Oppportunistic Fluid Antenna Multiple Access
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Abstract—Multiple access can be realized by utilizing the spatial moments of deep fades, using fluid antennas. The interference immunity for fluid antenna multiple access (FAMA), nevertheless, comes with the requirement of a large number of ports at each user. To alleviate this, we study the synergy between opportunistic scheduling and FAMA. A large pool of users permits selection of favourable users for FAMA and decreases the outage probability at each selected user. Our objective is to characterize the benefits of opportunistic scheduling in FAMA. In particular, we derive the multiplexing gain of the opportunistic FAMA network in closed form and upper bound the required number of users in the pool to achieve a given multiplexing gain. Also, we find a lower bound on the required outage probability at each user for achieving a given network multiplexing gain, from which the advantage of opportunistic scheduling is illustrated. In addition, we investigate the rate of increase of the multiplexing gain with respect to the number of users in the pool, and derive a tight approximation to the multiplexing gain, expressed in closed form. As a key result of our analysis, we obtain an operating condition on the product of the number of users in the pool and the number of ports at each fluid antenna that ensures a high multiplexing gain. Numerical results demonstrate clear benefits of opportunistic scheduling in FAMA networks, and corroborate our analytical results.

Index Terms—Fluid antenna systems, Multiple Access, Multiplexing gain, Opportunistic scheduling, Spatial multiplexing.

I. INTRODUCTION

A. Context

With several generational changes, mobile communications has evolved into multi-functional communications over air that can combine with new features such as wireless power transfer [1], physical-layer security [2], mobile edge computing [3] and more. The past years have also seen overwhelming interest in applying machine learning methods in wireless communication systems, e.g., [4]–[7]. Despite the changes in emphasis on the applications, the push for greater spectral and energy efficiency has never changed.

The IMT-2020 Standard was motivated to meet the demands in three use cases with eight capabilities. One of the use cases is massive machine type communication (mMTC) which, for 5G, should provide connection density of $10^6$ devices per km$^2$ [8]. Planning ahead of time, the ITU-R Working Party 5D has already begun to develop a new draft Recommendation “IMT Vision for 2030 and beyond”. Though there has not been any consensus of what 6G will be, researchers around the world have speculated on the key performance indicators (KPIs) of 6G [9], [10]. To enable massive connectivity, multiple access technologies that allow an enormous number of users to share the same radio resource unit are of utmost importance.

Multiuser multiple-input multiple-output (MIMO), with its latest form as massive MIMO in 5G, has been a key enabling multiple access technology [11], [12]. By exploiting the channel state information (CSI) at the base station (BS), MIMO is able to differentiate and distribute the multiuser signals to the user equipments (UEs) in the downlink without interference. Nevertheless, the degree of freedom (DoF) is determined by the number of antennas at the BS, which is set by the standard and cannot be upgraded easily, not to mention the complexity and overhead associated with the CSI acquisition process and beamforming optimization. Another prevalent multiple access technology is non-orthogonal multiple access (NOMA) which promotes to overlap users even more beyond the capability of MIMO but requires successive interference cancellation (SIC) to be employed at the UEs [13], [14]. While NOMA is being portrayed as a massive connectivity technology, ironically, the majority of research is limited to only two-user cases.

Recently, it is proposed that spatial moments of deep fades can be exploited to eliminate interference and enable multiple access by a position-adjustable fluid antenna at a UE [15]. The idea is that signals intended for different UEs transmitted by distributed BS antennas undergo independent fading in space and hence, a UE equipped with high-resolution fluid antenna can access the deep fades suffered by its interference signals in space for multiple access without precoding nor SIC. Fluid antenna offers the possibility of massive connectivity with UE-led simple signal processing. Presumably, the network capacity can be increased if the resolution of fluid antenna at the UE side improves with time. It was reported that many users could be accommodated on the same time-frequency channel if the fluid antenna adapts its position on a symbol-by-symbol basis, an approach referred to as fast fluid antenna multiple access (f-FAMA) [16], [17] whereas supporting a few users on the same channel was still possible even if the fluid antenna only switched its position once in a coherence period, referred to as slow fluid antenna multiple access (s-FAMA) [18]–[20].

Fluid antennas can be implemented using liquid-based radiating structures [21], [22] or reconfigurable RF pixels [23]–[25]. Recent studies have seen many liquid-based reconfigurable antennas being designed, e.g., [26]–[30]. Latest devel-
The channels, thereby relaxing the required number of ports of the fluid antenna at each UE for multiple access.

B. Objectives

In this paper, we consider the opportunistic FAMA network where $U$ best users are selected out of a pool of $M \geq U$ users based on their signal-to-interference ratios (SIRs) for FAMA. Our main objective is to quantify the benefits of opportunistic scheduling in FAMA by studying the multiplexing gain of the network, $m$. Lower bounds of the multiplexing gain will be obtained to gain insight on how the multiplexing gain scales with $M$ and how $M$ eases the requirement of the number of ports, $N$, for the fluid antenna at each UE for suppressing the interference to obtain a given performance. We will consider both $f$-FAMA [16] and $s$-FAMA [18] in the analysis.

C. Contributions and Summary of Results

Our contributions include a number of analytical results for the performance of the opportunistic FAMA network. We use order statistics to obtain the average network outage rate and thus the multiplexing gain when the outage probability of the fluid antenna, $p$, is given. Considering the case when $M_p$ and $M(1-p)$ are reasonably large, Gaussian distribution is used to approximate the expression of the multiplexing gain, from which we investigate the impact of various system parameters such as the number of users in the pool, $M$, the number of selected users, $U$, the number of ports at each fluid antenna, $N$, and the size of the fluid antenna, $W\lambda$ where $W$ denotes the normalized length and $\lambda$ is the wavelength.

Before proceeding, we here provide an overview of the material covered in this paper and highlight our key findings for the opportunistic FAMA systems.

- First, the average outage rate of the opportunistic FAMA network can be expressed as

$$C = \sum_{u=M-U+1}^{M} I_{1-p}(M - u + 1, u) \log_2(1 + \gamma),$$

in which $I_x(a, b)$ denotes the regularized incomplete beta function, and $\gamma$ denotes the SIR threshold. Furthermore, the multiplexing gain can be found as

$$m = \sum_{u=M-U+1}^{M} I_{1-p}(M - u + 1, u),$$

which, for large $M$, can be approximated by

$$m \approx \sum_{u=M-U+1}^{M} \left[ 1 - \frac{1}{2} \text{erfc} \left( \frac{u - 1 - M_p}{\sqrt{2M_p(1-p)}} \right) \right],$$

where $\text{erfc}(\cdot)$ is the complementary error function.

- For $0.0786U < m < 0.9214U$, it can be shown that the required outage probability, $p$, achieved by the fluid antenna at each UE is lower bounded by

$$p \geq 1 - \frac{1}{M} \left( U + \alpha \sqrt{2U} \right),$$

where $\alpha$ is a constant.

1A ‘port’ refers to a fixed position to which the fluid radiating element can be switched. The number of ports dictates the resolution of a fluid antenna. Fig. 1 shows the surface-wave based fluid antenna in [32] with $N$ ports.

Fig. 1. The surface-wave based fluid antenna in [32].
where $\alpha \triangleq \operatorname{erfc}^{-1}\left(2 - \frac{m}{M}\right)$ specifies the requirement of the multiplexing gain of the network.\footnote{Note that $\alpha$ does not have a convenient physical meaning but is defined mainly to simplify the expressions. With the range of the multiplexing gain $0.0786U < m < 0.9214U$ that $\alpha$ is defined, we have $-1 < \alpha < 1$.} Clearly, $M$ has the role of reducing the burden of the fluid antenna when attempting to meet a given multiplexing gain, $m$.

