

Book Review for RME:

'Connecting Mathematics and Mathematics Education' by Erich Christian E. Ch. Wittmann
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E. Ch. Wittmann, E. C. (2021). Connecting Mathematics and Mathematics Education: Collected Papers on Mathematics Education as a Design Science (p. 318). Springer Nature.

This book is based on works of Erich Christian Wittmann and his major publications (single authored but also co-authored) since the early 1970s to 2019, presented chronologically to reflect the author's historical progress of his didactic thinking about the teaching of mathematics, in his quest to contribute to establishing mathematics education as a discipline in its own right.

Wittmann studied mathematics and physics at the University of Erlangen (1959 – 1964), followed by practical training for the teaching profession (1964 – 1966). He then received a PhD with a dissertation in group theory in 1967 and after four years of research in mathematics, his interests turned towards mathematics education, much influenced by an encounter with Hans Freudenthal in 1966. As a full professor of mathematics education at the University of Dortmund (1970-2004), Wittmann's work has been heavily influenced by great minds such as John Dewey, Johannes Kuehnel, Jean Piaget, Hans Freudenthal, and Heinrich Winter.

It is important to note that Wittmann transitioned from 'mathematics' to 'mathematics education' at a time when mathematics education as a young discipline was under considerable pressure, with scholars of established disciplines displaying ignorance and acting toward it in unfair ways. While Wittmann recognised that mathematics education (or 'didactics of mathematics' as more referred to in this book) could not exist and grow without close relationships to related areas such as mathematics, psychology, and pedagogy, he was also aware of and warned of the risk of adopting standards, methods and research contexts from these other well-established disciplines. His life-long professional quest thus became that of working towards establishing mathematics education as a discipline in itself, and the discipline of mathematics education as a research domain.

Overview of the book

The book follows chronologically Wittmann's efforts towards conceiving mathematics education as a 'design science'. Wittmann does this by drawing on 12 of his main papers, which can be classified into four categories, as explained by the author himself in the Preface of the book: 1. papers that explain the *design science* approach and elaborate on the specific status of mathematics education as a systemic-evolutionary design science; 2. papers that describe how to design *teaching units*, later referred to as *learning environments*, in particular the structure-genetic didactical analyses approaches which are implicit in developing *substantial learning environments*; 3. papers that deal with teacher education, where learning environments are proposed to and have been shown to be a very effective tool for introducing student teachers to elementary mathematics, to principles of mathematics teaching, and to the curriculum, and finally 4. papers that use non-symbolic means of representation as a

crucial element for designing substantial learning environments which are both mathematically sound and adapted to students' prior knowledge.

About the book content

The book comprises of 14 chapters. In Chapter 1 we learn that Wittmann's view of mathematics education as a *design science* was inspired by Simon's (1970) view of design sciences as sciences concerned with the construction of artefacts intended for specific purposes. For developing mathematics education as a design science, it was thus crucial to find ways of how 'design' on the one hand and 'empirical research' on the other, can be related to one another, and Wittmann proposed a specific approach, namely empirical research centred around *teaching units*.

The design science approach to mathematics education was thus born from the intention to support teachers by providing them with knowledge for organizing learning processes in the form of elaborated teaching units. As described in Chapter 2, in 1984 Wittmann proposed to consider *teaching units* as the artefacts of mathematics education, and he later referred to these as *learning environments*. Wittmann draws on Brousseau's (1997) five 'didactical situations': instruction, action, formulation, validation, and institutionalization, to explain the interplay between the teacher's interventions and students' activities. These units are not intended to be detailed or complete lesson units, but environments where didactical knowledge is organised in an effective way for mathematics teaching. According to Wittmann, these elaborated learning environments contain the essential information for teaching, and 'the reflective teacher' (Schön, 1983) will not stick to them, but use them as springboard for exploring other learning environments and transferring what they have learned in order to make sense of new situations. Wittmann exemplifies such effective organisation by describing in detail a teaching unit on 'Calculating with Remainders' in the second part of Chapter 1. Based on Brousseau's theory of didactical situations, he proposes a specific format for such units which involves: brief information on objectives (O), teaching materials (M), mathematical problems arising from the context of the unit (P), and the mostly mathematical, sometimes psychological background of the unit (B).

