Active RIS Assisted Rate-Splitting Multiple Access Network: Spectral and Energy Efficiency Tradeoff


Abstract—With the increasing demand of high data rate and massive access in both ultra-dense and industrial Internet-of-things networks, spectral efficiency (SE) and energy efficiency (EE) are regarded as two important and inter-related performance metrics for future networks. In this paper, we investigate a novel integration of rate-splitting multiple access (RSMA) and reconfigurable intelligent surface (RIS) into cellular systems to achieve a desirable tradeoff between SE and EE. Different from the commonly used passive RIS, we adopt reflection elements with active load to improve a newly defined metric, called resource efficiency (RE), which is capable of striking a balance between SE and EE. This paper focuses on the RE optimization by jointly designing the base station (BS) transmit precoding and RIS beamforming (BF) while guaranteeing the transmit and forward power budgets of the BS and RIS, respectively. To efficiently tackle the challenges for solving the RE maximization problem due to its fractional objective function, coupled optimization variables, and discrete coefficient constraint, the formulated nonconvex problem is solved by proposing a two-stage optimization framework. For the outer stage problem, a quadratic transformation is used to recast the fractional objective into a linear form, and a closed-form solution is obtained by using auxiliary variables. For the inner stage problem, the system sum rate is approximated into a linear function. Then, an alternating optimization (AO) algorithm is proposed to optimize the BS precoding and RIS BF iteratively, by utilizing the penalty dual decomposition (PDD) method. Simulation results demonstrate the superiority of the proposed design compared to other benchmarks.

Index Terms—Rate-splitting multiple access, RIS, active load, tradeoff, resource efficiency.

I. INTRODUCTION

Rate-splitting multiple access (RSMA) technique has been considered as a powerful multiple access strategy and interference management technique for the sixth generation (6G) wireless networks [1]. Particularly, the RSMA technique was firstly introduced in [2], which creatively separates the user messages into common and private parts, and encodes the former into common stream while encoding the latter into separate streams. Then, in [3] and [4], the authors utilized the RSMA in multiuser multiple-input single-output (MISO) networks and massive multiple-input multiple-output (MIMO) channels, respectively. The results of the above works confirmed that RSMA is a promising technique to reduce the inter-user interference and enhance the transmission robustness especially when channel state information (CSI) can not be fully attained at the transmitter in practice.

With these advantages, RSMA was systematically presented in [5], which is a more general case of the space-division multiple access (SDMA) or non-orthogonal multiple access (NOMA) scheme [6]. To be specific, RSMA utilizes linearly precoded rate-splitting (RS) at the transmitter and successive interference cancellation (SIC) at the receiver, which decodes part of the interference and treats the others as noise [7]. Existing works have revealed that RSMA outperforms the orthogonal multiple access (OMA), linear SDMA, and power-domain NOMA, in the aspects of the spectral efficiency (SE) [8]-[11], max-min fairness [12], [13], and energy efficiency (EE) [14], [15].

Meanwhile, following the breakthroughs on the fabrication of programmable metamaterials, reconfigurable intelligent surface (RIS) has emerged as an effective technique to enhance the SE, EE and network coverage, etc. [16]. RIS is a planar array with a large number of low-cost reflective elements, which reflect the incident signal to a desired direction via controlling the phase shifts [17]. Considering that RS only reflects the received signal without decoding and regenerating signals process, the power consumption and hardware cost of RIS are much lower than those of the conventional radio frequency relay [18]. In addition, RIS can be easily deployed on the facades of buildings, ceilings of factories and indoor spaces [19]. Due to the above advantages, RIS has attracted a great deal of research attention.

For example, L. Wei et al. in [20] proposed a factor decomposition-aided channel estimation technique for RIS-
assisted wireless networks. Then, the authors further developed a message passing algorithm to achieve joint channel estimation and signal recovery for RIS-aided networks in [21]. The authors in [22] studied the sum rate and fairness optimization in RIS-aided systems. Besides, [23] studied the fairness design in RIS-aided multi-user networks, where a majorization-maximization (MM)-based method was developed. Then, the work [24] proposed a two-timescale beamforming (BF) scheme for an RIS-enhanced wireless network, where the passive BF was designed with the statistical CSI assumption, while the active BF was designed using the instantaneous CSI. Recently, [25] studied the sum rate maximization design for multiuser MISO networks using deep reinforcement learning, and the work has been extended in [26] when considering multiple cascaded RISs. Moreover, [27] investigated the new fully-connected and group-connected RIS architectures based on reconfigurable impedance networks that can alter both the magnitudes and phases of the incident signals. At present, RIS-aided transmission design has been widely studied in various aspect, such as RSMA networks [28], NOMA networks [29], symbiotic radio networks [30], the full-duplex communication scenario [31], the physical-layer security scenario [32], the millimeter-wave networks [33], hybrid terrestrial-aerial networks [34], and unmanned aerial vehicle networks [35], etc.

Typically, for passive or nearly-passive RIS, the reflected signals have to go through a cascaded channel composed of the transmitter-RIS and RIS-receiver links, leading to serious degradation of the system performance caused by the double fading attenuation. To overcome this shortcoming, the authors in [36] proposed an active RIS architecture in which active load impedance is used by each reflection element. By converting direct current bias power into radio frequency power, the active element can directly amplify the incident signal [37]. Then, in [38], the authors considered the effect of active RIS in improving the achievable secrecy rate of cognitive satellite-terrestrial network. In [39], the authors studied the active RIS-enabled energy-constrained wireless network and showed the superiorities of the active RIS in supporting multiple energy-limited devices. In [40], the authors demonstrated that active RIS achieves better EE performance than passive RIS. Recently, the work in [41] suggested that active RIS outperforms passive RIS in terms of the weighted sum secrecy rate. The authors in [42] compared whether active RIS is superior to passive RIS or not under the same power budget, and revealed that active RIS is superior if the RIS power budget exceeds certain value.

Among the above works, the SE is a commonly used performance metric and usually formulated as the objective of optimization problems [24], [31]. On the other hand, the EE, which is defined as the ratio of the signal data rate to the total power consumption, is one of the key performance metrics in green communication oriented networks [18], [43]. Specifically, [44] investigated the EE optimization in satellite-terrestrial networks with RSMA, where a penalty-based method was proposed. In [45], the authors investigated the EE maximization in RIS-assisted downlink transmission, where a multi-objective optimization (MOO) framework was proposed. Then, [46] studied the EE optimization in RIS-assisted uplink network. It is noted that the above works mainly focused on the SE or EE optimization. However, SE would still increase while EE remains stable with the increasing power in a high power region. Therefore, the SE and EE are not linearly correlated and can not simultaneously increase or decrease with the changing transmit power [47]. Thus, a fundamental tradeoff between the above two metrics needs to be investigated for meeting various communication requirements in future networks.

To be specific, [48] studied the SE-EE tradeoff optimization in a downlink RSMA network. Recently, [49] studied the MOO for the SE-EE tradeoff in the RIS-assisted cognitive radio network. The authors in [50] proposed a new metric called resource efficiency (RE) in RIS-enabled networks to obtain the tradeoff between EE and SE, where an MM-based approach was developed. In addition, [51] investigated the fully-connected RIS-aided RSMA scheme, where the authors proposed a weighted minimum mean square error (WMMSE)-based method to optimize the transmit beamformer and the scattering matrix of the RIS. Then, the authors of [52] proposed the group-connected RIS-based RSMA design to achieve the tradeoff between the beam controlling accuracy and hardware complexity of fully connected RIS. However, the RE optimization in RIS-assisted RSMA networks has not been studied, and it is worth investigating whether active RIS outperforms passive RIS in terms of the RE.

Motivated by the above facts, we focus on the RE optimization in the RIS-assisted RSMA network in this paper. To the best of the authors’ knowledge, it is the first work to investigate the active RIS-enabled transmission design in RSMA system from the perspective of SE and EE tradeoff. Our main contributions are summarized as follows:

- We investigate a novel integration of RSMA and RIS into cellular systems in this paper, where the RIS with active reflection elements is deployed to mitigate the double fading effect and one-layer RSMA is adopted at the base station (BS) to suppress the inter-user interference. To achieve the SE-EE tradeoff, we optimize the RE metric by jointly designing the transmit precoding and reflecting BF, while guaranteeing transmit and forward power constraints of the BS and RIS, as well as the common message rate constraint.

