

Intelligent Fault Detection Algorithm Based on H_i/H_∞ Optimization and a Cascaded Neural Networks

Muzhuang Guo¹, Chen Guo², Chuang Zhang¹, Xinyu Zhang^{1*} and Yuanchang Liu^{3*}

¹Navigation College, Dalian Maritime University, Dalian, China

²Marine Engineering College, Dalian Maritime University, Dalian, China

³Department of Mechanical Engineering, University College London, London, UK

(* Corresponding authors: Yuanchang Liu, yuanchang.liu@ucl.ac.uk; Xinyu Zhang, zhangxy@dlnu.edu.cn)

Abstract

Autonomous surface ships (ASSs) have attracted attention owing to their ability to perform various tasks in complex and challenging aquatic environments without relying on a crew. However, they require reliable sensors to ensure navigational safety. In this study, a robust and intelligent fault detection algorithm was designed for the integrated navigation system of an ASS. First, a residual observer-based fault detection algorithm using H_i/H_∞ optimization is proposed to deal with process disturbances and measurement noise. Such noise can be modeled under the condition of a bounded l_2 -norm to account for the sensitivity and robustness of the residual observer against random noise with unknown properties. However, this fault detection algorithm is insensitive to soft faults, which manifest as noise characterized by a small amplitude and slow variation. Conventional strategies for evaluating the fault detection threshold rely on human experience, which is insufficiently sophisticated for fault detection. Therefore, a cascaded neural network is proposed for optimizing the fault detection algorithm when the amount of training data is limited. The cascaded neural network consists of a multi-feature time domain network, a frequency-domain fault detection network as well as a decision-level fusion network. The proposed algorithm was verified in simulations as well as on historical data collected from real ship sensors. The results demonstrated that the proposed algorithm offers intelligent fault detection, including soft faults, with a low false alarm rate for integrated navigation systems.

Key words: integrated navigation system, robust fault detection, H_i/H_∞ optimization, cascaded neural network

1. Introduction

According to Fan et al. (2020), an autonomous surface ship (ASS) can function independently of human input to varying degrees. An ASS comprises three primary systems: control, navigation, and propulsion. The navigation system feeds information on the motion state to the control system, which is critical for ensuring safe and reliable navigation. The integrated navigation system of an ASS includes the ship's inertial navigation system (SINS), a global navigation satellite system (GNSS), such as the Global Positioning System (GPS), the Doppler velocity log (DVL), and the navigation radar, each of which offers advantages and disadvantages. At present, improving the performance of a single navigation technology is inconvenient. A more common approach is to combine the information from multiple sensors to improve navigational accuracy. However, previous studies (Liu et al. 2016; Thombre et al. 2022) have shown that introducing additional sensors to increase the estimation robustness and accuracy of the integrated navigation system also considerably increases its complexity. If a fault occurs in the subsystems of the integrated navigation system, this could generate catastrophic effects through feedback control (Chen et al. 2020). Considering the complex operating environments of ASSs, robust state estimation algorithms have been proposed to account for unknown input noise. Accurately detecting sensor failures in the presence of uncertain inputs is an important research subject (Wang et al. 2022) with some studies already applied fault detection technology to ASS communication systems (Thombre et al. 2022). Consequently, this field has attracted considerable research attention and many fault detection methods have been proposed, which can be broadly classified as model-based or model-free (Gao et al. 2015; Gao, Cecati, & Ding 2015).

55 Model-based fault detection methods can be further divided into those based on a residual observer
56 (Miao et al. 2014) and those based on a state observer (Yu et al. 2021). These methods use a Kalman
57 filter to create state or information residual statistics and determine if an integrated navigation system
58 has a fault based on the probability distribution it follows. However, this algorithm is highly dependent
59 on the system and measurement noises satisfying the constraint of noncorrelated zero-mean white
60 Gaussian noise (Li et al. 2020), and it cannot guarantee that the evaluation function of the observer is
61 robust against uncertain noise and sensitive to faults. To solve this limitation, several researchers (Chen
62 et al. 2000; Liu et al. 2018; Khan et al. 2014) have applied H_i/H_∞ optimization to discrete-time nonlinear
63 systems. Zhong et al. (2016) proposed a fault detection algorithm for optimizing the SINS/GNSS
64 integrated navigation system in the presence of normally distributed disturbances. Liu et al. (2019)
65 proposed a new residual evaluation function for the H_i/H_∞ optimization of discrete-time nonlinear
66 systems. However, this residual observer-based extended H_i/H_∞ algorithm is restricted by the
67 linearization accuracy of the dynamic model for Taylor expansion in the extended Kalman filter
68 framework, which could result in a large state estimation error and high false alarm rate. Residual
69 observer-based methods are often limited by their dependence on known model parameters, and the
70 detection threshold is difficult to define, so they are often not sensitive enough to detect soft faults,
71 which are characterized by a slow change or small amplitude (Sun et al. 2021).

72
73 To address this limitation, data-driven fault detection algorithms have been extensively studied (Gao,
74 Cecati & Ding et al. 2015). Xi et al. (2018) showed that the fundamental principles of fault detection
75 algorithm based on signal processing include the following steps: (1) extracting the time or frequency
76 domain features from the output information of the system under various operating conditions, and (2)
77 using the extracted statistical eigenvalues for fault detection. Zhu et al. (2016) proposed a fault detection
78 function that comprises the predicted and actual innovations of the Kalman filter in addition to their
79 variance. Zhong et al. (2017) proposed a soft fault detection method that incorporates the least-squares
80 support vector machine (LS-SVM) regression theory into the autonomous integrity monitored
81 extrapolation framework. Yang et al. (2020) proposed a fault detection algorithm based on the
82 similarities between support vector machine (SVM) and principal component analysis. Zanolini et al.
83 (2012) proposed a method for the real-time detection of soft and hard faults in an integrated navigation
84 system. Several researchers have considered combining data-driven fault detection with a neural
85 network. Guo et al. (2018) proposed a convolutional neural network (CNN)-based algorithm for
86 detecting frequency-domain signal faults in unmanned aerial vehicle sensors. However, extracting
87 features from the original sensor signal depends on the data volume, and different models are needed
88 for different sensors. Thus, such methods lack generality and are difficult to be applied to the integrated
89 navigation systems of ASSs operating in a complex environment.

90
91 It can be summarized that the current research on fault detection of integrated navigation systems has
92 the following deficiencies. First, model-based fault detection is not sufficiently robust against unknown
93 system noises and statistical characteristics. Second, residual observer-based fault detection normally
94 relies on human experience to determine the threshold settings and is not sensitive to soft faults. Finally,
95 many researchers have proposed using deep learning to solve the above problems, but most built a
96 neural network for a single sensor, and training such networks requires a large amount of data.
97 Extracting a large number of fault samples from the integrated navigation systems of an ASS is
98 unrealistic.

99
100 Thus, accurate fault detection for the integrated navigation system of an ASS that does not rely on model
101 constraints, or a large amount of data is an important research challenge. In this study, a new fault
102 detection algorithm was developed using H_i/H_∞ optimization and multiple neural networks to have an
103 improved sensitivity to faults and robustness against unknown inputs. The main contributions of the
104 proposed algorithm are as follows:

- 105
106 1. The algorithm was designed according to the cubature Kalman filter (CKF) framework so that it is
107 theoretically unaffected by linearization of kinetic model in contrast to the extended H_i/H_∞ fault
108 detection algorithm.

110 2. The optimization by a cascaded neural network does not rely on empirical parameters such as the
 111 threshold of the evaluation function making the proposed detection algorithm have an improved
 112 sensitivity to soft faults.
 113

114 3. The proposed algorithm was verified through extensive simulations as well as real ship data. The
 115 fault detection process functions perform well in all cases where limited sample data are available.
 116

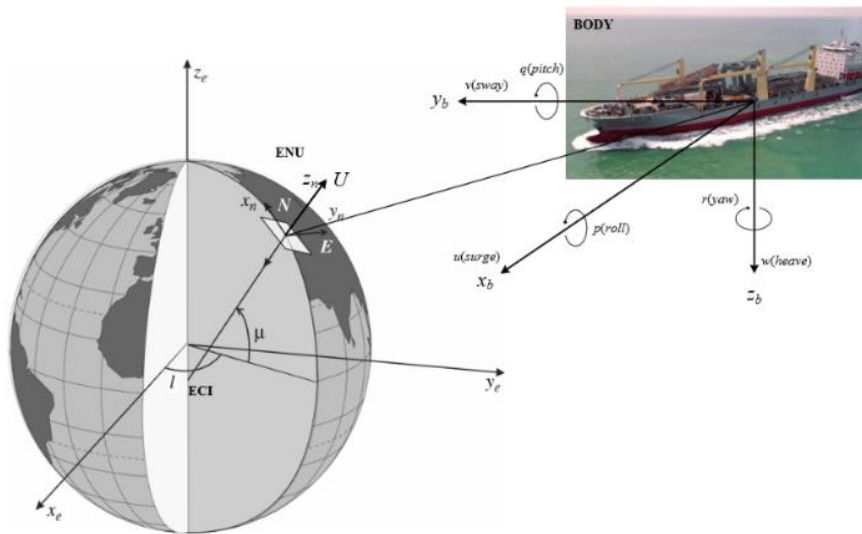
117 The rest of the paper is organized as follows. Section 2 introduces the commonly used error state and
 118 measurement equations for the integrated navigation systems of an ASS. Section 3 presents the
 119 proposed fault detection algorithm and the optimization using a cascaded neural network. Section 4
 120 presents the verification of the algorithm through simulations and historical ship data. Section 5
 121 concludes the paper.
 122

123 2. Integrated navigation system and its error model

124 The most commonly used sensors to measure the motion state of an ASS are SINS, GNSS, DVL, and
 125 the BeiDou Navigation Satellite System (COMPASS). The error models associated with these sensors
 126 have significant impact on the accuracy and performance of the entire integrated navigation system.
 127

128 2.1 Error state equation

129 MEMS-SINS comprises three microelectromechanical system (MEMS) gyroscopes and three
 130 accelerometers. The navigation coordinate system (n -frame) is defined according to the east–north–up
 131 system. The origin of the body frame (b -frame) is located at the centroid of the vessel. The Earth-
 132 centered Earth-fixed frame (ECEF, e -frame) and the inertial frame (ECI, i -frame) originate at the center
 133 of Earth. However, the i -frame does not rotate in lockstep with the fixed stars. Figure 1 shows the
 134 relationships between different frames.
 135



136 Figure 1. Reference frames and notation of variables for a vessel.
 137
 138

139 The nonlinear SINS error model can be expressed as follows:
 140

$$\begin{cases} \dot{\phi}^n = \phi^n \times \omega_m^n + \delta\omega_m^n - C_b^n \omega_{ib}^b - \varepsilon^n \\ \delta\dot{V}^n = -\phi^n \times f^n + C_b^n f^b + \delta V^n \times (2\omega_{ie}^n + \omega_{en}^e) \\ \quad + V^n \times (2\delta\omega_{ie}^n + \delta\omega_{en}^e) + \nabla^n \\ \delta\dot{P}^n = \delta V^n + \rho \times \delta P^n \end{cases} \quad (1)$$

142 where ϕ^n , δV^n , and δP^n are the attitude, velocity, and position error vectors, respectively, in the n -
 143

144 frame. ε^n and ∇^n are the vectors of the gyroscope and accelerometer noises, respectively.
 145 ω_{ab}^c ($a, b, c = i, e, n$) is the local Earth rotation rate in the b -frame with respect to the a -frame expressed in
 146 the c -frame (Wang et al. 2017). $\rho = [-\dot{L}, \dot{\lambda} \cos L, -\dot{\lambda} \sin L]^T$, where L , λ , and h are the latitude,
 147 longitude, and height, respectively. C_b^n is the attitude transformation matrix between the b -frame and n -
 148 frame (Wei et al. 2018). Thus, the continuous-time nonlinear state equation for an integrated navigation
 149 system can be expressed as follows:

$$150 \quad \dot{x}_t = f_c(x_t) + B_c \omega_t \quad (2)$$

151 where $x_t = [\delta P^n, \delta V^n, \phi^n, \varepsilon^n, \nabla^n]$ and $\omega_t = [\varepsilon^n, \nabla^n]$. f_c and B_c denote the nonlinear coupling system
 152 equations for transferring the state variables x_t and unknown noise ω_t in Equation (1).
 153

154 2.2 Measurement equations

155 The following discrete-time equation is usually used to express the measurement function:

$$156 \quad z_t = H_t x_t + v_t \quad (3)$$

157 where z_t is the measurement vector, H_t is the observation matrix, and v_t is the vector of the
 158 measurement noise. SINS can be used as the reference navigation system to ensure high precision. Thus,
 159 SINS/GNSS and SINS/DVL/COMPASS integrated navigation systems are often adopted to realize
 160 autonomy and accuracy.

