Received: 28 January 2022 | Accepted: 13 March 2023

DOI: 10.1111/poms.14002

POMS

ORIGINAL ARTICLE

# Designing shipping policies with top-up options to qualify for free delivery

Guang Li[1] | Lifei Sheng[2] | Dongyuan Zhan[3]

[1]Smith School of Business, Queen's University, Kingston, Ontario, Canada

[2]College of Business, University of Houston-Clear Lake, Houston, TX, USA

[3]School of Management, University College London, London, UK

**Correspondence**
Guang Li, Smith School of Business, Queen's University, Kingston, ON K7L 3N6, Canada.
Email: guang.li@queensu.ca

**Handling Editor**: Terry Taylor

**Abstract**

Motivated by the booming online grocery market and the extensive use of contingent free-shipping (CFS) policies in the e-grocery industry, we investigate the optimal CFS and pricing decisions for online grocers. Under a CFS policy, consumers enjoy free shipping for orders exceeding a certain threshold value; otherwise, they are charged a flat fee for orders below this threshold. We adopt a utility-based model to capture consumers' behavior of purchasing additional items to qualify for free shipping under a CFS policy and analyze its impact on policy structure and consumer surplus. We characterize the e-grocer's optimal pricing and CFS policy and find that consumer heterogeneity and demand distribution lead to different forms of the optimal shipping policy. When consumer heterogeneity is large enough, the optimal policy induces some consumers to top up and may allow some others to ship for free. In this case, the e-grocer can charge a high-profit margin. Otherwise, a top-up option is unnecessary, and a flat-rate shipping fee policy is optimal. Moreover, while the optimal policy never induces all consumers to top up when they are rational, it is possible to do so when consumers associate some psychological disutility with the shipping fee. Surprisingly, the total consumer surplus under the optimal policy may increase in the latter case. We further model a Stackelberg game between an e-grocer and an offline channel and find that the difference between the e-grocer's internal shipping cost and consumers' inconvenience cost of shopping offline is a main driver for market segmentation. Lastly, we show that a subscription-based free-shipping program, in addition to the jointly optimized CFS and pricing policy, cannot improve profits when consumers' order size and frequency are independent. Our findings help online grocers make operational and marketing decisions under the impact of consumers' top-up behavior.

**KEYWORDS**

contingent free-shipping policy, e-grocery, joint optimization, top-up behavior

## 1 | INTRODUCTION

E-commerce, one of the fastest-expanding industries in the global economy, is forecast to exceed \$8 trillion in sales in 2026 (eMarketer, 2022). The U.S. e-commerce sales reached \$601.7 billion in 2019 and steadily increased from 5.5% in Q1 2013 to 11.4% in Q4 2019 of the country's total retail sales (U.S. Census Bureau, 2019). Since 2020, the worldwide lockdown caused by the COVID-19 pandemic has catalyzed the growth of e-commerce. Analysis of the U.S. Department of Commerce data reveals that in 2020, consumer online spending experienced a 32.4% year-on-year growth (Ali, 2021). After 2020, the growth rate of the e-commerce market has adjusted back toward the pre-pandemic scale, giving rise to \$870.8 billion in sales in 2021 (U.S. Census Bureau, 2022). The U.S. retail e-commerce revenue is forecast to exceed \$1.7 trillion in 2027 (Statista, 2022).

One of the biggest winners in e-commerce amid the pandemic was the e-grocery industry. The onset of the COVID-19 pandemic and lockdown measures have accelerated

Accepted by Terry Taylor, after two revisions.

consumers' reliance on online grocery and permanently changed their shopping habits. Research indicates that this trend will continue even after the reopening of the economy (Aull et al., 2022). Indeed, the e-grocery market share in the United States surged from 3.4% in 2019 to 9.5% of all grocery sales in 2021 and is projected to reach 20.5% by 2026 (Acosta, 2021).

The transition to online grocery shopping brings about fundamental changes in the ways that grocery stores serve their customers. In contrast to traditional grocery stores, online grocers are responsible for delivering orders to their customers. The shipping and handling incur significant costs to the firms. Melton (2019) reports that most online grocers charge consumers only 80% of the overall delivery cost, and the current models of grocery delivery are simply not sustainable. It is no surprise that online grocers would like to pass on their internal shipping and handling costs to consumers as much as possible. For example, Whole Foods introduced free 2-hour delivery through Amazon Prime in 2017. However, starting on October 25, 2021, Whole Foods charges $9.95 for every delivery order to help cover operating costs associated with delivery.

The shipping policy implemented by an online grocer has a profound impact on consumer behavior. Market surveys show that shipping cost impacts the purchase decisions of up to 95% of U.S. online consumers, while 63% identify shipping cost as the primary reason for abandoning their shopping carts (Forter, 2019; Statista, 2018). There is little doubt, therefore, that shipping policy is one of the most important marketing and operations decisions for online grocers.

To attract and retain more consumers, and to alleviate the negative impact of shipping costs, online grocers adopt a variety of shipping policies. For example, Gopuff, a consumer goods and food delivery company operating in the United States and England, charges a flat fee of $1.95 for each delivery. Kroger Delivery and Walmart Grocery charge customers a flat-rate shipping fee that depends on consumer location and delivery speed. The majority of online grocers, such as Walgreens, Hungryroot, Yamibuy, and Weee, adopt a contingent free-shipping (CFS) policy. Under this policy, consumers incur no shipping charge as long as their order value exceeds a certain threshold; otherwise, they are charged a flat-rate shipping fee. Lewis (2006) shows that the CFS policy is the most effective in generating revenue for online grocers. This result is verified by a survey finding that 52% of online consumers add items to their shopping carts to reach the free-shipping threshold (Statista, 2018). In our paper, we refer to such an action as consumers' "top-up" behavior. We use "CFS policy" to broadly refer to the widely used contingent free-shipping policy, including the degenerate cases such as unconditional free shipping and flat-rate shipping policies.

Finding the optimal CFS policy, however, is not a simple task. In practice, online grocers (hereinafter, interchangeably referred to as e-grocers) adopt a broad range of CFS policy parameters (i.e., the free-shipping threshold and the below-threshold flat-rate shipping fee). For example, Walgreens offers free shipping on orders of $35 or more and charges a flat-rate fee of $5.99 otherwise. Hungryroot's free-shipping threshold and below-threshold flat-rate fee are $70 and $6.99, respectively. Two online grocers for Asian food, Yamibuy and Weee, use different policy parameters as well: the former charges $5.99 for orders below $49, whereas the latter charges $5 for orders below $35. In this paper, we establish an analytical model to understand the variety of shipping policies in the marketplace and offer a meaningful approach to designing shipping policies for online grocers.

The optimal CFS policy must balance the trade-offs between shipping revenue and additional sales generated by consumers' top-up behavior. A higher shipping fee certainly results in more shipping revenue from consumers who purchase below the free-shipping threshold. However, increasing the shipping fee will discourage some consumers from placing an order altogether, leading to a loss not only in the shipping revenue but also in the sale. On the other hand, a higher free-shipping threshold will motivate some consumers to order more but deter others from considering the top-up option, resulting in a smaller or zero basket size. Moreover, the effects of shipping fees and free-shipping thresholds are entangled: a higher shipping fee may encourage more consumers to top up, whereas a higher free-shipping threshold may encourage more consumers to pay the shipping fee. The interplay between shipping fees and consumers' top-up behavior deserves a rigorous investigation, especially when pricing is a joint decision.

In this paper, we consider an online grocer who optimizes decisions on its profit margin and CFS policy, characterized by a below-threshold flat-rate shipping fee and a free-shipping threshold, for a market with heterogeneous consumers. Given the grocer's CFS policy and profit margin, consumers make purchase decisions to maximize their net utility. Specifically, consumers can choose to make no purchase, purchase below the free-shipping threshold and pay a flat-rate shipping fee, or purchase no less than the threshold and enjoy free shipping. We aim to answer the following research questions:

(1) How does a CFS policy affect consumers' top-up behavior?
(2) What are an online grocer's optimal joint decisions on shipping policy and pricing in integrated marketing and operational planning?
(3) What is the impact of a CFS policy on the e-grocer's profit and consumer surplus?
(4) How does the competition from an offline channel affect the e-grocer's shipping policy and pricing decisions?

To the best of our knowledge, the joint optimization of all three decisions (i.e., the profit margin, the free-shipping threshold, and the below-threshold flat-rate shipping fee) has not been studied analytically in the existing literature. Our work helps understand the best balance among these three decisions. Moreover, our model is fairly general and can incorporate the impact of consumer irrationality around shipping fees and subscription-based free-shipping programs on the grocer's shipping policy and pricing decisions.

We summarize our main findings as follows:

(1) We characterize the structure of the optimal policy based on consumer heterogeneity and demand distribution. We find that even though it may be optimal to induce all consumers to pay a shipping fee, it is never optimal to induce all consumers to top up. Moreover, as consumer heterogeneity and the proportion of the high-valuation consumers increases, the optimal policy that covers both consumer segments is more likely to induce the high-valuation consumers to top up and the low-valuation consumers to pay a shipping fee.
(2) The optimal CFS policy may lead to different consumer surplus consequences. In particular, the optimal policy that charges all consumers a flat-rate shipping fee is the least effective in extracting consumer surplus as it does not discriminate between consumers; the optimal policy that allows some consumers to ship for free may hurt their surplus, as the policy enables the e-grocer to charge the highest profit margin.
(3) We consider a Stackelberg game where an offline channel acts as the first mover to compete with the e-grocer. We find that when the proportion of high-valuation consumers is large, the offline channel can price the e-grocer out of the market if the e-grocer's internal shipping cost is high relative to the consumers' inconvenience cost of shopping offline but shares the market with the e-grocer otherwise. In the latter case, the e-grocer serves the high-valuation consumers while the offline channel serves the low-valuation consumers.
(4) We are the first to incorporate consumers' psychological disutility of shipping fees analytically into the design of shipping policies. Interestingly, we find that, unlike the case without shipping fee disutility, it can be optimal for the e-grocer to induce all consumers to top up in the presence of the disutility. Such a psychological disutility, while always lowering the grocer's profit, can improve consumer surplus.
(5) We consider the profitability of subscription-based free-shipping programs and find that such a program is never profitable when consumers' order frequency and basket size are independent. When the order frequency and basket size are negatively correlated, introducing such a program in addition to the CFS policy can generate more profit for the grocer.

We organize the rest of the paper as follows. We review the relevant literature in Section 2. We describe the model in Section 3. We analyze the optimal policies for a market with two types of consumers and explore the implication of channel choice in Section 4. We explore extensions of our model in Section 5 and conclude the paper in Section 6. All technical proofs are presented in the Supporting Information.

## 2 | LITERATURE REVIEW

In this section, we review papers that are closely related to our work and highlight our contributions to each stream of the literature. We summarize and compare the different aspects of the most related literature and our work in Table 1.

The first stream of literature focuses on the effect of shipping fees on consumer purchase decisions. Brynjolfsson and Smith (2000) empirically demonstrate that online consumers are sensitive to shipping fees. Lewis (2006) and Lewis et al. (2006) are among the first to study the effect of shipping fees on order basket size. Through empirical analysis, they find that free shipping leads to greater order incidence but a smaller average basket size than does flat-rate shipping, whereas CFS policies that offer lower shipping fees on larger basket sizes lead to greater sales. Yang et al. (2005) examine the impact of CFS policies on consumer purchasing behavior and find that an increase in product price raises the probability of meeting the free-shipping threshold, thereby reducing the average shipping fee for repeat consumers. Xu (2016) uses transaction data in apparel retailing to study the effect of the free-shipping threshold on demand and product returns. Chen and Ngwe (2018) employ structural modeling and find that CFS policies promote consumer spending more across multiple product categories to meet the free-shipping threshold. Hemmati et al. (2021) empirically show that CFS policies induce "bubble purchases," where consumers top up to meet the free-shipping threshold and then return the unwanted products. While these papers focus on the impact of shipping policies on consumer behavior, our paper uses the insights from this literature to develop our consumer utility framework. We provide a complete characterization of the optimal pricing and CFS policy for online grocers through analytical modeling of consumer behavior.

Our research is also related to the partitioning mechanisms on pricing literature, which studies the impact of splitting the total purchase price into two or more parts on consumer behavior. Marketing research shows that consumers often do not make purchase decisions rationally—that is, based on the total price—when product prices and shipping and handling costs are charged separately. For example, Morwitz et al. (1998) find that consumers tend to overlook small shipping and handling costs, thereby discounting the total price. As a result, partitioned pricing can lead to higher consumer demand. In contrast, Thaler (1985) suggests that such a partitioning strategy creates a greater mental loss. Schindler et al. (2005) conduct a behavioral experiment to show that when consumers perceive the shipping charge as an alternative way to contribute to a retailer's profit, the partitioning strategy can result in reduced demand compared to a bundled pricing format. Gümüş et al. (2013) analyze the equilibrium of online retailers who adopt either a partitioned pricing or a bundled pricing format in an oligopolistic framework. Drawing insights from this literature stream, we capture consumers' top-up behavior to avoid the partitioned shipping fee induced by a CFS policy. We demonstrate the benefit of policies that promote consumers to top up for free shipping over the traditional shipping policies. In addition, we consider the case where the shipping fee creates a psychological disutility in consumer valuation and find that this disutility

**TABLE 1** Summary of related literature.

| | Leng and Becerril-Arreola (2010) | Becerril-Arreola et al. (2013) | Belavina et al. (2017) | Cachon et al. (2018) | Fang et al. (2021) | This paper |
|---|---|---|---|---|---|---|
| Profit margin/price | Endogenous | Endogenous | Exogenous | Endogenous | Endogenous | Endogenous |
| Free-shipping Threshold | Endogenous | Endogenous | Exogenous | Endogenous | Exogenous | Endogenous |
| Shipping fee | Exogenous | Exogenous | Endogenous | Exogenous | Exogenous | Endogenous |
| Competitor | Online | None | Offline | None | Online | Offline |
| Repeated purchase | Yes | No | Yes | No | Yes | Yes |
| Subscription | No | No | Yes | No | Yes | Yes |
| Product return | No | No | No | Yes | No | No |

further promotes the use of top-up policies and may improve consumer welfare.

