<td>127.71</td>
<td>2.46</td>
<td>200.73</td>
<td>4.21</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2759.63</td>
<td>125.85</td>
<td>2.42</td>
<td>198.87</td>
<td>4.18</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>3056.43</td>
<td>-17.53</td>
<td>1.74</td>
<td>55.48</td>
<td>3.83</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3056.56</td>
<td>-3.98</td>
<td>1.78</td>
<td>69.04</td>
<td>3.85</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3056.56</td>
<td>-17.74</td>
<td>1.89</td>
<td>55.28</td>
<td>3.90</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3057.45</td>
<td>31.93</td>
<td>2.28</td>
<td>104.94</td>
<td>4.10</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3057.57</td>
<td>51.65</td>
<td>2.21</td>
<td>124.66</td>
<td>4.06</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3057.64</td>
<td>65.14</td>
<td>2.03</td>
<td>138.45</td>
<td>3.97</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3057.73</td>
<td>77.23</td>
<td>2.13</td>
<td>150.58</td>
<td>4.02</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3058.45</td>
<td>133.55</td>
<td>1.72</td>
<td>206.56</td>
<td>3.82</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3058.66</td>
<td>122.37</td>
<td>1.98</td>
<td>195.34</td>
<td>3.95</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3059.53</td>
<td>0.45</td>
<td>1.43</td>
<td>73.55</td>
<td>3.70</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3059.63</td>
<td>-9.65</td>
<td>1.43</td>
<td>63.37</td>
<td>3.70</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3059.72</td>
<td>-17.37</td>
<td>2.09</td>
<td>55.64</td>
<td>4.00</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3059.72</td>
<td>-17.12</td>
<td>1.81</td>
<td>65.90</td>
<td>3.86</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3060.41</td>
<td>4.47</td>
<td>1.98</td>
<td>77.49</td>
<td>3.94</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3060.76</td>
<td>58.00</td>
<td>1.84</td>
<td>131.02</td>
<td>3.87</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3060.77</td>
<td>59.71</td>
<td>1.91</td>
<td>132.72</td>
<td>3.91</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3061.42</td>
<td>132.08</td>
<td>1.74</td>
<td>205.10</td>
<td>3.83</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3061.60</td>
<td>131.50</td>
<td>4.62</td>
<td>204.51</td>
<td>5.74</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3061.77</td>
<td>126.51</td>
<td>1.66</td>
<td>199.53</td>
<td>3.79</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
</tbody>
</table>

$^a$ Ritchey-Chretien Focus Spectrograph

$^b$ BL 380, 120 line/mm grating, used in 2nd order

$^c$ KPC 18C, 790 line/mm grating, used in 2nd order
Tab. B.20.: Radial Velocities of the target CLS 31. Absolute radial velocities were measured relative to the m1t3500R7900 synthetic stellar template. These data are formatted in the same way as Table B.1

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2455827</td>
<td>−44.24</td>
<td>5.20</td>
<td>−74.85</td>
<td>9.24</td>
<td>RC$^a$</td>
<td>BL 380$^b$</td>
</tr>
<tr>
<td>573.83</td>
<td>18.88</td>
<td>9.66</td>
<td>28.56</td>
<td>12.32</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>926.80</td>
<td>59.18</td>
<td>15.28</td>
<td>−15.34</td>
<td>10.72</td>
<td>KOSMOS</td>
<td>Blue VPH</td>
</tr>
<tr>
<td>1186.04</td>
<td>−19.60</td>
<td>8.48</td>
<td>−50.22</td>
<td>9.62</td>
<td>KOSMOS</td>
<td>Blue VPH</td>
</tr>
</tbody>
</table>

$^a$ Ritchey-Chretien Focus Spectrograph  
$^b$ BL 380, 120 line/mm grating, used in 2$^{nd}$ order  
$^c$ KPC 18C, 790 line/mm grating, used in 2$^{nd}$ order

Tab. B.21.: Radial Velocities of the target SDSS J110458.97+274311.8. Absolute radial velocities were measured relative to the m2t4500R7900 synthetic stellar template. These data are formatted in the same way as Table B.1

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2455827</td>
<td>8.09</td>
<td>1.48</td>
<td>60.51</td>
<td>2.73</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>505.54</td>
<td>8.77</td>
<td>2.94</td>
<td>61.19</td>
<td>3.73</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>510.52</td>
<td>13.64</td>
<td>3.92</td>
<td>66.06</td>
<td>4.54</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>925.74</td>
<td>20.66</td>
<td>2.03</td>
<td>73.09</td>
<td>3.07</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
</tbody>
</table>

Tab. B.22.: Radial Velocities of the target KA 2. Absolute radial velocities were measured relative to the m1t4000R7900 synthetic stellar template. These data are formatted in the same way as Table B.1

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2455827</td>
<td>17.29</td>
<td>1.50</td>
<td>38.43</td>
<td>3.42</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>505.63</td>
<td>5.29</td>
<td>14.40</td>
<td>15.86</td>
<td>14.73</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>925.74</td>
<td>12.34</td>
<td>10.73</td>
<td>33.49</td>
<td>11.16</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>1184.95</td>
<td>8.11</td>
<td>4.40</td>
<td>29.26</td>
<td>5.37</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>1187.00</td>
<td>9.10</td>
<td>4.10</td>
<td>30.24</td>
<td>5.13</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>2714.54</td>
<td>16.01</td>
<td>1.68</td>
<td>37.16</td>
<td>3.51</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2715.56</td>
<td>12.78</td>
<td>1.61</td>
<td>33.92</td>
<td>3.48</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2759.45</td>
<td>21.76</td>
<td>1.67</td>
<td>42.91</td>
<td>3.50</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2759.51</td>
<td>16.36</td>
<td>2.19</td>
<td>37.50</td>
<td>3.78</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2759.56</td>
<td>16.98</td>
<td>1.88</td>
<td>38.13</td>
<td>3.61</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>3057.67</td>
<td>21.55</td>
<td>3.38</td>
<td>42.70</td>
<td>4.57</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3057.67</td>
<td>18.84</td>
<td>3.63</td>
<td>39.98</td>
<td>4.76</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3060.61</td>
<td>18.19</td>
<td>3.21</td>
<td>39.34</td>
<td>4.45</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3060.61</td>
<td>18.22</td>
<td>1.09</td>
<td>39.37</td>
<td>3.27</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3060.72</td>
<td>22.03</td>
<td>2.33</td>
<td>43.18</td>
<td>3.86</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
</tbody>
</table>

$^a$ Ritchey-Chretien Focus Spectrograph  
$^c$ KPC 18C, 790 line/mm grating, used in 2$^{nd}$ order
Tab. B.23.: Radial Velocities of the target SDSS J112633.94+044137.7. Absolute radial velocities were measured relative to the m1t4500R7900 synthetic stellar template. These data are formatted in the same way as Table B.1

