

1 **A micromechanical investigation of sand–rubber mixtures using the discrete**
2 **element method**

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Abstract

Mixing soil or sand with tire rubber fibers or buffings is a practical and promising solution to the problem of global scrap tire pollution. However, sand–rubber mixtures exhibit unsatisfactory and complex mechanical properties in engineering applications due to the large deformation of rubber fibers. In this study, a detailed numerical approach to modeling mixtures of sand and rubber fibers via the discrete element method (DEM) was proposed, and the effect of rubber content on the macroscopic and microscopic mechanical behaviors of sand–rubber mixtures was investigated. Furthermore, the reinforcing mechanism by which rubber fibers contribute to the micromechanics of sand–rubber mixtures was explored to determine the optimum rubber content in terms of soil mechanical performance. Comparative analysis of the experimental and numerical results demonstrated the applicability and ability of the proposed DEM model and related parameters for modeling sand–rubber mixtures. Through investigation of the constitutive behaviors of sand–rubber mixtures with various rubber contents under triaxial compression, a rubber content of 20% was found to provide the best shear resistance in the critical state. The micromechanics of sand–rubber mixtures, namely particle kinematics, the interparticle coordination number and rubber fiber deformation, were investigated to demonstrate the specific reinforcing mechanism of rubber fibers with respect to improved soil performance. The resulting data strongly support the identified optimum rubber content for sand–rubber mixtures that will provide a valuable guidance to the relevant engineering applications.

Keywords: Sand–rubber mixture; Discrete element method; Local shear band; Coordination number; Rubber fiber deformation

48 **1 Introduction**

49 Environmental protection and construction material resource are important issues in
50 engineering geological fields. The development of the automotive industry has led to the
51 accumulation of an alarming volume of scrap tires worldwide. Most of these waste tires are
52 discarded in landfills or tire stockpiles, leading to serious environmental problems (Kawata et al.,
53 2008). Accordingly, there is an urgent global need for green and economical scrap tire recycling
54 options, such as the use of waste tire rubber-derived products as construction materials in
55 geotechnical and geological engineering applications.

56 Due to advantages such as light weight, low cost and easy processing, rubber-derived
57 products are used widely as subgrade and embankment fillings (Yoon et al., 2006; Edincliler et
58 al., 2010; Soleimanbeigi and Edil, 2015); to reinforce weak soil foundations (Humphrey, 2007;
59 Moghaddas Tafreshi et al., 2013 and 2019) and retaining walls (Ahn and Cheng, 2014; Reddy
60 and Krishna, 2015); and as alternative aggregates for light concretes (Batayneh et al., 2008; Liu
61 et al., 2012; Thomas and Gupta, 2016). Over the last two decades, these successful applications
62 have demonstrated the potential value of these rubber-derived products in the above mentioned
63 geotechnical construction applications and suggested a purpose for recycled waste tires.
64 However, as tire rubber shows large deformation, the mechanical performance (e.g., bearing
65 capacity and deformation) of rubber fills and soil–rubber mixtures in relevant engineering
66 applications has become a major concern (ASTM, 2008). Studies are urgently needed to
67 investigate the mechanical properties of soil–rubber mixtures and determine the optimum rubber
68 content in terms of mechanical performance.

69 According to practical engineering requirements, waste tires are usually manufactured into
70 rubber-derived products, such as rubber chips, rubber shreds, rubber crumbs and rubber fibers,

71 and then mixed with soils such as backfills to improve various soil properties. Through many
72 laboratory and in-situ experiments, researchers have found that these added rubber-derived
73 products can significantly improve the permeability, compressibility, expansibility, tensile
74 strengths and shear strengths of clayey and expansive soils (Cetin H. et al., 2006; Trouzine,
75 2012; Soltani et al., 2018; Abbaspour et al., 2020). However, the effectiveness of rubber-derived
76 products as a means of reinforcing sandy soils is still a controversial issue in research and
77 practical engineering. Recently, Tasalloti et al. (2021) made a comprehensive literature review
78 on the physical and mechanical properties of granulated rubber mixed with granular soils. The
79 latest results and findings were presented and discussed primarily in terms of effects of rubber
80 content and particle size ratio on compaction, permeability, strength and compression properties
81 along with dynamic and cyclic deformation characteristics of sand-rubber mixtures. These
82 experimental studies found that the addition of rubber shreds, chips and crumbs can increase
83 slightly the critical shear strength of sand–rubber mixtures (Attom, 2006; Anbazhagan et al.,
84 2017; Benessalah et al., 2019; Al-Rkaby, 2019). However, other studies observed the opposite
85 result, namely a decrease in the shear resistance of the mixtures with increased rubber content,
86 especially at the peak state (Cabalar, 2011; Noorzad and Raveshi, 2017; Balaban et al., 2019).

87 In light of the successful use of polypropylene fibers to improve soil properties (Consoli et
88 al., 2002; Cheng et al., 2020a), researchers and engineers have attempted to use rubber buffings
89 or fibers to reinforce sandy soils. According to recent publications, the addition of waste tire
90 textile fibers to various kinds of soils can effectively improve the geotechnical properties of
91 clayed and expansive soils, such as their tensile strength and hydraulic conductivity (Kalkan,
92 2013; Bekhiti et al., 2019; Narani et al., 2020), and those of sandy soils, such as their ductility,
93 peak strength and critical shear strength (Fu et al., 2017a and 2018; Shekhawat et al., 2018).

94 However, tire rubber fibers are more easily deformed than the polypropylene fibers commonly
95 used to reinforce soils. Therefore, soil–rubber mixtures have always exhibited complicated and
96 unsatisfactory mechanical performance (Akbulut et al., 2007; Santos et al., 2010; Fu et al.,
97 2018). In practical applications that use these mixtures, many important factors should be
98 considered, including the stress condition, content, length and aspect ratio of the rubber fibers.
99 Moreover, the large deformation of rubber-derived products leads to highly complex
100 mechanisms of interaction between the rubber and soil particles, which are very difficult to
101 explore using traditional experiments or in-situ testing.

102 As an alternative, the discrete element method (DEM) has become a powerful tool to
103 investigate the fundamental micromechanics of granular materials over the past four decades.
104 Recently, the DEM has been successfully applied to simulate the behaviors of sand–rubber
105 mixtures (Valdes and Evans, 2008; Lee et al., 2014). The numerical predictions of mechanical
106 behaviors presented consistent trends with the experiments, and further provided a full access to
107 the particle-scale information (e.g., particle kinematics and interparticle contact forces) within
108 sample deformation. By using DEM, Lopera Perez et al. (2016) investigated the effect of rubber
109 size on the behaviors of sand-rubber mixtures, and revealed the micromechanics underlying the
110 sand-rubber interaction in terms of coordination number, structural anisotropy, contribution of
111 contact type and contact force network. The same authors further extended their studies on the
112 liquefaction potential and critical state behaviors of soil-rubber mixtures at large strains (Lopera
113 Perez et al., 2017 and 2018). However, these DEM studies were performed using simple
114 spherical particles to simulate the behaviors of sand and rubber particles, and the pure rubber and
115 pure sand samples were separately calibrated using data found in existing publications.

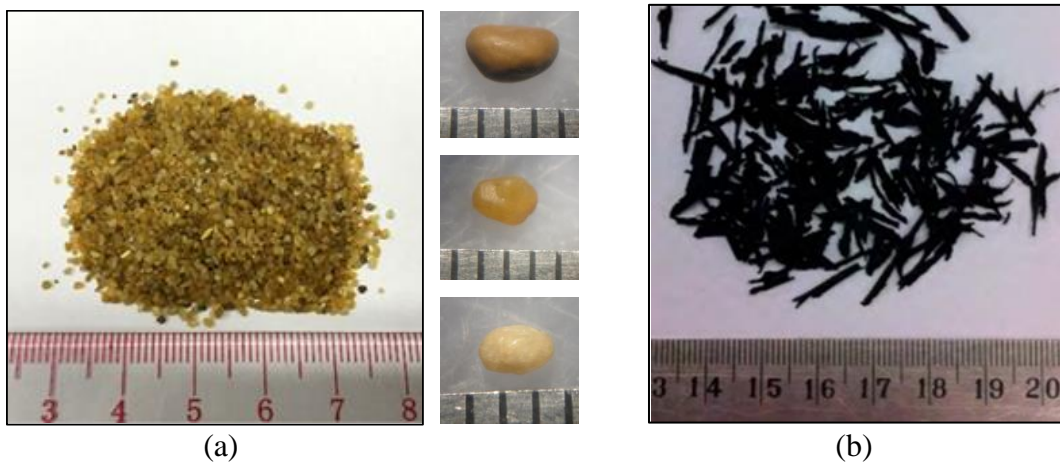
116 To overcome this problem, many researchers adopted the multi-sphere approach, namely
117 clump or cluster, to simulate the stiff sand or soft rubber particles with different shapes, e.g.
118 irregular sand grains and rubber shreds (Asadi et al., 2018a and 2018b), and realistic shapes
119 (Asadi et al., 2021); platy rubber chips but spherical sand particles (Zhang et al., 2021); irregular
120 gravel particles and cubic-like tire shreds (Chew et al., 2022). These DEM studies successfully
121 revealed the effects of rubber size, type, content and stress conditions on the mechanical
122 behaviors of sand–rubber mixtures, and further provided in-depth insights into the
123 micromechanics of the sand–rubber interactions. By carefully reviewing these literatures, rare
124 DEM study focused on the mixture of sand with rubber fiber. To the best of the authors’
125 knowledge, Gong et al. (2020) were the first to simulate the mixture of sand with rubber fiber by
126 spherical particles with rolling resistance and line-shaped clumps within DEM. However, the
127 reinforcing mechanism of the rubber fiber on soil strength, especially for its impact on the shear
128 banding development, was still not clear so far.