- To efficiently tackle the non-convex problems due to its fractional objective, coupled optimization variables, and discrete coefficient constraint, we propose a two-stage approach for the one-layer RSMA transceiver structure. For the outer stage optimization, a quadratic transformation is used to recast the fractional objective into a linear form and obtain a closed-form solution. While for the inner stage problem, we tackle the non-concave common and private signal rate by the first-order Taylor expansion method, which approximates the logarithmic objective into a quadratic function. Then, an alternating optimization (AO) algorithm is developed to obtain the BS precoding and reflective BF iteratively, by resorting to the penalty dual decomposition (PDD) method.

- The proposed algorithm can be extended to the two-layer
RSMA scenario, which is rarely investigated in open literature, and can also handle the common message rate constraint and effectively solve quadratically constrained quadratic program (QCQP) problems with fast convergence. Besides, the proposed algorithm is also applicable to passive-RIS design, by setting the zero noise power at the RIS and normalizing the amplitude of each reflection element. Thus, the proposed scheme is actually an unified optimization framework, which is suitable for various RIS-assisted RSMA networks.

- Simulation results verified the effectiveness of the proposed scheme and provide some insightful analysis: 1) RE is an effective metric to tradeoff the SE and EE performance by adjusting the weight; 2) RSMA outperforms other multiple access techniques such as SDMA; 3) active RIS obtain better RE performance than passive RIS, and the discrete coefficient is a better choice compared the continuous coefficient in terms of RE due to the lower power consumption of former.

The rest of this paper is organized as follows. Section II presents the system model and problem. Section III investigates the joint BF and reflecting coefficient design. Section IV extends the proposed design. Section V illustrates the simulations and section VI is the conclusion.

Notations: Throughout the paper, the upper case boldface letters and lower case boldface letters are used to represent matrices and vectors, respectively. $\text{Tr} (\Sigma)$, $\Sigma^{H}$, $\Sigma^{T}$, and $\Sigma^*$ denote the trace, the Hermitian transpose, the transpose, and the conjugate of matrix $\Sigma$, respectively. The diagonal matrix with diagonal elements $\sigma_{1}, \ldots, \sigma_{N}$ is denoted by $\text{Diag} (\sigma_{1}, \ldots, \sigma_{N})$. $\text{diag} (\Sigma)$ denotes the main diagonal element of matrix $\Sigma$. $\Re \{\cdot\}$, $|\cdot|$, and $\angle (\cdot)$ denote the real part, the modulus, and the angle of a complex number, respectively. $x \sim \mathcal{CN} (\sigma, \Sigma)$ denotes a circularly symmetric complex Gaussian (CSCG) random vector with mean $\sigma$ and covariance matrix $\Sigma$. In addition, $\|\cdot\|$ and $\|\cdot\|_{F}$ represent the Euclidean norm and the Frobenius norm, respectively. Besides, $\circ$ denotes the element-wise product.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We present the system model of the RSMA network and formulate the optimization problem.

A. Active RIS Model

The reflecting coefficient matrix of the active RIS is given by $\Phi = \text{Diag} (\phi_{1}, \ldots, \phi_{N_{c}}) \in \mathbb{C}^{N_{c} \times N_{c}}$, where the reflecting coefficient of the $n$-th element is denoted by $\phi_{n} = \alpha_{n} e^{j\theta_{n}}$, $n = 1, \ldots, N_{t}$, with $\alpha_{n}$ and $\theta_{n}$ being the amplitude and the phase within the intervals $\alpha_{n} \in [0, \alpha_{n, \text{max}})$, and $\theta_{n} \in [0, 2\pi)$, respectively. Here, $\alpha_{n, \text{max}}$ denotes the predetermined maximum amplitude of the active load for the $n$-th element. Different from the commonly used passive RIS, $\alpha_{n, \text{max}}$ is not less than 1 for active RIS. Besides, active RIS only utilizes power amplifiers and phase-shift circuits that control and reflect the signals. No dedicated digital-to-analog converters (DACs), analog-to-digital converters (ADCs) and mixers are required. In contrast, relays are usually equipped with these mentioned electronic components for transmission, and low-noise amplifiers for reception, which leads to higher hardware cost and power consumption than active RIS.

Due to the practical hardware conditions, $\alpha_{n}$ and $\theta_{n}$ can only take discrete values. Let $Q_{\alpha}$ and $Q_{\theta}$ denote the quantization bits for $\alpha_{n}$ and $\theta_{n}$, respectively. Then we have $\phi_{n} \in \mathcal{X}_{d} \triangleq \left\{ \phi_{n} \mid \phi_{n} = \alpha_{n} e^{j\theta_{n}}, \alpha_{n} \in \mathcal{S}_{\alpha}, \theta_{n} \in \mathcal{S}_{\theta} \right\}$, where $\mathcal{S}_{\alpha} \triangleq \left\{ 0, \frac{\alpha_{n, \text{max}}}{2^{Q_{\alpha} - 2}}, \ldots, \frac{2^{Q_{\alpha} - 3} - 3}{2^{Q_{\alpha} - 2}}, \ldots, \frac{\alpha_{n, \text{max}}}{2^{Q_{\alpha}} - 2}, \ldots, \frac{2^{Q_{\alpha} - 1} - 2}{2^{Q_{\alpha}} - 2} \right\}$ denotes the amplitude set, i.e., uniformly values $2^{Q_{\alpha}}$ points in $[0, \alpha_{n, \text{max}})$, and $\mathcal{S}_{\theta} \triangleq \left\{ 0, \frac{2\pi}{2^{Q_{\theta} - 2}}, \ldots, \frac{2^{Q_{\theta} - 1} - 2\pi}{2^{Q_{\theta}} - 2} \right\}$ denotes the phase set, i.e., $\theta_{n}$ are equally valued in $[0, 2\pi)$.

B. Signal Model

As depicted in Fig. 1, we consider a multiuser MISO network which consists of a BS and $L$ users, denoted as $\mathcal{L} \triangleq \{1, \ldots, L\}$. The BS and RIS are equipped with $N_{s}$ antennas and $N_{r}$ elements, respectively, while the users are single antenna node. The channel from BS to RIS, from RIS to the $l$-th user and from BS to the $l$-th user are denoted by $\mathbf{F} \in \mathbb{C}^{N_{t} \times N_{s}}$, $\mathbf{h}^{H}_{l} \in \mathbb{C}^{1 \times N_{r}}$, and $\mathbf{g}^{H}_{l} \in \mathbb{C}^{1 \times N_{r}}$, respectively. The instantaneous CSI for these links are available at the BS and RIS. For more details about the channel estimation technique for RIS-assisted networks, readers can refer to [20], [21]. In addition, there exists a RIS controller to adjust the amplitudes and phases of the reflection elements [17].

Inspired by [7], one-layer RSMA is an alternative and effective multiple access technique to manage the inter-user interference among these users with relatively low implementation complexity. Hence, we adopt the one-layer RSMA at the BS, where the transceiver architecture is shown in Fig. 2. Particularly, the BS adopts the message combiner and linear precoding to split the message $\mathbf{M}_{l}$ into two sections, namely, a common part $M_{c,l}$ and a private part $M_{p,l}$. $M_{c,1}, \ldots, M_{c,L}$ are combined into a common message $M_{c}$ and encoded into a common signal $s_{c}$ using a codebook shared by all users. On the contrary, $M_{p,1}, \ldots, M_{p,L}$ are separately
Messages for L users

Message splitting

Message combiner

Linear precoder

Decode

Encoder

Combine

Decode

Base station

User-1

Channel

Precoder

Combine

M-

User-L

Receivers

Fig. 2: Transceiver architecture of one-layer RSMA.