Moreover, our work is closely related to the literature on the design of shipping policies. Leng and Becerril-Arreola (2010) are the first to analyze the joint decisions of profit margin from the marketing function and the CFS threshold from the operations function. They assume that consumer heterogeneity is continuous and characterize consumers' optimal purchase amount using an analytical model and numerically find the retailer's optimal decisions in response to shipping fees, retailer's shipping subsidy, and consumer heterogeneity. Becerril-Arreola et al. (2013) further consider inventory decisions, in addition to the profit margin and the free-shipping threshold, in a two-stage process through a simulation study. However, the above two papers do not endogenize the important decision of shipping fee. Shao (2017) considers a supply chain with a supplier and competitive retailers and finds the retailers' equilibrium price and order quantity under the free-shipping and paid shipping policies. The paper also treats shipping fees as exogenous. Cachon et al. (2018) employ a data-driven analytical model to evaluate the profitability of a retailer's CFS policy and identify the best free-shipping threshold policy for the retailer. Their model accounts for consumers' top-up behavior and product returns. In contrast, our paper provides analytical solutions to the joint optimization of the CFS policy parameters (i.e., the flat-rate shipping fee and the free-shipping threshold) and the profit margin. We identify the regimes where the optimal policy discriminates consumers by inducing different ordering behavior and investigate how market conditions and the firm's internal logistical efficiency affect the e-grocer's decisions.

There has been a rich literature that analytically studies the competition between online and offline channels (see, e.g., Balasubramanian, 1998; Chun & Kim, 2005; Liu et al., 2006; Viswanathan, 2005). These papers incorporate factors such as offline transportation cost, online disutility cost, and different prices of online and offline retailers into their model and analyze how these factors affect consumers' channel choice. Forman et al. (2009) empirically examine the trade-offs between buying online and from a local brick-and-mortar store and provide evidence for the existence of physical transportation costs and online disutility costs. We consider the competition between an offline channel and an online grocer adopting a CFS policy. Similar to the above-mentioned papers, we assume that consumers incur an inconvenience cost when purchasing offline and a disutility cost when shopping online. We find that the competition from the offline channel further motivates the e-grocer to adopt a top-up policy and, surprisingly, may induce the e-grocer to raise the free-shipping threshold to compensate for a reduced profit margin and shipping fee.

Last but not least, subscription models have been generating growing attention in the operations management literature. Under such a model, consumers prepay a fixed membership fee and receive free shipping services for their subsequent orders. Belavina et al. (2017) compare the subscription model with the flat-rate shipping model in online grocery shopping in the presence of an offline channel. They find that the subscription model leads to more frequent orders with a smaller basket size, higher profitability, and lower food waste but higher delivery-related greenhouse gas emissions. Wang et al. (2019) study the impact of service subscriptions on product pricing and consumer surplus in both monopolistic and competitive settings. Fang et al. (2021) study the impact of subscription programs on e-tailers when consumers are independently heterogeneous in terms of shopping frequency and disutility in topping up for free shipping. They find that when the e-tailer can optimize the product prices and membership fee, the addition of a membership program is always beneficial. In stark contrast, we show that a subscription program cannot generate a greater profit when the e-grocer has already jointly optimized the profit margin and the shipping policy if consumers' order frequency and order quantity are independent. When order frequency and quantity are negatively correlated, a subscription program can be profitable.

# 3 | MODEL

## 3.1 | The online grocer's shipping policy

We describe each CFS policy using two parameters: a free-shipping threshold $\tau \in [0, \infty]$ and a fixed flat-rate fee

$S \in [0, \infty]$. Under a CFS policy, consumers qualify for free shipping if the total dollar value of their order exceeds $\tau$; otherwise, they have to pay the flat-rate shipping fee $S$ for each order. Our construction of the CFS policy encompasses various widely used shipping policies in practice, such as unconditional free shipping policy ($S = 0$ or equivalently $\tau = 0$) and flat-rate shipping policy ($\tau = \infty$ and $S > 0$). In addition, we assume that the e-grocer adopts a uniform profit margin $m \in [0, 1)$ across all its products. This assumption is applicable if we consider goods from similar product categories; see, for example, Anderson et al. (1992), Cachon and Kök (2007), Leng and Becerril-Arreola (2010), and Belavina et al. (2017). We present a set of pricing and CFS policy decisions by a triplet $(m, \tau, S)$. We consider the problem that the e-grocer jointly optimizes over *all three* decisions and refer to the triplet $(m, \tau, S)$ as a "policy" hereinafter.

Next, we define the basket size of an order. In our basic setting, we assume that each order consists of only a single type of product. Therefore, its basket size can simply be defined as the number of items in the order. In Supporting Information Section EC.1.1, we relax this assumption and consider the case where the e-grocer offers multiple horizontally differentiated products, such as apples and pears, and we show that our results are robust under this generalization.

We further assume that the product has a unit procurement cost. Therefore, an order with basket size $y$ is equivalent to an order that has a procurement cost of $y$ dollars. Let $x$ denote the total purchase price of an order. Since $y$ is equivalent to the procurement cost of the order and $m$ denotes the profit margin, we have $m = (x - y)/x$. That is, an order of dollar value $x$ is equivalent to a basket of goods with size $y = x(1 - m)$.

Lastly, let $c_{\mathrm{h}}$ denote the grocer's internal shipping and handling cost of every order placed by consumers. Since the variable part of the internal shipping and handling cost can be incorporated into the procurement cost (thus the profit), we assume that $c_{\mathrm{h}}$ is independent of the size or value of the order and is a measure of the grocer's logistical efficiency.

## 3.2 | The consumers' purchase decisions

We consider a market consisting of $\alpha \in [0, 1]$ proportion of high-type consumers (denoted by "$H$") and $1 - \alpha$ proportion of low-type consumers (denoted by "$L$"). Consumers of type $i$, $i \in \{L, H\}$, have valuation $u_i(y)$ upon consuming an order of basket size $y$. We assume that $u_i(y)$ has the following properties, which are consistent with most literature:

**Assumption 1.** *The function $u_i(y)$ is continuously differentiable, increasing, and concave. Moreover,* $u_i(0) = 0, \lim_{y\to 0} u_i'(y) > 1, \lim_{y\to+\infty} u_i'(y) = 0$.

Let $U_i(y)$ denote type $i$'s *consumer net utility* (hereinafter, interchangeably referred to as *consumer surplus*) in purchasing an order of basket size $y$. Given a policy characterized by $(m, \tau, S)$, $U_i(y)$ is defined as the difference between the consumption valuation $u_i(y)$ and the total payment, which includes $x$, the dollar value of the order, and the flat-rate shipping fee $S$ if the order does not qualify for free shipping (i.e., $x < \tau$). Recall that $y = (1 - m)x$. Equivalently, the dollar value of the order can be presented by $x = \frac{y}{1-m}$. Thus,

$$U_i(y) = \begin{cases} u_i(y) - \dfrac{y}{1-m} - S, & \text{if } \dfrac{y}{1-m} \in (0, \tau) \\ u_i(y) - \dfrac{y}{1-m}, & \text{otherwise} \end{cases}. \quad (1)$$

Let $y_i^{\mathrm{u}}(m)$ denote the optimal basket size of type $i$ consumers' order without considering the shipping fee, for any $m \in [0, 1)$. Equivalently, $y_i^{\mathrm{u}}(m)$ maximizes type $i$'s utility under a free shipping policy (i.e., $S = 0$ or $\tau = 0$). That is,

$$y_i^{\mathrm{u}}(m) = \arg\max_{y \geq 0} u_i(y) - \frac{y}{1-m}.$$

We refer to $y_i^{\mathrm{u}}(m)$ as type $i$'s *intrinsic basket size*. Assumption 1 implies that $u_i(y) - \frac{y}{1-m}$ is continuous, differentiable, and concave. Define $\overline{m}_i = 1 - \frac{1}{u_i'(0)}$. Then, $y_i^{\mathrm{u}}(m)$ is the unique (positive) solution to the first-order condition $u_i'(y_i^{\mathrm{u}}(m)) = \frac{1}{1-m}$ for $m \in [0, \overline{m}_i)$; otherwise, $y_i^{\mathrm{u}}(m) = 0$, implying that consumers do not purchase from the e-grocer if the products are priced exorbitantly high.

For future analysis, we define $y_i^0(m)$ as the *maximum basket size* that leads to zero consumer utility under a free shipping policy, that is,

$$y_i^0(m) = \max\left\{ y \geq 0 : u_i(y) - \frac{y}{1-m} = 0 \right\}.$$

Note that $y = 0$ is a trivial solution to this equation. In fact, for any $m \in [\overline{m}_i, 1)$, $y = 0$ is a unique solution to the equation. Thus, $y_i^0(m) = 0$ for $m \in [\overline{m}_i, 1)$. However, for $m \in [0, \overline{m}_i)$, $u_i(y)$ and $\frac{y}{1-m}$ have a unique positive intersection. That is, $y_i^0(m) > 0$. In addition, given the concavity of $u_i$, we have $y_i^0(m) > y_i^{\mathrm{u}}(m)$ and both terms are decreasing in $m \in [0, \overline{m}_i)$.

Furthermore, we define $w_i(y)$ as the social surplus associated with an individual type $i$ consumer whose basket size is $y$. In particular, the social surplus from a type $i$ consumer is the summation of the consumer net utility and the firm's profit. That is,

$$\begin{aligned} w_i(y) &= \underbrace{u_i(y) - \frac{y}{1-m} - S \times \mathbf{1}_{\frac{y}{1-m} \in (0,\tau)}}_{consumer\ net\ utility} \\ &\quad + \underbrace{m\frac{y}{1-m} + S \times \mathbf{1}_{\frac{y}{1-m} \in (0,\tau)} - c_{\mathrm{h}} \times \mathbf{1}_{y>0}}_{firm's\ profit} \\ &= u_i(y) - y - c_{\mathrm{h}} \times \mathbf{1}_{y>0} \end{aligned}$$

for $y \geq 0$, where $\mathbf{1}$ is the indicator function. Let $y_i^{\mathrm{w}}$ denote the basket size that maximizes the total social surplus of an order. We refer to $y_i^{\mathrm{w}}$ as the *socially optimal basket size* for type $i$ consumers. Formally, we have

$$y_i^{\mathrm{w}} = \arg\max_{y \geq 0} w_i(y).$$

Assumption 1 guarantees the existence and uniqueness of $y_i^{\mathrm{w}}$. In particular, $y_i^{\mathrm{w}}$ can be computed by solving the first-order condition $u_i'(y_i^{\mathrm{w}}) = 1$. Note that, unlike $y_i^{\mathrm{u}}(m)$ and $y_i^{0}(m)$, $y_i^{\mathrm{w}}$ is independent of the profit margin $m$. It is straightforward to verify that $y_i^{\mathrm{u}}(m) \leq y_i^{\mathrm{w}}$ for all $m \in [0, 1)$, yet $y_i^{0}(m)$ can be greater than or less than $y_i^{\mathrm{w}}$, depending on the magnitude of $m$.

The three basket sizes ($y_i^{\mathrm{u}}(m)$, $y_i^{0}(m)$, and $y_i^{\mathrm{w}}$) play important roles in characterizing the consumers' purchase behavior and the e-grocer's optimal policy. Given any policy $(m, \tau, S)$, type $i$ consumers decide how much to order to maximize their net utility $U_i(y)$ given in Equation (1) by taking one of the following four actions: (1) make no purchase, denoted by $\varnothing$; (2) purchase the intrinsic basket size $y_i^{\mathrm{u}}(m)$ and pay flat-rate shipping fee $S$, denoted by $s$; (3) top up the order value to the free-shipping threshold $\tau$, denoted by $t$; or (4) automatically qualify for free shipping with the intrinsic basket size, denoted by $f$. In the following discussion, we denote type $i$ consumers' *induced action* by $a_i$, where $a_i \in \{\varnothing, s, t, f\}$.

Specifically, when $\frac{y_i^{\mathrm{u}}(m)}{1-m} \geq \tau$, type $i$ consumers qualify for free shipping with their intrinsic basket size $y_i^{\mathrm{u}}(m)$. That is, $a_i = f$. Otherwise, $\frac{y_i^{\mathrm{u}}(m)}{1-m} < \tau$ and $a_i \in \{\varnothing, s, t\}$. In particular, if $a_i = s$, that is, type $i$ consumers choose to pay a shipping fee for intrinsic basket size $y_i^{\mathrm{u}}(m) > 0$, their net utility becomes $U_i(y_i^{\mathrm{u}}(m)) = u_i(y_i^{\mathrm{u}}(m)) - \frac{y_i^{\mathrm{u}}(m)}{1-m} - S$. If $a_i = t$, that is, the consumers choose to top up the order value to $\tau$, their net utility is $U_i((1-m)\tau) = u_i((1-m)\tau) - \tau$. If $a_i = \varnothing$, type $i$ consumers purchase nothing and get zero net utility. When $\frac{y_i^{\mathrm{u}}(m)}{1-m} < \tau$, consumers will choose the action $a_i \in \{\varnothing, s, t\}$ that leads to the highest net utility. If there is a tie between the net utilities of two choices, we assume that consumers always prefer the one with a greater basket size.

