acreg: Arbitrary Correlation Regression

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Abstract. We present acreg, a new Stata command that implements the *arbitrary* clustering correction of standard errors proposed in Colella et al. (2019).¹ Arbitrary here refers to the way observational units are correlated with each other: we impose no restrictions so that our approach can be used with a wide range of data. The command accommodates both cross-sectional and panel databases and allows the estimation of OLS and 2SLS coefficients, correcting standard errors in three environments: in a spatial setting using units' coordinates or distance between units; in a network setting starting from the adjacency matrix; and in a multi-way clustering framework taking multiple clustering variables as input. Distance and time cutoffs can be specified by the user and linear decay in time and space are also optional.

Keywords: acreg, inference, arbitrary correlation, geospatial data, network data

1 Introduction

Thanks to increasing computational power, databases have become more and more complex in the past decades. They nowadays embed convoluted correlation structures between observational units that were not common before. For example, fueled by the growing availability of geocoded data and the integration of geographic information systems (GIS) in the toolkit of economists, empirical works using spatial data are proliferating in fields like development economics, urban economics, and economic history. Other examples of *new* correlation structures pertain to network data: individuals are linked, and these links are now measurable through social networks, mobile data, co-working relations or co-authorships.

Statistical inference in these environments is challenging because the underlying data generating process is often unknown and researchers need to make assumptions on the relationship between observations. Available methods to address correlation between objects build on the sandwich-type variance covariance estimator proposed by White (1980). The most common approach is standard clustering (Cameron and Miller 2015)

^{1.} Our statistical package (acreg) can be downloaded at the following address https://acregstata.weebly.com.

that defines clusters as groups of linked observations that share a common characteristic. With spatial data, a frequently used approach has been developed by Conley (1999) who considers a circle around each unit within which the strength of the dependence between the unit and the surrounding ones is specified. In the case of network data, the practice is less developed; many studies simply do not correct for the potential correlation of unobserved shocks across linked observations.

In our companion paper Colella et al. (2019), we explore pitfalls and provide guidelines of conducting inference in complex settings, allowing for any type of topological and temporal dependence between observational units in large samples. Our *arbitrary clustering* approach builds on the seminal insight by White (1980), using estimated regression errors and knowledge on the clustering structure to re-construct estimates of the unknown elements of the sandwich formula. We perform extensive Monte Carlo simulations for both spatial and network data structures, e.g. U.S. counties and co-authorship in economics. Our simulation results show that arbitrary clustering inference dominates inference based on conventional estimators.

In this current article, we present our new user-written command acreg that implements the arbitrary clustering correction of standard errors proposed in Colella et al. (2019). We also provide several examples on how to use it. Our command accommodates OLS and 2SLS estimations and is designed to deal with network clustering and several clustered covariance matrix estimators (Bester et al. 2011), including multi-way clustering (Cameron et al. 2011), spatial clustering (Conley 1999) and HAC (Newey and West 1987).

In network settings, we are not aware of any existing Stata routine designed to correct standard errors starting from the knowledge of the binary links between observations. In spatial settings, there are three user-written Stata commands available (Conley 1999; Hsiang 2010 and Fetzer 2015): however, they suit only OLS estimation. In addition, they all have pre-set options that are not desirable in all settings. In particular the commands by Conley 1999 and Fetzer 2015 impose a linear decay in the correlation structure between units (*Bartlett*), while Hsiang 2010 and Fetzer 2015 set a time decay (HAC) as default.² In comparison with those commands, acreg is more flexible as it enables the users to freely set the type of correlation structure and decay across observations and time. Moreover, in presence of multiple cross-sectional observational units sharing the same geo-location, our command provides consistent standard errors, replicating the heteroskedasticity-robust standard errors from ivreg2 when the distance correction is set to 0, while the programs by Conley 1999; Hsiang 2010 and Fetzer 2015 do not. Stata 15 introduced a series of commands, named sp, to model spatial relations between objects using spatial autoregressive models (SAR). These models allow for spatial lags of the dependent variable, which modifies point estimates, or for spatial autocorrelation in the errors. The command closest to ours, spregress, allows only for heteroskedasticity-robust and asymptotic maximum-likelihood theory driven standard errors. Conversely, acreg does not modify the point estimates, but improve inference by computing standard errors corrected for spatial correlation.

 $\mathbf{2}$

^{2.} Conley 1999 allows to correct for only cross-sectional dependence and not time dependence.

The rest of this article is organized as follows. In section 2, we review the arbitrary clustering method developed in Colella et al. (2019). In section 3, we provide a detailed description of the syntax of *acreg*. In section 4, we offer an illustration of our command with several examples in the spatial and the network settings: we show how options of our command can be used to suit many models of correlation structure. Finally, in section 5, we conclude.

2 Estimator for the Variance-Covariance Matrix

Here we present the estimator of the variance-covariance (VCV) proposed in Colella et al. (2019). The proposed estimator builds on the seminal insight from White (1980) and can be seen as an extension of the one-way or multi-way clustering (Cameron et al. 2011) that includes also spatial clustering (Conley 1999).³

In our setting each observation can be correlated to any another and the strength of their correlation is a function of both time and distance. We define a matrix S containing information on cross-observation correlations in errors. With spatial data, S is built from information on the geographic distance between spatial units, e.g., regions, cities, and countries; while in a network context, it reflects the direct links between observations at different degrees. **acreg** computes the matrix S starting from the position of objects in space, using their coordinates, or from the link structure in a network; it also allows the user to define the matrix S to accommodate more complex correlation structures. Entries of the S matrix range from 0 to 1: this measure represents the strength of the correlation between two units and is inversely proportional to their distance. The diagonal of S is a vector of ones, reflecting the self-links.

