Distributed Bearing-Only Formation Control for Heterogeneous Nonlinear Multi-Robot Systems

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Abstract: This paper addresses the bearing-only formation tracking problem for heterogeneous nonlinear multi-robot systems. In contrast to position and distance-based formation algorithms, the robots can only measure the bearing information from their neighbors to achieve cooperation while the state information is unavailable. This characteristic is able to be implemented in the hardware to reduce the requirements of the sensors. We construct a compensation function in the proposed controller to eliminate the effect of the unknown nonlinear terms in the system. This compensation function is also based on bearing measurements, which guarantees that the overall controller is bearing-only. The stability of the proposed formation tracking strategy can be ensured by Lyapunov techniques. Moreover, we analyze the performance of the protocol for moving leaders, where the formation tracking error can be restricted in a bounded set. Finally, the simulation results are presented to validate the feasibility of the proposed algorithm for both fixed and moving leaders.

Keywords: Bearing measurements, formation tracking, nonlinear control, multi-robot systems.

1. INTRODUCTION

With the development of network science and technology, distributed control have been widely used in different applications, for instance, cooperative collision avoidance (Wang et al., 2020), autonomous vehicle platooning (Xie et al., 2022a; Pimentel et al., 2020; Xie et al., 2022b), manipulators (Liu et al., 2022, 2023), ecosystem hacking using micro-robot swarms (Stefanec et al., 2022), target surveillance and autonomous exploration (Li et al., 2020a). Formation control is a practical and effective approach to solving the problem of coordination and cooperation in multi-robot systems (MRS). Nevertheless, there are still many issues to be considered when implementing formation control techniques on real-world multi-robot systems, e.g., dealing with nonlinear dynamics to ensure the convergence and stability of the whole system.

One of the most common formation control protocols is based on position measurements, which means that the formation is controlled by measuring the relative position information of each agent's neighbors. For example, a novel distributed position-based method was proposed by Aranda et al. (2016) to make a set of agents achieve a desired rigid formation in a two-dimensional environment. In (Stacey and Mahony, 2015), a framework for formation control of dynamic robots was designed using partial measurements of relative position. In (Kang and Ahn, 2015), a distributed position-based formation control law was presented to solve the problem of a moving leader with unknown velocity in a multi-agent system. Wu et al. (2021) designed a SDP-based formation-containment protocol with input saturation. However, the agents should be relatively stationary when using position information for formation control, thereby making it difficult to estimate the position information of adjacent agents from different angles. Besides, the geometry of the agents must be known in order to observe the position information, which can be regarded as a limitation.

Another similar formation control protocol is on the basis of distance measurement, which means that the formation can be obtained by measuring the relative distance information of each agent's neighbors. Kang et al. (2016) developed a distributed distance-based formation control protocol for multi-agent systems, that globally keeps and obtains the desired formation in a two-dimensional space. A novel distance-based algorithm was proposed by Cao et al. (2011) to handle the formation of mobile agents when the agents are only able to measure the distances to their respective neighbors instead of the relative positions of their neighbours. In (Soares et al., 2013), a distance-based control strategy which estimates the formation heading and speed from the ranges of the two leader robots was

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introduced. Nevertheless, it is difficult to observe distance information in some special environments, such as underwater robots equipped with radar and sonar, which limits the practical application of formation control. Additionally, it is necessary to design a corresponding estimation algorithm in order to use the distance observation information. This will increase the computational cost of the MRS, especially when estimating the distance information of multiple robots at the same time.

To overcome these difficulties, a formation control protocol that is on the basis of only bearing measurement has attracted much attention. Different from position or distance measurement, bearing measurement only uses pure angle observation information without other estimation information such as distance or position. The bearing measurement information can be directly obtained from different angles using vision sensors, which will overcome the limitations of formation control based on distance or position observation information. In (Li et al., 2020b), a novel control law was proposed to accomplish target formations in finite time with bearing-only measurement. A bearing-based Henneberg construction with leader-first follower graphs was introduced in (Trinh et al., 2018) for formation forming of multiple agents in an arbitrary dimensional space. In (Wu et al., 2022), a mixed formation control design based on edge and bearing measurements was elucidated for networked multi-vehicle systems. However, the authors did not consider nonlinear systems or heterogeneous dynamics in the aforementioned works, which may be considered as a restriction when dealing with real robotic platforms.

