Can manipulatives benefit pupils' mathematical learning at Upper Key Stage Two?

Authors: Tom Cooper and Nasreen Majid

This research study examined teachers' and pupils' perceptions of the value of manipulatives in mathematics teaching and learning at Upper Key Stage Two. Although a wide range of research exists regarding the use of manipulatives in the primary mathematics classroom, there is less research into their effectiveness at Upper Key Stage Two. The methods used in this research were first, interviews with 12 teachers across four different schools, eight in Upper Key Stage Two and four in Key Stage One, to give as much breadth to the research as possible. Secondly, 14 Upper Key Stage Two pupils' views were gathered in focus groups to ascertain their perception of manipulatives. The pupil focus groups were chosen as a research method because very little research currently exists regarding pupils' perceptions of manipulatives.

Background on Manipulatives in Mathematics Education

Manipulatives can be defined as physical and concrete objects used to enhance pupils' mathematical learning (Kablan, 2016) and can be a means of representing abstract mathematical concepts concretely (Moyer, 2001). There is a wide variety of manipulatives used today in classrooms, such as Cuisenaire Rods, Unifix, Geoboards, Base-10 Blocks and Abacuses (Furner & Worrell, 2017).

There is relatively little emphasis on the use of manipulatives throughout the National Curriculum as a whole, with manipulatives only explicitly referred to at Key Stage One (DfE, 2013), even though the NCETM funded by the DfE, provides a range of ways in which manipulatives can be used across topics in the primary years (NCETM, 2019). Furthermore, Ofsted (2011) concluded that manipulatives are underused in schools and should be used in Key Stage Two as well as Key Stage One. The use of mathematics manipulatives is often associated with younger pupils (Chinn, 2012). Uribe-Flórez and Wilkins (2010) determined that the older the pupils being taught, the less likely the teachers were to use manipulatives as a teaching tool.

The English curriculum differs significantly from the Singapore curriculum (Ministry of Education, 2017). Singaporean mathematics advocates the use of manipulatives throughout every stage of primary school, employing a Concrete-Pictorial-Abstract (C-P-A) approach, based upon Bruner's (1966) modes of representation: the enactive mode, the iconic mode and the symbolic mode. The enactive mode involves pupils learning through actions (Bruner,

1966). In common with Bruner, many other theorists advocate their use (Furner & Worrell, 2017), including Liebeck (1990), Papert (1980), Piaget and Inhelder (1969), and Haylock and Manning (2014). It is worth noting that the C-P-A approach, which involves greater use of manipulatives, is becoming increasingly common in England (Boyd & Ash, 2018).

One approach which has advocated the use of manipulatives for over a hundred years is the Montessori approach (Marshall, 2017). Laski, Jor'dan, Daoust, and Murray (2015) have drawn four key messages from this approach. Firstly, manipulatives should be used regularly and over a significant period. Secondly, it is beneficial to start with a concrete representation before progressing to more abstract representations. Thirdly, the manipulatives used should not look like everyday items or have features which are irrelevant to mathematics. Finally, teachers should clearly explain links between the manipulative and the concept it represents.

Moyer (2001) found that teachers involved in the study stated that real mathematics learning did not occur as effectively with manipulatives and also said that many pupils just saw manipulatives as a fun break from regular mathematics routines (Moyer, 2001). Subsequently, Moyer and Jones' (2004) study found that providing pupils with the autonomy to select their manipulatives changed the way they approached their mathematics learning and allowed them to make sense of concepts independently.

Carbonneau, Marley and Selig's (2013) meta-analysis indicated that manipulatives have small to medium impact on pupils' mathematical understanding, in comparison to a purely abstract approach. It also suggests that it is not enough merely to introduce manipulatives into mathematics lessons and that there is a range of instructional variables to be considered, including the support provided, the effectiveness of a type of manipulative and the pupil's current ability.

From the research outlined above, it is clear that manipulatives can be employed effectively in the classroom to facilitate deep mathematical learning. However, this needs careful planning to sustain the long term use of manipulatives across the primary age range and not just consigned to KS1.