Consider n observations at each t instant of time T from the following linear model:

$$y = X\beta + \epsilon$$

where we observe each unit i several times in different periods t. y is a dependent variable, and X is a matrix of k linearly independent components. Note that X could include a long list of dummies for each unit, in case we are interested in the within estimates in a Panel dataset. The OLS estimator can be written as follow:

$$b_{OLS} = (X'X)^{-1}X'y$$

and the theoretical VCV of the b_{OLS} is:

$$VCV(b_{OLS}) = (X'X)^{-1}X'\Omega X (X'X)^{-1}$$

^{3.} We do not provide any theoretical or empirical validation of our approach here. In Colella et al. (2019) we show results of extensive Monte Carlo simulations based on real life data on U.S. metropolitan areas, or on co-authors in Economics. We show that our arbitrary clustering estimator of the VCV yields inference at the correct significance level in moderately sized samples, and it always dominates other commonly used approaches to inference. We provide guidance to the applied practitioners on how to cluster and to make reasonable assumptions on the error distribution in absence of prior knowledge about the data generating process.

where $\Omega \equiv E(\epsilon \epsilon' | X)$ is the unknown VCV of ϵ .

The VCV is estimated by the following sandwich estimator (White 1980):

$$\widehat{VCV}(b_{OLS}) = (X'X)^{-1}X'(S \times (ee'))X(X'X)^{-1}$$

where $e \equiv y - Xb_{OLS}$ represent the vector of residuals and where S is the pattern matrix, and \times is element-by-element matrix multiplication. The key element of this estimator is the middle part $X'(S \times (ee'))X$.

$$X'(S \times (ee'))X = \sum_{i=1}^{n} \sum_{t=1}^{T} \sum_{j=1}^{n} \sum_{s=1}^{T} x_{it} e_{it} e_{js} x'_{js} s_{itjs}$$

where x_{it} is the (column) vector of regressors, and x'_{it} is the row *it* in matrix X.

This framework can also be used in situations with endogeneity. We refer the reader to our paper (Colella et al. 2019) for an illustration of the 2SLS version of the estimator.

3 The acreg command

acreg requires the installation of the latest versions of ranktest, ivreg2 (Baum et al. 2003), and hdfe (Correia 2016). To check whether the most up-to-date versions of these packages are installed (and to install them if they are not), please type acregpackcheck after having installed acreg.

3.1 Syntax

acreg depvar [varlist1] [(varlist2 = varlist_iv)] [if] [in] [fweight pweight]

[, id(idvar) time(timevar) spatial network latitude(latitudevar) longitude(longitudevar) links_mat(varlist_links) dist_mat(varlist_distances) dist(#) lag(#) weights(varlist_weights) cluster(varlist_cluster) hac bartlett nbclust(#) pfe1(fe1var) pfe2(fe2var) correctr2 dropsingletons storeweights storedistances]

- *depvar* is the dependent variable.
- varlist1 is the list of exogenous variables.
- *varlist2* is the list of endogenous variables.
- *varlist_iv* is the list of exogenous variables used with varlist1 as instruments for varlist2.

3.2 Options

Panel

- *idvar* is the cross-sectional unit identifier; required in panel setting.
- *timevar* is the time unit variable; required in panel setting.

If *idvar* and *timevar* are not specified, the model is assumed to be cross sectional.

Spatial Environment

- **spatial** specifies that the environment is a spatial one; not required if arbitrary cluster correction is not performed or in the case it is if varlist_weights, varlist_cluster, or network option is specified.
- *latitudevar* is the variable containing the latitude of each observation in decimal degrees: range[-180.0, 180.0].
- *longitudevar* is the variable containing the longitude of each observation in decimal degrees: range[-180.0, 180.0].
- *varlist_distances* is the list of N variables containing bilateral distances between observations. In the spatial environment, bilateral distance is the spatial distance between observations, e.g., physical or travel distance between two locations. (In the network environment, it is the network distance between observations, i.e., the number of links along the shortest path between two nodes.)
- dist(#) specifies the distance cutoff beyond which the correlation between error term of two observations is assumed to be zero; required if latitude and longitude are specified or if dist_mat is specified. The distance cutoff is in kilometers if latitude and longitude are specified. It can be in any other meaningful metric if bilateral distances are specified.
- lag(#) specifies the time lag cutoff for observations with the same idvar; not required in cross-sectional environment; default is 0 in panel environment, i.e., when id and time options are specified. In panel environment when timecutoff is 0 or not specified, standard errors are clustered at id \times time level.

Network Environment

- **network** specifies that the environment is a network one; not required if arbitrary cluster correction is not performed and in the case it is if varlist_weights, varlist_cluster, or spatial option is specified.
- varlist_links is the list of N dummy variables specifying the links between observations, i.e., the adjacency matrix. The links between two units can change over time. However, if *distcutoff* is set to be greater than one: only the first observation in time of each individual will be used as input to compute the bilateral distance between two nodes.
- varlist_distances is the list of N variables containing bilateral distances between observations. In the network environment, bilateral distance is the network

distance between observations, i.e., the number of links along the shortest path between two nodes. (In the spatial environment, it is the spatial distance between observations, i.e., distance between two locations.)

- dist(#) specifies the distance cutoff (geodesic paths) beyond which the correlation between error term of two observations is assumed to be zero; required if dist_mat is specified; optional if links_mat is specified; default is 1 in the network environment. When links_mat is specified and distcutoff is greater than 1, acreg computes automatically the bilateral distance between two nodes.
- lag(#) specifies the time lag cutoff for observations with the same idvar; not required in cross-sectional environment, default in panel environment is 0, i.e., when id and time options are specified. In panel environment when timecutoff is 0 or not specified, standard errors are clustered at id \times time level.

Multiway Clustering Environment

• *varlist_cluster* is the list of variables to use for multiway clustered standard errors; not required if arbitrary cluster correction is not performed and in the case it is if spatial option, network option, or varlist_weights is specified.

Arbitrary Clustering Environment

• varlist_weights is the list of $N \times T$ variables containing the weights that will be used for error correction; not required if if spatial option, network option, or varlist_cluster is specified. The $N \times T$ variables need to follow the same order of the observations.

Correlation Structure

- hac reports Heteroskedasticity and Autocorrelation Corrected (HAC) standard errors; lagcutoff will be the temporal decay; requires id, time, and lagcutoff.
- **bartlett** imposes a distance linear decay between observations within the cutoff in the correlation structure.
- nbclust(#) is the number of clusters used to compute the Kleibergen-Paap statistic in case of arbitrary cluster correction; default is 100.

High-Dimensional Fixed Effects

- *fe1var* identifies the first high-dimensional fixed effects variable to be partialled out.
- *fe2var* identifies the second high-dimensional fixed effects variable to be partialled out.
- correctr2 when pfe1 or pfe2 are specified the r-squared is computed on the "partialled out sample". This option reports the correct r-squared, i.e. the prepartialling out r-squared. Not allowed with fweights.
- dropsingletons drops singleton groups when pfe1 (and pfe2) is (are) specified.