encoded into the private signals $s_1, \ldots, s_L$, which are only decoded by the specified user. Therefore, the whole signal $s = [s_c, s_1, \ldots, s_L]^T \in \mathbb{C}^{(L+1) \times 1}$ is generated for RSMA transmission. Then, $s$ is linearly precoded via a precoding matrix $W = [w_c, w_1, \ldots, w_L]^T \in \mathbb{C}^{N_s \times (L+1)}$ with $w_c$ and $w_l \in \mathbb{C}^{N_s \times 1}, \forall l \in \mathcal{L}$ being the corresponding precoding vector for $s_c$ and $s_l$, respectively. Thus, the transmitted signal is given as

$$x = w_c s_c + \sum_{l=1}^{L} w_l s_l. \quad (2)$$

Then, the received signal at the $l$-th user, $l \in \mathcal{L}$ is given by

$$y_l = (g_l^H + h_l^H \Phi F) x + h_l^H \Phi n_l + n_l, \quad (3)$$

where $n_l \sim \mathcal{CN}(0, \sigma_n^2 I)$ and $n_l \sim \mathcal{CN}(0, \sigma_l^2)$ denote the effective noise including the self-interference and the additive white Gaussian noise (AWGN) at the RIS and the AWGN at the $l$-th user, respectively. The $l$-th user first decodes $s_c$, into $\hat{s}_c$ by treating $s_l, \forall l \in \mathcal{L}$ as noise. Therefore, the signal-to-interference-plus-noise ratio (SINR) to decode $s_c$ at the $l$-th user is

$$\Gamma_{c,l} = \frac{\|\hat{h}_l^H w_c\|^2}{\sum_{i=1}^{L} \|\hat{h}_l^H w_i\|^2 + \|h_l^H \Phi\|^2 \sigma_n^2 + \sigma_l^2}, \quad (4)$$

where $\hat{h}_l = (g_l^H + h_l^H \Phi F)^H$ is the equivalent channel from BS to the $l$-th user.

After decoding and subtracting $s_c$, from $y_l$, the $l$-th user decodes $s_l$ by regarding other private signals $s_j, j \neq l, \forall j \in \mathcal{L}$ as the interference. Thus, the SINR to decode $s_l$ at the $l$-th user is expressed as

$$\Gamma_{p,l} = \frac{\|\hat{h}_l^H w_l\|^2}{\sum_{i=1, i \neq l}^{L} \|\hat{h}_l^H w_i\|^2 + \|h_l^H \Phi\|^2 \sigma_n^2 + \sigma_l^2}. \quad (5)$$

C. System SE and EE

The corresponding achievable rates of $s_c$ and $s_l$ at the $l$-th user are $R_{c,l} = \log_2 (1 + \Gamma_{c,l})$ and $R_{p,l} = \log_2 (1 + \Gamma_{p,l})$, respectively. To guarantee that all users can successfully decode $s_c$, $R_c$ must satisfy $R_c = \min \{R_{c,1}, \ldots, R_{c,L}\}$.

According to [7], the overall achievable rate of the network is the sum of $R_c$ and $R_{p,l}$, and is given by

$$\eta_{SE}(W, \Phi) = R_c + \sum_{l=1}^{L} R_{p,l} \quad \text{(bits/s/Hz)}. \quad (6)$$

Besides, the total power consumption is given by

$$P_{tot}(W, \Phi) = \varepsilon_s \|W\|_F^2 + \varepsilon_r \left(\left\|\Phi F W\right\|_F^2 + \sigma_n^2 \|\Phi\|_F^2\right) + P_s, \quad (7)$$

where $\varepsilon_s$ and $\varepsilon_r$ are the inverses of the power amplification coefficients at the BS and RIS, respectively, and $P_s$ denotes the total static power consumption which is independent of $\{W, \Phi\}$ and given by

$$P_s = N_s P_a + N_R (P_r + P_{DC}), \quad (8)$$

where $P_a$ denotes the power dissipation per antenna at the BS, $P_s$ is the static circuit power consumption at the BS, $P_r$ incorporates the power consumption of the circuit at each reflection element [50]. According to [18], $P_r$ are 1.5, 4.5, 6.0, and 7.8 mW for 3-, 4-, 5-, and 6-bit quantization of each element, respectively. Besides, $P_{DC}$ is the direct current biasing power consumption of a single reflection element [36].

Therefore, the EE is given by

$$\eta_{EE}(W, \Phi) = \frac{\eta_{SE}(W, \Phi)}{P_{tot}(W, \Phi)} \quad \text{(bits/Joule)}, \quad (9)$$

where $B$ is the system bandwidth.

D. Problem Formulation

Rather than considering SE or EE as the objective, we aim to obtain the SE-EE tradeoff by using a weighted sum representation, i.e., maximizing $(1 - \delta) \eta_{EE} + \delta \eta_{SE}, 0 \leq \delta \leq 1$. 

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Nevertheless, it is improper to directly combine \( \eta_{EE} \) and \( \eta_{SE} \) because they have different units. Instead, we study a system indicator called RE, which is given in [47], [50]:

\[
\eta_{RE}(W, \Phi) \triangleq \frac{\eta_{EE}(W, \Phi)}{B} + \frac{\beta \eta_{SE}(W, \Phi)}{P_{sum}} \quad \text{(bits/J/Hz)},
\]

where \( \beta > 0 \) is the weight used to denote the priorities of EE and SE. The denominator \( P_{sum} \) in the second term is fixed and given by

\[
P_{sum} = \varepsilon_s P_{s,max} + \varepsilon_r P_{r,max} + P_c. \quad \text{(11)}
\]

\( P_{sum} \) denotes the whole power budget of the considered network. By multiplying \( \eta_{EE} \) and \( \eta_{SE} \) with \( 1/B \) and \( 1/P_{sum} \), the units of the two terms in (10) are both normalized into bits/Joule/Hz. Moreover, letting \( \frac{\beta P_{tot}}{P_{sum}} = \delta/(1-\delta) \) and substituting it into (10), the maximization of \( \eta_{RE} \) is equivalent to that of \( (1-\delta) \eta_{EE} + \delta \eta_{SE}, 0 \leq \delta \leq 1 \).

Mathematically, the RE optimization is formulated as:

\[
\begin{align*}
\max_{W, \Phi} & \quad \eta_{RE}(W, \Phi) \\
\text{s.t.} & \quad ||W||_F^2 \leq P_{s,max}, \\
& \quad ||\Phi FW||_F^2 + \sigma^2 ||\Phi||_F^2 \leq P_{r,max}, \\
& \quad |||\Phi||_F^2 \leq \alpha_{n,max}, \phi_n \in \mathcal{X}_d, \forall n, \\
& \quad R_c \geq R_{c,min},
\end{align*}
\]

where \( P_{s,max} \) and \( P_{r,max} \) are the transmit and forward power budgets at the BS and RIS, respectively. Besides, \( [\Phi]_n \) denotes the \( n \)-th element of \( \Phi \). It is known that \( [\Phi]_n = \phi_n \). In addition, \( R_{c,min} \) is the pre-determined minimum common message rate. In fact, different SE-EE tradeoff designs can be realized by changing \( \beta \). For example, \( \eta_{RE} \) relaxes to \( \eta_{SE} \) when \( \beta \to \infty \) and relaxes to \( \eta_{EE} \) when \( \beta = 0 \) with bandwidth normalization.

Unfortunately, problem (12) is challenging to solve where the major difficulties can be summarized as follows: 1) It is complicated to tackle the coupled variables \( W \) and \( \Phi \) jointly, especially for the case of large \( N_s \) or \( N_r \); 2) \( \eta_{RE} \) is a transformation from \( \eta_{EE} \), which means that the optimization of the former is much more complex than the latter; 3) The optimization of \( \Phi \) with a discrete coefficient constraint is a mixed integer program, and is also non-convex.

In the next section, we aim to develop an efficient approach to handle (12).

### III. JOINT BS PRECODING AND RIS BF DESIGN

We first linearize the RE objective by the quadratic transformation method proposed in [54]. Next, an AO algorithm is proposed to do the optimization iteratively.

#### A. Quadratic Transformation to Fractional Programming

To be specific, by introducing a slack variable \( \kappa \in \mathbb{R} \), we equivalently recast (12) into a non-convex formulation as:

\[
\begin{align*}
\max_{W, \Phi, \kappa} & \quad f(W, \Phi, \kappa) = 2\kappa \sqrt{\eta_{SE}(W, \Phi)} \quad \text{(13a)} \\
& \quad - \kappa^2 P_{tot}(W, \Phi) + \frac{\beta \eta_{SE}(W, \Phi)}{P_{sum}} \\
\text{s.t.} & \quad (12b) - (12c),
\end{align*}
\]

To solve (13), we optimize the prime variables \( \{W, \Phi\} \) and the slack variable \( \kappa \) in an iterative manner. According to [54], given \( \{W, \Phi\} \), the optimal \( \kappa \) can be directly obtained as

\[
\kappa = \frac{\sqrt{\eta_{SE}(W, \Phi)}}{P_{tot}(W, \Phi)} \quad \text{(14)}
\]

In the following part, we deal with the optimization of \( \{W, \Phi\} \) with a given \( \kappa \).

