

Chapter 13

What Theories and Methodologies Are Appropriate for Studying Phenomena Related to Mathematics Curriculum Reforms?



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Curriculum is a contested ‘word’ and object for schools, society and for our civilisation. Curriculum has been changing its form and status over the years. As Artigue (2018) argues, curricula interact with the conditions and constraints of their functioning to catalyse change in the state of educational systems. Moreover, their design involves a diversity of institutions and agents, and their implementation an even greater number. Institutionally recognised curricula are eventually formalised through the corresponding texts and other resources, which may be understood as products of a complex and dynamic process impacted by many institutions and agents. Where there is some level of freedom available to schools and teachers, the transposition of a curriculum into school affords (and constraints) the range of possible dynamics for the teaching and learning of the discipline.

When curriculum reforms become the objects of study for research in mathematics education, a diversity of theoretical approaches emerges exposing and addressing different research problems linked to curriculum and curriculum reforms. In

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order to gain perspective on the notion of curriculum and to be able to address the variety of phenomena related to their design and implementation, theoretical approaches provide tools to model curriculum and their reforms. The approach to interpreting curriculum reforms, the kinds of entities that are taken into account, and the empirical domain that is considered as the minimal unit of analysis may vary significantly depending on the research framework chosen (Ernest, 2016). Choices about the elements of the unit of analysis can lead to entirely different research problems related to curriculum reforms, diversity of methodologies to address them, and give rise to different or even incommensurable findings.

The objective of this chapter is to give an overview of the current state of the art related to theoretical frameworks and the methodologies used to address phenomena related to school mathematics curriculum reforms. With this aim we begin by presenting an overview of the main theoretical frameworks discussed in the contributions to the ICMI Study 24 and beyond, extending this overview by considering wider research on this topic. This is followed by the analysis of a selection of case studies as representatives of most prominent lines of research about curriculum reforms, depending on which entities they are (or are not) questioning. It aims at identifying theoretical areas related to curriculum reforms, in this extensive territory for research, and at developing new insights that might catalyse further research.

First, we present an overview of the main theoretical approaches that have been used to address research problems of mathematics curriculum reforms, through our analysis of those discussed in the contributions to ICMI Study 24 and beyond. To facilitate this overview, we have opted to organise these frameworks around three main groups depending on their focus on: the conceptualisation of curriculum and its elements, the didactic process of creation and dissemination of curriculum reform, and on the communities involved in curriculum and the factors affecting their success. This is followed by a discussion of the tentative parallels detected among the theoretical approaches, which allow us to delimit certain lines of related research.

The next section presents a set of cases that illustrate the relations between theoretical frameworks and the methodologies offered to approach curriculum reform research. We distinguish between five main lines of research related to curriculum, depending on what is questioned and what is not. These lines are then further exemplified with case studies that we consider as representatives of each line of research. The chapter concludes with some reflections on the main contributions to the research domain of curriculum reforms within mathematics education, and includes some open questions for future research, with respect to the frameworks adopted and the methodological tools proposed for their analysis.

Overview of Theoretical Approaches for Analysing Phenomena Related to Mathematics Curriculum Reforms

Regarding theoretical approaches used in relation to mathematics curriculum reforms, we start by emphasising the diversity of the theoretical frameworks adopted in the papers discussed in the ICMI Study 24 and, in some cases, the difficulties in identifying the adoption of any specific theoretical approaches. This section aims to present the most prominent theoretical approaches discussed in the different themes. These are far from the only theoretical approaches that could be adopted, but we consider our analysis to be useful as it makes explicit the tools and methodologies the different approaches offer to analyse curriculum reforms.

We focus our overview on several such frameworks, each of them approaching curricula and curriculum reforms with different aims and ways of undertaking the analysis. Being conscious of the difficulty in comparing different theoretical approaches in mathematics education, which is not the purpose of this chapter, we have opted to organise them around three main groups according to their main focus. Firstly, we have a group which aims to provide elements to define and to conceptualise curriculum. Here, we consider the TIMSS Curriculum model (Mullis & Martin, 2015; Mullis, 2019) distinguishing different curricula (intended, implemented and attained), and the approach provided by Niss (2016) which adds some particular elements to the ‘curriculum’ definition.

Secondly, there are further general approaches that aim to analyse the epistemological and didactic process of delimiting the curriculum, including how curriculum reforms are transposed to different institutions for their interpretation, teaching and learning. These tend to include curriculum and curriculum reforms within a wider process of construction and dissemination of the knowledge to be taught and learnt in school institutions. Within this category, we consider the anthropological theory of the didactic (Chevallard, 1992), together with the theory of didactic transposition (Chevallard, 1985) and the didactic analysis curriculum model (Rico, 1997).

The third set of approaches address how cultural, social, contextual factors impinge on the possibilities for that transformation, constraining or supporting curriculum reforms. In this sense, we include more socio-cultural approaches focusing on the analysis of institutional facilitators impacting on the processes that different communities adopt for curriculum reforms, as well as the approaches that focus on identifying and analysing factors affecting the co-creation and implementation of curriculum reforms by the different communities involved.

Approaches Focusing on Curriculum Conceptualisation

Concerning curriculum conceptualisation approaches, the first approach we address is the one presented by Steiner at the Osnabrück meeting (1980), recovered by Travers (1992), and more recently adapted by Mullis and Martin (2015) for the

TIMSS (Trends in International Mathematics and Science Study) assessment framework. In TIMSS, curriculum is broadly defined, as the major organising concept engaged with when considering how educational opportunities are provided to students and the factors that influence how students use these opportunities (Mullis, 2019, p. 4). The TIMSS curriculum model distinguished between the *intended curriculum*, the *implemented curriculum*, and the *attained curriculum*, as three different entities (see Fig. 13.1).

The *intended curriculum* is given by a document that is typically written by staff of national education bodies. Such documents generally identify the expectations of skills, competences and knowledge that students are supposed to reach once the curriculum is developed and being implemented. The *implemented curriculum* is found in school or classroom contexts and refers to the teaching-learning processes that, in fact, occur in them. Finally, the *attained curriculum* focuses on the achievement and attitudes of the students as they are shown in their performances in the tasks and tests.