129 Accordingly, the objective of this study was to develop a detailed numerical method of
130 modeling mixtures of granular soils with tire rubber fibers in DEM. We also investigated the
131 effects of rubber content and confining pressure on the macroscopic and microscopic mechanical
132 behaviors of sand–rubber mixtures subjected to triaxial compression. The results of real
133 experiments and DEM simulations were initially subjected to comparative analysis to validate
134 and calibrate the DEM and DEM parameters proposed for modeling sand–rubber mixtures. The
135 micromechanical features of sand–rubber mixtures, specifically particle kinematics, the
136 interparticle coordination number and rubber fiber deformation, were investigated to identify the
137 specific mechanism by which the rubber fibers provided mechanical reinforcement. Finally, this
138 study aimed to determine the optimum rubber fiber content in terms of the macroscopic and

139 microscopic mechanical performance of granular soils, which will be applicable to future
140 engineering applications.

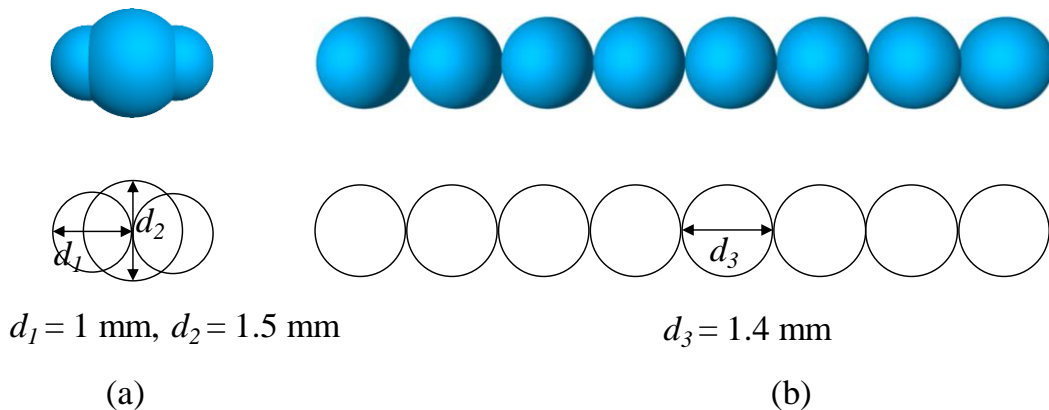
141 **2 Discrete element method modeling**

142 The base sand used in this study was Leighton Buzzard sand (LBS), a standard quartz
143 sand that deposits in shallow seas and estuarine environments. Due to its mineral composition
144 and geological origin, the particle morphology of LBS was characterized by sub-ellipsoid form,
145 rounded corner and smooth surface texture, as shown by Fig. 1(a). The sand sample was sieved
146 to obtain particles in the size range of 1.18 – 2.36 mm. As demonstrated in Fig. 1(b), the rubber
147 fibers used in this study had an average length of 11.0 mm and an approximate diameter of 1.4
148 mm. To determine the basic mechanical behaviors, a set of tests and a uniaxial tensile tests were
149 conducted on a pure sand sample and rubber fibers, respectively. The results of which were also
150 used to calibrate the microscopic parameters applied in the DEM simulations. The sand–rubber
151 mixture samples with rubber contents of 10% and 30% were subjected to triaxial testing to
152 validate the DEM model. All the experimental results are presented and discussed with the DEM
153 results in the following sections.



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156 Fig. 1 Photographs of the research materials: a) Leighton Buzzard sand, and b) rubber fibers

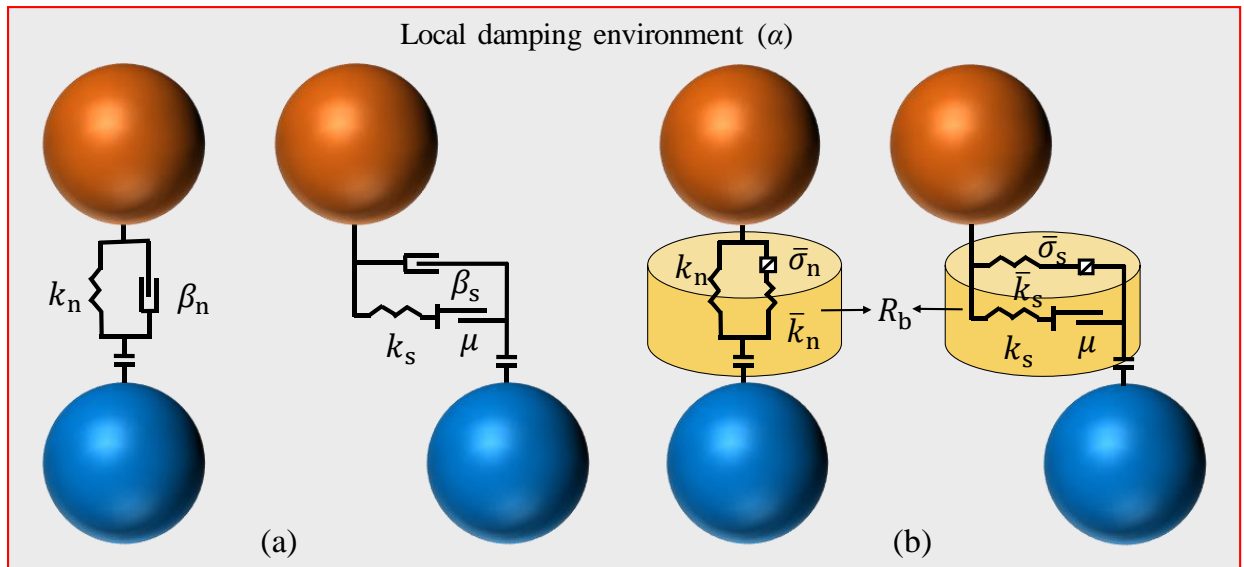
157 As the chosen LBS particles had sub-ellipsoid form and rounded and corners, a
 158 monotonic ellipsoid-shaped particle was established within the DEM framework using the clump
 159 logic, as demonstrated in Fig. 2(a). This particle shape was widely adopted by many DEM
 160 studies (Yang et al., 2017; Jiang et al., 2018 and Wu et al., 2020) to simulate a more realistic
 161 mechanical response of granular soils. The Clump logic can be used to generate particles with
 162 ideal shapes by bonding a small number of overlapping ball elements as rigid bodies without
 163 calculating the internal contact forces, thereby improving the computing efficiency of DEM
 164 simulations (Itasca, 2016). In contrast, as illustrated in Fig. 2(b), a cluster comprising a line of
 165 ball elements was used to model a rubber fiber. The artificial roughness of the produced rubber
 166 fiber was observed by using this modelling technique. Therefore, the friction coefficient of
 167 rubber fibers should be slightly adjusted within the calibration to compromise its influence. On
 168 account of the computation efficiency, a limitation of this study was the usage of the monotonic
 169 and simple shapes for the representation of the sand particle and rubber fiber.



