

The Lohe hoard Revisited

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1 Introduction

In 1948 Bengt Thordeman published in this journal his classic article on the huge hoard of mostly Swedish mark coins found in 1937 hidden in a city house in Stockholm. The house had belonged to the wealthy Lohe family,¹ who had partly made their fortune by money-lending.² The aim of this paper is to revisit Thordeman's work in the light of advances in numismatic understanding and method, and with the help of previously unpublished data on the content of the hoard. One might question the need to revisit such an old publication.³ Thordeman's article, as we shall show below, has been extremely influential and justifies, we feel, this review.

The hoard, which was concealed in c.1742, consisted of 18,216 coins of which 74% were one-, two-, four- or eight-mark silver issues of Karl XI (1660–97) to Fredrik I (1720–51) minted in Stockholm. Figure 1 shows the relevant denominations. The silver content of all denominations of Swedish mark coins was, in 1664 reduced by 8.8% resulting, in accordance with 'Gresham's Law', in a nearly total absence in the hoard of

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¹ Thordeman 1948.

² Berggrén 1937.

³ This paper is based on a preliminary paper written by Sven Aagaard and Michael Märcher presenting a critical review of Thordeman's law based on a rediscovered list of the Lohe hoard.

mark coins earlier than 1664: only 38 pieces from Kristina up to 1663 compared to 137 from 1664 alone.

The hoard also included silver vessels, a copper fish-kettle, a copper cauldron and other artefacts. The silver alone weighed 201kg. Unfortunately, Thordeman's paper does not provide a list of the coins within the hoard. In 1992 a typewritten list was discovered in the archives of the Kungliga Myntkabinettet, Stockholm, dated 1942 which refers to work undertaken in 1937 with (later?) handwritten corrections.⁴ We have included this data in Table 6. A full listing of the hoard derived from the 1942 list is presented in Appendix B.

Figure 1 about here.

Thordeman's observations in the second part of his article are those which have made the most impact. By comparing the contents of the hoard with existing mint records, Thordeman proposed that (a) coin hoards reflected the coins in circulation at the time and place where they were assembled, and the larger the hoard 'the better the agreement should be';⁵ and (b) due to a variety of factors including accidental loss, hoarding and export, a systematic proportion of coins in circulation are lost each year, and for this period of history that loss rate was about 2%. Thordeman wondered if his observations could be 'ascribed to some general law'.⁶ Thordeman's article was internationally well received, and was published in a slightly revised Swedish version in 1983.⁷

Subsequently, a number of authors have referred to this 'law' when arguing that hoards are representative of the relative sizes of issues in circulation.⁸ In Scandinavian

⁴ Rasmusson (1937) presents a condensed and preliminary list of the hoard. Later Källholm (1999, p. 80), published a list of the mark coins only from Karl XI, corresponding to the 1942 list followed by a concise list of other Swedish coins in the hoard.

⁵ Thordeman 1948, p. 200.

⁶ Thordeman 1948, p. 200.

⁷ Thordeman 1983.

⁸ e.g., Duncan-Jones 1994, p. 113; Haider 1996, p. 336; Noonan & Kovalev 2000, p. 254; Kuroda 2008, p. 9.

literature the term ‘Thordeman’s Law’ is often used without critical comment.⁹ The acceptance of this ‘law’ is perhaps best reflected by Michael Crawford who stated that the Lohe hoard ‘showed an almost perfect correlation between the numbers of coins of each year in the hoard and the numbers of coins known from mint records to have been issued each year. This remarkable hoard was very much in my mind when I wrote Roman Republican Coinage, as indeed it is in the mind of anyone well-informed in the field of quantitative studies of ancient coinages.’¹⁰ One aim of this paper is to highlight the complexities in this seemingly simple correlation between coins as struck and coins as hoarded.

Of course, the importance of Thordeman’s observations lie beyond mere numismatic curiosity. The size of coinage issues is a matter of great interest to economic historians, but one which is problematic when there are no surviving records from the mint. Although there have been many attempts to estimate the size of coinage issues from coin hoard data, such as the study by Crawford of the coinage of the Roman Republic noted above,¹¹ they have not been uncontroversial.¹² Hopkins’ use of Crawford’s figures to look at patterns of *Taxes and Trade* in the Roman world, used a figure of 2% loss to model the growth of the Roman Republican coinage pool, although in his case the figure was derived from the work of Patterson.¹³ Other authors have also noted this figure of 2%.¹⁴ It is this wider use of ‘Thordeman’s Law’ which motivated this re-examination.

In the following, we address three topics individually for the sake of clarity. Firstly, we examine Thordeman’s *method*. Secondly, we look at Thordeman’s ‘law’,

⁹ e.g., Tingström 1973, p. 110; Jensen 1979, p. 262; Malmer 1982, p. 59; Ingvaldsen 1991, p. 177.

¹⁰ Crawford 1979, p. 162. See also Crawford 1983, p. 211.

¹¹ Crawford 1974.

¹² e.g., Buttrey 1989.

¹³ Hopkins 1980; Patterson 1972.

¹⁴ e.g., Volk 1987, pp. 58–61, and references; Howgego 1992, note 14; de Callatay 1995, p. 292 and references; Kay 2014, p. 92.

subdividing this into an examination of the 2% loss rate, and an appraisal of how hoards do, or do not, represent the coinage in circulation.

2 The background to the hoard

Archival research has revealed that the house in Stockholm, Lille Nygatan 21, was inherited in 1742¹⁵ by Court Chamberlain Adolf Tobias Lohe (born 1682), who lived here with his sister Johanna Lohe (born 1698) until their deaths in 1759. Their personal lives are little known, but they are both described as eccentric and stingy, a devoted bachelor and spinster who collected valuable property.¹⁶ The house was originally bought in 1722 by their mother, Anna.¹⁷ She died in 1731 and left it to their elder brother Johan Fredric Lohe, who was elected in her will as head of the huge family company established by her husband, Chamberlain Johan Lohe. When he died in 1704 he was regarded as one of the richest persons in Sweden. His fortune was created partly through commerce and industrial activity and partly through money-lending.