- For $f$-FAMA with opportunistic scheduling, the number of ports at each UE’s fluid antenna to meet a prescribed multiplexing gain is upper bounded by

$$N \leq \frac{1}{M} \left[ \frac{(U - 1)\gamma}{1 - \mu^2} \right] (U + \alpha \sqrt{2U}),$$

where $\mu$ is the correlation parameter which depends on the size $W$ of the fluid antenna. For the $s$-FAMA case, the result becomes

$$N \leq \frac{1}{M} \left( \frac{\gamma}{1 - \mu^2} \right)^{U-1} (U + \alpha \sqrt{2U}).$$

For both FAMA scenarios, the results illustrate that the number of ports, $N$, is reduced by a factor of $M$.

- The rate of increase of the multiplexing gain, $m$, with respect to (w.r.t.) $M$ can be derived as

$$\frac{\partial m}{\partial M} = \frac{1}{2 \sqrt{2\pi}} \sum_{u=1}^{U} \frac{(1-p)M + u}{\sqrt{p(1-p)}} \left( e^{-\frac{[(1-p)(M-u)]^2}{2p}} \right).$$

For very large $M$, it can further be shown that

$$\frac{\partial m}{\partial M} \propto e^{-\frac{[(1-p)(M-u)]^2}{2p}} \sqrt{M}.$$

- A rule of thumb for operating the opportunistic FAMA network efficiently is that the product of the number of UEs in the pool $M$ and the number of ports at each UE’s fluid antenna, $N$, should satisfy

$$MN > \begin{cases} U(U-1)^\gamma & \text{if } f\text{-FAMA}, \\ U \left( \frac{\gamma}{1 - \mu^2} \right)^{U-1} & \text{if } s\text{-FAMA}. \end{cases}$$

- Finally, the multiplexing gain can be accurately approximated as

$$m \approx \begin{cases} \min \left\{ U, \frac{MN(1 - \mu^2)}{(U - 1)^\gamma} \right\} & \text{if } f\text{-FAMA}, \\ \min \left\{ U, MN \left( \frac{1 - \mu^2}{\gamma} \right)^{U-1} \right\} & \text{if } s\text{-FAMA}. \end{cases}$$

### D. Organization

The rest of the paper is organized as follows. In Section II, we introduce the model of the opportunistic FAMA network in the downlink and lay down our assumptions. Our main results will be presented in Section III while Section IV discusses the parameter selection. Section V then provides the numerical results that illustrate the benefits of the opportunistic FAMA networks. Finally, we conclude this paper in Section VI.

### II. NETWORK MODEL

#### A. FAMA

We consider a downlink system where a multi-antenna BS is transmitting messages to $U$ UEs on the same time-frequency resource unit. Each BS antenna is devoted to transmitting one UE’s signal and the BS employs standard, fixed antennas. By contrast, each UE is equipped with a fluid antenna with a linear structure of length $W\lambda$ where $\lambda$ denotes the wavelength. The fluid antenna has $N$ ports (see an example in Fig. 1), to which the fluid radiating element can be switched, making it possible to receive the signal at $N$ different, but close locations. The ports are distributed evenly over the space. For UE $u$, the received signal at the $k$-th port of fluid antenna is given by

$$r_k^{(u)} = s_u g_k^{(u,u)} + \sum_{\substack{\tilde{u}=1\atop\tilde{u} \neq u}}^U s_{\tilde{u}} \tilde{g}_k^{(u,u)} + \eta_k^{(u)},$$

in which $s_u$ represents the transmitted symbol for user $u$ with $E[|s_u|^2] = \sigma_u^2$, $\eta_k^{(u)}$ is the zero-mean complex additive white Gaussian noise (AWGN) at the $k$-th port with $E[|\eta_k^{(u)}|^2] = \sigma_\eta^2$, $g_k^{(u,u)}$ is the fading channel from the BS antenna dedicated for transmitting UE $\tilde{u}$’s signal to the $k$-th port of UE $u$, and $\tilde{g}_k^{(u,u)}$ denotes the overall sum-interference plus noise signal at a symbol instant. Here, the time index is omitted for conciseness.

The amplitude of the channel, $|g_k^{(u,u)}|$, is assumed Rayleigh distributed, with the probability density function (pdf)

$$p_{|g_k^{(u,u)}|}(r) = re^{-\frac{r^2}{\sigma_u^2}}, \text{ for } r \geq 0 \text{ with } E[|g_k^{(u,u)}|^2] = 2. \quad (2)$$

The average received signal-to-noise ratio (SNR) at each port is given by $\Gamma = \frac{2\sigma_u^2}{\sigma_\eta^2}$. As the ports can be arbitrarily close to each other, the channels $\{g_k^{(u,u)}\}_{\forall k}$ are correlated. To account for the channel correlation, we parameterize $g_k^{(u,u)}$, through a single correlation parameter $\mu$, as \footnote{Note that $\alpha$ does not have a convenient physical meaning but is defined mainly to simplify the expressions. With the range of the multiplexing gain $0.0786U < m < 0.9214U$ that $\alpha$ is defined, we have $-1 < \alpha < 1$.}

$$g_k^{(u,u)} = \left( \sqrt{1 - \mu^2} x_k^{(u,u)} + \mu x_0^{(u,u)} \right) \left( \sqrt{1 - \mu^2} y_k^{(u,u)} + \mu y_0^{(u,u)} \right), \quad k = 1, 2, \ldots, N, \quad (3)$$

where $x_0^{(u,u)}, \ldots, x_N^{(u,u)}, y_0^{(u,u)}, \ldots, y_N^{(u,u)}$ are all independent Gaussian random variables with zero mean and variance of 1. Using this model, the correlation among the ports is realized through the common random variables $x_0^{(u,u)}$ and $y_0^{(u,u)}$. The parameter $\mu$ serves to control the amount of spatial correlation between any two ports of the fluid antenna. Specifically, with a linear structure of $W\lambda$, $\mu$ can be chosen such that

$$\mu = \sqrt{2} \sqrt{1 - \frac{1}{2} \frac{3 \pi^2 W^2}{2}} - \frac{J_1(2\pi W)}{2\pi W}, \quad (4)$$

where $J_0(\cdot)$ denotes the generalized hypergeometric function and $J_1(\cdot)$ is the first-order Bessel function of the first kind.
Setting μ using (4) achieves the same mean correlation coefficient for an N-port linear structure of length $W\lambda$ [44, Theorem 1]. According to [44, Theorem 2], μ can be approximated as

$$\mu \approx \begin{cases} 1 - \frac{\pi^2W^2}{12}, & \text{for } W \leq 0.6, \\ \frac{1}{\sqrt{\pi}W}, & \text{for } W \geq 1. \end{cases}$$

(5)

Since the performance of the network will be interference-limited, we can drop the noise in our analysis. To benefit from the fluid antenna, UE $u$ can select the best port for maximizing the SIR for communications. For $f$-FAMA, it is proposed to track the sum-interference null on a symbol-by-symbol basis so that the selection can be done by [16, 17]

$$k^f_{\text{FAMA}} = \arg \max_{k \in \{1, \ldots, N\}} \left| \frac{g_k^{(u, u)}}{g_k^{(u)}} \right|^2.$$  

(6)

By contrast, $s$-FAMA only updates the ports of the users if the fading channels change. In this case, it chooses [18]

$$k^s_{\text{FAMA}} = \arg \max_{k \in \{1, \ldots, N\}} \left| \frac{g_k^{(u, u)}}{\sum_{v=1}^U g_v^{(u, u)}} \right|^2.$$  

(7)

The various challenges of performing (6) and (7) and how they may be addressed have been discussed in [15].