In this work, a cooperative bearing-only formation protocol is designed to deal with the heterogeneous MRS with nonlinear dynamics. The robots can only measure the bearing information from their neighbors while the position or distance measures are inaccessible. Moreover, the heterogeneous nonlinear function is included in the system. The stability of the proposed strategy can be guaranteed via Lyapunov techniques. Furthermore, we also discuss the robustness of the controller for moving leaders, which is more practical in real applications. Finally, the simulations are presented to verify the effectiveness of the proposed algorithm. The contribution of this paper can be summarized as:

- A cooperative bearing-only formation strategy is proposed for nonlinear heterogeneous multi-robot networks. Compared with traditional position-based and distance-base coordination methods, the coordinated movement of each robot merely requires the relative bearings from their neighbors, which significantly reduces the sensing requirements.
- A novel compensation term based on bearing measurements is introduced in the proposed controller. The compensation function is able to eliminate the effect of the unknown nonlinear dynamics in the system without position and distance measurements.Different from the works presented in (Wu et al., 2023) and (Zhao et al., 2019), the dynamics of the agents considered in this work could be nonlinear and heterogeneous.
- Moreover, the nonzero velocities of the leader is also considered in this research. The formation error can be guaranteed in a bounded set under the proposed protocol for moving leaders.

The rest of the paper proceeds as follows. In Section 2, the preliminaries and the problem description are introduced. In Section 3, the cooperative bearing-only formation scheme is proposed and the stability analysis of the controller is presented by Lyapunov method. The proposed results are extended to deal with moving leaders. Simulation results are shown in Section 4 to verify the feasibility of the proposed algorithm. Section 5 concludes the paper.

2. PRELIMINARIES AND PROBLEM DESCRIPTIONS

2.1 Graph Theory and Notions

Consider a group of n cooperative mobile robots with n_l leaders and n_f followers ($n_l + n_f = n$). Define the position of each robot as $x_i(t) \in \mathbb{R}^p$ ($i \in \{1, 2, \cdots, n\}$) at time t, and $x(t) = [x_1(t)^T, \cdots, x_{n_l}(t)^T, \cdots, x_n(t)^T]^T$. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ denote the interaction topology among the robots, where the vertex and the edge set of each robot are represented by $\mathcal{V} = \{v_1, \ldots, v_{n_l}, \cdots, v_n\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ respectively. We claim $(i, j) \in \mathcal{E}$ if the communication can be transmitted from the ith robot to the jth robot. Then, the neighbor set of the ith robot can be expressed as $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$. It holds that $(j, i) \in \mathcal{E} \Leftrightarrow (i, j) \in \mathcal{E}$ under an undirected graph $\mathcal{G}$.

Suppose edge (i, j) represent the kth unditected edge and there exists m undirected edges ($|\mathcal{E}| = m$) among the graph $\mathcal{G}$. Define the oriented graph as the assignment of an orientation for all undirected edges in $\mathcal{G}$. The incidence matrix $H = [H]_{ki} \in \mathbb{R}^{m\times n}$ of the oriented graph with the kith entry can be defined as

$$[H]_{ki} = \begin{cases} -1, & i \text{ is the tail of } k\text{th edge,} \\ 1, & i \text{ is the head of } k\text{th edge} \\ 0, & \text{otherwise.} \end{cases}$$

The edge and bearing vectors of the kth edge (i, j) is defined, respectively, as

$$y_{ij} = y_k = x_j - x_i, \quad z_{ij} = z_k = \frac{y_{ij}}{\|y_{ij}\|}, \tag{1}$$

where $\|\cdot\|$ represents the Euclidean norm of a vector or the spectral norm of a matrix. The unit vector z_{ij} denotes the relative bearing of x_j with respect to x_i. Define $\tilde{H} = H \bigotimes I_p$, where $\bigotimes$ denotes the Kronecker product. From the definition of the incidence matrix H, it is easily to obtain that $y = \tilde{H}x$, where $y = [y_1(t)^T, \cdots, y_m(t)^T]^T$, and $I_p \in \mathbb{R}^{p\times p}$ denotes the identity matrix.

