# Exploring the Relationship Between Traffic, Topology and Throughput: Towards a Traffic-Optimal Optical Network Topology Design [Invited]

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Compiled March 6, 2023

The design of optical networks for maximum throughput, under diverse traffic demands, is a longstanding NP-hard problem. In this paper, by parameterising the relationship between the network topology and traffic demand, a novel polynomial-time objective function, the demand weighted cost (DWC) is introduced and evaluated for different scale networks and diverse traffic scenarios. It is shown that the proposed DWC is highly correlated to network throughput, while speeding up the topology evaluation process up by  $\sim$ 5 – 6 orders of magnitude. The DWC was then applied as the optimisation target with 3 different topology optimisation algorithms (DWC-selection, genetic algorithm and the our novel hierarchical topology design), achieving 90% and 460% throughput increases, on average, for small-scale (14-node) and large-scale (100-node) topology designs, respectively. The proposed methods have the potential of maximising throughput in the design of future optical network topologies. © 2023 Optical Society of America

http://dx.doi.org/10.1364/ao.XX.XXXXXX

# 1. INTRODUCTION

Optical communication networks underpin the global communication infrastructure and support the exponential growth in demand for data transmission, for all application scenarios, including telecommunications, the Internet and datacentres [1]. Using wavelength division multiplexing (WDM), optical networks provide high-capacity, low-latency and cost-effective solutions compared to traditional electronic packet switching networks at all time- and distance-scales. This is because they do not require any optical–electronic–optical (O–E–O) conversion, electrical processing and buffering between source and destination nodes.

However, designing an optical network fulfilling the application requirements of throughput, latency, resilience is not trivial. Both the features and constraints of the traffic and the network need to be considered simultaneously to ensure the network resources match the demanded traffic. One of the most important constraints in optical networks is the physical topology, which determines the available routes between nodes, and further impacts the performance (including throughput, latency, resilience) and the costs of building and managing the network.

Physical topology design has been a long-standing NP-hard optimisation problem for the most commonly used network performance objectives since optical networking emerged [2]. Previous work has mainly focused on minimising network cost [3–5], wavelength requirements [6], blocking rate [7, 8], power consumption and resilience [9] or a combination of these goals. Since the computational complexity of topology design exponentially increases with the number of nodes in the network, heuristic algorithms are often used. These include greedy [10], cut-saturation [4, 5] and branch exchange [11], as well as metaheuristics including genetic algorithms (GA) [3, 6, 12], particle swarm optimization (PSO) [13] and simulated annealing [14] to find near-optimal solutions in feasible time.

To fulfill the growing and changing bandwidth requirements in optical networks, maximising network throughput - defined as the total achievable bitrate of all lightpaths serving a given traffic demand - is a key optimisation target in network design. Thus, it is important to include this target in topology design to support the bandwidth hungry applications, e.g. cloud service, virtual reality (VR) [15]. In addition, most of the previous topology design methods considered both the network costs and performance by analysing small-scale networks (e.g. 6-node [9], 10-node [7, 16], 14-node [8]). However, the size of both core and access optical networks can reach more than 100-nodes (e.g. UK core network [17], US coronet [18]) and thus, the design of

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large-scale (e.g. 100-node) optical networks, considering both network cost and performance, needs to be explored.

The challenge of designing networks with maximum throughput is two-fold, with both stages involving NP-hard computational problems: (i) the evaluation of network throughput for a given topology and demand, where conventional methods such as integer linear programming (ILP) are too computationally complex and (ii) the search for a near-optimal topology, in a huge solution space of possible topologies, given a number of nodes, total edge length and demand, with large number of candidate solutions. Both of these problems lead to infeasible computation times, let alone when both need to be considered.

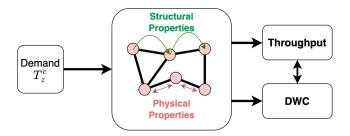
To tackle problem (i) previously surrogate models that looked to learn the relationships between optimisation objective and the graph were proposed, such as using a artificial neural network (ANN) [19] or more recently a graph neural network (GNN) [20]. Alternatively, we recently proposed a analytical computationally-efficient optimisation objective, termed demand weighted cost (DWC), with the aim of maximising network throughput [21]. The proposed DWC is a network metric, to parameterise the relationship between network topology and traffic demand with polynomial computational complexity. We showed that the DWC is highly correlated to network throughput and describes how well the network resources match the demand distribution. However, the proposed DWC metric and the topology design methods need to be further evaluated in different node-scale network designs with generalised traffic distributions.

In this paper, the concept of demand weighted cost is described and the relationship between it and throughput in various node-scale networks is explored. The DWC is applied as the optimisation objective in both small-scale (14 node) and large-scale (100 node) optical network topology design problems. The DWC's performance, as an optimisation objective, is investigated by designing topologies in three ways: (i) by selecting the topologies with the best objectives from a large number of solutions, created via three generative graph models: Barabasi-Albert (BA) [22], signal-to-noise ratio aware Barabasi-Albert (SNR-BA) [23, 24] and Prufer sequence (PS) [6] (ii) a objective function within genetic algorithms (iii) a objective function within a novel hierarchical topology design (HTD) method. The topologies are designed by optimising the DWC values of the graphs, however the maximum achievable throughput of the networks is used to validate their *real* performance.

The rest of the paper is organised as follows. Section 2 describes the methods used in estimating the network throughput for different node-scale networks, as well as introduces the proposed DWC metric and its relationship to throughput. Section 3 introduces the physical topology design problem along with the 3 types of proposed topology design algorithms. Section 4 presents both the large and small scale topology design results and analysis with key conclusions described in Section 5.

# 2. THROUGHPUT ANALYSIS AND ESTIMATION

To reduce the NP-hard complexity of maximum achievable throughput estimation [25] in the topology design process, we explored the relationship between topology properties, traffic demand and network throughput as shown in Fig. 1, where it can be seen that traffic is an input to the topology, which has structural (how the nodes are connected) and physical (how far away nodes are from each other) properties, that then govern the maximum achievable throughput. The aim is to find a lowcomplexity topology design target that mimics the maximum achievable throughput.