170 Fig. 2 Discrete element modeling of a sand particle and a rubber fiber: a) a sand particle with a
 171 diameter of 1.5 mm, and b) a rubber fiber with a diameter of 1.4 mm and a length of 11.2 mm
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 174 As illustrated in Fig. 3, a linear contact model and a parallel-bond contact model were
 175 adopted to define the interparticle and intraparticle interactions, respectively. Specifically, the

176 linear contact model (Fig. 3(a)) was defined for the interparticle contacts of the sand–sand (SS),
177 sand–rubber (SR) and rubber–rubber (RR). In this model, the contact stiffness is the harmonic
178 average between the stiffnesses of two contacted particles, and the contact force has a linear
179 correlation with the relative displacement between the contacted particles. Asadi et al. (2018a
180 and 2018b) indicated that the Hertz model was the best choice to calculate the contact stiffness
181 between the sand and rubber particles with large difference in stiffness. For the consideration of
182 the compatibility with the parallel-bond model, the simple harmonic average for the contact
183 stiffness was still adopted, but a more careful calibration was conducted to the error. Owing to
184 the diverse material stiffness of the sand and rubber, the interparticle contact stiffness between
185 SS, SR and RR particles were different, thereby resulting in various interparticle contact
186 deformations. Besides, the contact-slip law was defined to determine the shear slip between the
187 contacted particles along the tangential direction, in which the interparticle friction coefficient
188 was the minimum friction coefficient of the contacted particles. For the intraparticle contacts
189 inside a rubber cluster, the parallel-bond contact model was defined, which can be assumed as a
190 certain thickness of cement with a tensile and shear strength, as demonstrated in Fig. 3(b). This
191 model allowed for the modeling of compressive, tensile and bending deformation and the elastic
192 springback of the rubber fiber under different loading conditions (Itasca, 2016). The validation
193 and calibration of the DEM model and the mentioned parameters are discussed in section 3.



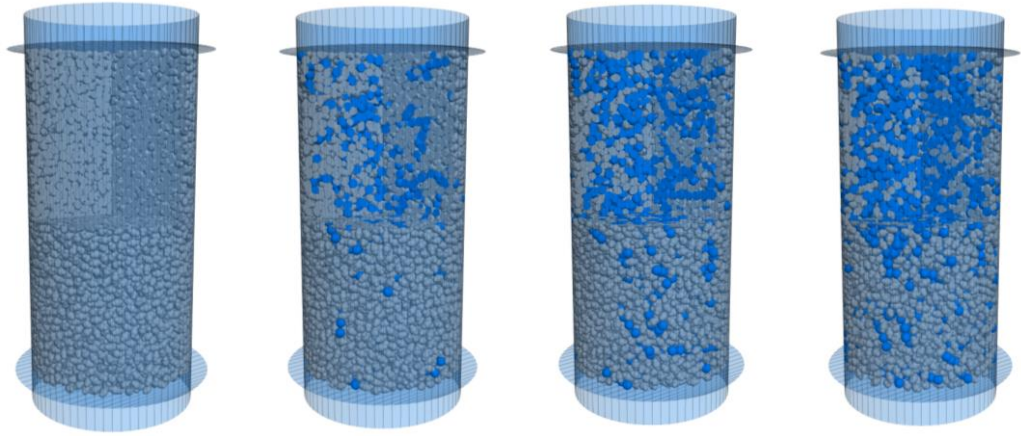
normal
 shear
 normal
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Spring
 Slider
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 Bond
 Gap
 Local damping

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195 Fig. 3 Diagram of the adopted interparticle contact models: a) linear contact model, and b)
196 parallel-bond contact model.
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198 As the aim of this work was to investigate the effects of rubber fiber content on the
199 macroscopic and microscopic mechanical behaviors of sand–rubber mixtures and to determine
200 the optimum rubber content in terms of soil performance, rubber fiber contents of 10%, 20% and
201 30% by weight were selected to represent the range of contents commonly used to prepare these
202 sand–rubber mixtures. To increase the computing efficiency of DEM simulations, the size of the
203 triaxial samples was scaled to a diameter of 30 mm and a height of 60 mm. Within each sample,
204 the sand particles were of a single grading, with a mean particle size of 1.5 mm, and the rubber
205 fibers had a uniform length and diameter as in Fig. 2(b). To generate triaxial samples using the
206 DEM, the initial void ratio was set to 0.55 for all of the samples. Therefore, the numbers of sand
207 particles and rubber fibers within each sample were calculated according to their volume
208 fraction.

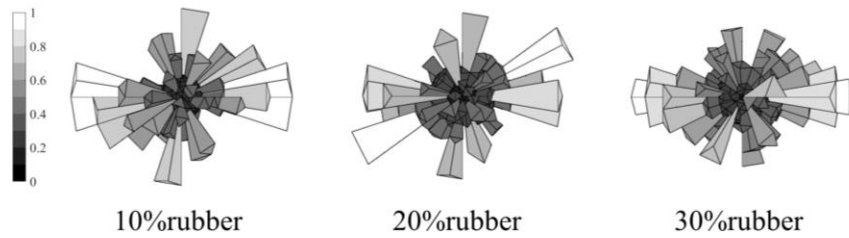
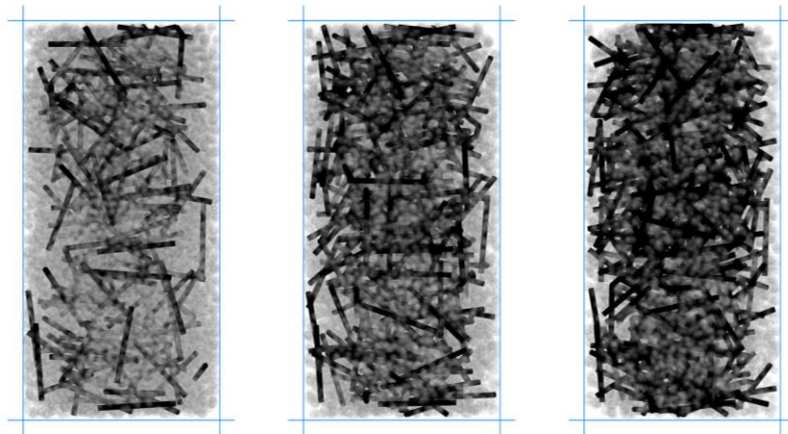
209 Using an intrinsic function ‘clump template’ in the commercial DEM platform PFC^{3D}
210 (Itasca, 2016), each sand–rubber mixture sample was created by randomly generating rigid
211 clumps of the sand particle and rubber fiber inside a cylindrical boundary. The clump system was
212 then solved to reach an initial equilibrium. Subsequently, all the clumps of the rubber fiber were
213 released to deformable clusters with parallel-bonds. The servo-control mechanism was then
214 utilized to reach the initial stress state of the sample with a given confining pressure. Figure 4
215 shows the generated DEM samples of pure sand and sand–rubber mixtures with various rubber
216 contents. Figure 5 highlights the spatial distributions of the rubber fibers, accompanying with the
217 polar distributions of the fiber orientation, inside the sand-rubber mixture samples. Overall, the
218 sand particles and rubber fibers were uniformly distributed in the sample space, indicating good
219 homogeneity of the generated samples. A slight anisotropy of the fiber orientation along the
220 radial direction can be observed within the samples. It was reasonable that the fiber is more
221 likely to rotate towards the vertical direction under compression. Based on the current DEM
222 configuration, the time step of the system was approximately 2×10^{-7} s, and the average DEM
223 computation time of the sand-rubber mixtures was approximately 19 h using 6 cores, each with a
224 3.2 GHz intel Core i7-8700 CPU.