## 3.3 | The online grocer's problem

We consider the problem when the e-grocer jointly optimizes over $(m, \tau, S)$ to maximize its total profit. This scenario applies when the e-grocer can make simultaneous pricing and shipping policy decisions through interdepartmental coordination. Denote the e-grocer's total expected profit by $\Pi(m, \tau, S)$ and the profit from a type $i$ consumer by $\pi_i(m, \tau, S)$. Normalizing the total market size to one, we have

$$\Pi(m, \tau, S) = \alpha \pi_H(m, \tau, S) + (1-\alpha)\pi_L(m, \tau, S).$$

Given a policy $(m, \tau, S)$, the internal per-order shipping cost $c_{\mathrm{h}}$, and an order of a positive dollar value $x$ (with a corresponding basket size $y = (1-m)x$), the online grocer earns a profit of $mx + S - c_{\mathrm{h}}$ if the order does not qualify for free shipping (i.e., $0 < x < \tau$). Otherwise, the grocer's profit is $mx - c_{\mathrm{h}}$. Incorporating consumer choice and applying the relationship $y = (1-m)x$, we can write the online grocer's profit $\pi_i(m, \tau, S)$ from a type $i$ consumer as

$$\pi_i(m, \tau, S) = \begin{cases} 0, & \text{if } a_i = \varnothing \\ \dfrac{m y_i^{\mathrm{u}}(m)}{1-m} + S - c_{\mathrm{h}}, & \text{if } a_i = s \\ m\tau - c_{\mathrm{h}}, & \text{if } a_i = t \\ \dfrac{m y_i^{\mathrm{u}}(m)}{1-m} - c_{\mathrm{h}}, & \text{if } a_i = f \end{cases}. \tag{2}$$

We state the following assumption on $c_{\mathrm{h}}$ to ensure that the e-grocer can earn a nonnegative profit from any type $i$ consumers to exclude trivial cases.

**Assumption 2.** $0 \leq c_{\mathrm{h}} \leq \min_i\{u_i(y_i^{\mathrm{w}}) - y_i^{\mathrm{w}}\}$.

Assumption 2 guarantees that the social surplus associated with any type $i$ consumer cannot be negative. Once $c_{\mathrm{h}}$ exceeds $u_i(y_i^{\mathrm{w}}) - y_i^{\mathrm{w}}$, the e-grocer will not target type $i$ consumers because doing so always results in a negative profit. This implies that e-grocer will either target the other type of consumers only or exit the market, making the analysis trivial. In Section 4.2.2, we relax this assumption and explore the impact of $c_{\mathrm{h}}$ on the e-grocer's optimal shipping policy.

Clearly, the grocer's profit increases with $m$, $\tau$, or $S$, ceteris paribus. However, the interdependence between the online grocer's policy decisions and the consumer's choice of basket size makes joint optimization challenging and worth investigation. We analyze the optimal shipping policies in the following section.

# 4 | ANALYSIS AND RESULTS

We first analyze the optimal policy for a general valuation function $u_i(y)$ in Section 4.1. Then, we derive more structural results and managerial insights when $u_i(y)$ is a square root function of $y$ in Section 4.2. Lastly, we explore the optimal decisions when the e-grocer competes with an offline channel in Section 4.3. We denote the optimal policy by $(m^*, \tau^*, S^*)$ and the corresponding profit by $\Pi^* = \Pi(m^*, \tau^*, S^*)$.

## 4.1 | Optimal policy under general valuation functions

In this section, we consider any general valuation function $u_i(y)$ that satisfies Assumption 1.

### 4.1.1 | Homogeneous consumers

We start the analysis by looking into the scenario when there is only one type of consumers, that is, $\alpha = 0$ or 1. Before stating the main result, we establish a few key intuitions. First, when consumers are homogeneous, the optimal policy $(m^*, \tau^*, S^*)$ (if exists) must induce the consumers to either purchase an order of size $y_i^{\mathsf{u}}(m^*)$ and pay the flat-rate shipping fee $S^*$ (i.e., action $s$) or to top up the order value to the free-shipping threshold $\tau^*$ (i.e., action $t$). From now on, we name a policy that induces action $s$ as a "flat-rate" policy and a policy that induces action $t$ as a "top-up" policy. Second, the optimal policy should extract *all* consumer surplus when there is a single type of consumer. Hence, the grocer's profit should coincide with the social surplus under the optimal policy. We provide a complete characterization of the optimal policies in the following proposition.

**Proposition 1** (Optimal policy with homogeneous consumers). *There are two types of optimal shipping policies for homogeneous consumers:*

(1) *a zero-margin flat-rate policy that induces action s with* $m^* = 0, \tau^* = \infty$, *and* $S^* = u_i(y_i^{\mathsf{w}}) - y_i^{\mathsf{w}}$;
(2) *a top-up policy that induces action t with* $m^* = 1 - \frac{y_i^{\mathsf{w}}}{u_i(y_i^{\mathsf{w}})}$, $\tau^* = u_i(y^{\mathsf{w}})$, *and* $S^* = \infty$.

*Moreover, under any optimal policy, the consumers order a basket of size* $y_i^{\mathsf{w}}$. *The online grocer always earns the maximal social welfare, that is,* $\Pi^* = w(y_i^{\mathsf{w}})$.

Proposition 1 indicates that the online grocer has two ways to induce homogeneous consumers to purchase the socially optimal basket size $y_i^{\mathsf{w}}$. The first is to adopt a zero-margin flat-rate policy. Note that the consumers' intrinsic basket size coincides with the socially optimal basket size at $m = 0$ (i.e., $y_i^{\mathsf{u}}(0) = y_i^{\mathsf{w}}$). Then, the e-grocer can set a sufficiently large $\tau$ to deter consumers from topping up their order and set an appropriate $S$ to extract all consumer surplus. The second option is to employ a top-up policy. The e-grocer can induce consumers to top up to the socially optimal basket size $y_i^{\mathsf{w}}$ by setting threshold $\tau = \frac{y_i^{\mathsf{w}}}{1-m}$. Then, the e-grocer can set a sufficiently large $S$ to prevent consumers from paying the shipping fee and set an appropriate profit margin $m$ to extract all consumer surplus. Under both policies, the online grocer employs all three levers $(m, \tau, S)$ to extract the entire consumer surplus and earn the highest possible profit of $w(y^{\mathsf{w}})$. Moreover, since the flat-rate policies are equally optimal as the top-up policies, we conclude that policies with top-up options are not necessary when consumers are homogeneous.

Lastly, it is worth noting that the optimal policies are not unique. For example, under the zero-margin flat-rate policy, the threshold $\tau^*$ can take any value exceeding $y_i^{0}(0) = y_i^{\mathsf{w}}$ at $m^* = 0$; under the top-up policy, the shipping fee $S^*$ can take any value exceeding $u_i(y_i^{\mathsf{u}}(m^*)) - \frac{y_i^{\mathsf{u}}(m^*)}{1-m^*} = 0$ at $m^* = 1 - \frac{y_i^{\mathsf{w}}}{u_i(y^{\mathsf{w}})}$. Without loss of generality, we set $\tau^* = \infty$ under the zero-margin flat-rate policy and $S^* = \infty$ under the top-up policy in Proposition 1.

### 4.1.2 | Heterogeneous consumers

We now consider the scenario when the market consists of both high- and low-type consumers. We assume that the high-type consumers have a higher consumption valuation than the low-type consumers. To be more precise, we state the following assumption regarding $u_i(y)$:

**Assumption 3.** *For all* $y > 0$, $u_H(y) > u_L(y)$ *and* $u'_H(y) \geq u'_L(y)$.

Under the optimal policy, the e-grocer must extract all consumer surplus from at least one type of consumer. Otherwise, the e-grocer can always increase the profit margin $m$ and/or the CFS policy parameters $(\tau, S)$ to improve its profit. Assumption 3 implies that the high-type consumers can obtain a strictly higher net utility than the low-type by simply choosing the same basket size as the low-type consumers. Since the high-type consumers can make better decisions than simply mimicking the low-type's behavior, we conclude that the high-type consumers always obtain a strictly higher net utility than the low-type as long as they make a purchase. Therefore, the optimal policy must extract all surplus from *exactly* one type of consumer. It either extracts all surplus from the high-type consumers (in this case, the low-type will not purchase to avoid a negative surplus) or extracts all surplus from the low-type and leaves the high-type some positive surplus.

Hereinafter, we refer to a policy that induces only the high-type consumers to purchase as a *high-coverage policy* (HCP). In contrast, we refer to a policy that induces both types of consumers to purchase as a *full-coverage policy* (FCP). We further refer to the "best" HCP (FCP) as the one that maximizes the grocer's total profit among the set of HCPs (FCPs).

**Lemma 1.** *The optimal policy* must be *either the best HCP or the best FCP.*

Let $(m_H, \tau_H, S_H)$ and $(m_F, \tau_F, S_F)$ denote the best HCP and the best FCP, respectively. Lemma 1 indicates that it suffices for us to compare the e-grocer's profits under the two candidate policies, $(m_H, \tau_H, S_H)$ and $(m_F, \tau_F, S_F)$. The optimal policy must be the one with a higher profit. Next, we aim to characterize the best HCP and the best FCP.

The best HCP should be similar to the optimal policy for homogeneous consumers in Proposition 1. We state this result in the next corollary.

**Corollary 2** (Best HCP with heterogeneous consumers). *The best HCP is*

(1) *a zero-margin flat-rate policy with* $m_H = 0$, $\tau_H = \infty$, *and* $S_H = u_H(y_H^{\mathsf{w}}) - y_H^{\mathsf{w}}$*; or, equivalently,*
(2) *a top-up policy with* $m_H = 1 - \frac{y_H^{\mathsf{w}}}{u_H(y_H^{\mathsf{w}})}$, $\tau_H = u_H(y_H^{\mathsf{w}})$, *and* $S_H = \infty$.

*Under the best HCP policy, the high-type consumers order the socially optimal basket size* $y_H^{\mathsf{w}}$ *and the low-type consumers do not purchase. The e-grocer earns a profit of* $\alpha w_H(y_H^{\mathsf{w}})$.

It turns out to be challenging to provide a complete characterization of the best FCP under the joint optimization over $(m, \tau, S)$. Nevertheless, we can start by considering the set of possible consumer action pairs under a policy. Let $(a_H, a_L)$ be the induced consumer action pair, where $a_i \in \{\emptyset, s, t, f\}$ denotes type $i$ consumers' induced action. As the high- and low-type of consumers may behave differently, there exist 16 possible consumer action pairs. The next lemma shows that some of these action pairs are infeasible.

**Lemma 2.** *We define a partial ordering among the four actions:* $\emptyset \prec s \prec t \prec f$. *Under any policy* $(m, \tau, S)$, *the high-type consumers' induced action* $a_H$ *has no lower order than the low-type consumers' induced action* $a_L$. *That is,* $a_H \succeq a_L$.

Lemma 2 has several implications. First, as discussed earlier, the high-type consumers will always purchase a positive basket size if the low-type choose to do so. In addition, if the low-type qualify for free shipping with their intrinsic basket size, so do the high-type consumers. More interestingly, Lemma 2 suggests that no policy can induce the high-type to pay shipping fees but the low-type to top up. That is, the action pair $(s, t)$ is infeasible. In other words, as long as the low-type choose the top-up action, the high-type will also top up if $\tau > \frac{y_H^{\mathsf{u}}(m)}{1-m}$ or automatically qualify for free shipping if $\tau \leq \frac{y_H^{\mathsf{u}}(m)}{1-m}$, no matter how the e-grocer manipulates the value of $S$. Thanks to Lemma 2, we can rule out several infeasible consumer action pairs in the search for the optimal policy.

Next, we argue that some action pairs, though feasible, cannot occur under the optimal policy. For example, it is never optimal to allow both types of consumers to ship for free. It is also not optimal to induce the -low-type to pay shipping fees, but the high-type to ship for free because the e-grocer can always raise the free-shipping threshold to force the high-type to pay shipping fees and earn a greater profit. Thus, we can further narrow down the search for optimal policies by removing the suboptimal action pairs. The following lemma shows that we only need to focus on *four* candidate action pairs under the best FCP policy.

**Lemma 3.** *Under the best FCP policy,* $(a_H, a_L) \in \{(s, s), (t, s), (t, t), (f, t)\}$.

We remark that, for a market with homogeneous consumers, it is never optimal to allow consumers to ship for free. However, this result no longer holds when there are multiple types of consumers. When consumers have heterogeneous valuations, allowing the high-type to enjoy free shipping can be optimal when the low-type is induced to top up. Although the e-grocer can set a larger $\tau$ to induce the high-type consumers to top up as well, doing so would cause the low-type to suffer from a negative net utility. Hence, the low-type consumers will end up with no purchase. Once the marginal loss in profit from the low-type consumers outweighs the marginal gain in profit from the high-type consumers, the e-grocer would rather let the high-type ship for free while inducing the low-type to top up. We will illustrate this scenario using a parametric example in Section 4.2.

Define $H_{a_H}L_{a_L}$ as *the best policy* among the set of policies that induce consumer action $(a_H, a_L)$, where $(a_H, a_L) \in \{(s, s), (t, s), (t, t), (f, t)\}$. For example, $H_tL_s$ is the best policy that induces the high-type to top up and the low-type to pay a shipping fee. We refer to the "best" policy as the one that maximizes the e-grocer's total profit. In addition, we denote the e-grocer's profit under $H_{a_H}L_{a_L}$ by $\Pi_{H_{a_H}L_{a_L}}$. As discussed in Section 3.2, $H_tL_t$ and $H_fL_t$ are mutually exclusive. When either $H_tL_t$ or $H_fL_t$ does not exist, we simply let the corresponding profit be zero.

Lemma 3 implies that the best FCP should be one of the four candidate policies: $H_sL_s$, $H_tL_s$, $H_tL_t$, or $H_fL_t$. It follows that $\Pi(m_F, \tau_F, S_F) = \max\{\Pi_{H_sL_s}, \Pi_{H_tL_s}, \Pi_{H_tL_t}, \Pi_{H_fL_t}\}$. Surprisingly, we find that $H_tL_t$ that induces all consumers to top up is *always suboptimal*. We state this result in the following proposition.

**Proposition 3** (Suboptimality of $H_tL_t$ with a variable margin). *If* $H_tL_t$ *exists, it is dominated by* $H_sL_s$ *and can never be the optimal policy.*

The intuition of Proposition 3 is as follows: $H_tL_t$ does not discriminate between consumers at all as it forces both types of consumers to order the same basket size without paying shipping fees. On the other hand, $H_sL_s$ allows consumers to purchase different basket sizes while paying the same shipping fee. Thus, the e-grocer is better off through basket size discrimination under $H_sL_s$.