The disposition of the fortune was, after the death of Anna, followed by a long series of disputes in the family ending in the strange exodus from Sweden in 1740 by Johan Fredric, who eventually died abroad in 1741. As the Lohe hoard contains coins from 1741, he could not himself have concealed it.¹⁸ Therefore, the hoard was most likely concealed by the siblings, probably shortly after they moved to the house in 1742 or just after.

Nothing indicates that Adolf Tobias Lohe or his sister had earned the fortune by themselves, so it seems likely, that the majority of the coins in the hoard were inherited from their brother Johan Fredric. The younger coins from 1741¹⁹ were placed together on the top of the hoard in the big copper kettle as were numerous 10 öre pieces struck

¹⁵ Adolph Tobias Lohe obtained access to the part of the house, in Lilla Nygatan 5 where the hoard was found, in March 1742 before the administration of the estate (after Johan Fredric) was finished. Later he received the rest of the house (Hedström 1937b).

¹⁶ Berggrén 1937, p. 61–2

¹⁷ Berggrén 1937, p. 54

¹⁸ Hedström 1937b; Thordeman 1948, note 14.

¹⁹ Seven pieces of one rigsdaler dating to 1741.

in 1739–40.²⁰ The youngest coins in the hoard are most likely those that were earned by the siblings and placed in the kettle by them.

As none of the siblings apparently kept any important position in the family's financial activities, one possible explanation is that the coins older than 1741 were assembled by their elder brother, Johan Fredric, shortly before he fled from Sweden. As there were other brothers and sisters living, the main part of the hoard is probably only part of the fortune left by Johan Fredric.

The threat of war with Russia in 1741²¹ may have forced the siblings to bury their goods under the stone flooring in one of the rooms in their house in Nygatan. It remains a mystery why the hoard was seemingly left untouched, but the siblings probably had income from other sources that fulfilled their daily needs in the 1740s and 1750s.

3 Thordeman's method

Thordeman's paper outlines the series of steps he went through in his analysis, each accompanied by a graph. It is worth looking at each of these steps in turn in the light of methodological developments.

Firstly, it is important to note some aspects of the mint data. For the period 1664–97 the records provide figures for the value of eight-mark coins struck, but combine the figures for the four-, two- and one-mark coins. Only from 1698 are the values of the smaller denominations given individually. This aspect of the data had an impact on what Thordeman was able to do. To aid the discussion here we have given the detailed mint figures in Tables 4–5 in Appendix A.²²

In his Diagram A, Thordeman plots the total value of the one-, two- and four-mark coins in the Lohe hoard as a line graph. In the same diagram he also plots the total value of the same denominations struck by the mint. In order to do this, he uses two different y-axes to bring the lines to roughly the same scale. He omits the rarer eight-

²⁰ 1739: 1,124 ten öre coins; 1740: 1,648 ten öre coins (Rasmusson 1937, p. 39).

²¹ A smaller hoard consisting of 454 silver coins was found in a nearby house in 1947, also with the latest coin from 1741 (Thordeman 1948, p. 196).

²² Data from *Kungliga Mynt- och Justeringsverket* 1913, pp. 86–116; reprinted by Wallroth 1918, pp. 89–131.

mark coins as he felt that the 'irregularity of their issue did not make them suitable for a statistical investigation...' ²³ Indeed, the 1942 listing shows only 101 eight-mark coins out of the 15,329 mark coins in the hoard. One weakness of Thordeman's paper is that he generally slipped into talking about *numbers* of coins, whereas his figures were, by necessity, the *value* of those coins.

Thordeman was not satisfied by this diagram, and indeed statisticians decry the use of dual y-axis graphs, and so he decided to draw a second diagram based on percentages. His Diagram B scaled the figures to a percentage of the year with the highest values for both the mint and the hoard, which in both cases was 1694. Again, however, we have a slight wrinkle as regards the eight-mark coins. The total value of coins for 1694 in the hoard was 4,100 marks but excluding the eight-mark coins leaves a total of 3,972. Thordeman decided to scale his diagram with the maximum year set to $3,972/4,100 \times 100 = 97\%$. Similarly, for the mint figures he calculates the percentage for the lower denomination coins of the total, *i.e.*, $5,997,462/6,402,918 \times 100 = 94\%$. The resulting diagram shows a remarkably close match between the hoard and the mint figures (reproduced here as closely as possible from currently available data, Fig. 2a).

Figure 2 somewhere near here.

Two questions arise. Firstly, why the need to scale the percentages at all? An identical graph, in terms of the shape of the curves, would be obtained by using straight percentages, albeit with smaller figures on the y-axis. Secondly, why include the eight-mark coins in the total for 1694, but exclude them from the calculated percentages? This leads to a small offset between the hoard figures and those for the mint. It would be more consistent to either exclude them completely, or to include them. As it is, the eight-mark coins were only struck in 17 years of the 77 under consideration here, and in ten of those years they accounted for less than 6% of the total value of coins struck. The largest exception was 1698 when 19.2% of the value of coins struck was in eight-mark pieces.

Diagram C takes the figures from B, and plots how the hoard deviates from the expected mint figures (Fig. 2b). Thordeman notes how the hoard values lie below the

²³ Thordeman 1948, p. 198.

abscissa for the early years and lie generally above the abscissa for the later years. Thordeman, rightly, interprets this as a result of the systematic loss of coins from the coinage pool. Again, we must remember that these deviations are scaled in terms of the most common year. Using straight percentages gives us deviations between -1.56% and $+1.22\%$.

Thordeman's Diagram D is an attempt to derive the loss rate. The data line in Figure 2c is calculated by taking the differences between the hoard and the mint figures plotted in C, and then expressing this as a percentage of the higher of the two percentages for the value of marks struck in a particular year from the hoard and the mint. So, for 1664, the deviation plotted in C is -7.5% . The value of the marks as a percentage of 1694, as used in his Diagram B, is 14.0% for the mint figures and 6.5% for the hoard. The figure plotted in Diagram D is, therefore $-7.5/14.0 \times 100 = -53.4\%$. We have provided these deviation figures in Table 6 in Appendix A.