Secondly, inspired by Kilpatrick's (1994) definition of the term 'curriculum' as, "an amalgam of goals, content, instruction and materials" (p. 7), Niss (2016) extended the framework of Mullis and Martin by proposing a definition of curriculum, with respect to a given educational setting, as a *vector* with six entries: goals; content; materials; forms of teaching; students' activities; assessment. According to the author, analysing an existing curriculum in a given educational setting then amounts to specifying each of these six components. Furthermore, implementing a given curriculum amounts to specifying it, as well as to carrying it out, i.e. putting all the six components into practice (Niss, 2018, p. 70).

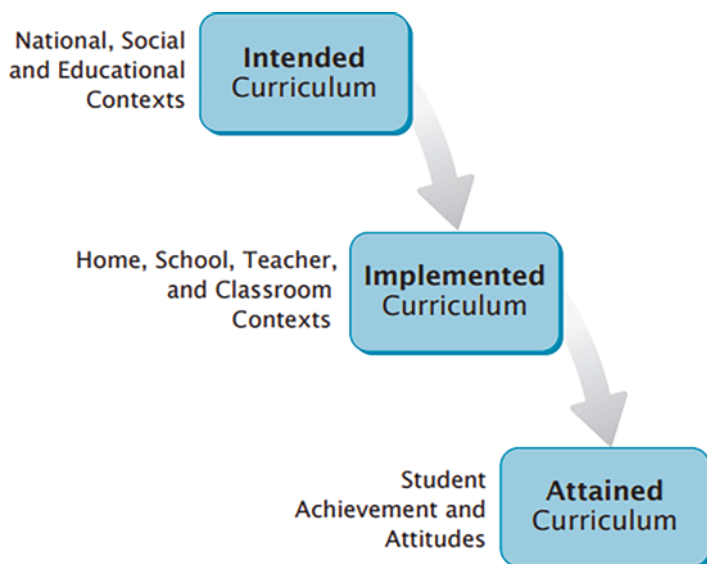


Fig. 13.1 The TIMSS curriculum model (Mullis, 2019, p. 4)

In addition, different agents intervene in the curriculum definition and, in particular, in defining the six entries distinguished. Each agent has more or less impact on some of these entries. For instance, curriculum authorities tend to retain control of the curricular goals, content and the summative components of assessment, at 'intended' level. But sometimes, these authorities may devolve some or total control of the other components (materials, forms of teaching and student activities) as well as the formative components of assessment, to external agents (textbook writers, assessment developers) and educators at the local level (schools, consultants, teachers). The Niss model, therefore, incorporates more aspects of the entire 'curriculum system' than the TIMSS model. Below, it will be seen that much recent work at least implicitly focuses on the latter, at some stage (usually 'intended' or 'enacted': we notice that few recent curriculum studies focus on the *attained* curriculum).

Approaches Focusing on the Didactic Process of Creation and Dissemination of Curriculum Reforms

Related to the second type of approaches, curriculum (as the intended scope of teaching and learning, at each level) is analysed by placing it in a wider didactic process of definition and transposition among different institutions. In this context, the object of study is similar to what defines curriculum in the previous sub-section, though the curricular documents, classroom analyses, etc. are compared and contrasted to mathematics in other institutions, epistemological analyses of content and cultural analyses. Agents and institutions affecting the development of curricular documents are included as objects of study.

In this sub-section, we first have the *anthropological theory of the didactic* (ATD) (Chevallard, 1992) which provides an approach to curriculum reforms by exploring the institutions and their dynamics involved in curriculum definition and dissemination. In particular, this approach refers to the notion of *ecology* (Chevallard, 2002) or *ecological analysis*: the study of the conditions that can facilitate, and the constraints that can limit, the teaching and learning practices. In particular, the conditions and constraints under which curricula are defined, reformed, and transposed are objects of analysis in order to understand the 'perturbations' of the ecological setting for teaching practices. As Artigue (2018) explains in her plenary presentation at ICMI Study 24 (see Conference Proceedings):

To question the implementation of curriculum reforms, which determines their success or failure, is therefore to try to understand the functioning of these particular dynamic systems in the face of the ecological disruption that is always a curriculum reform, and the means used to regulate these dynamics. [...] I consider curriculum reforms as ecological disruptions of education systems and the analysis of their implementation and effects as the study of the responses to these disruptions. (p. 43)

More concrete is the analysis of the *didactic transposition process* of curriculum reforms (see Fig. 13.2). One of the main contributions of the theory of didactic



Fig. 13.2 The didactic transposition process. (Chevallard, 1985)

transposition (Chevallard, 1985; Bosch & Gascón, 2006) is taking into account that, in order to analyse what knowledge can be taught and learnt, it is necessary to consider its institutional origin and the conditions and constraints for dissemination. This knowledge undergoes transformations from its production as *scholarly knowledge* to *knowledge to be taught* and, when it is transposed to school institutions and to particular classrooms, as *taught knowledge* and as *learned knowledge* by the community of study involved. Analysing a curriculum reform requires taking into account a diversity of institutions (and agents who occupy different institutional positions) for its (re-)definition and implementation.

As explained more recently by Chevallard (2018), in order to gain perspective on the notion of curriculum, we have to look at the curricular conundrum from the point of view of *society* as a whole. While society is made up of persons and of institutions, “institutional positions are thus the alpha and the omega of the curriculum issue” (p. 214).

The ATD adds another important tool for curriculum analysis: the notion of *praxeology* that appears as the basic unit into which one can analyse human action at large and, in particular, mathematical knowledge and practices. A praxeology is understood as an entity formed by four components: a type of tasks, a set of techniques, a technological discourse, and a theory; it is particularly useful as it provides a unitary vision of different activities. Praxeologies do not emerge suddenly, but are the result of ongoing processes, with complex dynamics, which require analysing what is happening in different institutions setting up the *knowledge to be taught*, and through curriculum and curriculum reforms.

To describe the set of conditions favouring and the constraints hindering the dissemination of certain praxeologies, another important tool in the ATD is the level of didactic co-determinacy (Chevallard, 2002) (see Fig. 13.3). This has been used as a methodological tool for *ecological analysis*, and to illustrate at which level, including those outside school systems, different conditions and constraints appear to support or limit curriculum reforms and their dissemination.