Storing

- **storeweights** stores the computed weights used to correct the VCV for arbitrary cluster correlation as a matrix under the name weightsmat, which may be used as input for the option varlist_weights; optional only if spatial option, network option, or varlist_cluster is specified.
- **storedistances** stores the computed distances used to correct the VCV for arbitrary cluster correlation as a matrix under the name distancesmat, which may be used as input for the option varlist_distances; optional only if spatial option or network is specified and varlist_distances is not specified.

3.3 Stored results

acreg stores the following in e():

Scalars

- e(N) number of observations
- e(mss) model sum of squares (centered)
- e(mssu) model sum of squares (uncentered)
- e(rss) residual sum of squares
- e(tss) total sum of squares (centered)
- e(tssu) total sum of squares (uncentered)
- e(r2) centered R2 (1-rss/tss)
- e(r2u) uncentered R2
- e(widstat) Kleibergen-Paap Wald rk F statistic

Matrices

- e(b) coefficient vector
- e(V) corrected variance-covariance matrix of the estimators

Functions

• e(sample) marks estimation sample

4 Examples

We illustrate the use of our command in four environments: spatial and network settings, both in cross-sectional and panel contexts. In every environment, we estimate the same equation imposing different assumptions on the error correlation structure: iid, standard clustering, and arbitrary clustering.

4.1 Spatial Environment, Cross-Sectional Setting

For this example we use the data on the homicides in southern states of the U.S. homicide_1960_1990.dta available at the STATA website. Data contain, among others, the county-level homicide rate per year per 100,000 persons (hrate), the population in logs (ln_population), the logarithm of the average income (ln_income), the unemployment rate (unemployment), and the average age (age). This dataset is an extract of the data originally used by Messner et al. (1999) and concerns 4 different periods (1960, 1970, 1980, 1990). We consider only the cross-sectional database for 1990 and we estimate the effect of *income* on *homicide rate*, controlling for *population*, and *age*. For the sake of illustration, we claim that income is endogenous and we assume that unemployment is a valid instrument for it. Figure 1 shows the spatial dependency of the outcome variable, the endogenous regressor, and the instrument.

We first estimate the model assuming that observations' errors are uncorrelated.⁴

. acreg hrate : HETEROSKEDASTIC No HAC Correct: No Absorbed FE: Included instru- Instrumented: : Excluded instru- Kleibergen-Paap Total (centered Total (uncenter Residual SS	CITY ROBUST ST ion s uments: ln_pop ln_income uments: unempl p rk Wald F st d) SS = 6 red) SS = 1	CANDARD ERRO Dulation age Loyment	RS	Nu Ce	mber of obs =	0.1079
hrate	Coefficient	Std. err.	Z	P> z	[95% conf.	interval]
ln_income ln_population age _cons	-8.822082 1.404433 281615 94.4605	.050726	-6.51 5.07 -5.55 7.60		-11.47766 .861622 381036 70.10091	-6.166507 1.947244 1821939 118.8201

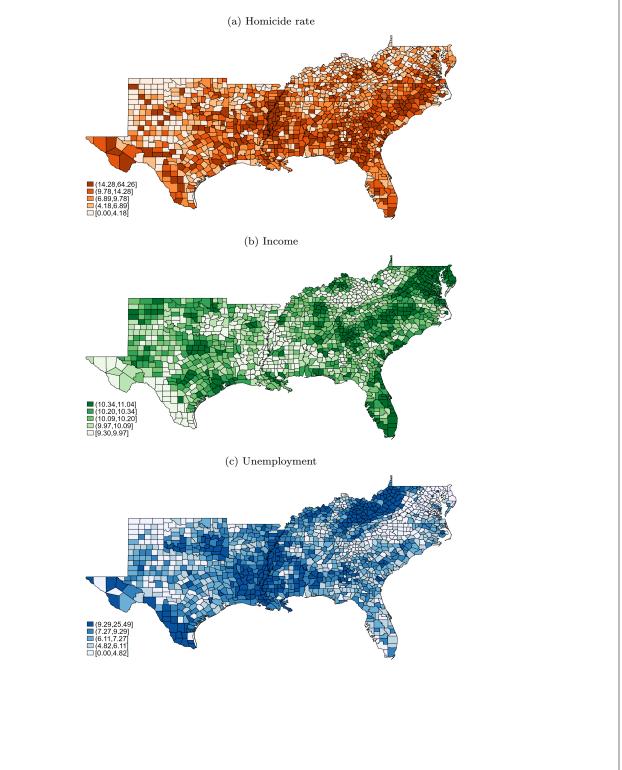
We now estimate the model above clustering standard errors by state.⁵

. acreg hrate ln_population age (ln_income=unemployment), cluster(sfips)
MULTIWAY CLUSTERING CORRECTION
Cluster variable(s): sfips
No HAC Correction
No Absorbed FEs
Included instruments: ln_population age
Instrumented: ln_income
Excluded instruments: unemployment
Kleibergen-Paap rk Wald F statistic: 143.959

^{4.} This is equivalent to using ivreg2 (Baum et al. 2003) and the following syntax: ivreg2 hrate ln_population age (ln_income=unemployment), robust

^{5.} This is equivalent to using ivreg2 (Baum et al. 2003) and the following syntax: ivreg2 hrate ln_population age (ln_income=unemployment), cluster(sfips). We are aware that the number of states (clusters) is small and inference would suffer from it, but this is irrelevant for the scope of this exercise.

Figure 1: Homicide rate, log income, and unemployment in 1990 for southern U.S. counties



Total (centered Total (uncente: Residual SS	red) SS = 1	59908.59003 198667.4579 52363.84851		Ce	mber of obs = ntered R2 = .centered R2 =	0.1079
hrate	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
ln_income ln_population age _cons	-8.822082 1.404433 281615 94.4605	1.801762 .3090553 .1303804 17.89048	-4.90 4.54 -2.16 5.28	0.000 0.000 0.031 0.000	-12.35347 .7986955 5371558 59.3958	-5.290693 2.01017 0260741 129.5252

We now estimate the model above using a spatial correction following Conley (1999), with a threshold of 100 kilometers. This means that the error of each county is assumed to be correlated with the counties that are located within a radius of 100 kilometers from it.