**Fig. 1.** Relationship between topology properties, traffic demand and throughput

A normalised traffic demand matrix, in terms of connections requested ( $T^C$ ) is used to describe the network traffic demand, where  $T_z^C$  represents the normalised demand value between node pair *z* and  $\sum_z T_z^C = 1$ .

An ILP model is used to quantify the maximum achievable throughput of small-scale networks, described in section 2A. For large-scale networks (e.g. 60-100 nodes), a First-Fit k-Shortest-Path (FF-kSP) heuristic [25] combined with a demand sequential loading (DSL) scheme is used to estimate the maximum achievable throughput, as described in section 2B. To calculate the final maximum achievable throughput, a closed-form Gaussian-Noise (GN) model was used, as described in 2C. Finally in section 2D the demand weighted cost (DWC) is introduced by parameterising the relationship between the traffic demand and network topology. The relationship between DWC and throughput is then explored in various node-scales and traffic distributions. In addition, the computational efficiency of calculating the DWC is compared with that of calculating throughput via routing and wavelength assignment optimisation, i.e. ILP and heuristics.

#### A. ILP model for throughput estimation

The ILP model used to evaluate the maximum achievable network throughput in small-scale networks is defined as follows. In a network with the set of nodes *N* and the set of edges *E*, with each edge carrying a set of *W* wavelengths, we assume a set of *Z* node pairs that need to be connected via a routing and wavelength assignment (RWA) solution using a set of *K* paths (*K* shortest paths weighted by physical distances in km). The size of *K* was kept at 20 (|K| = 20). The decision variable  $\delta_{w,k,z}$  – with  $w \in W$ ,  $k \in K$ ,  $z \in Z$  – is able to fully define the RWA of a network, following Eq. (1).

$$\delta_{w,k,z} = \begin{cases} 1 & \text{if } (k, w) \text{ are path and wavelength} \\ & \text{assigned for node pair } z \\ 0 & \text{otherwise} \end{cases}$$
(1)

Another integer decision variable is the throughput indicator M, which determines how many demand requests, i.e. the number of lightpaths between node pair  $z(\lceil M \cdot T_z^C \rceil)$ , the network can accommodate. The ILP objective function is to maximise M.

The first constraint in the ILP model, is that the number of established lightpaths (represented by the decision variable  $\delta_{w,k,z}$ ) needs to be equal to the connection demands, described in Eq. (2).

$$\sum_{w \in W} \sum_{k \in K} \delta_{w,k,z} = \lceil M \cdot T_z^C \rceil \quad \forall z \in Z$$
(2)

Another constraint is that no two lightpaths can share a wavelength on any given edge *j*. Therefore, the variable  $I(j \in k)$  is defined to be 1 when edge *j* is in path *k* and 0 otherwise. Using this the wavelength uniqueness can be constrained as in Eq. (3).

$$\sum_{z \in \mathbb{Z}} \sum_{k \in K} \delta_{w,k,z} I(j \in k) \le 1 \quad \forall j \in E \quad \forall w \in W$$
(3)

The time to solve the ILP scales exponentially with the size of the network due to the NP-hard nature of the problem [25]. Thus, we only used the ILP to calculate the throughput in the smallscale networks; for larger networks (60-100 nodes) we applied heuristic algorithms, as described in the following section.

#### B. Heuristic for throughput estimation

As the ILP is not scalable for calculating throughput in largescale networks (e.g. 60-100 nodes), we used the FF-kSP algorithm [25] combined with a demand sequential loading (DSL) scheme to estimate the throughput in this work. More specifically, after obtaining the RWA solution of the current traffic by FF-kSP, we continue to add the load (according to  $T^{C}$ ) to the network until blocking occurs. The process of FF-kSP combined with DSL is described in Algorithm 1.

#### Algorithm 1. Demand sequential loading algorithm

**Input**: Network topology *G*, normalised traffic matrix  $T^C$ , wavelength number *W* 

```
Output: The final RWA solution RWA<sub>f</sub>.
 1: M_0 = 0
 2: step = 100
 3: t = 0
 4: for i \leq n do
        while RWA blocking free do
 5:
            M_t = M_{t-1} + step
 6:
            T_r = \lceil M_t \cdot T^C \rceil
 7:
            RWA_t = F_R(G, T_r, RWA_{t-1})
 8:
            t = t + 1
 9:
        M_t = M_t - step
10:
        step = \lceil step/2 \rceil
11:
        t = t + 1
12:
13: RWA_f = RWA_t
```

Here the same objective throughput indicator (M) is maximised as in the ILP; it is initialised as 0 and increased by *step* in each iteration. The total traffic matrix ( $T_r$ ), in terms of lightpaths, is calculated according to the normalised traffic matrix ( $T^C$ ) and the current M. The routing function (FR) takes the inputs of the topology (G), total traffic matrix (Tr) and the previous RWA to calculate a new RWA using FF-kSP, which chooses the path that can be allocated on the lowest indexed wavelength. The previous RWA is used to reduce the amount of computation the routing function has to perform at each iteration, however already allocated paths cannot be changed down the line. Once blocking occurs, the step is halved and the network is loaded again until the next blocking occurs. The process iterates n times, after which the final RWA is obtained. In this work, we set n = 6, as more would have negligible impact to the result.

### C. Physical Layer Impairements (PLI) Model

After obtaining the RWA solution from the ILP or FF-kSP combined with DSL, we calculated the signal-to-noise ratio (SNR) for each wavelength and edge. This was estimated using a 3

closed form Gaussian noise (GN) physical layer impairments (PLI) model [26]. Using the RWA solution, we take the inverse of the SNR (NSR) and sum the individual NSR values along the lightpath to calculate the total SNR value for this lightpath. Following this, the Shannon equation [27] is applied to calculate the network throughput.

For the physical layer, we assumed a full C-band (1530-1570 nm) transmission with 156 wavelengths (32 GHz Nyquistspaced) on all fibre links. Furthermore, we assumed multiples of 80km standard single mode fibre spans, with fibre attenuation coefficient 0.2dB/km, chromatic dispersion coefficient  $18ps/(nm \cdot km)$  and nonlinear coefficient  $1.2(W \cdot km)^{-1}$ . Erbium doped fibre amplifiers (EDFA) are assumed to be deployed between the spans, all having a noise figure of 4dB. Colourless, directionless and contentionless, reconfigurable optical adddrop multiplexers (CDC-ROADM) are deployed at all switching nodes.