Proposition 3 implies that the best FCP is either $H_fL_t$, $H_tL_s$, or $H_sL_s$. To be consistent, we express the best HCP as $H_tL_\emptyset$ and $H_sL_\emptyset$. In light of the discussions above, we can narrow down the candidates for the optimal policy to the following five policies: $H_fL_t$, $H_tL_s$, $H_sL_s$, $H_tL_\emptyset$, and $H_sL_\emptyset$. Theoretically, we can find the optimal policy by comparing the profits under these five candidate policies. That is, $\Pi(m^*, \tau^*, S^*) = \max\{\Pi_{H_sL_s}, \Pi_{H_tL_s}, \Pi_{H_fL_t}, \Pi_{H_tL_\emptyset}, \Pi_{H_sL_\emptyset}\}$. However, the comparisons are intractable with general valuation functions $u_H(\cdot)$ and $u_L(\cdot)$. We illustrate and offer more insights into the optimal policy using parametric valuation functions in the following section.

## 4.2 | Optimal policy under square root valuations

In this section, we employ a square root valuation function. The square root utility is one of the commonly used utility functions in the literature; see, for example, Basu et al. (1985), Chung (1994), and Leng and Becerril-Arreola (2010). Specifically, let $u_i(y) = \sqrt{k_i y}$ for $i \in \{L, H\}$, where $k_i$ measures how many type $i$ consumers value the size of their basket. The larger the $k_i$, the more valuable each unit basket size is to the consumers, and the greater the intrinsic basket size. We assume that $k_H > k_L$, which is consistent with Assumption 3. We characterize the optimal policies in Section 4.2.1. We explore the impact of market conditions and the e-grocer's internal shipping cost on the optimal policy and investigate the implications of top-up policies for the e-grocer's revenue and consumer surplus in Section 4.2.2. We consider the revenue implications of more complicated CFS policies in Supporting Information Section EC.1.2.

### 4.2.1 | Optimal policy structure

Given the square root valuation function, we can compute the intrinsic basket size $y_i^{\mathsf{u}}(m) = \frac{(1-m)^2 k_i}{4}$, the maximum affordable basket size $y_i^{0}(m) = (1-m)^2 k_i$, and the socially optimal basket size $y_i^{\mathsf{w}} = \frac{k_i}{4}$ for type $i$ consumers, where $i \in \{L, H\}$. The social surplus associated with the above basket sizes is $w_i(y_i^{\mathsf{u}}(m)) = \frac{(1-m)(1+m)k_i}{4} - c_{\mathsf{h}}$, $w_i(y_i^{0}(m)) = m(1-m)k_i - c_{\mathsf{h}}$, and $w_i(y_i^{\mathsf{w}}) = \frac{k_i}{4} - c_{\mathsf{h}}$, respectively. By Assumption 2, we focus on $c_{\mathsf{h}} \le \frac{k_L}{4}$ to ensure that the e-grocer can earn a nonnegative profit from both types of consumers.

Recall that $H_{a_H}L_{a_L}$ is *the best policy* among the set of policies that induce the high-type consumers to choose action $a_H$ and the low-type to choose action $a_L$. Following Corollary 2, the best HCP $(m_H, \tau_H, S_H)$ is $H_sL_\varnothing$ with $m_H = 0$, $\tau_H = \infty$, and $S_H = \frac{k_H}{4}$ or $H_tL_\varnothing$ with $m_H = \frac{1}{2}$, $\tau_H = \frac{k_H}{2}$, and $S_H = \infty$. Since $H_tL_\varnothing$ and $H_sL_\varnothing$ are equivalently the best HCP, hereinafter we refer to these two policies as $H_{only}$. The e-grocer's profit under the best HCP then becomes

$$\Pi(m_H, \tau_H, S_H) = \Pi_{H_{only}} = \alpha\left(\frac{k_H}{4} - c_{\mathsf{h}}\right). \tag{3}$$

Per discussions in Section 4.1.2, the best FCP must be one of the following three policies: $H_sL_s$, $H_tL_s$, and $H_fL_t$, where the corresponding profits are given by

$$\Pi_{H_sL_s} = \frac{(\alpha(k_H - k_L) + 2k_L)^2}{16(\alpha(k_H - k_L) + k_L)} - c_{\mathsf{h}}, \tag{4a}$$

$$\Pi_{H_tL_s} = \frac{(\alpha(k_H + 2\sqrt{k_H k_L} - k_L) + 2k_L)^2}{16(\alpha(k_H + 2\sqrt{k_H k_L}) + k_L)} - c_{\mathsf{h}}, \tag{4b}$$

$$\Pi_{H_fL_t} = \begin{cases} \alpha\dfrac{k_H}{16} + (1-\alpha)\dfrac{k_L}{4} - c_{\mathsf{h}}, & \text{if } k_H \ge 4k_L \\ \dfrac{(2(1-\alpha)k_L + \alpha\sqrt{k_H k_L})^2}{16 k_L} - c_{\mathsf{h}}, & \text{if } k_H < 4k_L \end{cases}. \tag{4c}$$

We defer the derivation of equations in (4) to Supporting Information Section EC.2.1.

We are interested in understanding when the best HCP ($H_{only}$) or the best FCP ($H_sL_s$, $H_tL_s$, or $H_fL_t$) becomes optimal. Notice that Equations (3) and (4) are functions of $k_H/k_L$ and $\alpha$, which capture two distinct features of the market. In particular, $\alpha$ depicts the demand distribution in terms of consumer basket size (e.g., household size), whereas $k_H/k_L$ represents consumer heterogeneity in terms of the disparity between the valuations of high- and low-type consumers. Therefore, these two parameters should have differing impacts on the e-grocer's optimal shipping policy.

Intuitively, we expect the best HCP to be optimal only when $\alpha$ is sufficiently large. The following theorem shows the existence and uniqueness of thresholds $F_\alpha^i$ $(i = 1, 2, 3)$ on $\alpha$ as functions of $\frac{k_H}{k_L}$ and $\frac{c_{\mathsf{h}}}{k_L}$. We defer the expressions of the thresholds to Supporting Information Section EC.2.1. Given any set of $(\frac{k_H}{k_L}, \frac{c_{\mathsf{h}}}{k_L})$, the optimal policy solely depends on the magnitude of $\alpha$ with respect to the thresholds $F_\alpha^i$. We provide a complete structural characterization of the optimal policy $(m^*, S^*, \tau^*)$ in Theorem 4.

**Theorem 4** (Optimal policy with heterogeneous consumers). *The optimal policy must be one of policies $H_{only}$, $H_tL_s$, $H_sL_s$, and $H_fL_t$. In particular, for $c_{\mathsf{h}} \in [0, \frac{k_L}{4}]$, there exist thresholds $0 \le F_\alpha^3 \le F_\alpha^2 \le F_\alpha^1 < 1$ as functions of $\frac{k_H}{k_L}$ and $\frac{c_{\mathsf{h}}}{k_L}$ such that*

1. *if $\alpha \in (F_\alpha^1, 1]$, then $H_{only}$ is the optimal policy;*
2. *if $\alpha \in (F_\alpha^2, F_\alpha^1]$, then $H_tL_s$ is the optimal policy;*
3. *if $\alpha \in (F_\alpha^3, F_\alpha^2]$, then $H_sL_s$ is the optimal policy;*
4. *if $\alpha \in [0, F_\alpha^3]$, then $H_fL_t$ is the optimal policy.*

Figure 1 illustrates the optimal policy. Theorem 4 indicates that $H_fL_t$ that focuses on extracting surplus from the low-type consumers is optimal only when $\alpha$ is sufficiently low, that is, when low-type consumers dominate the market, and when $\frac{k_H}{k_L} \ge 4$. When $\alpha$ is moderate, the e-grocer needs to balance the profits from both types of consumers. In particular, the flat-rate policy $H_sL_s$ outperforms the other policies when $\alpha \in [F_\alpha^3, F_\alpha^2)$. When $\alpha$ is even higher, that is, $\alpha \in [F_\alpha^2, F_\alpha^1)$, the e-grocer is better off under $H_tL_s$, which further discriminates between consumers by inducing them to choose different shipping options: the high-type will top up, whereas the low-type will pay a shipping fee. Lastly, Theorem 4 confirms our intuition that $H_{only}$ is optimal only when $\alpha$ is sufficiently high.

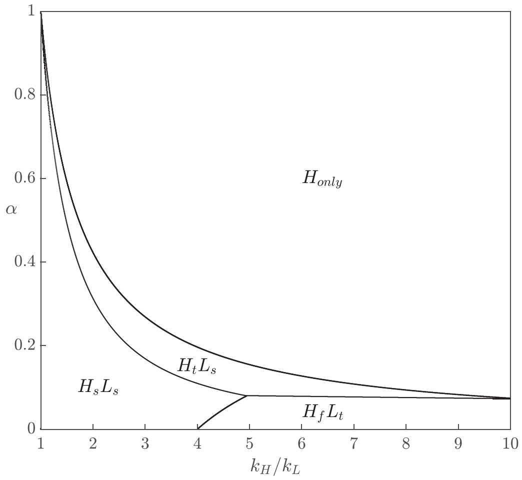



**FIGURE 1** Optimal policy with respect to $\alpha$ and $k_H/k_L$ for $c_h = 0.1k_L$.

We remark that some of the thresholds may coincide with each other. For example, in the proof of Theorem 4, we show that $F_\alpha^3 = 0$ if and only if $\frac{k_H}{k_L} < 4$. When $F_\alpha^3 = 0$, the interval $[0, F_\alpha^3)$ will disappear, meaning that $H_fL_t$ can no longer be optimal. It is also possible that $F_\alpha^2 = F_\alpha^3$ or $F_\alpha^2 = F_\alpha^1$. If this happens, the interval $[F_\alpha^3, F_\alpha^2)$ or $[F_\alpha^2, F_\alpha^1)$ disappears and the corresponding policy ($H_sL_s$ or $H_tL_s$) cannot be optimal.

To sum up, our results suggest that only when there exist a relatively large number of low-type consumers, that is, $\alpha$ is relatively small, it is worthwhile for the e-grocer to serve both low-type and high-type consumers. Moreover, we find that a top-up policy such as $H_tL_s$ is not needed in a market with low heterogeneity (i.e., a small $k_H/k_L$), given that the flat-rate shipping policy $H_sL_s$ is already optimal. However, as the consumers' heterogeneity increases, $H_tL_s$ eventually outperforms $H_sL_s$ and becomes the optimal policy.

### 4.2.2 | Discussion

In this section, we perform sensitivity analysis with regard to consumer heterogeneity ($k_H/k_L$), demand distribution ($\alpha$), and the e-grocer's internal shipping cost ($c_h$). We then discuss the implications of having a top-up option on the e-grocer's revenue and consumer surplus.

*Impact of consumer heterogeneity and demand distribution*

We analyze how consumer heterogeneity ($k_H/k_L$) and demand distribution ($\alpha$) impact the optimal policy. The next result shows how the thresholds $F_\alpha^i$'s change with consumer heterogeneity.

**Proposition 5.** *$F_\alpha^1$ and $F_\alpha^2$ decrease with $k_H/k_L$; if $F_\alpha^3 < F_\alpha^2$, then $F_\alpha^3$ increases with $k_H/k_L$.*

By Theorem 4, the thresholds $F_\alpha^1$, $F_\alpha^2$, and $F_\alpha^3$ determine the structure of the optimal shipping policy. Proportion 5 states that $F_\alpha^1$ decreases with $k_H/k_L$, suggesting that $H_{only}$ becomes more preferable as consumers become more heterogeneous. It makes sense as a larger $k_H/k_L$ (i.e., $k_H$ is getting larger with respect to $k_L$) motivates the e-grocer to focus more on the high-type consumers.

Proportion 5 further implies that as consumer heterogeneity increases, the flat-rate shipping policy $H_sL_s$ is less likely to be optimal. Note that $H_sL_s$ induces both types of consumer to take the same action (i.e., pay a shipping fee). As consumer heterogeneity becomes greater, the e-grocer can benefit from inducing the two types of consumers to behave differently. Doing so enables the e-grocer to extract more consumer surplus. This explains why the top-up policies $H_tL_s$ and $H_fL_t$ become more desirable when consumers are more heterogeneous.

Next, we are interested in the impact of $\alpha$ and $k_H/k_L$ on the pricing and shipping policy decisions. Proposition 6 shows how the profit margin changes with $\alpha$ and $k_H/k_L$. As shown at the beginning of this section, the profit margin is either 0 or $\frac{1}{2}$ under $H_{only}$. Therefore, we focus on the case that the optimal policy is the best FCP.

**Proposition 6** (Profit margin under the best FCP). *When the optimal policy is an FCP, the optimal profit margin $m^*$ increases with $k_H/k_L$. In addition, if $k_H/k_L < 4$, $m^*$ increases with $\alpha$; otherwise, $m^*$ is nonmonotone in $\alpha$.*

Proposition 6 suggests that the more heterogeneous the consumers are, the higher profit margin the e-grocer can charge. In addition, because shipping revenue and sales revenue are complementary, we can show that the flat-rate shipping policy $H_sL_s$ has the smallest profit margin. The e-grocer can charge a higher profit margin under $H_tL_s$, where only the low-type consumers pay a shipping fee. Lastly, since no consumer is induced to pay a shipping fee under $H_fL_t$, the e-grocer can charge the highest margin. We have mentioned earlier that $F_\alpha^3 = 0$ if and only if $k_H/k_L \leq 4$. Thus, if $k_H/k_L \leq 4$, as $\alpha$ increases from 0, the optimal policy switches from is $H_sL_s$ to $H_tL_s$. In this case, the optimal profit margin $m^*$ increases with $\alpha$. However, if $k_H/k_L > 4$, the optimal policy is initially $H_fL_t$ and changes to $H_sL_s$ and then to $H_tL_s$, which causes the nonmonotonicity of $m^*$ in $\alpha$.

Finally, we examine the impact of $\alpha$ and $k_H/k_L$ on the optimal policy parameters.