Number of sand particles:	12459	7477	5312	3935
Number of rubber fibers:	0	194	312	397
Initial void ratio:	0.64	0.79	0.85	0.96
	(a)	(b)	(c)	(d)

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Fig. 4 DEM samples of pure sand and sand–rubber mixtures: a) pure sand, b) 10% rubber content, c) 20% rubber content, d) 30% rubber content

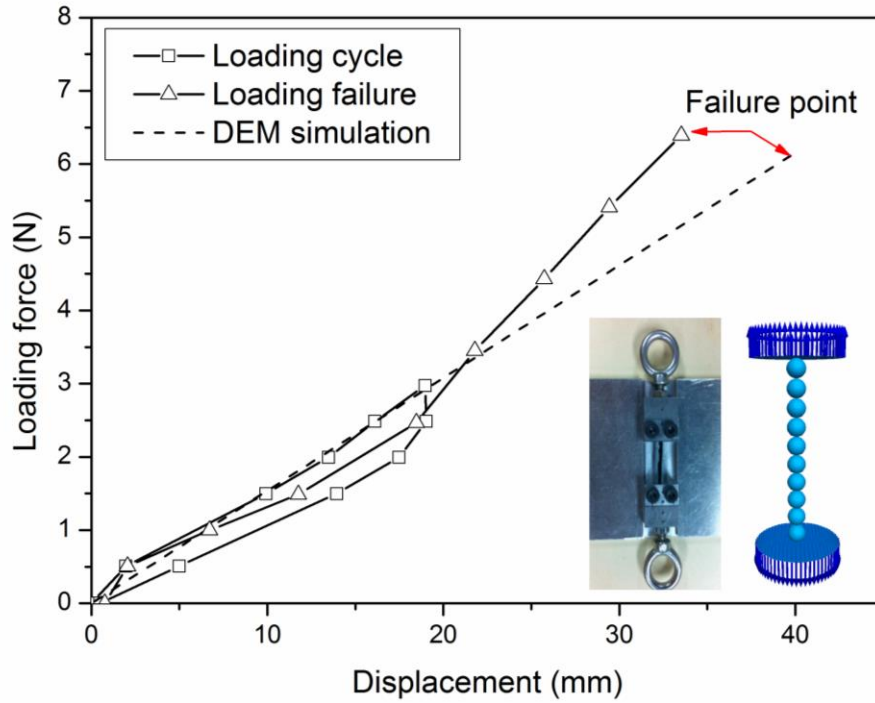


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Fig. 5 Spatial distributions of the rubber fibers, accompanying with the polar distributions of the fiber orientation, inside the sand-rubber mixture samples

232 **3 Calibration and validation**

233 A conventional trial-and-error method based on comparisons between DEM simulations
234 and real experiments was used to calibrate the DEM parameters. Briefly, the target mechanical
235 behaviors identified in the DEM simulations were matched with those in the corresponding
236 experiments to the maximum extent by continuously modifying the main controlling parameters
237 in the DEM (Fu et al., 2017b; Coetzee, 2017). Specifically, for a rubber fiber cluster, the contact
238 stiffness of the ball elements and the parallel-bond strength are the most important determinants
239 of deformation and fracture properties (Itasca, 2016). Therefore, uniaxial tensile tests of a rubber
240 fiber cluster (see Fig. 2(b)) were simulated and the results were compared with the experimental
241 data. Fig. 6 shows the comparison of the data generated by the DEM simulation and experiment
242 using the calibrated DEM parameters summarized in Table 1. By default, the local damping
243 coefficient was set to 0.7, while the viscous coefficient was set to zero in the whole study, to
244 guarantee the quasi-static modelling of DEM (Itasca, 2016). This figure shows good agreement
245 between the numerical and experimental results, as both indicated an approximately linear
246 relationship between tensile force and displacement at the earlier loading stage and a close
247 failure point of the loading curve. Therefore, it was reasonable to use a line cluster of bonded
248 ball elements and associated DEM parameters to simulate the rubber fibers.



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Fig. 6 Relationship between the loading force and displacement of the rubber fiber according to DEM and experimental results

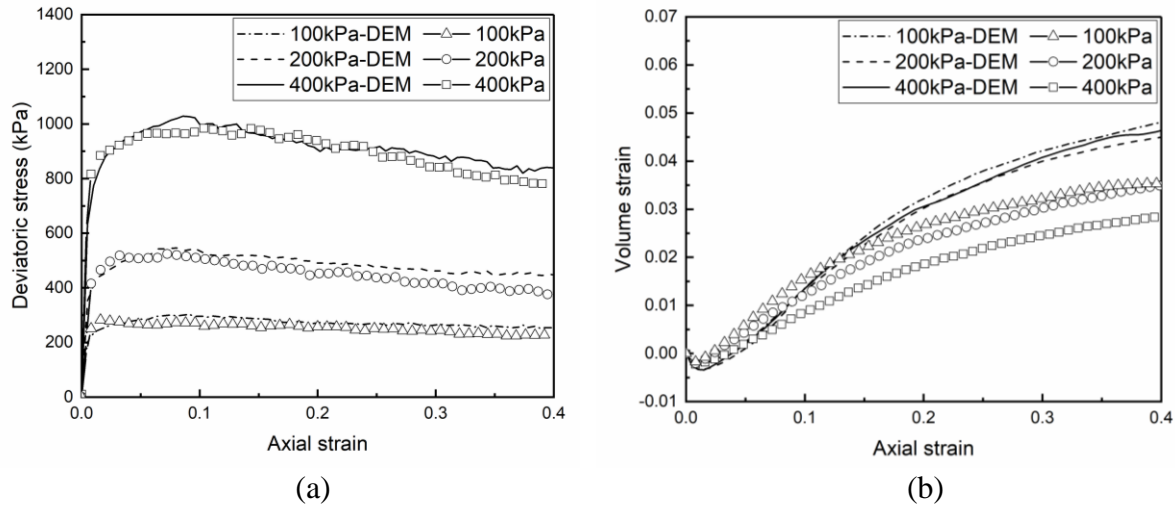
253 Table 1 DEM parameters adopted in this study

Items	Parameter	value
Sand particle (ball element)	Density (kg/m ³)	2630
	Mean size (mm)	1.5
	Normal/shear stiffness (N/m)	5.0×10 ⁶
	Interparticle friction coefficient	0.35
Rubber fiber (ball element and parallel-bond)	Density (kg/m ³)	1150
	Diameter and length (mm)	1.4, 11.2
	Normal/shear stiffness of ball elements (N/m)	2.5×10 ⁴
	Interparticle friction coefficient of ball elements	0.6
	Normal/shear stiffness of parallel-bonds (N/m ³)	9×10 ⁸
	Normal/shear strength of parallel-bonds (N/m ²)	4×10 ⁶
	Ratio of bond radius to ball radius	1.0
Wall	Normal stiffness of top and bottom walls (N/m)	1×10 ⁵
	Normal stiffness of cylindrical wall (N/m)	10 ⁷
	Friction coefficient	0.0
System	Local damping coefficient	0.7
	Viscous damping coefficient	0

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256 Initially, the pure sand samples were subjected to a set of triaxial tests to calibrate the
257 DEM parameters for the sand particles. Figure 6 demonstrates the experimental and DEM
258 findings corresponding to the macroscopic mechanical responses of pure sand samples subjected
259 to triaxial compression under different confining pressures, using the DEM parameters
260 summarized in Table 1. The comparison clearly indicated good agreement between the DEM
261 results and the experimental results, especially for the relationship between the deviatoric stress
262 and axial strain (Fig. 7(a)). The stress–strain curves generated from both the experimental and
263 the DEM results exhibit obvious peak and strain-softening behaviors, which are highly consistent
264 with the expected mechanics of pure sands. Regarding the relationship between volumetric strain
265 and axial strain (Fig. 7(b)), a minor difference between the experimental and DEM results was
266 observed at large strain stages, which might be due to the idealized particle shape and rigid servo
267 boundary adopted in the DEM simulations. Kozicki et al. (2014) and Zhou et al. (2013) found
268 that the irregular particle shape and rigid servo boundary tended to overestimate the volumetric
269 strain of a sample within a DEM simulation compared with the regular particle shape and
270 flexible servo boundary. Generally, the experimental and DEM results demonstrated a highly
271 consistent tendency for shear-induced compression at pre-peak stress states and shear-induced
272 dilation at post-peak stress states. Overall, the DEM and adopted DEM parameters were found to
273 be capable of modeling the constitutive behavior of the pure sand sample.



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Fig. 7 Comparative results of triaxial experiments and DEM simulations: a) deviatoric stress versus axial strain; b) volumetric strain versus axial strain (triaxial test data from Fu et al., 2017a)

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Finally, triaxial testing on sand–rubber mixtures with a rubber contents of 10% and 30%

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were simulated to validate further the use of the DEM and adopted DEM parameters to simulate

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sand–rubber mixtures. Figures 8 and 9 show the mechanical response of the 10% and 30%

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mixtures subjected to triaxial compression under different confining pressures, based on the

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laboratory experiments and DEM simulations. Generally, comparative analysis of the results

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indicated that the mechanical behaviors determined from the DEM simulations agree well with

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those identified in the laboratory experiments, especially for the stress-strain curves of the 10%

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mixture (Fig. 8(a)). Moreover, the relationship between volumetric strain and axial strain

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exhibited a consistent trend in both the experimental and the DEM results, especially for the 30%

