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Research Paper

Correlations in operational risk stress testing: use and abuse

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ABSTRACT

Correlations between operational risk loss severity, frequency and economic factors have been used as a de facto tool to assess economic and regulatory capital since 1990. We demonstrate, using data from a single retail bank, that such correlations do not apply universally, and that projections of capital requirements are subject to wide error margins. Some correlations can be explained in terms of data trends. Given worldwide regulatory requirements to assess the resilience of financial institutions to economic shocks, an alternative to using correlations that makes use of economic data is proposed. The proposal is consistent with a much broader interpretation of capital allocation than has applied to date. Evidence that the Covid-19 pandemic had minimal effect on operational risk losses in 2020 is presented and the effect of model risk is emphasized. Our results show that the existence or otherwise of significant correlations depends on the regression model used, whether data series show trends, the time window concerned, geographical location and the type of financial institution.

Keywords: loss distribution; stress testing; stress index; Covid-19; Comprehensive Capital Analysis and Review (CCAR); model risk.

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1 INTRODUCTION

The need for banks to be resilient to adverse change due to economic circumstances has been a principle, and indeed a regulatory requirement, for many years. As a result, stress testing has become a routine component of banking operations. The principles involved are specified in central bank publications such as Bank of England (2019b), European Banking Authority (2019) and Board of Governors of the Federal Reserve System (2019).

Those references concentrate on the principles of stressing credit, market and operational risk losses with respect to changes in economic factors such as gross domestic product (GDP), employment and interest rates. However, very little is said about precise methodologies for doing so. For many years there has been an assumption that operational risk losses are correlated with economic factors. This is explicit in the case of the US stress test for evaluating capital adequacy. We demonstrate, using our own evidence and evidence from previous research, that it is not safe to rely on correlations between economic factors and operational risk losses. Conceptually, our view of stress testing is therefore much wider than the view demanded by current regulations. We summarize it in Box 1.

Associations between operational risk losses and economic factors tend to refer to loss severity rather than loss frequency. Often the term "severity" is not explicit. In this paper we concentrate on severity because it is what determines total loss. Frequency associations are mentioned briefly because of their importance in assessing risk mitigation.

1.1 The effect of model risk

Many regression models are available (linear, nonlinear, parametric, etc), and we emphasize the effect of particular regression models in the sections that follow. The results depend heavily on which model is used. Therefore, this topic is very subject to model risk. Other influential modeling factors are the time windows concerned and whether data series show trends.

1.2 The structure of this paper

Section 2 reviews prior research on correlation between operational risk losses and economic factors. The methodology used to analyze the relationship between economic factors and operational risk is introduced in Section 3. Numerical results are then presented in Section 4. A proposal is then made in Section 5 to develop a stress testing process that does not rely on correlations. The implications are evaluated in Section 6.

BOX 1 Additional stress testing requirements in this paper.

There is a need to retain sufficient capital to withstand shocks to the banking system. The capital calculation should be done in a statistically sound way that does not make undue assumptions. The amount of regulatory capital retained should be geared to the degree of stress. In principle, any operational risk capital generated can be used to mitigate credit or market risk losses, if required.

1.3 Nomenclature

Throughout this paper, the following terms are used.

- "Capital" refers to regulatory capital as specified in the internal capital adequacy assessment process.
- The seven Basel Committee on Banking Supervision (BCBS) risk classes we use are
  1. internal fraud (IF);
  2. external fraud (EF);
  3. employment practices and workplace safety (EPWS);
  4. clients, products and business practices (CPBP);
  5. damage to physical assets (DPA);
  6. business disruption and systems failures (BDSF); and
  7. execution, delivery, and process management (EDPM).
- "All" denotes a risk class that aggregates all BCBS risk classes.
- "Non-CPBP" is a risk class that aggregates all BCBS risk classes except CPBP.

2 LITERATURE REVIEW

The literature review comprises two parts. The first part is a summary of regulator and other guidance on stress testing. The second part deals with the correlation of operational risk (OpRisk) losses with economic factors, and it is prompted by economic data supplied by the Financial Conduct Authority (the UK regulator).

2.1 Stress testing requirements

The background to the current stress testing environment is that regulatory authorities want to be assured of the resilience of the banking system to an economic downturn.

The following quote is from the introduction to the Bank of England's (2019b) stress test guidance (in particular, the terms "forward-looking" and "quantitative" in the quote are important for the analysis given in the present paper):

> The main purpose of the stress-testing framework is to provide a forward-looking, quantitative assessment of the capital adequacy of the UK banking system as a whole, and individual institutions within it.

For the United Kingdom, the Bank of England (BoE) provides stress test documentation that gives the background for the most recently completed UK stress test (see, for example, the documentation for the 2018 and 2019 stress tests (Bank of England 2018, 2019b)). The same principle applies to the European Union (EU) stress test, governed by the European Central Bank (ECB) (see, for example, European Banking Authority 2019). The main documentation should state the principal economic factors that should be used for such assessments, outline baseline and stressed scenarios and provide a general introduction to the use of capital buffers as a means of mitigating economic stress. The principal economic factors listed are GDP, residential property prices, commercial real estate prices, the unemployment rate, foreign exchange rates and interest rates. There is an assumption that these factors should affect bank activity in general, but there is no specific mention of any relationship with OpRisk losses. The point is rather to ensure that the capital position is such that a bank can withstand an economic shock. The effect of economic factors should therefore be seen in the context of significant factors that contribute to the Tier 1 capital buffer (which, in effect, represents risk-weighted assets). Although OpRisk losses contribute to the value of risk-weighted assets, no specific link between OpRisk losses and economic factors is suggested in the stress test documentation, but such documents do include the more general proposition that fraud may be related to business cycles and that banks may reduce transaction volume in stressful economic periods, leading to fewer OpRisk losses. This view is in partial agreement with Chernobai *et al* (2011), which determined that "changes in market conditions" are responsible for only 13% of OpRisk events.

Stress testing in the United States has a similar aim to that in the European Union and the United Kingdom, and it is governed by the Dodd–Frank Act Stress Test (DFAST) methodology, which is described as "a forward-looking quantitative evaluation of the impact of stressful economic and financial market conditions on firms' capital" (Board of Governors of the Federal Reserve System 2019). Hypothetical stressful economic conditions are developed by the US Federal Reserve Board (Fed), and the results are used in the Comprehensive Capital Analysis and Review (CCAR) to evaluate capital adequacy. There are two essential differences between the approach of the United States and that of the European Union and the United Kingdom. First, the United States makes explicit use of correlation models (see Section 2.3.4). Second, the stress test model is administered and run by the Fed itself, using data supplied by regulated firms.

2.2 Existing stress testing approaches

Current approaches to stress testing are not well defined, but they do fit into broad categories. The categorization presented in Axtria (2014) partitions approaches into those that are "parameter stressing" and those that are "risk driver stressing". Parameter stressing involves direct manipulation of the quantity being stressed. In risk driver stressing, elements that determine the degree of risk are identified and stressed in order to verify the contingent stress on model parameters.

An alternative classification splits methodologies into those that are "top down" and those that are "bottom up". This categorization refers to the role of the regulator, and an explanation may be found in Basel Committee on Banking Supervision (2017). In the top-down approach, the regulator collects data from firms and uses regulator-developed scenarios and models. Firms may never even know how the regulator's stress test was done. In the bottom-up approach, firms run their own models (possibly using regulator-developed scenarios). Individual approaches may be very diverse, which is something regulators try to avoid.

The methodological approach of Drehmann (2009) lists four stages: generating scenarios, developing risk factors, calculating exposures and measuring the resulting risk. It expresses the general idea that stress tests should stress existing data and observe the effect of the stress on that data.

An alternative stress testing approach is reverse stress testing. Details may be found in, for example, Grundke (2011). Instead of applying a calculation method to data (usually to scenarios) and then calculating the effect on a target metric (in our case, value-at-risk[1]), a desired level of stress in the target metric is decided in advance, and the necessary data transformations to achieve that stress level are carried out.

2.3 Economic factor correlation

This subsection of the literature review deals specifically with correlation of economic factors with OpRisk losses. Very little has been reported since around 2016 (but see Section 2.3.3), and practitioners have had to rely on older studies. The research presented in this subsection presents findings from typical UK retail banks. The economic periods before and after the 2007–9 global financial crisis proved to be very different. Therefore, we consider them separately, starting with the period

[1] This should be taken to mean value-at-risk at 99.9%.

before the crisis. Research has concentrated on severity-related effects (ie, monetary loss), but in Section 3.6 we also comment on frequency effects.

The Covid-19 pandemic has also had a marked effect on economic conditions, and its effect on OpRisk has (so far) been unexpected (see Section 4.8).

2.3.1 Older studies: precrisis

Allen and Bali (2007) looked for correlations between cyclical economic factors (mainly GDP and unemployment) and OpRisk capital for financial institutions, including banks. They suggested that capital reserves increase with a downturn in the economy, which is followed by elevated levels of industrial production. Capital also increases when unemployment rates decrease at the end of an economic boom. Specifically, Allen and Bali found that, out of 43 economic variables that affect economic cycles, only 4 were not significantly correlated with catastrophic risk losses (characterized as a combination of losses due to market risk, credit risk and OpRisk). However, they did not use OpRisk data directly, because they considered different types of financial institution, some of which did not maintain an OpRisk database. Having removed the effects of market and credit risk using a multivariate regression on equity returns, they assumed OpRisk losses to a residual. Consequently, their results are less powerful than they would have been had direct OpRisk data been available.

2.3.2 Older studies: during and after the crisis

Moosa (2011) assumed that unemployment (measured by data sourced from the International Monetary Fund (IMF)) is the only relevant economic determinant of OpRisk losses. This is hard to justify, as he argued that credit card fraud is more prevalent when consumer spending is strong, that rogue trading (ie, IF) is more prevalent when financial markets are booming and that legal action due to employment practices (ie, EPWS) is more likely when the economy is in recession. Loss events from Fitch and IMF unemployment figures for 1990–2007 were used, so the data period and source are very different from ours. Moosa specifically sought relationships between cyclical unemployment figures and OpRisk losses. His results include plots comparing an unemployment time series with log(loss severity) and with log(loss frequency). The peaks and troughs of the plots generally coincide, but significant correlation between OpRisk losses and unemployment was not demonstrated.