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2455827</td>
<td>15.38</td>
<td>1.48</td>
<td>−6.52</td>
<td>2.72</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>505.65</td>
<td>10.98</td>
<td>1.48</td>
<td>−6.52</td>
<td>2.72</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>925.76</td>
<td>6.12</td>
<td>1.48</td>
<td>−6.52</td>
<td>2.72</td>
<td>KPC 18Cc</td>
<td></td>
</tr>
<tr>
<td>1184.98</td>
<td>3.86</td>
<td>1.48</td>
<td>−6.52</td>
<td>2.72</td>
<td>KPC 18Cc</td>
<td></td>
</tr>
<tr>
<td>1548.22</td>
<td>−0.47</td>
<td>3.86</td>
<td>−22.37</td>
<td>4.48</td>
<td>KOSMOS Blue VPH</td>
<td></td>
</tr>
</tbody>
</table>

*a* Ritchey-Chretien Focus Spectrograph  
*c* KPC 18C, 790 line/mm grating, used in 2nd order

Tab. B.24.: Radial Velocities of the target SDSS J120024.09+381720.3. Absolute radial velocities were measured relative to the m2t4000R7900 synthetic stellar template. These data are formatted in the same way as Table B.1

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2455827</td>
<td>10.34</td>
<td>1.48</td>
<td>60.23</td>
<td>7.20</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>503.71</td>
<td>5.87</td>
<td>2.18</td>
<td>55.77</td>
<td>7.38</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>504.72</td>
<td>7.83</td>
<td>1.58</td>
<td>57.73</td>
<td>7.23</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1548.24</td>
<td>−8.71</td>
<td>9.77</td>
<td>41.19</td>
<td>12.05</td>
<td>KOSMOS Blue VPH</td>
<td></td>
</tr>
<tr>
<td>1564.58</td>
<td>20.48</td>
<td>1.24</td>
<td>70.38</td>
<td>7.16</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2715.52</td>
<td>30.77</td>
<td>1.28</td>
<td>80.66</td>
<td>7.17</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
</tbody>
</table>
Tab. B.25.: Radial Velocities of the target CLS 50. Absolute radial velocities were measured relative to the m2t4500R7900 synthetic stellar template. These data are formatted in the same way as Table B.1

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2455827</td>
<td>503.74</td>
<td>−0.39</td>
<td>1.82</td>
<td>162.41</td>
<td>4.48</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>504.77</td>
<td>−3.68</td>
<td>2.33</td>
<td>159.13</td>
<td>4.71</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>505.68</td>
<td>−6.40</td>
<td>1.77</td>
<td>156.41</td>
<td>4.46</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>572.88</td>
<td>−1.62</td>
<td>1.93</td>
<td>161.18</td>
<td>4.52</td>
<td>RC$^a$ BL 380 $^b$</td>
</tr>
<tr>
<td></td>
<td>925.78</td>
<td>−2.84</td>
<td>2.95</td>
<td>159.97</td>
<td>5.04</td>
<td>RC$^a$ KPC 18C$^c$</td>
</tr>
<tr>
<td></td>
<td>1180.7</td>
<td>3.25</td>
<td>1.57</td>
<td>166.05</td>
<td>4.38</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>1185.01</td>
<td>4.56</td>
<td>6.40</td>
<td>167.37</td>
<td>7.59</td>
<td>RC$^a$ KPC 18C$^c$</td>
</tr>
<tr>
<td></td>
<td>1548.25</td>
<td>−15.33</td>
<td>4.63</td>
<td>147.47</td>
<td>6.18</td>
<td>KOSMOS Blue VPH</td>
</tr>
<tr>
<td></td>
<td>2714.74</td>
<td>4.25</td>
<td>1.48</td>
<td>167.05</td>
<td>4.35</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>2715.59</td>
<td>6.78</td>
<td>1.75</td>
<td>169.58</td>
<td>4.45</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>3057.51</td>
<td>11.91</td>
<td>3.21</td>
<td>174.72</td>
<td>5.20</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3057.59</td>
<td>12.98</td>
<td>2.62</td>
<td>175.79</td>
<td>4.86</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3057.6</td>
<td>12.11</td>
<td>2.43</td>
<td>174.91</td>
<td>4.76</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3057.75</td>
<td>15.06</td>
<td>2.46</td>
<td>177.86</td>
<td>4.77</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3058.52</td>
<td>10.68</td>
<td>3.52</td>
<td>173.48</td>
<td>5.39</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3058.52</td>
<td>13.5</td>
<td>1.97</td>
<td>176.30</td>
<td>4.54</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3058.76</td>
<td>12.78</td>
<td>1.09</td>
<td>175.58</td>
<td>4.23</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3059.61</td>
<td>12.85</td>
<td>2.34</td>
<td>175.66</td>
<td>4.71</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3059.76</td>
<td>14.89</td>
<td>2.23</td>
<td>177.69</td>
<td>4.66</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3060.51</td>
<td>10.75</td>
<td>1.87</td>
<td>173.55</td>
<td>4.50</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3060.78</td>
<td>13.15</td>
<td>2.57</td>
<td>175.96</td>
<td>4.83</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3061.53</td>
<td>10.65</td>
<td>2.57</td>
<td>173.45</td>
<td>4.83</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3061.78</td>
<td>13.72</td>
<td>2.83</td>
<td>176.52</td>
<td>4.97</td>
<td>ISIS red arm R1200R</td>
</tr>
</tbody>
</table>

$^a$ Ritchey-Chretien Focus Spectrograph
$^b$ BL 380, 120 line/mm grating, used in 2$^{nd}$ order
$^c$ KPC 18C, 790 line/mm grating, used in 2$^{nd}$ order

Tab. B.26.: Radial Velocities of the target SDSS J122328+353251.9. Absolute radial velocities were measured relative to the m2t4500R7900 synthetic stellar template. These data are formatted in the same way as Table B.1

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2455827</td>
<td>503.76</td>
<td>12.63</td>
<td>1.48</td>
<td>66.30</td>
<td>5.18</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>505.71</td>
<td>4.25</td>
<td>1.48</td>
<td>167.05</td>
<td>4.35</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>506.75</td>
<td>3.94</td>
<td>1.90</td>
<td>57.60</td>
<td>5.31</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>1180.74</td>
<td>10.44</td>
<td>2.03</td>
<td>64.11</td>
<td>5.36</td>
<td>ISIS blue arm R1200B</td>
</tr>
</tbody>
</table>

Tab. B.27.: Radial Velocities of the target SDSS J130744.53+600903.7. Absolute radial velocities were measured relative to the m2t4500R7900 synthetic stellar template. These data are formatted in the same way as Table B.1