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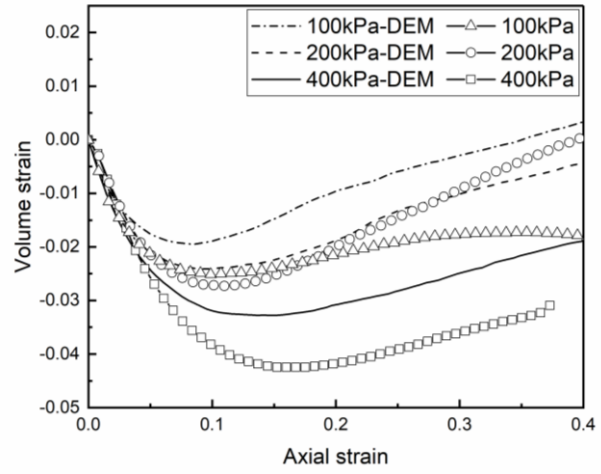
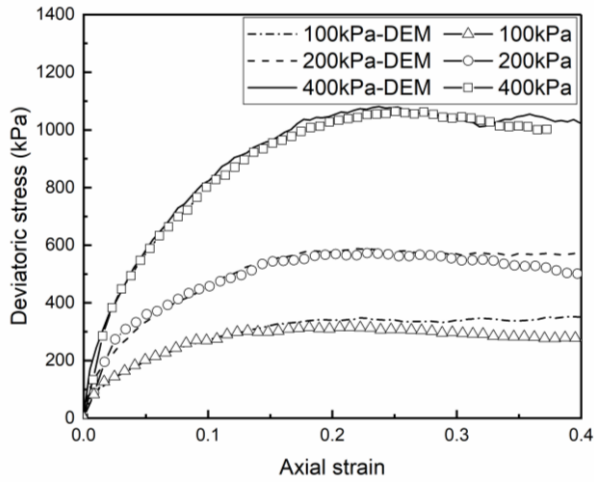
mixture (Fig. 9(b)). Conclusively, the proposed DEM model and adopted DEM parameters were

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shown to be robust and optimal for simulating the mechanical behaviors of sand–rubber

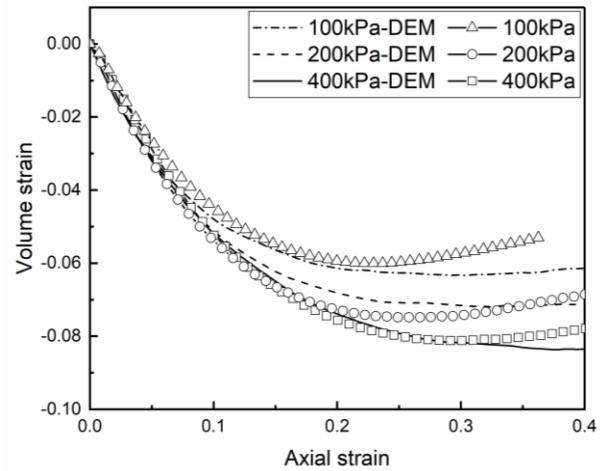
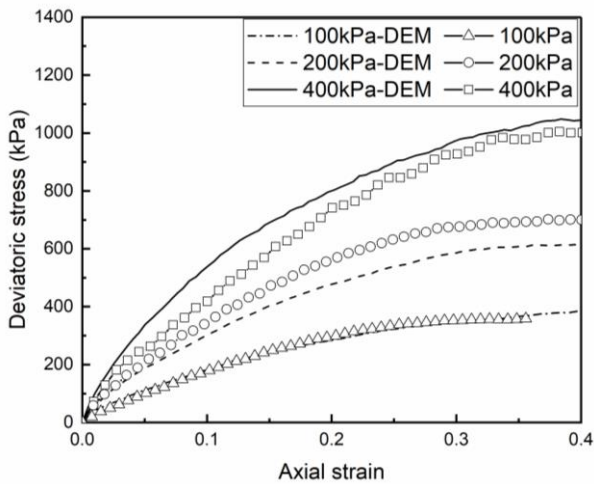
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mixtures.



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Fig. 8 Comparative results of triaxial tests and DEM simulations of a 10% mixture: a) deviatoric stress versus axial strain, b) volumetric strain versus axial strain (triaxial test data from Fu et al., 2017a)



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Fig. 9 Comparative results of triaxial tests and DEM simulations of a 30% mixture: a) deviatoric stress versus axial strain, b) volumetric strain versus axial strain (triaxial test data from Fu et al., 2017a)

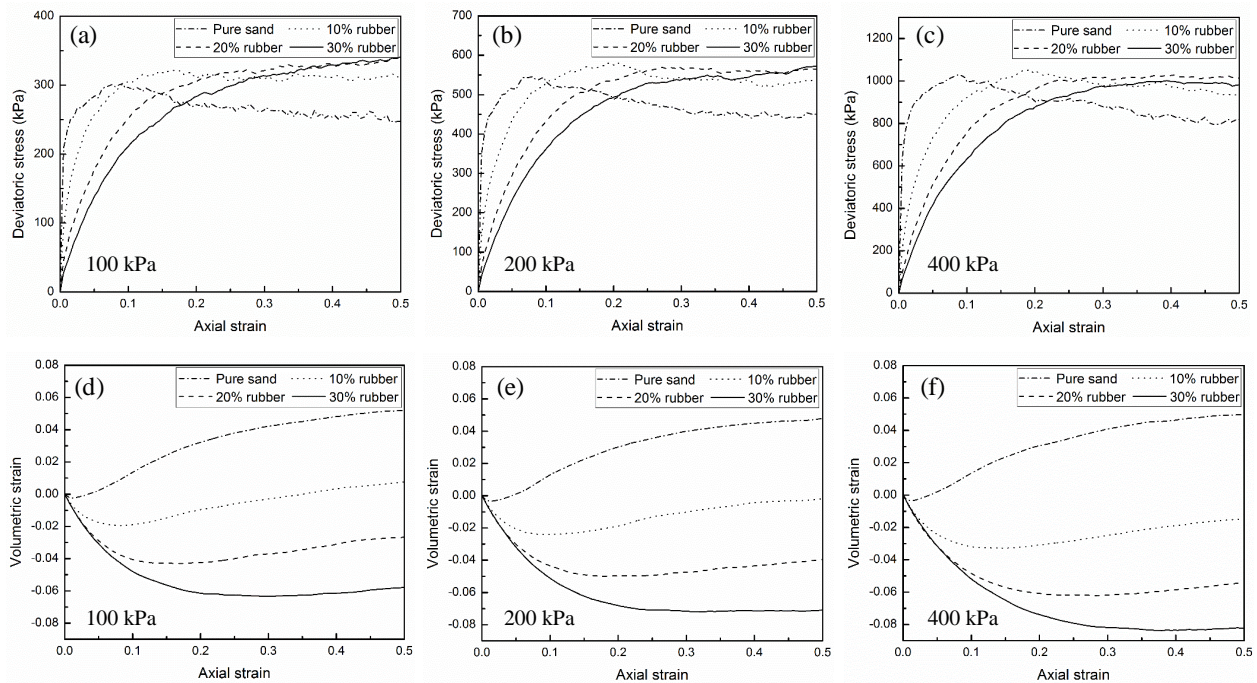
304 **4 Results and discussion**

305 **4.1 Macroscopic mechanical behaviors**

306 To investigate the effect of rubber content on the macroscopic and microscopic
307 mechanical behaviors of sand–rubber mixtures, an additional DEM simulation was conducted on
308 sand–rubber mixture samples with rubber content of 20%. Figure 10 shows the macroscopic
309 mechanical responses of the sand–rubber mixtures subjected to triaxial compression under
310 various confining pressures. Overall, the pure sand samples show a clear deviatoric stress peak
311 and a strain-softening behavior post-peak until the critical state, whereas all of the sand–rubber
312 mixture samples exhibited a continuous strain-hardening behavior at large strains until reaching
313 the critical state, especially for the lowest confining pressure of 100 kPa (Fig. 10(a)).
314 Correspondingly, the pure sand samples show a significant volumetric dilation post-peak until
315 reaching the critical state, whereas all of the sand–rubber mixture samples exhibited continuous
316 volumetric compression at large strains until reaching the final critical state.