#### D. Demand weighted cost

To reduce the computational complexity in estimating network throughput, the goal of creating a new network metric was to parametrise the relationship between topology and demand, and find an approximate-linear relationship to network throughput. By weighting the communication costs of the node pairs with the traffic demand between them, we proposed a new parameter, termed demand weighted cost (DWC), first introduced in [21].

As the throughput depends on both structural and physical properties of the network, we included both in the definition of communication cost ( $C_z$ ) between a certain node pair z. To see how the structural and physical properties affect the throughput we used two different definitions of  $C_z$ . The first, in Eq. (4) is the sum of the the shortest path length and the minimum hop number between a given node pair z. The second uses the network physical connectivity ( $\alpha = \frac{2E}{N(N-1)}$ ) [28] to weight the two terms, as in Eq. (5), where  $\alpha$  is the network physical connectivity,  $L_z$  is the shortest path physical length and  $H_z$  is the number of hops in the shortest path. Calculating the Pearson's correlation coefficient ( $\rho$ ) between the total communication cost ( $\sum_{z} C_{z}$ ) and throughput quantifies how well each definition correlates with the actual maximum achievable throughput. Here an ILP was used to calculate the maximum achievable throughput, introduced in section 2A, for the small scale graphs (N = 14) and FF-kSP with DSL, as introduced in section 2B, was used for larger scales (N = 100).

$$C_z = L_z + H_z \tag{4}$$

$$C_z = \alpha \cdot L_z + (1 - \alpha) \cdot H_z \tag{5}$$

We calculated the Pearson correlation coefficient ( $\rho$ ) of both definitions in 4,000 14-node and 100-node topologies, 2,000 at each node-scale. The  $\rho$  values of the two definitions are -0.914/-0.917 (14-node) and -0.832/-0.856 (100-node), respectively. Thus, the second communication cost definition has higher correlation to throughput and we use it in the following paper. After defining the communication cost, the traffic demand is used to weight this cost and the DWC is defined as in Eq. (6).

$$DWC = \sum_{z \in Z} T_z^C \cdot (\alpha \cdot L_z + (1 - \alpha) \cdot H_z)$$
(6)

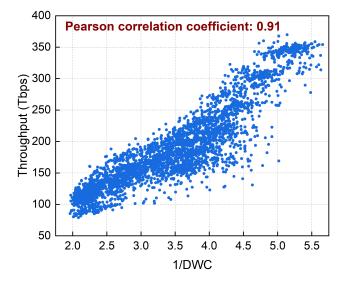
To explore the relationship between the proposed DWC metric and the network throughput under different topology and demand scenarios, 3,600 14-node 21-edge topologies were generated via the Erdos Renyi (ER) [29], BA [22] and SNR-BA [23, 24] graph generative models, 1,200 topologies of each kind. The ER model decides whether an edge exists between a certain node pair by independent Bernoulli distributions, while the other two models consider the node degrees and signal-to-noise ratios to determine the probability of adding an edge. Due to the fact that the ER model does not always generate connected-graphs, we used the other two models in our design method described in section 3A.

We first investigated the impact of demand distribution. As an example, a distance-based skew is introduced in the traffic matrix, using Eq. (7). This is only intended to make a comparison to uniform traffic, other methods of introducing traffic skew can also be considered [30].

$$T_{ij}^{C} = \frac{1}{\left(\frac{D(i,j)}{\sum_{k \in \mathcal{N}} D(i,k)}\right)^{\gamma}}$$
(7)

Here D(i, j) is the distance between node *i* and *j* and the traffic skew factor ( $\gamma$ ) weighs how heavily the demand is skewed. When  $\gamma$  increases, the traffic becomes more localised, node pairs with shorter distances have heavier loads.

The throughput for the 3,600 generated 14-node topologies were calculated and are plotted against their inverse DWC value in Fig. 2. It can be seen that in these small-scale networks, the relationship is approximately linear, so an increase in 1/DWC indicates that networks have a higher throughput. The Pearson correlation coefficient ( $\rho$ ) between 1/DWC and throughput reaches 0.91.



**Fig. 2.** Throughput vs DWC in 14-node topologies

To verify this relationship in different node-scale networks, 10,000 topologies were generated via the SNR-BA generative graph model [24], with 60-100 nodes, 2,000 for each node scale. In these networks, the Pearson correlation coefficient ( $\rho$ ) between 1/DWC and throughput remains above 0.969 as shown in Tab. 1, which further verifies that the relationship between DWC and throughput also holds in large-scale networks.

These results show that DWC is a robust topology design metric, especially as it is computationally simple. The only significant time contribution is the calculation of the shortest paths 4

Table 1. Pearson correlation coefficient in different node scales

Ν	60	70	80	90	100
ρ	0.969	0.970	0.972	0.974	0.972

for all node pairs, in our case using the Floyd–Warshall algorithm [31]. Thus, the computational complexity of evaluating a topology using the DWC metric instead of calculating throughput using the ILP model, introduced in Section 2A, reduces from  $O(2^{D \cdot E \cdot W})$  [32] to  $O(N^3)$ , where *D* is the total demand number and  $D \propto N^2$ . The computation time of DWC and throughput was compared using a server with 2 x Intel(R) Xeon(R) CPU E5-2660 v3 @ 2.60GHz and 256Gb RAM. The average computation time of DWC and throughput (calculated by ILP) in 14-node networks was 0.0023s and 5107.04s, respectively, which shows the potential of reducing computing time by some 6 orders of magnitude.

In larger scale networks with 60-100 nodes, since the ILP does not scale, the throughput was calculated using the FF-kSP heuristic combined with the DSL algorithm. Even using the heuristic algorithm, the complexity is still  $O(RKN^3(E + Nlog(N)))$ , where *R* is the number of rounds of adding network demand, *K* is the number of paths considered, and Yen's algorithm [33] was used to calculate the k-shortest paths. The actual average computing time for DWC and throughput (for this heuristic) in different node-scale networks is plotted in Fig. 3. The gap between calculating DWC and throughput remains around 5 orders of magnitude.