161 For the SINS/GNSS integrated navigation system, the measurement vector z_t^{sg} comprises the difference
 162 between the position and velocity obtained using SINS and GNSS (Shen et al. 2019). v_t^{sg} is the
 163 measurement noise of the SINS/GNSS integrated navigation system:
 164

$$165 \quad z_t^{sg} = \begin{bmatrix} L_t^{SINS} - L_t^{GNSS} \\ \lambda_t^{SINS} - \lambda_t^{GNSS} \\ h_t^{SINS} - h_t^{GNSS} \\ V_{E,t}^{SINS} - V_{E,t}^{GNSS} \\ V_{N,t}^{SINS} - V_{N,t}^{GNSS} \\ V_{H,t}^{SINS} - V_{H,t}^{GNSS} \end{bmatrix} \quad (4)$$

166 The measurement equation for the SINS/GNSS integrated navigation system is as follows:
 167

$$168 \quad z_t^{sg} = \begin{bmatrix} I_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 9} \\ 0_{3 \times 3} & I_{3 \times 3} & 0_{3 \times 9} \end{bmatrix} x_t + v_t^{sg} \quad (5)$$

169 For the SINS/DVL/COMPASS integrated navigation system, the measurement information comprises
 170 the difference between the east–north velocities and headings obtained using SINS, DVL, and
 171 COMPASS:
 172

$$173 \quad z_t^{sdc} = \begin{bmatrix} V_{E,t}^{SINS} - V_{E,t}^{DVL} \\ V_{N,t}^{SINS} - V_{N,t}^{DVL} \\ \psi_t^{SINS} - \psi_t^{COMPASS} \end{bmatrix} \quad (6)$$

174 Thus, the measurement equation for the SINS/DVL/COMPASS integrated navigation system is as
 175

183 follows:

184

$$185 \quad z_t^{sdc} = \begin{bmatrix} 0_{1 \times 2} & 1 & 0_{1 \times 3} & 0_{1 \times 3} & 0_{1 \times 3} \\ 0_{3 \times 2} & 0_{3 \times 1} & I_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix} x_t + v_t^{sdc} \quad (7)$$

186

187 v_t^{sdc} is the measurement noise of the SINS/DVL/COMPASS integrated navigation system.

188

189 2.3 Integrated navigation system based on the cubature Kalman filter

190 For an integrated navigation system comprising the error state system of Equation (2) and measurement

191 system of Equation (3), Fig. 2 shows the estimation error of the motion state and process of correcting

192 the SINS output. The system noise is $\omega_k \sim N(0, Q_k)$, and the random measurement noise is $v_k \sim N(0, R_k)$.

193 Therefore, a Gaussian filter with a Kalman filter structure can be used to process the nonlinear discrete

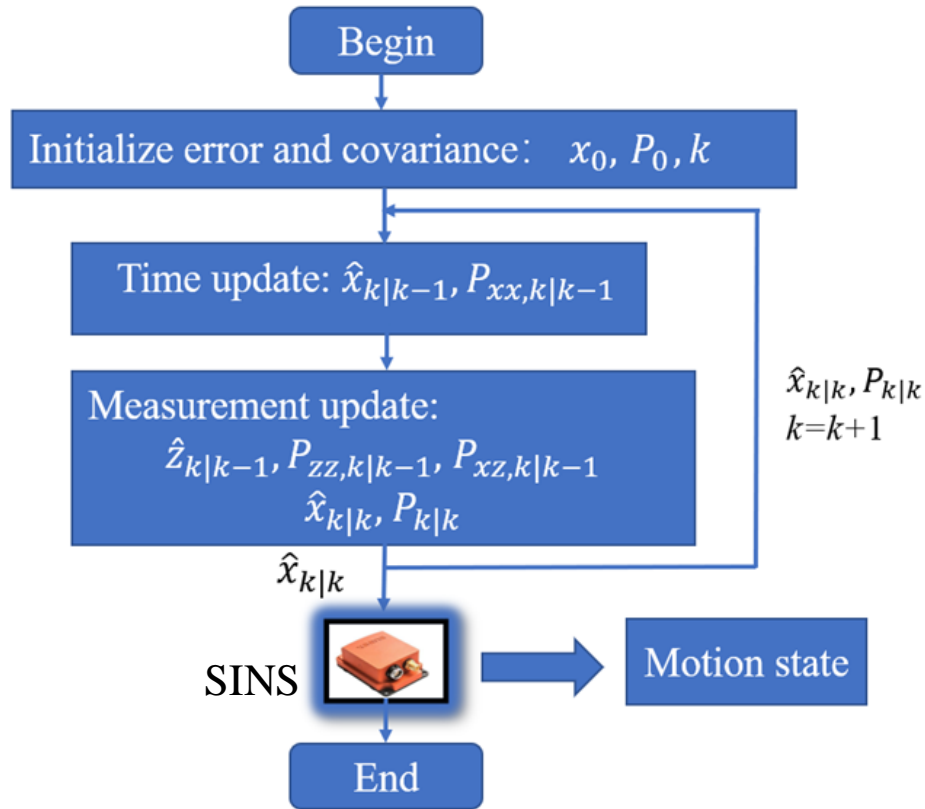
194 integrated system at step k in the state estimation task while the time and measurement are being

195 updated. The details of the CKF algorithm for the integrated navigation system is given in Appendix I.

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Figure 2. CKF-based integrated navigation system.

201

202 When an ASS is cruising at sea, ω_k and v_k are affected by the complex environment, such as thermal

203 changes and electromagnetic interference. The properties of random errors may be uncertain and

204 variable, which contradicts the requirement of the Kalman filter for zero-mean Gaussian white noise.

205 This random error will lead to significant measurement error or even failure of the angular velocity and

206 acceleration, which will affect one-step state prediction ($\hat{x}_{k|k-1}$) and prediction variance ($P_{xx,k|k-1}$) in the

207 time update. Furthermore, GNSS and DVL frequently encounter measurement outliers, which will lead

208 to errors in measurement matrix (z_k) at k step and affect the filter gain. This in turn reduces the state

209 estimation accuracy. The above faults eventually affect state prediction covariance matrix ($P_{k|k}$) and

210 will contaminate the output of the integrated navigation system in step $k+1$.

211 3. Proposed fault detection algorithm

212 In addition to accurate motion state estimation, the increased demand for reliability has increased the
 213 importance of fault detection for the integrated navigation system of an ASS. Therefore a new fault
 214 detection algorithm has been proposed with the details described below.

216 3.1 Residual observer-based fault detection using CKF and H_2/H_∞ optimization

217 To account for the sensitivity to faults and robustness against unknown random noise of the residual
 218 observer, a fault detection algorithm is proposed for a time-varying nonlinear integrated navigation
 219 system using H_2/H_∞ optimization. Much of the theory behind the proposed algorithm is based on the
 220 research of Zhong et al. (2015). The fault case for a nonlinear state-space model of integrated navigation
 221 system can be expressed as

$$222 \begin{cases} \dot{x}_t = f_c(x_t) + B_c \omega_t + B_{cf,t} f_{1,t} \\ z_t = H_t x_t + v_t + D_{cf,t} f_{2,t} \end{cases} \quad (8)$$

224 where $f_{1,t}$ and $f_{2,t}$ are the SINS and measurement faults, respectively. Unlike Zhong et al. (2015, 2016),
 225 the proposed algorithm adopts CKF to satisfy the accuracy and robustness requirements for motion state
 226 estimation of a strongly nonlinear integrated navigation system. The first step of designing a residual
 227 observer-based fault detection filter is the discrete linearization of f_c as:

$$229 \begin{aligned} f_c(x_t) &= A_k x_k + \xi_k \\ \{\hat{A}_k\} &= \arg \min \frac{1}{2n} \sum_{k=1}^{2n} \xi_k^T \xi_k \end{aligned} \quad (9)$$

231 where $\xi_k = \gamma_k - A_k \chi_k$ and γ_k are the $2n$ sampling result of a one-step state prediction. Then,
 232 $A_k = (\chi_k \chi_k^T)^{-1} (\chi_k \gamma_k^T)$ can be obtained using culture point sampling. A_k is the result from the
 233 linearization of f_c . d_k and f_k are assumed to be $l_2[0, N]$ -norm bounded uncertain disturbances and
 234 measurement noise in step k .

235 Thus, the nonlinear discrete-time dynamic system of Equation (8) can be rewritten as follows:

$$237 \begin{cases} x_{k+1} = A_k x_k + B_c d_k + B_f f_k + \xi_k \\ z_{k+1} = H_k x_k + D_c d_k + D_f f_k \end{cases} \quad (10)$$

240 where $f_k = [f_{1,t}^T, f_{2,t}^T]^T$, $B_f = [B_{cf,t}, 0]$, and $D_f = [0, D_{cf,t}]$. The following observer-based fault detection
 241 function is then used to generate residuals:

$$242 \begin{cases} \hat{x}_{k+1} = A_k \hat{x}_k + K_k \tilde{y}_k + \xi_k \\ \tilde{y}_k = z_k - H_k \hat{x}_k \\ r_k = W_k \tilde{y}_k \end{cases} \quad (11)$$

243 where r_k is the residual, K_k is the gain matrix, and W_k is the postfilter. Note that \hat{x}_k is the state
 244 estimation. If $A_{dk} = A_k - K_k H_k$, $B_{dk} = B_c - K_k D_c$, $B_{fk} = B_f - K_k D_f$, and $D_{dk} = D_c$, then the state
 245 estimation error $\tilde{x}_k = x_k - \hat{x}_k$ can be obtained as follows:

$$249 \begin{cases} \tilde{x}_{k+1} = A_{dk} \tilde{x}_k + B_{dk} d_k + B_f f_k \\ \tilde{y}_k = H_k \tilde{x}_k + D_{dk} d_k + D_f f_k \\ r_k = W_k \tilde{y}_k \end{cases} \quad (12)$$

251 The following is defined:

252

$$\|G_{rd}\|_{\infty[0,N]}^2 = \sup_{f_i=0} \frac{\sum_{k=0}^N \|r_k\|_2^2}{\sum_{k=0}^N \|d_k\|_2^2 + \|\tilde{x}_0\|_2^2}$$

$$253 \quad \|G_{rf}\|_{\infty[0,N]}^2 = \sup_{\tilde{x}_0=0, d_k=0} \frac{\sum_{k=0}^N \|r_k\|_2^2}{\sum_{k=0}^N \|f_k\|_2^2} \quad (13)$$

$$\|G_{rf}\|_{-[0,N]}^2 = \inf_{\tilde{x}_0=0, d_k=0} \frac{\sum_{k=0}^N \|r_k\|_2^2}{\sum_{k=0}^N \|f_k\|_2^2}$$

254

255 where $\|G_{rd}\|_{\infty[0,N]}^2$ is the robustness to unknown inputs and $\|G_{rf}\|_{\infty[0,N]}^2$, $\|G_{rf}\|_{-[0,N]}^2$ are the best and worst-
 256 case sensitivities, respectively, of the residual to faults.

