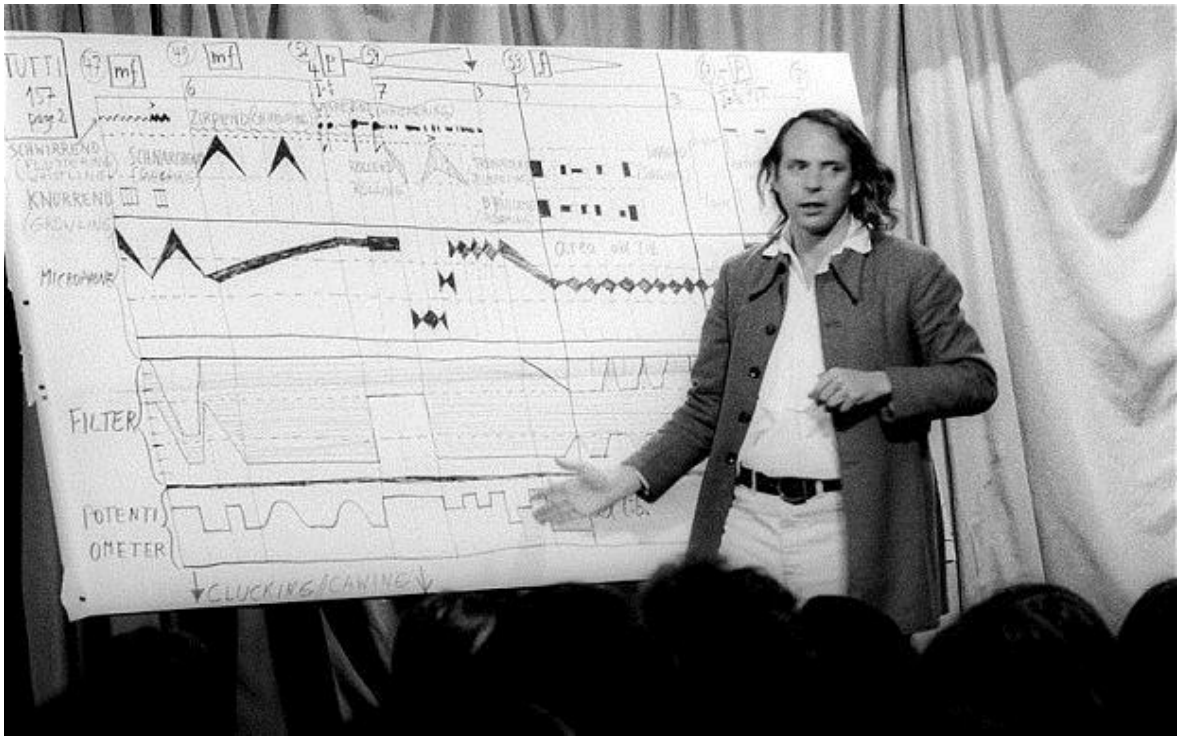
European Music Portfolio (EMP) – Maths: 'Sounding Ways into Mathematics'

Teacher's Handbook

Lifelong Learning Programme

Mathematician or musician?



Teacher's Handbook

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Introduction

> Rhythm depends on arithmetic, harmony draws from basic numerical relationships, and the development of musical themes reflects the world of symmetry and geometry. As Stravinsky once said: "The musician should find in mathematics a study as useful to him as the learning of another language is to a poet. Mathematics swims seductively just below the surface."
>
> (du Sautoy, 2011)

The project 'European Music Portfolio – Sounding Ways into Mathematics' (EMP-Maths) aims to combine music and mathematics in ways that highlight the common features of both.

Exploring links between mathematics and music is not something new:

> Mathematics, like music, is worth doing for its own sake…This is not to deny the great usefulness of mathematics; this very usefulness, however, tends to conceal and disguise the cultural aspect of mathematics. The role of music suffers no such distortion, for it is clearly an art whose exercise enriches composer, performer, and audience; music does not need to be justified by its contribution to some other aspect of human existence. Nobody asks, after listening to a Beethoven symphony, 'What is the use of that?' Moreover, mathematics does not gain in utility by having its inherent worth ignored – on the contrary, an appreciation of mathematics and an understanding of its inherent quality and dynamic are necessary in order to be able to apply it effectively.
>
> (Hilton, cited in Gullberg, 1997, p. xvii).

Learning and teaching that combine different disciplines often create new approaches to solving problems and give new insights into materials for all involved. Established paths can be abandoned, especially those that are often tainted by negative emotions, to make way for new and better ones.

The combination of music and mathematics in the EMP-Maths project provides a creative approach to developing learning and understanding in both subjects. This approach can support children to develop a deeper understanding by providing them with opportunities to engage with low threshold high ceiling activities which are challenging, thought-provoking and engaging.

Musical Lives and Musical Play

Music plays an important part in both our everyday experiences and the more pivotal moments of our lives. All documented societies practice something we can recognise as being music (Blacking, 1995). It is a 'complex and universal social behaviour' (Cross, 2001:95) that serves different functions according to the context and the needs of the participants. From the mundane to the spectacular, the small scale of the bedtime lullaby to mass religious, sporting or ceremonial occasions, music is present. Music lives within people and their cultures (Blacking 1976). As Welch argues:

> We are musical: it is part of our basic human design. The human brain has specialist areas whose prime functions are networked for musical processing. Also, we are musically educated, in the sense that we acquire sophisticated musical behaviours from pre-birth through encultured experience...our basic neuropsychobiological design enables us to make sense of, and find significance in, that patterns of sound that are organised as music within our culture.
>
> (Welch, 2005, p.117)

We begin our aural awareness of the musical world during the final trimester of pregnancy (Kisilevsky, Hains, Jacquet, Granier-Deferre,and Lecanuet 2004) and infants are born 'primed for music' (see, for example, Trehub, Schellenberg and Hill, 1997; Trevarthen 1999 and Papousek 1996) demonstrating an early ability to listen, produce and physically move to sound (Cross, 2001:96). Describing music in such broad terms as sound, or 'humanly organised sound' (Blacking 1976:3) may help both teachers and children to appreciate that the definition of music is in the control of the performer, composer or listener. Music is

what we decide it sounds like: it is personal to us and yet shared across geographical, spatial and political boundaries. Playing with sound, and through play, developing effective control of sound, is key to the arguments outlined below.