<pre>. acreg hrate ln_population age (ln_income=unemployment), /// > spatial latitude(_CX) longitude(_CY) dist(100) SPATIAL CORRECTION DistCutoff: 100 LagCutoff: 0 No HAC Correction No Absorbed FEs Included instruments: ln_population age Instrumented: ln_income Excluded instruments: unemployment Kleibergen-Paap rk Wald F statistic: 112.917 Number of obs = 1412</pre>							
Total (centered				Ce	ntered R2 =	0.1079	
Total (uncentered) SS = 198667.4579 Uncentered R2 = 0.680 Residual SS = 62363.84851 <td>0.6861</td>						0.6861	
hrate	Coefficient	Std. err.	z	P> z	[95% conf.	interval]	
ln_income	-8.822082		-3.74	0.000	-13.44298	-4.201183	
ln_population	1.404433						
age		.109112					
_cons	94.4605	21.86325	4.32	0.000	51.60932	137.3117	

Additional options

<u>Thresholds</u>. If we want to account for correlation between counties at a greater distance, we can increase the distance cutoff using the dist() option. In the following example we allow for a radius of 200 kilometers.

- . acreg hrate ln_population age (ln_income=unemployment), ///
- > spatial latitude(_CX) longitude(_CY) dist(200)
- . estimates store sp1

Bartlett. In previous examples the matrix used for the computation of the variance covariance matrix is binary: for each county pair, it contains 1 if they are located within

the distance threshold from each other and 0 otherwise. *acreg* allows for weights in the matrix to linearly decrease as the distance between units increases. To do that we only need to add the option **bartlett** to the syntax.

. acreg hrate ln_population age (ln_income=unemployment), ///

- > spatial latitude(_CX) longitude(_CY) dist(200) bartlett
- . estimates store sp2

Partial out high dimensional fixed effects. acreg allows for adding high dimensional fixed effects and partial them out, using the hdfe command by Correia (2016). Up to two fixed effects variables can be specified through the options pfe1() and pfe2(). In the example below we estimate the previous model adding state fixed effects.

. acreg hrate ln_population age (ln_income=unemployment), ///
> spatial latitude(_CX) longitude(_CY) dist(100) pfe1(sfips)

. estimates store sp3

The following code reports the result of the three estimations in this subsection:

. esttab sp1 sp2 sp3, cells(b se) keep(ln_income ln_population age) mtitles(spatial bartlett FE)

	(1)	(2)	(3)
	spatial	bartlett	FE
	b/se	b/se	b/se
ln_income	-8.822082	-8.822082	-13.88229
	2.733507	2.313018	1.835268
ln_populat~n	1.404433	1.404433	1.649735
	.4834539	.4388646	.4000578
age	281615	281615	178832
	.1223503	.1015135	.0960779
N	1412	1412	1412

4.2 Spatial Environment, Panel Setting

For this example we use the same database we used in the previous section: homicide_1960_1990.dta. We estimate again the effect of *income* on *homicide rate*, controlling for *population* and *age* and we assume that unemployment is a valid instrument for it. Compared to the previous section, we now use all four waves of the dataset.

Pooled model

We first consider a pooled model in which we do not include any Random or Fixed Effects. We first estimate the model assuming that observations' errors are uncorrelated.⁶

^{6.} This is equivalent to using ivreg2 (Baum et al. 2003) and the following syntax: ivreg2 hrate ln_population age (ln_income=unemployment), robust

<pre>. acreg hrate ln_population age (ln_income=unemployment) HETEROSKEDASTICITY ROBUST STANDARD ERRORS No HAC Correction No Absorbed FEs Included instruments: ln_population age Instrumented: ln_income Excluded instruments: unemployment Kleibergen-Paap rk Wald F statistic: 289.132 Number of obs = 5648 Total (centered) SS = 286387.1082 Centered R2 = -0.0447 Total (uncentered) SS = 781008.6785 Uncentered R2 = 0.6169 Residual SS = 299188.6495</pre>						
hrate	Coefficient	Std. err.	Z	P> z	[95% conf.	interval]
ln_income ln_population age _cons	3.83872 4411802 4626917 -7.265041		4.91 -2.24 -7.26 -1.76		2.306947 8270955 5875425 -15.35191	

We now estimate the same model, but we use the panel feature of acreg to account for autocorrelation between observations from the same county over time.⁷ We assume no correlation across counties. We specify the option id() with the county id, the option time() with the year variable and the option lag() with a number greater or equal than the maximum lag between observations, which in this case is $30.^{8}$

```
. acreg hrate ln_population age (ln_income=unemployment), id(_ID) time(year) lagcut(30)
TEMPORAL CORRECTION
No HAC Correction
No Absorbed FEs
Included instruments: ln_population age
Instrumented: ln income
Excluded instruments: unemployment
Kleibergen-Paap rk Wald F statistic: 210.438
                                                       Number of obs =
                                                                            5648
                                                       Centered R2 = -0.0447
Total (centered) SS
                           286387.1082
                        =
                           781008.6785
Total (uncentered) SS
                        =
                                                       Uncentered R2 =
                                                                         0.6169
Residual SS
                        =
                           299188.6495
        hrate
                Coefficient Std. err.
                                             z
                                                  P>|z|
                                                            [95% conf. interval]
    ln income
                   3.83872
                               .921289
                                           4.17
                                                  0.000
                                                            2.033027
                                                                         5.644414
ln_population
                 -.4411802
                              .2513095
                                          -1.76
                                                  0.079
                                                            -.9337379
                                                                        .0513774
                  -.4626917
                              .0787756
                                          -5.87
                                                  0.000
                                                            -.617089
                                                                        -.3082943
          age
        cons
                 -7.265041
                             4.832603
                                          -1.50
                                                  0.133
                                                           -16.73677
                                                                        2,206687
```

We now extend the model above, which takes into account autocorrelation over time, by adding the spatial correction proposed by Conley (1999), with a threshold of 100 kilometers. This means that the error term of each county at a given year is assumed

^{7.} Note that the estimation of the betas does not change with respect to the previous model, **acreg** is only used for the computation of the standard errors.