#### B. SE Approximation

In the previous subsection, we have transformed the fractional programming to a relatively simple formulation. However, given \( \kappa \), (13) is still unsolvable since \( \eta_{SE} \) is non-concave with respect to (w.r.t) \( W \) and \( \Phi \). Besides, the discrete coefficient constraint is also non-convex. To overcome this, in the following, we design a lower bound of \( \eta_{SE} \). Specifically, at the \( t \)-th iteration, around the given point \( \{W(t), \Phi(t)\} \), the following lemma is useful to approximate \( R_{c,t} \) and \( R_{p,t} \).

**Lemma 1** [56]: For any \( \delta \) and \( \gamma \), we have

\[
\log_2 \left( 1 + \frac{|\delta|^2}{\gamma} \right) \geq \log_2 \left( 1 + \frac{|\delta|^2}{\bar{\gamma}} \right) - \frac{|\delta|^2}{\bar{\gamma} \ln 2}
\]

\[
+ 2R \left( \frac{\delta^* \delta}{\bar{\gamma} \ln 2} - \frac{|\delta|^2 (\gamma + |\delta|^2)}{\bar{\gamma} \ln 2}, \right)
\]

where \( \bar{\delta}, \bar{\gamma} \) are fixed points.

Based on Lemma 1, \( R_{c,t} \) can be lower bounded as

\[
R_{c,t} \geq \log_2 \left( 1 + \frac{|c_{l,t}^* a_t|^2}{b_{l,t}^2} \right) - \frac{|a_{l,t}^2| b_{l,t}^2}{b_{l,t}^2 + |a_{l,t}^2|^2} + 2R \left( \frac{|c_{l,t}^* a_t|^2}{b_{l,t}^2} \right)
\]

\[
+ \frac{2R \left( \frac{|c_{l,t}^* a_t|^2}{b_{l,t}^2} \right)}{b_{l,t}^2 + |a_{l,t}^2|^2} \cdot \left( \frac{|c_{l,t}^* a_t|^2}{b_{l,t}^2} \right)
\]

\[
\geq \log_2 \left( 1 + \frac{|c_{l,t}^* a_t|^2}{b_{l,t}^2 - |c_{l,t}^* a_t|^2} \right) - \frac{|c_{l,t}^* a_t|^2}{b_{l,t}^2 - |c_{l,t}^* a_t|^2} + 2R \left( \frac{|c_{l,t}^* a_t|^2}{b_{l,t}^2} \right)
\]

\[
+ \frac{2R \left( \frac{|c_{l,t}^* a_t|^2}{b_{l,t}^2} \right)}{b_{l,t}^2 - |c_{l,t}^* a_t|^2} \cdot \left( \frac{|c_{l,t}^* a_t|^2}{b_{l,t}^2} \right)
\]

\[
\geq \log_2 \left( 1 + \frac{|c_{l,t}^* a_t|^2}{b_{l,t}^2} \right) - \frac{|c_{l,t}^* a_t|^2}{b_{l,t}^2} + 2R \left( \frac{|c_{l,t}^* a_t|^2}{b_{l,t}^2} \right)
\]

\[
+ \frac{2R \left( \frac{|c_{l,t}^* a_t|^2}{b_{l,t}^2} \right)}{b_{l,t}^2} \cdot \left( \frac{|c_{l,t}^* a_t|^2}{b_{l,t}^2} \right)
\]

where \( a_{l,t} = (g_{l,t}^H + h_{l,t}^H \Phi(t) F) w_{l,t} \), \( b_{l,t} = (g_{l,t}^H + h_{l,t}^H \Phi(t) F) w_{l,t} + \sigma_{l,t}^2 + \sigma_{l,t}^2 \), and \( c_{l,t} = (g_{l,t}^H + h_{l,t}^H \Phi(t) F) w_{l,t} \), \( \Phi(t) \) w_{l,t}.

Based on the above reformulation, an approximated version of \( \eta_{SE}(W, \Phi) \), which is denoted as \( \tilde{\eta}_{SE}(W, \Phi) \), is achieved.
by solving the following problem

\[
\max_{\mathbf{W}, \Phi} R_c + \sum_{l=1}^{L} \left\{ 2R \left\{ \left( \mathbf{c}_l^{(t)} \right)^* \left( \mathbf{g}_l^H + \mathbf{h}_l^H \mathbf{F} \mathbf{F} \right) \mathbf{w}_l \right\} + \log_2 \left( 1 + \frac{\left| \mathbf{c}_l^{(t)} \right|^2}{b_l^{(t)} - \left| \mathbf{c}_l^{(t)} \right|^2} \right) - \frac{\left| \mathbf{c}_l^{(t)} \right|^2}{b_l^{(t)} - \left| \mathbf{c}_l^{(t)} \right|^2} \ln 2 \right\} \right\}
\]

(18a)

\[
\begin{align*}
\text{s.t.} & \quad R_c \leq \frac{2R \left\{ \left( \mathbf{a}_l^{(t)} \right)^* \left( \mathbf{g}_l^H + \mathbf{h}_l^H \mathbf{F} \mathbf{F} \right) \mathbf{w}_l \right\}}{b_l^{(t)} \ln 2} \\
& \quad \log_2 \left( 1 + \frac{\left| \mathbf{a}_l^{(t)} \right|^2}{b_l^{(t)} - \left| \mathbf{a}_l^{(t)} \right|^2} \right) - \frac{\left| \mathbf{a}_l^{(t)} \right|^2}{b_l^{(t)} - \left| \mathbf{a}_l^{(t)} \right|^2} \ln 2 + \frac{1 + \sigma_l^2}{b_l^{(t)} + \left| \mathbf{a}_l^{(t)} \right|^2} \\
& \quad - \frac{\left| \mathbf{a}_l^{(t)} \right|^2}{b_l^{(t)} + \left| \mathbf{a}_l^{(t)} \right|^2} \ln 2 \right\} \\
& \quad \left\{ \left( \mathbf{a}_l^{(t)} \right)^* \left( \mathbf{g}_l^H + \mathbf{h}_l^H \mathbf{F} \mathbf{F} \right) \mathbf{w}_l \right\} + \log_2 \left( 1 + \frac{\left| \mathbf{a}_l^{(t)} \right|^2}{b_l^{(t)} - \left| \mathbf{a}_l^{(t)} \right|^2} \right) - \frac{\left| \mathbf{a}_l^{(t)} \right|^2}{b_l^{(t)} - \left| \mathbf{a}_l^{(t)} \right|^2} \ln 2 \right\} \right\} \\
& \quad \left\{ \left( \mathbf{w}_l \mathbf{P} \mathbf{w}_l + \mathbf{p}_l \right) \right\} \right\}
\]

(18b) – (18c)

In fact, an effective way to handle the SE optimization of the RSMA network is by solving (18). Next, we will tackle the RE optimization with the assistance of \( \eta_{\text{SE}} (\mathbf{W}, \Phi) \). However, (18) is not jointly convex w.r.t. \( \mathbf{W} \) and \( \Phi \). Fortunately, (18) can be decoupled into two subproblems. Then, the solution to (18) can be obtained in an alternating method. Next, we will focus on the formulation and solution of the subproblems.

C. Precoding Optimization

Here, we optimize \( \mathbf{W} \) with fixed \( \Phi \). According to (18) and after some mathematical manipulations, the subproblem w.r.t. \( \mathbf{W} \) is given as

\[
\max_{\mathbf{W}} 2\kappa \sqrt{R_c + \sum_{l=1}^{L} \left\{ 2R \left\{ \mathbf{p}_l^H \mathbf{w}_l \right\} - \mathbf{w}_l^H \mathbf{P} \mathbf{w}_l + \mathbf{p}_l \right\} + \kappa^2 P_{\text{tot}} (\mathbf{W}, \Phi) - \beta \left( \mathbf{R}_c + \sum_{l=1}^{L} \left\{ 2R \left\{ \mathbf{p}_l^H \mathbf{w}_l \right\} - \mathbf{w}_l^H \mathbf{P} \mathbf{w}_l + \mathbf{p}_l \right\} \right) \right\}
\]

(19a)

\[
\begin{align*}
\text{s.t.} & \quad R_c \leq -\text{Tr} \left( \mathbf{Q} \mathbf{W} \mathbf{W}^H \right) + 2R \left\{ \mathbf{q}_l^H \mathbf{w}_l \right\} + \mathbf{q}_l, \forall l, \left(12b\right), \left(12c\right), \left(12e\right),
\end{align*}
\]

(19b)

where \( \left\{ \mathbf{P}, \mathbf{p}_l, \mathbf{p}_l, \mathbf{Q}, \mathbf{q}_l, \mathbf{q}_l \right\} \) are respectively, given by (20) in the next page.