This section seeks to link this early predisposition to sound with the sorts of activities that children are likely to encounter in their first formal learning environments, and, later, to the specific musical activities that are more likely to lead to cognitive benefits in the mathematical domain.

Musical Learning

When we try to conceptualise how a musical activity might help our students to develop mathematically, it is important to balance a broad approach to musical learning with a particular awareness of those aspects that are more likely to facilitate cognitive transfer. It is most likely that children will develop their mathematical understanding by education *in* music and education *through* music. Research findings suggest that activities within school music lessons such as listening or moving to music, revealed no cognitive benefits (Mehr, Schachner, Katz, and Spelke, 2013). Whilst these activities are an important part of a rounded music education, they can also distance students' ability to 'do' music: to physically participate in the creation of sound. By contrast, the published research concerning those particular elements of music education that might be of most benefit, have focussed on instrumental learning (see, for example, Gouzouasis, Guhn, and Kishor, 2007; Fitzpatrick, 2006; Wetter, Koerner and Schwaninger, 2009; Schellenberg, 2004, 2006). More specifically, these findings are based on studies that involved children learning musical skills.

Musical Skills

Music can be thought of as set of core elements that includes rhythm and melody. We can think of rhythm as sound manipulated by time. The children can learn to control the time at which events occur and thereby change the rhythm within a piece. At the early stages of development this might include moving to a pulse, clapping rhythms and playing percussion instruments. Similarly, we can think of the melody as sound manipulated by pitch

frequencies. Children can learn to control the pitch frequency (or the highness and lowness of sounds) through singing in the first instance. The symbolisation of sound through notation can be a useful progression, but needs to follow the physical experience of 'playing' with sound. Musical notation, as and when it is introduced to the children, is spatial, representing fro example time, duration, dynamic and pitch. Playing from any score (including graphic scores) requires that the child translates a series of visual stimuli into physical responses and can benefit visuomotor coordination (Brown, Sherrill and Gench, 1981).

Mathematical learning

What are the best methods for teaching mathematics? This has been a question that educationalists and others have been trying to answer for hundreds of years. However, according to Swan, Lacey and Mann (2008), there are no best methods! The reasons for this are not surprising - children learn in many different ways and mathematics itself is a complex subject. The researchers were interested in looking at what types of learning opportunities were most effective and they came up with four common areas. These were:

- Understanding concepts and interpreting representations;
- Developing strategies to investigate novel problems;
- Developing fluency in calculation methods and recall of number facts; and
- Realising the power of mathematics and its role in society at large.[2]

Swan et al. (2008) then considered which teaching strategies best fostered these types of learning. They found that it was important for children to be provided with opportunities for mathematically rich conversations with their teachers and their peers. They also found that open-ended problems that provided challenge were the best for supporting children to develop their mathematical thinking and understanding. Askew, Rhodes, Brown, Wiliam and Johnson (1997) noticed that the most effective teachers were those that supported children

to make connections within mathematics, rather than those teachers who relied more either on a transmission (teacher-led) approach, or a discovery (child-led) approach.

There is also evidence that making maths meaningful for children can be very powerful (van den Heuvel-Panhuizen, 2001). This could mean, for example, teaching mathematics through stories, through music, or through play. Mathematics in the Netherlands is underpinned by this belief, in order to help children to understand the big ideas in mathematics. It is very important to differentiate this from providing children with 'real world' examples (such as problems involving household bills and banking interest rates), which are often not meaningful to children (as not directly relevant to their 'real world' experience) and can actually have a negative effect on attainment (Hattie, 2009).

Throughout Europe, there is a widespread acknowledgement of a tension between the learning of mathematical knowledge and the development of more fundamental mathematical skills (upon which the knowledge should, of course, be based). If we want to see higher attainment and more positive attitudes, then we need to ensure that we promote active learning and critical thinking in the teaching of mathematics (European Commission, 2011).

According to a recent study, at policy level central education authorities have some influence on the use of particular teaching methods. Across much of Europe, teaching methods are centrally prescribed or recommended in the majority of countries. In contrast, in Germany and the Netherlands, teachers or schools are only provided with central support in the form of web-based and other resources; and in five countries (Italy, Hungary, the Netherlands, Sweden and Iceland), teachers do not receive any guidelines and it is up to them to choose which methods to use. However, even within these apparently varied contexts, the themes and issues are not that dissimilar.

Low achievement in mathematics

Low achievement in mathematics is a problem for many children across Europe, although less than half of the countries in Europe have investigated the problem in depth (European Commission, 2011). Many reports link mathematics achievement with factors such as socio-

economic conditions, education of parents and student motivation. Research evidence on effective educational measures to tackle low achievement underlines the importance of:

- Laying the foundations for mathematics learning as early as pre-Primary level;
- Providing individual support to tackle difficulties as and when they occur;
- Increasing motivation by ensuring that links are established with other subjects;
- Making connections with everyday life; and
- Involving parents with their children's mathematics education.

Mathematics curricula in Europe

During the last 10 years, most countries in Europe have revised their mathematics curricula to focus more on skills and competences and less on content (European Commission, 2011). Many countries in Europe have reduced the mathematics content of the curriculum, in order to be able to make more cross-curricular links and to focus more on problem solving and the application of knowledge.[17] The exception to this is in England, where the recently updated 2014 National Curriculum for Mathematics has the teaching of algorithms and the knowledge of number facts as central to each yearly target. Even here, though, these requirements are situated within the context of a general statement for the curriculum to foster mathematical reasoning and problem solving skills (DfE, 2014).