An important performance metric for the FAMA network is the outage probability of the resulting SIR of the selected port for a typical user. In our model, the UEs are independent and identically distributed (i.i.d.) and hence have the average outage probability, $p$, which specifies the interference immunity of a typical UE. By definition, we have

$$p = E_u \left[ \operatorname{Prob} \left( \text{SIR}_u = \max_{k \in \{1, \ldots, N\}} \text{SIR}_{u, k} < \gamma \right) \right].$$  

(8)

in which γ denotes the SIR threshold and SIR$_{u, k}$ denotes the SIR at the $k$-th port of UE $u$. As such, SIR$_u$ represents the resulting SIR at the best port at UE $u$. The outage probability expressions for the $f$-FAMA and $s$-FAMA cases can be found as (9) [44, (17)] and (10) [18, (21)], respectively, where $I_k(\cdot)$ is the modified Bessel function of the first kind, $Q_m(\cdot)$ is the generalized Marcum-Q function, $(n)$ is the Pochhammer symbol, and $\Gamma(n) = (n-1)!$ is the gamma function.

For the typical case $W > 1$, the outage probability expressions can be simplified using first-order approximations, [45, Theorem 1] and [18, Theorem 3] so that

$$p \approx \begin{cases} \left[ 1 - \frac{N}{U} \left( \frac{1 - \mu^2}{U - 1} \right) \right]^{+} & \text{if } f\text{-FAMA}, \\ \left[ 1 - \frac{N}{\gamma} \left( \frac{1 - \mu^2}{\gamma} \right) U - 1 \right]^{+} & \text{if } s\text{-FAMA}, \end{cases}$$  

(11)

where $[a]^{+} = \max(0, a)$ and $\mu^2 \approx \frac{1}{W}$ can be used.

3While SIR indeed means the signal to interference ratio averaged over data in $s$-FAMA, for the case of $f$-FAMA, we have abused the notation ‘SIR’ to include the meaning of the ratio of the instantaneous energy of the desired signal to that of the sum-interference plus noise signal.

**B. Opportunistic Scheduling**

We consider that the $U$ users selected for communications on the same time-frequency resource unit are based on their SIRs, see Fig. 2. In particular, it is assumed that there are $M > U$ UEs in the pool and their SIRs are ranked, i.e.,

$$\text{SIR}^{(1)} \leq \text{SIR}^{(2)} \leq \cdots \leq \text{SIR}^{(M)},$$  

(12)

in which SIR$_{(u)}$ represents the SIR of the best port of the $u$-th weakest UE (or the $(M - u + 1)$-th strongest UE) in the pool. In particular, the strongest $U < M$ UEs are selected to communicate at any given time. Therefore, we have

$$\text{SIR}^{(M-U+1)} \leq \text{SIR}^{(M-U+2)} \leq \cdots \leq \text{SIR}^{(M)}.$$  

(13)

As in previous work [16, 18], [44], [45], the UEs’ messages are transmitted with a common fixed coding rate specified by the SIR target $\gamma$, i.e., $R = \log_2(1 + \gamma)$. As a consequence, we define a UE’s ‘capacity’ as the average outage rate as follows.

**Definition 1**: The capacity for UE $u$ is defined as

$$C_u = \left[1 - \operatorname{Prob} (\text{SIR}_u < \gamma)\right] \log_2(1 + \gamma).$$  

(14)

Opportunistic scheduling for selecting the best $U$ users from a pool of $M$ users is well motivated to

$$\max_{j(U)} \sum_{u=1}^U C_{j(u)},$$  

(15)

by finding the function $j(\cdot)$ that selects the best users, where $C_{j(u)}$ represents the average outage rate for the $u$-th strongest user. Note that we have $\text{SIR}^{(M-U+1)} = \text{SIR}_{j(u)}$.

In general, the SIR of a UE depends on other selected users which means that in theory, all $\binom{M}{U}$ combinations will need to be checked in order to truly find the $U$ strongest UEs. This nonetheless would be practically impossible to do. Fortunately, in FAMA, no BS preprocessing specific to the selected users is needed, meaning that the outage probability performance of any selected UE does not depend on any other selected UEs. As a result, a UE can establish the confidence level on whether it is ranked in the top $U$ in the pool given the UE’s SIR. Thus, it is possible for the BS to sequentially turn on and off UEs until the top UEs are identified. While such selection method is possible and deserves further investigation, in this paper, we will always assume that the strongest $U$ UEs are selected.

It is worth pointing out that FAMA does expect each UE’s channel envelope not to change too fast to work. Each UE will need to find and switch to the best port for communications. If the CSI changes too quickly due to high mobility, then it would be difficult for a UE to figure out the best port in time to avoid interference although recent studies have led to efficient port selection algorithms for $f$-FAMA [17] and $s$-FAMA systems [19]. If opportunistic scheduling is to be used with FAMA, then it is even more reasonable to consider low-mobility UEs because the $U$ best UEs might change over time. That said, since we target massive connectivity scenarios, it makes sense to consider primarily low-mobility UEs as traditionally about 70—80% of mobile data traffic is generated indoors (including traffic served by in-building systems). In addition, for high-mobility UEs, multiple access techniques such as interleave division multiple access (IDMA) [46] or code division multiple


\[ p = \int_{0}^{\infty} e^{-z} \int_{0}^{\infty} e^{-t} \left[ 1 + \left( \frac{(U - 1)\gamma}{(U - 1)\gamma + 1} \right) e^{-\left(\frac{(U - 1)\gamma + 1}{(U - 1)\gamma + 1}\right)\left(\frac{2\mu^2}{1 - \mu^2}\sqrt{z+t}\right)} I_{0}\left(\frac{\sqrt{(U - 1)\gamma}}{(U - 1)\gamma + 1} \left(\frac{2\mu^2}{1 - \mu^2}\right) \sqrt{z}\right) - Q_1\left(\frac{1}{\sqrt{(U - 1)\gamma + 1}} \sqrt{\frac{2\mu^2}{1 - \mu^2} \sqrt{z}}\right) \right] dtdz \] \]  

(9)

\[ p = \int_{0}^{\infty} \int_{0}^{\infty} \frac{\mu^2}{2U} e^{-\frac{x+y}{2\mu}} \left\{ Q_{U-1}\left(\frac{\mu}{1 - \mu^2}\sqrt{\frac{\gamma^2}{\gamma + 1} - \frac{\mu}{1 - \mu^2}\sqrt{\gamma + 1}}\right) \right. \\
- \left( \frac{1}{\gamma + 1} \right) e^{-\frac{\mu^2}{2(1 - \mu^2)} (\frac{r}{\gamma + 1})} \times \\
\left. \sum_{k=0}^{U-2} \sum_{j=0}^{U-k-2} \frac{(U - (j + k) - 1)_j}{j!} \left( \frac{\gamma}{r} \right)^k \gamma^j r^{-\frac{k-2}{2}} I_{j+k}\left(\frac{\mu^2}{1 - \mu^2} (\gamma + 1)\right) \right\} \right] \]  

\[ \int_{0}^{\infty} \int_{0}^{\infty} \frac{\mu^2}{2U} e^{-\frac{x+y}{2\mu}} \left\{ Q_{U-1}\left(\frac{\mu}{1 - \mu^2}\sqrt{\frac{\gamma^2}{\gamma + 1} - \frac{\mu}{1 - \mu^2}\sqrt{\gamma + 1}}\right) \right. \\
- \left( \frac{1}{\gamma + 1} \right) e^{-\frac{\mu^2}{2(1 - \mu^2)} (\frac{r}{\gamma + 1})} \times \\
\left. \sum_{k=0}^{U-2} \sum_{j=0}^{U-k-2} \frac{(U - (j + k) - 1)_j}{j!} \left( \frac{\gamma}{r} \right)^k \gamma^j r^{-\frac{k-2}{2}} I_{j+k}\left(\frac{\mu^2}{1 - \mu^2} (\gamma + 1)\right) \right\} \right] \]  

\[ \int_{0}^{\infty} \int_{0}^{\infty} \frac{\mu^2}{2U} e^{-\frac{x+y}{2\mu}} \left\{ Q_{U-1}\left(\frac{\mu}{1 - \mu^2}\sqrt{\frac{\gamma^2}{\gamma + 1} - \frac{\mu}{1 - \mu^2}\sqrt{\gamma + 1}}\right) \right. \\
- \left( \frac{1}{\gamma + 1} \right) e^{-\frac{\mu^2}{2(1 - \mu^2)} (\frac{r}{\gamma + 1})} \times \\
\left. \sum_{k=0}^{U-2} \sum_{j=0}^{U-k-2} \frac{(U - (j + k) - 1)_j}{j!} \left( \frac{\gamma}{r} \right)^k \gamma^j r^{-\frac{k-2}{2}} I_{j+k}\left(\frac{\mu^2}{1 - \mu^2} (\gamma + 1)\right) \right\} \right] \]  

access (CDMA) [47] are well known to work well. In fact, the low-mobility case which suffers from slow channel fading has long been regarded as the more challenging case to tackle.