Main Findings from Research: Teacher Perceptions

In the interviews and focus groups conducted, it was found that the frequency of use of manipulatives seemed to be quite inconsistent across schools at Upper Key Stage Two. From this small snapshot of primary schools, it seems that manipulatives are used much more regularly in Key Stage One than in Upper Key Stage Two, with some schools using manipulatives much more frequently than others. The lack of consistency regarding their use

in Upper Key Stage Two does suggest that Ofsted's (2011) findings, which state that manipulatives are under-used in schools, are well-founded.

All the teachers interviewed stated that manipulatives could be of value to all primary-aged pupils. 10 of the 12 teachers said that manipulatives were particularly useful in Key Stage One but could still be valuable further up the school. The overall trend established from the research appeared to suggest that manipulatives can be used effectively with all primary-aged pupils but are particularly essential with the younger pupils when they are starting on their mathematical journeys.

A multitude of different benefits of using manipulatives in teaching mathematics was cited, with the most popular being: pupils can appreciate more fully the mathematics behind what they are doing; manipulatives provide a variety of experiences, which provide a foundation for the abstract, which, in turn, allows for progression; manipulatives could be used to help pupils who are struggling to develop conceptual understanding of core mathematical ideas; they could be used to aid reasoning; manipulatives can enable pupils to explain their answers.

There was a wide range of disadvantages shared, with the most popular being organisation of the resources, and more specifically, having sufficient resources for each pupil and the setting-up of the resources. The other particularly prevalent disadvantage was the low-level disruption and fidgeting that can occur when manipulatives are introduced into the lesson. The time available to cover all the curriculum content was another disadvantage cited frequently, as using manipulatives can often be a slower process. Other factors that prevented manipulatives from being used to a greater degree were the personal experience and subject knowledge of some of the teachers and the fact that pupils were unable to use them in their SATs (Standardised Assessment Tests).

The most popular manipulatives used in Key Stage One were Numicon, dienes (base-10 blocks), bead strings, counters, Unifix and money. In Upper Key Stage Two, dienes, counters, bead strings, Unifix and place value counters were the most popular. From this research, it appears that dienes, Unifix and counters are the most universally used across the whole primary school age range. Numicon was overall the most popular in Key Stage One, and dienes in Upper Key Stage Two.

The teachers interviewed cited a wide variety of topics where manipulatives were particularly beneficial. In Key Stage One, the most popular topics were number (specifically involving the four operations) and place value. In Upper Key Stage Two, both these topics were also cited regularly, with telling the time, fractions and decimals also referenced multiple times.

All 12 teachers commented on the potential value of the C-P-A approach, as employed in Singapore, in supporting learners at Upper Key Stage Two (Ministry of Education, 2017). The vast majority believed that the C-P-A approach played a key role in facilitating all pupils' progression in understanding mathematical concepts.

Main Findings from Research: Pupils' Perceptions

Pupils stated that the use of manipulatives in their lessons was infrequent on the whole, with four out of six focus groups collectively saying that they were used infrequently. Findings from one focus group with a group of Year 6s concluded that manipulatives were used approximately once a term and another group agreed that they used them sometimes. In common with the responses given by the teachers, it appears that the use of manipulatives in teaching at Upper Key Stage Two was inconsistent.

15 of the 16 learners interviewed stated that they enjoyed using manipulatives. 10 of the 16 learners felt that manipulatives were useful in their mathematics learning. Four of the pupils who did not concur with this viewpoint stated that it depended on the topic and the resource that was being used in the lessons, but that they could be useful. Only two pupils said that manipulatives were not useful at this stage of their learning, with one saying that they were more beneficial when they were younger and the other saying that there would be no point using them, as it would not make sense to start using them at this point in primary school when they had hardly used them previously.

A wide variety of advantages of using manipulatives were mentioned by the pupils. The most regularly cited advantage was the fact that using manipulatives was a fun way of learning mathematics. Other advantages stated were: that using manipulatives made it easier to work out the answers to questions; that manipulatives helped them to see the relationships between mathematical concepts; they helped them to check their answers; to get better at mathematics; and finally, that they help them with their learning. Over 20 additional advantages, were also provided by pupils, which suggested that from their perspective, there are many reasons why using manipulatives is beneficial, with the vast majority of these advantages referring to ways in which manipulatives help them in their mathematics learning and are not purely used for 'fun'.