257

258 If $\phi_{k,j} = A_{dk}A_{dk-1}\dots A_{dj}$, $\phi_{j,j} = I$ as per Zhong et al. (2010), then Equation (12) can be rewritten as follows:

259

$$260 \quad r_N = \Gamma_0 \tilde{x}_0 + \Gamma_d d_N + \Gamma_f f_N \quad (14)$$

261

262 where

263

$$\Gamma_0 = \begin{bmatrix} W_0 H_0 \\ W_1 H_1 \phi_{1,0} \\ \vdots \\ W_N H_N \phi_{N,0} \end{bmatrix}$$

264

265

$$\Gamma_d = [\Gamma_{d(k,j)}]_{(N+1) \times (N+1)}$$

266 If $k < j$, then $\Gamma_{d(k,k)} = W_k D_{dk}$ and $\Gamma_{d(k,j)} = 0$. If $k > j$, then $\Gamma_{d(k,j)} = W_k H_k \phi_{k-1,j} B_{jd}$. Similarly, Γ_f can be
 267 used to replace D_f and B_f in Equation (12). Then, Equation (13) can be rewritten as follows:

268

$$269 \quad \max \frac{\|G_{rf}\|_{\infty[0,N]}}{\|G_{rd}\|_{\infty[0,N]}}, \max \frac{\|G_{rf}\|_{-[0,N]}}{\|G_{rd}\|_{\infty[0,N]}} \quad (15)$$

270

271 To satisfy Equation (15), both postfilter (W_k) and gain (K_k) can be approximated as follows:

272

$$273 \quad \begin{cases} K_k = (A_k P_k H_k^T + B_c D_c^T) R_{\tilde{y}k}^{-1} \\ R_{\tilde{y}k} = H_k P_k H_k^T + D_c D_c^T \\ P_{k+1} = A_k P_k A_k^T + B_c B_c^T - K_k R_{\tilde{y}k} K_k^{-1} \\ W_k = R_{\tilde{y}k}^{-1} \end{cases} \quad (16)$$

274

275 The following equation is commonly used to evaluate residual observer-based fault detection filters:

276

$$277 \quad J_N = 1/N \sum_{k=N}^k r_k r_k^T \quad (17)$$

278

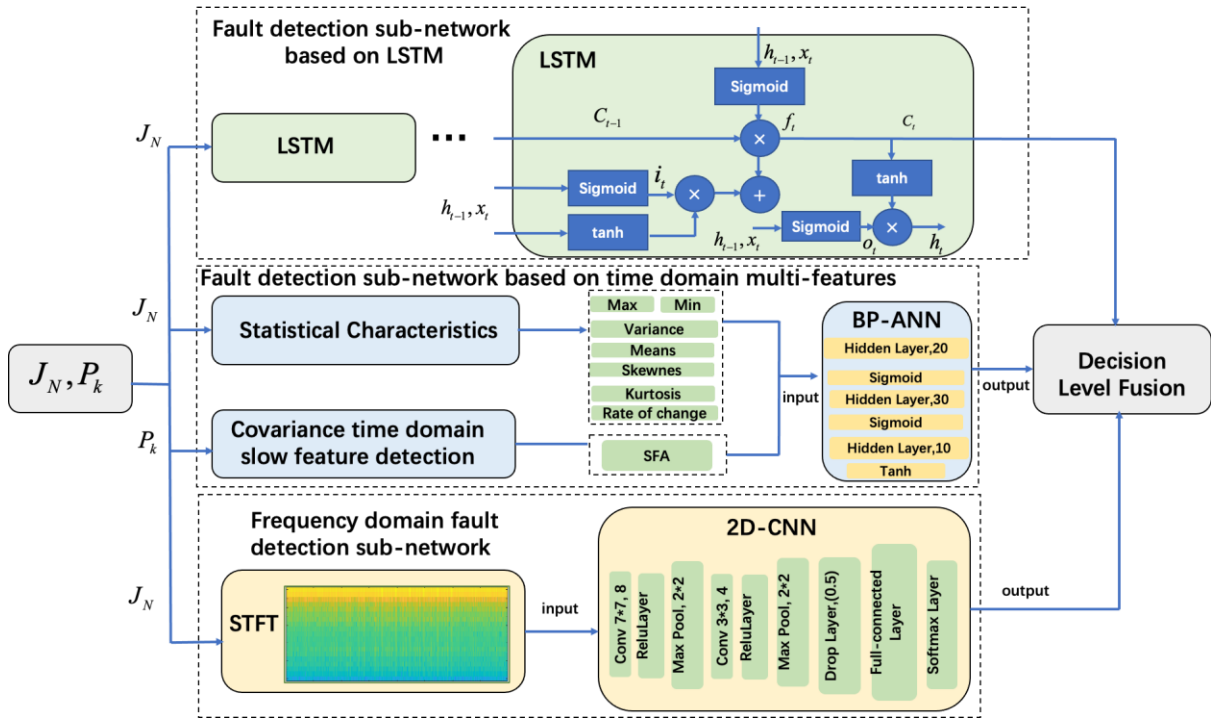
279 where N is the sliding window width. A fault alarm is triggered when J_N exceeds a predefined
 280 threshold. Algorithm 1 summarizes the proposed fault detection algorithm based on CKF and H_i/H_∞
 281 optimization for an integrated navigation system subject to energy-bounded random errors and

282 measurement noises.
 283
 284

Algorithm 1 Proposed fault detection algorithm

- Step 1: Set step $k = 0$, $\hat{x}_0 = 0_{1 \times 15}$;
 - Step 2: Obtain the linearized system matrix A_k and ξ_k with (9);
 - Step 3: Calculate L_k and W_k using (16);
 - Step 4: Calculate \hat{x}_{k+1} and r_{k+1} using (11);
 - Step 5: Calculate J_N with (17);
 - Step 6: Choose a threshold and comparing with (17) to detect faults;
 - Output: Boolean value of whether the fault exists
-

285
 286 **3.2 Optimization of the fault detection algorithm using a cascaded neural network**
 287 The proposed algorithm offers the advantages of sensitivity to faults and robustness against unknown
 288 inputs. Consequently, it can obtain reliable fault detection results. However, it is based on a residual
 289 observer, which is insensitive to soft faults. This affects the one-step state prediction of the Kalman
 290 filter and makes the state estimation track the fault values. Thus, the residual error changes are not
 291 evident. Moreover, the limitations of the sliding window width and the difficulty in determining the J_N
 292 threshold in Equation (17) need to be considered. Thus, Fig. 3 shows a framework of using a cascaded
 293 neural network to optimize the proposed fault detection algorithm.
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295
 296
 297
 298 Figure 3. Framework for optimizing the proposed fault detection algorithm using a cascaded neural
 299 network.
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 301
 302
 303
 304
 305

Table 1. Features extracted from the residual J_N signal

Features	Description
Mean	$\mu = 1/N \sum_n^{n+N} x_n$
Max	$\text{Max}(x_n)$
Min	$\text{Min}(x_n)$
Variance	$\sigma^2 = 1/N \sum_n^{n+N} (x_n - \mu)^2$
Skewness	$1/N \sigma^3 \sum_n^{n+N} (x_n - \mu)^3$
Rate of change	$\max(\sum_n^{n+N} (x_{n+1} - x_n))$
kurtosis	$1/N \sigma^4 \sum_n^{n+N} (x_n - \mu)^4$

307

308

309 To address this limitation, time-domain features can be extracted for the state estimation error
 310 covariance P_k , which is more severely affected by faults in the state estimation of the integrated
 311 navigation system. This is particularly helpful for the attitude, which has weak observability. Slow
 312 feature analysis (SFA) was used to extract the time-domain features of the covariance P_k owing to its
 313 ability to extract slowly changing fault signal features in a high-dimensional input. The following set
 314 of equations $f_j()$ is used to identify slowly varying components $y_j(k)$ from P_k :

315

$$316 \quad y_j(k) = f_j(P_k) \quad (18)$$

317

318 where $f_j(P_k) = [f_1(P_k), f_2(P_k), \dots, f_m(P_k)]$ is a set of proper functions. The primary objective function of
 319 SFA is given by

320

$$321 \quad \min \Delta y_j(k) = \min \langle \dot{y}_j^2(k) \rangle \quad (19)$$

322

323 under the constraints of

324

$$\begin{aligned}
 325 \quad & \langle y_j(k) \rangle = 0 \quad (\text{zero mean}) \\
 & \langle y_j^2(k) \rangle = 1 \quad (\text{unit variance}) \\
 & \forall i < j, \langle y_j(k) y_i(k) \rangle = 0 \quad (\text{decorrelation})
 \end{aligned} \quad (20)$$

326

327 where $\dot{y}_j(k)$ is the first derivative of $y_j(k)$ at transient step k and $\langle \cdot \rangle$ is the mean value of the signal.

328 However, obtaining the nonlinear mapping $f_j()$ directly is difficult. To address this limitation, a kernel

329 transformation $S_{ij} = s(P_k(i), P_k(j))$ is used on the inputs P_k to determine the kernel characteristics,

330 where $i, j = 1, \dots, n$ denote the n th-dimensional input signals (Du et al. 2019). The kernel transformation

331 vector can be determined as follows:

332

$$333 \quad m\lambda\alpha = \tilde{S}\alpha \quad (21)$$

334

335 The slowest- and second-slowest features correspond to the smallest and second-smallest values,

336 respectively, of λ . m is the number of samples, and the new principal component features of the kernel

337 can be acquired by

338

$$v'_{kj} = \sum_{i=1}^d \alpha_i^j \tilde{S}(P_k, P_k(i)) \quad (22)$$

where v'_k is zero-mean uncorrelated vector. To satisfy the constraints of Equation (20), an additional transformation should be applied to v'_k that does not change the mean, variance:

$$y_k = G^T v'_k \quad (23)$$

The objective of Equation (19) can be realized by transforming it into a problem of solving eigenvalues to obtain G (Zhang et al. 2021). Then, the optimal solution is given by $G^T = [g_{,0}, g_{,1}, \dots, g_{,d-1}]$, where the order of eigenvalues is from small to big.

In addition to the time-domain fault features extracted from LSTM and prior knowledge, the frequency-domain characteristics of the original signal are important for determining the signal quality. Thus, a sub-CNN network was adopted for frequency-domain fault detection based on short-time Fourier transform (STFT). The mathematical expression of the STFT is as follows:

$$ST(\omega, \tau) = \int f(t) g^*(t - \tau) e^{-j\omega t} dt \quad (24)$$

where $*$ is a conjugate symbol and $g(t)$ is the Hamming window function (Tao et al. 2019). To avoid the influence of extreme values on the learning efficiency of the neural network, the wavelet transform is used to extract the envelope of the signal change trend before the frequency-domain feature extraction. Because the noise of an integrated navigation system is relatively stable under normal operating conditions, the quality of the integrated navigation system can be measured using the fundamental and harmonic waves of J_N . The content of the fundamental wave can accurately reflect the noise distribution of SINS and external measurement sensors, i.e., ideal periodic noise. The harmonic wave reflects the fault and additional noise based on an ideal periodic signal because the solution period of an integrated navigation system is a fixed and nonstationary signal. Then, multidimensional frequency-domain characteristics extracted by STFT are substituted into the CNN for training (Guo et al. 2018). Finally, the training results of the LSTM-based fault detection subnetwork, multi-feature time-domain fault detection subnetwork, and frequency-domain fault detection subnetwork are used as inputs to the backpropagation (BP)-based decision-level fusion network to achieve online fault detection.

As shown in Fig. 3, the normalized J_N is substituted into the LSTM-based fault detection subnetwork comprising ten LSTM neuron units as training data. The BP-ANN in the multi feature time-domain fault detection subnetwork has a structure of 30 hidden layers and 10 output layers. The 2D-CNN in the frequency-domain fault detection subnetwork has the following structure: one input layer connected to eight 7×7 convolutional layers, a 2×2 max pooling layer, four 3×3 convolutional layers, another 2×2 maximum pooling layer, and final output to two full connection layers. The detection results of the three networks are integrated in the decision-level fusion network.

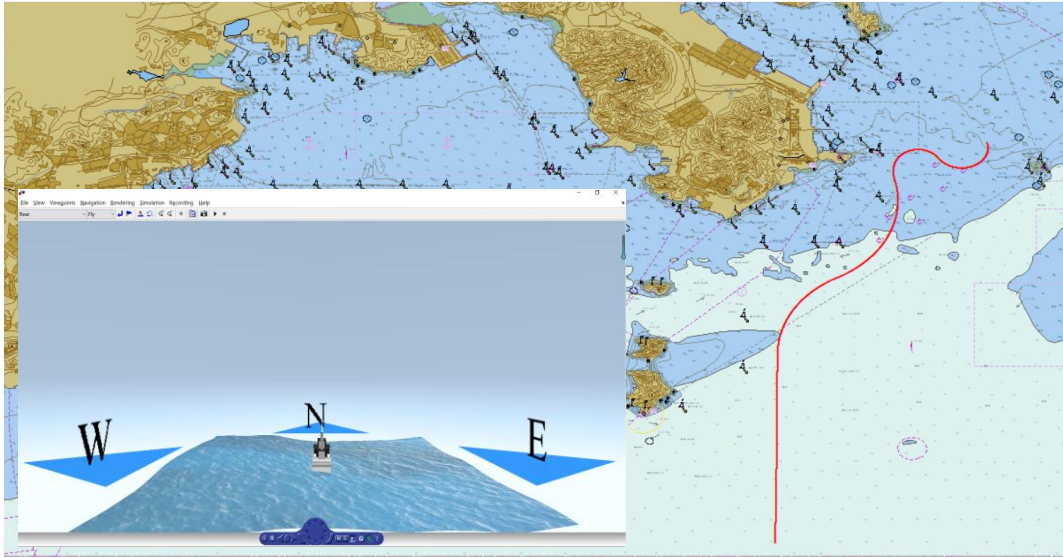
4. Verification

4.1 Simulation

Simulations were performed using the XSENS Mti-710 inertial measurement unit (IMU). The gyroscope and accelerometer have in-run bias stabilities of $10^\circ/\text{h}$ and $15 \mu\text{g}$, respectively, and noise densities of $0.01^\circ/\text{s}/\sqrt{\text{Hz}}$ and $60 \mu\text{g}/\sqrt{\text{Hz}}$, respectively. As the external measurement unit, the GNSS of the integrated navigation system had a positioning error of 5 m and velocity error of 0.4 m/s. The simulation error data of Mti-g-710 were taken from the instruction manual, and the simulation errors of GNSS and DVL were within reasonable ranges (Jin et al. 2022).