^{8.} This is equivalent to using ivreg2 (Baum et al. 2003) and the following syntax: ivreg2 hrate ln_population age (ln_income=unemployment), cluster(_ID). Alternatively, using acreg and the following syntax: acreg hrate ln_population age (ln_income=unemployment), cluster(_ID)

to be correlated with those of all the counties that are located within a radius of 100 kilometers from it observed at the same year while simultaneously correcting for autocorrelation over time for each county. Note that correlation between near counties but observed at different point in time is assumed to be zero.

. acreg hrate ln_population age (ln_income=unemployment), id(_ID) time(year) lagcut(30) /// $\,$ spatial latitude(_CX) longitude(_CY) dist(100) SPATIAL CORRECTION DistCutoff: 100 LagCutoff: 30 No HAC Correction No Absorbed FEs Included instruments: ln_population age Instrumented: ln_income Excluded instruments: unemployment Kleibergen-Paap rk Wald F statistic: 24.838 Number of obs = 5648 Centered R2 = -0.0447Total (centered) SS = 286387.1082 Total (uncentered) SS = 781008.6785 Uncentered R2 = 0.6169 Residual SS 299188.6495 = hrate Coefficient Std. err. P>|z| [95% conf. interval] z 3.83872 1.810937 0.034 7.388092 ln_income 2.12 .2893488 -.4411802.3176528 .3871668 -1.140.254 -1.200013ln_population age -.4626917 .1425257 -3.25 0.001 -.742037 -.1833464 -7.265041 9.814094 -0.74 0.459 -26.50031 11.97023 _cons

FE model

In the following example we replicate the previous model, accounting for both spatial and temporal correlation, but we add to the specification the *county fixed effects* using the option pfe1.

```
. acreg hrate ln_population age (ln_income=unemployment), id(_ID) time(year) lagcut(30) ///
>
          spatial latitude(_CX) longitude(_CY) dist(100) pfe1(_ID)
SPATIAL CORRECTION
DistCutoff: 100
LagCutoff: 30
No HAC Correction
Absorbed FE: _ID
Included instruments: ln_population age
Instrumented: ln_income
Excluded instruments: unemployment
Kleibergen-Paap rk Wald F statistic:
                                     49.605
                                                       Number of obs =
                                                                            5648
                                                                          0.0175
Total (centered) SS
                           144755.2058
                                                       Centered R2
                                                                    =
Total (uncentered) SS
                           144755.2058
                                                       Uncentered R2 =
                                                                         0.0175
                        =
Residual SS
                        -
                           142223.0274
        hrate
                Coefficient Std. err.
                                             z
                                                  P>|z|
                                                            [95% conf. interval]
                   .2588154
                             1,149746
                                           0.23
                                                  0.822
                                                           -1.994645
                                                                        2.512276
   In income
ln_population
                 -1.630949
                             1.740873
                                          -0.94
                                                  0.349
                                                           -5.042997
                                                                         1.781099
                  .1466193
                             .2006033
                                           0.73
                                                  0.465
                                                           -.2465559
                                                                         .5397944
          age
```

_cons | -1.31e-17 .1743959 -0.00 1.000 -.3418097 .3418097

nb: total SS, model and R2s are after partialling out. To get the corrected ones use the option correctr2

We now add to the previous model also time fixed effects using the option pfe2.

. acreg hrate ln_population age (ln_income=unemployment), id(_ID) time(year) lagcut(30) /// spatial latitude(_CX) longitude(_CY) dist(100) pfe1(_ID) pfe2(year) SPATIAL CORRECTION DistCutoff: 100 LagCutoff: 30 No HAC Correction Absorbed FE: _ID and year Included instruments: ln_population age Instrumented: ln_income Excluded instruments: unemployment Kleibergen-Paap rk Wald F statistic: 3.895 Number of obs = 5648 Total (centered) SS = 136166.339 Centered R2 = -0.0793Total (uncentered) SS 136166.339 Uncentered R2 = -0.0793Residual SS = 146961.8234 [95% conf. interval] hrate Coefficient Std. err. P>|z|z ln_income -13.30126 17.5969 -0.76 0.450 -47.7905521.18803 -1.6026952.253785 -6.0200332.814642 -0.710.477 ln_population age .0038921 .0937463 0.04 0.967 -.1798472.1876314 -1.11e-15 .128699 -0.00 1.000 -.2522454 .2522454 cons

nb: total SS, model and R2s are after partialling out.

To get the corrected ones use the option correctr2

Additional Options

<u>Thresholds</u>. Now we account for spatial correlation between observations of the same year without accounting for any temporal correlation. We do this by setting the lagcutoff at 0.9

. acreg hrate ln_population age (ln_income=unemployment), id(_ID) time(year) lagcut(0) ///

- > spatial latitude(_CX) longitude(_CY) dist(100)
- . estimates store spp1

Now we account for spatial correlation between observations of the same year, and also for temporal correlation between observations from the same county, but only between neighbor decades, i.e. two observations from the same county are assumed to be correlated only if they are observed with less than 10-year difference.¹⁰ We do that by setting the *laqcutoff* equal to 10.

^{9.} Note that the result will be different than the one obtained in the cross sectional environment (acreg hrate ln_population age (ln_income=unemployment), spatial latitude(_CX) longitude(_CY) dist(100)) because the spatial correlation is assumed to be present only between observations from the same year.

^{10.} Note that this would allow an observation's error term to be correlated with all other observations within 10-year lag and 10-year lead from the same county.

- . acreg hrate ln_population age (ln_income=unemployment), id(_ID) time(year) lagcut(10) ///
- > spatial latitude(_CX) longitude(_CY) dist(100)
- . estimates store spp2

<u>HAC.</u> In the previous examples the matrix used for the computation of the variance covariance matrix is binary. We can use the option **hac** to have a linear decay in time and compute Heteroscedasticity-Autocorrelation-Consistent standard errors, following Newey and West (1987).

- . acreg hrate ln_population age (ln_income=unemployment), id(_ID) time(year) lagcut(30) ///
- > spatial latitude(_CX) longitude(_CY) dist(100) hac
- . estimates store spp3

The following code reports the result of the three estimations in this subsection.