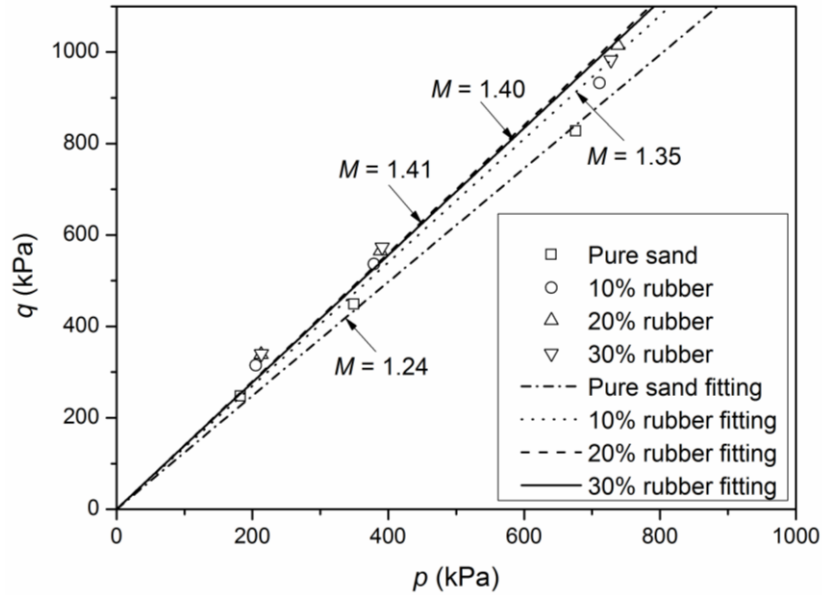
317 Specifically, as the rubber content increased, the sand–rubber mixture samples tended to
318 become increasingly compressible under shear stress. Accordingly, the stiffness pre-peak
319 decreased significantly and the required axial strain to reach the peak deviatoric stress increased
320 dramatically. These results agree well with the findings of the authors' previous experimental
321 research (Fu et al., 2014 and 2017a). The high level of compressibility of the sand–rubber
322 mixtures can be attributed mainly to the low level of material stiffness of the rubber fibers.
323 Generally, rubber fibers appear to enhance the critical state shear resistance of a sand–rubber
324 mixture. However, an increased rubber content seems to have limited improvement of the critical
325 state shear resistance, especially under higher confining stresses (Fig. 10(c)). This inference was
326 illustrated using the critical state data and the fitted lines in the spaces of deviatoric stress (q) and

327 mean effective stress (p) plotted for all of the investigated samples, as shown in Fig. 11. The
 328 slope of the fitted critical state line (M) was used to better assess the reinforcing effect of rubber
 329 fibers in sand–rubber mixtures. Compared with the pure sand samples, the sand–rubber mixture
 330 samples yielded higher values of M , demonstrating the reinforcement effect of the rubber fibers.
 331 However, the values of M calculated for the sand–rubber mixture samples with rubber contents
 332 of 20% and 30% were extremely similar to each other, indicating that this increase in the rubber
 333 content did not significantly contribute to soil reinforcement. This finding agrees well with the
 334 experimental result obtained by Mashiri et al. (2015). Conclusively, given the observed reduction
 335 in soil stiffness, a rubber content of 20% is recommended to improve the mechanical
 336 performance of sand–rubber fiber mixtures.



337
 338 Fig. 10 Macroscopic mechanical behaviors of sand–rubber mixture samples subjected to triaxial
 339 compression under various confining pressures: a) – c) deviatoric stress versus axial strain for
 340 confining pressures of 100, 200 and 400 kPa; d) - f) volumetric strain for confining pressures of
 341 100, 200 and 400 kPa

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343 Fig. 11 Critical state data and fitted lines in p - q space for various sand–rubber mixtures
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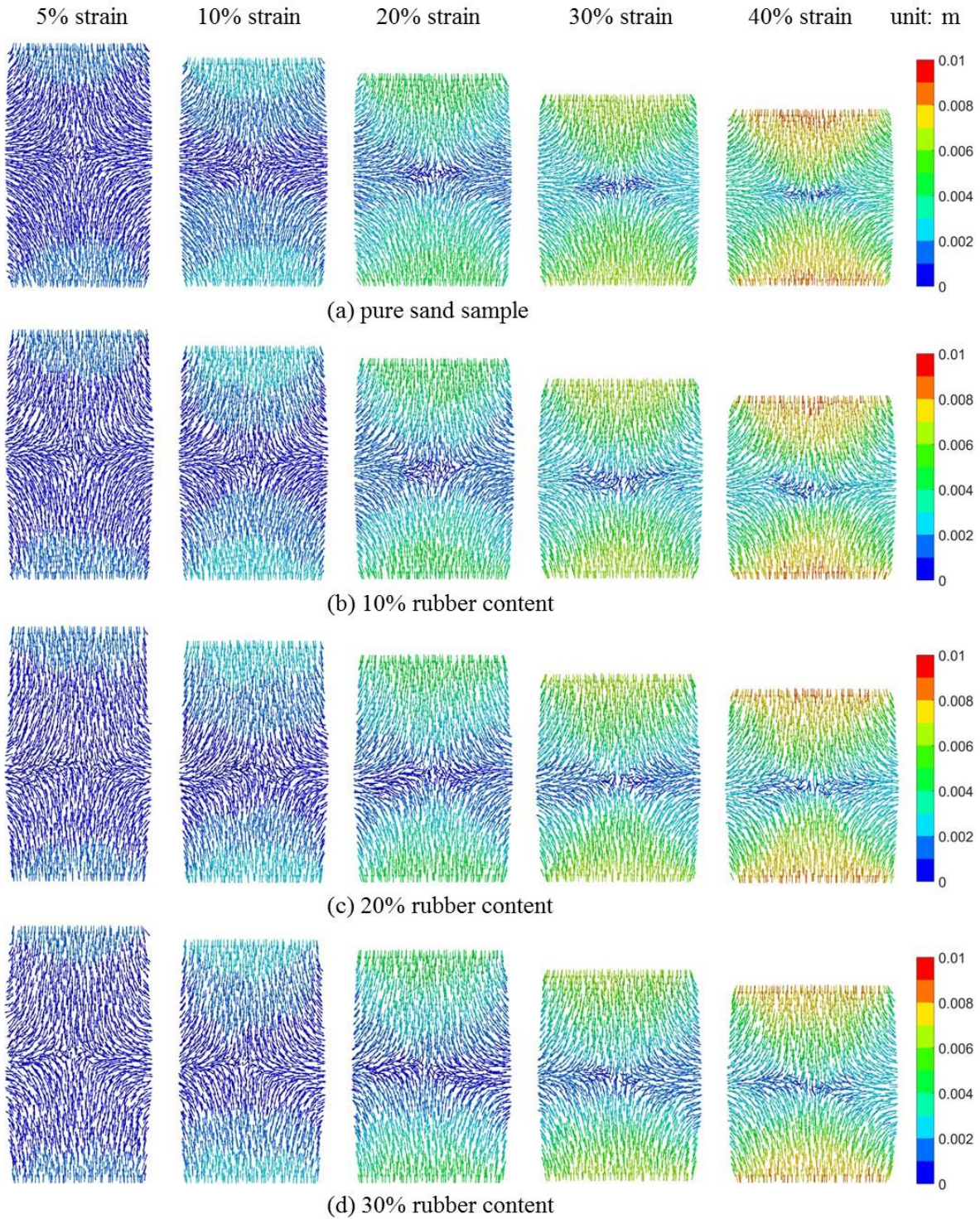
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346 4.2 Microscopic mechanical behaviors

347 To understand the mechanism by which rubber fibers reinforce a sand–rubber mixture,
 348 micromechanical features, namely particle kinematics, the interparticle coordination number and
 349 rubber fiber deformation, were investigated and are discussed in this section.