Across Europe, mathematics curricula can be broken down into five key areas:

- Understanding mathematical concepts and principles
- Reasoning mathematically.
- Mastering basic skills and procedures (but with understanding, not learning by rote)
- Applying mathematics in real-life contexts
- Communicating about mathematics

Music Curricula in Europe

Many of the music curricula from Western European countries are based upon three key areas including: (i) composing, (ii) performing, as well as (iii) listening and appraising. Despite these common areas, the level of prescriptive guidance can differ widely, as for example, in 2013 the National Curriculum for England was further revised (DfE, 2013). The specification was greatly reduced, providing instead a framework of guiding aims that an individual school or music teacher could develop. These guiding aims include: (i) performance; in which the child should be supported so as to listen to, review and evaluate music from a range of historical periods, genres, styles and traditions, including the works of great composers and musicians ; (ii) singing and the use of the voice; so that the child is able to create and compose music on their own and with others, have the opportunity to learn a musical instrument, use technology appropriately and have the opportunity to progress to the next level of excellence; and to be able to (iii) understand and explore how music is created, produced and communicated, including the inter-related dimensions of pitch, duration, tempo, timbre, texture, structure and appropriate musical notations (DfE, 2013).

Integrating music and mathematics in the classroom

Having an overview of mathematics curricula across Europe provides a very useful way of seeing where music can be pivotal for effective mathematics teaching and learning. In very general terms, the context in which mathematics is taught is key to fostering motivation, interest and learning. Music can provide such a context, given that teachers are supported to have both the mathematical knowledge and the musical knowledge, to best exploit this opportunity. The understanding of mathematical concepts and principles, which is key to all the mathematics curricula, involves an understanding that mathematics, at all levels, is usually concerned with an understanding of patterns and relationships. This fits very well within the context of music and, at a very basic level, is the reason that music is often used to support the rote learning of number facts. However, if music is used in conjunction with mathematics, more fundamental relationships can be developed within, for example, the contexts of geometry, number and algebra.

In reviewing the curricula for music and mathematics across Europe, we have tried to consider the bigger picture and look for the common strands where the two subjects overlap. Within the context of this big picture, we can begin to consider how this can be developed to ensure a true integration of the teaching of music and mathematics, where by teaching the subjects as a whole, the understanding of each will be enhanced.

By taking the common themes of:

- Understanding music in terms of aspects such as rhythm, pitch, duration, dynamics, tempo, texture, structure
- Notation
- Composition and improvisation
- Performance
- Listening

We will demonstrate how music and mathematics can be united.

Understand music in terms of aspects such as: rhythm, pitch, duration, dynamics, tempo, texture and structure.

These aspects should be integrated into composition, improvisation and performance, but are identified here in order to make specific links with the mathematics curriculum. In addition to the broader areas of reasoning mathematically, mastering basic skills and procedures, applying mathematics in real-life contexts and communicating about mathematics, these could include:

- Count forwards and backwards (in ones, twos, etc.)
- Make connections between number patterns
- Measure and understand time intervals in hours, minutes and seconds
- Compare and sequence intervals of time
- Compare durations of events
- Use non-standard units of measure

- Use position direction and movement, including half, quarter and three quarter turns (rotations); movement in straight lines (translations); line symmetry in 2-D (reflections)
- Order and arrange combinations of objects in patterns and sequences
- Recognise and use linear number sequences
- Generate and use linear number sequences
- Generalise number patterns
- Recognise proportionality
- Identify and use halves and quarters of shapes and objects
- Add and subtract fractions with the same denominator (e.g. ⅛ + ⅜)
- Add and subtract fractions with different denominators (e.g. ½ + ⅔)
- Solve problems involving multiplication and division

Notation

Musical play can be used to support children in their understanding of specific elements such as structure, rhythm and pitch, as well as invented notation (Lee and Lin, 2013). Such invented notations may be used to document a composition, measure musical understanding (Gromko, 1994) or act as an intermediary device for both storage and later retrieval of ideas (Lee, 2013). The use of notation and the symbolisation of sound allow the child to visualise both the musical and mathematical processes at work. Over time, the child can experience the similarities between the notation of sound (musical properties) and the abstract notation used in mathematics (mathematical properties).

In addition to the broader areas of reasoning mathematically, mastering basic skills and procedures, applying mathematics in real-life contexts and communicating about mathematics, notation can be associated with:

- Identify and represent numbers using objects or pictorial representations
- Use non-standard units
- Identify and represent numbers using alternative representations
- Describe position direction and movement, including half, quarter and three quarter turns (rotations); movement in straight lines (translations); line symmetry in 2-D (reflections)

- Use a variety of language to describe and understand multiplication and division
- Order and arrange combinations of objects in patterns and sequences
- Recognize and describe linear number sequences
- Generate and describe linear number sequences
- Generalize number patterns

Composition and Improvisation

Broadly, composition gives children the opportunity to explore pattern, structure, layers and transformation of sound from the inside. By creating music, children are able to explore the patterns, structures, transformations, and layers of a piece of music first hand. Improvisation, as both an activity in its own right and a stimulus for composition (Koutsoudipou, 2005) is an important approach to sound exploration. Whether as an individual or as part of a group, children move through different stages during which they create and evaluate sounds in relation to their current musical knowledge and the demands of the task (Fautley, 2005).

Links with the mathematics curriculum could include all of the above and more!

Performance

Put simply, performance encompasses all activities where the children produce music, whether this is using the voice or instruments or found sound. Group size may very from the individual to the whole class whilst the aim of the activity is generally to develop musical fluency (Finney, 2000). With particular reference to singing, research suggests that the melody should be learnt separately to the lyrics (Heddon, 2012). This enables the child to gain a deeper understanding of musical construction and pattern making, which, in turn supports the internalisation of melody.

Links with the mathematics curriculum could include all of the above and more!

Listening

Listening is an integral part of improvisation, composition and performance, but is often identified as a distinct musical activity. Activities that focus on listening can support children in their understanding of musical construction and expression through exposure to a wide

variety of musical features. This, in turn enables children to create idea banks of possible approaches for their own compositions. Analysing the music that children listen to using graphic and invented notations can support the understanding of form and identification of patterns (Reybrouck, Verschaffel and Lauwerier, 2009).

Examples of activities

This teacher's handbook includes six examples of activities for you to try.