Define the scale of the formation in the system as

$$\mathbb{S}(t) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} \|x_i - \bar{x}\|^2} = \frac{\|x - \mathbf{1}_n \bigotimes \bar{x}\|}{\sqrt{n}} \ ,$$

where $\bar{x} = \frac{1}{n}(\mathbf{1}_n \bigotimes I_p)^T x$ represents the centroid of the formation.

Let $x^* = [x_1^{*T}, \cdots, x_n^{*T}]^T$, $y^* = [y_1^{*T}, \cdots, y_m^{*T}]^T$, and $z^* = [z_1^{*T}, \cdots, z_m^{*T}]^T$ be the configuration, edge and bearing vectors of the desired formation, respectively. The *bearing Laplacian matrix* $\mathcal{B} \in \mathbb{R}^{dN\times dN}$ is demonstrated to characterize the properties of a formation. The (i,j) th block of $\mathcal{B}$ is expressed as (Zhao and Zelazo, 2016)

$$[\mathcal{B}]_{ij} = \begin{cases} \mathbf{0_{p\times p}}, & i \neq j,\ (i,j) \notin \mathcal{E}, \\ z_{ij}^* {z_{ij}^*}^T - I_p, & i \neq j,\ (i,j) \in \mathcal{E}, \\ \sum\limits_{k\in\mathcal{N}_i} (I_p - z_{ik}^* {z_{ik}^*}^T), & i = j,\ i \in \mathcal{V}. \end{cases}$$

It is obvious that $\mathcal{B}p^* = \mathcal{B}(\mathbf{1}_n \bigotimes I_p) = 0$, and $\mathcal{B} \geq 0$, where $\mathbf{1}_n = [1, 1, \cdots, 1]^T$. The partition of the bearing Laplacian matrix according to the leaders and followers can be written as

$$\mathcal{B} = \begin{bmatrix} \mathcal{B}_{ll} & \mathcal{B}_{lf} \\ \mathcal{B}_{lf}^\top & \mathcal{B}_{ff} \end{bmatrix},$$

where $\mathcal{B}_{ff} \in \mathbb{R}^{pn_f \times pn_f}$. The following lemma reveals that $\mathcal{B}_{ff}$ plays a key role to ensure the uniqueness of the target formation.

Lemma 1. (Zhao and Zelazo, 2016) The target formation can be uniquely determined by the positions of the leaders $\{x_i^*\}_{i\in\mathcal{V}_l}$ and the bearing vectors $\{z_{ij}^*\}_{(i,j)\in\mathcal{E}} \Leftrightarrow \mathcal{B}_{ff}$ is non-singular.

2.2 Problem Descriptions

In this paper, we mainly focus on nonlinear heterogeneous MRS. Suppose the leaders are fixed ($\dot{x}_i(t) = 0,\ \forall i \in \mathcal{V}_l$), the dynamics of the ith follower robot can be written as

$$\dot{x}_i = \psi_i(x_i(t)) + u_i(t), \quad \forall i \in \mathcal{V}_f \tag{2}$$

where $\psi_i(\cdot) \in \mathbb{R}^p$ denotes the unknown nonlinear continuous function for each robot. We can imply that the nonlinear MRS is heterogeneous since ψ_i is different for each robot. $u_i(t) \in \mathbb{R}^p$ represents the control input for the ith robot generated by bearing measurements. The main problem of the work can be expressed as

Problem: Design the cooperative formation strategies for each follower robot merely based on bearing vectors $\{z_{ij}\}_{j\in\mathcal{N}_i}$ such that all the robots will converge to the target formation x^*.

