Dynamic Channel Modelling for Multimode Fibre Links in All Linear Coupling Regimes

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Abstract *We introduce a new drift model for slow environmental perturbation affecting modal mixture and walk-off. Mode coupling strength asymmetries between mode pairs of different mode groups are preserved while transmission matrix is decorrelated. Large impact on channel equalization performance is observed for an SVD-MIMO system. ©2023 The Author(s)*

Introduction

One of the potential technologies to fulfil the increased demand of information transfer is the use of space division multiplexing (SDM) [1, 2]. Multimode fibre based SDM uses N spatial modes to increase the capacity approximately by N-times as compared to single mode fibre [3, 4]. Increase in the capacity also leads to increase in equalisation complexity as data in the multimode fibre experiences group delay spread due to the differential mode delay and linear mode coupling [5]. Several works model linear mode coupling assuming that it arises from the small deviation in the core-cladding boundary given deformations during fabrication and/or installation process [5, 6].

Environmental effects such as vibration or temperature fluctuations, affect the linear coupling between the modes [7-9]. Thus, the study of these effects in the transmission system is required to understand the effect of slow and fast time varying perturbations in real-field deployment where the channel is quite not stable [10-14]. Recently, the effect of environmental perturbations on multimode SDM transmission systems was studied for channels in strong coupling regime scenario [14, 15]. As we show further on, for the weak and intermediate coupling regime, this model enhances the background coupling between all pairs of modes including for mode pairs from non-adjacent mode groups, which is not desirable. Therefore, a stochastic model taking account of channel drift keeping the modal coupling behaviour same and closely matches with the fibre in real-time is required. In this work, we develop a fibre model accounting the effect of slow time varying perturbations (drift), extending our existing linear coupling model [6].

Modelling of Multimode Fibre Channel

For the unperturbed dielectric waveguide, the propagating field can be expressed as a linear combination of the ideal modes $\boldsymbol{E}(x,y,z,t) = \sum A_m(z,t)\boldsymbol{E}_m(x,y)$ where m is the mode index, $A_m(z,t)$ is the slowly varying mode field envelope and $E_m(x,y)$ is the ideal electric field distribution. In the presence of a time-invariant dielectric perturbation $\Delta\varepsilon(x,y,z)$, the coupling between the ideal modes is described by the following coupled-mode equations [16-18]:

$$\partial_z A_m(z,t) = -j\Big[\beta_m(z) + \alpha_m(z)\cdot\boldsymbol{\sigma} + \sum K_{m,n}(z)\Big]A_m(z,t) \quad (1)$$

$$K_{m,n}(z) = \omega\varepsilon_0/4\iint \Delta\varepsilon(x,y,z)E_m^{*}\cdot E_n\, dxdy \quad (2)$$

where β_m is the propagation constant of mode m, $\alpha_m(z)\cdot\boldsymbol{\sigma}$ [7, 9, 16] accounts for the polarization rotation of mode m with $\alpha_m = \theta_m\hat{a}$, a θ rotation of $[0,\pi)$ over a vector $\hat{a}$ in an unit sphere, $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3,)$ is a tensor of the Pauli spin matrices, and $K_{m,n}$ are the coupling coefficients given by the area integral of the inner product of the electrical fields of mode m and mode n, over the area where the permittivity perturbation $\Delta\varepsilon(x,y,z) \neq 0$.

Considering a multi-section approach with β_m and $K_{m,n}$ constant over an arbitrary small length dz, the solution of (1), for a time-invariant $A_m(z=0,t)$, is given by the exponential matrix [17]:

$$\mathbf{A}(z+dz) = \mathrm{expm}\Big(-j\big[\mathbf{K}(z) + \mathbf{R}(z) + \boldsymbol{\beta}(z)\big]dz\Big) \quad (3)$$

where $\mathbf{A} = [A_1, A_2,..., A_{2N}]^{T}$, $\mathbf{R}$ is a block diagonal matrix with a sequence of 2×2 N submatrices $(\alpha_m(z)\cdot\boldsymbol{\sigma}(z))$ along the diagonal, $\boldsymbol{\beta}$ is a diagonal matrix containing the propagation constants β_m, and $\mathbf{K}$ is a matrix whose elements are $K_{m,n}$. In the following, the β_m and $K_{m,n}$ correspond to those for the refractive index profiles optimised for low modal delay in [19]. β_m are kept constant over all fibre sections while $K_{m,n}$ changes given a random radial and azimuth offset in each section, following [6]. $\mathbf{K}$ is calculated for the case in which E_m and E_n in (2) are co-polarized following [6, 7, 16].