The propeller of the ASS was assumed to rotate at a constant speed in the face of a 3 m/s west wind and 1 m/s crosscurrent with an initial speed of 15.5 knots and heading of 000° . The heading was fine-tuned

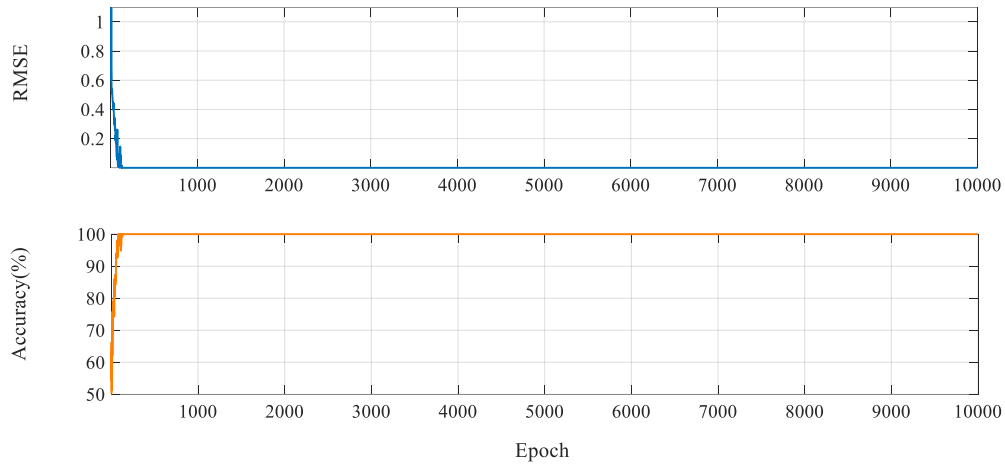
390 during 0–800 s. Then, the rudder angle was adjusted to 10° and -10° at 1000–1400 s and 1400–2000 s,
 391 respectively, and to 25° and -25° at 2000–2500 s and 2500–3000 s, respectively. Figure 4 shows the
 392 simulated motion trajectory.
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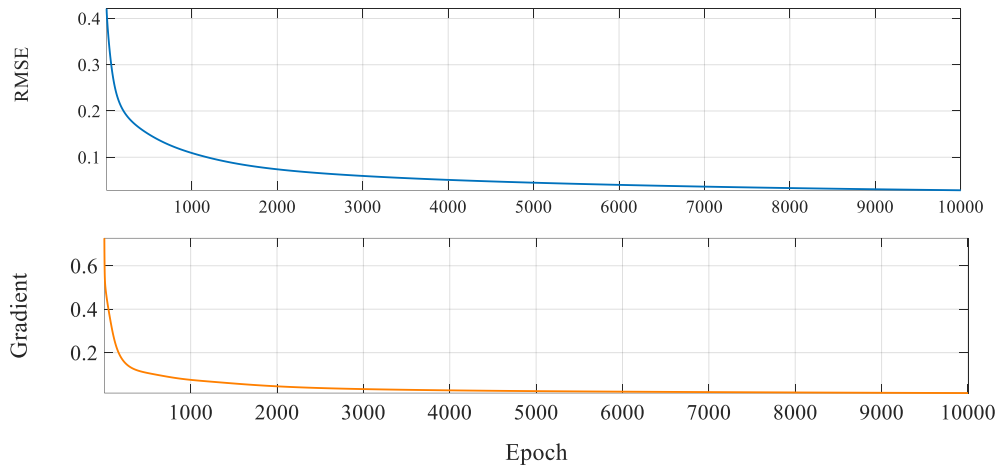
394
 395 Figure 4. Diagram of the ship motion trajectory and simulation system interface.
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397 Various simulation cases were input with different J_N and P_k under normal and fault conditions. The
 398 different subnetworks were pretrained 10,000 times, and the decision-level fusion network was trained
 399 22,000 times. The batch size was set to 50 based on the loss values and accuracy from the cross-
 400 experiments. The learning rate was set to be 0.0001. The training goal (i.e., minimum mean square error,
 401 MMSE) was set as 10^{-4} for all subnetworks except the multi-feature time-domain fault detection
 402 subnetwork. This subnetwork was set to a learning rate of 0.001 and an MMSE of 10^{-6} . For the
 403 frequency-domain fault detection subnetwork, the packet loss rate was set as 0.5 to avoid overfitting.
 404 For the decision-level fusion network, the training rate and MMSE were set the same as for the multi-
 405 feature time-domain fault detection subnetwork.
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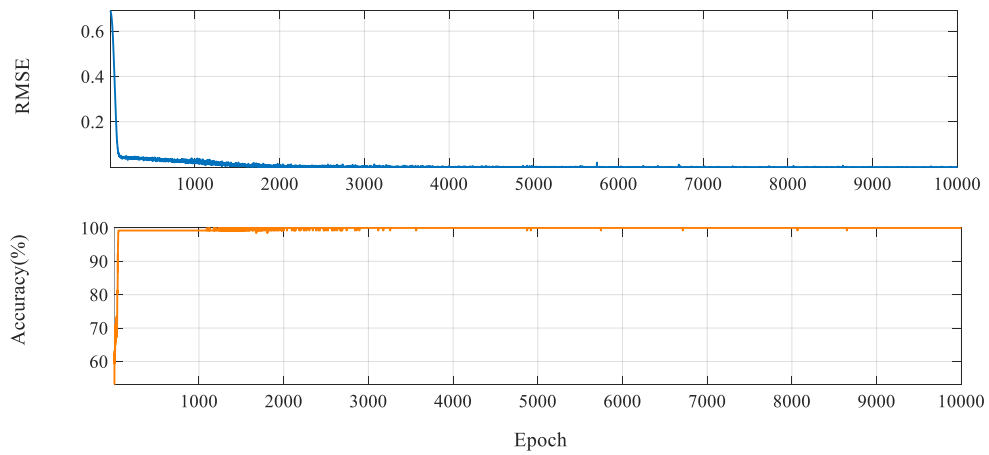
407 The training results are shown in Fig. 5. For the LSTM-based fault detection subnetwork [Fig. 5(a)] and
 408 CNN-based frequency-domain fault detection subnetwork [Fig. 5(c)], the error of the loss function
 409 gradually converged and the classification accuracy gradually improved, which indicates that the
 410 network performances tended to be stable. For the multi-feature time-domain fault detection
 411 subnetwork [Fig. 5(b)] and BP-based decision-level fusion network [Fig. 5(d)], the gradient and RMSE
 412 gradually decreased, which indicates that the trained networks tended to be stable. After all iterations,
 413 the three subnetworks had accuracy rates of 90.21%, 95.24%, and 99.32%, and the decision-level fusion
 414 network had an accuracy rate of 99.6%.
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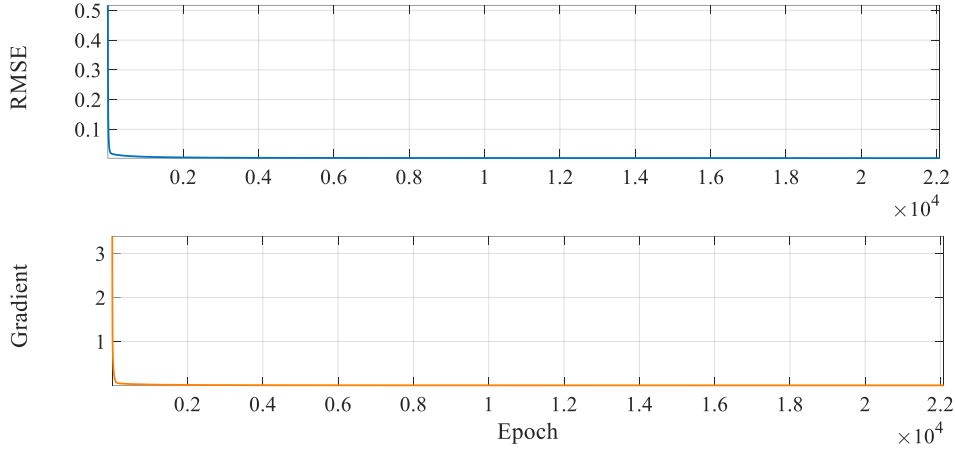
(a) LSTM-based fault detection subnetwork.



(b) Multi-feature time-domain fault detection subnetwork.



(c) Frequency-domain fault detection subnetwork.



(d) Decision-level fusion network.

Figure 5. Training results of the subnetworks.

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Various faults that may occur in ship sensors were simulated to confirm the effectiveness of the proposed algorithm. The SINS/GNSS integrated navigation system was considered as a research object to verify the effectiveness of the SINS/DVL/COMPASS integrated navigation system. External measurement faults primarily manifest as signal interruptions caused by interference or shielding as well as hard faults with increased errors in a short time period. Temperature, violent motion, and electromagnetic interference all affect the IMU primarily through soft faults that cause errors to accumulate over time. Table 2 summarizes the simulated hard and soft faults added to the SINS/GNSS integrated navigation system, where δP_{GNSS} and δV_{GNSS} are the position and velocity errors of GNSS, and ε_k is the gyroscope error of the IMU. It is worth noting that the faults simulated in this paper mainly include the hard faults, which can be denoted as a step-function type commonly seen in GNSS and DVL output data. The additive noise failure caused by noise drift and the additive Gaussian measurement noise can be regarded as soft failure, which comes with increasing error variance and often occur in SINS and GNSS.

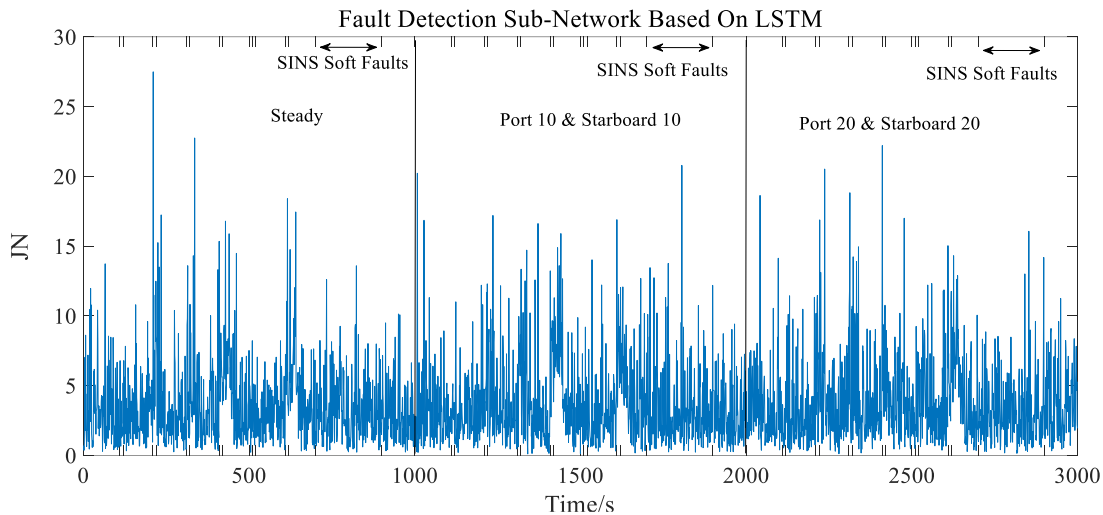
Table 2. Simulated hard and soft faults added to the SINS/GNSS integrated navigation system

	Time	Faults	Types
1	110 s < t < 120 s	$\delta P_{GNSS} = 1.5 * \delta P_{GNSS}$	Soft faults
	1110 s < t < 1120 s	$\delta P_{GNSS} = 2 * \delta P_{GNSS}$	
	2110 s < t < 2120 s		
2	210 s < t < 220 s	$\delta V_{GNSS} = 1.5 * \delta V_{GNSS}$	
	1210 s < t < 1220 s	$\delta V_{GNSS} = 2 * \delta V_{GNSS}$	
	2210 s < t < 2220 s		
3	310 s < t < 320 s	$\delta V_{GNSS} = 1.5 * \delta V_{GNSS}$	Hard faults
	1310 s < t < 1320 s	$\delta P_{GNSS} = 1.5 * \delta P_{GNSS}$	
	2310 s < t < 2320 s	$\delta V_{GNSS} = 2 * \delta V_{GNSS}$	
		$\delta P_{GNSS} = 2 * \delta P_{GNSS}$	
4	410 s < t < 420 s	$\delta V_{GNSS} = \delta V_{GNSS} + 1$ (m/s)	
	1410 s < t < 1420 s	$\delta V_{GNSS} = \delta V_{GNSS} + 2$ (m/s)	
	2410 s < t < 2420 s		
5	510 s < t < 520 s	$\delta P_{GNSS} = \delta P_{GNSS} + 3$ (m)	
	1510 s < t < 1520 s	$\delta P_{GNSS} = \delta P_{GNSS} + 6$ (m)	
	2510 s < t < 2520 s		
6	610 s < t < 620 s	$\delta P_{GNSS} = \delta P_{GNSS} + 3$ (m)	
	1610 s < t < 1620 s	$\delta V_{GNSS} = \delta V_{GNSS} + 1$ (m/s)	
	2610 s < t < 2620 s	$\delta P_{GNSS} = \delta P_{GNSS} + 6$ (m)	
		$\delta V_{GNSS} = \delta V_{GNSS} + 2$ (m/s)	

