Chapter 12
Coherence and Relevance of Materials and Technologies to Support Mathematics Curriculum Reforms

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In this chapter, we explore the role of the coherence and relevance of curriculum materials and technologies that support mathematics curriculum reforms. Taken together, we conceptualise those as (material) resources for teaching and learning. Although it is in many ways productive to conceive of, for example, student reasoning, or teacher knowledge of mathematics, as a resource for teaching and learning, we restrict ourselves here to physical (including digital) resources. Those include curriculum-related texts, whether digital or printed (and the former can be responsive), physical or virtual manipulatives that can range from plastic teddy bears through 'base ten' representations to mechanical simulations, and generic or subject-specific digital software that allows manipulation of mathematical representations such as graphs or geometric figures. In this sense, digital technologies, although offering distinctive affordances and constraints, can be construed as particular cases of curriculum materials, and here we consider them as such.

We take *coherence* of resources to refer to their internal and mathematical alignment, but also to the alignment of their designed use with the intended curriculum. We argue that curriculum materials and technologies used should also be *relevant* to the needs of the user, whether teacher or student, as well as to the intended curriculum and its valued uses. Otherwise, the user will not fully engage with the materials or technologies in the ways designed, which is likely to result in incoherence with the designer's curriculum-related intentions. McCallum (2018) argues that both coherence and relevance are necessary for learner meaning-making.

Below, we outline some general context in the field. We follow that with analyses of some recent developments around the globe, and use those 'case studies' to identify some of the ways in which curriculum resources, understood as above, can support both the coherence and the relevance of curriculum reform. We consider the

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constraints on that objective, and threats to its effectiveness, and conclude by drawing out some key messages for stakeholders and future research.

Background: Curriculum Resources Supporting Reform

Niss (2016) defines 'curriculum' for an educational setting, as a "vector with six entries – goals, content, materials, forms of teaching, student activities and assessments"(p. 240). These could be regarded as key components of an enacted curriculum, especially within the context of curriculum reforms. Similarly, Schmidt and Prawat (2006) talk about a 'curriculum system' as meaning much more than the intended totality of intended experience and learning within a formal educational environment, and including all major players, artefacts or identifiable capacities that have the potential to impinge on student experience in and related to the classroom: the written intended curriculum, the values and resources prevalent in surrounding communities at a variety of scales, the assessment system, available curriculum materials of whatever sort, teacher capacity – here, for change: their skills, knowledge and affect (Golding, 2017).

Of course, these elements are not independent, so that teacher capacity, for example, can be enhanced by engagement with suitable curriculum resources; in many cases the intended curriculum is built on teacher or other community input and so values, etc. Importantly, Schmidt and Prawat (2006) argue for the need for a deep-seated coherence of all aspects of the curriculum system if curriculum reform aspirations are to be met, since each has the potential to undermine or to support the achievement of that aspiration. Curriculum resources, then, are one critical aspect of the curriculum system.

We also note, though, that curriculum reform enactment is inherently contextually-bound and socially enacted (Gerrard & Farrell, 2013; Ball et al., 2012); and that, further, Supovitz and Weinbaum (2008), in the context of ambitious espoused change in the USA, identify persistent 'iterative refraction' of key messages at successive layers of interpretation from curriculum document writers to students in the classroom. Even if the system as a whole appears coherent, it is naïve to assume that central determination of curriculum intentions effectively leads to an experienced curriculum which exactly implements that which is envisaged, no matter how curriculum agency is framed within different societies and at different scales. We therefore find it helpful to talk in terms of aiming for (a range of) 'valid curriculum enactments', rather than for a single definitive such enactment.

We include in our considerations all textual resources, whether intended for teachers or students: textbooks, workbooks, teacher guides, often communicating curricular intentions, corresponding instructional plans, and support for enacting those. We do know these can influence what and how mathematics is taught, conveying specific views of mathematics and its organisation. Nico and Crespo (2006) show curricular materials can play a significant role in (elementary pre-service) teachers' learning. Remillard (2005) studied the textbook use of practising primary

teachers in the US engaging with large-scale professional change towards 'reform' curricula. She showed that the curriculum experienced by pupils showed significant variation, depending on teacher knowledge, beliefs about mathematics, students and about how students learn, and other teacher orientations towards the materials. Not only did many 'reform-oriented' materials place an emphasis on pedagogical guidance, promoting teaching practices that for many teachers required considerable re-orientation, but many teachers then used them in ways that undermined authors' intentions.

Importantly, Stein and Kaufman (2010) found that teachers who engaged with descriptions that articulated the central mathematical ideas of a lesson were more likely to enact tasks in ways that reflected the intentions of the curriculum. Remillard, Harris and Agodini (2014) further showed that different sets of primary age reform curriculum materials developed to align with different theories of learning varied significantly in instructional approach, mathematical emphasis (the mathematics knowledge and practice that are valued and the quality and treatment of mathematics in the curriculum) and support for teachers, and that this led to significantly different learning outcomes even after just one year of use of such materials.