Chernobai *et al* (2011) examined data from the Algo FIRST database from Algorithmics (Fitch) and expressed a general view that OpRisk events are exacerbated by lax risk controls. They examined OpRisk frequency correlations with equities, disposable income, GDP and corporate bond spreads. Significant correlations were found between the following pairs: EF and equities; CPBP and equities; CPBP and disposable income; "others" and equities; and "others" and GDP. All risk events together were significantly correlated with disposable income, equities and GDP.

Abdymomunov (2014) reported evidence that OpRisk losses are negatively correlated with macroeconomic growth for CPBP and EDPM (which comprise 90% of all losses he considered). He used aggregated data from the United States for the period 2003–12, and he linked log(GDP) with OpRisk losses. He found the following: total losses are negatively correlated to GDP, but the correlation is statistically non-significant; CPBP and EDPM are positively correlated with total assets; and EF and EPWS tail losses are negatively correlated with GDP, and the correlations are significant. Abdymomunov noted that results were different if data from 2000–2 was included. It is not clear whether the results of the study were obtained by extracting components of a multivariate regression or from individual regressions. Abdymomunov's study introduces further variations in data type, period and model, so his results cannot be generalized.

Cope and coworkers carried out three relevant studies. Cope and Antonini (2008), which predates the 2008 peak of the global financial crisis, found "little evidence" of strong correlations between OpRisk losses and economic factors but "slight evidence" of tail dependence for quarterly loss aggregations. Cope and Carrivick (2013) has particular significance in the context of tail dependence and makes several important points. Cope and Carrivick studied the overall effects of the global financial crisis on OpRisk loss frequency and severity, using the 2002–11 Operational Risk-data Exchange Association (ORX) data set for North America and Western Europe. They estimated unrealized losses using a time-dependent survival curve. Their analysis avoided stating any explicit relationship between OpRisk losses and economic factors. It merely highlighted a change in the pattern of OpRisk losses during and around a perceived time of economic stress. Effectively, correlations were implied. Their main findings were as follows.

- There were increases in OpRisk losses during the 2008 crisis period, mostly for EF and EDPM. Enhanced losses were detected in CPBP but to a limited extent.

- There was a decrease in IF during the same period relative to the preceding six years.

- In each risk category considered, there was a notable decline in OpRisk losses between 2002 and 2011, with an upward jump in 2008. Cope and Carrivick concluded that, although the effects of an economic shock might be severe, its effect on OpRisk losses is not persistent.

The final point is important, because it shows that the characteristics of OpRisk data change with time. Local characteristics predominate. Cope and Carrivick's study provides the motivation for our view of stress testing, which is summarized in Box 1.

The third relevant study by Cope and coworkers found correlations in various contexts, some of which are peripheral to the main thrust of our paper. Cope *et al* (2012) considers the effect of changes in GDP per capita and in unemployment on environmental, legal and regulatory factors. Out of the 22 factors analyzed therein, only 2 correspond to the economic factors specified by Bank of England (2019b). Like Cope and Carrivick (2013), Cope *et al* also used ORX data, and for a similar period (2002–10). The methodology used was to formulate a multivariate regression of log(loss) against a vector of environmental variables (including economic ones) and also against a vector of idiosyncratic factors. Cope *et al* stated whether correlations were significant (at the 95% level or higher). The first economic factor, GDP, had a statistically significant positive correlation with risk classes EF and EPWS, while for the second economic factor, unemployment, no significant correlation was found. The wide scope of the data they used makes it difficult to draw any general conclusions, as the countries concerned have disparate economic climates. It is possible that the significant correlation with EPWS arose because employment practices in the countries concerned (such as Brazil) result in greater costs than in the European Union and the United Kingdom due to litigation and compensation.

2.3.3 More recent research

United Kingdom. The BoE "Financial stability report" publications for 2018, 2019 and 2020 (Bank of England 2018, 2019a, 2020) all gave the same overall view of the resilience of the UK banking system to stressed economic conditions. A link between OpRisk losses and economic stress was implied, but correlation was not. Overall, both Tier 1 capital ratios and leverage ratios for UK banks have increased since 2017, when they were judged sufficient to cope with severe stress. The implication for OpRisk capital is that reserves have been increasing and perhaps do not need to increase further. The impact of economic stress is therefore expected to be minimal. However, there may be a need to increase overall reserves, and an OpRisk capital calculation can assist that. Two particular types of stress were highlighted: cyber crime and Brexit. Cyber risk is cited as a key source of nonsystemic risk. A lack of historical data has forced stress tests to use expert judgment to calibrate scenarios, so a large element of subjectivity is involved. The BoE has also focused on methods to minimize the effect of Brexit on disruption to financial services. No adverse consequences for OpRisk due to Brexit were anticipated, as capitalization was considered to be adequate. These reports, however were released before the 2020 UK–EU trade deal, which excluded financial services.

European Union. Similar to the BoE in the United Kingdom, the ECB aims to assess the resilience of EU banks under a common macroeconomic baseline scenario and a common adverse scenario. The ECB requires estimates of future OpRisk losses under both scenarios (European Banking Authority 2019), but there is no indication of how that should be done. The associated details for the adverse scenario indicate that worsening economic conditions would affect credit and market risks most. There are hints in the paragraphs on conduct risk in European Banking Authority (2019) that it would be valuable to consider events as sources of risk stress. Users are simply directed to use their own methodologies.

United States. Abdymomunov *et al* (2020) studied correlations between OpRisk losses of 38 large US banks (with assets of at least US$50 billion) and five US-relevant macroeconomic factors plus the first principal component of those factors (termed ME). They reported significant correlations with economic conditions in the United States, driven by the high frequency and severity of tail events. In particular, there was a general increase in OpRisk losses when the economy was deteriorating, but not in benign times. Loss occurrence was associated with market volatility. Specifically, there was a significant (at least 5%) correlation using the ME economic indicator with all BCBS risk classes except EPWS and DPA. Stronger correlations were found if only tail events were considered. Control regressions indicated that correlations persist over time, contradicting the result of Cope and Carrivick (2013).

There are important differences between the treatments used by Abdymomunov *et al* (2020) and ours. These are itemized below, and explain many contradictions in the respective findings.

- Economic factors are set against log(loss) rather than loss, which we use. Further, for any given risk class, log(loss) in any time period is standardized with respect to all losses for that risk class. Therefore, the treatment of loss is essentially different.

- The results could differ depending on which correlation measure is used. The precise measure is not stated in Abdymomunov *et al* (2020). Using rank correlation (Spearman or Kendall) should preserve the result of the log transformation, but using product moment (Pearson) correlation will not.

- There is a difference in the risk class mix. CPBP represents nearly 80% of total severity (50% in our case), and EF and EDPM frequencies are more dominant.

- The date range used was 2000–13 (and thus included the 2007–9 global financial crisis), whereas our data were mostly post 2013. The analysis in Abdymomunov *et al* (2020) uses the occurrence date for aggregation, but, oddly, correlations are calculated using the accounting date.

- A different selection of economic indicators was used in Abdymomunov *et al* (2020) (appropriate for the US), so the economic–OpRisk comparison represents an essentially different economy.
- A lower modeling threshold of US$20 000 was used in Abdymomunov *et al* (2020), whereas we have used £1000.[2] Our reduced threshold admits losses that may reduce the extent of correlations.

2.3.4 The Fed's regression model

The degree of detail in the stress testing documentation from the Fed (Board of Governors of the Federal Reserve System 2019) is very different to that of their European counterparts. The DFAST specifies details of the loss distribution model to be used and, significantly, how a regression of OpRisk losses against economic features should be done.

The first component of the OpRisk loss distribution model is quite standard: the distribution of a sum of independent and identically distributed random variables (each representing an individual loss), where the number of random variables summed is also a random variable (see Klugman *et al* 2004, Chapter 6).

The second component is a historical simulation model to predict future losses. The intention is to account for large and infrequent OpRisk losses by projecting loss frequency and severity separately. Klugman *et al* (2004) adopted a frequency–severity combination using either Panjer's recursion method, fast Fourier transforms or a numerical function-approximation method. The linear discriminant analysis (LDA) method is now more common, since it can be applied more generally (at the expense of taking much more time). The evidence for correlation significance in the Fed's documentation appears to come from an early paper (DeFontnouvelle *et al* 2006), in which a model based on a logit generalized Pareto distribution was proposed as a good estimator of loss severity for non-US data. However, reliable and comprehensive OpRisk data were not available to DeFontnouvelle *et al*, who instead used third-party vendor data (from OpRisk Analytics and OpVantage), which were restricted to losses exceeding US$1 million. The data covered all industries and was publicly sourced from news reports, court filings and Securities and Exchange Commission filings. The data restrictions and the time period (before the 2007–9 global financial crisis) cast doubt on the general validity of this correlation.

The third component of the Fed's model is a linear regression. The explanatory variables in this model are a set of macroeconomic features that measure economic activity, financial conditions and the interest rate environment. Specifically,

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[2] This threshold reflects a wish to include classes of losses that are important for retail operations, such as credit card fraud.

the following are stated explicitly for OpRisk: the BBB corporate yield, the House Price Index, the Market Volatility Index, the 10-year Treasury yield and the unemployment rate. Other explanatory variables are specified, but are directed at credit and market risk. The first principal component of the explanatory variables is calculated, and a function of that first principal component is used as the regressor. The regressand is an industry aggregate loss divided by industry aggregate assets in each quarter (ie, not a bank's OpRisk losses directly).

The Fed's approach raises several problems, which will be discussed in Section 6. For now, it is sufficient to note that the Fed considers that OpRisk losses and economic factors can be linked using a linear regression. However, that view is contradicted by Curti *et al* (2019), who say that banks "have struggled to find meaningful relationships between operational losses and the macroeconomy". They reason that large and infrequent loss events dominate OpRisk exposure, and that data sets are too often too small. The data stamp attached to such events is also not well defined, because there can be long delays before an event is even detected or between a detection date and a date on which a payout is actually made. A potential solution to these problems is to use financial statement benchmarks (those suggested are total assets, risk-weighted assets and gross income) as a proxy for losses.