III. Multiplexing Gain Analysis

In this section, we present our main results that characterizes the performance of opportunistic FAMA networks. We mainly focus upon the multiplexing gain analysis because it uncovers the capacity scaling achievable from integrating opportunistic scheduling and FAMA. Following the UE’s capacity definition, we define the network capacity below.

Definition 2: The network capacity is defined as

\[ C = \sum_{u=M-U+1}^{M} (1 - q_u) \log_2 (1 + \gamma), \]  

(16)

where

\[ q_u = \text{Prob} (\text{SIR}^{(u)} < \gamma) \]  

(17)

denotes the outage probability of the \( u \)-th weakest UE.

Note that \( C \) in (16) uses the fact that the \( U \) strongest UEs are selected for FAMA communications.

The following theorem presents the expression for \( q_u \).

Theorem 1: The SIR outage probability with an SIR threshold, \( \gamma \), for the \( u \)-th weakest FAMA user is given by

\[ q_u = \sum_{k=u}^{M} \binom{M}{k} p^k (1 - p)^{M-k}, \]  

(18)

where \( p \) is given by (9) for \( f \)-FAMA and (10) for \( s \)-FAMA. Furthermore, \( q_u \) can be rewritten as

\[ q_u = I_{p}(u, M - u + 1) \]  

(19)

where \( I_{p}(a, b) \) is the regularized incomplete beta function.

Proof: Given the SIR outage probability for a typical UE and using order statistics [48] get the result (18). Then (a) and (b) in (19) come directly from the properties of the probability of a binomial random variable.

Corollary 1: The network outage rate of the opportunistic FAMA network is given by

\[ C = \sum_{u=M-U+1}^{M} I_{1-p}(M - u + 1, u) \log_2 (1 + \gamma). \]  

Proof: Combining (16) and (19) gets the result.

Theorem 2: The multiplexing gain, \( m \), of the opportunistic FAMA network is given by

\[ m = \sum_{u=M-U+1}^{M} I_{1-p}(M - u + 1, u) \]  

(20)

\[ m \geq U I_{1-p}(U, M - U + 1) \]  

(21a)

\[ \geq U I_{1-p}(U, M - U + 1) \]  

(21b)

\[ \geq \frac{1}{2} I_{p}(M-U) \]  

(21c)

where \( \text{erfc}(\cdot) \) is the complementary error function.

Proof: As the multiplexing gain measures the capacity scaling of the network, \( m \) is obtained by dividing \( C \) in (20) by the coding rate of a single user with zero outage probability because of the absence of interference, which gives (21a). The result (21b) is derived by approximating the binomial random variable using a Gaussian random variable if \( M \) is large such that \( Mp > 5 \) and \( M(1 - p) > 5 \). The lower bound (21c) is obtained by replacing all the terms in (21b) by the probability of the weakest UE out of \( U \) strongest UEs. Finally, (21d) is due to Gaussian approximation, which completes the proof.

The next theorem provides a conservative estimate on the required number of UEs in the selection pool, \( M \) for achieving particular regions of the multiplexing gain, \( m \).

Theorem 3: To achieve a given multiplexing gain, \( m \), which is specified by the parameter

\[ \alpha \triangleq \text{erfc}^{-1}\left(\frac{2}{m}\right), \]  

(22)
the required number of users in the pool, $M$, is bounded by
\begin{equation}
M \leq \frac{U + \alpha^2 p - \sqrt{(\alpha^2 p)(\alpha^2 p + 2U)}}{1-p} \quad \text{for } 0 \leq m \leq \frac{U}{2},
\end{equation}
and for $\frac{U}{2} \leq m \leq U$, we have
\begin{equation}
\frac{U}{1-p} \leq M \leq \frac{U + \alpha^2 p + \sqrt{(\alpha^2 p)(\alpha^2 p + 2U)}}{1-p}.
\end{equation}

**Proof:** See Appendix A.

Theorem 3 gives a conservative estimate on upper bounding $M$ for achieving a given multiplexing gain $m$. An observation of the results reveals that a smaller $p$ will have a smaller upper bound on $M$ as expected. The following theorem attempts to understand the requirement on the interference immunity of the fluid antenna at a typical UE. In so doing, we illustrate the benefits of $M$ in the opportunistic FAMA network.

**Theorem 4:** To achieve a given multiplexing gain such that
\begin{equation}
0.0786U \approx \left[1 - \frac{\text{erfc}(-1)}{2}\right] U < m < \left[1 - \frac{\text{erfc}(1)}{2}\right] U \approx 0.9214U,
\end{equation}
the outage probability, $p$, of each of the UEs in the opportunistic FAMA network can be lower bounded by
\begin{equation}
p \geq 1 - \frac{1}{M}(U + \alpha \sqrt{2U}).
\end{equation}

**Proof:** See Appendix B.

Notice that the above theorem is based on the lower bound (21d), meaning that (26) cannot be very accurate and should somewhat be conservative. Thus, the importance of Theorem 4 lies in that it illustrates the impact of $M$ on the requirement of the interference immunity at the fluid antenna of each user. In particular, we see that as $M$ increases, the right hand side of (26) approaches one, which indicates that each UE is left with not much to do in eliminating the multiuser interference and the capacity scaling will be very good. On the other hand, it is true that Theorem 4 does not cover the full range of $m$ in the analysis. Nonetheless, when $m < 0.0786U$, the network would not be functioning properly at all and this case will not be useful. On the other hand, since the result in Theorem 4 is derived from (21d), the actual achievable multiplexing gain, $m$, will exceed the range beyond $m > 0.9214U$.

**Theorem 5:** For $f$-FAMA, the required number of ports, $N$, at the fluid antenna at each UE is upper bounded by
\begin{equation}
N \leq \frac{1}{M} \left[\frac{(U-1)\gamma}{1 - \mu^2}\right] (U + \alpha \sqrt{2U}),
\end{equation}
in order to meet a given multiplexing gain requirement specified by $\alpha$. For $W \geq 1$, the result can be expressed as
\begin{equation}
N \leq \frac{1}{M} \left[\frac{(U-1)\gamma}{1 - \frac{1}{W}}\right] \left(U + \text{erfc}^{-1}\left[2 \left(1 - \frac{m}{U}\right)\right] \sqrt{2U}\right).
\end{equation}

**Proof:** Using (11) and applying Theorem 4, we have
\begin{equation}
1 - \frac{N(1 - \mu^2)}{(U-1)\gamma} \geq 1 - \frac{1}{M}(U + \alpha \sqrt{2U}),
\end{equation}
which after some rearrangement, then gives (27). Note that the operation \([\cdot]^+\) in (11) is omitted as we assume the scenario \(N < \frac{(U-1)\gamma}{1-\mu^2}\). Finally, (28) is obtained after we have employed the definition of \(\alpha\) and used the approximation (5).