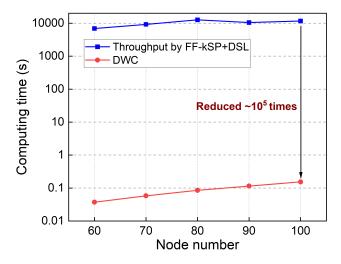


Fig. 3. Computing time for large scale networks

Based on the above analysis, DWC can be used as a computationally efficient objective that approximates the throughput performance of a network in physical topology design. The next section describes how the DWC metric can be incorporated in network topology design.

# 3. DWC IN TOPOLOGY DESIGN

We formulate the physical topology design problem by starting with a set of nodes (*N*), their positions ( $n_i = (x_i, y_i)$ ), the total edge length limit ( $L_{max}$ ) and the normalised traffic demand matrix ( $T^C$ ), where the goal is to design a bi-connected topology

(fulfilling the resilience requirement) and maximise the network throughput.

This problem can be expressed as an integer non-linear programming (INLP) problem by setting decision variables as to whether an edge between a given node pair exists. This can be expressed by the binary variable  $a_{ij}$ , which is 1 when edge ijexists and 0 when not. Ensuring the connectivity of the graph introduces the nonlinear constraint of that det(L) = 0, where Lis the laplacian matrix of the graph defined in Eq. (8) [34].

$$L = \begin{bmatrix} \sum_{i} a_{1i} - a_{11} & -a_{12} & \dots & -a_{1n} \\ -a_{21} & \sum_{i} a_{2i} - a_{22} & \dots & -a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n1} & -a_{n2} & \dots & \sum_{i} a_{ni} - a_{nn} \end{bmatrix}$$
(8)

Since there is no universal solution for such INLP problem, in this work we applied the heuristic/meta-heuristic search algorithms to find a near-optimal solution instead. Following the analysis in the previous section, the topology design objective of minimising DWC is used instead of maximising throughput. The proposed topology design algorithms are described below.

# A. DWC-selection methods

The first, and most intuitive, topology design method is to generate a large number of topologies using graph generative models (i.e. BA, SNR-BA and PS), then select one or multiple of them, according to an objective, which in this case is to minimise DWC, with the goal of maximising throughput. This is referred to as the DWC-selection method. The sections 3A.1, 3A.2 and 3A.3 introduce the graph generative models that are used.

# A.1. Barabasi-Albert (BA) [22]

The BA model starts with two nodes connected by an edge, after which nodes are sequentially added and a set number of *m* edges are connected to every newly added node. The edges are chosen given probability p(i, j) corresponding to Eq. (9), where *i* is the newly added node and *j* is an existing node in the graph,  $d_i$  donates the degree of node *i*.

$$p(i,j) = \frac{d_j}{\sum\limits_{k \in \mathcal{N}} d_k}$$
(9)

The probability p(i, j) is determined only by the sum of all the degrees in the graph currently and the degree of the node *j*. The model creates large connected hubs within the network, appropriate for modeling different types of real networks, including social and communication networks. However, it does not consider the physical properties of the network, which is key in optical networks.

#### A.2. Signal-to-Noise Ratio Aware Barabasi-Albert (SNR-BA) [23, 24]

The SNR-BA model is extended from the BA model, and takes into account the physical properties of real networks, and the associated linear and nonlinear optical fibre distortions. The signal-to-noise ratio (SNR) of the lightpaths are used to weight the probability of the edge existence as shown in Eq. (10), where a parameter  $\theta$  is used to determine how heavily to weight the physical properties within the graph generation.

$$P_{\text{SNR-BA}}(i,j) = \left(\frac{\text{SNR}(i,j)}{\sum_{k \in N} \text{SNR}(i,k)}\right)^{\theta} \cdot \frac{d_j}{\sum_{k \in N} d_k}, \quad (10)$$

This model generates graphs which are structurally most similar to real optical core networks compared to ER and BA models.

# A.3. Prufer Sequence (PS) [6]

The Prufer sequence was originally used for generating tree topologies. It creates an *N*-node tree topology with an (N - 2)-integer sequence, whose values are selected from [1, N]. By connecting the leaf nodes of the generated tree, a bi-connected topology is generated. The detailed topology generation process is described in Algorithm 2.

**Algorithm 2.** Prufer Sequence bi-connected topology generation

- 1. Randomly generate a (N 2)-integer Prufer sequence,  $P = \{s_1, s_2, \cdots, s_{N-2} | s_i \in [1, N]\}.$
- 2. Add all leaf nodes  $Q = \{q_i | q_i \in [1, N] \text{ and } q_i \notin P\}$  into the eligible list.
- 3. Scan along *P*, for each  $s_i \in P$ , find the lowest indexed node *j* in the eligible list, add an edge  $(s_i, j)$  to the topology. If  $s_i \notin \{s_{i+1} \cdots s_{N-2}\}$ , add  $s_i$  into the eligible list.
- 4. After step 3, only two nodes are eligible, add an edge between them.
- 5. Add edges between adjacent leaf-nodes (add edges  $\{e_{q_i,q_{i+1}} | q_i \in Q\}$ ).
- Check the total edge length *L*, if *L* ≤ *L<sub>max</sub>* then the generation process ends; otherwise restart from step 1.

A 6-node bi-connected topology generated via the PS method is shown in Fig. 4. In this example, we set the  $P = \{2, 2, 2, 1\}$ . After the tree topology is generated according to the PS method, additional edges (red edges in Fig. 4) are added to connect the leaf nodes to achieve a bi-connected topology.

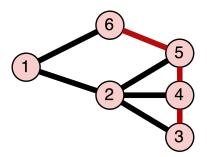


Fig. 4. A 6-node Prufer sequence based topology

The PS model provides a computationally efficient way of finding *N*-node bi-connected topologies, especially when the network is large (e.g. N > 50) and sparse (e.g.  $\alpha < 0.2$ ), compared to other graph generative models such as ER, BA and SNR-BA, which do not always generate bi-connected topologies.