In times of curriculum reform, then, textual materials have potential to deeply inform and influence teacher – and student – practice and thinking, particularly when reforms involve changes in learning approach or mathematical priorities or paradigm: they can directly communicate key fundamental principles intended to be then interpreted, and embedded in classrooms. However, Drake and Sherin (2006) also argue that teachers' narrative identities as learners and teachers of mathematics, which incorporate their past experiences with curriculum and with teaching, fundamentally frame the ways in which they use and adapt a new and challenging mathematics curriculum.

Ideally, then, if curriculum materials are to fully inform classroom enactment that is coherent with intentions, they should contain additional supports, communicating to teachers likely student thinking and misconceptions, key mathematical ideas, and the rationale behind particular design decisions, as well as the range of possible teacher and learner roles within that. Davis and Krajcik (2005) refer to such materials as *educative* because they aim to support teachers in developing practice aligned with curriculum intentions. It is important to note that there is comparatively little evidence around the impact of textual materials on student mathematical functioning or affect.

Of course, textual curriculum materials vary in quality and in appropriateness for a particular context, and so they, in common with all other curriculum materials, are dependent on teacher (and student) choices to realise their potential for supporting curriculum change. Oates (2014) concludes his review of printed textual resources by arguing that the highest quality materials reviewed (judged to be most effective for supporting enactment of curriculum intentions):

- were underpinned by well-grounded learning and subject-specific content theory;
- included coherent learning progressions within and across the subject;
- stimulated and supported learner reflection;

- featured varied application of concepts and principles – 'expansive application'; and
- controlled surface and structural features of texts to ensure consistency with underpinning learning theory.

Such resources cannot be developed overnight, which is one tension in any curriculum reform enacted on a short timescale, such as the English 2014 national curriculum outlined below.

Teacher agency, and the choices and learning opportunities available to teachers through the use of curriculum resources, are analysed by Remillard and colleagues (2009) in terms of their 'structure, look, voice, medium and genre', with corresponding messages for how teachers are positioned in relation to materials and so, how they are likely to interpret texts. Even so, these authors show that teachers working with the same materials might focus on very different 'reading' of the text for activities, for script or for 'big ideas'. For students, teachers mediate curriculum material use both directly and indirectly – but, for example, Rezat (2009) shows student response to, and use of, texts impact also on the choices made by teachers.

Gueudet and Trouche (2009) harness Rabardel's (1995) development of 'instrumentation' to develop a theory of 'documentation work' that encompasses the complex and interactive ways in which teachers, as individuals and groups, come to work with the range of curriculum-related resources, arguing that these are strongly intertwined with teachers' professional development, and therefore, far from static – and also foregrounding the interaction of teachers with resources, that can symbiotically transform. Research on student interaction with curriculum resources, and particularly textbooks, is much less well-developed, although there is a corpus developed around the impact on thinking of student interaction with digital resources, which could equally be conceptualised as instrumentation work.

To date, in general, though, we know less about the particular curriculum reform-supportive potential of digital texts and blended learning. They have potential benefits of easy updating and other editing, and for users, of availability anywhere there is web access. However, we have much less evidence of the potential impact on teachers and learners of their selection, use and shaping in pedagogical discourse. Gould (2011) uses examples from both printed and electronic textbooks to discuss how educational design features can help align the medium of presentation with the content, emphasising that digital texts can provide *different* affordances and constraints in learning mathematics.

The range of curriculum texts, then, have the potential to communicate curriculum in ways that support teacher sense-making of, and adaptation to, reform intentions, especially if they are also educative in nature. Where the communicated 'intended curriculum' does not encompass all aspects of Niss's vector, or of Schmidt and Prawat's 'curriculum system', texts have the potential to offer definition or concretisation of the curriculum – and might be used to do so even where such 'official' interpretation exists elsewhere. The corpus described above, though, shows curriculum work with resources is a complex and highly contextualised process.

The potential of curriculum texts is centrally important to many recent reforms across the globe, where the high-level 'doing mathematics' tasks often valued by twenty-first century intended curricula require deeply informed, selective and creative thinking, frequently prompting student anxiety and opening up classroom discourse in ways that are challenging for many mathematics teachers to manage, especially if they are unfamiliar with such approaches. Related curriculum link-making, designed to deepen conceptual grasp, is perhaps less demanding on teacher skills and subject-specific knowledge, but still more so than the more procedural approaches common in many classrooms historically. Further, curriculum texts necessarily reflect a particular philosophical and/or theoretical approach to enactment, and for coherent messages to learners, it is important both that these are made explicit and that teachers align their understanding and enactment with the espoused approach at a fairly deep level.