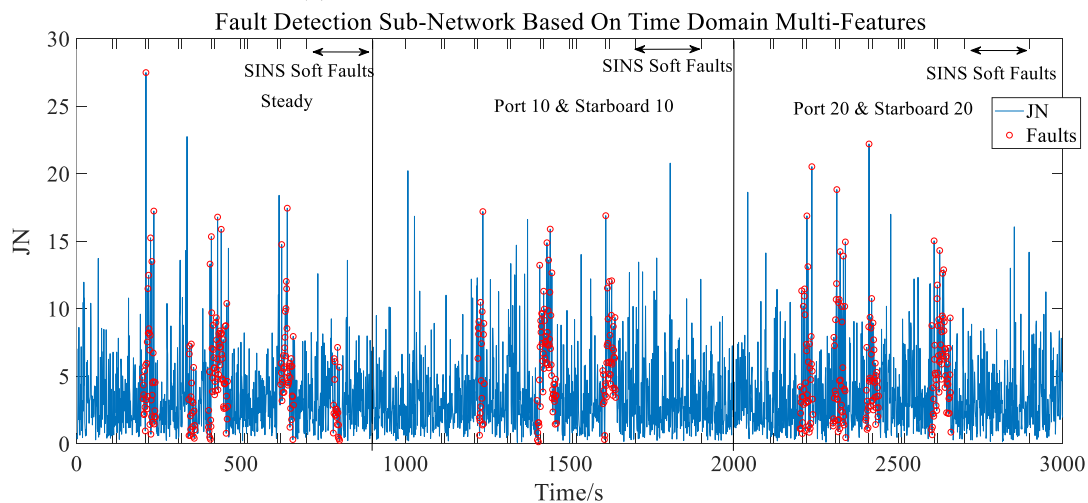
7	700 s < t < 800 s	$\varepsilon_k = \varepsilon_k + (0.036^\circ/\text{h}) * t$	Soft faults
	1700 s < t < 1800 s		
	2700 s < t < 2800 s		
8	800 s < t < 900s	$\varepsilon_k = \varepsilon_k + (0.18^\circ/\text{h}) * t$	
	1800 s < t < 1900 s		
	2800 s < t < 2900 s		

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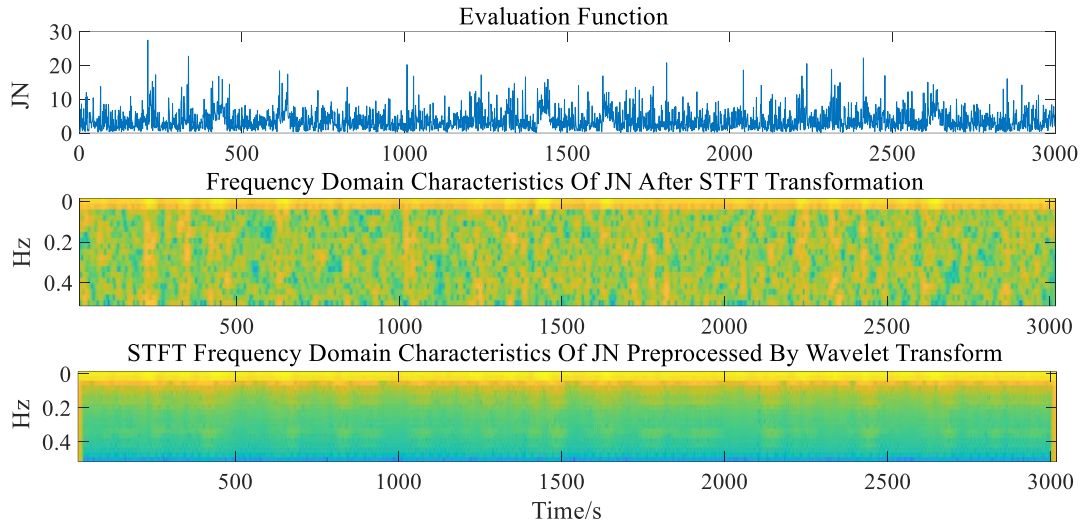
Figure 6 shows the detection results for the minor faults in Table 2 when the detection threshold and sliding window width were disregarded. Figure 6(a) shows that neither GNSS nor SINS faults were detected by the LSTM-based fault detection subnetwork. This demonstrates that a neural network alone is not ideal for detecting the original residual time-domain signal of an integrated navigation system and that the fault detection capability seems weak. Figure 6(b) shows that the multi-feature time-domain fault detection subnetwork did not detect some faults when the fluctuation of J_N was unclear. Some SINS soft faults were detected in the steady state but were not obvious during the turning phase. This is because ship maneuvers are conducive to the convergence of state errors in the integrated navigation system, which results in small residuals and unclear covariance characteristics. Figure 6(c) shows the features extracted by STFT, and Fig. 6(d) shows the results of the frequency-domain fault detection subnetwork. Although additional faults, including SINS soft faults, were detected, there were several false alarms, particularly during large changes in course. As shown in Fig. 6(e), the decision-level fusion network combined the advantages of the time-domain statistical characteristics with frequency-domain networks, which not only increased the fault detection accuracy but also decreased the false alarm rate.



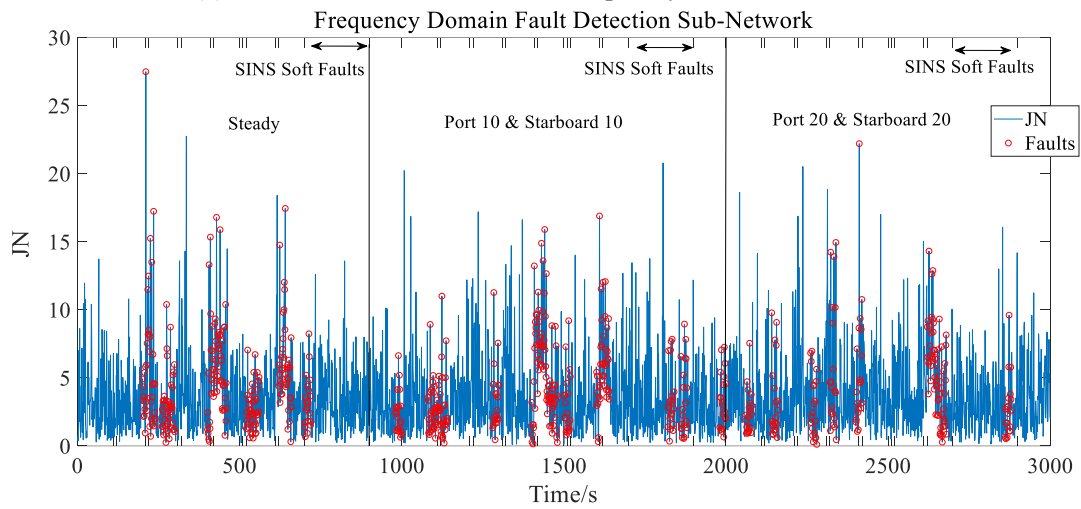
(a) LSTM-based fault detection subnetwork.



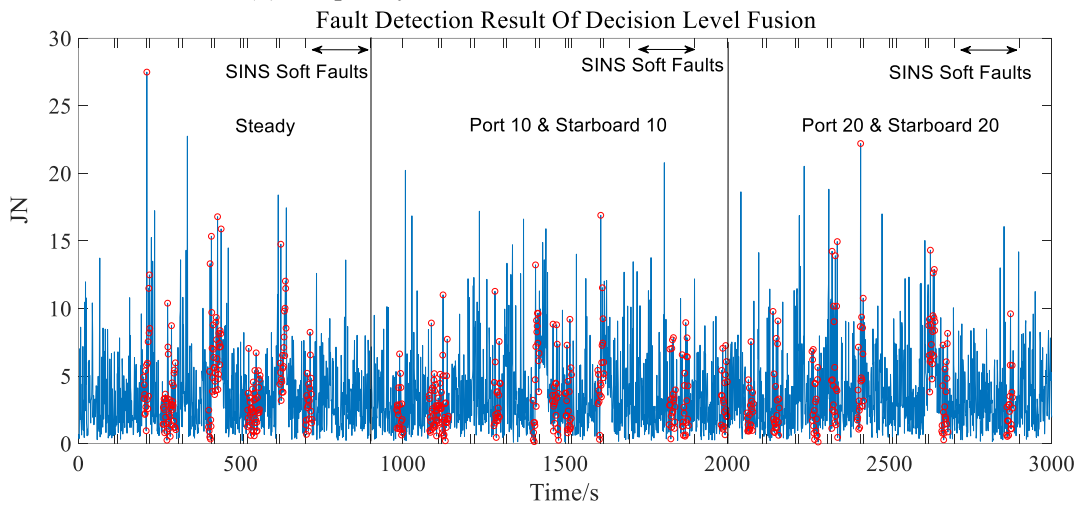
(b) Multi feature time-domain fault detection subnetwork.



(c) STFT extraction of residual frequency-domain features.



(d) Frequency-domain fault detection subnetwork.



(e) Decision-level fusion network.

Figure 6. Minor fault detection results of the proposed algorithm with neural network optimization.

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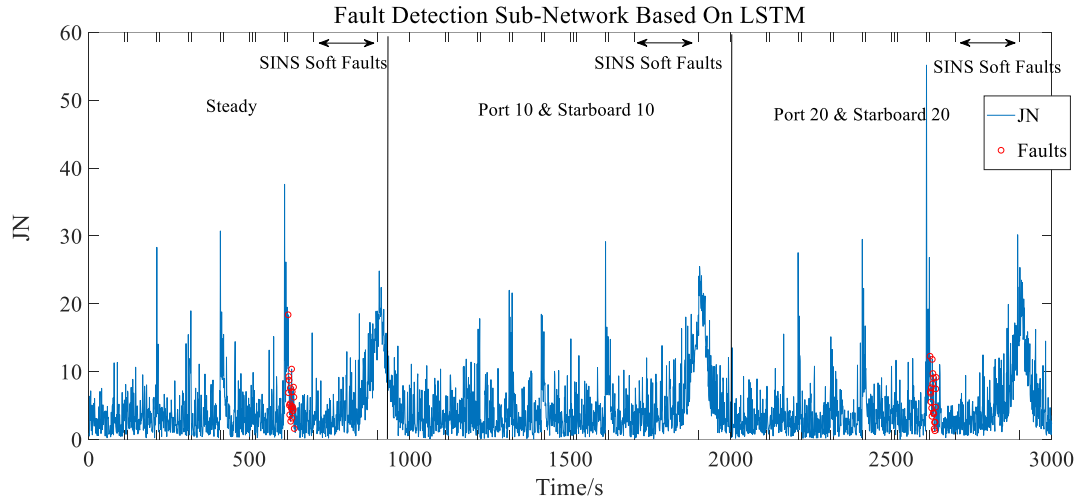
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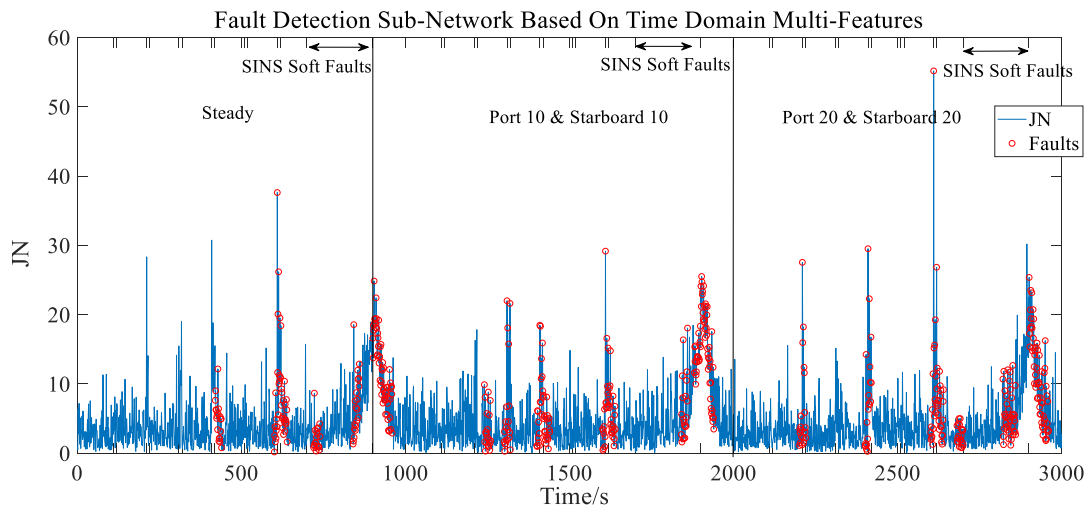
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Figure 7 shows the detection results for the major fault cases in Table 2. In contrast to the results for the minor fault cases, Fig. 7(a) shows that the LSTM-based fault detection subnetwork detected a small number of GNSS velocity and position faults. However, the effectiveness was not ideal. Figure 7(b) shows that the multi-feature time-domain fault detection subnetwork detected additional faults during