A further approach to curriculum reform is the *didactic analysis curriculum model* (Rico, 1997), which emerged from reviewing and articulating some classical curricular documents (e.g. Stenhouse, 1981; Steiner, 1980; Howson et al., 1981; Romberg, 1992) to elaborate a framework based on both dimensions and levels. This approach distinguishes four levels that expand *from* the particular actions in the *classroom* (first level), the *school system* (second level), *academic disciplines* (third level) and, finally, culminating in a more generic fourth, *teleological level*. Table 13.1

Fig. 13.3 Scale of levels of didactic co-determinacy

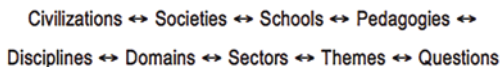


Table 13.1 The didactic analysis curriculum model (Rico, 1997)

	Cognitive dimension	Cultural dimension	Normative dimension	Social dimension
First level. Action in the classroom. Curriculum is assumed as a work planned by the teacher, based on the influences of the other levels.	Objectives	Contents	Methodology	Evaluation
Second level. School system. Curriculum is a planning instrument for the school system.	Pupils	Knowledge	Teachers	School
Third level. Academic disciplines. Curriculum is devised from disciplinary and erudite reflection, in which different academic disciplines approach and study its theoretical foundations and its technical implementation.	Learning theories	Mathematics, epistemology, history	Pedagogy	Sociology
Fourth level. Teleological. Curriculum is based on the different types of general goals: Cognitive, cultural, ethical, and social.	Training and development goals	Cultural and conceptual goals	Ethical and political goals	Social and utilitarian goals

outlines these four levels and dimensions considered and identifies the curricular elements – organisers – considered in each of these levels.

Socio-Cultural Approaches Focusing on the Conditions for Communities in Curriculum Reforms

The third type of theoretical framework focuses on the socio-cultural approaches to the construction and success of adoption of curriculum reforms. On the one hand, Boero (2018) presents the use of a framework derived from Habermas’ elaboration on rationality to deal with the cultural–epistemological orientation of curricular reforms. From this approach, it is proposed to look at the relations between the universal character of mathematics, and the cultures of the contexts where mathematics is taught and of those who are taught. The specific tools proposed aim to analyse the salient characters of different traditions and cultural practices, to identify contact points and differences among them, and to establish relationships between disciplinary culture of mathematics and other cultures, particularly when

implementing curricular content (such as teaching and learning modelling, or proof and proving).

On the other hand, other important socio-cultural frameworks may also be highlighted, more focus on the communities collaborating in curriculum reforms and on the conditions facilitating their success. Firstly, the *boundary-crossing approach* (Akkerman & Bakker, 2011) is used as a framework to analyse the collaboration and interaction of different communities, such as the communities of mathematicians, educational researchers and/or school teachers, often part of curriculum reform committees. Sometimes, this first approach is complemented by a framework providing tools for the analysis of discourses emerged in the context of different communities involved.

This is the case of the commognitive theory (Sfard, 2008), a theory based on the notion of commognition, which is premised on the conceptualisation of thinking as one's communication with oneself. The main objects of commognitive research are mathematical discourses, and more specifically the development of mathematical discourses. Within this theory, learning is a form of communication activity that can be conceived as inherently collective, or social, more than an individual phenomenon (Sfard, 2020).

Last but not least, we may mention the framework proposed by Memon (1997) who focuses on identifying the factors that are *enablers* or *inhibitors* of successful curriculum reforms. These factors are classified depending on whether they concern curriculum – primarily the intended curriculum, but also with some attention to other of the Niss components of curriculum that are about implementation – instruction or organisational conditions.

The author presents a number of inhibitors affecting curriculum change. These are divided into three categories. Curriculum factors include mismatch between the official and realised curriculum, not taking the needs of the teachers into consideration, external imposed innovation etc. Instructional factors cover elements such as students' interest, mismatch between a teacher's belief system and curriculum, how to create motivation and engagement, professional development and more. The last category is organisational factors covering influence of political leaders and bureaucracy, resources and physical facilities, communities of participation and other supportive structures. For a full list of factors, see Memon (1997).

Summary of Theoretical Frameworks Used to Address Curriculum Reforms

Above, we have briefly presented some of the most prominent theoretical approaches, organised around three main groups according to their focus. First, we have those which aim to provide definition and conceptualisation of curriculum; second, the more general approaches focused on the analysis of the epistemological and didactic processes used to scope the curriculum, and how curriculum reforms are

transposed to the different institutions for their definition, teaching and learning; and third, approaches focusing on how cultural, social, contextual factors impinge on the possibilities for that transformation, constraining or supporting curriculum reforms. As stated, our aim is not to compare the different theoretical approaches but to understand what these approaches aim to question, and what they do not, in curriculum reforms, so as to later be able to select some representative case studies to be described in more detail.

With this purpose, we introduce Fig. 13.4. which shows some tentative parallels among some of the theoretical approaches previously introduced. In particular, this figure presents the parallels among three of them: the TIMSS Curriculum model (Travers, 1992; Mullis, 2019), the theory of didactic transposition (Chevallard, 1985), and the didactic analysis curriculum model (Rico, 1997). There could certainly be other frameworks to include here, when looking for these tentative parallels, such as the one proposed by Niss (2018), but we have focused on these three particular theoretical frameworks as particularly useful to describe and delimit the lines of research on curriculum reforms in the next section.