Such texts then have the potential to support longitudinal coherence of the experienced curriculum for learners. In early stages of enactment, curriculum materials can carry a considerable share of the instructional load, but after initial engagement with tasks, and for embedded coherence, we know that positive engagement of the teacher with the text, if necessary as learner, become critical (Fullan, 2004). Without that, there is a risk that enactment remains only superficially coherent with curriculum intentions, and in particular with teacher meaning-making of key mathematical concepts and/or processes poorly aligned. Such issues are exemplified below.

Non-textual Resources

There is evidence that the deliberate harnessing of non-textual resources such as concrete manipulatives, has the potential to impact the formation and retention of mathematical concepts and procedures, particularly if careful bridging to symbolic and abstract thinking is supported (Carbonneau et al., 2013). There is a challenge, however, in transferring learning associated with manipulatives to the abstract concepts they represent (Nunes et al., 2009), so that it is helpful to frame such use within curriculum documentation as the Australian curriculum does with technology, in terms of learners engaging with ideas 'both with and without manipulatives': Coles and Sinclair (2019) frame this as engaging with "symbolically structured environments" (p. 470). However, little literature focuses on the role of such materials to support curriculum reform. Exceptions include, for example, that dealing with Singapore's post-1981 primary curriculum, developed with a key focus on a Concrete-Pictorial-Abstract approach to the teaching and learning of mathematics, in which concrete materials play a key role in the intended curriculum throughout the primary phase (Kaur, 2014), and Nigeria's Millennium Development Goal-centred redevelopment of promoted pedagogy (Adeniyi et al., 2013).

Digital manipulatives can go beyond dynamic digital representations of concrete manipulables, whose physical forms are only slowly manipulable, to include for example graphical representations. Suh, Moyer and Heo (2005) suggest that by

allowing students to manipulate digital objects to test hypotheses and experiment with ideas, the virtual manipulatives may more closely model the dynamic nature of thinking, which, in turn, may enhance students' thinking and creativity; however, Hunt and colleagues (2011) offer evidence that digital manipulatives may instead be differently, rather than preferentially, supportive of learning. Both sets of authors note that it is important to facilitate connection-making between different modes of representation of mathematical concepts so as to develop students' representational fluency. Some recently-developed curricula, such as that described by Kaur (2014) or the NCTM (2000) 'Principles and standards for school mathematics', explicitly provide for digital manipulatives in the authors' communicated visions of twenty-first century mathematics teaching and learning enriched by informed use of educational technology, and embrace such approaches as key to supporting meaning-making.

The Role of Resources in Recent Curriculum Reforms

We now exemplify the use of materials in recent curriculum reform initiatives to support consideration of how their coherence and relevance might function to enhance, or sometimes undermine, valid enactment of reformed curricula – and what the constraints might be. To do so, we draw on five case studies of recent reforms. The first three focus on top-down national-scale intentions in England, Mexico and Vietnam, respectively. **England** operates under a centralised, politically-controlled curriculum with distributed, market-driven provision of resources and high-stakes assessments. Golding (2018) shows that although serious attempts at systemic curriculum coherence have been made, the coherence achieved was fragile. **Mexico** also has a centrally controlled system, and has in the last 30 years achieved extension of universal education from primary to age 15. For the 1993 national curriculum, materials and teacher development were coherent with curriculum intentions. However, many teachers were not fully equipped to meet the demands of the mathematically more ambitious curriculum introduced from 2011, and did not in 2018 generally have curriculum resource support coherent with intentions; further, it appears learner performance may have dropped (Hoyos et al., 2018). In **Vietnam**, espousal of comparably ambitious curricula at university entrance level was not supported by production of (centrally accredited) textbooks coherent with that, in a culture historically dependent on textbooks and with conservative pedagogic traditions (Trung & Phat, 2018): early outcomes were unsurprisingly incoherent with intentions.

'Curricular reforms' are often generally conceptualised as centralised, large-scale initiatives, but there are also promising approaches and material developments that are bottom-up, capitalising on the intrinsic relevance to participants of such initiatives. Bonissoni and colleagues (2018) focus on local bottom-up development of pedagogy for teaching fractions in **Italy**, tackled using novel disciplinary approaches for which naïve concrete representations are central, offering key

relevance to learners. This exposes tensions between relevance and coherence – across the mathematics curriculum and with the parent discipline. In a larger-scale approach to curriculum development, Barabash (2018) focuses on a national curriculum initiative in **Israel**, where early careful approaches to the development of renewed approaches to geometry, together with expansion of school mathematics epistemologies, have led to collaborative design of supportive digital resources, instructive exemplification and draft assessments. Even though there was a measured and deliberate introduction of the reforms, this initiative produced tensions in coherence in the absence of timely planned teacher development and textbooks.