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2455827</td>
<td>503.80</td>
<td>1.12</td>
<td>11.34</td>
<td>43.34</td>
<td>12.02</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>504.75</td>
<td>4.59</td>
<td>1.48</td>
<td>46.82</td>
<td>4.25</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>2155.38</td>
<td>3.96</td>
<td>6.15</td>
<td>46.19</td>
<td>7.33</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>2156.38</td>
<td>6.68</td>
<td>3.46</td>
<td>48.91</td>
<td>5.28</td>
<td>ISIS blue arm R1200B</td>
</tr>
</tbody>
</table>
Tab. B.28.: Radial Velocities of the target SBSS 1310+561. Absolute radial velocities were measured relative to the m0t3500R6700 synthetic stellar template. These data are formatted in the same way as Table B.1

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>505.75</td>
<td>7.34</td>
<td>1.43</td>
<td>-29.10</td>
<td>3.87</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>510.77</td>
<td>4.45</td>
<td>1.65</td>
<td>-26.21</td>
<td>3.95</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>572.83</td>
<td>1.04</td>
<td>1.54</td>
<td>-22.79</td>
<td>3.83</td>
<td>RC$^a$</td>
<td>BL 380$^b$</td>
</tr>
<tr>
<td>573.93</td>
<td>8.11</td>
<td>1.52</td>
<td>-79.86</td>
<td>3.90</td>
<td>RC$^a$</td>
<td>BL 380$^b$</td>
</tr>
<tr>
<td>688.37</td>
<td>7.09</td>
<td>1.57</td>
<td>-28.85</td>
<td>3.92</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>925.80</td>
<td>77.72</td>
<td>3.88</td>
<td>-99.47</td>
<td>5.29</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>1185.04</td>
<td>12.89</td>
<td>3.78</td>
<td>-34.65</td>
<td>5.22</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>1187.04</td>
<td>100.55</td>
<td>4.43</td>
<td>-122.30</td>
<td>5.70</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>2155.37</td>
<td>9.03</td>
<td>1.53</td>
<td>-30.78</td>
<td>3.91</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2156.36</td>
<td>12.55</td>
<td>1.78</td>
<td>-34.30</td>
<td>4.01</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2714.64</td>
<td>1.69</td>
<td>1.52</td>
<td>-20.06</td>
<td>3.90</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2715.54</td>
<td>40.40</td>
<td>1.77</td>
<td>-62.15</td>
<td>4.01</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2715.61</td>
<td>43.23</td>
<td>1.87</td>
<td>-64.99</td>
<td>4.05</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2715.71</td>
<td>49.54</td>
<td>1.63</td>
<td>-71.29</td>
<td>3.95</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2759.59</td>
<td>24.55</td>
<td>1.69</td>
<td>-46.30</td>
<td>3.97</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2759.67</td>
<td>27.21</td>
<td>1.88</td>
<td>-48.97</td>
<td>4.06</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2759.74</td>
<td>29.88</td>
<td>2.10</td>
<td>-51.64</td>
<td>4.16</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>3056.51</td>
<td>12.74</td>
<td>1.85</td>
<td>-34.49</td>
<td>4.04</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3056.62</td>
<td>19.31</td>
<td>2.09</td>
<td>-41.06</td>
<td>4.15</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3057.54</td>
<td>73.15</td>
<td>1.55</td>
<td>-94.90</td>
<td>3.91</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3057.62</td>
<td>79.13</td>
<td>1.55</td>
<td>-100.88</td>
<td>3.91</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3057.62</td>
<td>77.95</td>
<td>1.48</td>
<td>-99.71</td>
<td>3.89</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3057.77</td>
<td>83.72</td>
<td>1.61</td>
<td>-105.47</td>
<td>3.94</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3058.54</td>
<td>91.38</td>
<td>1.51</td>
<td>-112.87</td>
<td>3.90</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3058.78</td>
<td>81.40</td>
<td>1.57</td>
<td>-103.15</td>
<td>3.92</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3059.59</td>
<td>30.65</td>
<td>2.00</td>
<td>-52.40</td>
<td>4.11</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3059.78</td>
<td>22.52</td>
<td>2.67</td>
<td>-44.28</td>
<td>4.48</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3060.57</td>
<td>3.08</td>
<td>2.04</td>
<td>-24.83</td>
<td>4.13</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3060.57</td>
<td>2.04</td>
<td>1.86</td>
<td>-23.80</td>
<td>4.05</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3060.79</td>
<td>5.75</td>
<td>2.00</td>
<td>-27.51</td>
<td>4.11</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3060.79</td>
<td>6.00</td>
<td>3.38</td>
<td>-27.76</td>
<td>4.93</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3061.62</td>
<td>52.56</td>
<td>1.40</td>
<td>-74.32</td>
<td>3.86</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3061.62</td>
<td>51.80</td>
<td>1.47</td>
<td>-73.56</td>
<td>3.88</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
<tr>
<td>3061.79</td>
<td>63.55</td>
<td>1.58</td>
<td>-85.31</td>
<td>3.92</td>
<td>ISIS red arm</td>
<td>R1200R</td>
</tr>
</tbody>
</table>

$^a$ Ritchey-Chretien Focus Spectrograph
$^b$ BL 380, 120 line/mm grating, used in 2nd order
$^c$ KPC 18C, 790 line/mm grating, used in 2nd order
Tab. B.29.: Radial Velocities of the target SDSS J145725.86+234125.4. Absolute radial velocities were measured relative to the m2t3500R7900 synthetic stellar template. These data are formatted in the same way as Table B.1.

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2455827$</td>
<td>687.54</td>
<td>-24.63</td>
<td>1.66</td>
<td>122.49</td>
<td>4.93</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>688.49</td>
<td>-22.84</td>
<td>1.50</td>
<td>124.29</td>
<td>4.88</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>1449.34</td>
<td>-18.94</td>
<td>3.52</td>
<td>128.19</td>
<td>5.83</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>2155.40</td>
<td>-20.09</td>
<td>3.12</td>
<td>127.03</td>
<td>5.60</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>2156.40</td>
<td>-19.61</td>
<td>3.27</td>
<td>127.52</td>
<td>5.68</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>2714.76</td>
<td>-20.27</td>
<td>3.44</td>
<td>126.85</td>
<td>5.78</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>2759.65</td>
<td>-20.50</td>
<td>2.43</td>
<td>126.62</td>
<td>5.24</td>
<td>ISIS blue arm R1200B</td>
</tr>
</tbody>
</table>

Tab. B.30.: Radial Velocities of the target CBS 311. Absolute radial velocities were measured relative to the m2t3750R6700 synthetic stellar template. These data are formatted in the same way as Table B.1.