In the following, crosstalk (XT) refers to the ratio between the mode set of interest $\sum_m P_m$ and the set of interfering modes $\sum_v P_v$, this is $XT = (\sum_v P_v / \sum_m P_m)$. For example, when using independent polarization diverse coherent receiver for each mode, one can define both polarizations of mode m as the set of interest, and all the others as aggressors. We follow this case.

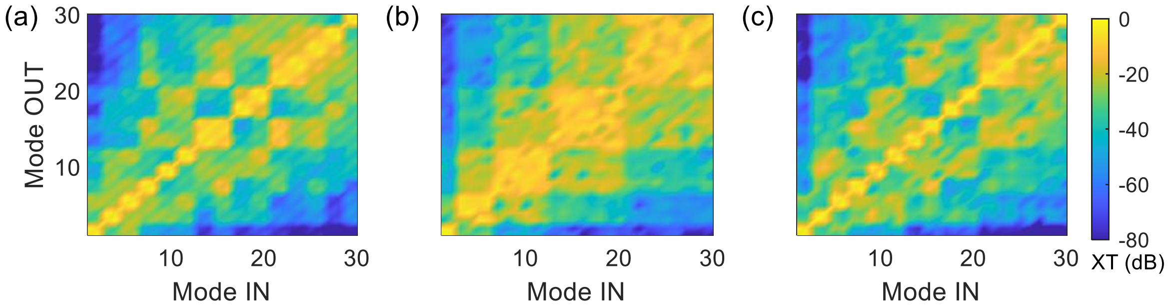

Fig. 1. Power coupling between modes (a) without drift (b) with HoP-drift, (c) with $\Delta\beta$-drift model for a 15-mode fibre. Fibre crosstalk strength is -40 dB/km and $\delta t/T_{env}$ =1.

Incorporating Modal Dynamics

The slow drift perturbation model proposed in [20] considers a channel in the strong coupling regime. The drifted channel is described by expm(M_{sh} + $(\kappa)^{1/2}M_{pert}\cdot\delta t/T_{env}\cdot$) where M_{sh} and M_{per} are random skew-Hermitian matrices for mode coupling and drift perturbation, respectively. M_{sh} and M_{per} have element-wise independent complex Gaussian random variables, and κ is the perturbation variance to achieve decorrelation when the change in time δt equals the characteristic time T_{env}. We refer to this model as homogenous perturbation drift model (HoP-drift). The HoP model has been proven reliable for fibres in the strong coupling regime [15]. However, in the weak-to-intermediate coupling regime the variance of the elements of M_{sh} is not homogeneous as crosstalk is strongest for modes in the same group, followed by coupling between modes of adjacent groups. Thus, applying a homogeneous perturbation artificially introduces coupling between sets of modes whose coupling strength should have remain negligible otherwise.

Here, we propose a model where perturbation is applied only to β_m keeping $K_{m,n}$ unchanged. In this way, we are accounting for slow drift perturbations such as temperature drift that are known to lead to changes in refractive index η that impact $\beta_m \approx \eta\cdot 2\pi/\lambda$ directly. However, $K_{m,n}$ is mostly determined by relative refractive index changes, see (2), that follow the difference in thermo-optic coefficient between SiO_2 and Ge and/or F. The model proposed here, refered to as $\Delta\beta$-drift model, can be described as:

$$\mathbf{A}_{\Delta\beta\text{-drift}}(z+dz,t+dt) = \mathrm{expm}\left(-j\left[\mathbf{K}+\mathbf{R}+(\boldsymbol{\beta}+\Delta\boldsymbol{\beta}(\kappa)^{1/2}\delta t/T_{env})\right]dz\right) \quad (4)$$

where $\Delta\boldsymbol{\beta}$ is a diagonal matrix containing the perturbation in the propagation constants whose elements are independent real Gaussian random variables of unit variance, and κ controls the perturbation strength. Here, κ is determined when drift is applied to all sections of the fibre.