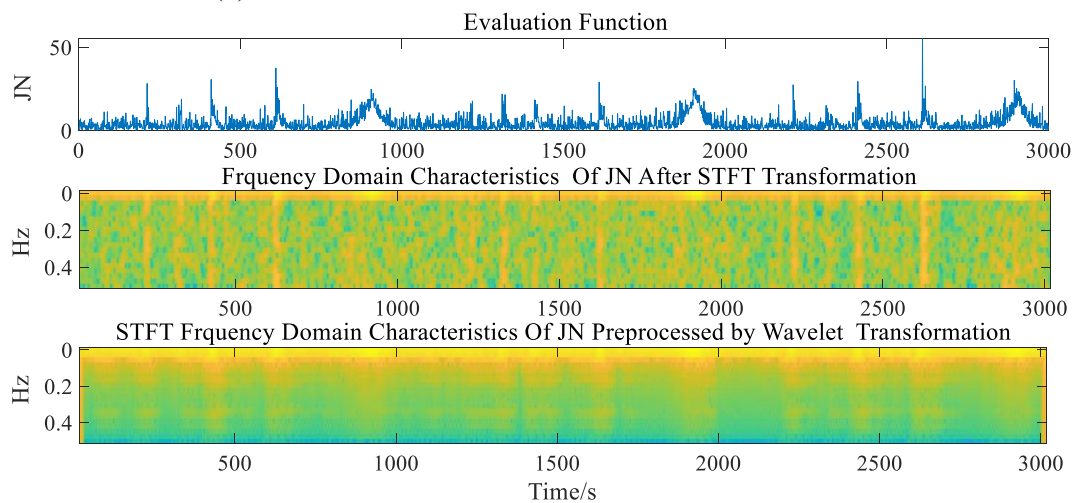
452 the course alteration stage, but there were some false alarms. Figure 7(c) shows the STFT extraction of
 453 the frequency-domain signal J_N , and Fig. 7(d) shows the results of the frequency-domain
 454 fault detection subnetwork. There were fewer false alarms and a higher degree of accuracy when compared
 455 to the multi feature time-domain fault detection subnetwork. Figure 7(e) shows the results of the
 456 decision-level fusion network, which were similar to the results shown in Fig. 7(d).
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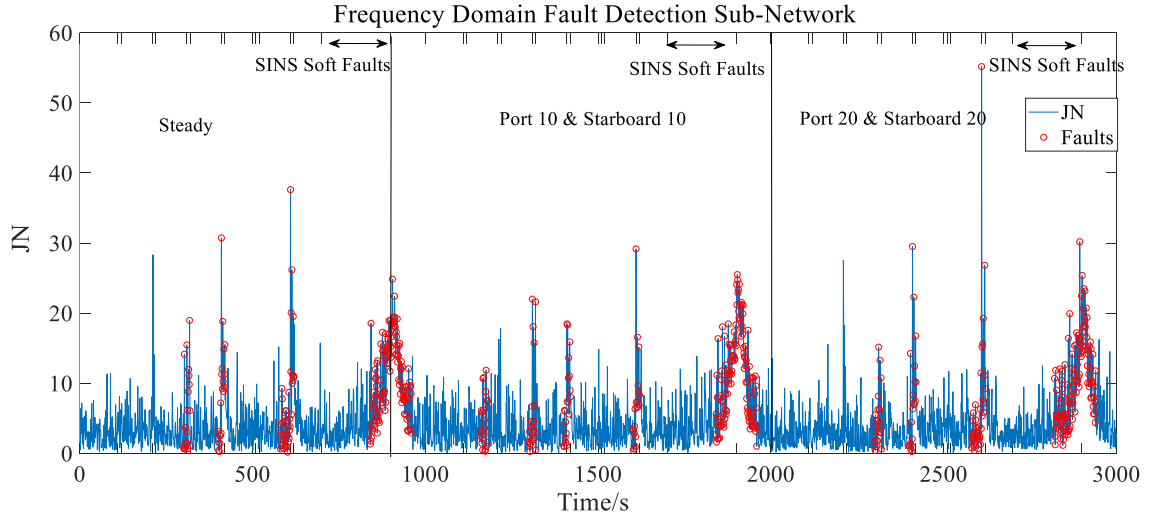
(a) LSTM-based fault detection subnetwork.



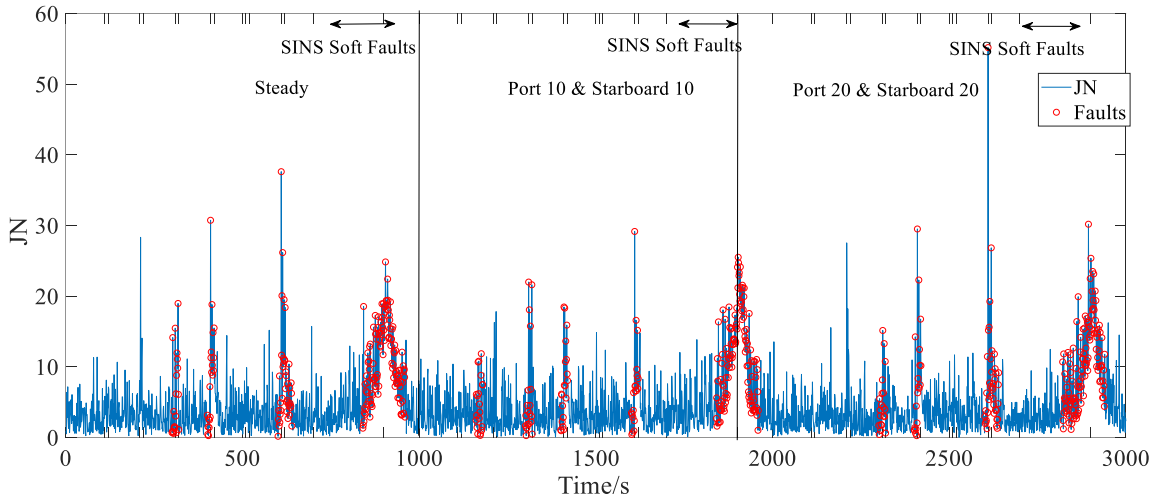
(b) Multi feature time-domain fault detection subnetwork.



(c) STFT extraction of residual frequency-domain features.



(d) Frequency-domain fault detection subnetwork.
Fault Detection Result Of Decision Level Fusion



(e) Decision-level fusion network.

Figure 7. Major fault detection results of the proposed algorithm with neural network optimization.

458 In summary, the proposed fault detection algorithm optimized using a cascaded neural network was
 459 capable of detecting faults regardless of the various motion states of the ASS without relying on an
 460 empirical threshold and designed sliding window width. The LSTM-based fault detection subnetwork
 461 had a low degree of generality, a low detection rate, and a negligible effect on the results of the decision-
 462 level fusion network. The multi-feature time-domain fault detection subnetwork missed minor faults
 463 but had a minimal false alarm rate. The frequency-domain fault detection subnetwork detected most
 464 faults, including SINS soft faults, but had a high false alarm rate. The decision-level fusion network
 465 combined the advantages of the three subnetworks by detecting most faults, including soft faults, but
 466 also realizing a low false alarm rate. Thus, the optimization of the proposed algorithm was verified. It
 467 is worth noting that it is difficult to determine which period the failure occurred based on the values of
 468 the vertical axis of the blue residual lines based on the traditional residual observer-based fault detection
 469 algorithms. According to previous studies (Fei et al. 2021, Liang et al, 2021, Oh et al. 2022, Zammali
 470 et al. 2021) limited by the algorithm principle, the fault detection algorithms based on residual observer
 471 are generally insensitive to gradual soft faults. Simply optimizing the residual parameters cannot make
 472 it sensitive to soft faults. In this paper, due to a lack of a large number of real ship data, the cascaded
 473 network is used to optimize it, and the result of soft fault detection is better than the traditional algorithm.
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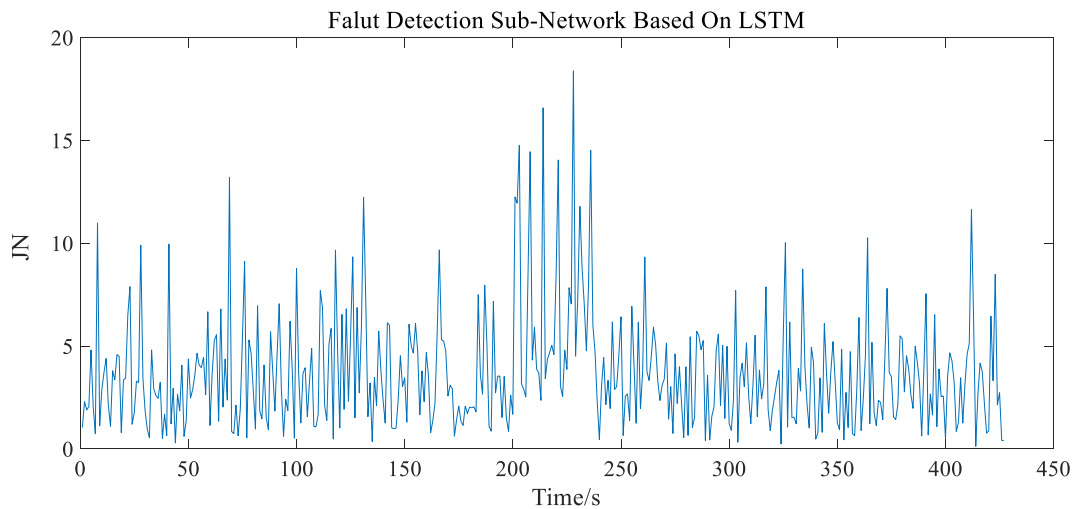
477 **4.2 Experimental result and discussion**

478 To confirm the effectiveness of the proposed algorithm in practical applications, real historical ship data
 479 were applied to the simulation-trained model. Faults were introduced into the historical sensor data to
 480 determine whether they could be detected using the proposed algorithm with various integrated
 481 navigation systems. The historical data were obtained from an experiment conducted using ASS 320 at
 482 Lingshui Port in Dalian, China. The experimental historical track included straight lines and turns. The
 483 experimental site is 170 m in length and 70 m in width. The bay is not considerably affected by tidal
 484 waves, and the current is slow. An Mti-G-710, a Lidar, cameras, a 4G communication antenna, and a
 485 5.8G WIFI antenna were installed on top of ASS 320. Moreover, ASS 320 was equipped with
 486 COMPASS and DVL, as shown in Fig. 8. To accurately simulate interference during actual navigation,
 487 the speed measured using DVL was subjected to sinusoidal function interference with an amplitude of
 488 0.5 m/s. The experimental data covered a period of 437 s. Then, $0.18^\circ/\text{h}$ SINS soft faults were added at
 489 101–150 s, and 1 m/s of GNSS velocity faults and 1 m/s of DVL faults were added at 201–230 s and
 490 301–330 s, respectively. The proposed algorithm was then applied to fault detection for verification.
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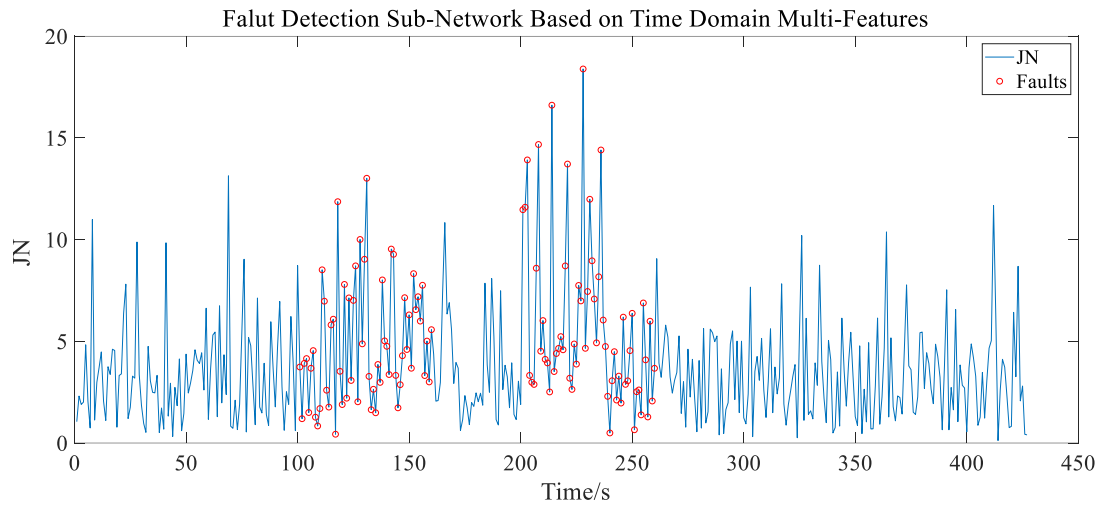