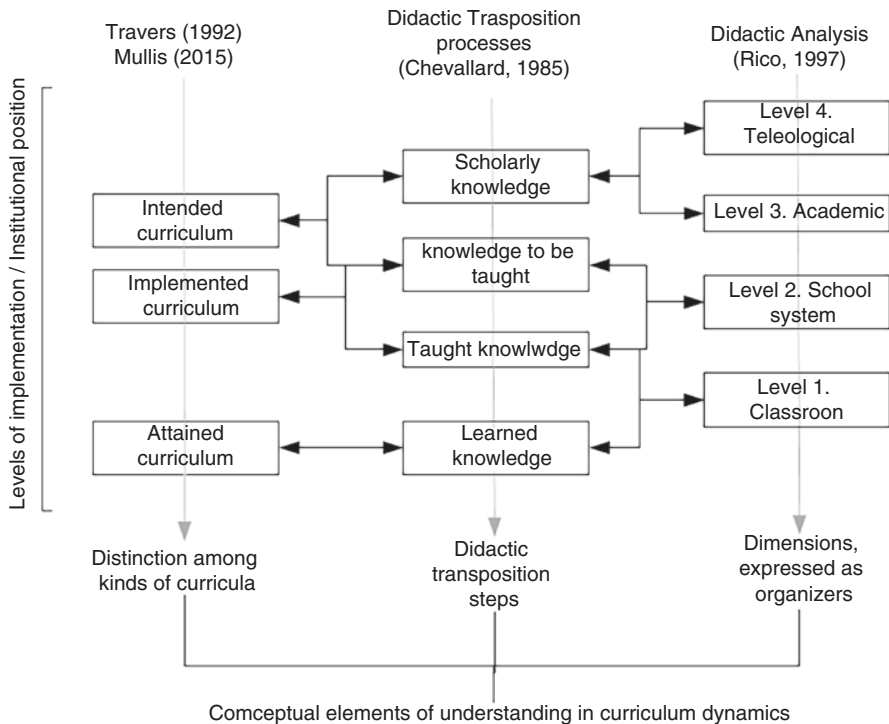


Fig. 13.4 Tentative parallelisms among some of the frameworks used to address curriculum reforms

The figure can be read both horizontally and vertically. A vertical reading allows identification of different levels of curriculum definition and/or implementation or institutional positions for each theoretical approach. A horizontal reading places some of the theoretical approaches previously described with some of their central theoretical constructs used to approach curriculum reforms. This horizontal reading also provides insights about possible relationships about these theoretical constructs. The arrows show relationships among some elements of the linked levels. For instance, the mathematics goals expressed in the curricular documents is an element that is usually included in: the ministerial curriculum (Niss, 2018), the intended curriculum (Mullis, 2019), the knowledge to be taught (Chevallard, 1985), and the school system level (Rico, 1997).

Another more general example could be developed when one refers to the “intended curriculum” (Mullis, 2019) which, in their definition, most of the time becomes the responsibility of the “scholarly institutions” (Chevallard, 1985) and the agents and institutions responsible (the “noosphere”) for agreeing the “knowledge to be taught”. In this process, the teleological and academic dimensions (Rico, 1997) emerge when the curriculum is analysed in relation to its cultural, social or disciplinary goals. But, its final form (in most of the countries) is the “official curriculum” whose authority lies beyond teachers’ community or students. This is the official ministerial (or other authorities’) curriculum (Niss, 2018) that is used then to regulate schools’, teachers’ and students’ practice.

This figure has helped us to stress some possible parallelisms that will undoubtedly need further research. But, more importantly for this chapter, this figure is used to delimit certain lines of research about curriculum reforms. More concretely, we distinguish five lines of research, depending on the choices these lines make about: (1) the kind(s) of curriculum taken as object of study: the intended, implemented and/or attained curricula; (2) the institutions considered in the delimitation of curricular knowledge: the scholarly institutions, the ‘noosphere’, the school institutions (and the classrooms) and/or the particular community of study (teacher/s with student/s); (3) the curricular elements considered at the teleological, academic, school and/or classroom levels. In the next section, not only these lines of research are presented, but also the selection of some case studies has allowed us to look at the particular research questions addressed, the unit of analysis considered, and the methodological choices and tools to problematise curricula reforms.

Research Questions About Curriculum Reforms, Unit of Analysis and Methodologies for Curricular Analysis

When curriculum reforms become the objects of study and research, the diversity of theoretical approaches that can be adopted inevitably delimit the unit of analysis taken into account. Hence there can be significant variation depending on the

theoretical framework and foci that are chosen. This delimitation can include different choices concerning the kind of curriculum considered, communities and institutions taken into consideration, and the curricular elements considered.

In this section, we distinguish among the main *lines of research* related to curriculum reforms that we have detected depending on what it is that they question and what they do not. In particular, we have identified the following lines of research, which are then further exemplified with some particular case studies that we consider as representatives of each line of research.

RL1: Research line questioning the *intended curriculum* through the interaction between the *scholarly knowledge* and the *knowledge to be taught*.

RL2: Research line questioning the selection and elaboration of the *knowledge to be taught* and of the resulting *intended curriculum* transposed to school systems.

RL3: Research line questioning the conditions under which *curriculum reforms* are *implemented*, through what means, under which constraints.

RL4: Research line questioning *teachers' actions on curriculum design* and *student attainment*: how the implementation of curriculum is planned and works in classrooms.

RL5: Research line questioning *communities* involved in curriculum reforms.

Table 13.2 summarises the research papers chosen, in correspondence to which research line, making also reference to the research framework(s) used.

Taking these particular cases, we aim to describe several aspects that characterise each particular area of research. In particular, and in order to unify their description, we focus on detecting: (1) the particular research questions addressed; (2) the unit of analysis considered and the particular empirical data taken into account; (3) the methodological choices and tools; (4) results and answers to the research questions.

Table 13.2 Case studies selected in relation to each research line

Research line	Paper(s) considered as case studies	Theoretical framework
RL1	Wijayanti and Bosch (2018)	ATD
RL2	Modeste (2018)	ATD
RL3	Hoyos et al. (2018) Lozano et al. (2018)	TIMSS curriculum model
RL4	Olsher and Yerushalmy (2018)	Didactic metadata
RL4	Ferretti et al. (2018)	TIMSS curriculum model
RL5	O'Meara et al. (2018)	Enablers and inhibitors impacting curricular reform
RL5	Pinto and Cooper (2018)	Commognitive theory Boundary crossing

Research Questioning the Intended Curriculum Through the Interaction Between the Scholarly Knowledge and the Knowledge to Be Taught

This first line of research focuses on the higher level of objects to be studied such as the *intended curriculum*, and the interaction between the *scholarly knowledge* and curriculum materials as ministerial documents and textbooks (see Fig. 13.5).