To the jagged data line in Diagram D Thordeman adds a trend line with a 2% change per annum. Unfortunately, he does not explain how he came to that figure.²⁴ The figures cited by Volk are, however, those for the trend line and not the data (*cf.* Table 6 here) and cannot therefore explain how that trend line was derived. As a result, we have undertaken a linear regression, the line from which is plotted in Figure 2c.²⁵

Thordeman's last diagram, E, uses his 2% line as the abscissa and plots the differences above and below that line. In Figure 2d we have closely reproduced this graph by using the regression line as the abscissa and plotting the variation around it. We would expect this variation — known as the residuals — to be random. From Figures 2c–2d, however, we can see that the pattern seems more structured, and hints at something more complex happening in the data, to which we will return.

There are two problems with this. Firstly, the observant will notice that Thordeman's diagrams D–E end at 1722, not at 1738, the date of the latest marks in the hoard. Plotting all the figures produces Figure 3. Clearly the neat pattern seen in the

²⁴ Volk suggests that he simply took the difference between 1664 (-62%) and 1722 ($+54\%$) and divided it by the number of years ($116/58 = 2$) (Volk 1987, p. 147).

²⁵ This has a high correlation of $r = 0.93$ and provides a figure of 1.85 for the slope, slightly less than Thordeman's 2% . We prefer, however, to estimate the loss rate differently, see below.

earlier years falls apart, and is presumably why Thordeman omitted these years. This break-down can be partly explained by Thordeman's method which over-emphasises variation in small issues. Thus, in 1738 a mere 56,524 marks-worth of the relevant coinage was struck, just 0.11% of the total production, or if we assume a 2% loss rate, 0.27% of the coinage in circulation in 1738. The Lohe hoard has, however, no coins of that date. Using straight percentages this seems irrelevantly small but Thordeman's method results in a -100% deviation. In addition, variation in the most recent issues — discussed at some length below — serves to exacerbate the problem.

Figure 3 somewhere near here.

Secondly, why should the deviations calculated in this manner reflect the annual overall loss rate? To examine this, we calculated a 'perfect' Lohe hoard where each year represented the same percentage as the value of the 1–4 mark coins in circulation in 1741 calculated on the basis of a 2% per annum loss rate. We then went through the same procedure used to calculate the deviation line in Thordeman's Diagram D. This is shown in Figure 4 as the dashed line alongside the actual deviations. The break-down in the neat pattern after c.1718 is still clear. Additionally, the 'perfect' line is actually a slight S-shape, and thus estimating a linear loss rate from this is problematic.

Figure 4 somewhere near here.

As a result of the various problems discussed above, we feel that Thordeman's method is flawed and should be rejected. There are other ways of comparing hoards to mint figures and calculating the loss rate as demonstrated below.

4 Thordeman's 'Law'

Thordeman's 'Law' breaks down into two distinct parts which for clarity we will discuss separately. The first part is that a coin hoard will represent the local coinage pool from which it was withdrawn, and the larger the coin hoard the more accurate that representation will be. The second part is that the composition of the global coinage pool is a reflection of coins as struck modified by a loss rate. We will examine the loss rate first.

4.1 The loss rate

Once coinage has been released into circulation, a certain proportion each year will be removed from the coinage pool due to hoarding, recycling, or simple loss. This

loss rate has also been called the decay, wastage or attrition rate.²⁶ Obviously, the longer an issue has been in circulation, the greater the proportion that has been lost.

As a result, older issues are poorly represented in the global coinage pool compared to newer issues. This loss rate is, in fact, a simple compound depreciation rate familiar to accountants and can be defined as

$$x = y \times (1 - d)^i$$

where y is the number of coins originally struck, d is the loss rate expressed as a proportion and i is the number of years since the issue was released into the coinage pool.²⁷

There have been a number of attempts to calculate this loss rate, or assess the impact of different rates on hoard compositions. Volk reviewed the topic in detail and estimated the loss rate for the Roman Republic at between 2.75% and 3%.²⁸ Patterson's study looked at loss rates for a variety of periods and places and obtained estimates from between 1% and 4%,²⁹ whereas Preston's study for the Roman Republic produced estimates between 2.5% and 4%.³⁰ Iossif's paper uses a value of 1.5%, but also assesses the impact of different rates.³¹ Lockyear's study, based on a simulation approach similar to that adopted here, suggested that an average rate of 2% was appropriate for Roman silver *denarii*.³² An important point made by both Volk and Lockyear is that it is likely that the decay rate will vary with the denomination under consideration.³³

²⁶ Buttrey 1993; Duncan-Jones 1999; Iossif 2015; Lockyear 1991, 2007; Volk 1987.

²⁷ See also Iossif 2015, p. 239, note 21.

²⁸ Volk 1987, p. 181.

²⁹ Patterson 1972.

³⁰ Preston 1983.

³¹ Iossif 2015.

³² Lockyear 1999, p. 236. Note that the estimates produced by Duncan-Jones 1999 fail to account for the growth of the coinage pool, as he himself admits (p. 252).

³³ Lockyear 1999, p. 240; Volk 1987, p. 145.

We have adopted a relatively simple approach to determining the loss rate, identical to that used previously.³⁴ For each year of minting, we can calculate the compound depreciation between that year and 1741, the closing date of the hoard in terms of coinage.³⁵ Thus, for the value of the 1–4 mark coins struck in 1664 and using a 2% loss rate we get:

$$897,541 \times (1 - 0.02)^{77} = 189,436$$

We do this for every year of minting until we have the percentage that each year's coinage forms in 1741. This allows us to plot the coinage population for 1741 as a cumulative percentage curve (see Figure 5) which can be compared to the curve for the Lohe hoard.³⁶ To see how well the calculated population curve compares to the Lohe hoard, we can calculate the Kolmogorov-Smirnov statistic D_{\max} between the two curves. D_{\max} is the largest absolute deviation between the two curves. The smaller the value of D_{\max} , the better the fit.³⁷ By varying the loss rate, and calculating D_{\max} between each curve and the hoard, we can find the closest fit. For the value of the mark coins struck and the hoard, this comes to 2.12% per annum (see Table 1).³⁸

Figure 5 somewhere near here.