**Proposition 7** (Shipping fee and free-shipping threshold under FCP). (*a*) *The optimal shipping fee $S^*$ strictly decreases with $\alpha$ and $k_H$ under $H_tL_s$ and $H_sL_s$.* (*b*) *The optimal free-shipping threshold $\tau^*$ strictly decreases with $\alpha$ but strictly increases with $k_H$ under $H_tL_s$; $\tau^*$ is a constant under $H_fL_t$.*

As discussed earlier, shipping revenue is complementary to sales revenue. Proposition 6 implies that $m^*$ increases with $\alpha$ and $k_H$ under $H_tL_s$ and $H_sL_s$. As a result, the optimal shipping fee $S^*$ decreases with $\alpha$ and $k_H$. On the other hand, since a higher $m^*$ discourages consumers from topping up, the e-grocer has to lower the free-shipping threshold to make the policy applicable. Thus, $\tau^*$ decreases with $\alpha$ under $H_tL_s$. Note that $k_H$ measures the valuation of high-type consumers. Under $H_tL_s$, the high-type consumers are induced to top up. Thus, the higher the $k_H$, the greater the free-shipping threshold the e-grocer can set for the high-type consumers. The positive impact of an increased $k_H$ offsets the negative impact of an increased profit margin on the free-shipping threshold. Hence, $\tau^*$ increases with $k_H$ under $H_tL_s$. However, under $H_fL_t$, since the high-type consumers already qualify for free shipping, a higher $k_H$ has no impact on the free-shipping threshold. In this case, $\tau^*$ is chosen to extract all surplus from the low-type and is therefore unrelated to demand distribution $\alpha$.

*Impact of internal shipping cost*

Next, we explore the role of the e-grocer's internal shipping cost in the e-grocer's policy design. The next proposition shows the impact of $c_{\mathsf{h}}$ on the optimal policy.

**Proposition 8.** *The thresholds $F_\alpha^1$, $F_\alpha^2$, and $F_\alpha^3$ decrease with $c_{\mathsf{h}}$ for $c_{\mathsf{h}} \le \frac{k_L}{4}$ and $F_\alpha^1 = F_\alpha^2 = F_\alpha^3 = 0$ for $c_{\mathsf{h}} \in (\frac{k_L}{4}, \frac{k_H}{4}]$.*

Previously, we assumed $c_{\mathsf{h}} \le \frac{k_L}{4}$ to ensure that the e-grocer can earn a nonnegative profit from the low-type consumers. Proposition 8 implies that the e-grocer should focus more on the high-type consumers when the internal shipping cost is higher. That is, $H_{only}$ is more likely to become optimal. Moreover, when $c_{\mathsf{h}}$ is even higher (i.e., $c_{\mathsf{h}} \in (\frac{k_L}{4}, \frac{k_H}{4}]$), the e-grocer should give up the low-type consumers and the optimal policy must be $H_{only}$. Correspondingly, $F_\alpha^1 = F_\alpha^2 = F_\alpha^3 = 0$. If $c_{\mathsf{h}} > \frac{k_H}{4}$, the internal shipping cost is too high for the e-grocer to run a profitable business.

We are also interested in whether the firm can recover the internal shipping and handling cost from the shipping revenue. We state the results in the following proposition.

**Proposition 9** (Role of internal shipping cost). *When a flat-rate policy ($H_sL_s$ or $H_sL_\varnothing$) is optimal, the shipping revenue always exceeds the internal shipping cost; when $H_tL_s$ is optimal, there exists a threshold $\hat{F}_\alpha \in [F_\alpha^2, F_\alpha^1]$ such that the shipping revenue cannot recover the internal shipping cost for $\alpha \in (\hat{F}_\alpha, F_\alpha^1]$. Moreover, $\hat{F}_\alpha$ is strictly decreasing in $c_{\mathsf{h}}$ for $\hat{F}_\alpha \in (F_\alpha^1, F_\alpha^2)$.*

We illustrate the result in Figure 2. The shaded region in Figure 2 indicates the case where the e-grocer's shipping revenue is less than its internal shipping cost. Proposition 9 states that when the firm employs a flat-rate shipping policy, its shipping revenue is always sufficient to cover the internal shipping cost. Otherwise, the e-grocer cannot be profitable. However, when the optimal policy is $H_tL_s$, the firm may

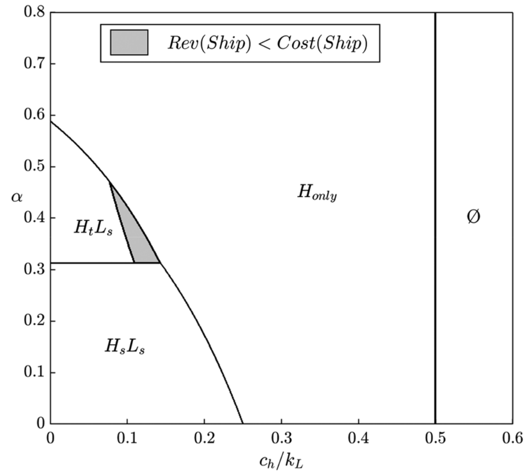



**FIGURE 2** Optimal policy with respect to $\alpha$ and $c_{\mathsf{h}}/k_L$ for $k_H = 2k_L$. "∅" denotes the region where the firm's profit is negative under any shipping policy.

suffer a loss in shipping in order to capitalize on the high-type consumers' top-up behavior. This happens when $\alpha$ falls into the region $(\hat{F}_\alpha, F_\alpha^1]$ with $\hat{F}_\alpha \in (F_\alpha^1, F_\alpha^2)$. Notice that as $\alpha$ increases, the proportion of low-type consumers decreases at the same time, and so does the shipping revenue collected from the low-type consumers. Hence, the shipping revenue may be lower than the shipping cost once $\alpha$ becomes sufficiently large. In contrast to Leng and Becerril-Arreola (2010) that do not endogenize the shipping fee decisions and assume a linear relationship between the firm's shipping cost and shipping revenue, we identify the conditions under which the firm can recover its internal shipping cost by endogenizing the shipping fee decision.

*Value of top-up policies/option*

We have seen that the CFS policies with top-up options, including $H_tL_s$ and $H_fL_t$, can be strictly more profitable than other policies. It would be interesting to ask: *How much benefit can consumers' top-up behavior bring compared with some benchmark policies?* Specifically, we compare the optimal CFS policy with two benchmarks: the (optimal) flat-rate policy $H_sL_s$ and the unconditional free shipping policy $H_fL_f$.

Figure 3 presents the ratios $\frac{\Pi^* - \Pi_{H_sL_s}}{\Pi_{H_sL_s}}$ (dashed line) and $\frac{\Pi^* - \Pi_{H_fL_f}}{\Pi_{H_fL_f}}$ (solid line), where $\Pi^*$ is the e-grocer's profit under the optimal policy and $\Pi_j$ is the profit under policy $j$, $j \in \{H_sL_s, H_fL_f\}$. We observe that, by using a slightly more sophisticated shipping policy and providing consumers with an option to top up their orders for free shipping, the e-grocer can boost its profit by as high as 150% compared to the simple flat-rate shipping policy. This result is particularly pronounced when both consumer heterogeneity and the proportion of high-type consumers are high. Moreover, the optimal policy could triple or even quadruple the e-grocer's profit, compared to the free shipping policy $H_fL_f$. That is, the e-grocer could lose substantial profit if it forgoes the entire

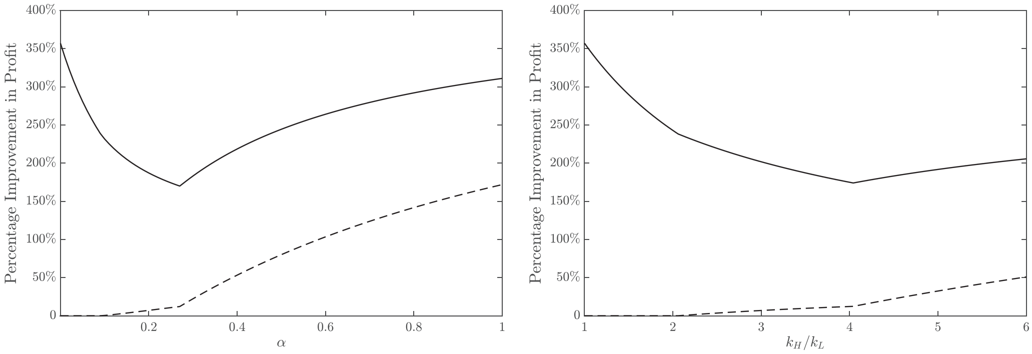



**FIGURE 3** Percentage improvement in profit between the optimal CFS policy and $H_sL_s$ (dashed line) and between the optimal CFS policy and $H_fL_f$ (solid line) with respect to $\alpha$ for $\frac{k_H}{k_L} = 4.5$ and $\frac{c_h}{k_L} = 0.01$ (left) and with respect to $\frac{k_H}{k_L}$ for $\alpha = 0.3$ and $\frac{c_h}{k_L} = 0.01$ (right).

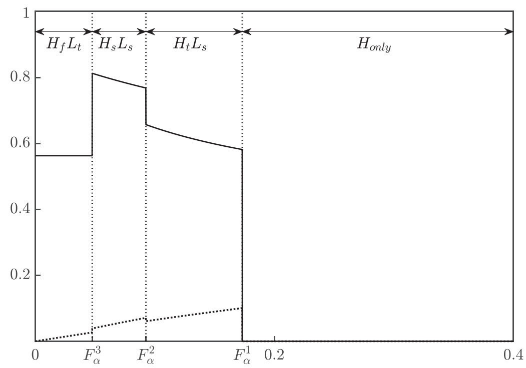



**FIGURE 4** A representative high-type consumer's net utility (solid line) and the total consumer surplus (dotted line) with respect to $\alpha$ for $k_L = 1$, $\frac{k_H}{k_L} = 4.5$, and $\frac{c_h}{k_L} = 0.1$.

shipping revenue by always allowing consumers to ship for free. Furthermore, the improvement in profit will be magnified when the shipping and handling cost $c_h$ increases. In sum, the CFS policy with a top-up option enables the e-grocer to improve its profitability significantly.

*Consumer surplus*

Lastly, we explore the capability of the optimal policy in extracting consumer surplus. Clearly, such capability depends on the form of the policy, which, in turn, depends on $\alpha$, as depicted in Proposition 9. Figure 4 presents the surplus (i.e., the net utility) of each high-type consumer (solid line) and the total consumer surplus (dotted line) with respect to $\alpha$.

*Surprisingly, allowing free shipping may hurt the consumer surplus but benefit the e-grocer*. In particular, when $\alpha$ is close to 0, the low-type consumers dominate the market, and it is easier for the e-grocer to extract consumer surplus using $H_fL_t$. Similarly, when $\alpha$ is sufficiently large, $H_{only}$ becomes optimal, and the high-type's net utility drops to zero. When $\alpha$ is moderate (i.e., consumers are more mixed), it becomes more difficult for the e-grocer to extract surplus, thereby leaving more surplus to the high-type under $H_sL_s$ or $H_tL_s$. Unlike $H_tL_s$, $H_sL_s$ does not discriminate between consumers well as it charges both types of consumers the same shipping fee. Therefore, $H_sL_s$ is the least effective in extracting consumer surplus. This variation in the capability of different forms of policies in extracting consumer surplus helps explain the nonmonotonicity and discontinuity in both the individual high-type's surplus and the total consumer surplus in Figure 4.

## 4.3 | Channel choice

In this section, we assume that there exists an offline channel (i.e., a brick and mortar grocery store) that competes with the e-grocer. We study the competition between this offline channel and the e-grocer. We continue to use the square root valuation function in this section.

First, we derive the consumer net utility of purchasing via the offline channel. We assume that the offline channel has a profit margin of $\hat{m}$. We further assume that a consumer incurs an inconvenience fixed cost $F$ (such as traveling or waiting-in-line cost) when purchasing via the offline channel. Without loss of generality, $F$ can also denote the difference between the offline inconvenience cost and the online disutility cost (e.g., the cost of being unable to inspect the products in person).

Let $U_{i,o}(y)$ be type $i$'s consumer net utility in purchasing an order of basket size $y$ via the offline channel. Then, we have

$$U_{i,o}(y) = \sqrt{k_i y} - \frac{y}{1-\hat{m}} - F. \tag{5}$$

Let $U_{i,o}^{\max}$ denote the maximum net utility a type $i$ consumer can have via the offline channel. Clearly, the maximizer of $U_{i,o}(y)$ is $y_{i,o}^{\mathrm{u}} = \frac{(1-\hat{m})^2 k_i}{4}$. Consequently, $U_{i,o}^{\max} = \frac{(1-\hat{m})k_i}{4} - F$.

In reality, e-grocers may adjust prices more often than offline grocers due to operational efficiency. For example, Hillen and Fedoseeva (2021) find that Amazon Fresh frequently adjusts the prices of food products, while Whole Foods continues to apply the traditional "sticky" retail pricing scheme despite the acquisition by Amazon. Therefore, we model the competition between an e-grocer and an offline channel by a Stackelberg game. Specifically, as the leader, the offline channel first sets the profit margin $\hat{m}$. Subsequently, the e-grocer, as the follower, determines its profit margin and shipping policy. Finally, the consumers choose which channel to purchase from and the basket size.

We solve the problem backwards. We denote the equilibrium profit margin of the offline channel by $\hat{m}^\star$ and the associated profit by $\Pi_o^\star$. We further denote the equilibrium decisions of the e-grocer by $(m^\star, \tau^\star, S^\star)$ and the e-grocer's equilibrium profit by $\Pi^\star$.