2.3.5 Regression analyses: summary

Our overall conclusion from the summaries in Section 2.3 is that OpRisk loss is event driven with respect to economic factors and that it is unsafe to rely on correlations. Our own evidence to that effect will be presented in Section 4. A more in-depth discussion of the implications follows in Section 6. Such an event-driven relationship adds impetus to the idea that economic factors could be used to inflate losses. The strategy to do this will be developed in Section 5.

3 METHODOLOGY

In this section we present the details of our own correlation tests, starting with the data components used.

3.1 OpRisk data

The data used in this analysis was extracted from the OpRisk database of the UK retail arm of a multinational bank and covers the period from January 2010 to June 2020. The bank's major competitors are rival banks whose branches are established in most UK towns and cities. In terms of market capital,[3] the bank ranks as

[3] This is as recorded on the London Stock Exchange in September 2021. See, for example, www.londonstockexchange.com/stock/LLOY/lloyds-banking-group-plc/company-page.

medium-to-large compared with its competitors, and it is therefore representative of UK retail banking and also of similar banks in mainland Europe.[4] The data extraction was based on the accounting date of risk events, rather than their occurrence or detection date. The bank's use of the accounting date in this way is consistent with its treatment of credit and market risk and has been agreed with the UK regulator.

The period covered by the data includes the first six months of the Covid-19 pandemic in 2020, and the effects of the pandemic are discussed in Section 4.8. Individual losses were aggregated by day. The seven BCBS risk classes are represented, including CPBP for completeness, even though CPBP is formally treated in a different way in the UK stress test.

3.1.1 Tail losses and the 2007–9 global financial crisis

Overall our data set contains no specific links between losses and the 2007–9 crisis. Other authors, however, have noted that losses due to the 2007–9 global financial crisis have been recorded in the years following the crisis.

Using ORX data covering North America and Western Europe for the period 2002–11, Cope and Carrivick (2013) observed falling mean losses during the precrisis period of 2002–6, followed by a rise to a peak in 2008–9 and a subsequent reduction between 2009 and 2011. They noted an increase in precrisis frequency for EDPM and EF in particular, as well as a decrease in severity for IF and EPWS during the crisis period. Thereafter, loss frequency decreased to precrisis levels. Cagan (2009), Anderson *et al* (2012) and Jobst (2010) all argued that a failure of operational control through lax lending strategies and subsequent product monitoring deficiencies precipitated the crisis. Cagan noted that US OpRisk losses increased fourfold between 2007 and 2008. Jobst considered inadequate OpRisk control to amplify systemic risk, and thereby implied that, by having contributed to the financial crisis, it had the potential to perpetuate it. Hess (2011) linked most financial crisis losses to the "trading and sales" and "retail brokerage" business lines, which are precisely those that are most affected by inadequate business controls. Chernobai *et al* (2011) gave the specific example of a US$7.2 billion unauthorized trading loss at Société Générale in 2008, which was attributed to a lack of internal control. Regulation post 2009 has tightened OpRisk controls, and thereby reduced OpRisk losses, as evidenced by reduced the frequency and severity of such losses after 2009.

For our data, tail losses are not concentrated in any part of the modeling period and are unlikely to be attributable to the 2007–9 global financial crisis. During 2011–12 there was an increase in the volume of small to medium-sized losses for IF and EF.

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[4] The relevant market capitalizations are £24.6 billion for Natwest, £30.8 billion for Lloyds, £31.2 billion for Barclays, £45.7 billion for Santander, £78.5 billion for HSBC, €5.6 billion for ABN AMRO, €21.8 billion for Deutsche Bank and €37.7 billion for Crédit Agricole.

IF and EF volumes were much lower in 2013–15, and rose again in 2016. Although about half of the larger IF losses have occurrence dates in 2012, almost all of the larger EF losses have occurrence dates in that year. The largest contribution to the tail is a payment protection insurance (PPI) provision (risk class: CPBP), which was established in 2011. CPBP regulatory capital consequently constitutes approximately 50% of the total capital. Very few small CPBP losses were recorded in 2011–16, and CPBP volume increased after 2016 as a result of improved data collection procedures.

In contrast, DPA, EPWS and BDSF represent very small components of the total capital, with occurrence dates spread throughout the entire data window. There is a concentration of large EDPM losses detected in 2014. While many EDPM losses have occurrence dates prior to 2000, their occurrence dates are spread throughout the entire data window.

Appendix A online gives the descriptive statistics of the data used in this study, analyzed by BCBS risk class.

3.2 Economic data

The BoE provides historical and projected (up to the year 2042) economic data in conjunction with documentation in Bank of England (2019b), specifically for use in stress testing. The BoE view is that sufficient capital reserves should be held to "withstand extreme market shocks". The implication is that an economic shock can result in additional losses for financial institutions. That view is quite plausible for credit and market risk, but less so for OpRisk. For 2019 the BoE provided two sets of economic time series: "base scenario" and "annual cyclical scenario", which we designate "Base" and "ACS", respectively. The former gives projected economic data for mild stress, and the latter represents a more severe stress. Multiple economic variables are provided, and practitioners are expected to choose the most appropriate ones.

We have selected variables that are UK-relevant to match our UK OpRisk data: specifically,

- UK.Real.GDP,
- UK.CPI,
- GBPEUR,
- GBPUSD,
- UK.Unemployment.Rate,
- UK.Corporate.Profits,

- UK.Household.Income,
- UK.Residential.Property,
- UK.Equity.Prices,
- Bank.Rate,
- Sterling.IG.Corp.Bond.Spread,
- Secured.Lending.Individuals,
- Consumer.Credit.Individuals,
- Oil.Price and
- Volatility.Index.

In addition to the selected economic data, lagged economic data with lags of three, six, nine and twelve months were considered. The autocorrelations for each economic factor were significant, so the lagged data were effectively repeats of the nonlagged data. Therefore, lagged data were omitted from the correlations.

3.3 Implementation

All calculations were done using R software on an i7 Windows processor with 16 GB random access memory. Particular use was made of the `loess` package for nonlinear regressions.

3.4 Correlation details

The purpose of our correlation analyses is not just to establish whether significant correlations exist, but also to make predictions of OpRisk for the upcoming year. The BoE data supplied are intended to be used to predict OpRisk losses for the period from January 2019. Correlations were calculated using the BoE and OpRisk data for the ten-year period prior to January 2019. The BoE asks for a three-year prediction, but that is an unreasonably long period given the amount of data available (ten years) and we also considered that, in principle, it was similarly unreasonable to predict five years into the future. Therefore, we calculated one-year predictions, which are presented in Section 4, and we discuss their implications in Section 6.

3.4.1 Autocorrelation

The purpose of an autocorrelation analysis is to justify the use of only nonlagged economic data. Autocorrelations for each economic factor were tested, using lags of up to four three-month periods, which is more than sufficient to cover a one-year time horizon. The maximum significant correlation (at 95%) was noted for each factor.

3.4.2 Univariate correlation

The data for each BCBS risk class were paired with the data for each economic factor, and the Spearman rank correlation coefficient was calculated for each pair. The plot of the loss data against time is clearly nonlinear for all risk classes, so Pearson correlation is less appropriate than (Spearman or Kendall) rank correlation. Rank correlation is also less affected by outliers. An authoritative discussion may be found in DeGroot and Schervish (2012). We note that, informally, regulators agree with this view.

The statistical model for the linear univariate correlation is given as follows:

$$L_i \sim E_{jk} + \varepsilon_{ijk}, \quad i = 1, \dots, 9,\ j = 1, \dots, J,\ k = 1, 2, \tag{3.1}$$

where L_i are the OpRisk losses for the seven BCBS risk classes i (augmented by the aggregates non-CPBP and All), aggregated by quarter, and E_{jk} are the BoE economic factors listed in Section 3.2 for economic scenario k (either Base or ACS). More generally, selections from these variables, and combinations of them, can be made. There are J economic variates in all. The ε_{ijk} term gives stochastic errors.

It is difficult to suggest a nonlinear data transformation that would be appropriate for all risk classes. Many authors (see, for example, Abdymomunov 2014; Abdymomunov *et al* 2020; Cope *et al* 2012; Moosa 2011) have chosen the transformation $L_i \to \log(L_i)$. In order to illustrate a simple nonlinear model, we have applied the same transformation. Many other nonlinear models are possible. In particular, there are almost countless combinations of the E_{jk} variables. Our simple univariate nonlinear correlation model is given as follows:

$$\log(L_i) \sim E_{jk} + \varepsilon_{ijk}, \quad i = 1, \dots, 9,\ j = 1, \dots, J,\ k = 1, 2. \tag{3.2}$$

For linear regressions, the null hypothesis for the theoretical Spearman correlation coefficient ρ is $\rho = 0$ and the alternative hypothesis is $\rho \neq 0$. For n ranks, the significance of a calculated correlation coefficient r is assessed using a Fisher transformation:

$$z = \frac{n-3}{2} \ln\left(\frac{r+1}{r-1}\right) \sim N(0, 1).$$

Alternatively, a t-test with $n - 2$ degrees of freedom can be used:

$$t = \left| r \sqrt{\frac{n-2}{1-r^2}} \right|.$$

3.4.3 Multivariate correlation

In contrast to using a single economic factor, subsets or combinations of the economic factors can be used. The following equation is a base linear multivariate statistical model that uses all the economic factors:

$$L_i \sim \sum_{j=1}^{J} E_{jk} + \varepsilon_{ijk}, \quad i = 1, \dots, 9,\ j = 1, \dots, J,\ k = 1, 2. \tag{3.3}$$

For each risk class i, a multivariate correlation coefficient L_i can be extracted. Significance (and hence a p-value) can be assessed using an F-test with $J - 1$ model degrees of freedom and $N - J$ error degrees of freedom, where N is the number of observations (ie, quarters in this analysis).[5] The null hypothesis for the coefficients β_{jk} corresponding to the terms E_{jk} is $\beta_{jk} \neq \beta_{j'k}$, $j \neq j'$. The alternative hypothesis is $\beta_{jk} = 0$ for at least one j.

3.4.4 Multivariate regression and prediction

The multivariate regressions measure the response of the loss data regressand to the economic regressors. Effectively they are a combination of responses to each individual regressor, assessed either linearly or nonlinearly. The predictions are all based on multivariate analysis but the same type of calculation can be applied to any combination of regressors.