Finally, one challenge associated with more teacher-led bottom-up curriculum material selection, is that of maintaining coherence of the *experienced* curriculum, and Olsher and Yerushalmy (2018) address the development of digital tools to 'tag' and monitor the curricular profile (content and processes) of the selections made, so that for well-informed teachers, the mathematical coherence of the resultant planned curriculum can be monitored and sustained. Visnovska, Cobb and Dean (2012) evidence just how ambitious a task that is.

We now analyse the above situations in greater detail, as 'case studies' of differently coherent resources intended to support reform in five jurisdictions: the categorisations suggested are subjective, but intended to point to the complexity of understanding such relationships.

Case Study 1: England's National Curriculum and Related Post-16 mathematics Provision from 2014 (Top-Down, Time-Pressured Reform with Initial Attempted Coherence)

This centrally-developed, highly aspirational curriculum drew on studies of curricula in high-performing jurisdictions. It features a renewed emphasis on deep conceptual fluency, mathematical reasoning and problem-solving, arguably intrinsically of more relevance to a technology-rich century, than a curriculum focused on facts and procedures. For upper secondary students, changes included attempts to further enhance relevance through mandatory engagement with data handling software to study a 'large data set'. The range of mathematical intentions espoused are widely-valued though there were concerns about speed and scale of introduction, and ambition: a new curriculum for all English 5–16-year-olds was introduced over two academic years, after just 2 years' central planning and preparation, and no time for piloting of curriculum teaching, resource support, or assessment, and very little for teacher professional development.

In England, both assessment and curriculum material provision operate in a market, with assessment heavily constrained by the government-funded body responsible for assessment. Reasonable scale 2–3-year longitudinal studies of the impact of assessments and curriculum materials developed by the major provider, and analysed in Golding (2018), show initial resources were, despite the challenges,

coherent with, and supportive of in-classroom progress towards, curriculum intentions. Teachers had to make considerable investment in areas of content and process unfamiliar to them, but were supported in doing so by the materials, which included deeply 'teacher educative' (Davis & Krajcik, 2005) elements such as consideration of necessary prerequisites, of common student questions and misconceptions, and pointers to the related mathematical progression and links. There was mixed reception and use of digital elements of the materials by both teachers and learners.

Over time, many teachers were able to make good progress towards the changes envisaged in teaching and learning, and reported that the focus materials helped them do so in valid ways that economised on preparation time. Learner progress towards confident mathematical engagement post-16 was closely correlated with differential teacher curriculum enactment, though few relatively weak students appeared to thrive in early enactment. Continued progress towards curriculum-aligned teaching and learning at scale, though, appeared fragile, threatened by market-driven assessment, speed of introduction, and high-stakes outcome metrics that mean teachers and students commonly privilege curriculum interpretation in assessment-related resources over that in curriculum-aligned support materials (Golding, 2018). There is a clear threat to sustained coherence of the curriculum system in such marketized and high-stakes assessment contexts.

Case Study 2: Curriculum reforms in Mexico, 1993 to 2011 (Top-Down, Variably Coherent Curriculum System Reform)

Mexico's compulsory education was extended from age 12 to age 15 in 1992; new curricula followed in 1993 and again in 2011. The intended changes over that time have much in common with those described for England, above: a move towards greater emphasis on key mathematical processes of problem-solving and reasoning, with their associated communication, together with flexible, integrated mastery of core knowledge and techniques. For example, in 2011:

It is expected that students develop the following mathematical competencies:

- solving problems autonomously;
- communicating mathematical information;
- validating procedures and results;
- efficient handling of techniques.

The promoted teaching approach is constructivist, building on Brousseau's work and supporting progression from concrete to abstract. Mexican policy is for free distribution of one set of official textbooks for each grade. For the 1993 curriculum, an official mathematics textbook was produced, together with a series of activity books and an 'educative' teacher's guide. Hoyos et al. (2018) argue that in general, in 1993 these materials were coherent with the theoretical approach and content adopted in the written intended curriculum, as was the associated in-service teacher

development provided, and that these together supported enactment of the reform intentions. In contrast, for the 2011 curriculum reform, no such teacher development was offered, and the new curriculum materials were not entirely coherent with curriculum intentions, being also less transparent in their support for teachers.

Between the two written curricula there appears little change in the content targeted, and where there are changes made, their alignment with enhanced curriculum intentions is now always clear: for example, under addition and subtraction of fractions there is a move to drop specific mention of manipulatives and of games to underpin meaning-making. 'Informal procedures' are introduced, but their use not followed through. Importantly, for the 2011 curriculum there was no large-scale official textbook series produced, and no national teacher development programme to support practices coherent with curriculum intentions. Teachers consequently had to adapt practice and textbook use to accommodate new emphases. Far from being 'educative' for the new curriculum in Davis and Krajcik's (2005) terms, such work requires sophisticated and subject-knowledgeable instrumentation.