To deal with the problem, we propose the following assumptions

Assumption 1 : The unknown nonlinear function $\psi_i(\cdot)$ is upper-bounded by a continuous function $\tilde{\psi}(t)$ which is known. That is to say, $\|\psi_i(\cdot)\| \leq \tilde{\psi}(t)$.

Assumption 2 : The formation scale $\mathbb{S}(t)$ is upper-bounded from the initial scale. In another words, $\mathbb{S}(t) \leq \mathbb{S}(0) = \mathbb{S}_0$, $\forall\ t \geq 0$.

Assumption 3 : The desired formation is unique. i.e., $\lambda_{\min}(\mathcal{B}_{ff}) > 0$, where $\lambda_{\min}(\mathcal{B}_{ff})$ denotes the minimum eigenvalue of $\mathcal{B}_{ff}$.

Assumption 4 : There is no collision between each robot during the formation task. i.e., $\|y_k\| > \sigma$, $\forall k \in \{1, 2, \cdots, m\}$, where σ is a positive constant.

Assumption 4 ensures that the bearing vectors generated by each pair of the neighbour are always well defined during the formation framework (Zhao and Zelazo, 2015; Zhao et al., 2019).

3. MAIN RESULTS

3.1 Bearing-Only Formation Protocol for Followers

In this section, we present a novel formation algorithm to solve the problem proposed in Section 2.2. The controller of the ith follower robot is designed as

$$u_i(t) = \sum_{j\in\mathcal{N}_i} (a\xi_{ij} + b\Psi_i(\xi_{ij})), \tag{3}$$

where $\xi_{ij} = \xi_k = z_{ij} - z_{ij}^*$ denotes the bearing error of the kth undirected edge (i,j), a and b are controller gains which should be defined later, and

$$\Psi_i(\xi_{ij}) = \begin{cases} \dfrac{\xi_{ij}}{\|\xi_{ij}\|^2}\tilde{\psi}^2(t), & \text{when } \xi_{ij} \neq 0 \\ 0, & \text{when } \xi_{ij} = 0. \end{cases}$$

Denote $e_i = x_i - x_i^*$ as the formation error of the ith robot, and $e = [e_1^T, \cdots, e_n^T]^T$. Let $\xi = [\xi_1^T, \cdots, \xi_m^T]^T$, some lemmas should be introduced before we show the main result

Lemma 2. (Zhao et al., 2019): Suppose Assumption 4 holds, we have

$$x^T \tilde{H}^T \xi \geq 0 \tag{4}$$

$$(x^*)^T \tilde{H}^T \xi \leq 0 \tag{5}$$

$$e^T \tilde{H}^T \xi \geq 0 \tag{6}$$

Lemma 3. (Zhao et al., 2019): Suppose Assumption 4 holds, we have

$$\frac{e^T \mathcal{B} e}{\max_k \|y_k\|} = \frac{x^T \mathcal{B} x}{\max_k \|y_k\|} \leq 2x^T \tilde{H}^T \xi\,. \tag{7}$$

Now, we present the stability analysis of the proposed controller as the following theorem,

Theorem 1. Under Assumption 1-4, the formation tracking error of the nonlinear heterogeneous MRS (2) converges to zero exponentially for the fixed leaders by implementing the controller (3) if the control gains a and b are selected to satisfy

$$ab > \frac{2n^2 \mathbb{S}_0}{m\sigma\lambda_{\min}(\mathcal{B}_{ff})}. \tag{8}$$

Proof. It is obvious that $e_i = 0,\ \forall i \in \mathcal{V}_l$ since the leaders are stationary. Hence, we can rewritten the formation error as $e = [0, e_f^T]^T$, where $e_f = [e_{n_l+1}^T, \cdots, e_n^T]^T$. Let $\Gamma = \begin{bmatrix} 0 & 0 \\ 0 & I_{pn_f} \end{bmatrix}$, it can be obtained that

$$e^T \Gamma = e^T \tag{9}$$

and

$$\begin{aligned} e^T \mathcal{B} e &= e_f^T \mathcal{B}_{ff} e_f \\ &\geq \lambda_{\min}(\mathcal{B}_{ff}) e_f^T e_f \\ &= \lambda_{\min}(\mathcal{B}_{ff}) e^T e. \end{aligned} \tag{10}$$