Two multivariate regression approaches were used to assess the effect of all the economic factors on a single risk class. Both use the locally estimated scatterplot smoothing (LOESS) method, and they can be used to approximate a linear regression by choosing a suitable LOESS "span" parameter, s. LOESS is a useful method if no suitable nonlinear relationship can be found for any particular risk class. The symbol $\mathfrak{L}(s)$ denotes the LOESS fit process with span s.

The statistical model for the nonlinear regressions is given as follows:

$$\begin{aligned} L &= \mathfrak{L}(s)(L_i \sim E_{jk}), \quad i = 1, \dots, 9,\ j = 1, \dots, 15,\ k = 1, 2, \\ R_i &= \sqrt{\left(1 - \frac{L_i^{\mathrm{r}}}{\sum(L_i)}\right)}, \\ \tilde{L}_i &= R_i L_i^{\mathrm{p}}, \end{aligned} \tag{3.4}$$

with the final prediction being a mean of the individual predictors, weighted by the sum of the pseudo-R^2 statistics:

$$\hat{L}_i = \frac{\tilde{L}_i}{\sum(R_i)}. \tag{3.5}$$

[5] See, for example, https://en.wikipedia.org/wiki/Regression_analysis (accessed June 8, 2022).

The calculation is complicated by the lack of a formal R^2 statistic for empirical regression processes (see Hu *et al* 2006). A broad equivalent, pseudo-R^2, has to be defined. We used the Cox and Snell (1989) version here. The LOESS predictor and residuals for L_i are denoted by L_i^{p} and L_i^{r}, respectively. Both are properties of $\mathfrak{L}(s)$. The variable $\tilde{L}_i$ is a scaled loss derived from L_i, as defined in (3.4).

Using LOESS presents problems in assessing goodness-of-fit. Jacoby (2000) recommends using the properties of residuals. Specifically, if we use the LOESS predictors for losses L_i and $\hat{L}_i$, $i = 1, \dots, n$, and we denote the number of degrees of freedom of the LOESS fit by d_{L}, then the statistic

$$F = \left(\frac{\sum_{i=1}^{n} L_i^2}{\sum_{i=1}^{n} (L_i - \hat{L}_i)^2} - 1\right)\left(\frac{n - d_{\mathrm{L}}}{d_{\mathrm{L}} - 1}\right)$$

has an $F(n - d_{\mathrm{L}}, d_{\mathrm{L}} - 1)$ distribution. Significance can therefore be assessed using percentage points on the appropriate F-distribution. The value of d_{L} is not straightforward. It should correspond approximately to the degree of a potential polynomial fit to the data. We used the number of regressors in all cases, since the fitted LOESS profiles are nonsmooth. Again, see Jacoby (2000) for a full discussion on this point. For consistency, the linear fit is quantified in the same way.

3.5 Frequency correlation

Frequency correlations were calculated using only the linear univariate and multivariate methods, since frequency is considered less important than severity from an accounting viewpoint. The statistical models are the same as those detailed in (3.1) and (3.3). The results are presented in Section 4.6.

3.6 Correlation persistence

The conclusion from the literature review in Section 2.3 was that, if correlations between economic factors and OpRisk losses exist, they do not persist. To test this with our historical data, a sequence of five-year rolling windows were set up, and correlations were tested on each. Correlations of both loss severity and frequency were tested against the economic factors. With 15 historical economic factors (identical for the Base and ACS cases) and 23 five-year windows, a total of 345 correlations were possible per risk class. Only the All risk class was used, in order to reflect the most general results.

3.7 Inappropriate correlations

Many correlation results in the context of OpRisk can be explained by an analysis of trending time series. To illustrate this point, we measured the correlation of our

TABLE 1 Autocorrelation extent.

Economic factor	Significant three-month periods	
	Base	ACS
UK.Real.GDP	10	10
UK.CPI	8	8
UK.Unemployment.Rate	10	6
UK.Corporate.Profits	10	9
UK.Household.Income	9	9
UK.Residential.Property	10	9
UK.Equity.Prices	8	4
Bank.Rate	12	4
Sterling.IG.Corp.Bond.Spread	4	3
Secured.Lending.Individuals	10	6
Consumer.Credit.Individuals	9	7
Oil.Price	7	7
Volatility.Index	4	3
GBPEUR	5	4
GBPUSD	8	6

The two right-hand columns show the maximum number of three-month periods for which autocorrelation is significant at 95% for the corresponding economic factor in the left-hand column.

OpRisk data with all 71 commodities listed in the World Bank commodity database.[6] The commodities listed there include, for example, crude oil, natural gas, coffee, meat, cocoa and groundnuts. None have a clear economic link to OpRisk, which is why an analysis of the World Bank data is so insightful. The results are given in Section 4.9.

4 RESULTS

In this section we report the results of correlations and regressions applied to loss data together with the Base and ACS economic scenario data.

4.1 Autocorrelations

Table 1 shows the number of significant (at 95%) autocorrelations for each economic regressor for both the Base and the ACS data. In all cases for the Base data, there are

[6] World Bank "Monthly Prices" monthly commodities markets data 2020. URL: www.worldbank .org/en/research/commodity-markets (accessed June 8, 2022).

TABLE 2 Univariate correlations: identical linear and nonlinear cases.

Significance level	Number of correlations: Base	ACS
Not significant	59	64
5–10%	10	5
1–5%	15	14
Less than 1%	21	22

no more than four significant autocorrelations (representing four three-month periods). For ACS data, only one economic factor has less than four significant autocorrelations. Therefore, we were able to omit lagged data, as the lagged data effectively reiterate the nonlagged data.

4.2 Univariate severity correlation results

Using rank correlation preserves the result of correlations after applying a nonlinear transformation. There is therefore only one set of results for the univariate linear case (loss) and the univariate nonlinear case (log(loss)). Table 2 summarizes the results of the (Spearman) severity correlation for each of the 15 economic factors with each of the seven risk classes. The total number of significant correlations at 1%, 5% and 10% were noted, as were non-significant correlations. Most correlations are not significant. Correlations at 1% and 5% are clustered in risk classes IF, EF and EPWS. There are fewer significant correlations for EPWS when using ACS data. Further, if losses from all risk classes are aggregated, there are no significant correlations with any economic factor, which is surprising given the IF, EF and EPWS results.

Appendix B online contains more precise details of the correlations summarized in Table 2. There are separate tables in that appendix for the Base scenario and the ACS.

4.3 Multivariate linear severity correlation results

Table 3 shows the results of using all economic variables as regressors, with OpRisk losses as the regressand. Equation (3.3) is the relevant statistical model. The significant correlations for IF and EF agree with the equivalent nonlinear regression significances in Table 4. There are differences for other risk classes.

4.4 Multivariate regressions and predictions

In principle, multivariate regressions and projections should provide a more comprehensive view of the relationship between a set of losses and all economic factors

TABLE 3 Multivariate linear correlations.

Risk class	Base		ACS	
	R-value	p-value	R-value	p-value
IF	0.880	0.000***	0.833	0.001***
EF	0.837	0.001***	0.807	0.004***
EPWS	0.571	0.630	0.568	0.643
CPBP	0.574	0.617	0.545	0.731
DPA	0.584	0.576	0.587	0.565
BDSF	0.664	0.236	0.748	0.038**
EDPM	0.682	0.173	0.593	0.536
Non-CPBP	0.597	0.517	0.558	0.682
All	0.587	0.563	0.553	0.704

***1% significance. **5% significance.

than univariate methods would. It should be noted that both sets of economic data are highly correlated. The correlations can thus bias economic–OpRisk correlation results. This problem is addressed in Section 4.5. Using all the economic features can be insightful because, in some cases, including the variables that (because they are significantly correlated with others) are nearly redundant can overemphasize their contribution. In the simpler case, more highly significant levels of economic–OpRisk correlations might be expected.

Prior to giving specific one-year projected losses, we illustrate general points regarding the linear and nonlinear multivariate regressions with particular examples. Figure 1 compares linear and nonlinear projections for IF using the Base economic scenario, which is intended to model continuation of current economic conditions. The dashed vertical lines in Figure 1 indicate the historic–projection data boundary: historical data are shown to the left of the line and projections appear to the right.

In the nonlinear case, predictions were made using LOESS estimation, which provides sufficient sensitivity to local variation in the data profile, as well as a degree of smoothing. LOESS is able to accommodate any data profile and is distribution-free. Consequently, the need to find a nonlinear fit that is appropriate to all risk class profiles is eliminated. Additionally, LOESS lag is much less than that for moving average smoothing methods, and LOESS has a convenient built-in implementation in R. Cubic splines would achieve the same effect, but they are less flexible in modeling profiles with a high degree of curvature. The degree of smoothing is governed by a LOESS span parameter, which was set to $\frac{2}{3}$ in all cases. That value is a compromise between preserving the profile of the original loss data and not being overinfluenced by outliers. Figure 1 illustrates the following general characteristics.

FIGURE 1 Contrasting Base (a) linear and (b) nonlinear projections for IF.

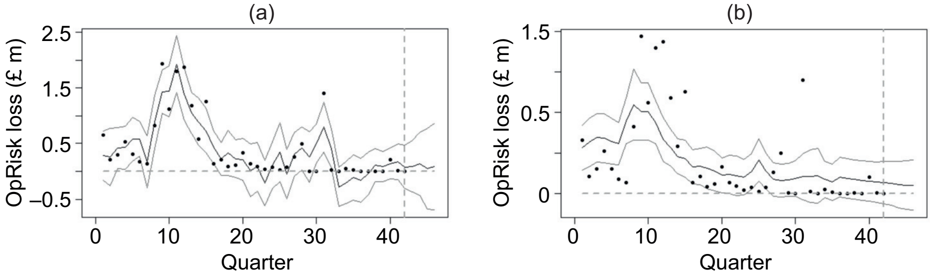
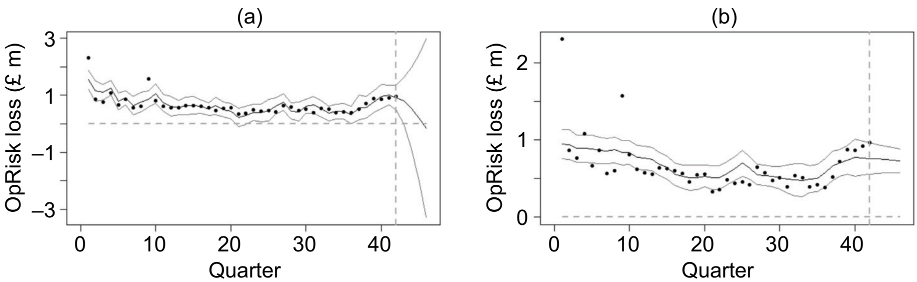


FIGURE 2 Contrasting ACS (a) linear and (b) nonlinear projections for EF.