Table 1 somewhere near here.

The fit between the Lohe hoard and the mint figures is astounding, but this should not blind us to the main weakness in both Thordeman's analysis and ours, namely that we are using the *value* of the mark coins struck, not the *numbers* of coins struck. Our depreciation rate is that for the *value* of the one- to four-mark coinage in

³⁴ Lockyear 1999.

³⁵ As no mark coins were struck after 1738, choosing any date after that would make no difference in the relative proportions of issues.

³⁶ In these graphs the first data point represents the coinage minted in 1664, the second point that for 1664 and 1665 combined, the third for 1664–66 and so on. The final point will be 100% and must be in the final year of minting (i.e., 1738) even though the figures are those calculated for 1741. Cumulative percentage curves have been used previously in numismatics, e.g., by Ryan 1988.

³⁷ See Lockyear 2018, pp. 130–1 for a description of this measure.

³⁸ The first author has developed a simple Excel spreadsheet in which one can simply change the decay rate and automatically obtain the calculated coinage pools for the period 1664–1741.

circulation, not for the individual denominations. It seems inherently likely that the decay rate will vary between denominations.

4.2 Representative hoards

That coin hoards can represent the coinage in the circulation pool, and that larger hoards *should be* a closer reflection of the coinage pool than smaller ones, is a specific instance of the basic laws of sampling.³⁹ Sampling theory, however, insists that for a sample to be representative of the overall population, it must be a *random* sample.⁴⁰ For hoards, it is clear that not all are random samples of the coinage in circulation. Hoarders may deliberately exclude debased coins, or only collect a single denomination. A good example is the large Brogade hoard found in a house in the town of Køge south of Copenhagen that contained, amongst 763 Danish four-mark coins, more than 150 freshly minted and die identical four-mark coins from 1671 which must have been delivered to the owner more or less directly from the mint in Copenhagen.⁴¹ It is with this in mind that Reece, in the absence of mint figures, compares groups of hoards in order to what define a 'normal' hoard should look like.⁴²

Within hoard studies is the idea that hoards can be 'emergency' or 'savings' hoards. In the former case, the coins have been collected at one moment in time, and in the latter case over a period of time. These terms are not ideal as they conflate the *reasons* for hoarding with the *time span* over which the coins were collected. In his study on the impact of these factors on coin hoard compositions, Lockyear replaced those terms with Type 1 and Type 2, but they have not been adopted.⁴³ In this paper we will refer to these two theoretical hoard types as Type 1 ('emergency') and Type 2 ('savings'). By definition, a hoard which is a simple random sample will be a Type 1 ('emergency') hoard. Lockyear has argued that hoards of Roman Republican *denarii* can be regarded, on the whole, as random selections of the *denarii* in circulation because of

³⁹ Orton 2000.

⁴⁰ There are many ways of selecting an appropriate random sample but for our purposes these need not detain us.

⁴¹ Aagaard 1992–3; Kromann 1989.

⁴² Reece 1974, 1980, 1981.

⁴³ Lockyear 1991, p. 196.

the consistent structure of those hoards.⁴⁴ This randomness is in part due to the fact that the coins were struck at a consistent weight and purity over the majority of the period he studied.

The Lohe hoard is a good example of this terminological problem. Thordeman is aware⁴⁵ that the manner in which a hoard is collected will impact its composition. Earlier in the paper Thordeman suggested the hoard was deposited in 1743 due to reversals in the war between Sweden and Russia and an uprising in Stockholm.⁴⁶ In that sense, it is an 'emergency' hoard, but it is unlikely that it represents a random selection of coins in circulation selected at one moment in time.⁴⁷ One can see that the selection of coins in the hoard is not entirely random by looking at the denominations from 1698 onwards which we discuss in detail below.

Lockyear has shown that large hoards are often rather different to other hoards of the same date and appear unusual.⁴⁸ Iossif omitted two very large hoards from his study "because they introduced considerable 'noise' in the results."⁴⁹ We can explain this deviation from sampling theory by thinking about the processes of coin hoard formation. Whereas a small hoard might represent a few days takings at a bar, how would a very large hoard come about? How did the Brescello hoard, with its 80,000 gold coins weighing two-thirds of a tonne, come to be collected, deposited and lost?⁵⁰

Two recent Roman Imperial hoards can illustrate the point. The Frome hoard consists of 52,503 coins. Careful excavation of the hoard in layers on site has revealed that the latest coins, struck by Carausius, were in a group in the middle of the hoard. That, and the sheer weight of coins in a relatively fragile pot, led Moorhead to suggest that the hoard was created by multiple additions of smaller hoards into the buried

⁴⁴ Lockyear 2007, pp. 28, 171–2.

⁴⁵ Thordeman 1948, p. 200, note 22.

⁴⁶ Thordeman 1948, p. 196.

⁴⁷ Volk (1987, pp. 159–65) provides an extended discussion of the nature of the hoard.

⁴⁸ Lockyear 2007, p. 172.

⁴⁹ Iossif 2015, p. 236.

⁵⁰ Lockyear 2007, p. 29.

vessel.⁵¹ The Beau Street hoard consisted of 17,577 coins. It was block-lifted and excavated in a conservation laboratory. The coins had been buried in eight leather bags in a wooden chest. One of the bags contained mainly earlier silver *denarii*, three contained early silver radiates and four were of late debased radiates.⁵² These bags were clearly deposited at the same time, but how were the contents collected? Are the silver *denarii* an earlier hoard added to the crate, or the deliberate selection of high-value silver coins that were, at the time of deposition, a tiny fraction of the circulating coinage pool?