The following examples are all broken down into four main parts:

Overview	Prerequisite skills	Implementation	Variations
• Title • Topic • Keywords • Short description • topics covered in music and maths	• Prerequisites in maths • Prerequisites in music • Connections between math and musc	• Aims • Group size • Timescale • Implementation of the activity • Resources	• Variations • Further approaches in music • Further approaches in maths

1 Sounding ways around school

Topic

'Sounding ways around school' is about soundscapes, their relations and their possible representations.

Keywords

Soundscapes (acoustic environment), listening, timelines, relations

Short description

In this activity, learners will listen to sounds from the school environment, allocate them to a timeline, and explore soundscapes on their own.

Assignment to the collection of subjects/the core of music and maths

Music: Appreciation of music and aural awareness through listening; differentiated perception of sounds; ability to describe sounds and noises according to various aspects; recognising the volatility of sounds and noises; graphic notation

Maths: Geometry (length, transformation); measurement (length); numbers (estimation and comparison); spatial orientation; temporal orientation; orders; relations (and/or, before, after, simultaneous, etc.); and the set theory

Prerequisite skills

Prerequisite skills in maths

Basic skills in spatial orientation and estimation of time and distances

Preprequisite skills in music

Basic skills in aural awareness of surrounding sounds

Connections between maths and music

Listening to a soundwalk and recognising the recorded sounds connects spatial orientation and the estimation of time and distances with aural awareness of the sounds of the environment.

The allocation of a sound/event to a certain time is related to distance/time allocation in maths. Making clusters according to different criteria (distance,

source, duration, intensity) leads to some aspects of set theory.

Implementation of the activity

Aims

To improve the learner's listening skills and ability to describe sounds. Develop an understanding that sounds are often momentary and that perceptions and memories of sounds are subjective. Make proper use of the timeline, and group sounds in sets according to different criteria. Find orders (closest to furthest, loudest to softest, first to last, etc.).

Group size

Whole class

Timescale

30 minutes

Activity

Preparation*:* The teacher records the noises of a soundwalk (for a definition, see the resources) in the surroundings of the school. (When wearing 'loud' shoes, floors and rooms will sound in combination with the many other sounds and noises of the surroundings.)

1. In the class, the students listen carefully to the recording. As well as listening, they write down or draw what they think they hear on the recording.
2. Collect answers on cards and discuss them with the class. Sort them in different ways (source, shape, distance, loudness, etc.) by making clusters and bringing them into relation with each other.
3. Allocate the sounds with the learners on a timeline, represented on the board or on the floor, with a line or on a string and clothes pegs. The discussion can start with the order of the sounds, and later on there can be a discussion of how much time there is between the different events.
4. Do the same soundwalk with the learners (this can also be done on another day).

Resources

Own recording, preferably of a soundwalk around the school (we strongly suggest that this walk is no longer than two minutes)

Recording devices (mobile phone apps, audio recorders, etc.) ('soundOscope' is our recommended app for mobile devices)

Variations

Make another recording (or have one made by the learners) and compare it to the first soundwalk. What is new, what is the same, and what has changed? Try to fit the new sounds and noises into the first timeline.

Groups of learners create/install a soundwalk near the school and expand and explore it according to the standard approach (e.g. under other time and weather conditions).

In higher grades, GPS navigation devices that track and later display a route (e.g. on an online map like Google Maps) can be used.

Share your soundwalk(s) with classes from other schools.

Further approaches in music

Combine the various sounds with a musical score and play them with instruments. Use individual sounds as samples to create a rhythm.

Invent a notation in order to describe sounds. Invent different, suitable signs for different sounds and their development.

Record typical sounds. Who knows the places/sounds near the school, in the neighbourhood, and in the city? Using this material, draw up a quiz or an orientation game, which could include other classes and/or parents.

Further approaches in maths

Learners draw maps of the soundwalks and compare them.

Prepare a map and divide it into areas that are connected by pathways. Learners then try to find a path on the map that allows them to cross every pathway just once. Alternatively, they find the shortest way to cross every area in the school grounds. Afterwards, they make a recording of this path.

Measure the distances at which the fountain, the street or the school bell can (still) be heard under different conditions (weather, noise, time).

Collect and identify the sounds of a specific soundwalk over a longer period, and organise them into sets. Some of them will be totally different, while others might overlap, i.e. a frequently used road and cars are human inventions, whereas a stream is natural. Both the stream and the road noise are continuous if you don’t move.

Record the same soundwalks while varying the pace with hard shoe soles. Stop, and then walk back.

2 Jump the rhythm: multiplicative relationships and metre

Topic

This activity uses physical embodiment, timbre and metre to encourage children to use pattern and rhythm to develop a deeper understanding of multiplicative relationships.

Keywords

Metre, rhythm, multiplicative relationships

Short description

By counting metre aloud in a circle, combined with elements of body percussion, children further develop their understanding of multiplicative relationships. Both musical metre and multiplicative relationships will be emphasised in this activity.

Topics covered in music and maths

Music: Pulse, metre and rhythm; practical music making

Maths: Reason mathematically and make connections; communicate mathematical ideas; numerical relationships – multiplication, estimation

Prerequisite skills

Prerequisite skills in maths

Understanding of addition, multiplication, patterns

Prerequisite skills in music

Physical coordination (clapping/stamping), pulse

Connections between maths and music

Multiplicative relationships and musical metre

Implementation of the activity

Aims

Children's understanding of multiplicative relationships and musical metre are developed through group work and active engagement

Group size

Whole class

Timescale

20+ minutes

Activity

- Stand in a circle with the children. Explain that each child will say only one number from 1 to 4 as you go round the class. Now, start with the child on your left and go round the class, counting 1, 2, 3, 4; continue until every child in the circle has said a number (1, 2, 3, 4, 1, 2, 3, 4, etc.). Repeat this process and get a rhythm going.
- Once the children have got the hang of this, you are going to add some body percussion. Ask the children who are number 1 to clap upon hearing their number, and ask the children who are number 4 to stamp. When you go round the class, do you end on number 4? Can the children explain why this happens?
- It is most likely that the round did not end on 4. If this is the case, ask the children to predict how many times they would have to go round in order to end on a 4? Try it and see what happens.
- Now, try the same activity with different numbers (e.g. 1, 2, 3, 4, 5 or 1, 2, 3). It is important that the children are encouraged to predict what will happen and why before trying out the activity. Were they right?
- What do the children notice about the different metres? Are there some metres that they prefer? Why is this?