As a consequence, only the best prepared and mathematically-knowledgeable primary teachers, skilled at developing their own materials for the class, could undertake the new approach with clarity, in ways coherent with the approaches intended. Hoyos and colleagues (2018) suggest that the enhanced aspirations of eh 2011 curriculum, consequently resulted in rather poorer quality mathematics curriculum experiences for man children in less privileged (in terms of teacher preparedness) classrooms. It is striking that in PISA assessments of learner cohorts spanning this change, the percentage of Mexican students that in PISA 2009 were *below level 2* (i.e. attaining the level 1 or zero) was 51%, rising to 57% in PISA 2015, perhaps evidencing an early increase in the proportion of Mexican students in the poorest levels of performance, though in comparatively early days of the intended reform. (Hoyos et al., 2018).

Case Study 3: University Entrance Curriculum reform in Vietnam (Top-Down, Not Yet Coherent with Supporting School Curriculum Materials)

As described in Chap. 10, Trung and Phat (2018) present a Vietnamese central intention to move towards, again, a greater valuing of conceptual mastery and engagement with mathematical processes. As elsewhere, such intentions bring with them challenges for teachers in valid enactment. They describe a cultural norm, and dominant approach, for mathematics teachers in Vietnam of a focus on procedure and memorisation, coupled with close adherence to content presented in textbooks – and available textbooks have not yet moved to align well with curriculum intentions. In parallel, the high-stakes university entrance examination has become the *de facto* high school graduation examination, yet the conceptually-oriented questions common in that examination since 2017 are poorly represented in approved

textbooks. Specific examples illustrate the dissonance between assessment and curriculum/textbooks. In this case the assessment would appear to be coherent with the intended curriculum – it is the textbooks that are yet to be similarly developed. However, the result for students – and teachers – is that curriculum intentions are not coherent with, and so well supported by, available resources.

Case Study 4: Bottom-Up Italian Development of a Radical Approach to the Fraction-Related Curriculum (Focus on Use of Naïve Concrete Materials to Build Relevance, But with Exposed Tensions for Coherence)

The development of multiple conceptualisations of fractions and related operations is widely recognised as problematic and so is well-represented in the literature, though with few clear pathways to meaning-making at scale. Radical, if yet small-scale, approaches therefore have potential to inform pedagogical approaches that can be taken to scale. The approach of Bonissoni and colleagues (2018) harnesses the familiar natural division of egg boxes of various sizes, so improving relevance and authenticity for grade 3/4 learners via 'intuitive representation' as opposed to 'primitive intuition'. It uses the comparison of 'number of sweets' with 'number of complete egg boxes' to provide an ordered pair identified as a fraction, so privileging the Pythagorean concept of ratio (logos). The approach derives from the historical evolution of the concept of fraction, introducing a mega-concept of fraction from which different sub-constructs are then interwoven. The range of sub-constructs introduced is therefore intrinsically internally coherent. As yet the intervention is only small-scale, and coherence with existing teacher conceptualisations of fractions, and their current didactic practices, have still to be worked through.

This work highlights persistent tensions between relevance and coherence in this context, given also the naïve conceptions of fractions as part-whole that children bring with them to school, and the challenge in relating the promoted representations to later mathematical conceptions. The approach has high relevance, but is not entirely coherent with some of the mathematical structures targeted later in the curriculum.

Case Study 5: Reform of the Israeli Intermediate Geometry Curriculum (Negotiated, Measured Building of Systemic Coherence and Relevance)

This initiative is discussed in more detail in Chap. 10, but we point to it here as an example of the time and co-ordinated effort that is needed to develop a fully coherent curriculum, even for a limited grade and student population target.

A measured and collaborative Israeli curriculum review began in 2014–2015 and continues, led by accountable program committees comprised of mathematicians, mathematics educators, Ministry of Education subject representatives and curriculum specialists, and experienced mathematics teachers. Together they have produced a new geometry curriculum for the second quartile of students. It integrates analytic geometry, trigonometry, and synthetic geometry, linking mathematical rigour with the development of intuition and valid visualisation-based reasoning, embracing possibilities created by dynamic geometry environments (DGEs), and applying ideas of experimental mathematics to high-school geometry. Sets of examination questions coherent with those intentions, together with curriculum enrichment examples, have been produced and exemplified. The committee is now seeking to develop coherent textbook and software, as well as appropriate teacher development opportunities, in the time to first curriculum enactment in 2021, and is confident that the approach adopted will result in a coherent and stable curriculum system (Barabash, 2018).