We have, therefore, a paradox: the Lohe hoard is huge and is therefore extremely unlikely to be a 'simple random sample' of the coins in circulation. There is, however, an astonishingly close match between the theoretical population profile and the hoard profile. One possible explanation is that as money-lenders the hoard could have been created by requiring payment from many small debtors, who perhaps had to procure money by selling goods, which would then accurately reflect the distribution of the individual issues of mark coins in the circulation in Stockholm and its catchment area. An alternative explanation can be offered by examining the impact of the pattern of production on variability in the coinage pool. Lockyear has demonstrated the impact of this factor elsewhere but it is worth reiterating the issues again here.⁵³

The coinage pool can be seen as a dynamic system with variable inputs (new coins added to the pool), circulation, and constant outputs (coins lost and so on). Hoards, as samples of that system, reflect these aspects of it. Lockyear divided hoard profiles into three zones:⁵⁴

1. The fall-out zone which represents the oldest coins which have largely fallen out of circulation and are thus rare;
2. The homogenous zone which represents the coins which have been circulating for sufficient time to be reasonably evenly spread across the coinage pool; and

⁵¹ Moorhead et al. 2010.

⁵² Ghey 2014.

⁵³ e.g., Lockyear 2007, pp. 216–24.

⁵⁴ Lockyear 1993.

3. the erratic zone where differences in the local coinage pools can result in marked differences between hoards.

More recently, Wythe has coined the somewhat more memorable terms: the tail, torso and nose of a hoard.⁵⁵

Reece noted that the variation between coin hoard profiles was often largely due to variation in the 'nose' of the hoard.⁵⁶ The causes of this variation was investigated subsequently by Lockyear.⁵⁷ One cause of the variation is the pattern of coinage production and supply.

To illustrate this, imagine an area with nine regions, represented in Figure 6 by the small grid. In both scenarios, A and B, there are 900 units⁵⁸ of coinage in the global coinage pool evenly distributed across the nine regions. There are, therefore, 100 units per region. Let us call these Type B coins. In scenario A, at time T1, a new issue of coinage — Type A — is released into the NW region of our coinage pool. This new issue is 9 units in size, thus 1% of the existing overall coinage pool. At the time of issue, the local coinage pool for the NW region has all the new Type A coins, forming 8.25% of that local pool. After some time has passed, at time T2, the new issues have started to spread out into the neighbouring regions until at time T3 they are evenly distributed. Note how the percentage of new coins in the local pool changes (shown in Fig. 6, line A%). In scenario B, 135 units of new Type A coins are struck and released into the NW region. Although this is only 15% of the existing global coinage pool, it forms 57.4% of the NW province's local coinage pool at time T1, and still forms 40.3% of that pool at time T2. We can see, therefore, that large issues of coinage can create enormous differences between hoards, principally in the 'nose'. Conversely, periods of relatively low coin issue leads to minimal differences between hoards.

Figure 6 somewhere near here

Obviously, any hoard is unlikely to reflect *exactly* the local coinage pool. Random variation will result in differences between the true figures and the estimates provided by the hoard. By using the *standard error of the proportion* we can see for any given

⁵⁵ Wythe 2015, pp. 170–1.

⁵⁶ Reece 1974.

⁵⁷ Lockyear 1989, 1991, 1993.

⁵⁸ These 'units' are arbitrary and can be multiplied by any large number one wishes.

hoard size and proportion what the range of values of the underlying population are likely to be. In Table 2 we can see the figures for a hypothetical hoard of 100 coins which contain Type A and Type B coins. If a hoard contained eight coins of Type A, the true population percentage has a 95% (19 in 20) chance of being in the range 3.5% to 15.2%. If our hypothetical hoard had 40 Type A coins, the underlying value is likely to be in the range 30.3% to 50.3%. This illustrates one aspect of the standard error of the proportion, namely that the closer the proportion gets to 50%, the larger the standard error. Going back to our scenarios A and B, hoards of 100 coins in scenario A at time T1 will show moderate inter-hoard variation, but by time T3 this will be quite small. Conversely, hoards of 100 coins in scenario B could easily have as many as 70% new coins at time T1, and even at time T3 will show, purely due to random variation, greater inter-hoard variation than scenario A.

Table 2 somewhere near here.

In the real world, of course, our hoards are of widely differing sizes. We can do a similar experiment to see the impact of hoard sizes. Figure 7 shows the upper and lower 95% limits for hoards from 5 to 500 coins in which 20% are Type A. As can be seen, for a hoard of only five coins the underlying population is likely to be in the range 0.5–71.6%. These limits rapidly come down so that for hoards of 30 coins the limits are 7.7–38.6% and for hoards of 500 coins it is 16.6–23.8%.

Figure 7 somewhere near here.

The last point to note is the impact of all this on the *rest* of the hoard profile. In our two scenarios the coinage pools were identical before the new issue of coinage. At time T1, however, the Type B coins in scenario A still form 91.75% of the local pool in the NW corner ‘province’, but only form 42.6% in scenario B. In terms of actual numbers of coins, however, they are identical. This problem is known by statisticians as ‘closure’. Reece dealt with this problem of variation in the ‘nose’ of hoards impacting the figures for the whole hoard by omitting the most recent coins from his calculations.⁵⁹

We can see the impact of this variation in a study recently published by John Creighton.⁶⁰ In that study we can see that average percentage of coins of Groups K and L

⁵⁹ Reece 1980, 1981.

⁶⁰ Creighton 2014.

(AD 161–197) is never more than 12.8% whereas the average for period M is 47.2% in ad 215.⁶¹ Creighton’s groups M and N were struck pretty much at the same time (AD 193–212 compared to AD 196–217) so we should perhaps add the 10.7% of Group N coins to the Group M giving 57.89% in total. We would, therefore, expect low inter-hoard variability in the period AD 160–200 but high variability in the early third century. This is indeed the pattern we can see in Creighton’s Figure 6. We have, therefore, to modify his interpretation diagram⁶² in the following manner:

Observation

Hoards with variable composition

Interpretation

Recent issues form a large proportion of the global coinage pool.

Hoards with uniform composition

Recent issues form a small proportion of the global coinage pool.