			-
	(1)	(2)	(3)
	lag0	lag10	hac
	b/se	b/se	b/se
ln_income	3.83872	3.83872	3.83872
	1.743993	1.801373	1.785354
ln_populat~n	4411802	4411802	4411802
	.3542752	.377059	.3727145
age	4626917	4626917	4626917
	.1347804	.1403627	.139132
N	5648	5648	5648

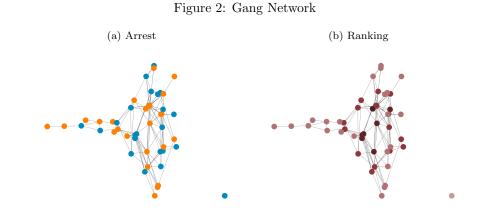
. esttab spp1 spp2 spp3, cells(b se) keep(ln_income ln_population age) mtitles(lag0 lag10 hac)

4.3 Network Environment, Cross-sectional Setting

For this example we use a dataset of co-offending in a London-based youth gang. Data were collected by James Densley and Thomas Grund. The data have been used in Grund and Densley (2012) and Grund and Densley (2015). Information on 54 individuals are reported, two individuals are recorded to be linked if they committed at least a crime together. Data contains, among others, the age (Age), the birthplace (Birthplace), the number of arrests (Arrests), the number of convictions (Convictions), and the position in the gang's internal hierarchy (Ranking). The symmetric binary links constituting the co-offending network are stored in 54 variables (_net2_1-_net2_54). Figure 2 presents the distribution of the variables Arrest and Ranking within the network. In this example we want to estimate the effect of ranking on arrests, controlling for age, residence, and birthplace FEs.

The code below is necessary to load the dataset (webnuse gang), load the network (nwload gang) and replace the diagonal of the adjacency matrix with ones (*the loop*). This is needed because the original database does not contains self-links.

. webnwuse gang



Notes: In panel (a), blue dots represent arrested people. In panel (b), darker red dots identify a greater position in the ranking.

```
Loading successful
```

```
(2 networks)
gang_valued
gang
. nwload gang
. forvalues j = 1(1)54 {
2. qui replace _net2_`j` = 1 in `j`
3. }
```

We first estimate the model assuming that observations' errors are uncorrelated.¹¹

```
. acreg Arrest Ranking Age Residence i.Birthplace
HETEROSKEDASTICITY ROBUST STANDARD ERRORS
No HAC Correction
No Absorbed FEs
Included instruments: Ranking Age Residence 1b.Birthplace 2.Birthplace 3.Birthplace 4.Birthplace
                                                       Number of obs =
                                                                              54
                            2196.537037
Total (centered) SS
                                                       Centered R2
                                                                          0.2442
                                                                     =
                         =
Total (uncentered) SS
                                   7497
                                                       Uncentered R2 =
                                                                          0.7786
Residual SS
                            1660.198039
               Coefficient
                                                 P>|z|
                                                            [95% conf. interval]
     Arrests
                            Std. err.
                                            z
     Ranking
                -2.168476
                             .8207074
                                         -2.64
                                                 0.008
                                                           -3.777033
                                                                       -.5599192
                             .3094139
                 .7665194
                                          2.48
                                                 0.013
                                                            .1600793
                                                                        1.372959
         Age
   Residence
                -1.534665
                             1.561649
                                         -0.98
                                                 0.326
                                                            -4.59544
                                                                        1.526111
  Birthplace
  Caribbean
                        0
                            (empty)
East Africa
                            2.869505
                                                 0.930
                                                            -5.87643
                                                                        5.371822
                 -.2523035
                                         -0.09
```

11. This is equivalent to using ivreg2 (Baum et al. 2003) and the following syntax: ivreg2 Arrest Ranking Age Residence i.Birthplace, robust

UK West Africa		2.228246 2.012521		0.753 0.685	-3.666016 -3.127297	5.068548 4.76164
_cons	2.317286	7.506876	0.31	0.758	-12.39592	17.03049

We now estimate the same model using the standard-error correction proposed in our paper (Colella et al. 2019). We assume that the error term of each observation is correlated with that of another if they are linked in the network. To implement this in acreg, we provide the variables containing the adjacency matrix as input in the links_mat option and set *distcutoff* equal to 1.

```
. acreg Arrest Ranking Age Residence i.Birthplace, network links_mat(_net2_*) dist(1)
NETWORK CORRECTION
DistCutoff: 1
LagCutoff: 0
No HAC Correction
No Absorbed FEs
Included instruments: Ranking Age Residence 1b.Birthplace 2.Birthplace 3.Birthplace 4.Birthplace
                                                       Number of obs =
                                                                              54
                            2196.537037
                                                        Centered R2
                                                                          0.2442
Total (centered) SS
Total (uncentered) SS
                                   7497
                                                       Uncentered R2 =
                                                                          0.7786
                         =
Residual SS
                         =
                            1660.198039
     Arrests
               Coefficient
                            Std. err.
                                            z
                                                 P>|z|
                                                            [95% conf. interval]
                                                           -3.566407
                 -2.168476
                             .7132431
                                         -3.04
                                                 0.002
                                                                       -.7705455
     Ranking
                  .7665194
                             .3730319
                                          2.05
                                                 0.040
                                                            .0353904
                                                                        1.497648
         Age
   Residence
                 -1.534665
                            1.618858
                                         -0.95
                                                 0.343
                                                           -4.707568
                                                                        1.638239
```

Birthplace Caribbean	0	(empty)				
East Africa	2523035	2.258789	-0.11	0.911	-4.679449	4.174842
UK	.7012659	2.984775	0.23	0.814	-5.148785	6.551317
West Africa	.8171717	2.260143	0.36	0.718	-3.612627	5.24697
_cons	2.317286	7.825902	0.30	0.767	-13.0212	17.65577

Additional Options

Accounting for degree grater than one. Each node of a network has a certain number of links that connects it to other nodes. This number is called the degree k of a node. acreg allows the user to account for correlation between two observations that are not necessarily directly linked but are linked through other observations. Starting from the same 0-1 adjacency matrix used in the previous example, we now want to allow for correlation also between individuals that are linked through another individual (degree 2). To do that we will use the same syntax but we change the *distcutoff* to 2.