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2455827$</td>
<td>687.46</td>
<td>-4.78</td>
<td>13.67</td>
<td>93.48</td>
<td>20.44</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>688.42</td>
<td>-11.42</td>
<td>1.77</td>
<td>86.84</td>
<td>15.30</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>2155.42</td>
<td>42.12</td>
<td>22.31</td>
<td>140.38</td>
<td>26.99</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>2156.42</td>
<td>12.62</td>
<td>29.57</td>
<td>110.88</td>
<td>33.24</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>2181.38</td>
<td>54.78</td>
<td>12.46</td>
<td>153.04</td>
<td>19.65</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>2714.62</td>
<td>27.11</td>
<td>30.18</td>
<td>125.37</td>
<td>33.79</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>2715.69</td>
<td>1.91</td>
<td>21.89</td>
<td>100.17</td>
<td>26.64</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>2715.77</td>
<td>24.68</td>
<td>54.82</td>
<td>122.94</td>
<td>56.89</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>2759.61</td>
<td>32.68</td>
<td>18.03</td>
<td>130.94</td>
<td>23.58</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>2759.73</td>
<td>23.52</td>
<td>31.51</td>
<td>121.78</td>
<td>34.98</td>
<td>ISIS blue arm R1200B</td>
</tr>
<tr>
<td></td>
<td>3058.64</td>
<td>-18.49</td>
<td>1.81</td>
<td>79.76</td>
<td>15.30</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3058.73</td>
<td>38.79</td>
<td>1.53</td>
<td>137.05</td>
<td>15.27</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3059.70</td>
<td>29.45</td>
<td>2.55</td>
<td>127.71</td>
<td>15.41</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3060.62</td>
<td>56.71</td>
<td>3.99</td>
<td>154.97</td>
<td>15.71</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3060.63</td>
<td>-15.85</td>
<td>6.56</td>
<td>82.356</td>
<td>16.55</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3061.63</td>
<td>-5.44</td>
<td>5.41</td>
<td>92.82</td>
<td>16.13</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3061.74</td>
<td>8.43</td>
<td>29.17</td>
<td>133.54</td>
<td>15.37</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3061.75</td>
<td>2.69</td>
<td>29.95</td>
<td>127.8</td>
<td>15.51</td>
<td>ISIS red arm R1200R</td>
</tr>
</tbody>
</table>

Tab. B.31.: Radial Velocities of the target SDSS J154859.72+341821.7. Absolute radial velocities were measured relative to the m2t3500R7900 synthetic stellar template. These data are formatted in the same way as Table B.1.

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2455827$</td>
<td>3058.71</td>
<td>18.47</td>
<td>1.09</td>
<td>60.39</td>
<td>6.54</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3059.67</td>
<td>138.88</td>
<td>2.92</td>
<td>180.80</td>
<td>7.08</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3059.68</td>
<td>135.3</td>
<td>2.01</td>
<td>177.23</td>
<td>6.75</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3059.74</td>
<td>133.21</td>
<td>2.95</td>
<td>175.13</td>
<td>7.09</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3060.64</td>
<td>-7.15</td>
<td>3.61</td>
<td>34.77</td>
<td>7.39</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3060.74</td>
<td>-9.30</td>
<td>2.58</td>
<td>32.62</td>
<td>6.95</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3060.74</td>
<td>-12.67</td>
<td>2.33</td>
<td>29.26</td>
<td>6.86</td>
<td>ISIS red arm R1200R</td>
</tr>
<tr>
<td></td>
<td>3061.72</td>
<td>52.93</td>
<td>2.81</td>
<td>94.85</td>
<td>7.03</td>
<td>ISIS red arm R1200R</td>
</tr>
</tbody>
</table>
Tab. B.32.: Radial Velocities of the target CLS 96. Absolute radial velocities were measured relative to the m2t4000R7900 synthetic stellar template. These data are formatted in the same way as Table B.1

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>573.95</td>
<td>-20.80</td>
<td>2.65</td>
<td>-234.72</td>
<td>12.43</td>
<td>RC$^a$</td>
<td>BL 380$^b$</td>
</tr>
<tr>
<td>687.49</td>
<td>-22.66</td>
<td>1.37</td>
<td>-236.58</td>
<td>12.22</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>688.44</td>
<td>-19.82</td>
<td>1.48</td>
<td>-233.74</td>
<td>12.23</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1011.57</td>
<td>-11.70</td>
<td>2.16</td>
<td>-225.62</td>
<td>12.33</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2155.44</td>
<td>-14.31</td>
<td>1.78</td>
<td>-228.23</td>
<td>12.27</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2156.44</td>
<td>-12.37</td>
<td>1.48</td>
<td>-226.29</td>
<td>12.23</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2514.50</td>
<td>-15.23</td>
<td>1.74</td>
<td>-229.15</td>
<td>12.26</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2715.74</td>
<td>-20.78</td>
<td>1.68</td>
<td>-234.70</td>
<td>12.26</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
</tbody>
</table>

$^a$ Ritchey-Chretien Focus Spectrograph  
$^b$ BL 380, 120 line/mm grating, used in 2nd order

Tab. B.33.: Radial Velocities of the target LP 225-12. Absolute radial velocities were measured relative to the m2t3750R7900 synthetic stellar template. These data are formatted in the same way as Table B.1

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>573.98</td>
<td>-9.92</td>
<td>5.58</td>
<td>40.63</td>
<td>10.78</td>
<td>RC$^a$</td>
<td>BL 380$^b$</td>
</tr>
<tr>
<td>687.52</td>
<td>-13.78</td>
<td>1.28</td>
<td>36.77</td>
<td>9.31</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>688.47</td>
<td>-13.75</td>
<td>1.75</td>
<td>36.79</td>
<td>9.39</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1449.38</td>
<td>0.41</td>
<td>3.50</td>
<td>50.95</td>
<td>9.87</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2155.46</td>
<td>-6.31</td>
<td>3.63</td>
<td>44.24</td>
<td>9.91</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2156.46</td>
<td>-6.33</td>
<td>3.29</td>
<td>44.22</td>
<td>9.80</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2181.41</td>
<td>-7.69</td>
<td>2.75</td>
<td>42.86</td>
<td>9.63</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2514.51</td>
<td>2.78</td>
<td>3.73</td>
<td>53.32</td>
<td>9.95</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2715.76</td>
<td>2.85</td>
<td>3.82</td>
<td>53.40</td>
<td>9.98</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
</tbody>
</table>

$^a$ Ritchey-Chretien Focus Spectrograph  
$^b$ BL 380, 120 line/mm grating, used in 2nd order

Tab. B.34.: Radial Velocities of the target SDSS J184735.67+405944.1. Absolute radial velocities were measured relative to the m2t3500R7900 synthetic stellar template. These data are formatted in the same way as Table B.1