Resources

No additional resources are required.

Other considerations: This activity should be carried out in a room where the children have space to stand in a circle and then work in pairs.

Variations

You could ask the children to add in other elements of body percussion (e.g. slap, click) on numbers that are in between the first and last numbers.

The class could be divided into two or more groups and the activity could then be repeated with each group. What do they notice this time? Was it harder or easier?

Based on the first activity, child 2 and child 3 remain silent, but the group still need to keep time, so that the only sounds are for beats 1 and 4.

Further approaches in music
Children could create their own element of body percussion. Children could use instruments instead of numbers and body percussion. For more of a challenge, children could include rests in their performances.
Further approaches in maths
Activities based on multiples, factors, lowest common multiples and highest common factors Activities involving patterns and sequences Developing ideas of permutations and combinations

3 Clapping the lowest common multiple of 2, 3 and 5

Topic

'Clapping the lowest common multiple of 2, 3 and 5' is about using body percussion and different body timbres, superposition and connection in order to solve the following question: which is the lowest common multiple of the numbers 2, 3 and 5?

Keywords

Lowest common multiple, body percussion, body timbre

Short description

In this activity, students will learn three body percussion rhythm patterns, each one related to the numbers 2, 3 and 5. Then, they will play each pattern simultaneously, counting from 1 to 30 in order to find the lowest common multiple of numbers 2, 3 and 5. Listening to the different body timbres will let students discover not only the lowest common multiple, but also other multiples and the relations between these numbers.

Topics covered in music and maths

Music: Body percussion performance; ability to listen to the different body timbres; timbre recognition; follow the beat; rhythmic reading; rhythmic imitation; regular and rhythmic precision; and the ability to perform and listen to different sound plans at the same time

Maths: Use the reasoning and proof to find the lowest common multiple; order; relations; numbering; multiples (and divisors); and connections

Prerequisite skills

Prerequisite skills in maths

Basic skills in counting and numbering are required. It is not necessary to know the lowest common multiple; you can introduce it through the activity.

Prerequisite skills in music

Basic skills in rhythm patterns (reading and imitation) are required. It is not necessary to know body percussion; you can introduce it through the

activity.

Connections between maths and music

Performing a regular rhythm pattern by counting numbers (1–30) while following a beat is related to mathematical precision, relation, numbering, order and time.

Dividing the classroom group into three lines, each playing different rhythm patterns, and listening to body timbres is also related to simultaneity and the inter-relations between numbers (multiples and divisors).

In this activity, children should try to solve a mathematical question by listening to the same timbre, so it is necessary to follow the same beat and have rhythm precision.

The sound simultaneity is connected with the lowest common multiple of some numbers.

Implementation of the activity

Aims

To improve the performing and rhythm skills of the learners; to listen to and recognise the same timbre in order to get to the solution of a problem or question; to find the lowest common multiple and other multiples of some numbers; to follow the beat and rhythm with precision; and to perform rhythm patterns using body percussion

Group size

Whole class

Timescale

Two sessions to understand the whole of the activity. 30 minutes to find the lowest common multiple of two numbers (2/3; 2/5; 3/5) instead of three (2, 3, 5)

Activity

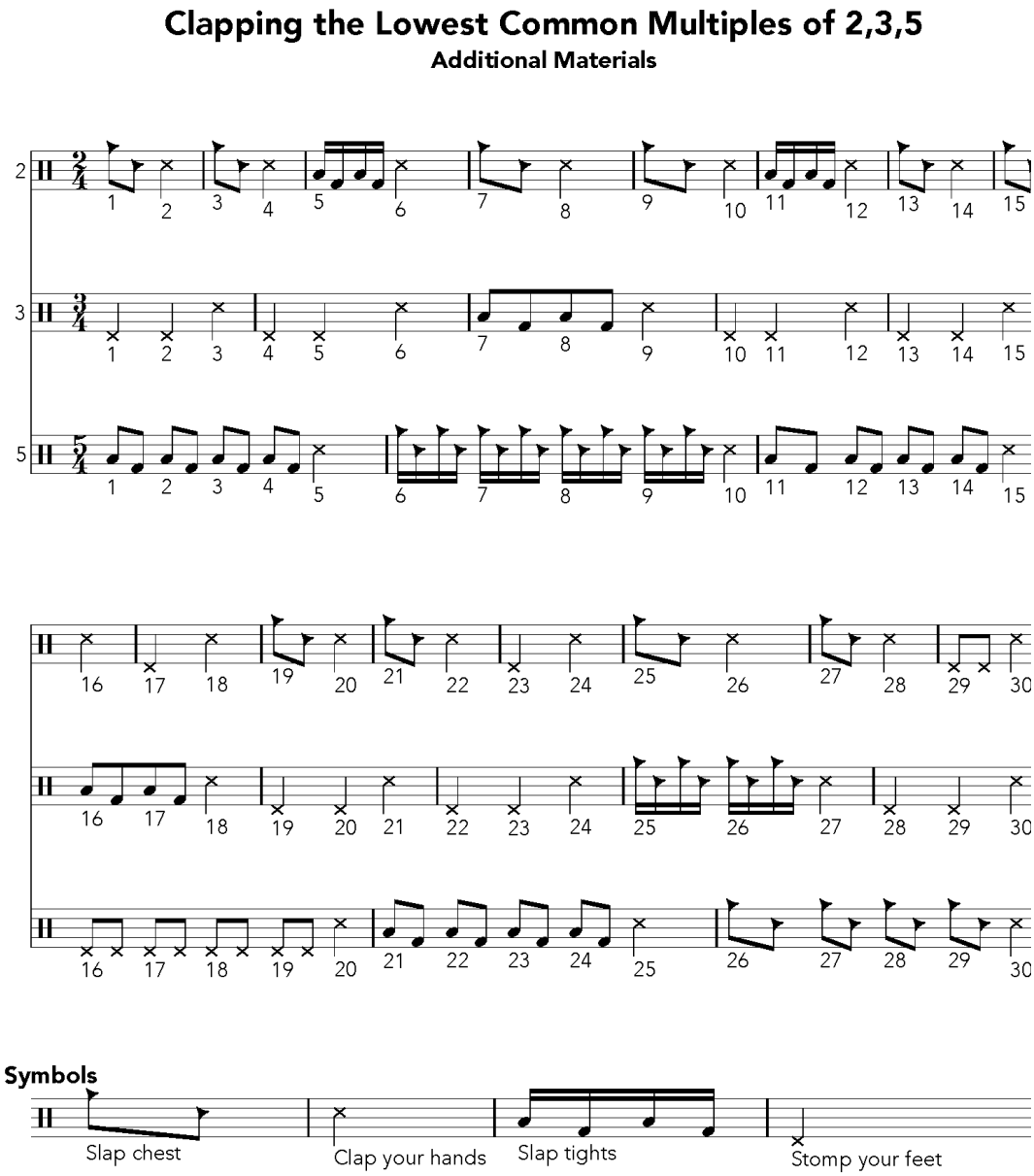
1. Start with the body percussion of number 2. The teacher introduces the children to a body percussion pattern of 30 beats (see materials) and explains the meaning of each symbol.