Case Study 6: Harnessing Technology to Improve Intended/ Enacted Curricular Coherence Across Domains and Levels of Teaching, in Bottom-Up Curriculum Development

This case study is rather different as it does not sit within a national reform context. In cultures where teachers commonly supplement any central resources with their own choices of digital or other materials, they need to be able to design curricular sequences skilfully: in particular, teachers need to be sensitive to aspects of curricular coherence, such as continuous mathematical progression, epistemological coherence and alignment with the Goals and Content of the intended national curricula. It should be noted that assumptions about teachers' capacity for such work have been problematised by e.g. Cobb (1999), who argues that the design of a coherent instructional sequence requires specialist support and development, even if teachers work collaboratively.

However, Olsher and Yerushalmy (2018) present tools which enable evaluation of the nature and balance of a collection of learning resources, developed to support teachers with a reasonable grasp of the discipline and its learning: a tagging tool that associates didactic metadata with individual learning resources, and a 'dashboard' representing didactic aspects of the curriculum, for visualising and navigating a tagged collection or textbook. This emerging work respects teachers' professional judgement of resources and promotes connections between teachers, researchers, administrators, authors on an equal footing in the processes of curriculum development; it offers a tool for evaluation and selection of available resources for teachers' identified purposes. It is suggested that the related teacherly judgment could be further developed through collaborative approaches to tagging.

Curriculum Materials: Affordances for Supporting Curriculum Reform Coherence and Relevance

These case studies, set in their wider supporting literature, show how curriculum materials can support relevance, including through specific, sometiomes naïve, manipulatives to support particular pedagogical purpose (for example, modelling concepts of fractions as numbers and fractions as division) (Bonissoni et al., 2018) and support for creation of pathways to mathematically coherent sequencing (Olsher & Yerushalmy, 2018). Curriculum materials are, moreover, widely used to further relevance of curriculum pathways for different groups of students (mathematical 'sense-making'– McCallum, 2018), for example as memorable representations, or by making links with personal or occupational pathways or of the societal, including scientific, world. This relevance might be in support of curricula which are 'nested' so that different students engage with nested subsets of material, though in principle with similar depths and breadths of the content tackled, such as in Singapore (Kaur, 2014) or England (Golding, 2018). Alternatively, resources might support different curricula for groups of students with differing post-school aspirations, as in Portugal (Carvalho e Silva, 2018).

The case studies above exemplify the positive benefits teachers can derive from engaging with curriculum resources that are well-aligned with curriculum intentions and are preferably also teacher-educative – provided the underlying curriculum is internally coherent, and coherent also with the mathematical and wider needs of the target students. The 2019 International Textbook Summit (Royal Society) suggested that such resources can contribute to good use of teacher time – and Golding (2018) found teachers of all phases of ages 5–18 claimed they saved planning time when they moved to working primarily with a single set of trusted resources, compared with selecting their own. They were therefore able to develop a better 'sense' of the intended curriculum and teach more coherent lesson sequences. Teachers did, though, note that making good use of educative resources demands an investment in getting to know the approach, the structure, and the dynamics of the resource. However, that investment supported their own professional development, particularly of subject-specific knowledge and pedagogic knowledge, as well as their confidence – again, supporting their capacity to teach in ways coherent with curriculum intentions.

For knowledgeable teachers, or groups within whom lies sufficient knowledge, there is a valuable teacherly role in involvement in the design of materials to support, or even drive, curriculum change and such development can be empowering, supporting a relevance sometimes harder to achieve in materials brought in from outside (Barabash, 2018; Bonissoni et al., 2018), although bringing with it also a challenge if there is a need to scale up from there, since any small-scale development is necessarily locally contextualised.

The range of such developments, then, potentially have educative purposes for both the teacher and the learner. In particular, recent work suggests curriculum-coherent, and particularly teacher-educative, materials can support curriculum

aspirations to educate young people for appreciation of wider societal challenges (Giménez & Zabala, 2018), for cross-curricular thinking (Lupiañez et al., 2018), and for purposeful engagement with twentieth century technologies (Barabash, 2018). A priori reasoning would suggest that resources aimed solely at students should similarly feature coherence with curriculum intentions, and relevance to the young people concerned and the related educational goals. However, as indicated above, there is not yet a well-developed body of work focused on school students' use of resources for learning mathematics.

Given the thrust of much current debate about the future of education in a technology-pervasive world, we give brief additional attention to the potential of digital technologies for supporting curricular coherence, and relevance to students' current and future needs.

Digital Tools Supporting Coherence and Relevance of Enacted Curricula

Purposively-integrated use of digital technologies clearly has the potential to complement traditional approaches and enhance relevance to students of the experienced curriculum for the twenty-first-century. These technologies can support curriculum-relevant computational thinking, and the acquisition, exploration, representation, interrogation and interpretation of a variety of real and realistic data, including large data sets or 'big data'. They offer a variety of modes of communication, teacher to/from student, student to student, or other, including globally, that can again enhance meaning-making and a variety of link-making across representations and conceptualisations. In so doing, digital technologies can bring external expertise into the classroom, and build wider digital literacy, potentially enhancing both curriculum coherence and its relevance to current wider issues.