Fig. 1 shows the 15-modes fibre transmission matrix for 1 km length with XT strength as -40 dB/km where each section length is considered as 10m, (a) without drift and (b) with HoP-drift and (c) with the proposed $\Delta\beta$-drift – considering $\delta t/T_{env}$ = 1. In Fig. 1 (b), the enhanced crosstalk is evident in overall and in particular for degenerate modes due to the homogeneous addition of perturbation in the HoP-drift. However, in Fig. 1 (c), it can be observed that the XT assymetries between and within mode groups remain unchanged with time despite having a decorrelated channel matrix.

To further investigate the proposed drift model, we analyse the crosstalk for a single section with a given core-cladding imperfection, for a 15-modes fibre considering $\delta t/T_{env}$ =0.005. Fig. 2 shows the evaluated XT for each mode m as a function of the normalized radial displacement (each point averaged over the azimuthal displacement and polarization rotation, as in [6]). We observe that with $\Delta\beta$-drift model, XT strength remains the same as for the case without drift for all the radial displacement values. Whereas, it can be observed that with the HoP-drift model, few modes have few orders of magnitude higher crosstalk than other modes, for small values of radial displacement. Near-degenerate modes are characterized by small propagation differences and small $K_{m,n}$, and therefore when applying homogeneous perturbation these will be most affected. This conveys that HoP-drift model is not suitable for the weak to intermediate coupling regimes. The change in the crosstalk distribution

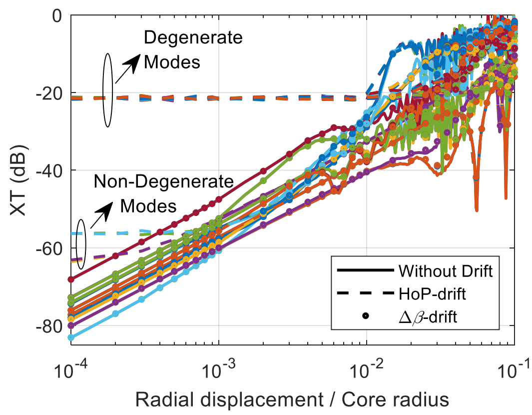

Fig. 2: XT as a function of the radial displacement *for each mode m* in a 15 mode fibre with $\delta t/T_{env}$ = 0.005 in the drift models.

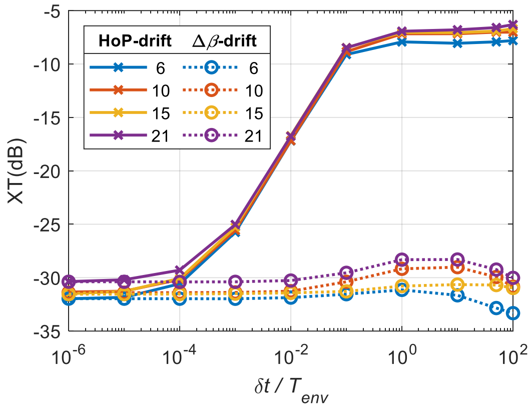

Fig. 3: *XT* at the output with drift models for fibres with {6,10,15,21} modes by varying $\delta t/T_{env}$ for the 1 km fibre of crosstalk strength -30 dB/km. *XT is averaged over all modes.*

with single step, in HoP-drift model, will keep propagating over the length of the fibre. Though, HoP-drift model follows the *XT* characteristic without drift for radial offset larger than 0.02.