While Wijayanti and Bosch (2018) focused on intended curriculum, scholarly knowledge or the teleological and academic level, their analysis does also point to challenges regarding implementations and teachers’ practices. In particular, they develop a didactic transposition analysis to understand why *proportionality* is currently proposed as a particular piece of knowledge to be taught in school. They analyse how this particular mathematical concept has been defined by mathematicians through history, as being part of arithmetic, algebra, geometry or linked to the notion of functions (all representing scholarly knowledge), to analyse which aspects were transposed to be taught, how, and to identify any incoherence that emerges from this process. As the authors describe, they address the following research questions:

How can the didactic transposition process explain its [proportionality] current form? Where does the current knowledge to be taught about proportionality come from? Why does it have the form it has? How has it been selected, designated, shaped, organized and arranged? What is its role in relation to the other pieces of mathematical knowledge? (Wijayanti & Bosch, 2018, p. 174)

These research questions are explicitly linked to the methodology used, which they denote ‘didactic transposition methodology’. They analyse the ‘habitats’ of the praxeological organisations where the concept of proportionality has existed in scholarly knowledge, as in Euler’s *Elements of Algebra*, and how it has been transposed into western mathematics education, the effects of the ‘New Math’ reform, and through to today’s teaching of the concept. The units of analysis include empirical data such as reform documents, textbooks and analyses of former reforms such

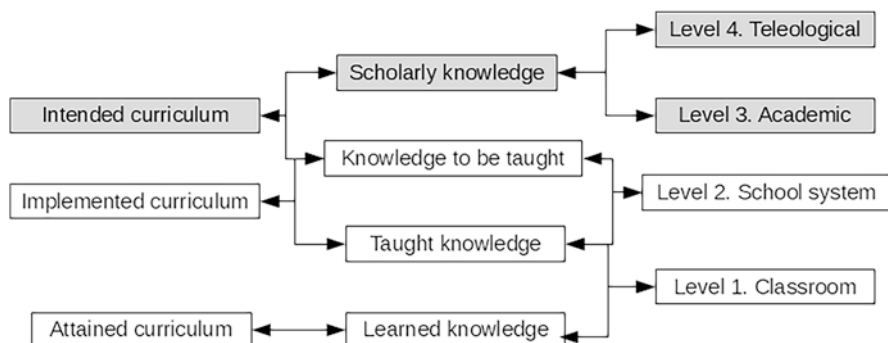


Fig. 13.5 Representation of what is questioned (in grey) in the first research line

as the New Math movement. In particular, the authors develop an analysis of the kind of mathematical praxeologies that exist concerning proportionality, both as a scholarly knowledge and as a knowledge that is planned to-be-taught. This epistemological analysis allows them to trace what, when and how the migration happened. As we see in the analysis of Artigue (2018), with the case of exponential functions in the French curriculum, the reforms have affected the praxeological organisation in which the curriculum sits, and exponential functions used to be linked with logarithms, but are now taught in relation to physics. The authors Wijayanti & Bosch (2018) conclude:

We are thus in front of blurred or hybrid organizations made up of pieces taken from different mathematical periods, mixing elements of different praxeologies that maintain redundancies and some incoherence in the kind of tools used. (p. 178)

Thus, the current teaching of proportionality is organised as entities drawing on elements from different former reforms that have quite different mathematical rationales. This results in the approach to teaching proportionality not being entirely mathematically coherent in its own right when compared to scholarly knowledge regarding proportionality. By carefully studying the historic development of the curricula, the authors manage to find the reasons for the current form of the praxeological organisation of proportionality.

Didactic transposition analysis has previously been used to analyse the notion of limit in Spanish upper secondary school. Findings indicated that often the practice block of the praxeological organisation was picked from one mathematical domain, whereas logos belonged to another (Barbé et al., 2005). In this case the incoherence is not historically based as in the work about proportionality by Wijayanti and Bosch, but rather stems from this fundamental disconnection arising from the transposition process.

Together, these findings constitute a genuine challenge for mathematics teachers needing to teach such inherently incoherent curriculum elements so that the mathematics still appears coherent and logical from the students' point of view.

Research Questioning the Selection and Elaboration of the Knowledge to Be Taught and of the Resulting Intended Curriculum Transposed to School Systems

The next line of research is also located at the higher levels of Fig. 13.4, though the main objects of study in this category are documents for implementation in terms of ministerial documents and teaching materials (see Fig. 13.6). The analysis focuses more on the agents and the transformations that the target knowledge undergoes when it is transformed from institutions producing it to the 'noosphere' and subsequently in agents' plans for framing its teaching in school systems.

Modeste (2018) presents one example with a clear research question, theoretical framework and methodology. This paper draws also on ATD when analysing the

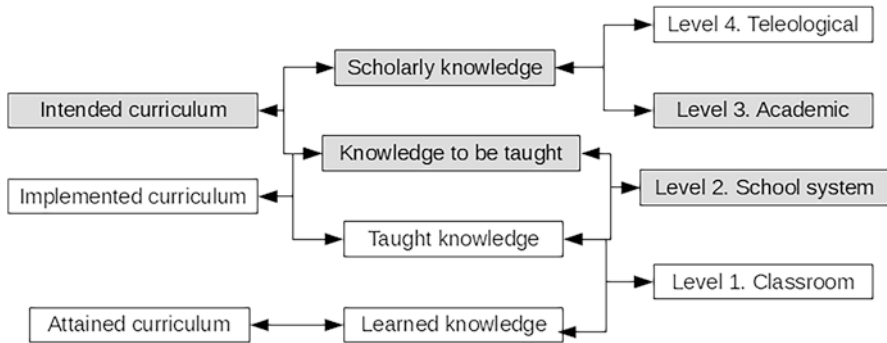


Fig. 13.6 Representation of what it is questioned (in grey) in the second research line

epistemological relations between scholarly knowledge of mathematics and that of computer science - and how that relationship is represented in curriculum documents. In particular, Modeste refers to praxeological organisations: how they are shaped and developed through didactic transposition processes and under which constraints and conditions these take place. He studies their ecology using the scale of levels of co-determinacy, where emphasis is put on the more generic levels beyond the mathematics discipline: that is, according to Chevallard (2002), the levels of the *society, school, pedagogy* and the interacting *disciplines*. The research questions pursued are:

What do Mathematics and Computer Science share as scientific disciplines and what kind of interactions between them can be developed in secondary school? How do the French curricula deal with this issue and in which direction are they developing? (Modeste, 2018, p. 277)