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>687.54</td>
<td>4.75</td>
<td>2.31</td>
<td>48.24</td>
<td>3.63</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>688.49</td>
<td>-6.68</td>
<td>1.64</td>
<td>36.81</td>
<td>3.25</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1126.33</td>
<td>6.87</td>
<td>2.89</td>
<td>50.36</td>
<td>4.03</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1449.40</td>
<td>-3.53</td>
<td>3.01</td>
<td>39.96</td>
<td>4.11</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1449.55</td>
<td>4.89</td>
<td>2.98</td>
<td>48.37</td>
<td>4.09</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2155.48</td>
<td>-12.45</td>
<td>3.05</td>
<td>31.03</td>
<td>4.14</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2156.48</td>
<td>-19.15</td>
<td>4.19</td>
<td>24.34</td>
<td>5.04</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2181.45</td>
<td>5.49</td>
<td>2.36</td>
<td>48.97</td>
<td>3.66</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2513.56</td>
<td>-14.69</td>
<td>2.57</td>
<td>28.80</td>
<td>3.80</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2514.53</td>
<td>-4.58</td>
<td>2.71</td>
<td>38.91</td>
<td>3.90</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2515.53</td>
<td>2.93</td>
<td>9.88</td>
<td>46.42</td>
<td>10.27</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2759.71</td>
<td>-6.11</td>
<td>3.49</td>
<td>37.38</td>
<td>4.48</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
</tbody>
</table>
Tab. B.35.: Radial Velocities of the target LSR 2105+2514. Absolute radial velocities were measured relative to the m2t3750R7900 synthetic stellar template. These data are formatted in the same way as Table B.1. The observation at JD2458342.57 was omitted from any analysis in this thesis due to poor signal.

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>687.58</td>
<td>-31.62</td>
<td>2.13</td>
<td>131.97</td>
<td>8.94</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>688.52</td>
<td>3.75</td>
<td>1.48</td>
<td>167.33</td>
<td>8.81</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1126.36</td>
<td>-1.53</td>
<td>4.53</td>
<td>162.05</td>
<td>9.80</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1185.56</td>
<td>-13.58</td>
<td>7.13</td>
<td>150.0</td>
<td>11.23</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>1449.43</td>
<td>-125.99</td>
<td>3.69</td>
<td>37.60</td>
<td>9.43</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1449.57</td>
<td>-137.58</td>
<td>11.83</td>
<td>26.01</td>
<td>14.68</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1546.82</td>
<td>-53.75</td>
<td>3.60</td>
<td>109.83</td>
<td>9.40</td>
<td>KOSMOS Blue VPH</td>
<td></td>
</tr>
<tr>
<td>1547.33</td>
<td>3.94</td>
<td>2.78</td>
<td>167.53</td>
<td>9.12</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1547.80</td>
<td>-3.19</td>
<td>3.97</td>
<td>160.39</td>
<td>9.55</td>
<td>KOSMOS Blue VPH</td>
<td></td>
</tr>
<tr>
<td>2155.56</td>
<td>-118.05</td>
<td>2.24</td>
<td>45.53</td>
<td>8.97</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2156.52</td>
<td>-53.73</td>
<td>2.66</td>
<td>109.85</td>
<td>9.08</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2180.53</td>
<td>-28.83</td>
<td>2.58</td>
<td>134.75</td>
<td>9.06</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2513.60</td>
<td>-6.67</td>
<td>3.17</td>
<td>156.91</td>
<td>9.24</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2514.56</td>
<td>-60.48</td>
<td>2.19</td>
<td>103.11</td>
<td>8.96</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2515.57</td>
<td>140.74</td>
<td>10.55</td>
<td>304.32</td>
<td>13.67</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2759.69</td>
<td>-125.02</td>
<td>3.79</td>
<td>38.56</td>
<td>9.47</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
</tbody>
</table>

$^a$ Ritchey-Chretien Focus Spectrograph  
$^c$ KPC 18C, 790 line/mm grating, used in 2$^{nd}$ order

---

Tab. B.36.: Radial Velocities of the target LP 758-43. Absolute radial velocities were measured relative to the m1t3750R6700 synthetic stellar template. These data are formatted in the same way as Table B.1

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>687.61</td>
<td>3.10</td>
<td>1.61</td>
<td>107.32</td>
<td>4.15</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>688.56</td>
<td>3.47</td>
<td>1.48</td>
<td>107.70</td>
<td>4.11</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1011.68</td>
<td>4.05</td>
<td>7.31</td>
<td>108.27</td>
<td>8.26</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1126.38</td>
<td>0.47</td>
<td>1.68</td>
<td>104.69</td>
<td>4.18</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1185.61</td>
<td>5.12</td>
<td>3.41</td>
<td>109.34</td>
<td>5.12</td>
<td>RC$^a$</td>
<td>KPC 18C$^c$</td>
</tr>
<tr>
<td>1449.45</td>
<td>-0.18</td>
<td>1.92</td>
<td>104.05</td>
<td>4.28</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2155.51</td>
<td>-3.98</td>
<td>1.45</td>
<td>100.25</td>
<td>4.09</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2156.51</td>
<td>-2.36</td>
<td>1.36</td>
<td>101.86</td>
<td>4.06</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2180.58</td>
<td>-4.95</td>
<td>1.56</td>
<td>99.27</td>
<td>4.13</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2513.59</td>
<td>3.16</td>
<td>1.88</td>
<td>107.38</td>
<td>4.26</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2515.61</td>
<td>4.53</td>
<td>12.07</td>
<td>108.75</td>
<td>12.67</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
</tbody>
</table>

$^a$ Ritchey-Chretien Focus Spectrograph  
$^c$ KPC 18C, 790 line/mm grating, used in 2$^{nd}$ order
Tab. B.37: Radial Velocities of the target SDSS J235443.13+362907.1. Absolute radial velocities were measured relative to the m2t4500R7900 synthetic stellar template. These data are formatted in the same way as Table B.1

<table>
<thead>
<tr>
<th>JD</th>
<th>$\Delta v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{\Delta v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>$v_{\text{rad}}$ (km s$^{-1}$)</th>
<th>$\sigma_{v_{\text{rad}}}$ (km s$^{-1}$)</th>
<th>Instrument</th>
<th>Grating</th>
</tr>
</thead>
<tbody>
<tr>
<td>1180.37</td>
<td>−21.34</td>
<td>3.51</td>
<td>164.58</td>
<td>5.85</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1449.47</td>
<td>−28.56</td>
<td>4.18</td>
<td>157.37</td>
<td>6.27</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1546.86</td>
<td>−39.85</td>
<td>3.19</td>
<td>146.07</td>
<td>5.66</td>
<td>KOSMOS Blue VPH</td>
<td></td>
</tr>
<tr>
<td>1547.38</td>
<td>−24.14</td>
<td>1.48</td>
<td>161.78</td>
<td>4.91</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>1547.84</td>
<td>−39.38</td>
<td>4.15</td>
<td>146.54</td>
<td>6.25</td>
<td>KOSMOS Blue VPH</td>
<td></td>
</tr>
<tr>
<td>2155.53</td>
<td>−25.94</td>
<td>2.12</td>
<td>159.98</td>
<td>5.14</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2156.55</td>
<td>−25.81</td>
<td>1.83</td>
<td>160.11</td>
<td>5.02</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2180.62</td>
<td>−23.35</td>
<td>2.61</td>
<td>162.57</td>
<td>5.36</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
<tr>
<td>2514.60</td>
<td>−24.81</td>
<td>1.90</td>
<td>161.11</td>
<td>5.05</td>
<td>ISIS blue arm</td>
<td>R1200B</td>
</tr>
</tbody>
</table>
Appendix: Posterior probability distribution functions determined by the JOKER for the dC star sample

C.1 Circular orbits

In the initial analysis using the JOKER, the eccentricity was fixed to zero meaning that only circular orbits were considered. Reducing the number of free parameters (by fixing eccentricity) results in a reduction in the computational time of the JOKER of around 15 per cent. Furthermore, this initial analysis was used to identify potentially interesting binary systems.