492
 493 Figure 8. Field test of ASS 320 and the experimental route.
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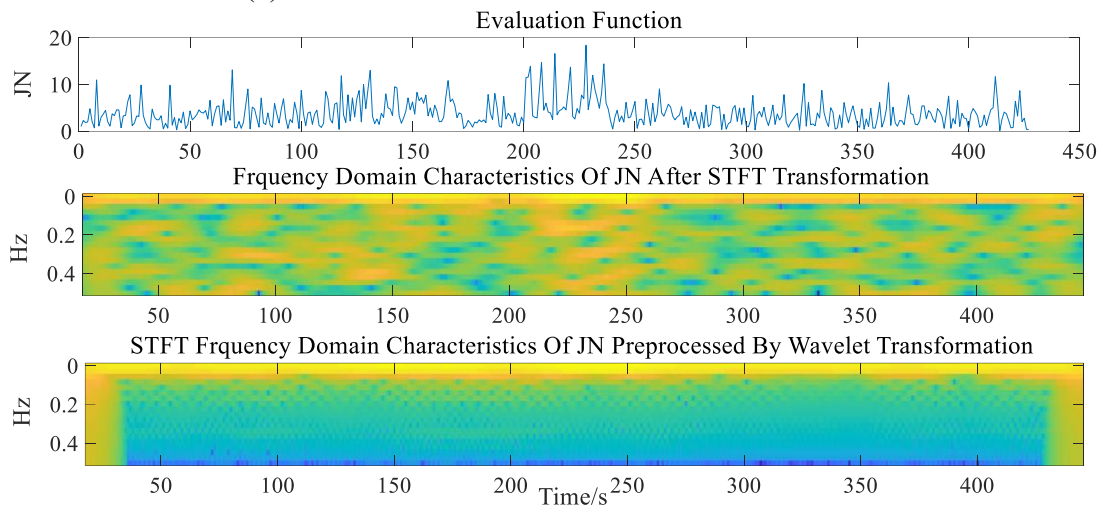
495 Figure 9 shows that the results of the SINS/GNSS integrated navigation system were comparable to the
 496 simulation results. Figure 9(a) shows that the LSTM-based fault detection subnetwork continued to
 497 perform poorly in terms of fault detection. Figure 9(b) shows that the multi-feature time-domain fault
 498 detection subnetwork detected most SINS soft faults between 100 and 150 s but had some false alarms
 499 after 250 s. Figure 9(c) shows the STFT extraction results, and Fig. 9(d) shows that the frequency-
 500 domain fault detection subnetwork had a large number of false alarms between 150 and 200 s. Figure
 501 9(e) shows that the decision-level fusion network not only achieved a lower false alarm rate than the
 502 frequency-domain fault detection subnetwork did but also successfully detected most faults, including
 503 SINS soft faults.
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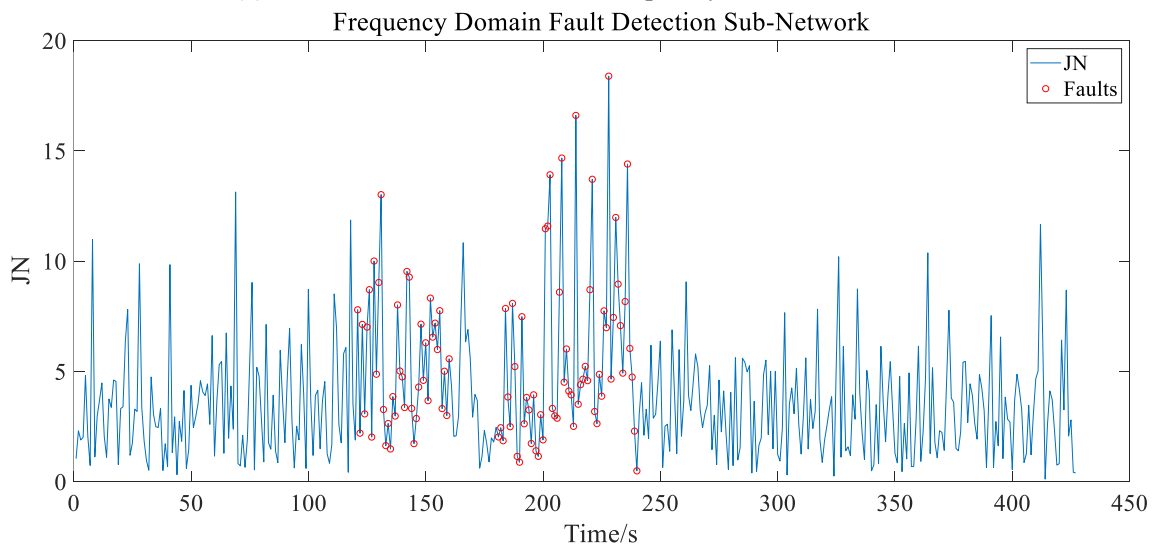
(a) LSTM-based fault detection subnetwork.



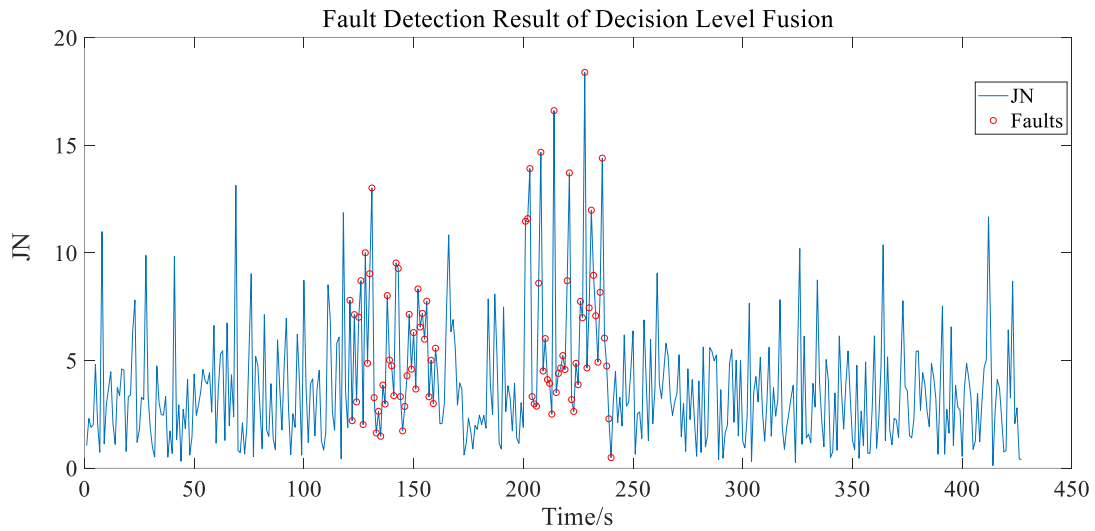
(b) Multi feature time-domain fault detection subnetwork.



(c) STFT extraction of residual frequency-domain features.



(d) Frequency-domain fault detection subnetwork.

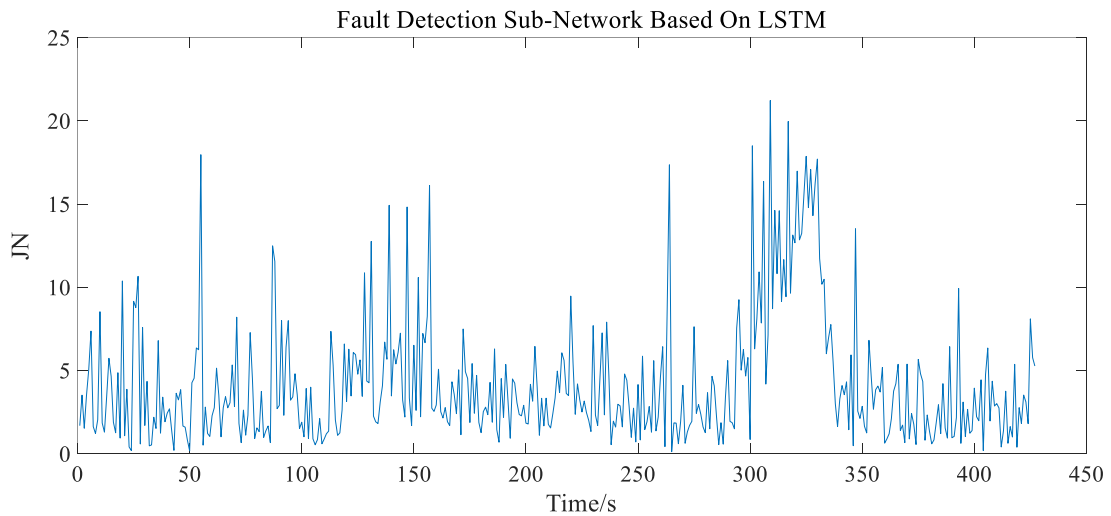


(e) Decision-level fusion network.

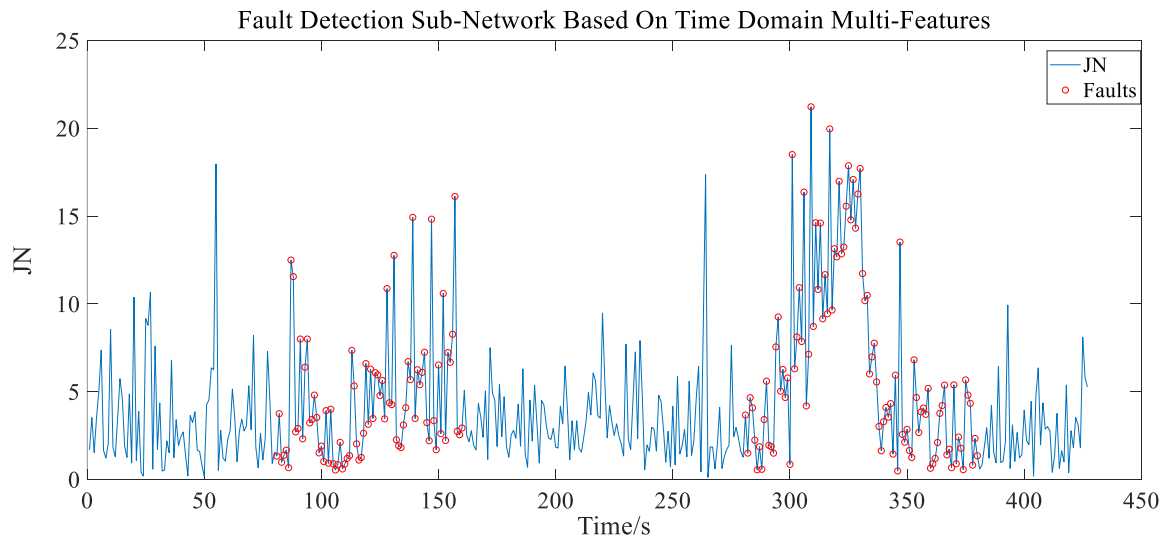
Figure 9. Fault detection results of the proposed algorithm with neural network optimization for the SINS/GNSS integrated navigation system of ASS 320.