The methodology adopted is an analysis of (historic and current) documents produced by the ‘noosphere’ regarding knowledge to be taught from computer science as integrated components of mathematics, e.g. algorithms and programming. The unit of analysis considered includes empirical data (evaluation reports, international reports including the ICMI study (Howson & Wilson, 1986) from the first steps of the didactic transposition to analyse what is (and what is not) finally transposed. Modeste exemplifies this by analysing specific pieces of knowledge such as algorithmic thinking in the intended curricula. The analysis allows him to conclude that:

Computer Science is still looking for its place in the curriculum, and questions the territories of other scientific disciplines. As we have seen, the interactions with Mathematics are important in scholarly knowledge. [...] In the noosphere, many actors influence the didactical transposition of Computer Science which has a direct impact on Mathematics curriculum in the French educational context. In our view, an important issue is the place that a curriculum can lead to the interactions between Mathematics and Computer Science. (p. 283)

Thus, the theoretical framework and methodology allow the author to point out factors and agents affecting the ecology of the teaching and learning of different elements of computer science in mathematics and in more independent course

elements of secondary education. Furthermore, the analytic tools point to national, as well as international trends, and how these are related.

Other researchers also analyse intentions for curriculum reform or its development in relation to curricular documents, taking both scholarly knowledge, epistemological aspects and the academic level into consideration when analysing the content of the curriculum. Not all are guided in their analysis by theoretical constructs as scale of levels of co-determinacy and didactic transposition, though their object of study is similar to those addressed by Modeste (2018).

For example, studies that focus on the *intended curriculum* mostly analyse the *knowledge to be taught* and how the ‘noosphere’ defines what may be taught in a particular school system. For instance, Barquero et al. (2018) turn to an institutional approach using the ATD when analysing how the notion of inquiry has become part of mathematics curriculum across European countries. Lupiáñez & Ruiz-Hidalgo (2018) base their work on the didactic analysis approach to analyse the key notions – specific abilities, processes and active contextualisation – that provide the structure of Costa Rica’s most recent curriculum reform. There are also studies that, while not working explicitly with a framework for the analysis of reform efforts, do consider aspects of the construction of mathematics curriculum framed as ‘new challenges’, as in Nguyen’s (2018) approach to analysing the teaching of mathematical modelling.

Research Questioning the Conditions Under Which Curriculum Reforms Are Implemented, Through What Means, Under Which Constraints

We now turn to lines of research in which the units of analysis considered are objects more located in the *implemented curriculum*, in relation to the *intended curriculum*. This third line of research takes into account elements related to the *knowledge to be taught* in school systems and to the *taught knowledge* in particular classroom contexts. The scholarly knowledge and the selections that are represented in the intended curriculum are not questioned or challenged by this research line (Fig. 13.7).

Hoyos et al. (2018), who use the TIMSS curriculum model, provide such a case study, presenting a comparative study of the mathematics curriculum of primary (elementary) school education in Mexico. By considering the distinctions between the intended, implemented and attained curriculum, the paper presents an analysis of empirical data included in official documents relating to two important periods of curriculum reforms, in 1993 and in 2009/2011.

For this analysis, the authors inquired into the characteristics and coherence among these different types of curriculum based on the previous work of Suurtamm et al. (2018), asking, for example, “How is the curriculum in Mexico organized?” and “What is the role of evaluation in the intended curriculum and in the enacted?”

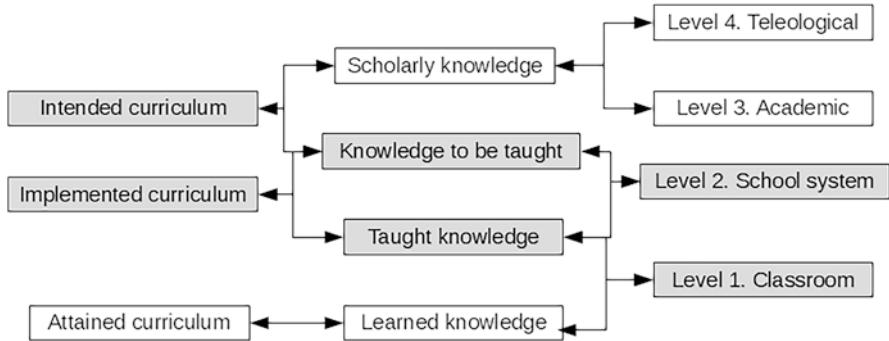


Fig. 13.7 Representation of what it is questioned (in grey) in the third research line

among others questions. They consider a broad unit of analysis including curriculum (general and theme-focused description), textbooks and evaluation results from PISA to analyse the impact of curriculum reforms and the possible connection between PISA results and curriculum reforms.

Specifically, they analyse the official curricula in 1993 and 2011, and focus on the general description of the curriculum and content description provided about adding fractions. They then consider some textbooks in order to analyse the implemented curriculum and contrast this with the approach of the intended curriculum in relation to the topic of adding fractions. Finally, they use selected data and results from the OECD's PISA 2003, PISA 2009 and PISA 2015 as an indication of the attained curriculum for Mexican students. What is noted is that there were minimal changes to the approach to mathematical concepts but an important change in the official discourse that the authors argue caused poorer levels of performance in PISA.

The applications of this broad approach are diverse. Other authors employ more specific frameworks for coherence (e.g. Golding, 2018) that evaluate the alignment between written curriculum, the available resources, the assessment system, and teachers' knowledge to facilitate these reforms, among other things. Giménez and Zabala (2018) combine theoretical approaches by presenting work on the design of a new curriculum from an interdisciplinary perspective, and offer several examples of projects that were consequently implemented in schools. Carvalho e Silva (2018) analyses the origin, rationale and development of the courses of mathematics applied to social sciences (MACS) in the Portuguese secondary school. As part of the analysis of the conditions created by several institutions that have enabled MACS to survive and thrive until the present (nearly two decades), the author discusses the role of the national examinations that may have put at risk the continuity of MACS courses.