Substituting the formation protocol (3) to the nonlinear heterogeneous MRS (2), then we present the compact form of (2) as

$$\dot{x} = -\Gamma \tilde{H}^T (a\xi + b\Psi(\xi)) + \psi(x). \tag{11}$$

where $\psi(x) = [0, \psi_{n_l+1}^T(x_{n_l+1}), \cdots, \psi_n^T(x_n)]^T$, and

$$\Psi(\xi)) = \tilde{\psi}^2(t) \left[\frac{\xi_1^T}{\|\xi_1\|^2}, \cdots, \frac{\xi_m^T}{\|\xi_m\|^2} \right]^T.$$

Choosing the Lyapunov candidate as

$$V = \frac{1}{2}e^T e. \tag{12}$$

The derivation of V can be expressed as

$$\begin{aligned}
\dot{V} &= e^T \dot{x} \\
&= -e^T \Gamma \tilde{H}^T (a\xi + b\Psi(\xi)) + e^T \psi(x) \\
&= -ae^T \tilde{H}^T \xi - be^T \tilde{H}^T \Psi(\xi) + e^T \psi(x) \\
&\leq -ax^T \tilde{H}^T \xi + \Omega \\
&\leq -\frac{ae^T \mathcal{B} e}{2\max_k \|y_k\|} + \Omega \\
&\leq -\frac{a\lambda_{\min}(\mathcal{B}_{ff})}{\max_k \|y_k\|} V + \Omega,
\end{aligned} \tag{13}$$

where

$$\Omega = -be^T \tilde{H}^T \Psi(\xi) + e^T \psi(x).$$

On the one hand, according to the definition of $\mathbb{S}_(t)$, we can imply that

$$\begin{aligned}
n^2 \mathbb{S}(t)^2 &= n \sum_{k=1}^{n} \|x_k - \bar{x}\|^2 \\
&\geq (\|x_i - \bar{x}\| + \sum_{k\in\mathcal{V}, k\neq i}^{n} \|x_k - \bar{x}\|)^2 \\
&\geq \|x_i - \bar{x}\|^2.
\end{aligned} \tag{14}$$

Combining with Assumption 2, it can be observed that

$$\begin{aligned}
\|y_k\| &= \|x_i - x_j\| \\
&= \|(x_i - \bar{x}) - (x_j - \bar{x})\| \\
&\leq \|x_i - \bar{x}\| + \|x_j - \bar{x}\| \\
&\leq 2n\mathbb{S}(t) \leq 2n\mathbb{S}_0 \,.
\end{aligned} \tag{15}$$

On the other hand, from Assumption 1 and 4, together with the average inequality, we have

$$\begin{aligned}
\Omega &= -b(y - y^*)^T \Psi(\xi) + e^T \psi(x) \\
&= -b \sum_{k=1}^{m} \frac{y_k^T z_k - y_k^T z_k^*}{\|z_k - z_k^*\|^2} \tilde{\psi}^2(t) + b \sum_{k=1}^{m} \frac{{y_k^*}^T z_k - {y_k^*}^T z_k^*}{\|z_k - z_k^*\|^2} \tilde{\psi}^2(t) \\
&\quad + e^T \psi(x) \\
&= -b \sum_{k=1}^{m} \frac{\|y_k\|(1 - z_k^T z_k^*)}{\|z_k - z_k^*\|^2} \tilde{\psi}^2(t) + b \sum_{k=1}^{m} \frac{\|y_k^*\|(z_k^T z_k^* - 1)}{\|z_k - z_k^*\|^2} \tilde{\psi}^2(t) \\
&\quad + e^T \psi(x) \\
&\leq -b \sum_{k=1}^{m} \frac{\|y_k\|}{2} \tilde{\psi}^2(t) + e^T \psi(x) \\
&\leq -b \sum_{k=1}^{m} \frac{\|y_k\|}{2} \tilde{\psi}^2(t) + \frac{n}{2bm\sigma} e^T e + \frac{bm\sigma}{2n} \|\psi(x)\|^2 \\
&\leq \frac{n}{2bm\sigma} e^T e - b \sum_{k=1}^{m} \frac{\|y_k\| - \sigma}{2} \tilde{\psi}^2(t) \leq \frac{n}{bm\sigma} V.
\end{aligned} \tag{16}$$