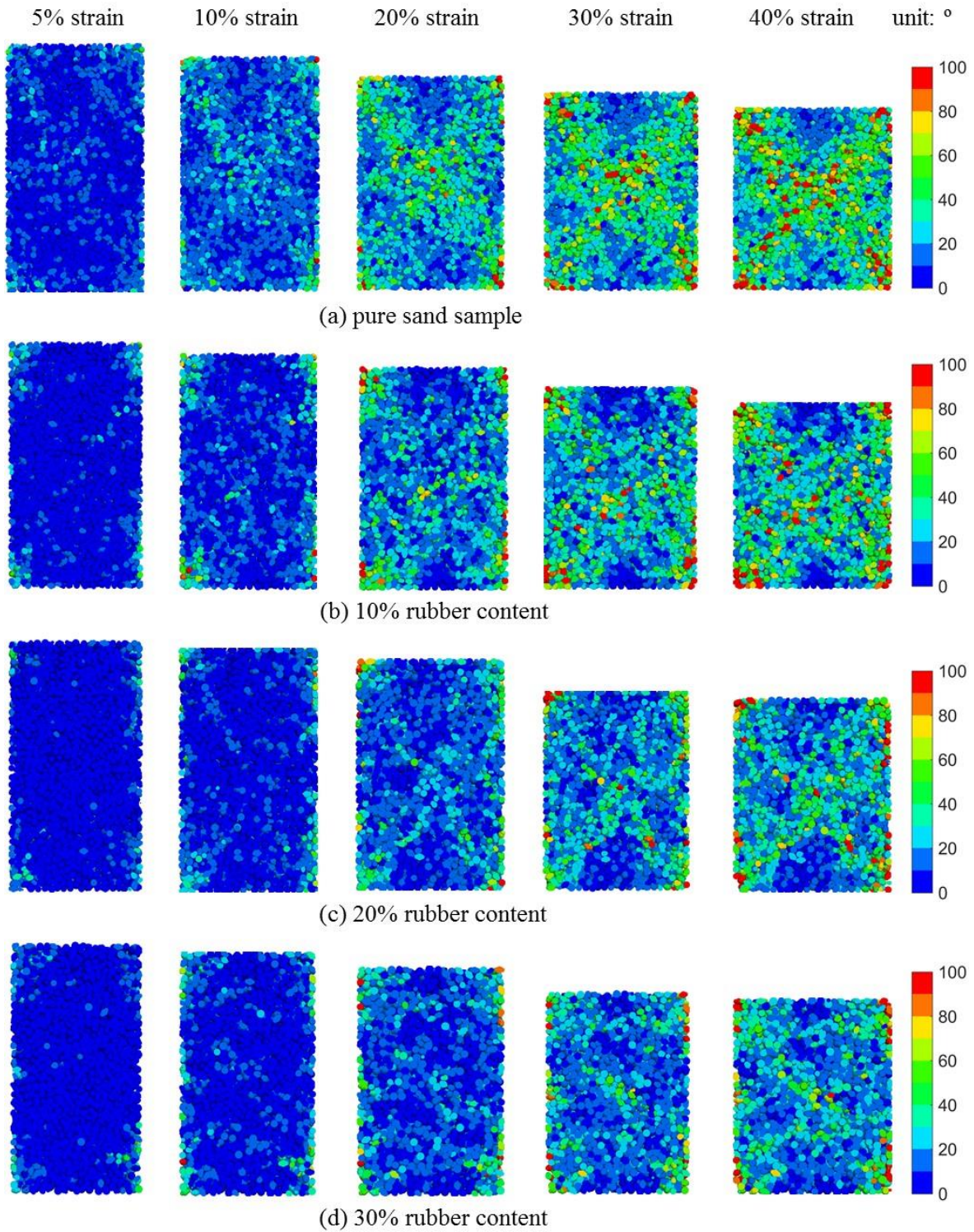
350 Within the DEM model, the data for particle kinematics, in terms of particle
 351 displacements and rotations, are extracted to highlight shear banding since these are regarded as
 352 important indicators revealing local shear failure mechanisms of granular materials (Zhou et al.
 353 2013; Wu et al., 2020). Figures 12 and 13 respectively demonstrate the evolution of the particle
 354 displacements and particle rotations of different samples as shearing developed. For both, a
 355 distinct localization phenomenon was observed within the pure sand sample post-peak (Figs.
 356 12(a) and 13(a)). This phenomenon became less apparent within the sand–rubber mixture
 357 samples as the rubber content increased, even at extremely large strains (Figs. 12(d) and 13(d)).
 358 This finding agrees well with the experimental results generated from a mini sand–rubber

359 mixture samples subjected to triaxial compression and observed using in-situ X-ray CT scanning
360 (Cheng et al., 2020b). The failure pattern of the sand–rubber mixture gradually shifts from a
361 local shear failure to an overall shear failure as the rubber content increases. This reveals an
362 essential reinforcing mechanism of rubber fibers within a sand–rubber mixture, which can be
363 considered a hysteresis effect that restrains the development of local shear failure.



364
 365 Fig. 12 Particle displacement fields within the vertical sections through the middle of the
 366 samples: a) pure sand sample; sand–rubber mixture samples with rubber contents of b) 10%, c)
 367 20% and d) 30%

368



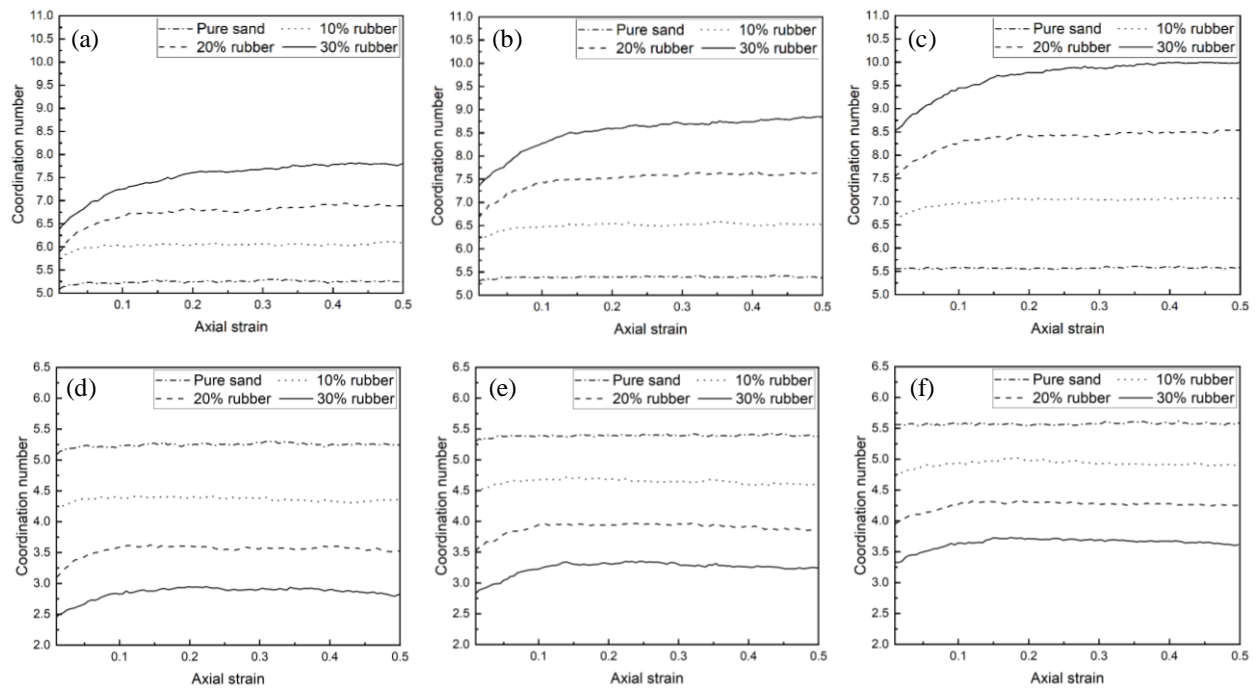
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Fig. 13 Particle rotation fields within the vertical sections through the middle of the samples: a) pure sand sample; sand–rubber mixture samples with rubber contents of b) 10%, c) 20% and d) 30%

373 To investigate the interparticle coordination number within the sand–rubber mixtures, the
374 overall interparticle contacts and interparticle contacts between sand particles were targeted,
375 mainly because these are the predominant components of the bearing structure of a particle
376 system. The interparticle coordination number was defined by the total contacts between the
377 comprised ball elements of the contacted particles. Figure 14 shows the development of the mean
378 coordination number of these two types of interparticle contacts within samples subjected to
379 triaxial compression under different confining pressures. Interestingly, in the pure sand sample,
380 the mean coordination number remained approximately constant throughout the shearing
381 process. In contrast, in the sand–rubber mixture samples, the mean coordination number of the
382 overall interparticle contacts gradually increased during the shearing process (Figs. 14(a) –
383 14(c)), and the mean coordination number of the sand–sand interparticle contacts initially
384 increased and then slightly decreased as shearing developed (Figs. 14(d) – 14(f)). The volumetric
385 compression of the sand–rubber mixtures contributed to the continuous increase of the
386 coordination number of the overall interparticle contacts within the sample. For the sand–sand
387 interparticle contacts within the sand–rubber mixture samples, the early volumetric compression
388 enhanced the coordination number; however, the deformed rubber was more likely to fill the
389 interparticle voids between the sand particles, thereby decreasing the coordination number
390 between the sand particles at large strains.

391 For all of the samples at any certain shearing strain level, with increasing rubber content,
392 the mean coordination number of the overall interparticle contacts increased significantly,
393 whereas the mean coordination number of the sand–sand interparticle contacts decreased. Here,
394 the increased rubber content significantly enhanced the overall interparticle contacts but
395 simultaneously weakened the sand–sand interparticle contacts. As the sand–sand interparticle

396 contacts play a major role in constructing the bearing structure of the particle system, whereas
 397 the sand–rubber and rubber–rubber interparticle contacts play only minor roles, the increased
 398 coordination number of the overall interparticle contacts and the decreased coordination number
 399 of the sand–sand interparticle contacts may indicate that the reinforcing effect of increased
 400 rubber content was compromised. This result provides an essential insight into the limited
 401 reinforcing quality of a sand–rubber mixture with a very high rubber contents.