Such learning environments are characterised by *substantial* mathematical content that goes beyond that of school level, and which also offers rich mathematical activities for (pre-service) teachers on a higher level. The empirical study described in Chapter 10 proved the high acceptance of these learning environments by student teachers, and Wittmann proposed they are used by teacher education programme. This work led to *substantial learning environments* being considered a very effective tool for introducing student teachers into elementary mathematics and into the principles of mathematics teaching. When designed appropriately, substantial learning environments, "provide opportunities for doing mathematics, for studying one's own learning processes and those of students, for evaluating different forms of social organization, and for planning, performing and analysing practical teaching" (p. 29).

The design of teaching units had never been a focus of research before. They were mainly used as incidental examples in investigating and presenting theoretical ideas. Similarly, the design of teaching tended to be considered as a mediocre task carried out by teachers and textbook authors, a task with no place under the research umbrella. Wittmann challenges this view and in Chapter 3 he presents an approach that tries to bridge this gap by means of

teaching units. It was only in 1995, following the gradual development of the design science approach, that Wittmann makes a concrete proposal for bridging the gap between theory and practice in mathematics education and for establishing a systemic relationship between researchers and teachers. In Chapter 9 Wittmann proposed to consider mathematics education as a design science, initially referred to as 'philosophy of teaching units', "with the intention to establish a sound methodological basis for a science of mathematics education that would guarantee a firm link between theory and practice and preserve the mathematically founded work achieved in curriculum development and teacher education by mathematics educators in the past" (p. 239).

One of the most important principles followed by Wittmann is that mathematics itself lies at the core of all work in mathematics education. To further strengthen the role of mathematics within the research in mathematics education, Wittmann worked on formulating the research method underlying the design of the learning environments. He called this method *structure-genetic didactical analysis* as described and exemplified in Chapter 13, where he combines a profound analysis of mathematical contents and processes with the design of substantial learning environments.

In Wittmann's view, such learning environments should form a major part of the professional training of teachers. Wittmann proposes that courses in elementary mathematics should be an indispensable part of the professional training of teachers. Such courses should start with mathematics: be explicitly related to the content of school mathematics, giving a coherent treatment of relevant parts of elementary algebra (number theory and combinatorics), elementary geometry, calculus, and elementary stochastics, but also going well beyond school mathematics in both depth and breadth; be rich in relationships to history, culture and real world; be organized in a genetic way; the style of these courses should be informal; should allow for a variety of teaching and learning formats, and contribute to a teacher's special mathematical expertise. Initially, Wittmann's view was that these courses in elementary mathematics should deal implicitly with the teaching of school mathematics, while didactical courses should be the ones addressing explicitly the teaching of school mathematics. However, drawing on his experiences of writing a textbook on elementary geometry for teacher training (Wittmann, 1987), Wittmann describes in Chapter 4 how this work contributed to the development of his new position whereby he became convinced that the development of courses on elementary mathematics "presupposes the conceptual reconstruction of elementary parts of mathematics from an educational point of view" (p. 58). As an illustration of what this could mean, Chapter 5 focuses on Wittmann's work on elaborating a central point of this kind of elementary mathematics, namely the notion of a *sound informal proof*. The author proposes ways for combating the formalistic viewpoints on proof, seen by many teachers and student teachers as the "official" philosophy of mathematics, and gives some examples which show that one such approach is to develop informal mathematics as a separate mathematical reasoning.

The relationship between research and practice has always been an important factor for Wittmann's engagement in teacher education, and hence Wittmann proposes that mathematics courses in teacher education should be *systematically related to substantial learning environments*. He proposes that what should count most for teacher preparation courses is not the didactical component, but the mathematical component, with mathematical activities that stimulate and provide student teachers with relevant

experiences in regard to learning processes, including learning challenges and potential ways to overcome them.

In Wittmann's conception, a substantial learning environment is not prescriptive, but left open, to allow for teachers to follow their students' ideas in trying to solve the problems. These units should offer flexibility to be used by teachers, and teachers need to be supported. A move away from viewing teaching as a transmission of information from the teacher to a passive student was needed, and therefore a re-conceptualisation of the learning environment as the productive achievement of the student who learns in social interaction with other students and the teacher was needed. This is exemplified in Chapter 8, where Wittmann introduces the project Mathe 2000 founded in 1987, which was intended as a developmental research project that was based on his conception of mathematics education as a design science. The chapter describes a specific feature of this project, namely the development of a *grammar of non-symbolic representations* and provides the reader with some further examples of teaching units to indicate how such number representations are no longer considered as tools of the teacher for transmitting knowledge, but as tools of the students for doing mathematics.