Note that with fixed \( \Phi \), \( P_{\text{tot}} (\mathbf{W}, \Phi) \) is convex w.r.t. \( \mathbf{W} \), and \( R_c + \sum_{l=1}^{L} \left\{ 2R \left\{ \mathbf{p}_l^H \mathbf{w}_l \right\} - \mathbf{w}_l^H \mathbf{P} \mathbf{w}_l + \mathbf{p}_l \right\} \) is a concave function w.r.t. \( \mathbf{W} \), so that the square-root is also concave. Thus, with fixed \( \kappa \) and \( \Phi \), (19) is convex w.r.t. \( \mathbf{W} \), which can be solved by the optimization toolbox CVX [55].
$$P = \sum_{l=1}^{L} \frac{|c_l(t)|^2}{b_l(t)} \left( g_l^H + h_l^H \Phi(t) F \right) \left( g_l^H + h_l^H \Phi(t) F \right)^H, \quad p_l = \frac{|c_l(t)|^2}{b_l(t)} \ln 2, $$

$$q_l = \log_2 \left( 1 + \frac{|a_l(t)|^2}{b_l(t)} \right) \frac{b_l(t)}{|a_l(t)|^2} \ln 2, \quad q_l = \frac{|a_l(t)|^2}{b_l(t)} \left( \| h_l^H \Phi(t) F \|^2 \sigma_r^2 + \sigma_i^2 \right), $$

$$Q_l = \frac{|a_l(t)|^2}{b_l(t)} \left( g_l^H + h_l^H \Phi(t) F \right) \left( g_l^H + h_l^H \Phi(t) F \right)^H, \quad q_l = \frac{|a_l(t)|^2}{b_l(t)} \left( \| h_l^H \Phi(t) F \|^2 \sigma_r^2 + \sigma_i^2 \right).$$

$$T = \sum_{l=1}^{L} \frac{(c_l(t))^*}{b_l(t)} F w_l(t)^H h_l^H \left( F \sum_{l=1}^{L} w_l(t)^H g_l^H h_l^H \right), \quad U = \sum_{l=1}^{L} \frac{|c_l(t)|^2}{b_l(t)} h_l h_l^H,$$

$$u = \sum_{l=1}^{L} \left\{ \log_2 \left( 1 + \frac{|c_l(t)|^2}{(b_l(t) - |c_l(t)|^2)} \right) - \frac{|c_l(t)|^2}{(b_l(t) - |c_l(t)|^2)} \ln 2 \right\} \frac{|c_l(t)|^2}{b_l(t)} \left( b_l^2 - |c_l(t)|^2 \right) \ln 2, \quad \frac{\sum_{l=1}^{L} |g_l h_l^H|}{b_l(t)} + 2 \Re \left\{ (c_l(t))^* g_l^T f_l(t) \right\} \ln 2, $$

$$D_l = \frac{(a_l(t))^*}{b_l(t)} F w_l(t)^H - \frac{|a_l(t)|^2}{b_l(t)} \left( F W_l(t) W_l(t)^H g_l^H h_l^H \right), \quad V_l = \frac{|a_l(t)|^2}{b_l(t)} h_l h_l^H,$$

$$d_l = \log_2 \left( 1 + \frac{|a_l(t)|^2}{b_l(t)} \right) \frac{b_l(t)}{|a_l(t)|^2} \ln 2 - \frac{|a_l(t)|^2}{b_l(t)} \left( \sigma_r^2 + |g_l W_l(t)|^2 \right) \ln 2, \quad \frac{\sum_{l=1}^{L} |g_l|}{b_l(t) + |a_l(t)|^2} + 2 \Re \left\{ (a_l(t))^* g_l^H f_l(t) \right\} \ln 2.$$

**augmented Lagrange (AL) of (25) can be obtained**

$$\min \phi, \omega, \lambda \quad \frac{-2 \kappa}{R_c + u + 2 \Re \left\{ \Phi^T \text{diag} (T) \right\} - \Phi^H \bar{U} \Phi} + \kappa^2 P_{ol} \left( W, \Phi \right) + \frac{1}{2 \varphi} \left\| \omega - \varphi \lambda \right\|^2, \quad \text{s.t.} \quad (25b), \quad \omega_n \in X_n, \forall n, \quad \lambda \in C^{M \times 1}, \quad \lambda \geq 0$$

(26a)

$$\min \phi, \omega, \lambda \quad \frac{-2 \kappa}{R_c + u + 2 \Re \left\{ \Phi^T \text{diag} (T) \right\} - \Phi^H \bar{U} \Phi} + \kappa^2 P_{ol} \left( W, \Phi \right) + \frac{1}{2 \varphi} \left\| \omega - \varphi \lambda \right\|^2, \quad \text{s.t.} \quad (25b), \quad \omega_n \in X_n, \forall n, \quad \lambda \in C^{M \times 1}, \quad \lambda \geq 0$$

(26b)

**The PDD procedure is composed of two stages. In the outer stage, we optimize \( \phi \) and \( \lambda \), while in the inner stage, we decouple (26) into two problems and obtain \( \phi \) and \( \omega \) alternately. First, we optimize \( \phi \), given \( \omega \). The problem is given as**

$$\min \phi \quad \frac{-2 \kappa}{R_c + u + 2 \Re \left\{ \Phi^T \text{diag} (T) \right\} - \Phi^H \bar{U} \Phi} + \kappa^2 P_{ol} \left( W, \Phi \right) + \frac{1}{2 \varphi} \left\| \omega - \varphi \lambda \right\|^2, \quad \text{s.t.} \quad (25b), \quad \omega_n \in X_n, \forall n, \quad \lambda \in C^{M \times 1}, \quad \lambda \geq 0$$

(27a)

$$\min \omega \quad \frac{-2 \kappa}{R_c + u + 2 \Re \left\{ \Phi^T \text{diag} (T) \right\} - \Phi^H \bar{U} \Phi} + \kappa^2 P_{ol} \left( W, \Phi \right) + \frac{1}{2 \varphi} \left\| \omega - \varphi \lambda \right\|^2, \quad \text{s.t.} \quad (25b), \quad \omega_n \in X_n, \forall n, \quad \lambda \in C^{M \times 1}, \quad \lambda \geq 0$$

(27b)

**Since \( \omega_n \) are decoupled from each other in (28), the optimal solution is \( \omega_n^* = \bar{\alpha}_n e^{i \bar{\theta}_n} \), where**

$$\bar{\alpha}_n = \arg \min_{\omega_n \in \mathbb{S}_n} \left\{ \alpha_n e^{i \theta_n} - \Phi^H \Phi \right\}, \quad \bar{\theta}_n = \arg \min_{\omega_n \in \mathbb{S}_n} \left\{ \theta_n - \lambda (\omega_n + \varphi \lambda_n) \right\}. \quad \text{respectively.}$$

The inner stage alternatively updates \( \phi \) and \( \omega \) until the stopping criterion is met. Then, for the outer stage iteration,
Algorithm 1 The PDD Algorithm.

1: Initialize \( \phi^{(0)}, \omega^{(0)}, \lambda^{(0)}, \varphi^{(0)} \), and set \( k = 1 \);
2: repeat
3: Set \( \phi^{(k-1,t)} = \phi^{(k-1)}, \omega^{(k-1,t)} = \omega^{(k-1)} \), and \( t = 0 \);
4: repeat
5: Obtain \( \phi^{(k-1,t+1)} \) via solving problem (27);
6: Obtain \( \omega^{(k-1,t+1)} \) via solving problem (28);
7: \( t \leftarrow t + 1 \);
8: until Convergence;
9: \( \phi^{(k)} \leftarrow \phi^{(k-1,t)} \), \( \omega^{(k)} \leftarrow \omega^{(k-1,t)} \);
10: \( \lambda^{(k)} \leftarrow \lambda^{(k-1)} + \frac{1}{\rho \varphi^{(k-1)}} \);
11: \( k \leftarrow k + 1 \);
12: until \( \| \phi^{(t)} - \omega^{(t)} \| \leq \epsilon_{1} \) or the maximum number of iteration is met.
13: Output \( \phi^{*} \).