- The nonlinear regression profile introduces a greater degree of smoothing than the linear regression profile. This is unsurprising given that LOESS is primarily a smoothing technique.
- Within the period covered by historical data, the 95% confidence bounds in both cases indicate negative predicted losses.
- In the projected data period (to the right of the vertical dashed lines), the confidence bounds diverge. Therefore, uncertainty in the projection increases with increasing time.
- The linear regression profile is much more sensitive to outliers than the nonlinear case.

TABLE 4 Regression significance of the Base and ACS economic scenarios.

Risk class	Base p-values and trends		ACS p-values and trends	
	Linear	Nonlinear	Linear	Nonlinear
IF	(–) 0***	(–) 0.0889*	(–) 0.0011***	(–) 0.1946
EF	(–) 0.0009***	(–) 0***	(–) 0.0043***	(–) 0***
EPWS	(–) 0.6298	(–) 0.2766	(–) 0.6435	(–) 0.3294
CPBP	(–) 0.6174	(–) 0.9388	(–) 0.7307	(–) 0.9668
DPA	(–) 0.5759	(–) 0.7831	(–) 0.5646	(–) 0.7957
BDSF	(+) 0.2363	(+) 0.5963	(+) 0.0377**	(+) 0.3916
EDPM	(0) 0.1730	(–) 0.1031	(0) 0.5358	(–) 0.0982*
Non-CPBP	(–) 0.5172	(–) 0.7516	(–) 0.6824	(–) 0.8247
All	(–) 0.5629	(–) 0.6783	(–) 0.7042	(–) 0.7604

***1% significance. **5% significance. *10% significance. Trends are indicated by + (positive), – (negative) and 0 (near zero).

The ACS economic scenario is intended to model stressed economic circumstances. Figure 2 shows a typical result. In the nonlinear case the span parameter was also set to $\frac{2}{3}$. The following general points should be noted.

- The linear projection (to the right of the dashed vertical line in Figure 2(a)) is downward-trending and projects a negative loss after one year. Such a projection is unreasonable for a stressed scenario.
- The confidence bounds in the linear projection splay rapidly, indicating a great deal of uncertainty in the projection.
- The projections in the two cases are contradictory, downward-trending in the linear case and approximately constant in the nonlinear case. Both are surprising given that toward the end of the historical data period (immediately to the left of the dashed vertical lines) there appears to be an uptrend in losses.

Tables 4 and 5 show summary results for the p-values and projections. Detailed results may be found in Appendix B online.

Table 4 shows the significance levels (p-values) of the linear and nonlinear data fits for the Base and ACS economic cases. The table points to an overall relationship between regressors and the regressand in each case. Significant correlations are indicated for IF and EF. However, these have to be seen in the context of both the resulting projections and the model parameters used to produce them (see Section 4.5.1). There are contradictions in the IF and EF cases between the linear and nonlinear models.

TABLE 5 Base one-year projections, upper confidence bounds and lower confidence bounds.

Risk class	Linear (% changes)			Nonlinear (% changes)		
	Projection	Upper	Lower	Projection	Upper	Lower
IF	–39	533	–613	–22	208	–252
EF	157	254	60	22	59	–14
EPWS	–164	388	–716	–19	114	–153
CPBP	54	661	–552	–38	182	–260
DPA	127	648	–393	–30	168	–228
BDSF	294	592	–4	46	133	–40
EDPM	–278	–55	–501	–23	37	–83
Non-CPBP	11	444	–422	–33	130	–196
All	2	437	–432	–32	125	–190

Entries are expressed as percentage changes relative to the mean value of historical losses for the most recent five years.

Most trends are negative (shown by (–) in Table 4). That is curious because it implies that OpRisk losses decrease in stressed economic circumstances. A plausible reason is reduced financial activity. Given any particular risk class apart from EDPM, the trend is independent of the economic data and of the regression model. For EDPM, the trend is marginally positive when using linear regression and marginally negative when using nonlinear regression.

Table 5 shows one-year projections under the Base economic scenario. The entries are expressed as percentage changes relative to the mean value of historical losses for the most recent five years. That five-year period provides a reasonable view of the ambient level of the most recent historical losses. The large percentage changes noted are not consistent with a scenario that should not reflect any significant economic stress. The same applies for negative projections. Rather, they are due to data volatility and the characteristics of the regression models. In general, the nonlinear model induces lower deviations from the ambient levels.

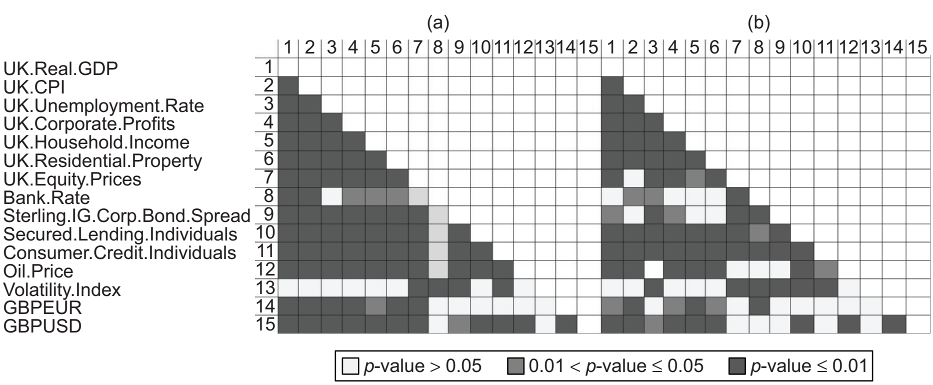
Table 6 shows the corresponding one-year projections under the ACS economic scenario. Large percentage changes in the linear projections are very much exaggerated compared with the Base scenario, although the same exaggeration is not so apparent for the nonlinear projections. Negative projections are not consistent with retaining more capital in preparation for economic hardship. In the context of the Covid-19 pandemic, there is some evidence of a downturn in activity, leading to reduced OpRisk losses (see Section 4.8). Such a downturn would not be consistent with retaining more capital in stressed times; although, in that case, reducing capital on the basis of a prediction would be unwise.

TABLE 6 ACS one-year projections, upper confidence bounds and lower confidence bounds.

Risk class	Linear (% changes)			Nonlinear (% changes)		
	Projection	Upper	Lower	Projection	Upper	Lower
IF	65	3532	−3400	201	396	6
EF	−127	415	−670	26	53	0
EPWS	161	3031	−2708	14	155	−126
CPBP	−1432	1789	−4655	10	235	−214
DPA	−982	1709	−3675	−23	169	−215
BDSF	−889	483	−2263	34	127	−59
EDPM	855	2126	−415	−34	22	−91
Non-CPBP	−910	1414	−3234	2	166	−161
All	−900	1418	−3220	7	169	−153

Entries are expressed as percentage changes relative to the mean value of historical losses for the most recent five years.

FIGURE 3 Heat map representation of economic correlations: (a) ACS and (b) Base.



4.5 Multivariate regressions from non-significantly correlated economic features

Correlations between the economic variables pose problems for determining economic–OpRisk correlations since they can overemphasize the contribution from certain variables. The heat maps in Figure 3 show that such correlations are extensive. The dark shaded cells in both parts of the figure represent pairs of economic factors for which the correlations are highly significant. The light shaded cells represent

TABLE 7 Regression significance of the Base and ACS economic scenarios using three non-significantly correlated economic components.

Risk class	Base p-values: T1		Base p-values: T2		ACS p-values: T1	
	Linear	Nonlinear	Linear	Nonlinear	Linear	Nonlinear
IF	0.002***	0***	0.952	0***	0.001***	0***
EF	0.085*	0***	0.04**	0***	0.002***	0***
EPWS	0.138	0***	0.741	0***	0.099*	0***
CPBP	0.645	0.002***	0.119	0.002***	0.781	0.005***
DPA	0.266	0.001***	0.631	0.001***	0.176	0.001***
BDSF	0.011**	0***	0.045**	0***	0.044**	0***
EDPM	0.089*	0***	0.138	0***	0.538	0***
Non-CPBP	0.566	0***	0.083*	0***	0.846	0.001***
All	0.595	0***	0.073*	0***	0.714	0***

***1% significance. **5% significance. *10% significance.

pairs of economic factors with non-significant correlations, and there are very few of them. We therefore seek to reduce correlations among the economic variables to a non-significant level by selecting a suitable subset.

Three methods were used to generate an appropriate subset of the economic variables. The first method was to (manually) identify groups of economic features with non-significant correlations. In the Base case a few pairs of features with non-significant correlations were apparent, but only two triples:

$$\text{T1} = (\text{GBPEUR}, \text{Volatility.Index}, \text{Oil.Price}),$$
$$\text{T2} = (\text{GBPEUR}, \text{Volatility.Index}, \text{Secured.Lending.Individuals}).$$

In the ACS case the only triple was T1. No subsets with more than three elements were found. The second method was the principal components method of Abdymomunov *et al* (2020), with the difference that the principal components that explained the bulk of the variance (in this case, 95%) were used, as well as only the first principal component. In both the Base and the ACS cases, four principal components were extracted, and those sets are denoted by PCA in this subsection. The third method follows that of Chernobai *et al* (2011). It is based on the idea of “internal control weaknesses” (ICWs), which in our case are economic features that could affect OpRisk losses. We have selected Volatility.Index, UK.Residential.Property and UK.Real.GDP, linked to CPBP, EDPM and EPWS, together with Sterling.IG.Corp.Bond.Spread, linked to IF and EF. These broadly mirror the ones used in Abdymomunov *et al* (2020). We refer to this as the ICW model.

TABLE 8 Regression significance of the Base and ACS economic scenarios using four PCA components.