The posterior probability distributions for the orbital period, velocity semi-amplitude, and systemic velocity are presented in Figure C.1. All systemic velocities are derived from the absolute velocities listed in Appendix B.
Fig. C.1: The posterior probability distribution functions are shown for the orbital period, velocity semi-amplitude, and systemic velocity for all 37 dC stars observed during the radial velocity survey, assuming that all orbits are circular. The name of each target is given for each subplot. While the analysis from the JOKER appears well constraining for the dC star SDSS J184735, it is actually a member of a triple system, and therefore, the orbital parameters reported here are likely incorrect. It is, however, included for completeness.
Fig. C.1. continued
**Fig. C.1.** continued
Fig. C.1. continued

C.1 Circular orbits
Fig. C.1. continued

Fig. C.1. continued
Fig. C.1. continued
$P \, [d] = 2.1023^{+0.0001}_{-0.0001}$

$SDSS \, J090128$

$K \, [\text{km s}^{-1}] = 23.64^{+0.99}_{-0.99}$

$\gamma \, [\text{km s}^{-1}] = 57.58^{+0.91}_{-0.90}$

$P \, [d] = 2.1023^{+0.0001}_{-0.0001}$

$SDSS \, J090302$

$K \, [\text{km s}^{-1}] = 21.27^{+18.79}_{-12.32}$

$\gamma \, [\text{km s}^{-1}] = 26.82^{+13.40}_{-12.00}$

Fig. C.1. continued
Fig. C.1. continued
Fig. C.1. continued

\[ P \text{[d]} = 1506.99^{+5164.09}_{-1416.03} \]

\[ K \text{[km s}^{-1}\text{]} = 4.77^{+5.50}_{-3.25} \]

\[ \gamma \text{[km s}^{-1}\text{]} = 16.20^{+3.63}_{-4.38} \]
Fig. C.1. continued

\[
P [d] = 0.203^{+0.001}_{-0.001}
\]

\[
SDSS J101548
\]

\[
K [\text{km s}^{-1}] = 18.61^{+2.52}_{-2.45}
\]

\[
\gamma [\text{km s}^{-1}] = -1.14^{+3.13}_{-3.45}
\]

Fig. C.1. continued

\[
P [d] = 3.147076^{+0.000009}_{-0.000009}
\]

\[
CLS 29
\]

\[
K [\text{km s}^{-1}] = 79.06^{+0.53}_{-0.54}
\]

\[
\gamma [\text{km s}^{-1}] = -19.86^{+0.65}_{-0.65}
\]

Fig. C.1. continued
Fig. C.1. continued

**CLS 31**

$P\ [d] = 7.68^{+16.26}_{-7.01}$

$K\ [\text{km s}^{-1}] = 62.66^{+24.88}_{-12.11}$

$\gamma\ [\text{km s}^{-1}] = 35.38^{+15.19}_{-15.54}$

**SDSS J110458**

$P\ [d] = 2762.32^{+3516.35}_{-2517.93}$

$K\ [\text{km s}^{-1}] = 10.83^{+6.57}_{-3.83}$

$\gamma\ [\text{km s}^{-1}] = -37.79^{+6.01}_{-4.64}$
Fig. C.1. continued
Fig. C.1. continued

[Graphs and data plots showing posterior probability distribution functions for SDSS J120024 and CLS 50]
\[ P \, [d] = 12.90^{+32.52}_{-11.75} \]

\[ K \, [\text{km s}^{-1}] = 19.03^{+31.53}_{-12.80} \]

\[ \gamma \, [\text{km s}^{-1}] = -46.12^{+15.45}_{-21.33} \]

\[ P \, [d] = 743.26^{+3981.36}_{-695.31} \]

\[ K \, [\text{km s}^{-1}] = 5.83^{+14.37}_{-4.37} \]

\[ \gamma \, [\text{km s}^{-1}] = -37.22^{+6.37}_{-4.29} \]

Fig. C.1. continued
**Fig. C.1.** continued

**SBSS 1310+561**

**K [km s\(^{-1}\)] = 46.73^{+0.56}_{-0.54}**

**γ [km s\(^{-1}\)] = -25.30^{+0.70}_{-0.68}**

**SDSS J145725**

**K [km s\(^{-1}\)] = 2.85^{+2.78}_{-1.71}**

**γ [km s\(^{-1}\)] = -168.99^{+1.70}_{-1.98}**
\begin{align*}
P \, [\text{d}] &= 0.187^{+0.002}_{-0.001} \\
K \, [\text{km s}^{-1}] &= 28.71^{+1.59}_{-1.57} \\
\gamma \, [\text{km s}^{-1}] &= -86.44^{+4.46}_{-6.24}
\end{align*}

Fig. C.1. continued

\begin{align*}
P \, [\text{d}] &= 2.816^{+0.021}_{-0.020} \\
K \, [\text{km s}^{-1}] &= 82.67^{+1.44}_{-1.42} \\
\gamma \, [\text{km s}^{-1}] &= 12.48^{+1.04}_{-1.01}
\end{align*}

Fig. C.1. continued
Fig. C.1. continued
\[ P [d] = 3.40^{+2.09}_{-2.89} \]

SDSS J184735

\[ K [\text{km s}^{-1}] = 11.82^{+1.66}_{-1.90} \]

\[ \gamma [\text{km s}^{-1}] = -48.04^{+1.48}_{-1.88} \]

\[ P [d] = 0.1548223^{+0.0000001}_{-0.0000001} \]

LSR J2105+2514

\[ K [\text{km s}^{-1}] = 65.77^{+1.21}_{-1.21} \]

\[ \gamma [\text{km s}^{-1}] = -215.86^{+1.14}_{-1.15} \]