Algorithm 2 Quadratic transformation algorithm for solving problem (12).

1: Solve (32) and obtain \( W^{(0)}, \Phi^{(0)} \), initialize \( \kappa^{(0)} \) and set \( k = 1 \);
2: repeat
3: Set \( W^{(k-1,t)} = W^{(k-1)}, \Phi^{(k-1,t)} = \Phi^{(k-1)} \), and \( t = 0 \);
4: repeat
5: Obtain \( W^{(k-1,t+1)} \) via solving problem (19);
6: Obtain \( \Phi^{(k-1,t+1)} \) via solving problem (24);
7: \( t \leftarrow t + 1 \);
8: until Convergence;
9: \( W^{(k)} \leftarrow W^{(k-1,t)}, \Phi^{(k)} \leftarrow \Phi^{(k-1,t)} \);
10: Update \( \kappa \) by (14);
11: \( k \leftarrow k + 1 \);
12: until \( \kappa^{(k)} - \kappa^{(k-1)} \leq \epsilon_{2} \) or the maximum number of iteration is reached.
13: Output \( W^{*}, \Phi^{*} \), and \( \kappa^{*} \).

E. Overall Algorithm and Analysis

Combining the proposed steps above, we obtain the integrated RE maximization approach in Algorithm 2, where \( \epsilon_{2} \) denotes the stopping threshold and the initial point \( \{ W^{(0)}, \Phi^{(0)} \} \) is set as the optimal solution of (32). In addition, we have the following Theorem.

Theorem 1: Algorithm 2 generates a convergent sequence of the objective values of (12).

Proof: Please refer to Appendix A.

Algorithm 2 can tackle the SE optimization problem which is relaxed from RE optimization problem.

Next, we calculate the computational complexity of Algorithm 2. Since the complexities of (28) and (29) can be omitted when compared with those of (19) and (27), the complexity of Algorithm 2 is mainly determined by (19) and (27). According to [57], the complexity for solving a QCQP problem is given by \( O \left( \sqrt{m} \left( mn^{2} + n^{3} \right) \ln \left( \frac{2N_{r}}{\epsilon} \right) \right) \), where \( m \) denotes the number of variables and \( n \) denotes the number of constraints, and \( \epsilon \) is the solution accuracy. Specifically, (19) has \( (3L + 2) \) quadratic constraints and the dimension of the variable is \( N_{s} \), while (27) has \( (N_{s} + L + 1) \) constraints and the dimension of the variable is \( N_{s} \). Thus, the complexities of (19) and (27) are given by \( O \left( \sqrt{N_{s}} \left( 9N_{s}L^{2} + 27L^{3} \right) \ln \left( \frac{2N_{r}}{\epsilon} \right) \right) \) and \( O \left( \sqrt{N_{r}} \left( N_{s}(N_{s} + L)^{2} + (N_{r} + L)^{3} \right) \ln \left( \frac{2N_{r}}{\epsilon} \right) \right) \), respectively. Therefore, the overall computational complexity of Algorithm 2 is given by

\[
C = O \left( T_{c} \max \left\{ \sqrt{N_{s}} \left( 9L^{2}(N_{s} + 3L) \ln \left( \frac{2N_{r}}{\epsilon} \right) \right) , \sqrt{N_{r}} \left( 2N_{s} + L \right) \left( N_{r} + L \right)^{2} \ln \left( \frac{2N_{r}}{\epsilon} \right) \right\} \right)
\]

(30)

where \( T_{c} \) denotes the search times for the updating of \( \kappa \) in the outer layer.

Thus, Algorithm 2 has polynomial time complexity, which is suitable for implementation.

F. Feasibility of (12)

Note that problem (12) with the common message rate constraint (12e) might be infeasible. It is necessary to study the feasibility of solving (12). Therefore, we solve the following problem to check the feasibility of (12):

\[
\max_{W, \Phi} \tau \text{ s.t. } \min_{l} R_{c,l} \leq \frac{1}{\epsilon} \left( a_{l}^{(t)} \right) (g_{l}^{H} + h_{l}^{H} \Phi F) w_{l}
\]

(31a)

(31b)

If the obtained optimal value of (31) is larger than or equal to \( R_{c,\min} \), then (12) is feasible to solve. Otherwise, (12) is infeasible. In fact, the previously proposed AO algorithm can be utilized to solve (31). Proceedings in a similar way as we obtained the lower bound of \( R_{c,l} \), an approximated version of (31) is obtained as follows

\[
\max_{W, \Phi} \tau \text{ s.t. } \tau \leq 2R \left\{ \left( a_{l}^{(t)} \right) (g_{l}^{H} + h_{l}^{H} \Phi F) w_{l} \right\} \ln 2
\]

(32a)

\[
- \frac{b_{l}^{(t)}}{b_{l}^{(t)}} \ln 2 \left( 1 + \frac{\sigma_{l}^{2}}{b_{l}^{(t)}} \right) + \log_{2} \left( 1 + \frac{b_{l}^{(t)}}{b_{l}^{(t)}} \right)
\]

(32b)

Then, an AO procedure is used to solve (32), which is similar to that in Algorithm 2. Thus, the feasibility of (12) can be checked.
IV. EXTENSION TO TWO-LAYER RSMA

Here, we extend the proposed design to the two-layer RSMA scenario, which is commonly used in the multi-group multi-cast communication scenario.

In a two-layer RSMA network, the $L$ users are separated into $I$ individual groups denoted as $\mathcal{I} = \{1, \ldots, I\}$ and group-$i$ contains $\mathcal{L}_i$ users with $\bigcup_{i \in \mathcal{I}} \mathcal{L}_i = \mathcal{L}$. User-$l$ splits its message $M_l$ into three sections, namely, an inter-group component $M_{l}^{I}$, an inner-group component $M_{l}^{c}$, and a private component $M_{l}^{p}$. Then, $\{M_{l}^{I} | l \in \mathcal{L}\}$ are wrapped into a common message $M_c$, which is encoded into a common signal $s_c$ using a codebook shared by all users and is decoded by all users. Then, $\{M_{l}^{c} | l \in \mathcal{L}_i\}$ are merged into a common message $M_{c,i}$, which is encoded into an inner-group common signal $s_{c,i}$ using a codebook shared by the users in group-$i$ and is decoded by these users. Finally, $\{M_{l}^{p} | l \in \mathcal{L}\}$ are independently encoded into $L$ signals $s_{p,1}, \ldots, s_{p,L}$, which are decoded by the specified users. The overall encoded streams $\mathbf{s} = [s_c, s_{c,1}, \ldots, s_{c,i}, s_{p,1}, \ldots, s_{p,L}]^T \in \mathbb{C}^{(L+I+1) \times 1}$ are linearly precoded with the precoding matrix $\mathbf{W} = [\mathbf{w}_c, \mathbf{w}_{c,1}, \ldots, \mathbf{w}_{c,i}, \mathbf{w}_{p,1}, \ldots, \mathbf{w}_{p,L}]^T \in \mathbb{C}^{N \times (L+I+1)}$. Hence, the signal sent from the BS is given as

$$\mathbf{x} = \mathbf{w}_c s_c + \sum_{i \in \mathcal{I}} \mathbf{w}_{c,i} s_{c,i} + \sum_{l \in \mathcal{L}} \mathbf{w}_{p,l} s_{p,l}. \tag{33}$$

Then, the received signal at user-$l$, $l \in \mathcal{L}$, is given as

$$y_l = \left(\mathbf{h}_l^H + \mathbf{h}_l^H \mathbf{\Phi} \mathbf{F}\right) \mathbf{x} + \mathbf{h}_l^H \mathbf{\Phi} \mathbf{n}_l + n_l. \tag{34}$$