Risk class	Base p-values		ACS p-values	
	Linear	Nonlinear	Linear	Nonlinear
IF	0.007***	0***	0.009***	0***
EF	0.008***	0***	0.002***	0***
EPWS	0.341	0.001***	0.331	0.001***
CPBP	0.436	0.064*	0.841	0.038***
DPA	0.669	0.002***	0.359	0.001***
BDSF	0.154	0.005***	0.085*	0.006***
EDPM	0.159	0***	0.219	0***
Non-CPBP	0.336	0.011**	0.828	0.005***
All	0.301	0.007***	0.759	0.004***

***1% significance. **5% significance. *10% significance.

Tables 7–9 show a marked distinction between the correlations for the linear and nonlinear multivariate regressions. The linear correlations are similar to those derived using all of the economic factors, whereas the nonlinear correlations for risk classes other than fraud are highly significant. Table 7 shows results for the linear and non-linear "triple" models T1 and T2. The linear model results are all broadly similar to the results obtained using all economic features (Table 4): the most apparent correlations are for IF and EF. In contrast to Table 4, all other BCBS risk classes show significant correlations when using the nonlinear model.

Table 8 shows the equivalent results for the PCA model (ie, the model with four principal components). The patterns observed for the T1 and T2 models are also apparent in the PCA model.

Table 9 shows the results for the ICW model suggested by Chernobai *et al* (2011). The results are similar to those from the T1, T2 and PCA models. The similarity is surprising, since some of the economic features that constitute the ICW model are significantly correlated. An example is the pair (UK.Residential.Property, UK.Real.GDP), for which the correlation coefficients are 0.958 and 0.946 for Base and ACS, respectively.

In an additional configuration, Abdymomunov *et al* (2020) also calculate principal components for their choice of economic factors, and they use only the first as a regressor. We have done the same, and the results vary depending on the model adopted. Abdymomunov *et al* (2020) record significant correlations for IF (***), EDPM (***), EF (**), CPBP (**) and BDSF (**). The asterisks have the same meaning as in the tables.

TABLE 9 Regression significance of the Base and ACS economic scenarios using four PCA components.

Risk class	Base p-values		ACS p-values	
	Linear	Nonlinear	Linear	Nonlinear
IF	0***	0***	0***	0***
EF	0.003***	0***	0***	0***
EPWS	0.423	0.001***	0.431	0.001***
CPBP	0.238	0.001***	0.877	0.05*
DPA	0.714	0.001***	0.823	0.005***
BDSF	0.037**	0.01***	0.018**	0.001***
EDPM	0.733	0***	0.629	0***
Non-CPBP	0.189	0***	0.91	0.008***
All	0.203	0***	0.882	0.006***

***1% significance. **5% significance. *10% significance.

The differences in results revealed by the models in this section indicate that factors other than correlations between economic factors are at play.

4.5.1 The effect of model parameters

Several points emerged from the preparation of Table 4 and of plots such as those in Figures 1 and 2.

- The LOESS F-test parameter (d_{L} in Section 3.4.4) is very influential in determining the correlation significance in the nonlinear multivariate regressions. It should be set to reflect the number of parameters needed to define the characteristics of the curve fitted. That is not possible for a nonparametric fit such as LOESS. An alternative characterization is to consider the number of turning points on the fitted curve, which is also difficult for LOESS. The choice of the number of economic factors is a reasonable approximation. Increasing d_{L} decreases the F-value and increases significance levels.
- More significant correlations can be generated by decreasing the LOESS span parameter. A value near zero implies a near-to-exact fit with emphasis on outliers, at the expense of smoothing. The value used ($\frac{2}{3}$) was a compromise between smoothing and reflecting the profile of the data.
- The linear and nonlinear models do not always agree on key decision points. Table 4, for example, shows disagreements on significance and trend.
- Negative confidence bounds indicate excessive uncertainty in the fits where they occur.

With a suitable choice of regression model, manipulation of model parameters can cause the significance of some results to be reversed. Particular examples are IF, EPWS and EDPM, which appear to be significant if unsuitable parameter values are set.

4.6 Frequency correlation results

A noticeable different between linear severity and frequency correlations is apparent. Multivariate results in the latter case are shown in Table 10. Significant frequency correlations are much more numerous, although it is curious that the pattern is different in the ACS case. The explanation lies in risk management. Risk mitigation and data collection methods improved greatly between 2012 and 2020, such that loss frequency decreased over this period. When such a trending series is matched with another trending series (including many of the BoE economic series), significant correlations are expected.

Table 11 summarizes the corresponding univariate frequency correlations. Again, there are many more univariate correlations in the case of frequency than in the case of severity (compare with Table 2). In both the Base and the ACS cases, most significant correlations appear in risk classes other than CPBP and DPA. They persist in the All aggregated risk class, but not in non-CPBP. The reasons given above apply in these cases too.

4.7 Persistence results

With the All risk class, the results in Table 12 were obtained. The table shows a striking distinction between correlation persistence for loss severity and for loss frequency.

Only a small percentage of severity correlations persist over the historical data period, whereas a high percentage of frequency correlations do. Correlations therefore depend heavily on the time period selected. There was no discernible pattern for the distribution of correlations among the economic factors. Many of the historical economic time series show a marked trend with respect to time, as do the aggregated loss frequency time series. In contrast, the aggregated severity-based time series do not. The results obtained can be explained by noting that associating two trending series inevitably results in a significant correlation.

4.8 The effect of Covid-19

The BCBS issued further guidance on measuring the resilience of the banking system to economic shocks, largely in response to the Covid-19 pandemic (Coelho and Prenio 2020). While the general measures proposed (robust risk management,

TABLE 10 Multivariate linear frequency correlations.

Risk class	Base R-value	Base p-value	ACS R-value	ACS p-value
IF	0.8768	0.0001***	0.8325	0.0011***
EF	0.8304	0.0013***	0.8068	0.0043***
EPWS	0.7588	0.0269**	0.568	0.6435
CPBP	0.816	0.0027**	0.5454	0.7307
DPA	0.7262	0.0678*	0.5866	0.5646
BDSF	0.866	0.0001***	0.7478	0.0377**
EDPM	0.7268	0.0668*	0.5931	0.5358
Non-CPBP	0.8992	0***	0.5583	0.6824
All	0.8291	0.0014***	0.5526	0.7042

***1% significance. **5% significance. *10% significance.

TABLE 11 Univariate linear frequency correlations.

Significance level (%)	Number of correlations: Base	Number of correlations: ACS
Not significant	31	40
5–10	5	6
1–5	5	5
<1	64	54

TABLE 12 Base one-year projections, upper confidence bounds and lower confidence bounds.

Significance level (%)	Severity	Frequency
<90	89	31.6
90	8.1	7.5
95	2.6	11.6
99	0.3	49.3

Entries are expressed as percentage changes relative to the mean value of historical losses for the most recent five years.

anticipation of the capital requirement, vulnerability assessment, etc) are sensible, the guidance remains notable in that it continues to not say how capital should be calculated.

Although the Covid-19 pandemic represents a very severe economic downturn, indications at the time of writing are that it will not inflate OpRisk losses. Risk.net (2020) reported on ORX data for the first half of 2020, showing that total OpRisk losses for financial firms had halved year-on-year (US$14.28 billion in 2019 and US$7.93 billion in 2020). Loss frequency also decreased year-on-year, by 35%. Notably, EF (mainly phishing) accounted for 55% of the total severity, but with reduced frequency. The most likely explanation for such a marked reduction in total OpRisk losses (in terms of both severity and frequency) is reduced transactional activity.

Aon (2020) provided a forecast for the total effect of the pandemic on 2020 OpRisk losses. Aon reported a significant correlation (with significance greater than 99%) between OpRisk losses greater than £1 million and the Chicago Board Options Exchange's Volatility Index (VIX), using a linear regression model. The intuition supporting this connection is that the volatility of financial markets tends to react rapidly in the event of crises. A doubling of OpRisk losses was predicted for 2020 – an implausible forecast, when viewed in light of observations for the first half of 2020. Given that these observations instead suggested a likely halving of OpRisk losses, this is another instance of the dangers of forecasting using regression.

Informally, for the data used in this study we noticed slight increases in EF due to scams and in CPBP due to customer complaints that internet banking has failed. There were EDPM costs due to additional provisions for working from home and BDSF costs due to additional protection measures. The result was a 25% increase in BDSF regulatory capital and little change in the regulatory capital for other risk classes. Total regulatory capital fell marginally when compared with pre-Covid-19 levels. That result was somewhat surprising, and it can be attributed to reduced economic activity. Overall, the main impact of Covid-19 was on credit risk, through impaired loans. These observations are consistent with those of KPMG (2020). KPMG noted that 28% (ie, a minority) of financial institutions experienced an increase in OpRisk frequency. Only 15% reported an increase in severity. More recently, the European Banking Authority issued guidance on specific Covid-19 items that should be included in OpRisk losses (European Banking Authority 2020). They recommended amortization of some provisions.

ORX data for 2021 did not differ significantly from previous years, indicating no large increases in Covid-19-related operational risk costs. We have noted two small increases in regulatory capital. The first was for EPWS, due to installation of protective screens in bank branches. The second was for BDSF, mainly because of staff shortages. The main impact of Covid-19 was on credit risk, where provisions for deferred loans had to be made.

TABLE 13 Commodity–operational risk correlations: percentages observed at the significance levels indicated.

Significance level (%)	% of significant correlations
<90	63.6
90	12.7
95	12.5
99	11.3

4.9 Inappropriate correlations results

The World Bank commodity database includes, for example, crude oil, bananas, silver, coffee, meat, cocoa and groundnuts. None have a clear economic link to OpRisk. Pairing each of the 71 commodities in the database with each of the seven BCBS risk classes listed in Section 1.3 resulted in 497 combinations to test. Risk classes All and non-CPBP were excluded so as to avoid double-counting. The results are shown in Table 13.