# B. Genetic algorithm with DWC

The second topology design method, explored in this work, uses the DWC as the objective function within a genetic algorithm (GA) framework [35]. The GA was selected over other

meta-heuristics and evolutionary strategies because of its fast convergence time to near-optimal solutions and capability to handle discrete problems [36, 37]. Fast convergence was necessary to be able to create and simulate large number of topologies with high fitness values.

Two types of individuals that describe a topology were implemented in the explored GA method. The first one uses a  $\frac{N^2-N}{2}$ dimensional vector (termed as topology vector in the following paper) consisting of the upper triangular elements of the graph adjacency matrix to describe a topology, while the second one uses a Prufer sequence (PS). The GA method involves a fixedsize population of individuals, where at each iteration a specific portion of the current population is chosen to be parents, where then crossover and mutation are sampled as discrete actions, as shown in Fig. 5. The topology vector based GA is able to search all kinds of topology structures, but it also cannot guarantee bi-connected graphs after crossover and mutation operations, which reduces the search efficiency. The Prufer sequence based GA, on the contrary, only generates graphs with specific structure (leaf-node-connected 'tree' topology), but the individuals will keep the bi-connected characteristic after crossover or mutation, enhancing the searching efficiency compared to the previous method, however reduces the variety of structures searched. Thus, the topology vector based GA and PS based GA are suitable for small and large scale network design, respectively. The detailed process of GA is shown in Fig. 5 and described in Algorithm 3.

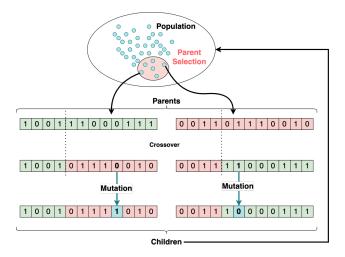


Fig. 5. Example of GA process in a single iteration

#### C. Hierarchical topology design

To design large-scale optical networks ( $N \ge 60$ ), we propose a new hierarchical topology design (HTD) method with the objective function of DWC. In this proposed method, the nodes are divided into different sub-networks according to their locations. Inter- and intra-sub-networks are designed independently to reduce the computational complexity. The detailed steps of HTD are described in Algorithm 4.

Figure 6 shows an example of using the HTD method to design a 14-node bi-connected network. As described in the Algorithm 4, the centres of the sub-networks are shown as red dots in the figure. All nodes are divided into 4 sub-networks according to their locations. The inter-sub-network edge numbers and the total edge length limits of the sub-networks are

#### Algorithm 3. Genetic algorithm with DWC

- Initialise the first generation individuals (topology vector or Prufer sequence).
- 2. Do while generation number not reached:
  - 2.1 Select a portion of individuals with smaller DWC values as parents.
  - 2.2 Uniformly crossover between parents to generate individuals for next generation
  - 2.3 Uniformly mutate the individuals for next generation
- 3. Select the topology with minimum DWC in the last generation.

determined by Eq. (13) and Eq. (15), respectively. Then the corresponding gate nodes connecting different sub-networks are determined by Eq. (14) and shown as blue dots. Finally, the interand intra-sub-network topologies are designed as Fig. 6 shows.

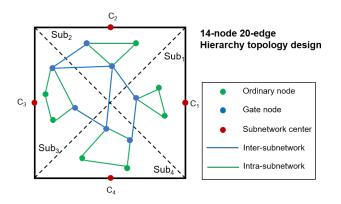


Fig. 6. A hierarchical topology design example

Using a the DWC-select, GA and HTD topology design methods, the performance of DWC as a topology optimisation objective is evaluated, to see whether it can maximise throughput. The next section describes the results obtained over a variety of topology sizes and traffic conditions.

# 4. TOPOLOGY DESIGN RESULTS AND DISCUSSION

## A. Small scale topology design (N = 14)

In this subsection, we describe the analysis to evaluate the performance of the proposed DWC-selection and topology vector based GA methods in small-scale topology design. We first generated 6 different traffic matrices, with the demand skew factors  $\gamma \in \{0.2 \cdot y | 0 \le y \le 5, y \in \mathbb{Z}\}$ . Then, we selected 200 topologies with minimum DWC, per traffic matrix, from the 10,000 topologies generated by each graph generative model, respectively. In addition, 200 graphs were also generated via the topology vector based GA, per traffic matrix. Lastly, 400 additional random BA and SNR-BA topologies were generated, used as baselines, 200 for each method. The node number and total edge length limit of the topologies to be designed were set to 14 and 42,000km respectively, according to the node locations, set to be the same as NSFNET (21 edges and 2000km per edge on average). The iteration number, population size, parent portion, crossover and

#### Algorithm 4. Hierarchical topology design

#### 1. Divide the nodes into different sub-networks:

- 1.1 Design the number of the sub-networks  $|N_{sub}|$  $(|N_{sub}| \le \sqrt{N}).$
- 1.2 Design the normalised centre coordinate of the subnetworks  $\{c_j : (x_j^c, y_j^c) | j \in |N_{sub}|\}$ . In this work we set:

$$x_j^c = \cos j \cdot \frac{2\pi}{|N_{sub}|} \tag{11}$$

$$y_j^c = \sin j \cdot \frac{2\pi}{|N_{sub}|} \tag{12}$$

- 1.3 Divide the nodes into sub-networks  $(N_{sub_i})$  according to their distances to the sub-network centres, select the nearest one (if  $k = \arg \min_i (D(n_i, c_j))$ , then  $i \in sub_k$ ).
- 1.4 Set a targeted edge number for inter-sub-network connections as  $E^{inter} = L_{max}/L_{ref} N$ , where  $L_{max}$  and  $L_{ref}$  are the total fibre length limit and reference length of an edge, respectively.