A complementary approach is taken by Lozano et al. (2018) who compare and contrast reform initiatives taking place in Mexico and England, particularly paying attention to the resources. Due to the twofold space of research in Mexico and England, as well as the intention of enriching their understanding of the curriculum

reforms in both countries, they adopt an approach in which multiple perspectives interact and they assume an enactivist approach to methodology. From this point of view, the authors first analyse innovation in resources both in Mexico and in England independently, obtaining patterns for each case. They then distil meta-themes, such as explicitness of the curriculum, innovative approaches to the teaching concepts, pedagogical aspects, and teachers’ autonomy, that allow them to compare and illuminate the changes in both countries.

Research Questioning Teachers’ Actions and Engagement with How the Implementation of Curriculum Is Planned and Works in Classrooms

In this sub-section, we discuss the implementation of curriculum in classrooms, considering two specific areas of application of this line of research. The first explores teachers’ impact on curriculum design, while the second focuses on students’ mathematical activity and includes the attained curriculum as part of its unit of analysis. The theoretical approaches for the first are usually focused on specific mathematical domains (Fig. 13.8).

Regarding teachers, Olsher & Yerushalmy (2018) provide a case study that focuses on teachers’ role in shifting from an intended curriculum designed by others to co-designing the intended curriculum to be implemented. The authors analyse aspects of teachers’ expertise for designing curricular sequences, such as sequencing that avoids gaps in the mathematical progression, consistent and balanced handling of mathematical objects, and coherence with national curricula. The authors underline three key actions for personalising and managing any curricular sequence and use of interactive textbooks: recognising aspects of affordances of metadata that characterise the resources, developing an awareness of the balance among the learning objects, and developing an awareness of the rationale of the sequencing. Their approach adopts methods based on technological tools: a tagging tool to associate

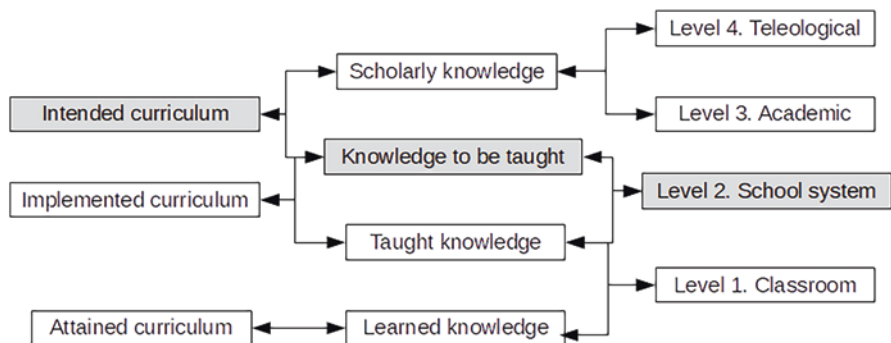


Fig. 13.8 Representation of what is questioned (in grey) in the fourth research line

metadata with individual learning resources, and a ‘dashboard’ for providing visual representations of didactic aspects of the intended curriculum, among other things.

In the discussion, the authors underline the role of contributions and methodologies from the domain of data analytics for several stakeholder groups – teachers, textbook authors, and policy makers. On the one hand, for the developers of learning resources there is the opportunity of realising that some tacit intentions should be better explained. On the other hand, teachers gain new insights in the author’s intentions. Olsher and Yerushalmy conclude the need for a more symmetrical approach between the variety of designer and practitioner communities that might better serve the evolving state of curriculum design.

Regarding student attainment, Ferretti et al. (2018) argue how the Italian standardised test can be used by teachers to interpret the intended curriculum. Though the authors do not explicitly mention the TIMSS Curriculum model, they locate the paper as linking intended and attained curriculum.

By means of a qualitative and quantitative analysis of the standardised tests, they foster discussion among teachers about the tasks’ features and possible student responses, showing that undertaking this kind of analysis can become a means for Italian teachers to engage with and reflect on the Italian Curriculum National Guidelines requirements. As a conclusion, the authors claim that Italian standardised assessment can be both a tool for policy makers for the acquisition of comparative information on students’ learning and also a vehicle for teachers to reflect on the goals for development of competencies as stated in the national guidelines.

Research Questioning Communities Involved in Curriculum Reforms¹

In this sub-section, we discuss Pinto and Cooper (2018) who provide a compelling case study in this line of research through their focus on analysing cases of cross-community interactions and collaboration in reform committees. The research question that guides their contribution is: how can members from different communities in mathematics education collaborate productively in curriculum and reform committees? More concretely, the authors reflect on the potential contributions that members from different communities in mathematics education make when taking part in curriculum reform committees that typically develop the intended curriculum. Their hypothesis is that cross-community collaboration significantly enriches the results of the discussion. However, such committees rarely capitalise on the opportunities of their diversity.

¹This sub-section analyses work with a particular socio-cultural orientation that does not itself lead to classification in terms of the diagram in Fig. 13.4. Hence, it does not include a diagrammatic representation of the approach.

Pinto and Cooper use the theoretical framework of commognitive theory (Sfard, 2008). They consider curricular discourse as the way in which individuals or communities communicate, think and act with regard to the mathematics curriculum. When committees work, the sociocultural differences of the comprising members are taken as differences in curricular discourse and provide commognitive conflicts. These conflicts force the individuals to make transitions and interactions across different points of view (boundary-crossing), which generate processes of learning by means of mechanisms of reflections and hybridisation. Though the work of reform and policy committees is usually confidential, the authors analyse data from various communities that are stakeholders in mathematics education. The findings show that boundary-crossing should be an explicit aim for committees, and the role of those participants that facilitate this boundary-crossing (named brokers) should be intentionally studied.

O'Meara et al. (2018) employ the framework of Memon (1997) to frame the barriers to a recent national mathematics curriculum reform in Ireland. The barriers were identified in three studies conducted locally to evaluate the implementation of Project Maths. Thus, 'Mind the Gap' sheds light on components that are 'Organisational Factors', the 'Time in Mathematics Education' (TiME) study investigates the 'Curriculum Factors', and the 'Teachers' Perception of Curriculum Reform' study mainly addresses components of the 'Instructional Factors'. All three studies are based on large online questionnaires distributed to 700 primary and 400 post-primary school teachers, exploring their experiences, viewpoints and beliefs. There is therefore a different unit of analysis from for the work of Pinto and Cooper described above, now focused on the primary and post-primary teacher communities.