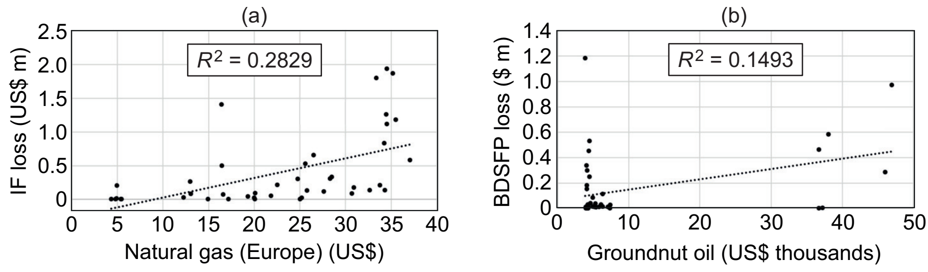
With a random choice of regressors, the expected figures at the significance levels indicated in Table 13 would be 84%, 10%, 5% and 1%. The variety and number of commodities in these lists suggests that there might be some sort of underlying factor, as yet unknown, that results in the correlations observed. Possibly there is a climatic reason. However, the observed results can more easily be explained by noting the following two points.

First, some commodity series show a trend. They would be expected to be significantly correlated with operational loss frequency, but not necessarily with severity, since loss frequency shows a trend but loss severity does not. IF, for example, shows a significant (at 99%) trend over time. Notably, 39 out of 71 commodities are correlated at 95% or more with IF, including bananas, rubber and tobacco.

Second, the BDSF series has a marked data concentration of low-value losses. This concentration can influence regressions considerably. BDSF is significantly correlated at 95% or more with 28 out of 71 commodities (notably phosphate and Colombo tea). No significant (at 90% or more) correlations were found for risk classes All or non-CPBP. Correlations are time-dependent, and differences were noted for 2019. Figure 4 shows a pair of regressions that illustrate the above points.

Significant correlations also result from pairing the IF data with any random time series that shows a significant trend. If the trend is removed, significant correlations disappear. This random-data test is an excellent illustration of the point that sensible regressors should be used, not just in the context of financial risk but also in any context.

FIGURE 4 Examples of loss–commodity significant correlation types.



(a) Two trending series. (b) Data concentration.

4.10 Further issues in OpRisk correlation

Grimwade (2018) has noted qualitatively that OpRisk losses can be correlated with market and credit risks. In turn, market and credit risks are much more affected by economic shocks, since they typically relate to the provision of products and services to clients in exchange for fees. Consequently, clients are then exposed to market and credit risks, which can result in financial loss. If there is any conduct or technical issue with those goods and services, CPBP and/or EDPM losses may increase. Fraud (IF and EF) is harder to explain. It may be loosely related to the business cycle. Anecdotally, fraud for some banks reduced during the 2007–9 financial crisis and increased marginally in 2020 during the Covid-19 pandemic. An analysis of causal effects linking economic factors and OpRisk losses is given in Grimwade (2020), although there is no formal correlation analysis.

Risk classes DPA and BDSF appear to be largely unaffected by economic cycles or shocks. Those risk classes are mainly driven by local events (accidents, power failure, etc).

5 A PROPOSAL FOR AN ALTERNATIVE STRESSED CAPITAL CALCULATION

In this section we summarize a way to make stress test predictions that is not dependent on correlations but is dependent on economic data. Specifically, the stress test is consistent with movements in economic time series. The idea is to compare predicted regulatory capital in stressed and unstressed conditions. This proposal is described in detail in Mitic (2021), where it is called the forward stress framework (FSF) method. Its premise is that financial institutions should accumulate capital in times of economic stress (and really, also in benign economic times) in order to prepare for unexpected losses. The implication is that it is reasonable to assume that economic conditions affect OpRisk, even if correlations are not apparent. The idea that capital should be used for any pressing purpose, even if it was not originally intended for that purpose (eg, capital that is nominally for OpRisk could be used to offset credit losses), is perhaps controversial. Credit losses are likely to be particularly significant as a result of the Covid-19 pandemic.

The FSF method works by tracking movements in economic factors as time advances, and it calculates an increase in capital accordingly. The movements in economic factors work in the following way. If $S_{i,t}$ is the value of economic factor i at time t, a risk factor

$$\lambda_{i,t} = 1 + \frac{S_{i,t} - S_{i,t-1}}{S_{i,t-1}}$$

can be calculated. A total risk factor λ_t then follows as a product of N such risk factors: $\lambda_t = \lambda_{1,t} \times \lambda_{2,t} \times \cdots \times \lambda_{N,t}$. In practice, an upper bound on $\lambda_{i,t}$ is necessary to prevent its value from "exploding". Mitic (2021) suggests an upper limit of 3, and that any such boundary should be subject to the discretion of the practitioner. The BoE economic data indicate that the individual contributions $\lambda_{i,t}$, $i = 1, \dots, N$, are actually much smaller. The values of λ_t are then used in the algorithm below.

The principal part of the calculation is a sequence of iterative operations, divided into two steps. First, let the interval $(t, t + l)$ define a window of length l quarters starting at time t. The first window covers the period $(0, l)$, and the window advances sequentially by one quarter at a time until, finally, no more historical data were available. In each of these windows a distribution can be fitted. A sequence of distribution parameters for windows $1, 2, 3, \dots$ is therefore built using historical data, and those parameters are then used to calculate capital for each window of length l. Using distribution parameters in this way allows the simulation of future distributions with associated capital calculations.

Second, the windows of length l advance into the future, and appropriate distribution parameters are estimated. Capital can be calculated and stressed using those parameters. Suppose that there are t historical windows, and that we want to project losses T' periods into the future. It is assumed that future estimates of economic factors are available, if needed. The sequence below gives some details on this second step.

STEP 1 (Using historical data only.) For times t from 1 to $T - l + 1$,

(a) using data for times t to $t + l - 1$, calculate distribution parameters p_t.

STEP 2 (Using historical and simulated data.) For times t from $T - l + 2$ to $T + T' - l + 1$,

(a) use all prior distribution parameters to estimate distribution parameters p_{t+1} for the last period $t + l - 1$ in the window;

(b) at time $t + l - 1$ draw a random sample, L_{t+1}, equal in size to the number of losses in the prior period $t - 1$, using p_{t+1} (these are the unstressed losses);

(c) apply stress to the sampled losses using λ_t, giving stressed losses L_{t+1}, where $L'_{t+1} = L_{t+1}\lambda_t$;

(d) calculate capitals for the period t to $t + l - 1$ for both L'_{t+1} and L_{t+1}.

The final results are the estimated stressed capitals for periods $T + 1$ to $T + T'$. Figure 5 shows the capital amounts for a sequence of successive five-year windows using historical data (step 1 above), along with predictions and confidence bounds (of one standard deviation) for the next five windows (step 2). The plots indicate the level of capital that should be set under the economic conditions specified. The Base and ACS economic scenarios are shown (losses L'_{t+1}), along with the "no stress" scenario (losses L_{t+1}).

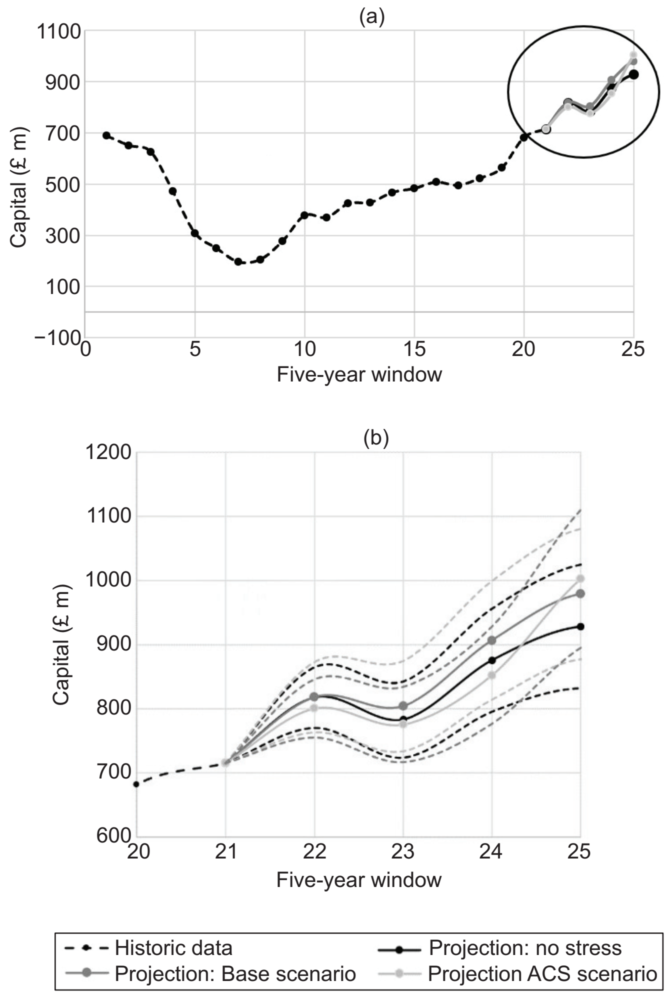
The plots show that both scenarios have little initial effect on the "no stress" projection. The Base scenario steadily shows increased capital relative to "no stress" as time advances. The ACS has a much greater effect than the Base scenario after the first year. Thereafter, the ACS effect becomes increasingly pronounced. Interestingly, the ACS predicts reduced capital in the first year. The confidence bounds are wide, but they can be used to vary the degree of capital held. For example, if it is felt that more capital should be retained, the upper bound could be used to calculate it.

This method is very flexible because any appropriate data series can be used to calculate stress factors. These can range from data that target particular risks (eg, cyber crime) to user-defined scenarios. Stress factors that target particular losses (eg, the largest ones) can be defined. The method also supports stresses to loss frequency as well as to loss severity.

6 DISCUSSION

The need for banks to acquire adequate reserves to be resilient to economic stress is clear. Indeed, it is a regulatory requirement. We have proposed in this paper that financial (principally market and credit) risk is affected by economic stress much more than nonfinancial (specifically operational) risk. Further, we suggest that it nevertheless makes prudential sense to also boost OpRisk reserves in times of stress. However, doing so by using regressions and correlations should take into consideration local conditions. The reasons are summarized below.

FIGURE 5 FSF method: five-year historical capitals, and future projections with confidence bounds (of one standard deviation) for five quarters (enlarged in part (b)).



(1) There is a lack of statistically significant correlations, and where they do occur, they may not persist with time.

(2) Considerable model risk is induced by the statistical methods used in correlation and regression analyses. We have highlighted particular dependencies on the linearity of the model used, and on the use of a nonparametric model (LOESS) to avoid dependence on any particular nonlinear relationship.