Fig. C.1. continued
$P \ [d] = 675.63^{+2019.36}_{-417.85}$

$K \ [\text{km s}^{-1}] = 4.32^{+1.59}_{-1.02}$

$\gamma \ [\text{km s}^{-1}] = -104.15^{+1.41}_{-0.95}$

$P \ [d] = 4162.63^{+3934.39}_{-3995.26}$

$K \ [\text{km s}^{-1}] = 2.90^{+4.53}_{-2.08}$

$\gamma \ [\text{km s}^{-1}] = -210.55^{+2.67}_{-2.80}$

Fig. C.1. continued
C.2 Eccentric orbits

The JOKER analysis was repeated for all targets in the radial velocity survey allowing the eccentricity to vary. As with the analysis assuming circular orbits, systemic velocities reported here are measured from synthetic stellar templates, and are hence, the true systemic velocities of the targets. The posterior probability distributions for the orbital period, velocity semi-amplitude, eccentricity, and systemic velocity are displayed for each target in Figure C.2.
Fig. C.2: The posterior probability distribution functions are shown for the orbital period, velocity semi-amplitude, eccentricity, and systemic velocity for all 37 dC stars observed during the radial velocity survey. The name of each target is given for each subplot. While the analysis from the joker appears well constraining for the dC star SDSS J184735, it is actually a member of a triple system, and therefore, the orbital parameters reported here are likely incorrect. It is, however, included for completeness.
Fig. C.2. continued
\[ P[d] = 1060.34^{+5840.01}_{-1035.13} \]

\[ K \text{ [km s}\,^{-1}] = 2.44^{+4.43}_{-1.76} \]

\[ e = 0.14^{+0.26}_{-0.15} \]

\[ \gamma \text{ [km s}\,^{-1}] = 191.44^{+1.39}_{-1.99} \]

\[ P[d] = 246.88^{+0.29}_{-0.35} \]

\[ K \text{ [km s}\,^{-1}] = 31.26^{+5.96}_{-4.29} \]

\[ e = 0.26^{+0.07}_{-0.06} \]

\[ \gamma \text{ [km s}\,^{-1}] = -57.25^{+1.82}_{-2.25} \]
Fig. C.2. continued

C.2 Eccentric orbits
**Fig. C.2. continued**

**SDSS J081807**

\[
P [d] = 211.80^{+1091.24}_{-200.63}
\]

\[
K \text{ [km s}^{-1}] = 15.49^{+18.39}_{-8.62}
\]

\[
e = 0.19^{+0.25}_{-0.14}
\]

\[
\gamma \text{ [km s}^{-1}] = -24.76^{+8.39}_{-8.92}
\]

**PG 0842+288**

\[
P [d] = 778.01^{+5897.97}_{-732.62}
\]

\[
K \text{ [km s}^{-1}] = 4.10^{+5.24}_{-2.80}
\]

\[
e = 0.18^{+0.26}_{-0.14}
\]

\[
\gamma \text{ [km s}^{-1}] = -31.09^{+2.70}_{-3.13}
\]

**Fig. C.2. continued**

---

**Chapter C**

Appendix: Posterior probability distribution functions of the dC star sample
Fig. C.2. continued

\[ P [d] = 1578.28^{+3958.93}_{-1454.42} \]

SDSS J084259

\[ K \text{ [km s}^{-1}] = 8.79^{+10.57}_{-6.17} \]

\[ e = 0.18^{+0.25}_{-0.14} \]

\[ \gamma \text{ [km s}^{-1}] = 10.83^{+7.46}_{-6.42} \]

Fig. C.2. continued

\[ P [d] = 2.0407^{+0.0001}_{-0.0007} \]

SDSS J090128

\[ K \text{ [km s}^{-1}] = 26.23^{+2.12}_{-1.07} \]

\[ e = 0.13^{+0.05}_{-0.07} \]

\[ \gamma \text{ [km s}^{-1}] = 56.46^{+1.91}_{-1.85} \]

Fig. C.2. continued
\[ P \text{[d]} = 210.19^{+1206.52}_{-299.60} \]

\[ K \text{[km s}^{-1}] = 22.50^{+21.42}_{-13.33} \]

\[ e = 0.17^{+0.25}_{-0.15} \]

\[ \gamma \text{[km s}^{-1}] = 26.67^{+14.24}_{-11.76} \]

\[ P \text{[d]} = 10.13^{+4.51}_{-3.67} \]

\[ K \text{[km s}^{-1}] = 66.61^{+45.41}_{-29.76} \]

\[ e = 0.16^{+0.20}_{-0.12} \]

\[ \gamma \text{[km s}^{-1}] = 52.81^{+27.65}_{-39.76} \]

Fig. C.2. continued
Fig. C.2. continued
Fig. C.2. continued
Fig. C.2. continued
\( P \ [d] = 2613.92^{+3420.61}_{-2183.55} \)  
\( SDSS \ J110458 \)

\( K \ [\text{km s}^{-1}] = 11.50^{+8.13}_{-4.53} \)
\( e = 0.10^{+0.24}_{-0.13} \)
\( \gamma \ [\text{km s}^{-1}] = -37.84^{+16.13}_{-4.90} \)

Fig. C.2. continued

\( P \ [d] = 1187.30^{+3733.18}_{-1132.40} \)  
\( KA \ 2 \)

\( K \ [\text{km s}^{-1}] = 4.09^{+5.72}_{-2.78} \)
\( e = 0.18^{+0.26}_{-0.14} \)
\( \gamma \ [\text{km s}^{-1}] = -6.39^{+2.23}_{-3.15} \)

Fig. C.2. continued
Fig. C.2. continued

\[ P [\text{d}] = 704.56^{+4705.71}_{-688.32} \]

SDSS J112633

\[ K [\text{km s}^{-1}] = 7.85^{+9.44}_{-5.42} \]

\[ e = 0.17^{+0.25}_{-0.13} \]

\[ \gamma [\text{km s}^{-1}] = 38.46^{+6.62}_{-5.49} \]

Fig. C.2. continued

\[ P [\text{d}] = 1174.05^{+4431.27}_{-1108.43} \]

SDSS J120024

\[ K [\text{km s}^{-1}] = 13.94^{+9.48}_{-5.11} \]

\[ e = 0.16^{+0.21}_{-0.13} \]

\[ \gamma [\text{km s}^{-1}] = -33.49^{+6.42}_{-5.79} \]

Fig. C.2. continued

C.2  Eccentric orbits  243
**Fig. C.2. continued**
Fig. C.2. continued
Fig. C.2. continued
Fig. C.2. continued

\begin{align*}
P \, [d] &= 2.77^{+0.06}_{-0.08} \\
SDSS \ J154859 \\
K \, [\text{km} \, \text{s}^{-1}] &= 78.07^{+5.69}_{-4.52} \\
e &= 0.13^{+0.13}_{-0.09} \\
\gamma \, [\text{km} \, \text{s}^{-1}] &= 10.85^{+2.14}_{-4.76}
\end{align*}

Fig. C.2. continued

\begin{align*}
P \, [d] &= 1105.14^{+2809.96}_{-1045.26} \\
CLS \ 96 \\
K \, [\text{km} \, \text{s}^{-1}] &= 6.46^{+4.60}_{-1.80} \\
e &= 0.20^{+0.24}_{-0.15} \\
\gamma \, [\text{km} \, \text{s}^{-1}] &= 196.98^{+1.80}_{-2.85}
\end{align*}
Fig. C.2. continued

Fig. C.2. continued
Fig. C.2. continued
$P[d] = 3945.90_{-3770.22}^{+4016.10}$

SDSS J235443

$P[d] = 3945.90_{-3770.22}^{+4016.10}$

$K \text{ [km s$^{-1}$]} = 3.27_{-2.33}^{+5.27}$

$e = 0.18_{-0.14}^{+0.26}$

$\gamma \text{ [km s$^{-1}$]} = -210.61_{-2.71}^{+2.59}$

**Fig. C.2.** continued
Appendix: Supplementary discussion on distance calculations

D.1 Covariances between photometric colours

The HRD prior used to infer distances to the dC stars assumes that astrometric data measured with *Gaia* eDR3 and photometric data measured by SDSS DR16 are independent. Furthermore, any apparent magnitude measured in a specific photometric band is independent of the apparent magnitude measured in the other photometric bands. However, because the HRD prior used to infer the distances to dC stars utilises three photometric colours with photometric bands in common, these colours are not independent and their covariances must be accounted for.