**Theorem 6:** For s-FAMA, the required number of ports, \(N\), at the fluid antenna at each UE is upper bounded by

\[
N \leq \frac{1}{M} \left( \frac{\gamma}{1-\mu^2} \right)^{U-1} (U + \alpha \sqrt{2U}),
\]

(30)
in order to meet a given multiplexing gain requirement specified by \(\alpha\). For \(W \geq 1\), the result can be expressed as

\[
N \leq \frac{1}{M} \left( \frac{\gamma}{1-\frac{\pi}{W}} \right)^{U-1} \left( U + \text{erf}^{-1} \left[ 2 \left( 1 - \frac{m}{U} \right) \right] \sqrt{2U} \right).
\]

(31)

**Proof:** The results can be proved similarly as before.

Apparently, both Theorem 5 and Theorem 6 reveal that the size of the fluid antenna, \(W\), clearly plays a major role in relieving the pressure of the fluid antenna at each UE, it is anticipated that increasing \(M\) increases the multiplexing gain of the opportunistic FAMA network when \(N\) is fixed. The following theorem characterizes the rate at which \(m\) is increased with \(M\).

**Theorem 7:** The rate of change of the multiplexing gain w.r.t. the number of users in the pool is given by

\[
\frac{\partial m}{\partial M} = \frac{1}{2\sqrt{2\pi}} \sum_{u=1}^{U} \left( 1 - \frac{M(u-1)}{M-1} \right) \frac{e^{-\frac{(1-p)^2M(u-1)^2}{2Mp(1-p)}}}{M^2}.
\]

(32)

**Proof:** We start by rewriting (21b) as

\[
m \approx \sum_{u=M-1}^{M} \left[ 1 - \frac{1}{2} \text{erfc} \left( \frac{u - 1 - Mp}{\sqrt{2Mp(1-p)}} \right) \right] \frac{M}{U} - \frac{1}{2} \sum_{u=1}^{U} \text{erfc} \left( \frac{(U-1)M - u}{\sqrt{2Mp(1-p)}} \right).
\]

(33)

Knowing that \(\frac{\partial \text{erfc}(x)}{\partial x} = -\frac{2e^{-x^2}}{\sqrt{\pi}}\), we can find that

\[
\frac{\partial}{\partial M} \left[ \text{erfc} \left( \frac{(1-p)M - u}{\sqrt{2Mp(1-p)}} \right) \right] = -\frac{(1-p)M + u e^{-\frac{(1-p)^2M(u-1)^2}{2Mp(1-p)^2}}}{\sqrt{2\pi} \sqrt{p(1-p)} M^2}.
\]

(34)

Differentiating (33) w.r.t. \(M\) and using (34) get (32).

**Corollary 2:** For very large \(M\), it can be found that

\[
\frac{\partial m}{\partial M} \approx \frac{1}{2\sqrt{2\pi}} \frac{1}{p} e^{\frac{(1-p)^2M-U}{2Mp(1-p)}}.
\]

(35)

**Proof:** The result can be obtained directly from (32) after considering \(M\) to be very large.

**Corollary 3:** If

\[
M > \frac{U}{1-p},
\]

(36)

then the increase in the multiplexing gain will plateau, and any further increase in \(M\) will have a diminishing return. Hence, (36) provides a useful condition to decide on the effective size of the fluid antenna, \(W\), for the best performance of the opportunistic FAMA network.

**Proof:** When (36) is held, it can be observed from (35) in Corollary 2 that \(\frac{\partial m}{\partial M} \approx 0\) and therefore, \(M\) saturates.

**Corollary 4:** For an effective operation of the opportunistic FAMA network, one should choose the parameters to satisfy

\[
MN > \begin{cases} 
U(U-1)\gamma \quad & \text{if } f\text{-FAMA}, \\
\frac{1}{1-\mu^2} \quad & \text{if } s\text{-FAMA}, 
\end{cases}
\]

(37)

For \(W \geq 1\), we can further express the result in terms of the size of the fluid antenna, \(W\), as

\[
MN > \begin{cases} 
U(U-1)\gamma \quad & \text{if } f\text{-FAMA}, \\
\frac{1}{1-\frac{\pi}{W}} \quad & \text{if } s\text{-FAMA}, 
\end{cases}
\]

(38)

**Proof:** The result (37) comes directly from the condition in Corollary 3 when the outage probability results in (11) are substituted. When \(W \geq 1\), we use (5) to get (38).

Corollary 4 provides a simple condition on the parameters of the opportunistic FAMA network that strikes a good balance between capacity performance and complexity. It inherently seeks to achieve the highest increase in capacity scaling as the number of users in the pool is increased.

**Corollary 5:** As \(M \to \infty\),

\[
m \to U.
\]

(39)

**Proof:** For very large \(M\), it can be seen that

\[
\text{erfc} \left( \frac{(1-p)M - u}{\sqrt{2Mp(1-p)}} \right) \approx \text{erfc} \left( \sqrt{\frac{1-p}{2p}} \sqrt{M} \right),
\]

(40)

which goes to 0 if \(M \to \infty\). Then taking the limit \(M \to \infty\) and using the above result in (33) obtains (39).

**Theorem 8:** For large \(M\), the multiplexing gain, \(m\), of the opportunistic FAMA network can be approximated by

\[
m \approx U \left[ 1 - \frac{1}{\sqrt{2\pi}} \sqrt{\frac{p}{1-p}} e^{-\frac{(1-p)^2M}{2p}} \right].
\]

(41)

**Proof:** For large \(x\), we have the Taylor series

\[
\text{erfc}(a\sqrt{x}) = \frac{e^{-a^2x}}{a \sqrt{x}} \times \left( x^{-0.5} - \frac{x^{-1.5}}{2a^2\sqrt{x}} + \frac{3x^{-2.5}}{4a^4\sqrt{x}} - \frac{15x^{-3.5}}{8a^6\sqrt{x}} + \cdots \right).
\]

(42)

Applying the above result while keeping only the term \(x^{-0.5}\) (i.e., for large \(M\)), we can get

\[
\text{erfc} \left( \sqrt{\frac{1-p}{2p}} \sqrt{M} \right) \approx \frac{1}{\sqrt{\pi}} \sqrt{\frac{2p}{1-p}} e^{-\frac{(1-p)^2M}{2p}}.
\]

(43)

Now, using this result in (33) obtains the desired result. Theorem 8 provides a succinct expression to illustrate how the multiplexing gain of the network grows with the number of
users in the pool when $M$ is very large. Despite its simplicity, it may not be accurate when $M$ is not much greater than $U$. Even if $M$ is indeed much greater than $U$, then $m \approx U$, which makes the expression (41) quite unnecessary. For this reason, the following theorem is presented to provide an estimate on $m$ for the case when $M$ is not much greater than $U$.

**Theorem 9:** For $M \geq U$, the multiplexing gain, $m$, of the opportunistic FAMA network can be approximated by (44) (see top of next page) in which $U_0 = U$ if $M \geq \frac{U}{1-p}$ and if $M < \frac{U}{1-p}$, then $U_0$ is selected such that

$$U \geq U_0 + 1 > (1-p)M > U_0 \text{ or } U_0 = \lfloor (1-p)M \rfloor,$$  \hspace{1cm} (45)

where $\lfloor z \rfloor$ returns the largest integer which is smaller than $z$.

**Proof:** See Appendix C.

**Theorem 10:** The multiplexing gain, $m$, of the opportunistic FAMA network can be accurately approximated by

$$m \approx \min \{U, M(1-p)\},$$  \hspace{1cm} (46)

where $m \approx U$ if $p < 1 - \frac{U}{MN}$ and $m \approx M(1-p)$ otherwise.

**Proof:** See Appendix D.

**Corollary 6:** In the linear region, the multiplexing gain, $m$, of the opportunistic FAMA network can be written as

$$m = \begin{cases} 
\frac{MN(1-\mu_2)}{(U-1)\gamma} & \text{if } f\text{-FAMA}, \\
MN \left(1 - \mu_2 \right)^{-1} & \text{if } s\text{-FAMA}.
\end{cases}$$  \hspace{1cm} (47)

**Proof:** Using (46) and (11) yields the result.

### IV. System Parameter Selection

The analysis in the previous section helps understand how various system parameters contribute to the overall capacity performance of opportunistic FAMA networks. Here, we discuss how the parameters may be chosen, with the help of the analytical results. We go through the key parameters below.