Within any given model, parameter selection can influence results greatly. LOESS analyses are vulnerable to the span parameter. In addition, if single economic factors are paired with single risk classes, correlation results can be contradictory.

(3) Predictions indicating lower or even negative capital reserves show that regressions should not be used as predictors. This is particularly true if predictions long into the future are needed. Use of a *de minimis* prediction is a possibility, although what that value should be is unclear.

(4) Wide error bands for predictions show that those predictions are not reliable.

(5) Any factor (economic or otherwise) that is chosen to be associated with OpRisk should be appropriate. The danger of choosing an inappropriate factor is well illustrated by an association with a trending random data series (mentioned in Section 3.7). Any conclusion derived from such an association would be false.

A major consequence of this summary is that the US CCAR stress test process, which relies on the existence of correlations (see Section 2.3.4), may not be applicable to other contexts. If correlations are missing or weak, an alternative must be found. Possibly, a proxy could be used to calculate a missing correlation. In an alternative approach, data accumulated nationally could be used to allocate capital requirements to individual banks, provided that the allocation respects the individual risk profiles of those banks. In that way it would provide an incentive for banks to mitigate risk.

The results of the predictions in Sections 4.2 and 4.4 show that making predictions for more than one year ahead is unreliable. In particular, the error margins for univariate predictions diverge rapidly over time. Predictions are ultimately based on regressions, which can be calculated for any data set without necessarily considering any significance calculation for the data. Given the volatility of OpRisk data, predictions too far into the future (ie, more than one year) should not be made, despite the BoE's directive to do so.

Using economic data as regressors also has its problems, as highlighted in Section 4.9. Regressors have to be chosen carefully, preferably with a causal basis. Choosing irrelevant regressors is easy to do, but the outcome depends on whether the time series used express trends. It is possible that some causal factor could be found to link changes in commodity prices with OpRisk losses, although it seems unlikely. Certainly, using random data exposes the weakness of reliance on regressions alone.

With the premise that it makes sense for banks to retain more capital in stressed economic times, we argue that it need not be calculated using correlations between

OpRisk and economic factors. Work on an alternative was introduced in Section 5. The principle is to mirror changes in economic conditions with corresponding changes in OpRisk capital in a statistically sound way. The main advantage of the proposed method is that it does not rely on correlations but still acknowledges that OpRisk capital should be inflated as a prudential measure in stressed economic times. It does so by mirroring the degree of economic stress. In principle, the question "What conditions affect OpRisk losses?" should be examined in detail. If a causal pathway linking factors such as economic conditions to OpRisk losses could be found, the case for assessing OpRisk stress on the basis of those factors would be strengthened considerably.

Finally, we summarize the main factors that can affect correlation analyses:

- geographical location;
- the size and type of the financial institution analyzed;
- the period covered by the data;
- data transformations and selection methods for economic factors;
- the statistical model used; and
- data preparation, including date selection (accounting, occurrence or discovery), aggregations, cleaning and collection thresholds.

DECLARATION OF INTEREST

The author reports no conflicts of interest. The author alone is responsible for the content and writing of the paper.

REFERENCES

Abdymomunov, A. (2014). Banking sector operational losses and macroeconomic environment. Working Paper, Social Science Research Network (https://doi.org/10.2139/ssrn.2504161).

Abdymomunov, A., Curti, F., and Mihov, A. (2020). US banking sector operational losses and the macroeconomic environment. *Journal of Money, Credit and Banking* **52**(1), 115–144 (https://doi.org/10.1111/jmcb.12661.).

Allen, L., and Bali, T. (2007). Cyclicality in catastrophic and operational risk measurements. *Journal of Banking and Finance* **31**(4), 1191–1235 (https://doi.org/10.1016/j.jbankfin.2006.10.007).

Anderson, L., Hager, D., and Maberg, S. (2012). The financial crisis in an operational risk management context: a review of causes and influencing factors. *Reliability Engineering and System Safety* **105**, 3–12 (https://doi.org/10.1016/j.ress.2011.09.005).

Aon (2020). Covid-19: implications for non-financial risk management. Report, Aon Insights. URL: https://bit.ly/3Efy4Rx.

Axtria (2014). Portfolio stress testing methodologies. White Paper, Axtria. URL: https://bit.ly/3Edy3NP.

Basel Committee on Banking Supervision (2017). Supervisory and bank stress testing: range of practices. Report, December, Bank for International Settlements. URL: www.bis.org/bcbs/publ/d427.pdf.

Bank of England (2018). Financial stability report. Report, June, Bank of England. URL: https://bit.ly/3NiKrzX.

Bank of England (2019a). Financial stability report, Financial Policy Committee record and stress testing results. Web Page, Bank of England. URL: www.bankofengland.co.uk/financial-stability-report/2019/december-2019.

Bank of England (2019b). Stress testing the UK banking system: 2019 guidance for participating banks and building societies. Guidance Document, March, Bank of England. URL: https://bit.ly/3arsCQG.

Bank of England (2020). Interim financial stability report. Report, May, Bank of England. URL: https://bit.ly/3thENWC.

Board of Governors of the Federal Reserve System (2019). Dodd–Frank Act stress test 2019: supervisory stress test methodology. Report, BGFRS, Washington, DC. URL: https://bit.ly/38YZUX6.

Cagan, P. (2009). Managing operational risk through the credit crisis. *Journal of Compliance, Risk and Opportunity* **3**(2), 19–26.

Chernobai, A., Jorion, P., and Yu, F. (2011). The determinants of operational risk in US financial institutions. *Journal of Financial and Quantitative Analysis* **46**(6), 1683–1725 (https://doi.org/10.1017/S0022109011000500).

Coelho, R., and Prenio, J. (2020). Covid-19 and operational resilience: addressing financial institutions' operational challenges in a pandemic. *FSI Brief*, No. 2, April, Bank for International Settlements. URL: www.bis.org/fsi/fsibriefs2.pdf.

Cope, E. W., and Antonini, G. (2008). Observed correlations and dependencies among operational losses in the ORX consortium database. *The Journal of Operational Risk* **3**(4), 47–74 (https://doi.org/10.21314/JOP.2008.052).

Cope, E. W., and Carrivick, L. (2013). Effects of the financial crisis on banking operational losses. *The Journal of Operational Risk* **8**(3), 3–29 (https://doi.org/10.21314/JOP.2013.125).

Cope, E. W., Piche, M. T., and Walter, J. S. (2012). Macroenvironmental determinants of operational loss severity. *Journal of Banking and Finance* **36**(5), 1362–1380 (https://doi.org/10.1016/j.jbankfin.2011.11.022).

Cox, D. R., and Snell, E. J. (1989). *Analysis of Binary Data*, 2nd edn. Chapman & Hall/CRC, Boca Raton, FL.

Curti, F., Migueis, M., and Stewart, R. (2019). Benchmarking operational risk stress testing models. Finance and Economics Discussion Series 2019-038, BGFRS, Washington, DC (https://doi.org/10.17016/FEDS.2019.038).

DeFontnouvelle, P., DeJesus-Rueff, V., Jordan, J. S., and Rosengren, E. S. (2006). Capital and risk: new evidence on implications of large operational losses. *Journal of Money, Credit and Banking* **38**(7), 1819–1846 (https://doi.org/10.1353/mcb.2006.0088).

DeGroot, M., and Schervish, M. (2012). *Probability and Statistics*, 4th edn. Pearson.

Drehmann, M. (2009). Macroeconomic stress testing banks: a survey of methodologies. In *Stress Testing the Banking System: Methodologies and Applications*, Quagliariello, M. (ed), Chapter 3, pp. 37–67. Cambridge University Press (https://doi.org/10.1017/CBO9780511635618.005).

European Banking Authority (2019). 2020 EU-wide stress test: methodological note. Guidance Document, June, EBA. URL: https://bit.ly/3M208Kg.

European Banking Authority (2020). EBA report on the implementation of selected Covid-19 policies. Report EBA/REP/2020/39, EBA. URL: https://bit.ly/3x7Hew9.

Grimwade, M. (2018). An alternative to SMA: using through the cycle loss data to propose an "hourglass" solution. *Journal of Risk Management in Financial Institutions* **11**(4), 361–380.

Grimwade, M. (2020). Ten laws of operational risk. *The Journal of Operational Risk* **15**(3), 1–53 (https://doi.org/10.21314/JOP.2020.244).

Grundke, P. (2011). Reverse stress tests with bottom-up approaches. *The Journal of Risk Model Validation* **5**(1), 71–90 (https://doi.org/10.21314/JRMV.2011.068).

Hess, C. (2011). The impact of the financial crisis on operational risk in the financial services industry: empirical evidence. *The Journal of Operational Risk* **6**(1), 23–35 (https://doi.org/10.21314/JOP.2011.087).

Hu, B., Shao, J., and Palta, M. (2006). PSEUDO-R2 in logistic regression model. *Statistica Sinica* **16**(3), 847–860.

Jacoby, W. (2000). Loess: a nonparametric, graphical tool for depicting relationships between variables. *Electoral Studies* **19**(4), 577–613 (https://doi.org/10.1016/S0261-3794(99)00028-1).

Jobst, A. (2010). The credit crisis and operational risk: implications for practitioners and regulators. *The Journal of Operational Risk* **5**(2), 43–62 (https://doi.org/10.21314/JOP.2010.075).

Klugman, S. A., Panjer, H. H., and Willmot, G. E. (2004). *Loss Models: From Data to Decisions*, 2nd edn. Wiley.

KPMG (2020). Risk and ICAAP benchmarking survey 2020 [Risk functions key to helping firms navigate the Covid-19 pandemic]. Press Release, November 26, KPMG. URL: https://bit.ly/3POwX0u (updated February 5, 2021).

Mitic, P. (2021). A framework for analysis and prediction of operational risk stress. *Mathematical and Computational Applications* **26**(1), Paper 19 (https://doi.org/10.3390/mca26010019).

Moosa, I. (2011). Operational risk as a function of the state of the economy. *Economic Modelling* **28**(5), 2137–2142 (https://doi.org/10.1016/j.econmod.2011.05.011).

Risk.net (2020). Op risk data: losses plummet during lockdown. ORX News Article (ed. L. Marshall), July 14, *Risk.net*. URL: https://bit.ly/3x1pFyX.