Given the offline channel's profit margin $\hat{m}$ and inconvenience cost $F$, the e-grocer determines a policy $(m, \tau, S)$ to maximize its own profit. The e-grocer's profit functions under various CFS policies have been given in Equations (3) and (4). As discussed in Section 3.2, if a consumer chooses to purchase online, the consumer either pays a flat-rate shipping fee $(a = s)$, or tops up the order to meet the free-shipping threshold $\tau$ $(a = t)$, or enjoys free shipping $(a = f)$. The corresponding net utility, denoted by $U_i^a, a \in \{s, t, f\}$, is characterized as follows:

$$U_i^a = \begin{cases} U_i^s = \dfrac{(1-m)k_i}{4} - S, & \text{if } \tau > \dfrac{(1-m)k_i}{4} \\ U_i^t = \sqrt{(1-m)\tau k_i} - \tau, & \text{if } \tau > \dfrac{(1-m)k_i}{4} \\ U_i^f = \dfrac{(1-m)k_i}{4}, & \text{if } \tau \le \dfrac{(1-m)k_i}{4} \end{cases}. \tag{6}$$

In the presence of the offline channel, the consumer chooses to shop online if and only if $\max\{U_i^s, U_i^t, U_i^f\} \ge U_{i,o}^{\max}$. Clearly, the competition from the offline channel makes the online channel less attractive if $U_{i,o}^{\max} > 0$. In response, the e-grocer should adjust its (original) shipping policy. The e-grocer's best response function can be derived similarly to the case without the offline channel, and we omit the details.

By anticipating the e-grocer's best response, the offline channel determines the profit margin $\hat{m}$ to maximize its profit. The offline channel's profit function can be written as $\Pi_o = \hat{m}\frac{y_o}{1-\hat{m}}$, where $y_o$ indicates the average basket size purchased from the offline channel. If the e-grocer can serve both consumer segments, then the offline channel has zero demand, and $y_o = 0$. If the e-grocer's best response policy is $H_{only}$ (i.e., the e-grocer only serves the high-type consumers), then the offline channel may earn a positive profit by serving the low-type consumers. In this case, $y_o = (1-\alpha)y_{L,o}^{\mathsf{u}} = \frac{(1-\alpha)(1-\hat{m})^2 k_L}{4}$. Of course, the offline channel can set a sufficiently low-profit margin to drive the e-grocer out of the market. In this case, no consumer prefers purchasing from the e-grocer, and the e-grocer has zero demand. Then, $y_o = \alpha y_{H,o}^{\mathsf{u}} + (1-\alpha)y_{L,o}^{\mathsf{u}} = \frac{(1-\hat{m})^2(\alpha k_H + (1-\alpha)k_L)}{4}$.

The following proposition characterizes the condition under which the e-grocer is either forced out of the market or its equilibrium policy is $H_{only}$.

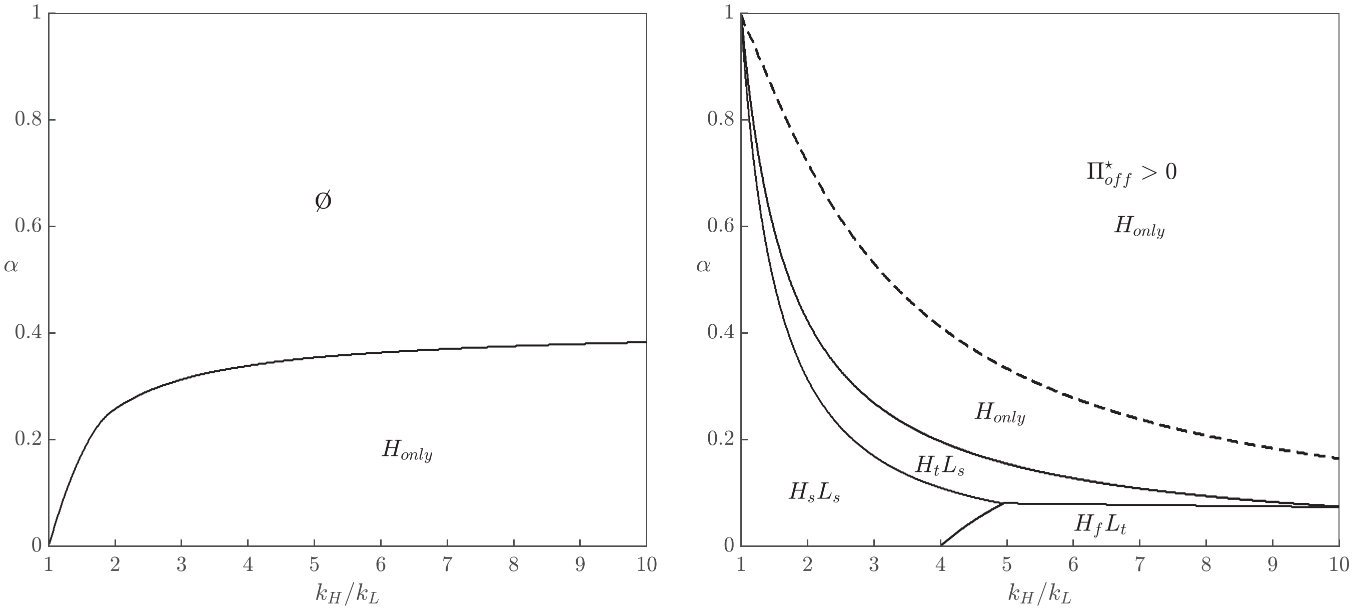
**Proposition 10.** *The e-grocer either serves no consumers or only the high-type consumers in equilibrium when $c_{\mathsf{h}} > F$.*

Proposition 10 suggests that the e-grocer cannot serve both types of consumers in equilibrium as long as $c_{\mathsf{h}} > F$. Specifically, a large internal shipping cost $c_{\mathsf{h}}$ prevents the e-grocer from profiting from both types of consumers. On the other hand, when the inconvenience cost $F$ is small, the offline channel is highly competitive. Thus, the offline channel can price low to attract the low-type consumers or even both consumer segments and drive the e-grocer out of the market.

Figure 5 depicts the e-grocer's equilibrium shipping policy. In Figure 5a, $F < c_{\mathsf{h}}$. We observe that the e-grocer either serves no consumer or only the high-type consumers in equilibrium, which confirms Proposition 10. Specifically, when $F$ is small, the offline channel serves the entire market when the proportion of high-type consumers $\alpha$ is significant. When $\alpha$ is sufficiently small, the offline channel is willing to give up the high-type segment, and the e-grocer's equilibrium policy is $H_{only}$.

As $F$ increases, the offline channel (e-grocer) becomes less (more) competitive. Once $F$ becomes sufficiently big, as shown in Figure 5b, the e-grocer can serve both types of consumers using an $H_sL_s$, $H_tL_s$, or $H_fL_t$ policy, leaving the offline channel zero demand (thereby zero profit) in equilibrium. Only if both demand distribution ($\alpha$) and consumer heterogeneity in valuation ($k_H/k_L$) are sufficiently large, the market is shared: the e-grocer serves the high-type consumers while the offline channel serves the low-type. This scenario is reflected by the upper right region above the dotted line in Figure 5b. In this case, the proportion of high-type consumers ($\alpha$) is significant. Moreover, due to their higher valuation of each unit basket size ($k_H/k_L$), the high-type consumers would purchase a much greater basket size than the low-type. As a result, the e-grocer would rather focus on the high-type consumers only and leave the low-type consumers to the offline channel.

Moreover, we compare Figure 5b with Figure 1 (the case without an offline channel). We observe that once $F$ becomes sufficiently big such that the e-grocer can serve both types of consumers, the presence of the offline channel does not affect the structure of the e-grocer's shipping policy. That is, the e-grocer's equilibrium shipping policy is *exactly the same as* its optimal policy without the offline competition. We further find in Figure 6 that when the e-grocer's equilibrium policy is $H_sL_s$, $H_tL_s$, or $H_fL_t$, the e-grocer's equilibrium policy parameters $(m^\star, \tau^\star, S^\star)$ are *exactly the same as* the optimal policy parameters $(m^*, \tau^*, S^*)$ without an offline channel. Only when the e-grocer's equilibrium policy is $H_{only}$, may the offline channel stay in the market and serve the low-type consumers. In response to the competition, the e-grocer reduces its profit margin and shipping fee under $H_sL_\varnothing$ or its free-shipping threshold under $H_tL_\varnothing$. This




(a) $F = 0.01k_L$ (b) $F = 0.1k_L$

**FIGURE 5** Comparison between the e-grocer's optimal policies for $c_h = 0.1k_L$. "∅" indicates the region that the e-grocer's profit is negative under any shipping policy. In (b), the offline channel earns a positive profit only in the region above the dashed line.

case also falls within the region above the dotted line in Figure 5b.

In sum, when the offline inconvenience cost $F$ is small, the offline channel is highly competitive so that it can either price the e-grocer out of the market or share the market with the e-grocer by giving up the high-type consumers. Correspondingly, the e-grocer either makes zero profit or adopts $H_{only}$, earning a positive profit from the high-type consumers. As $F$ increases, the e-grocer becomes more competitive and can eventually attract both consumer segments. When the e-grocer serves both types of consumers, the offline channel gets zero profit and has no impact on the e-grocer's policy decisions. The e-grocer's equilibrium policy is identical to its optimal policy without the offline channel. Only when both demand distribution ($\alpha$) and consumer heterogeneity in valuation ($k_H/k_L$) are sufficiently large, the e-grocer would prefer focusing on high-type consumers only. In this case, the offline channel can earn a positive profit by serving the low-type consumers.

## 5 | EXTENSIONS

We consider the following extensions in this section: the impact of consumers' psychological disutility about shipping fees in Section 5.1, and the profitability of subscription shipping programs in Section 5.2.

### 5.1 | Disutility with the shipping fee

Consumer surveys show that 74% of U.S. shoppers deem free shipping as the most critical factor when shopping online (UPS, 2017). Moreover, 35.7% of U.S. online shoppers' cart abandonments occur when consumers see shipping costs (Meola, 2016; Statista, 2018). These findings are consistent with the theory of Thaler (1985), who suggests that the price partitioning strategy creates a greater mental loss. It is reasonable to argue that most consumers associate a psychological disutility with their shopping experience if they have to pay a positive shipping fee.

In this section, we analyze the optimal CFS policy in the presence of the shipping fee disutility, denoted by $\mathcal{D}(S)$, in a market with heterogeneous consumers. We do not impose *any* structural restrictions on $\mathcal{D}(S)$, other than that $\mathcal{D}(S)$ is weakly increasing in $S$, $\mathcal{D}(S) > 0$ for all $S > 0$, and $\mathcal{D}(0) = 0$. This way, we can model various alternative forms of shipping fee disutility. For example, when $\mathcal{D}(S) = D$, all consumers experience the same fixed disutility in the mere presence of a shipping fee. When $\mathcal{D}(S) = D \cdot S$, the shipping fee disutility is proportional to $S$. Our main results and insights in this section are robust to the forms of disutility. Assuming the same square root valuation functions as in Section 4.2, the net utility of type $i$ consumers who choose action $a_i = s$ becomes $\frac{k_i(1-m)}{4} - S - \mathcal{D}(S)$, $i \in \{L, H\}$.

Before stating the main results in this section, we first illustrate how the optimal policy varies with $\mathcal{D}(S)$ using a numerical example. We consider the case where $\mathcal{D}(S) = D \cdot S$. Figure 7 plots the optimal shipping policy with respect to $\alpha$ and the ratio $\frac{k_H}{k_L}$, when $\frac{c_h}{k_L} = 0.1$. We set $D$ to different (positive) values in Figure 7a–c and use dotted lines to indicate the boundaries between different optimal policies in the benchmark case (i.e., $D = 0$), where we assume that consumers are fully rational with respect to shipping fees.

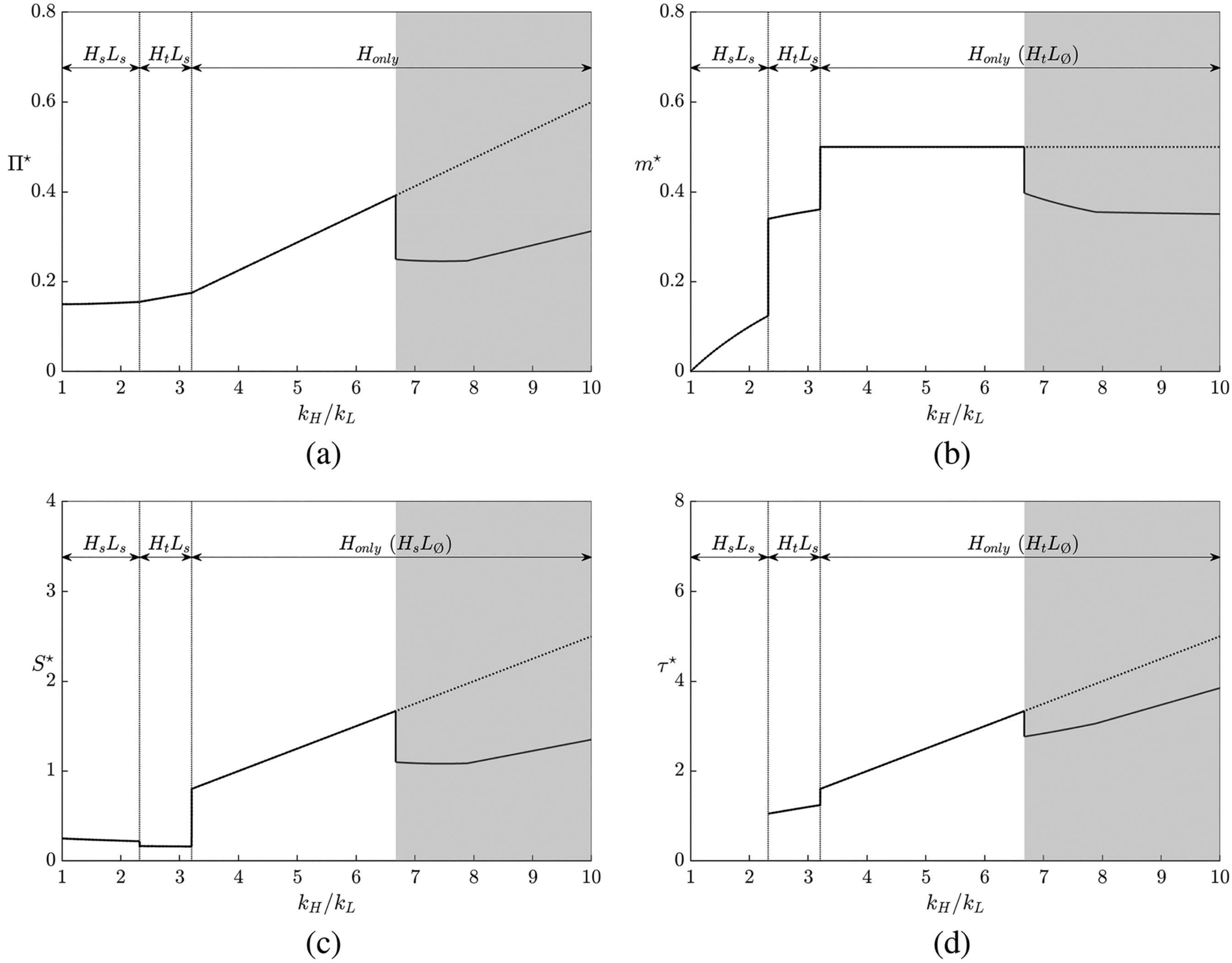


**FIGURE 6** The e-grocer's equilibrium profit and policy parameters with the offline channel (solid line) and without the offline channel (dotted line) for $k_L = 1, k_H = 4.5, \alpha = 0.25, c_{\mathsf{h}} = 0.1$, and $F = 0.1$. The gray region in each plot indicates that the market is shared between the e-grocer and the offline channel.