. acreg Arrest Ranking Age Residence i.Birthplace, network links_mat(_net2_*) dist(2)
. estimates store ne1

Bartlett. In previous examples the matrix used for the computation of the variance

covariance matrix is binary: it contains values 1 for each pair of individuals that are first or second degree linked, and zeros otherwise. *acreg* allows for weights in the matrix to linearly decreas as the as the network distance increases.¹² To do that in our sample, i.e., having ones for first degree linked observations and 0.5 for second degree ones we will use the option bartlett.

- . acreg Arrest Ranking Age Residence i.Birthplace, network links_mat(_net2_*) dist(2) ///
- > bartlett
- . estimates store ne2

Partial out high dimensional fixed effects. acreg allows for adding high dimensional fixed effects and partial them out, using the hdfe command by Correia (2016): up to two fixed effects variables can be specified through the options pfe1() and pfe2(). In the example below we estimate the previous model partialing out birthplace FEs instead of adding them as dummies in the main regression.

. acreg Arrest Ranking Age Residence, network links_mat(_net2_*) dist(1) pfe1(Birthplace) . estimates store ne3

The following code reports the result of the three estimations in this subsection.

	(1)	(2)	(3)
	degree2	bartlett	FE
	b/se	b/se	b/se
Ranking	-2.168476	-2.168476	-2.168476
	.4801238	.7688551	.7132431
Age	.7665194	.7665194	.7665194
	.4001636	.3427023	.3730319
Residence	-1.534665	-1.534665	-1.534665
	2.138931	1.590511	1.618858
۷	54	54	54

esttab ne1 ne2 ne3, cells(b se) keep(Ranking Age Residence) ///

4.4 **Network Environment**, Panel Setting

For this section we use an ad-hoc database that can be downloaded from our command's website. It is a balanced panel dataset of 1000 observations (NT) referring to 100 (N)individuals at 10 (T) points in time. Individuals are identified through the variable id, while time is identified through the variable time. Database also contains, among others, the following variables Y_it, X1_it, End_it, and IV_it. The symmetric binary links constituting the network are stored in 100 (N) variables (clus_1-clus_100). In this example we want to estimate the effect of End_it on Y_it, controlling for X_it. We

^{12.} With distance here we refer to the strength of the link: first degree is distance 1, second degree is distance 2, etc...

claim that End_{-it} is endogenous and we assume that IV_{-it} is a valid instrument for it.

Pooled model

We first consider a pooled model in which we do not include any Random or Fixed Effects. We first estimate the model assuming that observations' errors are uncorrelated.¹³

```
. use https://acregstata.weebly.com/uploads/2/9/1/6/29167217/acregfakedata.dta, clear
. acreg Y_it X1_it (Z_it=IV_it)
HETEROSKEDASTICITY ROBUST STANDARD ERRORS
No HAC Correction
No Absorbed FEs
Included instruments: X1_it
Instrumented: Z_it
Excluded instruments: IV it
Kleibergen-Paap rk Wald F statistic: 37.874
                                                       Number of obs =
                                                                            1000
Total (centered) SS
                                                       Centered R2 =
                         =
                                                                          0.4913
                           2834382,139
Total (uncentered) SS
                        =
                             4195421.4
                                                       Uncentered R2 =
                                                                          0.6563
Residual SS
                           1441795.144
        Y_it
               Coefficient Std. err.
                                                 P>|z|
                                                            [95% conf. interval]
                                            z
                  1.02863
                             .2409828
                                          4.27
        Z_{it}
                                                 0.000
                                                            .5563128
                                                                        1.500948
                 1,228864
                                                            .5780809
                                                                        1.879647
       X1_it
                             .3320382
                                          3.70
                                                 0.000
                 11.61852
                             3.013075
                                          3.86
                                                 0.000
                                                            5.713007
                                                                        17.52404
       cons
```

We now estimate the same model accounting for correlation between errors from observations of the same individual (id). We still assume that there is no correlation between individuals and do not consider the network structure yet. To do this, we use the panel features (options id() and time) and we set the lag() option to be greater than or equal to the maximum distance in time between observations, which in this case is 10.¹⁴

```
acreg Y_it X1_it (Z_it=IV_it), id(id) time(time) lag(10)
TEMPORAL CORRECTION
No HAC Correction
No Absorbed FEs
Included instruments: X1_it
Instrumented: Z_it
Excluded instruments: IV_it
Kleibergen-Paap rk Wald F statistic: 30.295
                                                      Number of obs =
                                                                           1000
Total (centered) SS
                        =
                           2834382.139
                                                      Centered R2
                                                                         0.4913
Total (uncentered) SS
                                                      Uncentered R2 =
                             4195421.4
                                                                        0.6563
                        =
                        =
Residual SS
                           1441795.144
               Coefficient Std. err.
                                                P>|z|
                                                           [95% conf. interval]
        Y it
                                           z
```

^{13.} This is equivalent to using ivreg2 (Baum et al. 2003) and the following syntax: ivreg2 Y_it X1_it (End_it=IV_it), robust

^{14.} This is equivalent to clustering by individuals using ivreg2 (Baum et al. 2003) and the following syntax: ivreg2 Y_it X1_it (End_it=IV_it), cluster(id), or acreg: acreg Y_it X1_it (End_it=IV_it), cluster(id)

Z_it	1.02863	.2720916	3.78	0.000	.4953406	1.56192
X1_it	1.228864	.3779895	3.25	0.001	.4880181	1.96971
_cons	11.61852	3.042037	3.82	0.000	5.656242	17.58081

We now estimate the model above adding to the temporal correlation the correction for network links as proposed in our paper (Colella et al. 2019). We assume that the error term of each individual is correlated with that of another individual observed in the same year if they are linked in the network while accounting for correlation between errors from observations of the same individual. To implement this in **acreg**, we provide the variables containing the adjacency matrix as input in the **links_mat** option and set *distcutoff* equal to $1.^{15}$ Note that correlation between linked individuals but observed at different point in time is still assumed to be null.