- **The number of ports at the fluid antenna of each UE, $N$**—This represents the resolution the fluid antenna has to experience the different fading variations in space. The larger the size, the higher the diversity and the better the performance at each UE.

- **The normalized size of the fluid antenna, $W$**—This represents the spatial coverage the fluid antenna has to experience the different fading variations in space. The larger the size, the higher the diversity of the channel and the better the performance at each UE.

- **The SIR threshold, $\gamma$**—This specifies the quality-of-service (QoS) of the UE that is related to the particular type of communication, and is usually given.

- **The number of UEs in the selection pool for scheduling, $M$**—This dictates the amount of multiuser diversity we can obtain and represents the total number of users that the network can access the physical layer performance measures at any one time for scheduling. The larger the value of $M$, the better the performance.

- **The number of selected UEs, $U$, for communication at any given time**—This represents the number of users to be selected for sharing the same time-frequency resource unit at each schedule. The larger the value of $U$, the more the interference each selected UE suffers but potentially the higher the network capacity if the fluid antenna at each UE is powerful enough to handle the interference.

Note that $N$ should be chosen as large as possible according to the hardware design and constraint while $W$ is another hardware parameter that again should be as large as possible depending on the available space of the device. Additionally, $\gamma$ is usually prescribed based on the required QoS of the UE. Furthermore, $M$ should be chosen as large as practically possible for the best performance regardless of the network conditions. Lastly, $U$ appears to be the trickiest parameter that should be optimized according to the interplay between the amount of interference suffered at each UE and the overall network capacity (or multiplexing gain).

The optimization of $U$ can be performed with the help of Corollary 4 and Corollary 5. We use $f$-FAMA as an example to show how this is conducted. First, according to Corollary 5, the maximum limit of the multiplexing gain is $U$ meaning that $U$ should be maximized if possible for greater network capacity. However, according to Corollary 4, given $M, N, \gamma$ and $W(\geq 1)$, $U$ should satisfy

$$MN > \frac{U(U-1)\gamma}{1-\frac{1}{\pi W}} \Rightarrow U^2 - U - \frac{MN}{\gamma} \left(1 - \frac{1}{\pi W}\right) < 0.$$  \hspace{1cm} (48)

As a result, $U$ should be found by

$$\max U \text{ s.t. } U^2 - U - \frac{MN}{\gamma} \left(1 - \frac{1}{\pi W}\right) < 0.$$  \hspace{1cm} (49)

It can be easily shown that

$$U_{\text{opt}} = \left[\frac{1 + \sqrt{4MN \left(1 - \frac{1}{\pi W}\right) + 1}}{2}\right].$$  \hspace{1cm} (50)

The case for $s$-FAMA can be addressed similarly.

### V. Numerical Results

Here, we provide the numerical results to evaluate the multiplexing gain performance of the opportunistic FAMA network and attempt to gain some useful insight on its operation. Note that the multiplexing gain, $m$, illustrates the capacity scaling of the network over a single-user system occupying the same bandwidth. This performance metric depends on three crucial parameters, $U, M$ and $p$ while $p$ is a function of the parameters of the fluid antenna at each UE such as its size $W$, the SIR threshold $\gamma$ and the number of ports $N$. Figs. 3 and 4 make observations on $m$ against the system parameters $U, M$ and $p$ while Fig. 5 attempts to unpack the impact of the parameters of the fluid antenna for both $f$-FAMA and $s$-FAMA cases.

In Figs. 3 and 4, the exact results (21a) and the approximations (33) and (44) are provided. The results illustrate that the results for (33) and (44) coincide, which indicates that the approximation (44) to (33) is extremely accurate whereas the accuracy of the approximations to (21a) improves greatly as $U$ increases. Even if $U$ is not so large (e.g., $U = 5, 10$), the approximations can pick up the trend of $m$ very well but they
tend to give a lower bound of the multiplexing gain. Also, we observe that as expected, the maximum multiplexing gain in all the cases is $U$. As $M$ increases, $m$ increases considerably demonstrating the effectiveness of opportunistic scheduling in FAMA. In a similar fashion, reducing $p$ using more powerful fluid antennas increases $m$ greatly. Thus the synergy between opportunistic scheduling and FAMA is significant. Moreover, a close observation of the results in Fig. 3 confirms that $m$ exhibits a linear relationship with $M$ and saturates sharply if $M > \frac{U}{1-p}$, as predicted in Corollary 3. For example, for the case $U = 100$, Corollary 3 predicts that $M > \frac{U}{1-p} = \frac{100}{0.5} = 200$ is the point at which $m$ begins to plateau, which is exactly what is observed in the figure. In other words, the result of Theorem 10 (or the approximation (46)) is validated.

Results of Fig. 4 are plotted against the outage probability at each fluid antenna in the $x$-axis. In the figure, we also include vertical lines that specify $p = 1 - \frac{M}{U}$, the point at which $m \approx M$ occurs as predicted by Corollary 3. Apparently, a larger $M$ can relax the requirement on $p$ for approaching $m \approx U$. The results reveal some attractive operations of the opportunistic FAMA network. For example, if $U = 5$ and $M = 8$, we can achieve $m \approx 4.5$ at $p = 0.375$ (achievable by a fluid antenna with 1000 ports using s-FAMA [18, Fig. 2]). On the other hand, if $U = 100$ and $M = 150$, then we can obtain $m \approx 98$ at $p = 0.3333$ (achievable by a fluid antenna with 1120 ports using f-FAMA). Lastly, the results in this figure again confirm that the approximations we have derived are accurate.

We conclude this section by examining the results in Fig. 5 in which the network multiplexing gain is plotted against the parameters of the fluid antenna at each UE for both f-FAMA (Fig. 5(a)) and s-FAMA (Fig. 5(b)) cases. The f-FAMA case refers to the more advanced and complex setup of fluid antenna (hence able to accommodate a large number of UEs; $U = 50$) while the s-FAMA case is a more practical setup (supporting a less number of UEs at much lower complexity; $U = 4$). As can be observed, if $U = M$, i.e., no opportunistic scheduling is employed, then it will require an extraordinarily large number of ports, $N$, to approach $m \approx U$, see Fig. 5(a)(i) & (b)(i). The increase in the number of users in the selection pool, $M$, helps greatly reduce the requirement on $N$, which can bring the required $N$ from thousands to hundreds. According to (38) in Corollary 4, $m \approx U$ if

$$N > \begin{cases} 
\frac{U(U - 1)}{M(1 - \frac{U}{\pi W})} & \text{if f-FAMA,} \\
U \left(\frac{\gamma}{1 - \frac{U}{\pi W}}\right)^{U - 1} & \text{if s-FAMA.}
\end{cases}$$

(51)

This predicts that for f-FAMA, if $U = 50$, $M = 110$, $\gamma = 5$dB and $W = 2$, then $m$ will begin to plateau when $N > 265$, which agrees with the numerical results. We notice that the prediction for s-FAMA is less accurate in the numerical results and this is because the analysis leading to (38) is based on the assumption that $\gamma$ is large and in the case of s-FAMA, we have chosen $\gamma = 5$dB and thus affected the accuracy.

Figs. 5(a)(i) & (b)(ii) investigate how the multiplexing gain varies when the SIR threshold changes. As expected, if the SIR threshold $\gamma$ increases, the multiplexing gain will go down for both f-FAMA and s-FAMA cases. Moreover, there is an SIR point after which the network multiplexing gain starts to drop considerably, as predicted by Corollary 4, i.e.,

$$\gamma > \begin{cases} 
\frac{MN (1 - \frac{1}{\pi W})}{U(U - 1)} & \text{if f-FAMA,} \\
\frac{MN^{\gamma}}{U} \left(1 - \frac{1}{\pi W}\right) & \text{if s-FAMA.}
\end{cases}$$

(52)

For example, in the f-FAMA case, when $M = 70$, $U = 50$, $N = 500$ and $W = 2$, it can be found from (52) that $\gamma = 10.8$dB is the breaking point where the multiplexing gain begins to substantially fall, which agrees very much with the results in Fig. 5(a)(ii). Similar but less accurate results can be seen for the s-FAMA case. For example, if $M = 5$, $U = 4$, $N = 100$ and $W = 5$, then (52) estimates that $\gamma = 6.7$dB will be the breaking point, which appears to be slightly larger than what is observed in Fig. 5(b)(ii).