### 2. Design the topology for inter-sub-network:

2.1 Calculate the edge numbers between the sub-networks (*E*<sup>*inter*</sup><sub>*ij*</sub> for sub-network *i* and *j*) according to the total traffic between them by Eq. (13).

$$E_{ij}^{inter} = \sum_{p \in sub_i, q \in sub_j} T_{pq}^{C} \cdot E^{inter} \quad \forall i, j \in |N_{sub}|$$
(13)

2.2 Decide the gate nodes (the nodes that connect to other sub-networks) for every sub-network by the traffic size.  $g_k^{ij}$  donates the  $k^{th}$  gate node in sub-network *i* which connects to sub-network *j*. It is determined by Eq. (14).

$$g_k^{ij} = \operatorname*{arg\,max}_{g \in sub_i} \sum_{q \in sub_j} T_{gq}^{\mathbf{C}} \quad \forall i, j \in |N_{sub}|$$
 (14)

2.3 Decide the edges between the gate nodes by their distances, select the edges with the minimum distances.

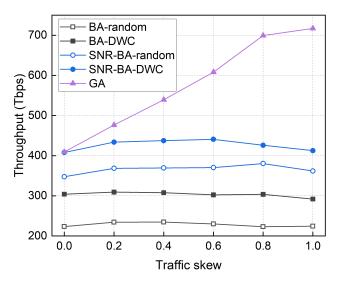
#### 3. Design the topologies for intra-sub-networks:

3.1 The total edge length of a intra-sub-network (*N<sub>subi</sub>*) is shown in Eq. (15), where *L<sup>inter</sup>* is the total length of inter-sub-network edges.

$$L_{max_i}^{intra} = \frac{N_{sub_i}}{N} * (L_{max} - L^{inter})$$
(15)

3.2 Design sub-network topologies in parallel using the topology vector based GA algorithm described in Section 3 B. mutation rate of the GA method were set to 100, 100, 30%, 80% and 10%, respectively, with values determined by grid-search.

For each of the topology design methods, the throughput for the 200 generated topologies per traffic skew was evaluated using the ILP and closed-form GN model and are plotted in Fig. 7. It can be seen that the average throughput of the BA and SNR-BA DWC-selection topologies outperformed the corresponding baseline topologies by 33% and 16% on average, respectively. The GA topologies achieved the highest throughput under all traffic matrices, with 90% and 57% throughput enhancement on average compared to the random BA and SNR-BA topologies. Compared to the DWC-selection methods, although the GA topologies have similar DWC values, they result in significantly higher throughput, especially for high skew traffic. The GA method is able to explore more diverse topology structures, not limited by the BA or SNR-BA methods, which make selections based on rigid probability constraints.

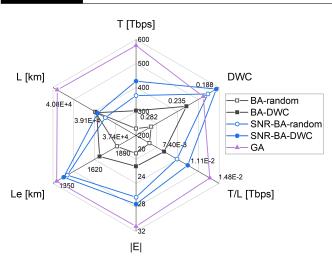


**Fig. 7.** Throughput of the designed 14-node topologies. The baseline topologies are denoted by the keyword 'random' and the DWC-selection topologies by 'DWC'. Each point represents the average throughput of 200 generated topologies.

To explore the characteristics of the designed topologies, we calculated the average throughput (T), throughput per km fibre (T/L), edge number (|E|), length per edge ( $L_e$ ), the total fibre length (L) and plotted these in Fig. 8. Compared to the baseline random BA and SNR-BA topologies, the GA and DWC-select topologies achieved 138% (GA compared to random BA) and 33% (BA-DWC compared to random BA) higher fibre deployment efficiency, represented by the throughput per km fibre (T/L), while still using roughly the same (maximum 5.5% difference) total fibre length (*L*). The DWC-select and GA methods tend to choose shorter edges, increasing the number of edges, compared to the baseline method. By setting the DWC as the optimisation target, the GA and the DWC-selection methods are able to make smarter choices where the edges are actually needed, according to the traffic matrix, achieving improved network structure and physical properties compared to the baseline methods.

# **B.** Large scale topology design (N = 100)

The large solution space of candidate topologies for 100-node networks meant that common graph generative models (ER, BA



**Fig. 8.** Characteristics of the designed 14-node topologies: throughput (*T*), throughput per km fibre (*T*/*L*), edge number (|E|), length per edge ( $L_e$ ) and the total fibre length (*L*). The average over all traffic skews ( $\gamma$ ) is taken.

and SNR-BA etc.) and the topology vector based GA could not find a bi-connected topology fulfilling the total edge length limit in a reasonable time. Thus, we implemented PS-based DWCselection, PS-based GA and the HTD method described in the previous section for the 100-node network design.

The node positions were chosen uniformly randomly over an area of the size of the north-American continent. The minimum distance between two nodes is set to 100 km to mimic the node distances in core networks. The total edge length limit ( $L_{max}$ ) is set to 280,000 km (set as the same of a reference topology with 140 edges, 2000km per edge on average) accordingly. For each traffic matrix, where the traffic skew factor  $\gamma \in \{0.2 \cdot y | 0 \le y \le 5, y \in \mathbb{Z}\}$ , we designed 600 topologies, 200 for each method (DWC-select, GA and HTD). 200 additional random PS topologies per-demand-matrix were used as baselines.

In the DWC-select and GA methods, we selected the best 200 topologies with minimum DWC from 10,000 topologies, generated by each method, respectively. The parameters of the GA method were set to be the same as in small-scale network design, avoiding long computation times. In the HTD method, the number of sub-networks ( $|N_{sub}|$ ) and the reference edge length ( $L_{ref}$ ) were set to 4 and 2,000 km, respectively, according to the generated node positions.

The average throughput values, under the 6 different traffic matrices ( $\gamma \in \{0.2 \cdot y | 0 \le y \le 5, y \in \mathbb{Z}\}$ ), evaluated via the FF-kSP and DSL method, are plotted in Fig. 9. It can be seen that PS-based topology generation methods are not sensitive to traffic skews and HTD topologies achieved the highest throughput under all traffic skews. Compared to the average throughput of random PS topologies, the DWC-select, GA and HTD topologies outperform them by 20%, 51% and 460%, respectively. They still however deploy approximately the same amount of fibre, with the maximum difference of 0.7% as shown in Fig. 10. Unlike the PS-based methods, that only generate "tree-based" topologies; the HTD method searches through a much larger solution space of feasible topologies, which is able to find the topologies with significantly smaller DWC as shown in Fig. 11. Moreover, the topology vector based GA, that is used within HTD for the sub-network design, achieves more significant throughput enhancement in high skew traffic conditions. Thus, the HTD

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method is able to achieve much higher throughputs compared with the PS-based methods as shown in Fig. 9.