Overall, pupils cited fewer disadvantages than their teachers. However, some pupils found that manipulatives were distracting or used as toys by some pupils; others felt that using manipulatives made them less independent if they felt they have to rely on them. Manipulatives were viewed by some children as less beneficial when they were older as the pupils felt them be unnecessary when they already knew how to work through their mathematical problem. Finally, and potentially an important pedagogical point emerging from

the research was that some pupils felt that using manipulatives sometimes made them feel muddled and confused.

Unifix appeared to be the most popular manipulative amongst pupils. The use of manipulatives was found to be particularly beneficial when dealing with fractions, decimals and place value and the four operations. These findings reflect the view of the teachers, with all of these topics proving to also be popular with teachers at Upper Key Stage Two.

Conclusion

It would appear from the research in this study, that a range of manipulatives can be both an enjoyable and valuable asset to mathematics lessons in a multitude of topics, with a wide range of advantages associated with their use, particularly when using manipulatives to introduce a new topic before progressing to a more abstract representation. Although a range of advantages exists, there are also some potential disadvantages explored here, particularly in terms of organisation and behaviour management. Careful planning to ensure the effective use of manipulatives in lessons is also necessary to ensure that children are not confused further. Careful consideration of these potential barriers to learning before teaching is important so that they can be minimised and so that the manipulatives can be used effectively.

References

Boyd, P & Ash, A. (2018). Mastery mathematics: changing teacher beliefs around in-class grouping and mindset. *Teaching and Teacher Education*, 75, 214-223.

Bruner, J.S. (1966). *Toward a Theory of Instruction.* Cambridge, Massachusetts: Harvard University.

Carbonneau, K.J., Marley, S.C., & Selig, J.P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology*, 105, 380-400. Doi: 10.1037/a0031084

Chinn, S. (2012). *The Trouble with Maths: A practical guide to helping learners with numeracy difficulties*. Oxon: Routledge.

Department for Education (DfE). (2013). *Mathematics programme of study: key stages 1 and 2*. Retrieved from https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/335158/PRIMARY_national_curriculum_-_Mathematics_220714.pdf

Furner, J.M., & Worrell, N.L. (2017). *The Importance of Using Manipulatives in Teaching Math Today. Transformations*, 3, 1-25.

Haylock, D., & Manning, R. (2014). *Mathematics Explained for Primary Teachers*. London: Sage Publications Ltd.

Kablan, Z. (2016). The effect of manipulatives on mathematics achievement across different learning styles. *Educational Psychology*, 36(2), 277-296. Doi: 10.1080/01443410.2014.946889

Laski, E.V., Jor'dan, J.R., Daoust, C., & Murray, A.K. (2015). What Makes Mathematics Manipulatives Effective? Lessons from Cognitive Science and Montessori Education. *SAGE Open*, 5, 1-8. Doi: 10.1177/2158244015589588

Liebeck, P. (1990). *How Children Learn Mathematics*. England: Penguin Books.

Marshall, C. (2017). Montessori education: A review of the evidence base. *NPJ Science of Learning*, 2, 1-9. Doi: 10.1038/s41539-017-0012-7.

Ministry of Education. (2017). *Mathematics Syllabus: Primary One to Five*. Retrieved from: https://www.moe.gov.sg/docs/defaultsource/document/education/syllabuses/sciences /files/primary_mathematics_syllabus _pri1_to_pri5.pdf

Moyer, P.S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics*, 47, 175-197.

Moyer, P. S., & Jones, M. G. (2004). Controlling choice: Teachers, students, and manipulatives in mathematics classrooms. *School Science and Mathematics*, 104, 16-31.

National Centre for Excellence in the Teaching of Mathematics (NCETM). (2019). *Mastery*. https://www.ncetm.org.uk/resources/47230

Ofsted. (2011). *[Withdrawn] Good practice in primary mathematics: evidence from 20 successful schools*. Retrieved from: https://www.gov.uk/government/publications/good-practice-in-primary-mathematicsevidence-from-successful-schools

Papert, S. (1980). *Mindstorms: Children, computers and powerful ideas*. Brighton: The Harvester Press Limited.

Piaget, J., & Inhelder, B. (1969). *The psychology of the child*. New York: Basic Books.

Uribe-Flórez, L.J., & Wilkins, J.L.M. (2010). *Elementary school teachers' manipulative use*. School Science and Mathematics, 110, 363-371. Doi: 10.1111/j.19498594.2010.00046.x