Finally, the results in Figs. 5(a)(iii) & (b)(iii) examine the change of multiplexing gain, $m$, against the size of the fluid antenna at each UE, $W$. By changing the subject of (38) in Corollary 4, it can easily be shown that if

$$W > \begin{cases} 
\frac{1}{\pi \left(1 - \gamma \left(\frac{U}{MN}\right)^{-\gamma}\right)} & \text{if f-FAMA,} \\
\frac{1}{\pi \left(1 - \gamma \left(\frac{U}{MN}\right)^{\gamma}\right)} & \text{if s-FAMA,}
\end{cases}$$

(53)

then $m \approx U$. Again, for the f-FAMA system, (53) is really accurate. When $M = 90$, $U = 50$, $\gamma = 10$dB and $N = 500$, (53) estimates that $W = 0.7$ is the point at which $m \approx U$, which agrees with the results in the figure.

VI. CONCLUSION

This paper attempted to combine opportunistic scheduling and FAMA for improved multiple access performance, where opportunism has helped alleviate greatly the burden of fluid antenna at each UE. Our objective was to quantify the benefits of the synergy between opportunistic scheduling and FAMA by analyzing the multiplexing gain of the network. This was
achieved by deriving various bounds and approximations on the multiplexing gain, which then facilitated the unpacking of the performance against key system parameters such as the number of UEs in the selection pool \( M \), the number of UEs selected to communicate on the same time-frequency channel \( U \) and the interference immunity at each fluid antenna \( p \). Both \( f \)-FAMA and \( s \)-FAMA systems were considered, with \( f \)-FAMA being the more powerful but less practical version than \( s \)-FAMA. Our analysis uncovered the rate of increase of the multiplexing gain \( m \) w.r.t. \( M \), from which we came up with a condition of the system parameters where the opportunistic FAMA network approached the maximum multiplexing gain \( m \approx U \). Numerical results were provided and discussed to gain useful insights of the operation of the opportunistic FAMA networks and the capability of the synergy was revealed.

APPENDICES

A. Proof of Theorem 3

To begin with, we rearrange (21d) to get

\[
\text{erfc}\left(\frac{(1-p)M - U}{\sqrt{2Mp(1-p)}}\right) \geq 2\left(1 - \frac{m}{U}\right).
\]

Now, consider the situation \( 0 \leq m \leq \frac{U}{2} \), which implies that \( 1 \leq 2 \left(1 - \frac{m}{U}\right) \leq 2 \) and \( \alpha \leq 0 \). Thus, we can rewrite (54) as

\[
\frac{(1-p)M - U}{\sqrt{2Mp(1-p)}} \leq \alpha = -|\alpha| \leq 0,
\]

which further implies that

\[
(1-p)M - U \leq 0 \Rightarrow M \leq \frac{U}{1-p}.
\]

It should be noted that in (55), the original inequality in (54) is flipped because \( \text{erfc}(\cdot) \) is a decreasing function. Squaring (55) on both sides (and flipping the inequality again because both sides are negative), we obtain

\[
f(M) = M^2 - \left(\frac{2}{1-p}\right)(U + \alpha^2p)M + \left(\frac{U}{1-p}\right)^2 \geq 0.
\]

The roots of \( f(M) = 0 \) can be easily found as

\[
M = \frac{U + \alpha^2p}{1-p} \pm \sqrt{(\alpha^2p)^2 + 2U(\alpha^2p + 2U)}.
\]

Denoting the smaller root as \( M_1 \) and the larger root as \( M_2 \), (57) requires that \( M \leq M_1 \) or \( M \geq M_2 \). We can see that

\[
M_1 = \frac{U + \alpha^2p}{1-p} - \sqrt{(\alpha^2p)^2 + 2U(\alpha^2p + 2U)} < \frac{U}{1-p}.
\]
and the dotted lines correspond to the expression (44). The approximation of (44) to (33) is again very accurate. As before, the solid lines refer to the exact results (21a) while the dashed lines are for the approximations (33) and the dotted lines correspond to the expression (44). The approximation of (44) to (33) is again very accurate.

B. Proof of Theorem 4

If \(0.0786U \leq m \leq \frac{U}{2}\), then it is easy to see that \(-1 < \alpha \leq 0\). Then we have \(\alpha^2p \leq 1\). Using (23), we get

\[
M \leq \frac{U + \alpha^2 p - \sqrt{(\alpha^2 p)(\alpha^2 p + 2U)}}{1 - p} \leq \frac{U - |\alpha|\sqrt{2U}}{1 - p},
\]

where (a) uses the fact that \(U \gg \alpha^2 p\). Simplifying (62) gives

\[
g_1(p) = p - \frac{|\alpha|\sqrt{2U}}{M} \sqrt{p} - \left(1 - \frac{U}{M}\right) \geq 0.
\]

The roots of \(g_1(p) = 0\) are given by

\[
\sqrt{p} = \frac{|\alpha|\sqrt{\frac{U}{2}}}{M} \pm \sqrt{\frac{\alpha^2 U}{M^2} - 1}\frac{U}{M}.
\]

Denoting the smaller root as \(\sqrt{p_1}\) and the larger root as \(\sqrt{p_2}\), we can easily see that \(\sqrt{p_1} < 0\) which is impossible and thus rejected. As a result, \(g_1(p) \geq 0\) gives the solution \(\sqrt{p} \geq \sqrt{p_2}\). For large \(M\), we can simplify

\[
\sqrt{\frac{\alpha^2 U}{2} + 1 - \frac{U}{M}} \approx \frac{1}{\sqrt{2M}} \left|\frac{U}{M} - \frac{\alpha^2}{2}\right| \frac{U}{M} \approx \frac{1}{\sqrt{2M}} \left(1 - \frac{U}{M}\right),
\]

where (a) uses \(M \gg \alpha^2\) and (b) is due to \(U < M\). Therefore, we have the lower bound

\[
\sqrt{p} \geq 1 - \left(\frac{U}{2M} - \frac{|\alpha|\sqrt{\frac{U}{2}}}{M}\right).
\]
Fig. 5. The multiplexing gain results of the opportunistic (a) $f$-FAMA and (b) $s$-FAMA networks against various different parameters such as (i) the number of ports, (ii) the SIR threshold and (iii) the size of fluid antenna at each UE. In the figures, only exact results (21a) using (9) and (10) are provided.

Squaring both sides of the above, employing the approximation $(1 - x)^2 \approx 1 - 2x$ for small $x$ and noting that $|\alpha| = -\alpha$ in this case, we obtain the result (26).

Now, consider the case $\frac{U}{2} \leq m < 0.9214U$. In this case, $0 \leq \alpha < 1$ and $\alpha = |\alpha|$. Again, we have $\alpha^2 p < 1 \ll U$. Using (24), we can get

$$M \leq \frac{U + \alpha^2 p + \sqrt{(\alpha^2 p)(\alpha^2 p + 2U)}}{1 - p},$$

which can be rewritten as

$$g_2(p) = p + \frac{|\alpha|\sqrt{2U}}{M} \sqrt{p} - \left(1 - \frac{U}{M}\right) \geq 0.$$  \hspace{1cm} (68)

Finding the two roots of $g_2(p) = 0$ and noting that the small root should be rejected, we then obtain

$$\sqrt{p} \geq 1 - \left(\frac{U}{2M} + \frac{|\alpha|}{M} \sqrt{\frac{U}{2}}\right).$$ \hspace{1cm} (69)

Finally, squaring both sides, utilizing the approximation $(1 - x)^2 \approx 1 - 2x$ for small $x$ and knowing that $|\alpha| = \alpha$, we again
derive the result (26), which completes the proof.