The importance of understanding these issues for the Lohe hoard lies in resolving the paradox of the hoard being enormous, and therefore unlikely to be a ‘simple random sample’, but actually being a close reflection of the composition of the coinage pool as seen in Figure 5. As can be seen in Table 6, very little mark coinage was struck after 1720. The hoard, however, was not deposited until c.1742. The composition of the global coinage pool in the period 1721–41 barely changed. To illustrate this we have plotted the coinage pools for five years between 1721 and 1741 in Figure 8. All the coin populations are almost identical and as a result the Lohe hoard could have been created by combining several smaller hoards collected at any time in the 15–20 years before the final closing date. We would not be able to see this easily in the data, and the net effect is to create one very large, essentially random, sample of coinage which reflects closely the structure of the coinage pool at that time. One should also note that the hoard was assembled in the same city as the principal mint — Stockholm — which would minimise the differences due to variable circulation patterns.

Figure 8 somewhere near here.

⁶¹ Figures from Creighton 2014, Appendix 3.

⁶² Creighton 2014, p. 133.

For comparison, let us look at the coinage pool for the period 1690–1710 when some very large issues were struck. As can be seen in Figure 9, there are huge differences between the coinage pools over that time span. A hoard which consisted of multiple smaller hoards collected at different dates over this time span would have a very strange composition compared to the coin population for its closing date. An example of this phenomenon from the Roman period is the Falkirk hoard which Richard Reece argued many years ago consisted of two separate ‘withdrawals’ from the coinage pool giving it a very unusual structure.⁶³

Figure 9 somewhere near here.

5 Beyond Thordeman

Up to now we have concentrated in examining Thordeman’s work in the light of more recent research. The Lohe hoard and the mint figures, however, do give us an opportunity to examine some of these issues in more detail.

In Thordeman’s original paper, he noted that the deviations he plotted followed ‘a certain undulation.’⁶⁴ A close examination of Figure 5 shows that the hoard profile lies below the mint at the start of the graph and above it towards the end. Figure 10 plots the deviation of the hoard from the coinage pool (*i.e.*, the depreciated mint figures) to make the pattern clearer. The zero line represents the coinage pool as a *cumulative percentage* as plotted in Fig 5. If a point for a specific hoard year is below the zero line it means that *cumulative* total up to that year is below the cumulative total for the coinage pool, and obviously *vice versa*. If a section of the curve is parallel to the zero line, those years have the same proportions as the coinage pool. If the section of the curve is rising, it means that section has higher proportions than the mint, and if it is falling, lower. Thus between 1706 and 1716 the hoard has higher proportions of coins of that period than the coinage pool. The cumulative total in 1706 is below the cumulative total for the coinage pool, but by 1716 it is now above the cumulative total.

Figure 10 somewhere near here.

⁶³ Reece 1980.

⁶⁴ Thordeman 1948, p. 199.

From this graph we can see that the pattern is rather complex. From 1664 until 1680 the hoard has less coinage than we would expect from the mint figures (in terms of value), and then from 1680 to 1694 it has more than the mint figures suggest. There is another downturn from 1694 to 1706, another upturn until 1716 and then a period of relatively stability (although we should remember that very little coinage was struck after 1722). What makes these very small deviations interesting — note that the entire range is only 3% — is the ‘undulation’ of them. A certain amount of random noise would be expected, but this should cause the deviations to be much less structured than we can see here. Thordeman wondered if they might be ‘periodical [sic.] changes in the economic conditions...’⁶⁵ Can we track down the causes more precisely?

Before continuing, we need to revisit the issue of sample size. Those familiar with the hypothesis-testing approach of classical statistics may be surprised that we have not employed this approach thus far. Indeed, the D_{max} statistic used above is normally the first step in the application of the Kolmogorov-Smirnov test. In this case, however, the sample size is so large that it is pretty much guaranteed that a ‘statistically significant’ result would be obtained. In recent decades the whole philosophy of the approach has been under debate, and we have to differentiate between *statistical* significance and *practical* significance.⁶⁶ In our case, the 1.77% maximum difference between the population and hoard profiles mentioned above (Table 1) may well be statistically significant but could well be of no practical significance.

Up until now we have been working with the value of coins because of the lack of detailed mint figures before 1698. From then onwards, however, we can examine the pattern in the three smaller denominations (see Tables 5– 6). The method employed is the same as previously, but this time the actual numbers of coins struck can be compared with the hoard. The figures for D_{max} are likely to be larger, partly because the numbers of coins involved is so much smaller⁶⁷ and partly because the relative size of each issue as struck will be larger. These two combine to increase the impact of variation in the ‘nose’ of the hoard.

⁶⁵ Thordeman 1948, p. 199.

⁶⁶ Chow 1996; Cumming & Calin-Jageman 2017; Wang 1993.

⁶⁷ For example, there are only 807 four-mark coins struck between 1698–1738 in the hoard.

For the one-mark coins, none were struck after 1721. Using all the coins struck between 1698–1721 we obtain a loss rate of 4.45%, much higher than for the value of all the coins we obtained earlier (Table 3). The two-mark coins by contrast have a loss rate of 2.14%, very similar to the overall rate. The four-mark coins, however, have a surprisingly low loss rate of only 0.96% and a relatively large Dmax value of 5.63%. Although the four-mark coins are the least numerous of the three denominations, 807 coins is still a large number. This large Dmax value is mainly due to the issues in the nose of the hoard. Four-mark coins struck in 1737–8 formed 6.6% of the four-mark coinage pool in 1741, but the Lohe hoard contained only 12 examples of coins from 1737 and none of 1738, only 1.5% of the four-mark coins.

In order to test the impact of omitting the most recent issues of coins,⁶⁸ the Dmax values for loss rates between 1–5% in 0.01% increments were calculated for a series of truncated date ranges starting with 1698–1713, then 1698–1714 and so on. The truncated date ranges with the best fit between the mint figures and the hoard is shown in the lower half of Table 3.⁶⁹

The figures for the four-mark coins, leaving out the last four issues, have a much improved Dmax figure, but a much higher loss rate of 3.93%. Looking at the graph (Figure 11c), we can see that the last four issues of four mark coins are under-represented in the Lohe hoard, and especially the last two. It seems that an average loss rate of 3.93% is more likely, although we must be aware this is based on only 19 years of issue.