They investigate teachers' perceptions of the recently reformed mathematics curriculum and identify any misalignments that exist between the beliefs held by teachers and the goals of the reformed curriculum. This study points to a reform effort that is reasonably coherent in terms of, for example, the six components of Niss' (2018) framework. It is noteworthy, however, that the implementation of the reformed curriculum was still challenged by too little attention paid to 'time' as an important component, a variable not explicitly considered in either the Memon (1997) framework or the other frameworks depicted in our Fig. 13.4. Their findings suggest that time is a critical and defining factor in the successful implication of curriculum reform. Teachers in the TiME study clearly indicated that time is impacting on their ability to implement the curriculum as intended, thus adversely impacting on students' opportunities to learn.

Conclusion and Key Messages

This chapter set out to provide an overview of the state of the art of theoretical frameworks and the associated methodologies used to address phenomena related to school mathematics curriculum reforms. Our approach has been to first identify the

main theoretical approaches (in the second section), and to organise those by the purposes to which they are typically put. We exemplify such purposes, and approaches, with case studies (in the third section) that show the diversity of lines of research addressing the complex reality of taking curriculum reforms as an object of study for research in mathematics education. We now summarise what we have found and critically analyse those findings to identify the further questions that arise.

The most striking finding from our work to identify theoretical approaches is that, whilst there are clear instances of rigorous and careful use of a theoretical framework to analyse curriculum reforms, such examples are in a distinct minority, both in the work presented to ICMI Study 24 – which has a focus on curriculum reforms – and beyond. In many instances we were unable to find explicit reference to any theory supporting researchers' analysis of curriculum reforms. In others there was what might be called a passing reference to theory without evidence of how it was applied in the analysis and the particular methodology followed.

We have started this chapter, in the second section, by presenting the most prominent theoretical approaches discussed in the different themes identified. We consider our analysis to be useful as it makes explicit the tools and methodologies the different approaches offer to analyse curriculum reforms. We present these theoretical frameworks according to three broad thematic categories, depending on their focus. First, we have those which aim to provide definition and conceptualisation of curriculum; second, the more general approaches focused on the analysis of the epistemological and didactic processes used to scope the curriculum, and how curriculum reforms are transposed to the different institutions for their definition, teaching and learning; and third, approaches focusing on how cultural, social, contextual factors impinge on the possibilities for that transformation, constraining or supporting curriculum reforms. Consideration of these three types of approaches led to the development of a schema (Fig. 13.4 in this chapter) that identifies 'levels of implementation/ institutional position' for each by reading vertically. The schema also identifies some tentative parallels or connections between the three broad categories of theoretical approaches by reading horizontally.

It is acknowledged that this schema represents a tentative means for a visual representation of the components of three quite diverse theoretical approaches and the connections between them, though we acknowledge there are more than three such we could have focused on. We have found it useful to describe the lines of research on curriculum reform in the previous section and, more concretely, to delimit the unit of analysis considered by the different case studies selected, with reference to that schema, validating its use in this chapter. Further work with, and development of, the schema may generate a more robust tool for identifying the components and connections when planning studies to address real research questions about mathematics curriculum reforms.

The current version is included also in the following explanation of the examples, but we have focus on these three particular theoretical frameworks as particularly useful to describe and delimit the lines of research on curriculum reforms we earlier analysed. The previous uses case studies to provide specific examples of

research on mathematics curriculum reform. The studies represented are diverse in many respects including geographic location, scale, specific focus and underpinning theoretical approach.

The case studies used are classified according to the line of research they represent: we have distinguished among five broad lines of research related to curriculum reforms, classified according to what it is questioned and what is not. We explore connections and coherence between specific components addressed in the case studies, within those broader research lines. These include the interaction between the scholarly knowledge and the knowledge to be taught in the context of the intended curriculum (RL1); between the knowledge to be taught and the resulting intended curriculum (RL2); in domains including the conditions under which curriculum reforms are implemented, with reference to means and constraints (RL3); teachers' actions on curriculum design and student attainment, with reference to how the implementation of curriculum is planned and works in classrooms (RL4); and the communities involved in curriculum reforms (RL5).

This is an indicative list of lines of research – it is likely that others are able to be identified. For example, the current international interest in comparative student achievement is likely to be generating research line(s) that consider the attained curriculum with others of the Niss components of curriculum. While some of the case studies refer to student attainment as evidence, the lack of representation of studies of student attainment in this collection could be an indication that such studies do not have a strong theoretical base and therefore lack scientific rigour.

The treatment of the main case studies follows the same pattern of identifying the specific research question(s) addressed, identifying the unit of analysis and data considered; outlining the methodological choices and tools used, and providing the key findings and answers to the research question(s). The diversity of the case studies included in the previous section is reflected in the wide range of findings about coherence (or lack of coherence) in mathematics curriculum reforms, aspects such as as challenges that are inherent in reforms, identification of unintended consequences, implications of teachers having an active role in reforms rather than being passive recipients, and lost opportunities, among others. Whilst such findings may well feature in studies that do not have a sound theoretical basis, the theoretical rigour of these studies should make them more credible and more worthy of attention.

In this chapter, we have only been able to scratch the surface of the intersections between mathematics curriculum reforms and the theories and methodologies used for studying them. As a result, the findings cannot be seen as more than indicative of some orientations that can be fruitful in studying reforms. In addition, we have developed a systematic approach that may well appeal to others who want to consider and learn from other analyses of curriculum reforms. Further use of the theoretical approaches will serve to validate, refine and extend the tools available for investigating and understanding phenomena related to mathematics curriculum reforms.

Despite these limitations, one finding that does stand out and is likely to be generally applicable is that – with notable exceptions, some of which are identified in this chapter – many reviews of mathematics curriculum reforms are not well supported by a clear theoretical basis that guides the methodology used. This necessarily limits the robustness of the analyses and, very likely, the impact of the work. The fact that this a-theoretical approach is so common can lead to the conclusion that many of those involved do not perceive having a well-defined theoretical framework as being important to their work – or that the theoretical framing is, unhelpfully, implicit only. The field of research on mathematics curriculum reform would be strengthened by an increased subjection to scrutiny of explicit theoretical underpinnings. The theoretically robust exceptions highlighted in this chapter and elsewhere can be considered examples of ‘good practice’ that can inform and set the scientific standard for future analyses of mathematics curriculum reforms.

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