C. Proof of Theorem 9

First, we recognize a tight approximation [49, (6)]
\[
\text{erfc}(x) \approx \left(1 - e^{-Ax^2}\right) \frac{1}{Bx}, \quad \text{for } x \geq 0,
\]
(70)
where \( A = 1.98 \) and \( B = 1.135 \). Note that when \( x = 0 \), the right hand side is \( \frac{A}{B}x \approx 1 \). When \( (1 - p)M \geq U \), the input argument to \( \text{erfc}(\cdot) \) of each term inside the summation in (33) is always positive. Then we can apply (70) to (33) so that
\[
m \approx U - \frac{1}{B} \sqrt{\frac{M(1-p)}{2\pi}} x \sum_{u=1}^U \left(1 - e^{-\left(\frac{(1-p)M}{\sqrt{2\pi}(1-p)}\right)^2} \right) e^{-\frac{(1-p)M - u}{2M(1-p)^2}} \frac{1}{(1-p)M - u},
\]
(71)
which is the same as (44).

Now, for the case \( (1 - p)M < U \) and \( u = \{1, \ldots , U\} \), we know that \( (1 - p)M - u \) can be positive or negative. By the definition of (45), \( U_0 \) is the number at which \( (1 - p)M - u \) switches its sign when \( u = U_0 \) changes to \( u = U_0 + 1 \). Thus, some of the terms inside the summation are positive and some negative. Hence, we split the summation into two sums, one for \( u \leq U_0 \) and another \( u \geq U_0 + 1 \). Thus, as we have (72) (see top of next page) in which (a) splits the summation, (b) treats the identity \( \text{erfc}(\cdot) = 2 - \text{erfc}(\cdot) \), (c) obtains after some simplifications, and (d) adopts the approximation (70). After some manipulations, (72) is expressed as (44).

D. Proof of Theorem 10

From Corollary 3, it is known that when \( p < 1 - \frac{U}{M} \) (i.e., \( M > \frac{U}{1-p} \)), \( m \approx U \). On the other hand, it is obvious that if \( p = 1 \), then \( m = 0 \). Also, the multiplexing gain is a decreasing function of \( p \). Thus, it is possible to approximate \( m \) by a linear function of \( p \) for \( p > 1 - \frac{U}{M} \), which suggests immediately that \( m = M(1-p) \). However, instead of simply accepting this approximation, we here aim to show that
\[
\frac{\partial m}{\partial p} \approx -M
\]
(73)
for most part of \( 1 - \frac{U}{M} < p \leq 1 \), which then implies that the approximation is very accurate.

We begin the proof by obtaining the derivative of (33) as
\[
\frac{\partial m}{\partial p} = -\frac{1}{2\sqrt{2\pi}M} \times \sum_{u=1}^U \frac{(1-p)M + (2p-1)u}{p(1-p)} e^{-\frac{(1-p)M - u^2}{2M(1-p)^2}}
\]
(74)
evaluate the derivative when \( p = 1 - \frac{U}{2M} \) is the mid-point in the interval \( [1 - \frac{U}{M}, 1] \), which gives (75) (see next page), where (a) is the result of direct substitution \( p = 1 - \frac{U}{2M} \), (b) uses the approximations \( 1 - \frac{U}{M} \approx 1 \) and \( 1 - \frac{U}{2M} \approx 1 \), and (c) separates it into two sums \( A \) and \( B \).

Before we proceed, we find the following integral useful:
\[
\int_{-\infty}^{\infty} e^{-x^2} dt = \sqrt{\pi},
\]
(76)
which can be shown by recognizing the total probability of a standard Gaussian random variable \( \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2} dx = 1 \).

Now, \( A \) can be derived as
\[
A = \frac{1}{2} \sum_{k=U/2}^{U-1} e^{-\frac{k^2}{2U}} \frac{1}{\sqrt{U}} \frac{1}{2} \int_{-\infty}^{\infty} e^{-x^2} dx = \frac{1}{\sqrt{2}}
\]
(77)
where (a) applies the change of variables \( k = \frac{U}{2} - u \), (b) treats \( \frac{1}{\sqrt{U}} dx \) and \( x = \frac{k}{\sqrt{2}} \), large \( U \) and then uses (76) to obtain the final result. Similarly, we can obtain \( B \) by
\[
B = \frac{1}{2} \sum_{k=-U/2}^{-1} e^{-\frac{k^2}{2U}} \frac{k}{\sqrt{U}} \times \int_{-\infty}^{x} e^{-x^2} dx \times \frac{1}{\sqrt{2}},
\]
(78)
where (a) uses the substitution \( k = \frac{U}{2} - u \), (b) splits the sum into two parts, (c) approximates the sums using integration when \( U \) is large and finally applies (76) to get the result. As a consequence, the derivative can be found as
\[
\frac{\partial m}{\partial p} = -M \left(\frac{\sqrt{\pi}}{2} + \frac{\sqrt{\pi}}{2}\right) = -M.
\]
(79)
Finally, the above analysis can be repeated if \( p = 1 - \frac{U}{2M} \pm \delta \) when \( \delta < \frac{U}{2M} \) is small, which completes the proof.

REFERENCES

\[ m \equiv U - \frac{1}{2} \sum_{u=1}^{U} \text{erfc} \left( \frac{(1-p)M - u}{\sqrt{2Mp(1-p)}} \right) + \sum_{u=U+1}^{U} \text{erfc} \left( \frac{(1-p)M - u}{\sqrt{2Mp(1-p)}} \right) \]

\[ b \equiv U - \frac{1}{2} \left\{ \sum_{u=1}^{U} \text{erfc} \left( \frac{(1-p)M - u}{\sqrt{2Mp(1-p)}} \right) + \sum_{u=U+1}^{U} \left[ 2 - \text{erfc} \left( \frac{u - (1-p)M}{\sqrt{2Mp(1-p)}} \right) \right] \right\} \]

\[ c \equiv U_0 - \frac{1}{2} \sum_{u=1}^{U_0} \left( 1 - e^{-\frac{A(1-p)M-u}{\sqrt{2Mp(1-p)}}} \right) e^{-\frac{|u-p|\sqrt{p(1-p)}}{2Mp(1-p)}} \]

\[ d \equiv U_0 - \frac{1}{B} \sqrt{\frac{Mp(1-p)}{2\pi}} \sum_{u=1}^{U_0} \left( 1 - e^{-\frac{A(1-p)M-u}{\sqrt{2Mp(1-p)}}} \right) e^{-\frac{|u-p|\sqrt{p(1-p)}}{2Mp(1-p)}} \]

\[ \frac{\partial m}{\partial p} \equiv - \frac{1}{2\sqrt{2\pi}M} \left[ \left( \frac{1}{2} - \frac{U}{2M} \right) \left( \frac{U}{2M} \right) \right]^{1.5} \sum_{u=1}^{U} \left[ 1 - \frac{U}{2} \right] e^{-\frac{(\frac{u}{M} - u)^2}{2\pi}} + \left[ 1 - \frac{U}{M} \right] \frac{U}{2} e^{-\frac{(\frac{u}{M} - u)^2}{2\pi}} \]

\[ e \equiv \frac{M}{\sqrt{\pi}} \left[ \frac{1}{2\sqrt{U}} \sum_{u=1}^{U} e^{-\frac{(\frac{u}{M} - u)^2}{2\pi}} + \frac{1}{\sqrt{U}} \sum_{u=1}^{U} e^{-\frac{(\frac{u}{M} - u)^2}{2\pi}} \right] \]

\[ \text{erfc} (x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} \, dt \]


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