User-$l$ ($l \in \mathcal{L}_i$) employs two layers of SIC to sequentially decode $s_{c,i}$, $s_{c,i},$ and $s_{p,l}$ with $s_c$ being decoded first, $s_{c,i}$ second, and followed by $s_{p,l}$. Then, the signal rates for decoding $s_c$, $s_{c,i}$, and $s_{p,l}$ at the $l$-th user are, respectively, given as

$$R_{c,l} = \log_2 \left(1 + \frac{\|\mathbf{h}_l^H \mathbf{w}_c\|^2}{\sum_{i \in \mathcal{I}} \|\mathbf{h}_l^H \mathbf{w}_{c,i}\|^2 + \sum_{j \in \mathcal{L}} \|\mathbf{h}_l^H \mathbf{w}_{p,j}\|^2 + \tilde{\sigma}_l^2}\right),$$

$$R_{c,i} = \log_2 \left(1 + \frac{\|\mathbf{h}_l^H \mathbf{w}_{c,i}\|^2}{\sum_{i' \in \mathcal{I}, i' \neq i} \|\mathbf{h}_l^H \mathbf{w}_{c,i'}\|^2 + \sum_{j \in \mathcal{L}} \|\mathbf{h}_l^H \mathbf{w}_{p,j}\|^2 + \tilde{\sigma}_l^2}\right),$$

$$R_{p,l} = \log_2 \left(1 + \frac{\|\mathbf{h}_l^H \mathbf{w}_{p,l}\|^2}{\sum_{i' \in \mathcal{I}, i' \neq i} \|\mathbf{h}_l^H \mathbf{w}_{c,i'}\|^2 + \sum_{j \in \mathcal{L}, j \neq l} \|\mathbf{h}_l^H \mathbf{w}_{p,j}\|^2 + \tilde{\sigma}_l^2}\right), \tag{35}$$

where $\tilde{\sigma}_l^2 = \|\mathbf{h}_l^H \mathbf{\Phi}\|^2 \sigma_r^2 + \sigma_l^2$.

Then, the message rates of $s_c$ and $s_{c,i}$ are given by

$$R_c = \min \left\{ R_{c,l} \mid l \in \mathcal{L} \right\},$$

$$R_{c,i} = \min \left\{ R_{c,i,l} \mid l \in \mathcal{L}_i \right\}, \tag{36}$$

Thus, the total achievable rate of the two-layer RSMA network is $R_c + \sum_{i=1}^I R_{c,i} + \sum_{l=1}^L R_{p,l}$ [7].

A two-layer RSMA transceiver architecture with 4 users is plot in Fig. 3, where user-1/2 is in group-1, user-3/4 is in group-2, respectively. $s_c$ is an inter-group common signal, while $s_{c,1/2}$ is the inner-group common signals for the users in group-1/2 only [7].

The RE optimization in two-layer RSMA is more complicated to handle than the counterpart in the one-layer RSMA. Fortunately, the proposed method can be used here with modifications. Firstly, we use the quadratic transformation to recast the fractional programming into a linear programming. Then,
in the outer stage optimization, we update $\kappa$ by using (14). While in the inner stage optimization, by utilizing Lemma 1 and introducing several slack variables, $\eta_{SE} (\mathbf{W}, \Phi)$ can be obtained by solving problem (37), with the relevant constants given in (38). Then, we further decouple (37) into two subproblems and obtain $\mathbf{W}$ and $\Phi$ using the corresponding methods. The whole procedure is similar to Algorithm 2. We omit the details due to space limitation.

\[ (12b) - (12e). \]

\[ (37d). \]

Here, we provide representative simulation results to verify the proposed design. As plot in Fig. 4, we assume one BS, one RIS, and 4 users, i.e., $L = 4$, without loss of generality. The BS and RIS are deployed at $(10 \text{ m}, 0 \text{ m}, 10 \text{ m})$ and $(0 \text{ m}, 50 \text{ m}, 10 \text{ m})$, while all users are randomly located in a circle with radius $5 \text{ m}$ and centered at $(10 \text{ m}, 50 \text{ m}, 1.5 \text{ m})$, respectively.

The following settings are adopted unless specified otherwise: $N_s = 8$, $N_r = 40$, and the maximum amplitude of the active RIS is $\alpha_{n_{\text{max}}} = 10, \forall n_{\text{r}}$ [36]. The common rate constraint is 2 bits/s/Hz [8]. Besides, the bandwidth is set as $B = 10 \text{ MHz}$ [47]. As for the power consumption model, we set $P_a = 30 \text{ dBm}$, $P_s = 40 \text{ dBm}$, $\epsilon_s = 1/0.9$ for the BS [47], and $P_r = -10 \text{ dBm}$, $P_{DC} = -5 \text{ dBm}$, $\epsilon_r = 1/0.8$ for the active RIS [36], respectively. Besides, the noise power is $\sigma^2 = -80 \text{ dBm}, \forall l$, and $\sigma^2 = -80 \text{ dBm}$ [40]. The path loss is $PL = PL_0 - 10\log_{10} \left( \frac{d}{d_{0}} \right)$, where $d$ indicates the link distance, and $\nu$ means the path loss exponent. Here, we set $PL_0 = -30 \text{ dB}$ and $d_{0} = 1 \text{ m}$. The exponents of the BS-users links and the RIS-related links are set as 4 and $2.2$ [32]. Besides, $F = \sqrt{\frac{r}{r+1}} F_{\text{LoS}} + \sqrt{\frac{1}{r+1}} F_{\text{NLoS}}$, with $r$ being

\[ a_i(t) = \left( g_i^H + h_i^H \Phi(t) F \right) w_c(t), \quad c_{i,l}(t) = \left( g_i^H + h_i^H \Phi(t) F \right) w_c(t), \quad x_l(t) = \left( g_i^H + h_i^H \Phi(t) F \right) w_p(t), \]

\[ b_l(t) = \sum_{i \in \mathcal{I}} \left| \left( g_i^H + h_i^H \Phi(t) F \right) w_c(t) \right|^2 + \sum_{j \in \mathcal{L}} \left| \left( g_i^H + h_i^H \Phi(t) F \right) w_p(t) \right|^2 + \left| h_l^H \Phi(t) \right|^2 \sigma^2 + \sigma^2, \]

\[ d_{i,l}(t) = \sum_{i' \in \mathcal{I}, \ i' \neq i} \left| \left( g_{i'}^H + h_{i'}^H \Phi(t) F \right) w_c(t') \right|^2 + \sum_{j \in \mathcal{L}} \left| \left( g_{i'}^H + h_{i'}^H \Phi(t) F \right) w_p(t') \right|^2 + \left| h_{i,l}^H \Phi(t) \right|^2 \sigma^2 + \sigma^2, \]

\[ y_l(t) = \sum_{i' \in \mathcal{I}, \ i' \neq i} \left| \left( g_{i'}^H + h_{i'}^H \Phi(t) F \right) w_c(t') \right|^2 + \sum_{j \in \mathcal{L}} \left| \left( g_{i'}^H + h_{i'}^H \Phi(t) F \right) w_p(t') \right|^2 + \left| h_{l}^H \Phi(t) \right|^2 \sigma^2 + \sigma^2. \]
the Rician factor. Here, $F^{\text{LoS}}$ denotes the line-of-sight (LoS) component and is given by $F^{\text{LoS}} = a_t a_r^H$. When a uniform planar array is utilized, $a_t$ is given as

$$a_t = \frac{1}{\sqrt{MN}}\begin{bmatrix} 1, \ldots, e^{j2\pi c(m\sin(\delta_t)\sin(\psi_t) + n\cos(\psi_t))}, \\ \ldots, e^{j2\pi c((M-1)\sin(\delta_t)\sin(\psi_t) + (N-1)\cos(\psi_t))}\end{bmatrix}^T$$

(39)

where $m$ and $n$ are the element indices in horizontal and vertical directions, $c$ is the normalized distance between adjacent elements, and $\delta_t$ and $\delta_q$ represent the azimuth and elevation angles, respectively. $a_r$ can be obtained similarly. $F^{\text{NLoS}}$ denotes the non-LoS component and is modeled as the Rayleigh variable. Besides, the solution accuracy is set as $\epsilon_1 = \epsilon_2 = 10^{-3}$ and the scaling factor is set as $\rho = 0.85$ [24].