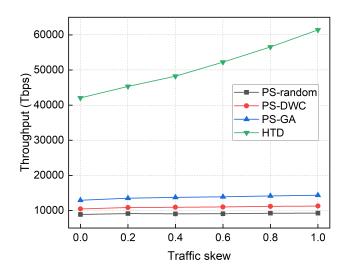


Fig. 9. Throughput of the designed 100-node networks

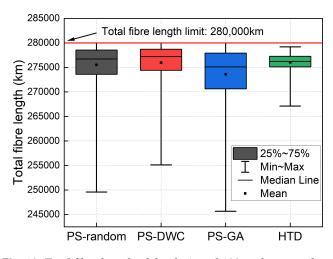


Fig. 10. Total fibre length of the designed 100-node networks

To investigate the characteristics of the designed topologies, we also calculated the average throughput (*T*), throughput per km fibre (T/L), edge number (|E|), length per edge  $(L_e)$  and the total fibre length (L) of these topologies and plotted them in Fig. 12. Compared to random PS topologies, the DWC-select, GA and HTD topologies achieved 20%, 52% and 459% higher average throughput per km fibre (T/L) respectively, which indicates the high efficiency of fibre deployment of the proposed methods. The average edge length  $(L_e)$  of DWC-select, GA and HTD topologies are 4.3%, 10.4% and 58% shorter, leading to the edge number (|E|) increment of 4.6%, 10.8% and 139%, respectively. The increased number of edges leads to the advantage of topology structure (improving physical connectivity) further leading to performance advantages. Compared to the PS-based topologies, the HTD method is able to explore more diverse topology structures: by splitting the large-scale design problem into multiple smaller scale problems and then using topology vector based GA (rather than Prufer-sequence based GA) to design them. This leads to significantly better topology optimisation for a given traffic matrix. Finally, it is worth acknowledging that the prufer

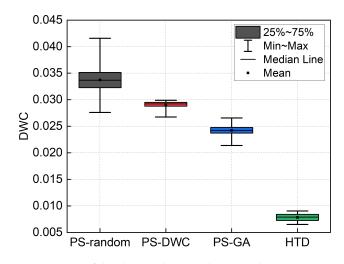
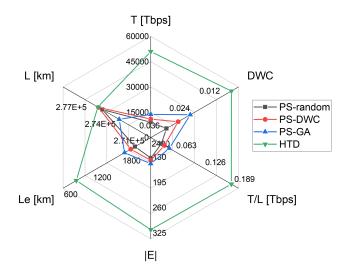


Fig. 11. DWC of the designed 100-node networks

sequence GA, generally is not able to generate topologies that hit the fibre limit. This is due to the lack of variety that the prufer sequence initialisation causes, as it only produces tree-based topologies.



**Fig. 12.** Characteristics of the designed 100-node topologies: throughput (*T*), throughput per km fibre (*T*/*L*), edge number (|E|), length per edge ( $L_e$ ) and the total fibre length (*L*). The average over all traffic skews ( $\gamma$ ) is taken.

# 5. CONCLUSIONS

This paper delves into the long-standing NP-hard problem of physical topology design for optical networks, with the target of maximising network throughput. This problem contains 2 NP-hard sub-problems: (i) evaluation of the throughput for a given topology and demand (ii) finding a near-optimal topology in a huge solution space.

In this paper, by parameterising the relationship between network topology and traffic demand, a novel polynomial complexity objective function for optical network topology design, the demand weighted cost (DWC) metric, is introduced. It was shown that there is an approximately inverse proportional relationship between DWC and throughput under different node-scale (14 9

– 100 nodes) networks and traffic scenarios, with the Pearson correlation coefficient above 0.91. The proposed DWC metric was used as a computationally effective objective (5 ~ 6 orders of magnitude speed-up) to evaluate topologies in the network design process. By implementing DWC within 3 polynomial-time topology optimisation methods, DWC-select, GA and HTD, 90% and 460% throughput enhancement were demonstrated for both small-scale (14-node) and large-scale (100-node) topology designs, respectively. Both the methods and the results reported in the paper can be used to develop traffic-tailored network topologies and/or adapt them to deliver bandwidth when and where it is needed, which is the key to intelligent optical network designs.

Further work is ongoing to expand the current topology design methods with combined optimisation targets such as average end-to-end latency, network resilience and more diverse traffic modelling.

# FUNDING

Financial support from UK EPSRC Doctoral Training Programme and the Programme Grant TRANSNET (EP/R035342/1) is gratefully acknowledged. Microsoft is thanked for the support under the 'Optics for the Cloud' programme. For the purpose of open access, the author has applied a Creative Commons Attribution (CC BY) licence to any Author Accepted Manuscript version arising.

## ACKNOWLEDGMENTS

Part of this work was presented at the International Conference on Optical Network Design and Modelling (ONDM) in 2022, with the paper entitled "Towards a Traffic-Optimal Large-Scale Optical Network Topology Design".

# REFERENCES

- 1. B. Mukherjee, I. Tomkos, M. Tornatore, P. Winzer, and Y. Zhao, *Springer Handbook of Optical Networks* (Springer Nature, 2020), p. 1.
- D. R. de Araujo, C. J. Bastos-Filho, and J. F. Martins-Filho, "An evolutionary approach with surrogate models and network science concepts to design optical networks," Eng. Appl. Artif. Intell. 43, 67–80 (2015).
- T. Fencl, P. Burget, and J. Bilek, "Network topology design," Control. Eng. Pract. 19, 1287–1296 (2011).
- G. Xiao, Y.-W. Leung, and K.-W. Hung, "Two-stage cut saturation algorithm for designing all-optical networks," IEEE Transactions on Commun. 49, 1102–1115 (2001).
- H. Liu and F. A. Tobagi, "Physical topology design for all-optical networks," Opt. Switch. Netw. 5, 219–231 (2008).
- Y. Xin, G. Rouskas, and H. Perros, "On the physical and logical topology design of large-scale optical networks," J. Light. Technol. 21, 904–915 (2003).
- D. A. R. Chaves, C. J. A. Bastos-Filho, and J. F. Martins-Filho, "Multiobjective physical topology design of all-optical networks considering qos and capex," in 2010 Conference on Optical Fiber Communication (OFC/NFOEC), collocated National Fiber Optic Engineers Conference, (2010), pp. 1–3.
- C. J. A. Bastos-Filho, D. R. B. Araujo, E. A. Barboza, D. A. R. Chaves, and J. F. Martins-Filho, "Design of transparent optical networks considering physical impairments, capex and energy consumption," in 2011 13th International Conference on Transparent Optical Networks, (2011), pp. 1–4.
- N. Dharmaweera, R. Parthiban, and Y. A. Sekercioglu, "Multi-constraint physical topology design for all optical networks," in 2011 18th International Conference on Telecommunications, (2011), pp. 463–469.