Fig. 3 shows the average *XT* with drift as a function of $\delta t/T_{env}$ for a length of 1 km with section length as 10 m and the fibre crosstalk strength as -30 dB/km. We observe that as $\delta t/T_{env}$ increases, *XT* remains at the crosstalk strength level inherent to the fibre for the $\Delta\beta$-drift model whereas with the HoP-drift model increases beyond this level as seen in Fig. 2. Moreover, for the $\Delta\beta$-drift model, we observe some fluctuations in the crosstalk values for higher values of time instants and it is due to the large $\Delta\beta$ perturbation upsetting the coupling of degenerate modes such as LP_{11a} and LP_{11b}.

In the transmission systems, channel equalisation will be necessary to unravel the coupling introduced by channel and recover the spatial tributaries launched. An ideal solution to interference-limited MIMO systems is the SVD approach – used here for simplicity without lack of generality. Further, we calculate the SVD of the channel without drift, and then equalize the drifted channel after a certain time while accounting for the fact that in the SVD approach channel state of information must be exchanged between the transmitter and the receiver.

Fig. 4 shows the residual XT for different $\delta t/T_{env}$ and observe that the HoP-drift model has higher residual XT than the $\Delta\beta$-drift model for all the time instants. We observe that the residual *XT* in the $\Delta\beta$-drift model saturates around the fibre crosstalk strength value for large time instants. The residual *XT* level is not following the number of modes, it would be unexpected otherwise since the fibres used here were optimised for minimum group delay [19] whereas the coupling is largely dependent on the difference in the propagation constant between modes. Fig. 5 shows the difference in *XT* residual between the HoP-drift model and the $\Delta\beta$-drift

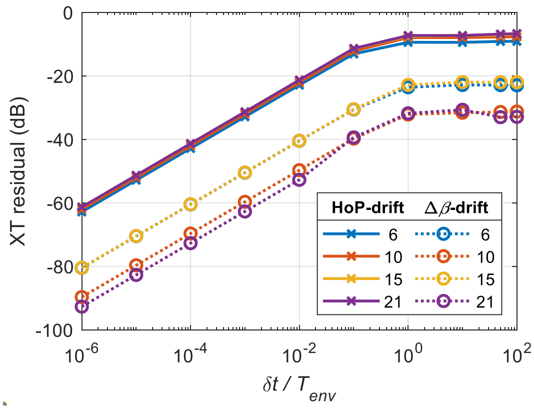

Fig. 4: *XT* residual *averaged over all modes* as a function of $\delta t/T_{env}$ for the 1 km fibre with crosstalk strength -30 dB/km.

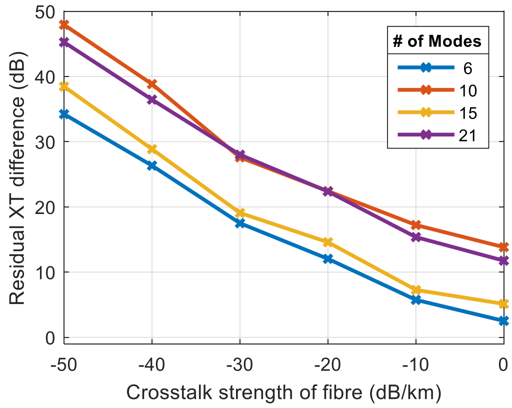

Fig. 5: Difference in residual *XT* between HoP-drift model and the $\Delta\beta$-drift model for different XT strength and number of modes in the fibre.

model varying the crosstalk strength of the fibre. We observe that the difference in residual *XT* decreases with the increase in the crosstalk strength of the fibre. This confirms that the performance of both the drift models coincide in the strong coupling regime, whereas there is a large difference in the weak to intermediate coupling regime.

Conclusions

We have introduced a drift model for multimode fibres that is applicable to all linear coupling regimes. Including, to the intermediate regime, that is most relevant to conventional fibres. The model proposed is shown to decorrelate the channel while keeping the mode coupling characteristic to all mode pairs. The model applicability is confirmed for an SVD-MIMO system.

Acknowledgements

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