   If the level of the students is sufficient, it is possible to learn the body's percussion pattern by reading it. Make sure to follow the beat by counting the numbers (from 1 to 30). The multiples of 2 (2, 4, 6, 8, until 30) should

coincide with the hand clapping. If the students cannot read the score, the teacher can teach it through imitation, so they can improve their rhythm memory. Once students learn the body percussion, they have to perform it while counting until 30 (following the beat).

2. Follow the same procedure with the body percussion of number 3 (see materials). Notice that now the hand clapping is based on multiples of number 3 (3, 6, 9, until 30).
3. The teacher divides the classroom group in two lines (face to face). One line performs the body percussion of number 2, and the other performs the body percussion of number 3. Each time there is a multiple of 2 and 3, the students will clap their hands at the same time. The first time that this happens, they find the lowest common multiple of numbers 2 and 3 (. At the end, we can list the common multiples we have found by listening to the same timbre (hand clapping).
4. The teacher can introduce the body percussion of number 5 (see materials) and try to find, following the same procedure, the lowest common multiple of 2, 3 and 5. Notice that in the body percussion for number 5 the multiples also coincide with the hand clapping. Facing two lines, students can follow the lowest common multiple of 2 and 5 (10) or 3 and 5 (15).
5. Finally, the teacher organises the students into three lines, two parallel and one perpendicular, and each line performs the body percussion of one number (2, 3, or 5). When all the students are clapping their hands at the same time, they will find the lowest common multiple of numbers 2, 3 and 5 (30). It is necessary to count until 30, following the beat in order to know in which number the three lines coincide.
6. The teacher can project the image of the three rhythm pattern overlapped (see materials) to show which numbers coincide with the hand clapping (so the common multiples of numbers 2, 3, and 5).

Resources



Variations

Variation #1: Instead of using body percussion, use the notes **do** (feet), **mi** (thighs), **so** (hands) and **do** (chest). The concept of chords appears in this variation. The procedure would be the same; however, instead of using body percussion rhythms patterns, use easy melodic patterns with the

note **so** in the multiples of each number.

Every time that the note **so** is sung, a multiple is found. While singing, it is not possible to say the numbers, so they can be written on the board and the teacher can point at them while following the beat, or a volunteer can do this or say them aloud while the rest of the students are singing the melodic pattern of one number (2, 3 or 5).

Variation #2: Using body percussion, the students are situated in a circle. They are doing a step towards the right side. Every time, a student performs the beat of a given rhythm pattern while saying aloud the number of the beat that it corresponds to. For instance, if they follow the rhythm pattern of number 2, they would realise that the students who said number 2, 4, 6 or 8 clapped their hands. So, they are the multiples of number 2. If we repeat the activity with the rhythm patterns of numbers 3 and 5 (starting with the same person every time), we could discover the common multiples of these numbers.

Further approaches in music

Students can create a more complicated body percussion rhythm pattern by changing the timbre of the multiples.

Students can create a melodic pattern for each number and write it down. Change the chord or change the note that corresponds to the multiples.

Change the parts of the body used in the body percussion.

Use instruments to perform each rhythm in order to have as many timbres as those used in the body percussion.

Further approaches in maths

Change the numbers and find their lowest common multiple and their other multiples.

If you are doing variation #2, try to build a perfect circle and talk about geometry.

4 Sounding numbers

Topic

The 'sounding numbers' activity is about the creation of different acoustic models of the natural numbers.

Keywords

Maths: Numbers, digits, the positional notation of a number in the decimal numeral system (advanced and shortened notation), decomposition of a number

Music: Rhythm, metre, metro-rhythm

Short description

In this activity, learners will invent different types of acoustic models for the natural numbers, and identify and write down *n*-digit natural numbers based on their acoustic representations.

Topics covered in music and maths

Music: Elements of music (pulse, rhythm); playing musical instruments and singing; rhythmic playing echo (imitation)

Maths: Numbers (natural numbers, place value); numeration; the positional notation of a number in the decimal numeral system

Prerequisite skills

Prerequisite skills in maths

Basic numeration skills – reading and writing natural numbers in the decimal numeral system, graphically representing *n*-digit numbers

Prerequisite skills in music

Knowing and understanding the principle of echos (body playing, children's rhythmical musical instruments)

Connections between maths and music

Listening to different types of sounds for units (tens, hundreds, etc.) and counting them to connect the abstract concepts of the decimal numeral system with the acoustic model of the number.

Implementation of the activity

Aims

To develop an understanding that the natural numbers could be represented in different ways (written notation, graphical representations (symbols), manipulation with small objects, acoustic models). Improve learners' skills when it comes to transforming a written model of numbers into an acoustic one, and vice versa.