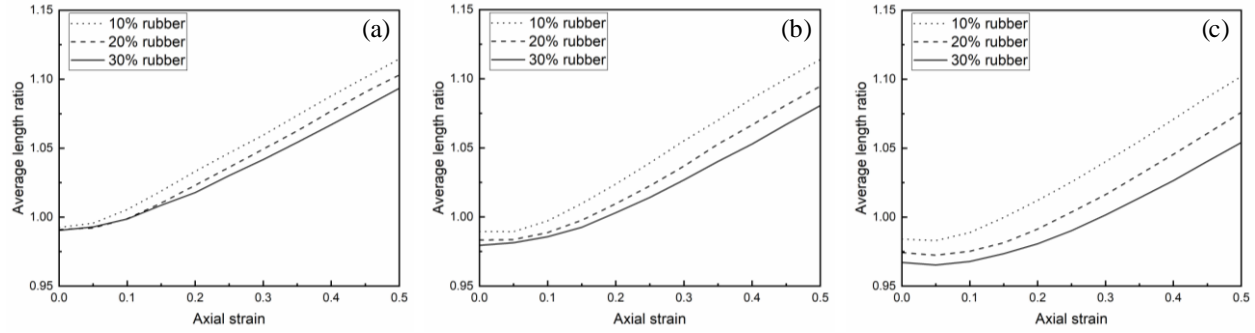
402
 403 Fig. 14 Development of the mean coordination numbers of two types of interparticle contacts: a)
 404 – c) overall interparticle contacts under confining pressures of 100, 200 and 400 kPa; d) – f)
 405 sand–sand interparticle contacts under confining pressures of 100, 200 and 400 kPa.

406

407 Because rubber is an easily deformed material, it was also important to investigate the
 408 deformation behaviors of rubber fibers within sand–rubber mixtures to gain novel insights into
 409 the reinforcing mechanisms of the rubber fibers. The ratio of the length of a deformed rubber
 410 fiber (I) to its original length (I_0) was defined to characterize the degree of deformation of the
 411 rubber fiber. A rubber fiber is compressed when I/I_0 is less than 1.0 and stretched when I/I_0 is

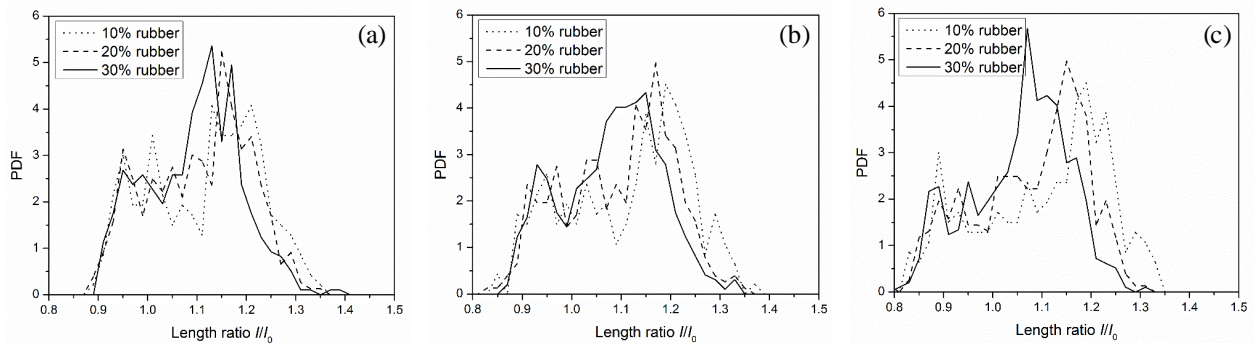
412 greater than 1.0. Figure 15 shows the development of the average length ratio of all of the rubber
413 fibers within a sand–rubber mixture under different confining pressures. The rubber fibers in the
414 observed mixtures were slightly compressed in the initial shearing stages and clearly stretched at
415 large strains. The axial strains at which the rubber fibers transformed from general compression
416 to general stretching tended to increase as the confining pressure increased.

417 An important observation is that the average length ratio of the rubber fibers significantly
418 increased with decreasing rubber content, especially for the sample under the highest confining
419 pressure (Fig. 15(c)). This indicates that the rubber fibers inside the sand–rubber mixture with a
420 lowest rubber content are more likely to stretch than when there is a higher rubber content. This
421 finding was illustrated by the probability density function (PDF) of the length ratios of all of the
422 rubber fibers within the various sand–rubber mixtures at the critical state in Fig. 16. In Fig. 16(c),
423 the value of the length ratio at the peak of the PDF curve is much greater for the sand–rubber
424 mixture with 10% rubber content (approximately 1.2) than for the mixture with 30% rubber
425 content (approximately 1.06). These findings reveal a crucial micromechanism of soil
426 reinforcement that relies on the deformation of rubber fibers. Compared with a sand–rubber
427 mixture with a higher rubber content, a mixture with a lower rubber content will exhibit better
428 rubber fiber deformation and thus better resist local shear failure. The results again indicate that
429 20% is the optimum rubber content with respect to the mechanical performance of a sand–rubber
430 mixture.



431
 432 Fig. 15 Development of the average length ratios of the rubber fibers within various sand–rubber
 433 mixtures under confining pressures of a) 100 kPa, b) 200 kPa and c) 400 kPa

434



435
 436 Fig. 16 Probability density function of the deformation ratios of all of the rubber fibers within
 437 various sand–rubber mixture samples in the critical state under confining pressures of a) 100
 438 kPa, b) 200 kPa and c) 400 kPa

439 **5 Conclusion**

440 In this study, a series of DEM simulations and calibrations of granular soils with tire
 441 rubber fibers were conducted, and the effect of rubber content on the macroscopic and
 442 microscopic behaviors of the sand-rubber mixtures was investigated. The specific reinforcing
 443 mechanism of rubber fibers within a sand–rubber mixtures was revealed by analyzing particle
 444 micromechanics and used to determine the optimum rubber content. The following major
 445 conclusions can be drawn.

- 446 (1) Within the DEM model, a rigid clump and a deformed cluster with parallel-bonded
 447 interparticle contacts can be used to model the mechanics of granular soils and tire rubber

448 fibers with small tensile deformation, respectively. Based on a comparative study of the
449 experimental and DEM results, the proposed DEM method and adopted DEM parameters
450 were shown to provide a robust and accurate simulation of the mechanical behaviors of a
451 sand–rubber mixtures subjected to triaxial compression.

452 (2) Both the experimental and the DEM results indicate that the rubber fibers had a significant
453 reinforcing effect on the shear resistance of the sand–rubber mixtures at the critical state.
454 However, increasing the rubber content from 20% to 30% resulted in only a limited
455 improvement of the shear resistance. The sand–rubber mixtures tended to become
456 increasingly compressible in shear at higher rubber contents, resulting in a continuous
457 decrease of stiffness pre-peak. Therefore, a sand–rubber mixture with a rubber content of
458 20% is recommended to give the best mechanical performance. This finding was consistent
459 with the experimental result by Mashiri et al. (2015), who found the sand-rubber mixture
460 with 20% tire chips has the largest shear strength.

461 (3) The addition of rubber fibers produced a hysteresis effect restraining the development of
462 local shear failure in the sand–rubber mixtures. However, due to the large deformations of
463 rubber fibers, increasing the rubber content significantly enhanced the overall interparticle
464 contacts but weakened the sand–sand interparticle contacts within the mixtures. Moreover,
465 rubber fibers in the mixture with a lower rubber content are more likely to be stretched,
466 indicating that rubber fibers show better deformation performance and greater shear failure
467 resistance at a lower rubber content. The particle micromechanics data generated in this study
468 provide an understanding of reinforcing mechanism of rubber fibers within a sand–rubber
469 mixture and conclusive evidence of the rubber content required to optimize soil performance.

470 Due to the limitation of the idealized particle shapes by conventional modelling
471 techniques, the future work will develop a more detailed DEM method to simulate the real
472 morphologies of rigid sand particles and deformed rubber fibers. Moreover, to verify the
473 obtained micromechanics from DEM, the in-situ experiment on sand-rubber mixtures subjected
474 to X-ray micro-CT scanning will be conducted along this research line.

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