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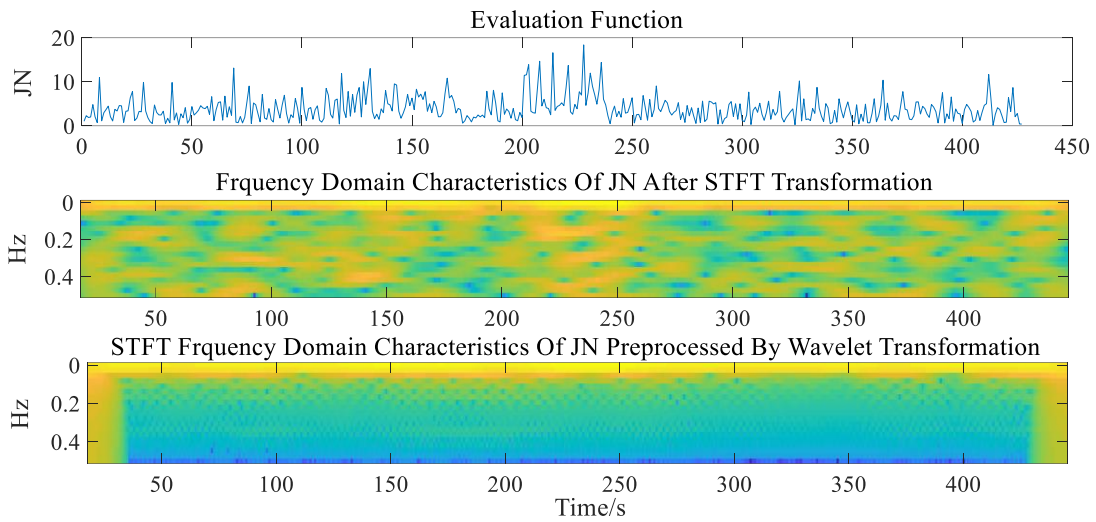
The proposed algorithm was adopted for the SINS/DVL/COMPASS integrated navigation system. Figure 10 shows that similar results were obtained as in the previous cases. Figure 10(b) shows that the multi-feature time-domain fault detection subnetwork had a higher number of false alarms. Figures 10(d) and (e) show that the decision-level fusion network and frequency-domain fault detection subnetwork had similar results and successfully detected SINS soft faults without obvious fluctuations in J_N and with a low number of false alarms.



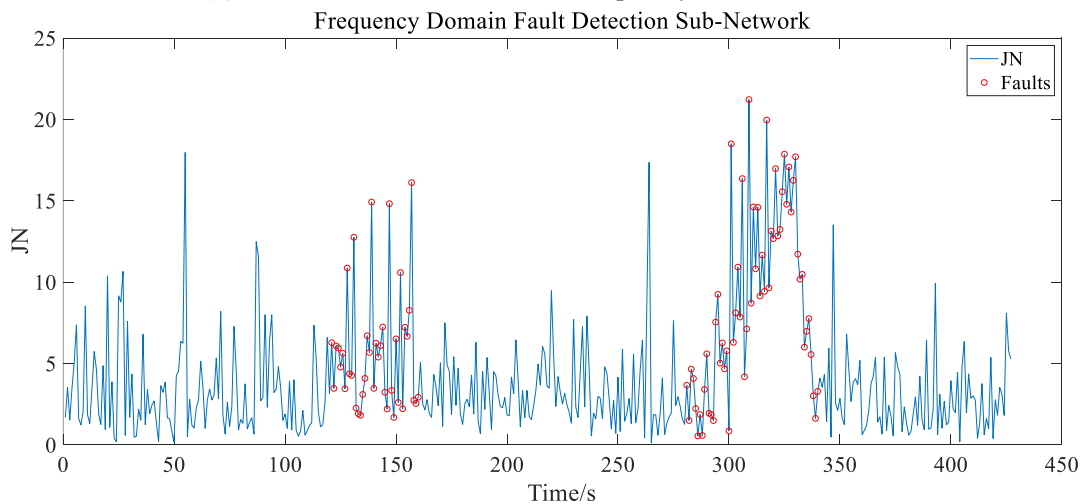
(a) LSTM-based fault detection subnetwork.



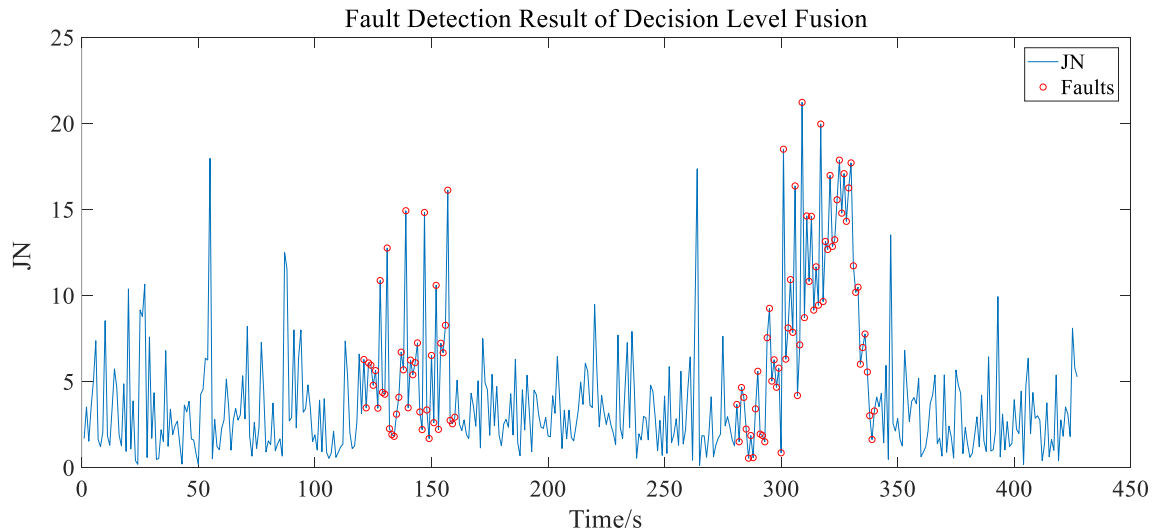
(b) Multi feature time-domain fault detection subnetwork.



(c) STFT extraction of residual frequency-domain features.



(d) Frequency-domain fault detection subnetwork.



(e) Decision-level fusion network.

Figure 10. Fault detection results of the proposed algorithm with neural network optimization for the SINS/DVL/COMPASS integrated navigation system of ASS 320.

513

514 The experimental results verified that the proposed algorithm could detect faults in the integrated
 515 navigation system of an ASS. The proposed algorithm was capable of detecting slowly changing SINS
 516 soft faults while retaining a low false alarm rate for SINS/GNSS and SINS/DVL/COMPASS integrated
 517 navigation systems. The LSTM-based fault detection subnetwork was not sensitive to such faults, but
 518 the frequency-domain fault detection subnetwork and multi feature time-domain fault detection
 519 subnetwork had significant impacts on the overall results. The multi feature time-domain fault detection
 520 subnetwork had a high false alarm rate for both integrated navigation systems.

521

522 5. Conclusion

523 Previous studies on the fault detection of integrated navigation systems for ASSs have shown that
 524 setting the fault detection threshold is highly dependent on experience, the sliding window width is
 525 difficult to design, SINS gradient soft faults are difficult to detect, and previous algorithms cannot be
 526 learned online. In this study, a fault detection algorithm was proposed to address the above issues. The
 527 proposed algorithm was optimized using a cascaded neural network comprising an LSTM-based fault
 528 detection subnetwork, multi feature time-domain fault detection subnetwork, and frequency-domain
 529 fault detection subnetwork. The algorithm uses residual and covariance information during the filtering
 530 process to eliminate the dependence of traditional model-based fault detection algorithms on empirical
 531 parameter settings. This solved the problem of the residual observer being insensitive to soft faults. The
 532 proposed algorithm was verified through simulations and historical data from a real ship. The LSTM-
 533 based fault detection subnetwork had limited effectiveness at detection of time-domain soft faults.
 534 Although the multi feature time-domain fault detection subnetwork was capable of detecting SINS soft
 535 faults, it had a high rate of missed detection. In contrast, the frequency-domain fault detection
 536 subnetwork had a high detection rate but also had a large number of false alarms. The results of the
 537 three subnetworks were output to the decision-level fusion network to obtain the optimal results. The
 538 results showed that the proposed algorithm is capable of not only detecting soft faults without relying
 539 on an empirical threshold but also reducing the rate of missed detections and false alarm for two
 540 integrated navigation systems (i.e., SINS/GNSS and SINS/DVL/COMPASS). The detection
 541 performance with the decision-level fusion network was better than that of a single subnetwork. This
 542 confirmed the effectiveness of the proposed algorithm and neural network optimization.

543

544 **Appendix I CKF algorithm for an integrated navigation system**

545 For the time update, the mean and variance of the posterior probability of the system under the
 546 measurement conditions are approximated by transforming $2n$ cubature points with equal weights $\frac{1}{2n}$
 547 using the nonlinear system equation $f_c(\cdot)$. The propagated i -th sampling points cubature points at step
 548 k can be computed as

$$549 \begin{aligned} \chi_{k-1}^{(i)} &= \sqrt{nP_{k-1}} + \hat{x}_{k-1} \quad (i = 1, 2, \dots, 2n) \\ \bar{\chi}_{k-1}^{(i)} &= f_c(\chi_{k-1}^{(i)}) \end{aligned} \quad (25)$$

550 The one-step state prediction $\hat{x}_{k|k-1}$ and prediction variance $P_{xx,k|k-1}$ are computed as

$$551 \begin{aligned} \hat{x}_{k|k-1} &= \frac{1}{2n} \sum_{i=1}^{2n} \bar{\chi}_{k-1}^{(i)} \\ P_{xx,k|k-1} &= \frac{1}{2n} \sum_{i=1}^{2n} \bar{\chi}_{k|k-1}^{(i)} (\bar{\chi}_{k|k-1}^{(i)})^T - (\hat{x}_{k|k-1})(\hat{x}_{k|k-1})^T + Q_{k+1} \end{aligned} \quad (26)$$

552 For the measurement update, the volume point is calculated according to the estimation $\hat{x}_{k|k-1}$ and
 553 variance $P_{xx,k|k-1}$ at time k . Q_k denotes the system noise covariance.

$$554 \begin{aligned} \xi_{k|k-1}^{(i)} &= \sqrt{nP_{k|k-1}} + \hat{x}_{k|k-1} \quad (i = 1, 2, \dots, 2n) \\ \bar{z}_{k|k-1}^{(i)} &= H_t(\xi_{k|k-1}^{(i)}) \end{aligned} \quad (27)$$

555 The measurement prediction $\hat{z}_{k|k-1}$, measurement prediction error variance (innovation variance)
 556 $P_{zz,k|k-1}$, and state measurement cross covariance $P_{xz,k|k-1}$ can be obtained as follows:

$$557 \begin{aligned} \hat{z}_{k|k-1} &= \frac{1}{2n} \sum_{i=1}^{2n} \bar{z}_{k|k-1}^{(i)} \\ P_{zz,k|k-1} &= \frac{1}{2n} \sum_{i=1}^{2n} \bar{z}_{k|k-1}^{(i)} (\bar{z}_{k|k-1}^{(i)})^T - \hat{z}_{k|k-1} (\hat{z}_{k|k-1})^T + R_k \\ P_{xz,k|k-1} &= \frac{1}{2n} \sum_{i=1}^{2n} \xi_{k|k-1}^{(i)} (\bar{z}_{k|k-1}^{(i)})^T - \hat{x}_{k|k-1} (\hat{z}_{k|k-1})^T \end{aligned} \quad (28)$$

558 The measurement update of the gain K_k , state prediction vector $\hat{x}_{k|k}$, and state prediction covariance
 559 matrix $P_{k|k}$ can be performed as follows:

$$560 \begin{aligned} K_k &= P_{xz,k|k-1} / P_{zz,k|k-1} \\ \hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_k (z_k - \hat{z}_{k|k-1}) \\ P_{k|k} &= P_{k|k-1} - K_k P_{zz,k|k-1} K_k^T \end{aligned} \quad (29)$$

561

562 **Appendix II List of notation**

563 Most of the symbols are introduced in detail in the main text. Some symbols with subscripts are further
564 explained below.

565 L_t^{SINS} : Latitude measured by SINS at time t .

566 λ_t^{SINS} : Longitude measured by SINS at time t .

567 h_t^{SINS} : Height measured by SINS at time t .

568 L_t^{GNSS} : Latitude measured by GNSS at time t .

569 λ_t^{GNSS} : Longitude measured by GNSS at time t .

570 h_t^{GNSS} : Height measured by GNSS at time t .

571 $V_{E,t}^{SINS}$: Eastern velocity measured by SINS at time t .

572 $V_{N,t}^{SINS}$: Northern velocity measured by SINS at time t .

573 $V_{H,t}^{SINS}$: Vertical velocity measured by SINS at time t .

574 $V_{E,t}^{GNSS}$: Eastern velocity measured by GNSS at time t .

575 $V_{N,t}^{GNSS}$: Northern velocity measured by GNSS at time t .

576 $V_{H,t}^{GNSS}$: Vertical velocity measured by GNSS at time t .

577 $V_{E,t}^{DVL}$: Eastern velocity measured by DVL at time t .

578 $V_{N,t}^{DVL}$: Northern velocity measured by DVL at time t .

579 ψ_t^{SINS} : Heading measured by SINS at time t .

580 $\psi_t^{COMPASS}$: Heading measured by compass at time t .

581 B_c : System noise transfer matrix.

582 $B_{cf,t}$: SINS fault transition matrix.

583 $D_{cf,t}$: Measurement fault transition matrix.

584 $\tilde{S}(\cdot)$: Function of the covariance inner product after whitening.

585

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588

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