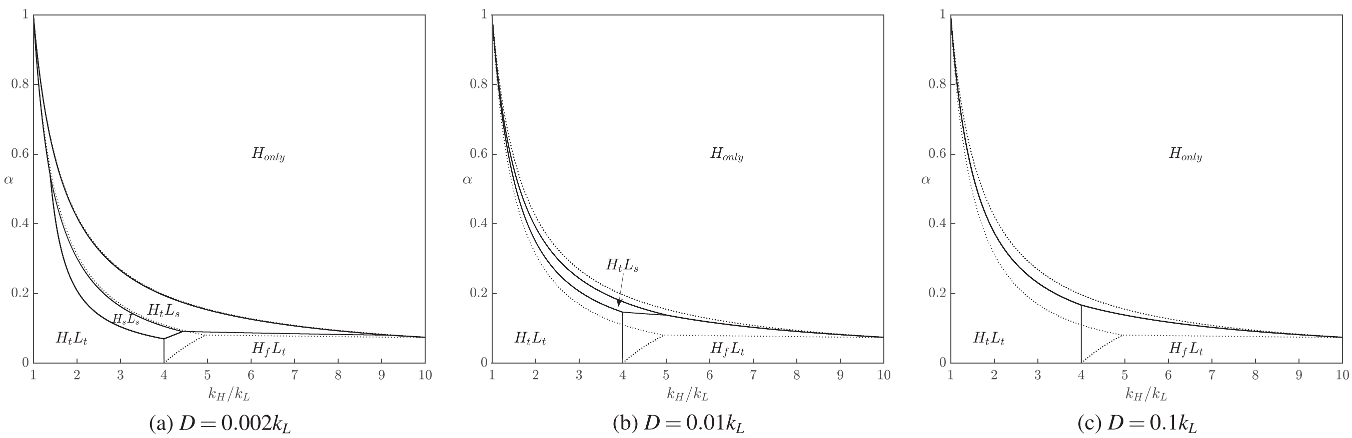


**FIGURE 7** Comparison between the optimal policies with and without the disutility $\mathcal{D}(S^*) = D \cdot S^*$ for $\frac{c_{\mathsf{h}}}{k_L} = 0.1$, where $S^*$ denote the flat-rate shipping fee in the optimal policy.

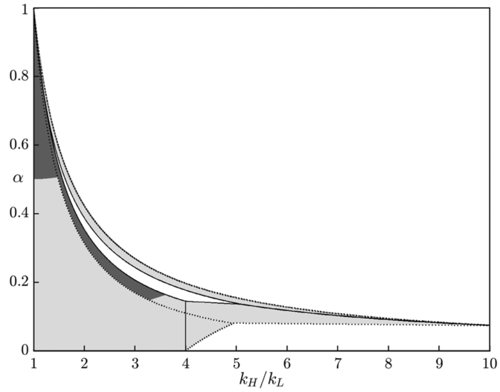



**FIGURE 8** High-type's consumer surplus under the optimal policy with and without the shipping fee disutility under the setting of $\mathcal{D}(S) = D$, $\frac{c_h}{k_L} = 0.1$ and $\frac{D}{k_L} = 0.015$.

Several phenomena deserve our attention. First, we observe from Figure 7a–c that $H_tL_t$ may become optimal in the presence of the shipping fee disutility. This is contrary to Proposition 3, which states that in the absence of the shipping fee disutility, $H_tL_t$ is dominated by $H_sL_s$ and thereby can never be optimal. Second, we observe that $H_tL_t$ and $H_fL_t$ become more attractive as $D$ increases. The region where either $H_tL_t$ or $H_fL_t$ is optimal gradually replaces those for $H_sL_s$ and $H_tL_s$. This observation implies that the shipping-fee disutility makes the top-up option more favorable to both types of consumers. We state these findings formally in the following proposition.

**Proposition 11** (Optimal policy with shipping-fee disutility).

(1) *When* $\frac{k_H}{k_L} \in (1,4)$, *$H_tL_t$ dominates $H_sL_s$ if* $\mathcal{D}(\check{S}) > \frac{\alpha^2(k_H-k_L)^2}{16(\alpha(k_H-k_L)+k_L)}$, *where $\check{S}$ is the shipping fee under $H_sL_s$.*
(2) *$F^1_\alpha$ is (weakly) decreasing in $\mathcal{D}(\cdot)$, while $F^3_\alpha$ is (weakly) increasing in $\mathcal{D}(\cdot)$.*

Proposition 11(1) shows that $H_tL_t$ could instead dominate the flat-rate policy $H_sL_s$ when the shipping-fee disutility is sufficiently large. Moreover, recall from Theorem 4 that $F^\alpha_1$ is the threshold between $H_{only}$ and $H_tL_s$, and $F^\alpha_3$ is the threshold between $H_sL_s$ and $H_fL_t$. Proposition 11(2) implies that as the shipping-fee disutility increases, the two policies $H_sL_s$ and $H_tL_s$, which induce at least one type of consumer to pay shipping fees, become less attractive in the presence of the shipping-fee disutility.

Lastly, we investigate the impact of the shipping fee disutility on consumer surplus. Interestingly, we find that the total consumer surplus may improve in the presence of the disutility. Note that the low-type consumers always get zero surpluses under the optimal policy. Thus, to illustrate our finding, we simply compare the high-type's consumer surplus under the optimal policy with and without disutility in Figure 8. The dark (light) gray area indicates the region where the high-type consumers get more (less) surplus in the presence of the shipping fee disutility. The nonshaded regions indicate that the high-type's surplus stays the same. The dotted lines represent the boundaries between different optimal policy structures in the benchmark case (i.e., $D = 0$).

Figure 8 shows that the high-type consumers may enjoy a higher surplus in the presence of the shipping fee disutility. This could happen especially when the optimal policy switches from $H_sL_s$ to $H_tL_t$ and $\alpha$ is large. Note that the optimal profit margin under $H_tL_t$ is a constant, while the optimal profit margin under $H_sL_s$ is increasing in $\alpha$. Thus, when $\alpha$ is large, consumers retain a higher surplus under $H_tL_t$ than under $H_sL_s$ even without the shipping fee disutility.

Our findings accentuate the advantage of CFS policies over flat-rate policies in the presence of the shipping-fee disutility. The CFS policies empower the e-grocer with more levers to extract greater consumer surplus by inducing consumers to top up their order size. Meanwhile, consumers may *benefit* from an e-grocer's CFS policy, as their surplus may increase in the presence of disutility.

## 5.2 | Subscription shipping programs

Many online grocers offer subscription service programs that provide members with unlimited free shipping for their online orders. For example, Instacart, an online grocery delivery company fast-growing during the pandemic, offers a subscription program called Instacart Express. Members pay \$99 per year for free shipping on eligible purchases. In September 2020, Walmart launched the Walmart Plus membership program which charges an annual fee of \$98 for grocery delivery.

In this section, we consider the impact of such subscription shipping programs on the e-grocer's shipping policy design. In particular, we consider the situation where an online grocer has four levers: the profit margin $m$, the flat-rate shipping fee $S$, the free-shipping threshold $\tau$, and the subscription membership fee $P$. We assume that the same margin $m$ applies to both members and nonmembers. This assumption is generally consistent with the reality, where the platform offers each product at the same price to all consumers at the same time.

One characteristic of online grocery shopping is that purchase frequency is usually independent of the purchase quantity. While purchase quantity typically depends on household size, purchase frequency can be relatively stable as a matter of routine or buying habits. For example, market surveys show that the majority of the U.S. households shop for groceries once or twice a week (Statista, 2020). To capture this feature, we consider purchase frequency as another dimension to model consumer heterogeneity in the subscription programs. We attempt to answer the following research question: *When can a subscription program improve the e-grocer's profit?*

To better illustrate our results and insights, we employ the same square root valuation function $u_i(y) = \sqrt{k_i y}$, $i \in \{L, H\}$, as before. In addition, we assume that consumers purchase from the online grocer at either a high-frequency $f_H$ or a low-frequency $f_L$. As a result, we consider four types of shoppers $(k_H,f_H)$, $(k_H,f_L)$, $(k_L,f_H)$, and $(k_L,f_L)$, with the corresponding proportions $\gamma_{k_H f_H}$, $\gamma_{k_H f_L}$, $\gamma_{k_L f_H}$, and $\gamma_{k_L f_L}$, respectively, where $\gamma_{k_H f_H} + \gamma_{k_H f_L} + \gamma_{k_L f_H} + \gamma_{k_L f_L} = 1$.

Type $(k_i,f_j)$ consumers can choose to join the subscription program by paying a lump sum fee $P$ and enjoy free shipping for all their subsequent orders. We denote this consumer action by $p$. If consumers do not choose $p$, they make decisions in response to a CFS policy, as described in Section 3.2. Let $a_{i,j} \in \{\emptyset, s, t, f, p\}$ denote the action that type $(k_i,f_j)$ consumers take. We capture the consumer net utility, denoted by $U_{(k_i,f_j)}(a_{i,j})$, under all possible scenarios as follows:

$$U_{(k_i,f_j)}(a_{i,j}) = \begin{cases} 0, & \text{if } a_{i,j} = \emptyset \\ f_j\left(\dfrac{k_i(1-m)}{4} - S \times \mathbf{1}_{\frac{k_i(1-m)}{4} < \tau}\right), & \text{if } a_{i,j} = s \\ f_j(\sqrt{k_i(1-m)\tau} - \tau), & \text{if } a_{i,j} = t \\ f_j \dfrac{k_i(1-m)}{4}, & \text{if } a_{i,j} = f \\ f_j \dfrac{k_i(1-m)}{4} - P, & \text{if } a_{i,j} = p \end{cases} \tag{7}$$

We find that the distribution of consumer types plays an important role in the profitability of a subscription program. We consider two scenarios to illustrate our results. First, we consider the case where the consumer distribution with respect to basket size is independent of that with respect to order frequency. Specifically, recall that $\alpha$ denotes the proportion of $k_H$ and $1-\alpha$ the proportion of $k_L$ consumers. We further let $\beta$ denote the proportion of $f_H$ consumers and $1-\beta$ the proportion of $f_L$ consumers. Due to the independence between basket size and order frequency, we have $\gamma_{k_H f_H} = \alpha\beta$, $\gamma_{k_H f_L} = \alpha(1-\beta)$, $\gamma_{k_L f_H} = (1-\alpha)\beta$, and $\gamma_{k_L f_L} = (1-\alpha)(1-\beta)$. The next proposition shows that the subscription program should not be introduced in this case.

**Proposition 12** (When subscription is unprofitable). *Suppose that the consumer distribution follows* $(\gamma_{k_H f_H}, \gamma_{k_H f_L}, \gamma_{k_L f_H}, \gamma_{k_L f_L}) = (\alpha\beta, \alpha(1-\beta), (1-\alpha)\beta, (1-\alpha)(1-\beta))$, *where* $\alpha, \beta \in (0,1)$. *Then, an e-grocer can never earn higher profits by introducing a subscription program.*

In the above setting, since the consumer distributions with respect to basket size and order frequency are unrelated, we can consider consumers with order frequencies $f_H$ and $f_L$ separately. Note that, if type $(k_i,f_j)$ consumers join the shipping program, their net utility can be rewritten as $f_j(\frac{k_i(1-m)}{4} - \frac{P}{f_j})$. In other words, we may view the term $\frac{P}{f_j}$ as an alternative "shipping fee" and correspondingly view the subscription program as an alternative policy $(m, \tau, \frac{P}{f_i})$. However, this alternative policy can never outperform the optimal CFS policy. Therefore, the e-grocer cannot increase its profit by introducing a subscription program.

Proposition 12 is in sharp contrast to Fang et al. (2021) who show that the introduction of a subscription service always improves an e-tailer's profit. The key reason is that in our model all the policy parameters $(m, \tau, S)$ are endogenous. Since the e-grocer already has three levers to manipulate, an additional lever such as a subscription service has a limited impact on the firm's profit. In contrast, Fang et al. (2021) show that the cost of subscription membership, as an additional decision variable to the pricing decision, is much more likely to improve the firm's profit.

Next, we consider the case where the consumer distributions with respect to basket size and order frequency are correlated. We show that introducing the subscription program in this case may improve the e-grocer's profit. To this end, we explore a special case where $\gamma_{k_H f_H} = 0$, $\gamma_{k_H f_L} = \gamma$, $\gamma_{k_L f_H} = (1-\gamma)$, and $\gamma_{k_L f_L} = 0$ for some $\gamma \in (0,1)$. That is, there are only two types of consumers under this setting. Type $(k_L,f_H)$ consumers have a smaller intrinsic basket size but a greater order frequency, and type $(k_H,f_L)$ consumers have a greater intrinsic basket size but a smaller order frequency. In this case, the e-grocer can be better off by introducing the subscription program, as stated in Proposition 13.