Figure 11 somewhere near here.

The truncated sequence for the one-mark coins has a better fit (Figure 11a) but only has a relatively small change in the loss rate, and remains the highest loss rate for the three denominations. The two-mark coins were the only denomination in continuous production from 1698 until 1737. Truncating just the final year from the calculations provided a much better fit (Figure 11b), and a modest change in the loss rate.

⁶⁸ As had Reece previously, Reece 1980, 1981.

⁶⁹ This was achieved by writing a script in the R statistical package (R Core Team 2018). The script is available from the first author.

The variation in the nose of the hoard is probably the result of the various factors discussed above, and the probable creation of the hoard from many smaller assemblages. The irregular production of the one- and four-mark coins adds to the uncertain patterning in the twenty years before the closing date of the hoard. It is notable that there is not a simple progression in loss rates between denominations, with the smallest and largest denominations having the highest loss rates. The short sequence on which these rates are based, however, must be kept in mind.

Having examined the pattern for each denomination, we can now compare the composite pattern for the mint and the hoard across all four denominations. Figure 12 shows the cumulative percentage line for the sum of the one-, two and four-mark coins in the Lohe hoard and the figures for those issues in 1741 depreciated using the loss-rates in the lower half of Table 3. We can see from this that the match between Lohe and the mint records is very close at first, but then the hoard line rises above that for the mint between 1705 and 1716, and then slowly converges with it again from c.1716 to 1738.

Above, we argued that we would not be able to see the differences between a Type 1 ('emergency') hoard and a Type 2 ('savings') hoard when recent issues formed a very small component of the coinage pool. In this case, however, the huge size of the hoard and the detailed mint records allow us to demonstrate that the hoard must be regarded as a Type 2 hoard and was probably made up of several lots collected over perhaps twenty years.

Can we see any patterning in the denominations selected for the hoard? It is quite clear that the eight-mark coins are under-represented in the hoard. Volk, building on the data provided by Sarvas, has shown that these coins are rarely hoarded within Sweden.⁷⁰ For the one- to four-mark coins, we can simply calculate the percentage of each denomination for each year from 1698–1737 in the hoard and compare those figures to the percentages struck. As the three denominations appear to have different loss rates, we have also compared the hoard figures to those for the struck coins depreciated by figures given in Table 3. A curious pattern emerges. The four-mark pieces are generally over-represented in the hoard with 15 of the 19 possible years having more coins than one would expect, and only four with less. Using the depreciated

⁷⁰ Volk 1987, p. 156; Sarvas 1969.

figures makes the pattern stronger with 17 years with more than expected quantities of four-mark coins. A similar pattern exists for the one-mark coins with 15 of the 23 possible years with too many coins, or 20 years using the depreciated figures. In contrast, the two-mark coins are under-represented with only five of 25 years having too many coins, or three using the depreciated figures. Why the four- and one-mark coins should have been preferentially selected over the two-mark ones is unknown, but it reflects the patterns seen in the loss rates discussed above.

Selection of specific denominations to be hoarded is a widely observed phenomenon. Roman Republican hoards of silver coins sometimes contain a selection of *quinarii* alongside the *denarii*, and sometimes do not. The Stockholm mint struck several minor silver denominations up to 1741 which are completely absent from the Lohe hoard. If the Lohe hoard is, as we argue, actually several hoards combined then variation in the representation of the one-, two- and four-mark coins *may* be a reflection of hoarding choices of the individuals who assembled the constituent parts of the combined hoards. The inclusion of large numbers of 10 öre coins from 1739–40 is at odds with the rest of the hoard and may be a reflection of the fact that they contained 3.12g of silver compared to the 1 mark coins which contained 3.61g of silver but had not been struck after 1721.⁷¹

6 Conclusions

Thordeman's paper remains a key work in the development of numismatic studies. His method, some 70 years or so later, is unnecessarily complex and over-emphasizes the variation in small issues of coinage. He also does not explain the key step which is how he derives the loss rate from his Diagram D. With the development of computers and statistical techniques, there are more viable alternatives for examining these themes of which we have used just one in this paper. It is also unfortunate that he did not publish the numbers on which his graphs are based which would have enabled others to follow-up on his work more easily.

Thordeman's 'law' consists of two parts: firstly, that *if* we can be confident that a hoard is essentially a random sample of coinage — or at least of a denomination — in circulation, it can be used to examine the relative sizes of coin issues in the circulation

⁷¹ Tingström 1963, pp. 73–4.

pool. The accuracy of those estimates depends on the sample size and can be calculated using standard statistical sampling methods.

Secondly, once an issue has been released into circulation, it will be subject to compound depreciation providing there are no events that would disrupt that pattern such as debasements, deliberate removals of coinage or monetary reforms. The rate of that depreciation will vary from place to place, time to time and even between denominations.

There are, however, several caveats to this 'law'.

1. Very large hoards are unlikely to be random samples. They might be gradual accumulations of coins, multiple smaller hoards combined, or be an official payment gone awry containing 'too much' new coinage. Other scenarios are possible. The composition of large hoards is often unusual due to these factors.
2. Even if a hoard can be considered a random sample, it will represent the composition of the *local* coinage pool, not the *global* coinage pool. This will result in the most recent coinage being over-represented in some areas and under-represented, or even missing altogether, in others.
3. If the most recent issues form a large proportion of the coinage pool, this will create large differences between local coinage pools which is reflected in large differences between hoards. Conversely, as with the Lohe hoard, if the most recent issues are relatively small, hoards will look very similar to each other. Also, it will be extremely difficult to differentiate between Type 1 ('emergency') and Type 2 ('savings') hoards.
4. As the standard error of the proportion increases the nearer the figure is to 50%, large issues will continue to create larger differences between hoards even after the coinage pool has evened out.
5. Partly due to the factors listed above, estimation of the loss rate in the absence of mint figures is difficult. We would argue that studies that rely on these uncertain depreciation rates should provide a *range* of likely answers, not a single figure.⁷²

⁷² For example, Lockyear (2007, pp. 243–5) in re-examining Hopkins' figures suggests that the quantity of Roman Republican *denarii* in circulation rose between five- and ten-fold in the period 157–50 BC.