Group size

Groups of four(+) students; or work in pairs

Timescale

20 minutes

Activity

- The teacher writes a 3-digit number in its decimal notation and in its graphical representation (e.g. 235, // --- +++++).
- The teacher then makes the sounding number using stamping (2x), slapping (3x) and clapping (5x). The next number is played and learners write it using digits or signs (graphical representation).
- Learners from each group invent sets of sounds in order to code an acoustic representation of the natural numbers (e.g. four 3-digit numbers). They can use different sounds (body playing, Orff instruments, spoons, etc.).
- The learners of the first group present (play) the numbers using their invented sounding code.
- The learners of the second group write the sounding numbers (or draw a graphical model of the numbers).
- Control of solution and discuss: what numbers were played (represented), and what types of coding were used?
- Discuss the advantages and disadvantages of different types of representation of natural numbers (graphical, auditory, decimal). Compare different representations of the numbers.

Resources

Paper, pens, Orff instruments

Variations

This activity can also be performed in pairs (cooperative education).

Any subjects or musical instruments (sticks, triangles, drums, mugs, cans, and pellets) can be chosen for presenting the sounds.

The signs of sounds will make learners play numbers using the sign models of numbers; for example, 235 and // --- +++++.

The activity can also be conducted with a group of older students, depending on the selected line of numbers (for example, over 1,000, 10,000, etc.).

There is space for creating different tasks and variants based on the abilities and skills of the target group. From the point of view of the target group, it is possible to flexibly adjust the tasks for any age group or line of numbers.

Further approaches in music

Invent different sounds for signs in graphical models of the natural numbers.

Invent a notation in order to write the number (units, tens, hundreds).

By using Orff instruments, make an acoustic model of numbers to create the rhythm.

Note values could represent the place value of digits in the decimal numeral system (e.g. quarter note = one, half note = ten, whole note = hundred).

Further approaches in maths

Through the regular application of the above-mentioned activity from the first year of secondary school, a new atypical model of the natural numbers is created, which is different from the concrete ones (abacus, cubes, subjects, graphic representation) that are usually used. During the process of realisation, it is necessary to develop one's own mental representation of a multi-digit number. A number of sounds are transformed into the symbol of a digit, which is kept in the memory and finally recorded using mathematical terminology. Conducting the above-mentioned activities develops higher cognitive processes and involves executive functions, especially the working memory and shifting.

5 Angle dances

Topic

Different types of angles are expressed with different arm and leg positions in a choreographed dance.

Keywords

Angles, body movements, patterns

Short description

A choreographed dance is to be developed in the learning environment. The dancers should express different types of angles through different arm and leg positions. The choreography is to be developed with small cardboard figures; later, the dancers are supposed to act out their development with matching music. The different positions of the legs and the arms have to be connected in a continuous flow. Depending on the knowledge of the participating person, different types of angles can be introduced via the use pictures in the learning environment.

Assignment to the collection of subjects/the core of music and maths

Communicate mathematical ideas using multiple representations; types of different angles; pattern recognition; connecting music with body movements; and physical responses to music

Prerequisite skills

Prerequisite skills in maths

Knowledge of what an angle is
Knowledge of the different types of angles: Right angle, acute angle, obtuse angle, straight line and that there are 360^{o}

Prerequisite skills in music

The dance is the centre of this learning environment. The expression of dance is created through the movement of the body and limbs (arms and legs).

Connections between maths and music

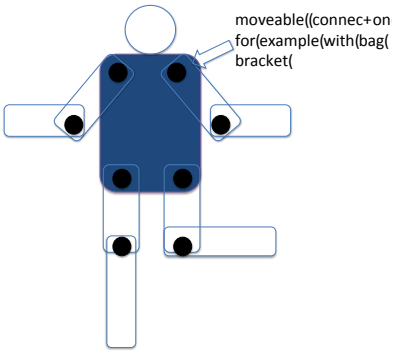
Music creates patterns of sound, which can be represented in angles and geometric figures.

Implementation of the activity

Aims
Identification of sound patterns in music -> moods Sound patterns shall be transformed into body movements (dance) Translation of moods (caused by music) into movements and therefore into geometric figures: in adequate types of angles
Group size
This depends on the age of the participants
Timescale
About three hours, including performance
Activity
At the beginning of the learning experience, there has to be clarity on which angles are which (right angle, acute angle, obtuse angle, straight angle, etc.). Children unfamiliar with the angles can be helped through the use of pictures. After having cleared this issue, one has to consider how to create the different types of angles with different arm and leg movements. These considerations can be met with cardboard figures. Some types of angles offer different possibilities. The expression through the dance can lead to special tensions when the arm and body movements have a special character. At the same time, one has to consider whether the arm and body movements can actually be copied with real arms and legs. After clearing the different possible positions, the choreography has to be created. For the choreography, one has to listen to the music that is presented and think about which sequences of angles match the music, showing the character of the music in a specific way.

Resources

It is recommended that the choreography be planned at a table. Therefore, cardboard figures should be used. The cardboard figures have movable knees and elbows (see picture). With the figures, different positions can be



considered dynamically. The final choreography can be documented with drawings.

Variations

Present different geometric figures in dances. Geometric figures can be presented by multiple persons. This way, the corners can be represented by single people and the edges by the connection of these people's hands and arms.

Further approaches in music

The quality of the music can be expressed with different geometric figures. For example, fast or high-pitched music could be expressed through triangles. Slower and more harmonious music could be represented with circles and regular polygons that move through the space. According to the music, different dances occur.

Further approaches in maths

This could provide an opportunity to analyse the properties of 2D and 3D shapes. This could be done by exploring the number and position of the edges and the corners. For example, a square could be made with four people holding hands, where their bodies represent the corners. This could then be transformed to create shapes such as parallelograms or rectangles by moving their arms to create the new shapes.

6 Twinkle, Twinkle Little Star

Topic

Use singing to explore symmetry, pattern, time and reflection.

Keywords

Rhythm, reflection, motif, reflection, transformation and symmetry

Short description

Children will explore what happens when they transform music. They will also discover that there are different patterns, depending on whether they focus on the rhythm or the musical score. This will help them to understand that if we focus on different aspects of a problem, there will be different solutions.