**Proposition 13** (When subscription is profitable). *Suppose that the consumer distribution follows* $(\gamma_{k_H f_H}, \gamma_{k_H f_L}, \gamma_{k_L f_H}, \gamma_{k_L f_L}) = (0, \gamma, 1-\gamma, 0)$ *for* $\gamma \in (0,1)$ *and* $k_H f_L > k_L f_H$. *When* $H_t L_s$ *is the optimal, an e-grocer can increase its profit by introducing a subscription program.*

The idea behind Proposition 13 is as follows. Suppose that the optimal subscription-free policy $(m^*, \tau^*, S^*)$ is in the form of $H_t L_s$: that is, the policy induces type $(k_H,f_L)$ consumers to top up and $(k_L,f_H)$ consumers to pay a shipping fee. Then, the e-grocer can induce type $(k_L,f_H)$ consumers to join the subscription program by setting $P = f_H S^*$. Doing so does not affect the e-grocer's profit from type $(k_L,f_H)$ consumers. Furthermore, the subscription program allows the e-grocer to raise the flat-rate shipping fee and the free-shipping threshold. By carefully choosing $S > S^*$ and $\tau > \tau^*$, the e-grocer can still induce $(k_H,f_L)$ consumers to top up, and thereby extracting more surplus from these consumers. Hence, the e-grocer can make a greater profit, thanks to the presence of the subscription program.

As we can see from the above two scenarios, the correlation of the consumer distributions with respect to shopping frequency and basket size plays an important role in the profitability of the subscription programs. Therefore, the online grocer should carefully examine the nature of its consumers' ordering behavior before introducing such a program.

## 6 | CONCLUSION AND FUTURE WORK

Shipping fee is one of the key factors that influence online shoppers' purchasing decisions. In this paper, we study online grocers' integrated contingent free-shipping policy and pricing decisions. We study two competing driving forces—the free-shipping threshold and the flat-rate shipping fee—that induce different consumer purchasing behaviors. In particular, a lower free-shipping threshold is more likely to induce consumers to top up their orders, while a lower flat-rate shipping fee hinders consumers from doing so. We characterize the optimal CFS policy and pricing decisions and the corresponding consumer surplus. Our work provides a relevant approach to understanding the phenomenon that different online grocers may adopt different forms of shipping policies and reveals important insights about online grocers' integrated operational and marketing decisions.

In particular, we find that CFS policies, via manipulating shipping fee and free-shipping threshold, serve as a more practical alternative to price discrimination for improving the firm's profit. When consumers are sufficiently heterogeneous, the e-grocer should adopt a CFS policy with a top-up option to induce differentiated buying behaviors, thereby improving its profitability. Otherwise, when consumers are more or less homogeneous, a simple flat-rate shipping policy is sufficient. We show that the form of the optimal CFS policy depends on both demand distribution and consumer heterogeneity. Moreover, we find that among different forms of CFS policies, the one that induces all consumers to pay shipping fees is the least effective in extracting consumer surplus. Surprisingly, more complicated shipping structures, such as a two-tier CFS shipping policy, do not always generate higher profits than simple CFS policies. This further justifies the popularity of simple CFS policies in practice.

In reality, many consumers associate a psychological disutility with paying shipping fees. We find that in the presence of this disutility, a policy that induces all consumers to top up may result in a win-win situation for the e-grocer and the consumers.

Lastly, we look into whether an e-grocer should introduce a subscription program that waives shipping fees for members on top of a CFS policy. We show that the profitability of a subscription program depends on the correlation between consumers' shopping frequency and basket size. Thus, online grocers should closely examine the nature of their products and consumer shopping behavior before introducing a subscription program.

Our paper focuses on a monopolistic online grocer who sells products with a homogeneous profit margin. One future research direction is to consider the competition between multiple e-grocers and investigate whether CFS policies would intensify or soften the competition. We believe that our study on the impact of consumers' top-up behavior serves as a first step for further analysis under the competitive settings, but a more complex model will be needed to avoid Bertrand competition. In addition, we would like to study the joint decisions on shipping policy and pricing for e-grocers such as Weee.com which sells products across a variety of categories with varying profit margins. Our current model needs to be carefully redesigned to ensure tractability. It will also be interesting to consider the design of delivery time and delivery window, which are two important features that influence consumer behavior in e-grocery. Last but not least, our model does not consider consumers' stockpiling and return decisions, which are less common in e-grocery. However, they serve as an interesting future research direction for e-tailing. Broadly speaking, we believe there are significant opportunities for employing analytical models to better understand firms' integrated pricing and shipping decisions.

## ACKNOWLEDGMENTS

We would like to express our gratitude to the Department Editor, Prof. Terry Taylor, the anonymous senior editor, and the reviewers for their constructive feedback and valuable suggestions. Their guidance has been instrumental in helping us improve the quality of this manuscript.

All authors contributed equally in this research.

## ORCID

*Guang Li* https://orcid.org/0000-0002-8549-5473
*Dongyuan Zhan* https://orcid.org/0000-0001-8242-1332

## REFERENCES

Acosta. (2021). *The growth of online grocery shopping shows no signs of slowing down*. https://www.acosta.com/news/new-acosta-report-explores-the-current-and-future-growth-of-online-grocery-shopping

Ali, F. (2021). *Charts: How e-commerce and small businesses were affected by COVID-19*. https://www.digitalcommerce360.com/2021/02/19/ecommerce-during-coronavirus-pandemic-in-charts/

Anderson, S. P., De Palma, A., & Thisse, J.-F. (1992). *Discrete choice theory of product differentiation*. MIT Press.

Aull, B., Coggins, B., Kohli, S., & Marohn, E. (2022). *The state of grocery in North America*. McKinsey & Company. https://www.mckinsey.com/industries/retail/our-insights/the-next-horizon-for-grocery-ecommerce-beyond-the-pandemic-bump

Balasubramanian, S. (1998). Mail versus mall: A strategic analysis of competition between direct marketers and conventional retailers. *Marketing Science*, *17*(3), 181–195.

Basu, A. K., Lal, R., Srinivasan, V., & Staelin, R. (1985). Salesforce compensation plans: An agency theoretic perspective. *Marketing Science*, *4*(4), 267–291.

Becerril-Arreola, R., Leng, M., & Parlar, M. (2013). Online retailers' promotional pricing, free-shipping threshold, and inventory decisions: A simulation-based analysis. *European Journal of Operational Research*, *230*(2), 272–283.

Belavina, E., Girotra, K., & Kabra, A. (2017). Online grocery retail: Revenue models and environmental impact. *Management Science*, *63*(6), 1781–1799.

Brynjolfsson, E., & Smith, M. D. (2000). Frictionless commerce? A comparison of internet and conventional retailers. *Management Science*, *46*(4), 563–585.

Cachon, G. P., Gallino, S., & Xu, J. (2018). *Free shipping is not free: A data-driven model to design free-shipping threshold policies* (Working Paper). Wharton School, University of Pennsylvania, Philadelphia, PA.

Cachon, G. P., & Kök, A. G. (2007). Category management and coordination in retail assortment planning in the presence of basket shopping consumers. *Management Science*, *53*(6), 934–951.

Chen, C., & Ngwe, D. (2018). *Shipping fees and product assortment in online retail* (Working Paper). Harvard Business School, Boston MA.

Chun, S.-H., & Kim, J.-C. (2005). Pricing strategies in B2C electronic commerce: Analytical and empirical approaches. *Decision Support Systems*, *40*(2), 375–388.

Chung, J. W. (1994). *Utility and production functions: Theory and applications*. Wiley-Blackwell.

eMarketer. (2022). *Worldwide ecommerce forecast update 2022*. https://www.insiderintelligence.com/content/worldwide-ecommerce-forecast-update-2022

Fang, Z., Ho, Y.-C., Tan, X., & Tan, Y. (2021). Show me the money: The economic impact of membership-based free shipping programs on e-tailers. *Information Systems Research*, *32*(4), 1115–1127.

Forman, C., Ghose, A., & Goldfarb, A. (2009). Competition between local and electronic markets: How the benefit of buying online depends on where you live. *Management Science*, *55*(1), 47–57.

Forter. (2019). Infographic: Why a friction-filled online checkout process causes shopping cart abandonment. https://www.forter.com/blog/infographic-customers-wont-tolerate-friction-filled-checkout/

Gümüş, M., Li, S., Oh, W., & Ray, S. (2013). Shipping fees or shipping free? A tale of two price partitioning strategies in online retailing. *Production and Operations Management*, *22*(4), 758–776.

Hemmati, S., Elmaghraby, W. J., Kabra, A., & Jain, N. (2021). *Contingent free shipping: Drivers of bubble purchases* (Working Paper). Robert H. Smith School of Business, University of Maryland, College Park, MD.

Hillen, J., & Fedoseeva, S. (2021). E-commerce and the end of price rigidity? *Journal of Business Research*, *125*, 63–73.

Leng, M., & Becerril-Arreola, R. (2010). Joint pricing and contingent free-shipping decisions in B2C transactions. *Production and Operations Management*, *19*(4), 390–405.

Lewis, M. (2006). The effect of shipping fees on customer acquisition, customer retention, and purchase quantities. *Journal of Retailing*, *82*(1), 13–23.

Lewis, M., Singh, V., & Fay, S. (2006). An empirical study of the impact of nonlinear shipping and handling fees on purchase incidence and expenditure decisions. *Marketing Science*, *25*(1), 51–64.

Liu, Y., Gupta, S., & Zhang, Z. J. (2006). Note on self-restraint as an online entry-deterrence strategy. *Management Science*, *52*(11), 1799–1809.

Melton, J. (2019). Fast grocery delivery can win customer loyalty, but executing on it can be costly. https://www.digitalcommerce360.com/2019/01/11/fast-delivery-can-win-loyalty-for-grocery-retailer-but-costs-are-unsustainable/

Meola, A. (2016). *E-commerce retailers are losing their customers because of this one critical mistake*. Business Insider. http://www.businessinsider.com/e-commerce-shoppers-abandon-carts-at-payment-stage-2016-3

Morwitz, V. G., Greenleaf, E. A., & Johnson, E. J. (1998). Divide and prosper: consumers' reactions to partitioned prices. *Journal of Marketing Research*, *35*, 453–463.

Schindler, R. M., Morrin, M., & Bechwati, N. N. (2005). Shipping charges and shipping-charge skepticism: Implications for direct marketers' pricing formats. *Journal of Interactive Marketing*, *19*(1), 41–53.

Shao, X.-F. (2017). Free or calculated shipping: Impact of delivery cost on supply chains moving to online retailing. *International Journal of Production Economics*, *191*, 267–277.

Statista. (2018). *Statista survey online-shopping in the U.S. 2018*. https://www.statista.com/forecasts/961961/statements-regarding-the-shipping-of-online-orders-for-us-consumers

Statista. (2020). *Consumers' weekly grocery shopping trips in the united states from 2006 to 2019*. https://www.statista.com/statistics/251728/weekly-number-of-us-grocery-shopping-trips-per-household/

Statista. (2022). *Retail e-commerce revenue in the united states from 2017 to 2027 (in billion U.S. dollars)*. https://www.statista.com/statistics/272391/us-retail-e-commerce-sales-forecast/

Thaler, R. (1985). Mental accounting and consumer choice. *Marketing Science*, *4*(3), 199–214.

UPS. (2017). *UPS pulse of the online shopper™*. https://www.ups.com/assets/resources/media/knowledge-center/ups-pulse-of-the-online-shopper.PDF

U.S. Census Bureau. (2019). *Quarterly retail e-commerce sales*. https://www2.census.gov/retail/releases/historical/ecomm/19q4.pdf

U.S. Census Bureau. (2022). Quarterly retail e-commerce sales. https://www2.census.gov/retail/releases/historical/ecomm/22q4.pdf

Viswanathan, S. (2005). Competing across technology-differentiated channels: The impact of network externalities and switching costs. *Management Science*, *51*(3), 483–496.

Wang, R., Dada, M., & Sahin, O. (2019). Pricing ancillary service subscriptions. *Management Science*, *65*(10), 4712–4732.

Xu, J. (2016). *Empirical studies in online retail operations and dynamic pricing* [Unpublished doctoral thesis]. University of Pennsylvania.

Yang, Y., Essegaier, S., & Bell, D. R. (2005). Free shipping and repeat buying on the internet: theory and evidence (Working Paper). Wharton School, University of Pennsylvania, Philadelphia, PA.

## SUPPORTING INFORMATION

Additional supporting information can be found online in the Supporting Information section at the end of this article.

**How to cite this article:** Li, G., Sheng, L., & Zhan, D. (2023). Designing shipping policies with top-up options to qualify for free delivery. *Production and Operations Management*, 1–19. https://doi.org/10.1111/poms.14002

## APPENDIX: LIST OF KEY NOTATIONS

| | |
|---|---|
| $y$ | basket size of an order |
| $x$ | total dollar value of an order; $x = \frac{y}{1-m}$ |
| $\tau$ | free shipping threshold |
| $S$ | below-threshold flat-rate shipping fee per order |
| $m$ | profit margin, $m \in [0, 1)$ |
| $c_{\mathrm{h}}$ | e-grocer's internal shipping and handling cost per order |
| $\Pi(\cdot)$ | e-grocer's profit |
| $u(\cdot)$ | consumer valuation of consuming the products in an order |
| $U(\cdot)$ | consumer net utility of an order |
| $w(\cdot)$ | social surplus of an order |
| $k$ | valuation of unit basket size |
| $a$ | consumer action |
| subscript $i$ | consumer type |
| superscript u | quantity corresponding to the intrinsic basket size that maximizes consumer net utility |
| superscript 0 | quantity corresponding to the maximum basket size that leads to zero consumer utility |
| superscript w | quantity corresponding to the socially optimal basket size that maximizes social surplus |
| $F$ | inconvenience cost of shopping offline |
| $\hat{m}$ | profit margin of the offline channel |
| $\mathcal{D}(S)$ | shipping fee disutility as a function of the flat-rate shipping fee |
| $f$ | order frequency |
| $P$ | membership fee of a subscription shipping program |
| $\alpha$ | proportion of high-$k$ consumers |
| $\beta$ | proportion of high-frequency consumers |
| $\gamma_{k_i f_j}$ | proportion of consumers who have basket size valuation $i$ order frequency $j$ |