Let $\text{Var()}$ and $\text{Cov()}$ denote the variance and covariance operators, respectively. Assuming two random variables $X$ and $Y$ that possess arbitrary distributions, a general result is:

$$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2\text{Cov}(X, Y) \quad (D.1)$$

Additionally, a general result for the random variables $W, X, Y, Z$ with arbitrary distributions that are multiplied by the random constants $a, b, c, d$, respectively, yields:

$$\text{Cov}(aX + bY, cW + dZ) = ac\text{Cov}(X, W) + ad\text{Cov}(X, Z) + bc\text{Cov}(Y, W) + bd\text{Cov}(Y, Z) \quad (D.2)$$

Thus, the covariance of two colours, $m_1 - m_2$ and $m_2 - m_3$, sharing a common photometric band, $m_2$, under the assumption that each band has been measured independently, can be given as:

$$\text{Cov}(m_1 - m_2, m_2 - m_3) = -\text{Var}(m_2) \quad (D.3)$$

Therefore, for the specific HRD prior that uses the $g - r, g - i$, and $r - i$ colours (such that $p = (g - r, g - i, r - i)$), the covariance matrix can be written as:
\[
C_p = \begin{bmatrix}
\sigma^2_x + \sigma^2_g & -\sigma^2_r & 0 \\
-\sigma^2_r & \sigma^2_g + \sigma^2_i & -\sigma^2_i \\
0 & -\sigma^2_i & \sigma^2_r + \sigma^2_i
\end{bmatrix}
\]  \hspace{1cm} (D.4)

where \( \sigma^2_x \) corresponds to the variance in photometric band, \( x \).

### D.2 Probability distribution of a sine wave

The calculation of orbital actions depends on the knowledge of the radial velocity of a star, or the systemic velocity (i.e. the radial velocity of the centre of mass) of a binary system. However, not all binary stars possess constrained orbital parameters, and hence, the systemic velocity of a system may not be known. For single-lined binaries (such as the dC stars) that have a few radial velocity measurements, it is possible to estimate the systemic velocity of the system, assuming that the orbit is circular.

Radial-velocity measurements of a particular star are time-series observations of the star’s velocity in the radial direction. If a star follows a circular orbit, time-series measurements of the star’s radial-velocity will form a sine wave. Therefore, to estimate the systemic velocity of a single-lined binary system in a circular orbit, it is necessary to inspect the probability distribution function of a sine wave.

Consider any function \( y = f(x) \). Because probabilities are defined as the area under the probability distribution function, any operation on a variable results in the modification of the probability distribution function. However, all probabilities must remain positive. Therefore, to ensure the area under the probability distribution function is positive, the probability distribution functions for \( x \) and \( y \), \( p(x) \) and \( p(y) \), respectively, can be given as:

\[
p(y) \mid dy = p(x) \mid dx
\]  \hspace{1cm} (D.5)

where the absolute value of the change in \( x \) and \( y \) is taken to ensure the area under the probability distribution function remains positive. Thus:

\[
p(y) = \frac{p(x)}{\left| \frac{dy}{dx} \right|}
\]  \hspace{1cm} (D.6)

Consider now the example of a sine wave that represents the radial-velocity curve of a single-lined binary with a circular orbit. In this example, time is represented on the \( x \)-axis.
Fig. D.1.: Left panel: The probability distribution function for the measured radial velocity at any given time, for a single-lined binary possessing a systemic velocity and velocity semi-amplitude of 50 km s$^{-1}$ and 10 km s$^{-1}$, respectively. Right panel: The cumulative distribution function of the radial velocity is shown for the same single-lined binary.

(thus $x = t$) and the radial velocity is represented by the $y$-axis (hence, $y = \sin t$). Therefore, the probability distribution function becomes:

$$p(y) = \frac{p(t)}{|\frac{dy}{dt} \sin t|} \quad (D.7)$$

Because $t$ is uniform over one cycle (e.g. $-\pi/2 \leq t \leq \pi/2$), the probability distribution function can be expressed as $p(t) = 1/\pi$. Therefore, the probability distribution function of the sine wave can now be written as:

$$p(y) = \frac{1}{\pi \cos t} \quad (D.8)$$

Consider now some radial velocity measurement of the binary, $R$, such that $R$ can be written as: $R = \sin t$ and $R$ is bound by: $y_{\text{min}} \leq R \leq y_{\text{max}}$ ($R$ exists on the radial-velocity curve of the binary). By substituting $R = \sin t$ into Equation D.8, the probability distribution function becomes:

$$p(R) = \frac{1}{\pi \sqrt{1 - R^2}} \quad (D.9)$$

$^{1}$using the identity: $\sin^2 a + \cos^2 a = 1$
The probability and cumulative distribution functions for a sine wave are shown in Figure D.1 for the specific example of a single-lined binary possessing a systemic velocity of $50 \text{ km s}^{-1}$ and velocity semi-amplitude of $10 \text{ km s}^{-1}$. The probability distribution function for the radial velocity is strongly peaked at the maximum displacement from the systemic velocity (in this example at $50 \pm 10 \text{ km s}^{-1}$), however, the cumulative distribution function shows that the systemic velocity corresponds to the median of the distribution. Therefore, while any one particular measurement of a binary’s radial velocity is likely to be at its maximum displacement from the systemic velocity, as the number of radial velocity measurements becomes large, the average of all radial velocity measurements tends towards the systemic velocity (assuming that the orbit is well sampled).

Radial velocity data used to calculate the orbital actions of the dC population were sourced from the SEGUE survey which measures radial velocities through cross-correlation to synthetic stellar templates similar to those of the target spectrum. Furthermore, for all SEGUE targets, multiple spectra are observed over multiple nights, often with large intervals between observations. The radial velocity value reported by the SEGUE team is therefore the weighted average of all of these individual observations per target. Thus, under the assumptions that the radial velocity measurements well-sample a binary’s orbit, and that the binary’s orbit is circular, the radial-velocity values reported by SEGUE can be assumed to be close to the true systemic velocity of the binary system.