Substituting (15) and (16) into (13), from (8), we have

$$\dot{V} \leq -\tilde{a}V < 0, \tag{17}$$

where

$$\tilde{a} = \frac{abm\sigma\lambda_{\min}(\mathcal{B}_{ff}) - 2n^2\mathbb{S}_0}{2bmn\sigma\mathbb{S}_0} > 0.$$

That is to say the formation error will converge to zero exponentially under the control strategy (3) with the exponential convergence rate equals to $\tilde{a}$. This completes the proof.

3.2 Convergence Analysis for Moving Leaders

In this section, the performance of the proposed controller (3) is considered if the leaders are not fixed. Suppose the trajectories of the leaders are described as $\dot{p}_i = v(t)$, $\forall i \in \mathcal{V}_l$, where $v(t)$ is the velocities of each leader which is bounded, i.e., $\|v\| \leq \tilde{v}$, where $\tilde{v}$ is a known positive constant. Then, it is easily to find that $\dot{x}_i^* = v$, $\forall i \in V$ (Zhao and Zelazo, 2016). The stability analysis of the proposed controller under moving leaders is shown in the following theorem

Theorem 2. Under Assumption 1-4, the formation error of the nonlinear heterogeneous MRS (2) converges to the bounded set

$$\mathcal{S} = \left\{ e : \|e\|^2 \leq \frac{4b^2m^2\sigma^2\tilde{v}^2}{n} \right\}.$$

for the moving leaders with the velocity v by implementing the controller (3) if the control gains a and b are selected to satisfy

$$ab > \frac{4n^2\mathbb{S}_0}{m\sigma\lambda_{\min}(\mathcal{B}_{ff})}. \tag{18}$$

Proof. Selecting the Lyapunov function as (12), similar to the analysis in Theorem 1, we can get

$$\begin{aligned}
\dot{V} &= e^T(\dot{x} - \mathbf{1}_n \otimes v) \\
&\leq -\frac{a\lambda_{\min}(\mathcal{B}_{ff})}{\max_k \|y_k\|} V + \Omega - e^T v \\
&\leq -\tilde{a}V - e^T v \\
&\leq -\frac{n}{bm\sigma} V - e^T v,
\end{aligned} \tag{19}$$

where the last inequity of (19) can be obtained from (18).

From average inequality, we have

$$\begin{aligned}
-e^T(\mathbf{1}_n \otimes v) &\leq \frac{n}{4bm\sigma}\|e\|^2 + \frac{bm\sigma}{n}\|\mathbf{1}_n \otimes v\|^2 \\
&= \frac{n}{2bm\sigma} V + bm\sigma\|v\|^2 \\
&\leq \frac{n}{2bm\sigma} V + bm\sigma\tilde{v}^2.
\end{aligned} \tag{20}$$

Combining (19) and (20), we can get

$$\dot{V} \leq -\frac{n}{2bm\sigma} V + bm\sigma\tilde{v}^2. \tag{21}$$

Define the bounded set $\mathcal{S}$ as

$$\mathcal{S} = \left\{ e : \|e\|^2 \leq \frac{4b^2m^2\sigma^2\tilde{v}^2}{n} \right\}.$$

Denote $\bar{\mathcal{S}}$ as supplementary set of $\mathcal{S}$. if $e \in \bar{\mathcal{S}}$, it can be observed that

$$\dot{V} \leq 0. \tag{22}$$

That is to say, when $t \to \infty$, it holds that

$$\|e\|^2 \leq \frac{4b^2m^2\sigma^2\tilde{v}^2}{n} \tag{23}$$

Hence, the formation tracking error will converge to the bounded set $\mathcal{S}$. This completes the proof.