Topics covered in music and maths

Music: Pulse and beat; practical music making; composition and improvisation using voice; appreciation of music; and aural awareness through listening and performance

Maths: Explore transformations, symmetries, patterns and sequences

Prerequisite skills

Prerequisite skills in maths

Experience of exploring patterns and sequences, as well as some experience of reflections

Prerequisite skills in music

Physical coordination (clapping/stamping), pulse, use of voice for singing, listening

Connection between maths and music

Patterns, sequences, and transformations

Implementation of the activity

Aims

Children will learn about symmetries, patterns and motifs in music and mathematics.

Group size

Whole-class and pair/group work

Timescale

20+ minutes

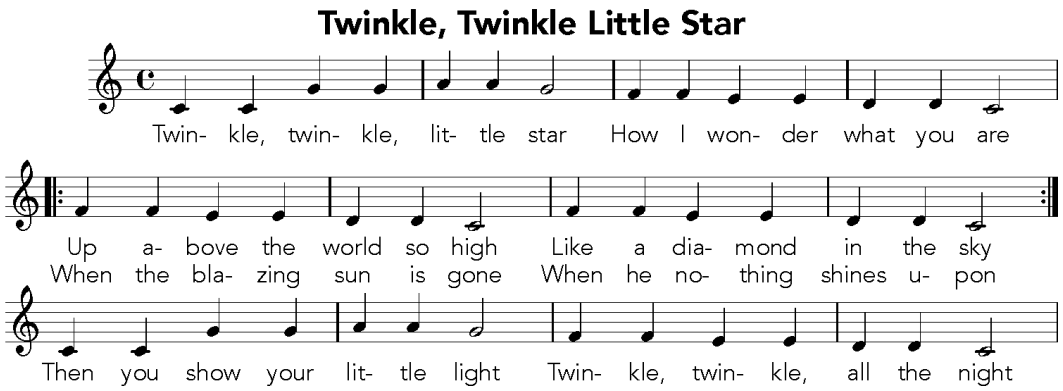
Activity

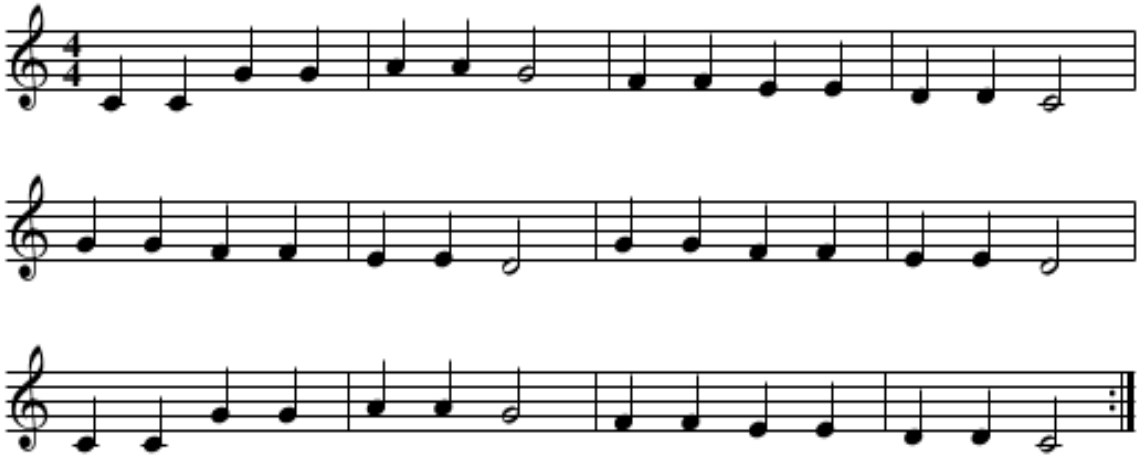
- Sing the song with the whole class a few times to make sure that the children become familiar with it. It may help to have the words on the board or on paper for the children to see. Ask the children if they notice any patterns or symmetries in the tune (rhythm patterns, melody patterns, form A-B-A).
- Draw the melody as lines, showing the ups and downs.
- Now, clap the rhythm with the children and ask what patterns they notice this time. Are they the same or different from the ones they noticed before?
- Next, ask the children to work in pairs or small groups. The children need to choose a motif from the song, using either the song, the tune or the rhythm. Ask the children to create their own notation to represent the motif. The children should then explore what happens when they reflect the motif and they should draw this reflection. The children may want to use mirrors to check that they have drawn their reflections correctly. Once this has been done, the children should practise singing or clapping their motif along with its reflection. It might be easier if the children try singing the tune without the words.

Materials, pictures, music – Material spatial arrangement

Mirrors, copies of the song

Other considerations: This activity should be carried out in a room where the children have space to stand in a circle. If there is a board, the children may not need copies of the song.





Variations

There are many different songs that could be chosen as a starting point, but it is important that they are songs that are very familiar to the children and have a simple structure.

Further approaches in music

Use different versions of the song, such as:

A, B, C (song);

Baa, Baa Black Sheep;

A vous dirais je maman (original version);

Mozart variations of the song;

Louis Armstrong's *What a Wonderful World* (inspired by the melody);
Choose a theme and present a new version of the song. For example:

I came into school today

And I shouted "Let's go play!"

Saw my friends and off we went

Round the playground, through the fence

I came into school today

And I shouted "Let's go play!"

Instruments could be used to explore different transformations.

Further approaches in maths

Work could be developed using other transformations (rotations and translations). Can we do the same things with rhythms and scores as we can with words?

The idea of using a motif and transforming it could also be explored using designs for wallpaper or wrapping paper. More traditional designs, such as those used in Islamic art and design, could also be explored.

This activity could also lead into work on combinations and permutations, and this might support work on fractions.

The ideas could be developed to include work on sequences.

Conclusions

We hope that you and your students have enjoyed the activities in this book and we hope that you have seen how exciting it is when students learn mathematics and music together. The potential for developing knowledge and understanding is greater when both mathematics and music are explored in ways that are both meaningful and engaging. We can 'sound ways into mathematics' and 'mathematise ways into sound' at the same time.

We would very much appreciate your feedback on these activities and we would be delighted if you would share your own activities with us.

Thank you for taking part and for having fun!

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