<pre>. acreg Y_it X1_it (Z_it=IV_it), id(id) time(time) lag(10) /// > network links_mat(clus*) dist(1) NETWORK CORRECTION DistCutoff: 1 LagCutoff: 10 No HAC Correction No Absorbed FEs Included instruments: X1_it Instrumented: Z_it Excluded instruments: IV_it</pre>	
Kleibergen-Paap rk Wald F statistic: 22.720 Number of obs =	1000
Total (centered) SS = 2834382.139 Centered R2 = 0	0.4913
Total (uncentered) SS = 4195421.4 Uncentered R2 = 0	0.6563
Residual SS = 1441795.144	
Y_it Coefficient Std. err. z P> z [95% conf. inte	erval]
Z_it 1.02863 .3842782 2.68 0.007 .2754589 1.	781802
-	109913
_cons 11.61852 4.743084 2.45 0.014 2.32225 24	0.9148

FE model

In the following example we replicate the previous model, accounting for both spatial and temporal correlation, but we add to the specification the *individual fixed effects* using the option pfe1.

^{15.} Note that the total number of observations in the database is NT (1000), but the total number of individuals is N (100). Since we are using the panel feature, **acreg** will require a link matrix formed by N variables, not NT.

Absorbed FE: : Included instr Instrumented: Excluded instr Kleibergen-Paa	ruments: X1_it Z_it ruments: IV_it		38.899			
	-r				Number of obs :	= 1000
Total (centere	ed) SS =	2331112.842	2		Centered R2	
Total (uncente	ered) SS =	2331112.842	2		Uncentered R2 :	= 0.4938
Residual SS	=	1180104.818	3			
Y_it	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
Z_it	1.368636	.346849	3.95	0.000	.6888244	2.048448
X1_it	.7942328	.3663375	2.17	0.030	.0762245	1.512241
_cons	9.58e-17	1.266864	0.00	1.000	-2.483007	2.483007

nb: total SS, model and R2s are after partialling out. To get the corrected ones use the option correctr2

We now add to the previous model also time fixed effects using the option pfe2.

Total (uncentered) SS = 2226516.365 Uncentered R2 = 0.4935 Residual SS = 1127664.807	<pre>. acreg Y_it X1_it (Z_it=IV_it), id(id) time(time) lag(10) /// > network links_mat(clus*) dist(1) pfe1(id) pfe2(time) NETWORK CORRECTION DistCutoff: 1 LagCutoff: 10 No HAC Correction Absorbed FE: id and time Included instruments: X1_it Instrumented: Z_it Excluded instruments: IV_it Kleibergen-Paap rk Wald F statistic: 39.988 Number of obs = 1000 Total (centered) SS = 2226516.365 Centered R2 = 0.4935</pre>								
	Y_it Coeff	icient Std. err	. z P> z	[95% conf. interval]					
Y_it Coefficient Std. err. z P> z [95% conf. interval]	X1_it .82	327506 .3119844 232877 .3574087 70e-17 .9797572	2.30 0.02	.1227796 1.523796					

nb: total SS, model and R2s are after partialling out. To get the corrected ones use the option correctr2

Additional Options

Thresholds. Now we still account for network correlation between observations of the same year, but we do not account for any kind of temporal correlation. We do that by setting the lagcutoff at 0.

```
. acreg Y_it X1_it (Z_it=IV_it), id(id) time(time) lag(0) ///
> network links_mat(clus*) dist(1)
```

```
. estimates store nep1
```

Now we account for network correlation between observations of the same year, and also for temporal correlation between observations from the same individual, but only if they were observed with a lag lower than 3 years. We do that by setting the *lagcutoff* equal to 3.

```
. acreg Y_it X1_it (Z_it=IV_it), id(id) time(time) lag(3) ///
> network links_mat(clus*) dist(1)
. estimates store nep2
```

<u>HAC.</u> In the previous examples the matrix used for the computation of the variance covariance matrix is binary. We can use the option **hac** to have a linear decay in time and compute Heteroscedasticity-Autocorrelation-Consistent standard errors, following Newey and West (1987).

```
. acreg Y_it X1_it (Z_it=IV_it), id(id) time(time) lag(3) ///
> network links_mat(clus*) dist(1) hac
. estimates store nep3
```

The following code reports the result of the three estimations in this subsection.

. esttab nep1 nep2 nep3, cells(b se) keep(X1_it Z_it) mtitles(lag0 lag10 hac)

	(1)	(2)	(3)
	lag0	lag10	hac
	b/se	b/se	b/se
Z_it	1.02863	1.02863	1.02863
	.3629168	.3783906	.3756538
X1_it	1.228864	1.228864	1.228864
	.4116362	.4578899	.4442984
N	1000	1000	1000

4.5 Multiway clustering

For this example we use again the data on the homicides in southern states of the U.S. homicide_1960_1990.dta available at the STATA website. As in section 4.1 we consider only the cross-sectional database for 1990 and we estimate the effect of *income* on *homicide rate*, controlling for *population*, and *age*. For the sake of illustration, we claim that income is endogenous and we assume that unemployment is a valid instrument for it.

In this section we illustrate the multiway clustering environment. We cluster standard errors following two dimensions: state and age.

```
    use http://www.stata-press.com/data/r15/homicide1990.dta , clear
(S.Messner et al.(2000), U.S southern county homicide rates in 1990)
    acreg hrate ln_population age (ln_income=unemployment), cluster(sfips age)
MULTIWAY CLUSTERING CORRECTION
```

Cluster variab No HAC Correct No Absorbed FE Included instr Instrumented: Excluded instr Kleibergen-Paap Total (centered Total (uncente: Residual SS	ion s uments: ln_poj ln_income uments: unemp p rk Wald F s d) SS = red) SS =	pulation age loyment		Ce	umber of obs = entered R2 = ncentered R2 =	1412 0.1079 0.6861
hrate	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]
ln_income ln_population age _cons	-8.822082 1.404433 281615 94.4605	1.818954 .2960066 .1315389 17.95901	-4.85 4.74 -2.14 5.26	0.000 0.000 0.032 0.000	-12.38717 .8242705 5394265 59.26148	-5.256998 1.984595 0238035 129.6595

5 Conclusion

In this article we presented the acreg Stata command: a new user-written routine that allows for standard error correction in OLS and 2SLS estimation of models with complex correlation structure. acreg can accommodate in a flexible way dependence of the errors between units in space or in a network and across time. This command includes most of the standard options present in previous commands to estimate regression coefficients. The correlation structure can be inputed by the user in a matrix form or built from information on the geographic distance between spatial units or from the links between observations. We also provide a broad collection of examples with both cross-section and panel data.

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