Golding (2018) evidences the use of text-hyperlinked sources for these purposes, enriching the meaning-making accessible to both teachers and students. In terms of internal mathematical coherence, dynamic software and bespoke digital packages can support inductive exploration and reasoning with curriculum concepts (Barabash, 2018), as well as independent and immediately responsive, non-judgmental self-assessment, and so ownership (and relevance) for learners. Additionally, responsive technologies can support increased sense-making of the experienced curriculum (Golding, 2018). However, a rapidly increasing body of work evidences that the conditions necessary to reliably achieve such desirable outcomes can be quite complex; for example, the TPACK framework (Mishra & Koehler, 2006) identifies the multiple knowledge bases on which effective teachers draw when they teach mathematics with technology.

For teachers, technological affordances have potential to support teacher subject-specific development, and so curriculum-coherent values and approaches, whether through engagement with professional development packages or software

principally aimed at students (Golding, 2018). They have a central place in twenty-first century bottom-up mathematics curriculum-making (Barabash, 2018), as well as supporting mathematical coherence of locally-developed curricula (Olsher & Yerushalmy, 2018).

Dynamic digital tools are sometimes included as part of approved curriculum material packages, as described, for example for fractions in Singapore, in Lee and Ferrucci (2012). Such packages appear to engage students in their learning, positively impact progress compared with non-manipulative use (sometimes, including comparison with concrete manipulatives), and can have a positive effect on narrowing the range of students' achievement, as well as supporting both thinking and creativity. However, there may still remain a novelty effect of such use, and large-scale studies often show mixed outcomes. Digital tools, then, *can* support both deductive and inductive approaches, as well as exploratory and experimental work, and bring with them very real benefits for increased student agency, engagement and meaning-making – but our characterisation of those aspects of tools and of teaching which are necessary for such benefits, is not yet well-developed in many instances. Consequently, digital tools often have real, though not always realised, potential for contributing to curriculum coherence, as well as to its relevance.

Curriculum Materials: Constraints for Supporting Curriculum Reform Coherence and Relevance

It is important to note that materials can also be constrained in their impact on experienced coherence – or relevance – if, for example, they are produced in haste, with inadequate investment of money, time or effort, or by resource developers, central or local, whose beliefs, attitudes, knowledge or curriculum-making skills are not fully coherent with curriculum intentions – or with extant teacher or student knowledge resource. Such limitations can lead to superficial, or worse, mathematically incoherent or irrelevant resources (Hoyos et al., 2018; Trung & Phat, 2018), or those which simply lack transparency of objectives or enactment intentions.

If materials are to fully support robust curriculum coherence, developers have to communicate with teachers and students – consistently and in depth – the full range of curriculum intentions, at all levels, in ways which are coherent and relevant to the range of end-users, teachers and students, in the range of target contexts. Given the aspirations of many current curriculum reforms, that is a complex and demanding task. Even then, there are threats from teacher enactment that is faithful to and perhaps unhelpfully reliant on the resource, possibly resulting in lack of flexibility/capacity to adjust to particular students' learning needs, or contributing to teacher de-professionalisation – or equally, from teachers (or local leaders) choosing to ignore or engage only superficially, with challenging messages conveyed therein.

Such responses are often related to educators' beliefs, which are slow to be influenced: curriculum reform without coherent surrounding community beliefs is

unlikely to prosper. Even given conscientious teacher investment in coming to know and appreciate the communicated philosophy and values, structure and approaches in a key curriculum-coherent resource, this will not bear fruit if other parts of the curriculum system, such as high-stakes assessment, do not also remain coherent with the intended curriculum (Golding, 2018).

There is an argument that if coherent materials are produced centrally, then teachers do not need to have the skills to develop their own detailed curriculum, but are freed up to develop detailed enactment at lesson and smaller granularity, harnessing their knowledge of individual and classes of learners: development of curriculum vision consistent with that of the curriculum resources, takes time and effort. Further, curriculum trust and curriculum vision are closely related, so that teachers need to have reason to have confidence in the resources they are expected to work with. Even then, more aspirational curricula can be subverted by teachers, e.g. choosing to reduce cognitive demand from that promoted by curriculum-coherent resources. Fundamentally, such approaches will falter if the resources used by teachers are not coherent with curriculum intentions (Trung & Phat, 2018). Others (e.g. Apple, 1990) argue that a fidelity approach may contribute to teacher de-professionalisation, undermining the affirming possibilities of effective teacher 'curriculum-making'. As above, there is also the view that 'educative' resources can constrain, for example by restricting the range of student responses to which teachers are sensitised. In contexts of high stakes assessment, supporting student attainment might involve sacrificing some professional status in relying heavily on texts – but equally, where the system is developed coherently, teachers can also be seen as designer of curriculum, using text as a tool (Golding, 2018), so much depends on the details of the contextualised enactment, and the informed capacity of teachers to move beyond what resources present as possibilities.