4. SIMULATION RESULTS

In this section, the simulation case studies are presented to verify the theoretical results for both fixed and moving leaders.

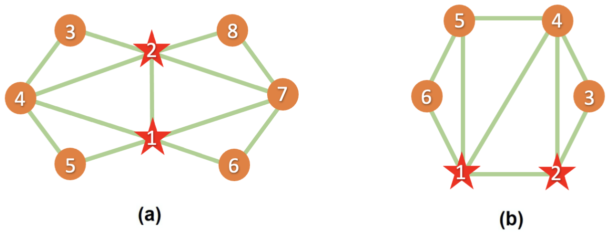

Fig. 1. Interaction topology of the MRS for (a) Case 1: Fixed leaders, and (b) Case 2: Moving leaders.

4.1 Case Study on Fixed Leaders

We first design the simulation to validate the proposed bearing-only formation protocol (3) on fixed leaders. Eight mobile robots, with two leaders and six followers, are applied to this case study. The interaction topology among the robots is appeared in Fig. 1 (a). The leaders are represented by two red stars labeled with 1 and 2, and the followers are denoted by six orange circles labeled from 3 to 8. The communications between each robot are denoted by green solid lines. These robots aim to attain a target formation as two pentagons together in 2D space. In robot dynamics, the nonlinear function for the ith follower robot is defined as

$$\psi_i(x_i(t)) = \begin{bmatrix} \dfrac{0.5sin(ix_{i1}(t))}{t+1} \\ \dfrac{0.5sin(ix_{i2}(t))}{t+1} \end{bmatrix}, \; i \in \mathcal{V}_f, \qquad (24)$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t)]^T$. It is easy to find that $\|\psi_i(x_i(t))\| \leq 1/(t+1)$, which satisfies Assumption 1. The MRS is heterogeneous since $\psi_i(x_i(t))$ is different for each follower. According to the condition in Theorem 1, we select the control gains as $a = 10$, and $b = 2$.

By implementing the bearing-only controller (3), the trajectories of the follower robots are elucidated in Fig. 2. We set the positions of the leaders (represented by two red stars) as $(3, 0)$ and $(6, 0)$. The initial states of the followers (represented by six circles with different colors) are chosen as $(15.9, 2)$, $(10, 2)$, $(-5, 1)$, $(-5.9, 3)$, $(4, -9)$, and $(14, -8)$. It can be observed that all the robots are able to form as the target formation under the proposed protocol. Fig. 3 reveals the time variation of the formation errors $\|e_i(t)\|$. We can conclude that all the formation errors of the followers will converge to zero during the formation task. Based on these result, the bearing-only formation task can be accomplished by the proposed strategy (3).

4.2 Case Study on Moving Leaders

In this section, we further verify the performance of the proposed protocol for moving leaders. Six robots, with two leaders and four followers are used in this task. Fig. 1 (b) displays the interaction topology between each robot. The velocities of the leaders are set as

$$\dot{x}_i(t) = \begin{bmatrix} 0.1 \\ 0.1sin(\frac{\pi}{80}t) + 0.01cos^5(\frac{\pi}{50}t) \end{bmatrix}, \quad i \in \mathcal{V}_l, \qquad (25)$$

where $\mathcal{V}_l = \{1, 2\}$. The nonlinear function $\psi_i(x_i(t))$ is chosen as (24). The control gains are set as $a = 15$, and $b = 1$, which satisfy the condition in Theorem 2.