Here, we compare the proposed design with several benchmarks: 1) the active RIS with continuous coefficient; 2) conventional single-connected passive RIS; 3) the active RIS-assisted SDMA scheme; 4) the fully-connected RIS [51]; 5) the group-connected RIS [52]. These schemes are labelled as “Proposed scheme, 3 bit”, “Proposed scheme, 4 bit”, “Continuous scheme”, “Passive RIS”, “SDMA scheme”, “Fully-connected RIS”, and “Group-connected RIS”, respectively, where 3 bit means $Q_{\alpha/\theta} = 3$ and 4 bit means $Q_{\alpha/\theta} = 4$, respectively.

### A. Convergence Behaviour

Firstly, the convergence of Algorithm 1 for different values of $N_s$, $N_r$, and $Q_{\alpha/\theta}$ is examined in Fig. 5. From this figure, it can be seen that the RE always increases with the number of iterations, and gradually converges within 20 iterations for various parameters, which demonstrates the efficiency of the PDD method.

Then, we study the convergence of Algorithm 2 for different values of $N_s$, $N_r$, and $Q_{\alpha/\theta}$. From Fig. 6, it can be seen that no matter what values of these parameters are selected, the RE increases with the number of iterations, and gradually converges almost within 20 iterations, which verifies the convergence behaviour of Algorithm 2.

### B. Performance Evaluation

Now, we evaluate the performances of difference schemes. Here, we set the sum of $P_{s,\text{max}}$ and $P_{r,\text{max}}$, which is denoted by $P_{\text{sum}}$, as a constant equals 10dBW for active RIS, unless specified otherwise. In addition, since the passive RIS has no transmit power consumption and no direct current biasing power consumption, we add the term $N_r P_{\text{DC}} + P_{r,\text{max}}/\epsilon_r$ to the transmit power budget at the BS when using the passive RIS for fair comparison. Thus, we can ensure that $P_{\text{sum}}$ is the same value regardless of the active RIS or passive RIS.

The SE performance achieved by Algorithm 2 is shown in Fig. 7, where we set $\delta = 1$. As expected, from Fig. 7, it can be seen that the SE performance always increases with $P_{s,\text{max}}$. 

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Fig. 4: Simulation scenario.

Fig. 5: Convergence of Algorithm 1.

Fig. 6: Convergence of Algorithm 2.

Fig. 7, where we set $\delta = 1$. As expected, from Fig. 7, it can be seen that the SE performance always increases with $P_{s,\text{max}}$. 

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and the active RIS with continuous coefficients attains the best performance. However, using the discrete coefficients with 3 or 4 bit resolutions performs very closely to the continuous coefficient case. In addition, it is seen that the active RIS significantly outperforms the other RIS-aided schemes due to its great capability of signal amplification. This observation indicates that active RIS design is effective to reduce the negative impact of the double fading, thus obtaining a higher SE. Besides, the RSMA scheme outperforms the SDMA scheme, since the RSMA technique splits the transmitted message in both the power domain and the spatial domain, and adjusts the split coefficient and power allocation to efficiently suppress the multiuser interference, and thus achieves better performance than the SDMA scheme.

Moreover, Fig. 9 demonstrates the impact of the weight \( \beta \) on the corresponding system EE and SE, with \( P_{r,\text{max}} = P_{s,\text{max}} = 0 \) dBW. It can be seen that increasing \( \beta \) improves system SE but reduces system EE. This is because that a larger \( \beta \) puts a higher priority on SE and thus allocates more power to maximize SE. On the other hand, when reducing \( \beta \), we obtain an improved EE but a reduced SE. Fig. 9 shows the ability of the proposed approach to balance the tradeoff between EE and SE by setting a proper \( \beta \).

Besides, we show the RE performances of these schemes versus \( N_r \) in Fig. 10, where we set \( \beta / P_{\text{sum}} = 1 \) and \( P_{s,\text{max}} = P_{r,\text{max}} = 0 \) dBW. From this figure, it can be seen that the RE first increases with \( N_r \), then decreases, which is quite different from the results in most related works where larger \( N_r \) leads to higher SE performance. This is because a larger \( N_r \) may lead to more power consumption at the RIS. Thus there exists a tradeoff in the RE performance w.r.t. \( N_r \).

Next, Fig. 11 plots the obtained RE versus BS-RIS distances, where we assume that the RIS moves along the \( y \)-axis from the BS to the users area. In this figure, it is seen that...
the active RIS scheme always outperforms the passive RIS scheme in the considered region. Moreover, for active RIS, the RE increases when RIS moves from the BS to the user area, while for passive RIS, the RE first decreases to a low point and then increases. Then, weather for active RIS or passive RIS, when RIS moves away from the user area, the RE decreases. Thus, deploying the active RIS near the users is beneficial to improve the RE.

Finally, we compare the RE performance of two-layer RSMA with the one-layer RSMA and SDMA schemes, where the 4 users are divided into 2 groups. The results are shown in Fig. 12, with $\beta/P_{\text{sum}} = 1$ and the other parameters are same as those in Figs. 7 and 8. From this figure, it can be seen that the two-layer RSMA outperforms the other benchmarks in terms of the RE.

VI. CONCLUSION

In this paper, we proposed a novel infrastructure of active RIS-assisted downlink RSMA network. By adopting the RE as the performance metric to achieve a tradeoff between the SE and EE performances, we formulated the RE optimization problem. By applying a two-stage optimization scheme and proposing an AO algorithm, the non-convex RE maximization problem was decomposed into a two-stage optimization problem, and solved to obtain the BS precoding and reflective BF alternatively by using the PDD method. Simulation results demonstrated that through reconfiguring the wireless communication environment, the active RIS can achieve adjustable tradeoff between the SE and EE, and the proposed active RIS-assisted RSMA scheme outperforms the benchmark schemes.

APPENDIX A

Proof of Theorem 1

Firstly, for the convergence of the quadratic transformation, [54, Theorem 3] has proved that if problem (13) is nondecreasing and concave, then the original problem (12) is guaranteed to converge. Thus, we mainly focus on the convergence of (13). Specifically, we denote the corresponding objective value of $\eta_{SE}(W, \Phi)$ in (13) and the objective of (18) as $\mu(W, \Phi)$ and $\mu(t)(W, \Phi)$, respectively, where $t$ denotes the number of iterations. Then, for any $W$ and $\Phi$, we have

$$\mu(W, \Phi) \geq \mu(t)(W, \Phi),$$

where $\{W(t), \Phi(t)\} \leftarrow \{W(t-1), \Phi(t-1)\}$. Based on this observation and the convexity of (19) and (24), we have

$$\mu(W(t+1), \Phi(t+1)) \geq \mu(t)(W(t+1), \Phi(t+1)),$$

where the second inequality holds true since both $\Phi(t+1), \Phi(t)$ are the optimal solutions and feasible points of (13). (41) suggests that $\{W(t+1), \Phi(t+1)\}$ is better than $\{W(t), \Phi(t)\}$. Moreover, the sequence $\{W(t), \Phi(t)\}$ is bounded by the constraints (12b)-(12d). Then, according to [56, Proposition 2], there exists a convergent subsequence $\{W(t_k), \Phi(t_k)\}$ with a limit point $\{W^*, \Phi^*\}$, i.e.,

$$\lim_{\gamma \rightarrow +\infty} \left[ \mu(W(t_k), \Phi(t_k)) - \mu(W^*, \Phi^*) \right] = 0.$$ (42)

For any $t$, there exists a $\gamma$ such that $t_\gamma \leq t \leq t_{\gamma+1}$, then we have

$$0 = \lim_{\gamma \rightarrow +\infty} \left[ \mu(W(t_k), \Phi(t_k)) - \mu(W^*, \Phi^*) \right] \leq \lim_{t \rightarrow +\infty} \left[ \mu(W(t), \Phi(t)) - \mu(W^*, \Phi^*) \right] \leq \lim_{\gamma \rightarrow +\infty} \left[ \mu(W(t_{\gamma+1})\Phi(t_{\gamma+1})) - \mu(W^*, \Phi^*) \right] = 0,$$

which means that $\lim_{t \rightarrow +\infty} \mu(W(t), \Phi(t)) = \mu(W^*, \Phi^*)$. Thus, the convergence of $\eta_{SE}(W, \Phi)$ has been proved. Besides, since $P_{\text{tot}}(W, \Phi)$ is bounded due to (12b)-(12d), (13) is guaranteed to converge, which completes the proof.

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