In relation to the use of digital technology tools for learning mathematics, we have identified their intrinsic relevance in educating for a digitally-immersed society, as well as a wide range of potentially highly impactful benefits for supporting coherence with curriculum intentions. However, in relation to e.g. dynamic graphing or geometry software, or for developing meaning-making in the use of data, there are demanding implications for teacher learning: of not only newer emphases in the curriculum and their pedagogies, perhaps harnessing technologies for problem solving or for interrogating and so interpreting data, but of the technological pedagogical needs of confidently, effectively, and safely, harnessing technology for such purposes (Mishrak & Koehler, 2006). Without that, benefits might be more about student engagement than mathematics learning that is fully coherent with curriculum intentions.

All curriculum resources then, are likely to have limited impact on coherence or relevance of the experienced curriculum, if there is rushed and/or superficial development, unclear or muted communication of curriculum intentions, or inadequate investment, either financially or in terms of teacher learning; if there are significant limitations to developer beliefs, attitudes, knowledge or skills in relation to curriculum aspirations – or if, for whatever reason, those responsible simply fail to choose to make use of the tools developed. To enhance buy-in more generally, there is a

need to balance fidelity of use with attention to the degree of teacher autonomy valued by, and appropriate to, teachers in that context. Finally, curriculum tools are likely to be optimally effective only if the whole system is coherent: for example, in a high-stakes assessment regime, teachers are likely to fully invest, and maintain engagement with, the potential of resources only while those are seen to be coherent with emerging assessments (Golding, 2018).

Conclusion and Key Messages

What, then, are the key messages from this overview of the role of curriculum resources in supporting a curriculum that is both relevant and coherent? First, no curriculum reform exists in a social or contextual vacuum, whatever its scale, so that resources can only be supportive if they are designed to function in the range of target contexts, including that of policy. Materials are part of a larger curriculum system, and the range of evidence we have seems to suggest that systemic coherence is a necessary condition for large-scale sustainability of curriculum enactment coherent with intentions. A key facet of that system is the teacher capacity – their knowledge, skills, and affect (Golding, 2017) – for the intended change.

Central, then, are transparent and detailed exemplification of novel content and/or intended pedagogical approaches and resource-linked messages around those, as well as opportunities framed to support related teacher development. Here, teacher-educative resources might have a central role. The effectiveness with which teachers use well-formulated curriculum-coherent materials will also depend on other macro social educational variables: in addition to the quality of the teachers that is crucial, the presence of appropriate classroom action quality assurance, and an effective control over educational materials, are needed: the presence in the system of inadequate materials can undermine choice and best use of good resources. This is not uncommon in developing countries or with relatively weaker educational systems (Royal Society, 2019). Additionally, we note (e.g. Barabash, 2018; Bonissoni et al., 2018) the potential for bottom-up curriculum reform, and for collaborative efforts – but also their potential constraints.

Once coherent resources are established, their *sustainability* depends not least on continuing and detailed monitoring for systemic coherence, if student experience is to maintain coherence with intentions even in, for example, high-stakes accountability regimes. While recognising the constraints on policymakers, we have seen above the cumulative threats to continued coherence, of tensions within the curriculum system, and of inadequate resourcing or rushed design. We have also seen that textual, manipulative and digital resources can all be harnessed to support increased relevance for students or society. Taken together, high quality curriculum resources have the potential to promote enhanced enactment, supporting teachers in focusing on detailed planning at lesson and smaller granularity, and harnessing their knowledge of individual and classes of learners.

It would seem that the development of high quality, teacher-educative resources coherent with curriculum intentions also has the potential to modify teacher

workload while simultaneously enhancing their potential for professionally-affirming, classroom- and wider-scale 'curriculum making'. On a student level, digital materials can, if used in ways coherent with intentions, support a range of mathematical meaning-making, and so relevance, that complements that available by other channels, but there is much that we have yet to understand about the affordances and constraints of digital materials. The knowledge base around school student use of mathematics curriculum resources in general is also under-developed, including in relation to student received coherence with curriculum intentions, and perceptions of relevance to their own current and future needs, warrants further work.

In conclusion, a range of evidence from across the world shows that deep systemic change at scale remains highly challenging, and resource-consuming in all aspects, so that collaborative and measured curriculum *evolution*, rather than *revolution*, has many advantages. The recent research cited then offers some pointers to the development and use of curriculum resources which can effectively support increased and sustained both relevance and coherence within globally aspirational curricula for the twenty-first century.

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