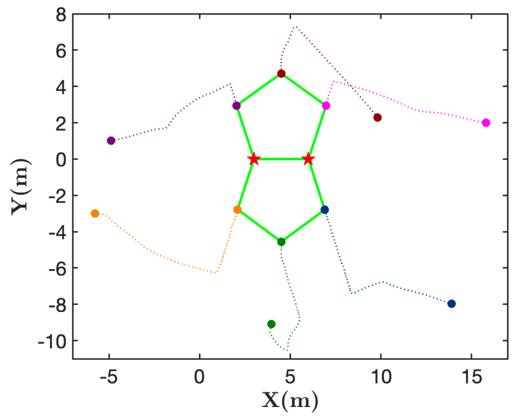

Fig. 2. Trajectories of the followers for fixed leaders (labeled by red stars).

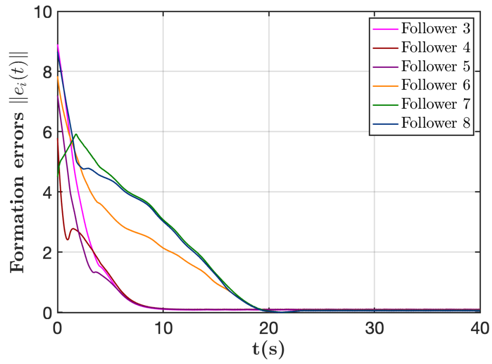

Fig. 3. Formation errors ($\|e_i(t)\|$) of the followers for fixed leaders.

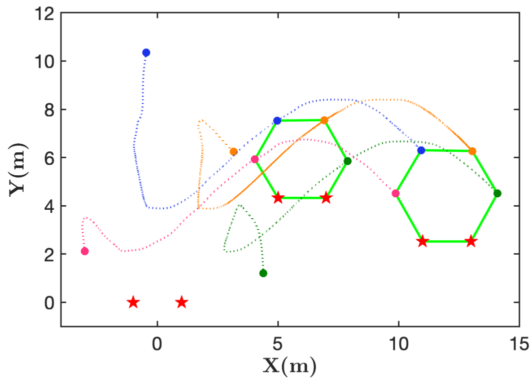

Fig. 4. Trajectories of the followers for moving leaders (labeled by red stars).

We select the initial positions of the leaders as $(-1, 0)$ and $(1, 0)$. Fig. 4 demonstrates the movement of each robot under the controller (3). Two red stars denote the moving leaders. The trajectories of the followers are represented by four dash lines with various colors. From Fig. 5, we can obtain that all the formation errors of the followers will converge to a bounded set (the boundary is denoted by a black dash line). To sum up, the formation error can be guaranteed in a bounded set by implementing the proposed bearing-only formation protocol, which validates the feasibility of the theoretical result.

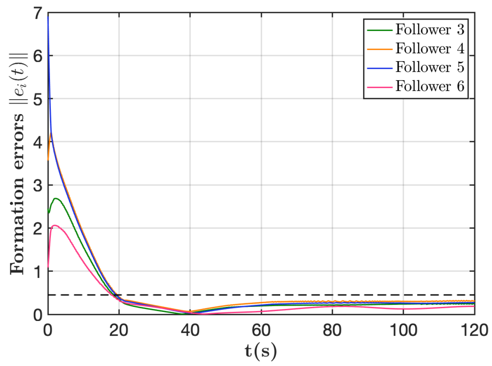

Fig. 5. Formation errors ($\|e_i(t)\|$) of the followers for moving leaders.

5. CONCLUSION

In this paper, the bearing-only formation tracking problem is addressed for heterogeneous nonlinear MRS. We propose a novel formation protocol for the follower robots based on bearing measurements to form the desired formation configuration. A compensation term is included in the controller to deal with the unknown nonlinear items in the system. By using the Lyapunov method, the formation tracking error will converge to zero exponentially under the proposed bearing-only algorithm. Furthermore, we extend the stability analysis of the proposed strategy on moving leaders, and the formation tracking error is able to be guaranteed in a bounded set. Simulation case studies are provided to verify the effectiveness of the theoretical results.

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