Wittmann admits to being an adherent of the genetic principle, whereby "the best way to understand a concept is to see how it originated from a rough idea and how it has been increasingly articulated, expanded, differentiated, and coordinated with other concepts in a continued process" (p. xi). He thus promotes a view of teaching mathematics that fosters a lively and dynamic picture of mathematics in contrast with the "anaemic skeleton usually presented to freshmen students" (p. 28), and such a view should be reflected in the design of the learning environments.

Moreover, the design of substantial learning environments in Wittmann's view requires the designer to know higher mathematics (such as the formal representation of the theory of residue class rings, for example). He proposes that such knowledge is a significant advantage for the designer and design of the learning environment, as *substantial* learning environments should be based on *substantial* mathematics beyond the school level. Offering such mathematical activities on a higher level, student teachers are empowered pedagogically, as they can learn to deal with elementary mathematical structures in a productive way. Indeed, in Chapter 7 Wittmann uses the famous Pythagorean theorem to illustrate a successful design of teaching when based on an integrative approach of mathematics and pedagogy.

Wittmann first introduces the suggestion to conceive mathematics education as a *design science* in Chapter 6, and he returns to this main argument in Chapter 14 in the context of new developments in the mathematics education all over the world. According to Wittmann, the design of these substantial learning environments, the empirical research, and the implementation of these environments are the main tasks of research in mathematics education. It is exactly these tasks that prove the autonomy of mathematics education as a discipline on its own!

Why read this book

Looking back at the whole book, this is a great contribution to scholarly work in mathematics education as it describes the author's journey on the timeline of **historical developments of**

mathematics education as a discipline. It is a must-read for researchers in mathematics education. Mathematics teacher educators will find the examples of the substantial learning environments insightful, providing food for thought, in particular the argument about the **higher mathematics knowledge of designers** of these environments.

As an adherent of the genetic principle, Wittmann seems to have applied this view to putting together this book. Terminology and concepts of his philosophical stance are introduced at various stages in the book, as they were encountered and developed by the author in their chronological order. Wittmann is convinced that this decision will stimulate the readers to critically examine and engage with his work. This is indeed the case, the drawback being that reading chapters not in their numerical order or reading a chapter as a stand-alone makes it harder for the reader to see the big picture and understand the terminology and/or argument put forward by the author.

The chapters are limited to the German context, where teacher training is undertaken as part of an undergraduate programme, where courses in mathematics, in education, and more recently courses that integrate mathematics and education are taught alongside each other. Nevertheless, the specific **examples of the substantial learning environments** are a valuable learning resource for mathematics educators and student teachers internationally, irrespective of the structure of their teacher training programmes.

Connecting 12 papers on different topics together must have been a challenge for the author; this will certainly provide a challenge for the reader of the book, as understanding how these papers/chapters fit together in the bigger puzzle (book) is not necessarily made explicit by the author. However, arranging chapters chronologically is also the strength of this book, which when reading from cover to cover provides the reader with an understanding of the development of Wittmann's conception of **mathematics education** as a *design science*, which cannot develop without close links to **mathematics**.

The book is published open access <https://link.springer.com/book/10.1007/978-3-030-61570-3>, meaning that its 14 chapters spreading over 328 pages can be downloaded for free, which is especially beneficial to those who do not have wide access to scientific books and journals.

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Cosette Crisan is an associate professor in mathematics education at UCL Institute of Education. Dr Crisan's research activities stem from her strong interest on how teachers' mathematical and pedagogical knowledge are intertwined and how they affect one's

teaching. In her recent work, Dr Crisan explored how engagement with a body of advanced mathematics knowledge has the potential to empower teachers not just conceptually, but also pedagogically, thus situating the study of advanced mathematics in relation not only to school mathematics content, but also its teaching. Her main research interest is situated at the nexus of mathematics education and digital technology, with a focus on investigating the use of educational technologies for the learning and teaching of mathematics at all stages of education.