- D. Zili, Y. Nenghai, and L. Zheng, "Designing fault tolerant networks topologies based on greedy algorithm," in 2008 Third International Conference on Dependability of Computer Systems DepCoS-RELCOMEX, (2008), pp. 227–234.
- M. Gerla and L. Kleinrock, "On the topological design of distributed computer networks," IEEE Transactions on Commun. 25, 48–60 (1977).
- K.-T. Ko, K.-S. Tang, C.-Y. Chan, K.-F. Man, and S. Kwong, "Using genetic algorithms to design mesh networks," Computer. **30**, 56–61 (1997).
- S. A. Khan and A. P. Engelbrecht, "A fuzzy particle swarm optimization algorithm for computer communication network topology design," Appl. Intell. 36, 161–177 (2012).
- F. Altiparmak, B. Dengiz, and A. E. Smith, "Optimal design of reliable computer networks: A comparison of metaheuristics," J. Heuristics 9, 471–487 (2003).
- Cisco, "Cisco visual networking index: Forecast and trends, 2017–2022 white paper," Tech. report (2019).
- K. Higashimori, F. Inuzuka, and T. Ohara, "Physical topology optimization for highly reliable and efficient wavelength-assignable optical networks," J. Opt. Commun. Netw. 14, 16–24 (2022).
- D. Payne, "Evolvable, sustainable, low power optical network architectures," in 2011 37th European Conference and Exhibition on Optical Communication, (2011), pp. 1–44.
- J. M. Simmons, *Optical Network Design and Planning*, Optical Networks (Springer International Publishing, Cham, 2014).
- D. R. de Araújo, C. J. Bastos-Filho, and J. F. Martins-Filho, "An evolutionary approach with surrogate models and network science concepts to design optical networks," Eng. Appl. Artif. Intell. 43, 67–80 (2015).
- R. Matzner, R. Luo, G. Zervas, and P. Bayvel, "Ultra-fast optical network throughput prediction using graph neural networks," in 2022 International Conference on Optical Network Design and Modeling (ONDM), (IEEE, 2022), pp. 1–3.
- R. Luo, R. Matzner, G. Zervas, and P. Bayvel, "Towards a traffic-optimal large-scale optical network topology design," in *2022 26th International Conference on Optical Network Design and Modelling (ONDM)*, (2022), pp. 1–3.
- A.-L. Barabasi and R. Albert, "Emergence of scaling in random networks," Science. 286, 509–512 (1999).
- P. Bayvel, R. Luo, R. Matzner, D. Semrau, and G. Zervas, "Intelligent design of optical networks: which topology features help maximise throughput in the nonlinear regime?" in 2020 European Conference on Optical Communications (ECOC), (IEEE, 2020), pp. 1–4.
- R. Matzner, D. Semrau, R. Luo, G. Zervas, and P. Bayvel, "Making intelligent topology design choices: understanding structural and physical property performance implications in optical networks," J. Opt. Commun. Netw. 13, D53–D67 (2021).
- R. J. Vincent, D. J. Ives, and S. J. Savory, "Scalable capacity estimation for nonlinear elastic all-optical core networks," J. Light. Technol. 37, 5380–5391 (2019).
- D. Semrau, R. I. Killey, and P. Bayvel, "A Closed-Form Approximation of the Gaussian Noise Model in the Presence of Inter-Channel Stimulated Raman Scattering," J. Light. Technol. **37**, 1924–1936 (2019).
- C. E. Shannon, "A Mathematical Theory of Communication," The Bell Syst. Tech. J. 27, 379–423 (1948).
- S. Baroni and P. Bayvel, "Wavelength requirements in arbitrarily connected wavelength-routed optical networks," J. Light. Technol. 15, 242– 251 (1997).
- P. Erdos and A. Renyi, "On the Evolution of Random Graphs," in Publication of the Mathematical Institute of the Hungarian Academy of Sciences, (1960), pp. 17–61.
- R. J. Gibbens, F. P. Kelly, and S. R. E. Turner, "Dynamic routing in multiparented networks," IEEE/ACM Transactions on Netw. 1, 261–270 (1993). Conference Name: IEEE/ACM Transactions on Networking.
- R. W. Floyd, "Algorithm 97: shortest path," Commun. ACM 5, 345 (1962).
- R. Luo, Y.-Z. Xu, R. Matzner, G. Zervas, D. Saad, and P. Bayvel, "Message passing: Towards low-complexity, global optimal routing and wavelength assignment solutions for optical networks," in 2022 Optical

Fiber Communications Conference and Exhibition (OFC), (2022), pp. 1–3.

- J. Y. Yen, "Finding the K Shortest Loopless Paths in a Network," Manag. Sci. 17, 712–716 (1971). Publisher: INFORMS.
- 34. F. R. Chung and F. C. Graham, *Spectral graph theory*, vol. 92 (American Mathematical Soc., 1997).
- 35. D. Whitley, "A genetic algorithm tutorial," Stat. Comput. 4, 65–85 (1994).
- A. Ottino, A. Saljoghei, T. Hayashi, T. Nakanishi, C. Kochis, P. De Dobbelaere, and G. Zervas, "Genetic algorithm optimization of multi core fibre transmission links based on silicon photonic transceivers," in 2019 Optical Fiber Communications Conference and Exhibition (OFC), (2019), pp. 1–3.
- S. Shabir and R. Singla, "A comparative study of genetic algorithm and the particle swarm optimization," Int. J. electrical engineering 9, 215–223 (2016).