We believe that understanding how the pattern of supply, circulation and loss, along with hoard size and the manner of hoard collection, impacts the composition of coin hoards is vital in order to meaningfully interpret the patterns we observe in our data.

In numismatics, rarely do we have mint figures to act as a check against our hoard data. To assume that hoards generally represent a random sample of coins in circulation is dangerous and wherever possible multiple hoards should be compared. Different denominations will be impacted by different factors, and so comparisons are likely to only be valid for a single denomination. Thordeman appreciated this as shown by his diagram G, which plots the distribution of Swedish one öre coins from 1633–1677 in relation to the mint records in seven hoards from different places in Scania in Southern Sweden. If such a fused ‘hoard’ representing a shorter mint period shows a good correspondence to the mint records — perhaps after a suitable adjustment for a yearly loss in circulation — we can have some confidence in our calculations. A lack of correlation is more problematic and we may have to identify aberrant hoards using alternative methods such as die studies.⁷³

Thordeman’s diagram G represents a promising basis to check and possibly supplement mint records, since it is based on 1) the same denomination and 2) almost contemporary hoards from different locations. Thordeman is apparently not aware of the fact that there might be significant geographical differences in the distribution of years of coin groups as a result of an uneven spread of newly minted coins as discussed in detail above.⁷⁴ This part of Thordeman’s work presents nevertheless valuable ideas such as using fused hoards to estimate/control mint records.

We would argue that Thordeman’s ‘law’ should be regarded as a ‘general principle’ which is subject to a large number of important caveats. The Lohe hoard provides us with an excellent opportunity to examine and illustrate these issues, but it is in many ways unique.

⁷³ Aagaard 1992–3; Aagaard 2004, pp. 201–2.

⁷⁴ Aagaard & Märcher 2016.

Figure Captions

Figure 1: Selected denominations of Swedish coins in the Lohe hoard. Drawn by Sven Aagaard using MDTM (Aagaard & Märcher 2015).

NB Size of the coins is given in the file names.

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(a) Diagram B.

(b) Diagram C.

(c) Diagram D.

(d) Diagram E.

Figure 2: Thordeman's Diagrams B–E redrawn from the current figures. NB. The trend lines used in Diagrams D–E are from our regression analysis and differ from Thordeman's.

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(a) Diagram D plotted to 1738.

(b) Diagram E plotted to 1738.

Figure 3: Thordeman's Graphs D–E with all the years until the end of mark production plotted. The trend line is from regression analysis and differs from Thordeman's trend line.

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Figure 4: Graph D-type deviations for the Lohe hoard (solid line) and a hoard which perfectly reflects the coin population in 1741 after a 2% depreciation (dashed line).

Figure 5: The Lohe hoard compared to the calculated coinage pool using a decay rate of 2.12%.

Figure 6: The impact of new coinage issues on the coinage pool (see text for details).

Figure 7: The upper and lower 95% confidence limits for the proportion of Type A coins in hoards of between 5 and 500 coins, where Type A forms 20% of the hoard.

Figure 8: Coinage populations for 1721, 1726, 1731, 1736 and 1741 calculated from the figures in Table 6 and using a 2.12% decay rate.

Figure 9: Coinage populations for 1690, 1695, 1700, 1705 and 1710 calculated from the figures in Table 6 and using a 2.12% decay rate.

Figure 10: The deviation of the hoard figures for 1–4 mark coins against the population profile as shown in Fig. 5.

(a) One-mark coins.

(b) Two-mark coins.

(c) Four-mark coins.

Figure 11: Cumulative percentage curves for individual denominations in the Lohe hoard and the coin populations based on the loss rates given in Table 3. The population curves are all depreciated until 1741 but reach 100% in the final year of production, or inclusion in the calculations.

Figure 12: Cumulative percentage curves of the total numbers of coins struck but depreciated by the best-fit loss rates compared to the Lohe hoard.

Table captions

Table 1: Comparison between the Lohe hoard and the calculated coin populations omitting the values for the eight-mark coins.

Table 2: Lower and upper 95% limits for hoards with 100 coins for the scenarios illustrated in Figure 6. See text for details.

Table 3: Loss rate calculations for the one-, two- and four-mark coins.

[in Appendix A]

Table 4: Mint figures for one- to eight-mark coins, 1664–97. †One carolin-daler is worth four marks. ‡There is a typographic error in the published record for this year with 366,346½ in one column and 366,364½ in the other. This is only a 72 mark difference and makes little practical difference.

Table 5: Mint figures for one- to eight-mark coins, 1698–1738. †One carolin-daler is worth four marks.

Please align columns with fractions on final digit.

Table 6: Mark coins minted between 1664–1738 in the Lohe hoard together with the mint records and the deviations for Thordeman's Diagrams C and D. The deviations are calculated from the current figures and may vary in detail from Thordeman's.

Table 7: Percentages of each denomination struck and in the hoard.

Appendix B The hoard

The listing below is based on an undated typewritten list from Kungliga Myntkabinetten, Stockholm from 1942 which refers to work from 1937 with (later?) handwritten corrections. In 1992, a new typewritten list dated 1981 was presented to the second author. For unknown reasons the number of coins in the new list was, for almost all years, lower than shown here.

Insert listing here

Foreign coins

Swedish Pomerania: $\frac{1}{3}$ thaler 1673

Denmark: 1 krone (= 4 mark) 1657

Wernigerode: 1 thaler 1624

Braunschweig-Lüneburg: 1 thaler 1667

Olmütz: 1 thaler 1695

The Netherlands: $\frac{1}{2}$ thaler 1620 and 1624

Spanish the Netherlands: $\frac{1}{4}$ patagon no year

Hungary: Thaler 1619, 1651, 1691 (3), 1692 (2), 1695

Elbing: 18 groschen no year

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