# Word by Word: Everyday Math Talk in the Homes of Hispanic Families 

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#### Abstract

Children learn math concepts long before they enter school. Across all cultures, children are exposed to number and spatial language to varying degrees during everyday home routines. Yet most studies of math talk occur in the lab and target nonHispanic, English-speaking families. We expanded inquiry to the spontaneous math language (i.e., number and spatial language) of Spanish-speaking mothers and their 1to 2-year-olds ( $N=50$ ) during home activities. Mothers varied enormously in their use of math language, and mother math language related to toddler math language, whereas mother non-math language did not. Children's math language both preceded and followed mother math talk, suggesting imitation and reinforcement as important processes in children's math language learning. Children also produced math language outside the context of mother input. Findings advance an understanding of children's early math language in natural settings and have implications for interventions aimed at promoting math skills in toddlers from diverse backgrounds.


## 152 Words

Keywords: math cognition, math language, Spanish-speaking families, mother-child communication, language development, dual language learners

## Introduction

Language is a powerful tool for communicating mathematical concepts. Adults regularly talk about quantities, numbers, and spatial relations-how many, how much, where things are, and so on-and children are beneficiaries of such information from early ages. Indeed, children's exposure to math words supports learning in STEMrelated areas (i.e., science, technology, engineering, and mathematics) (e.g., Casasola, 2008; Casasola \& Baghwat, 2007; Nguyen et al., 2016; Rubenstein \& Thompson, 2002), with children differing widely in their math skills already by preschool (e.g., Ramani et al., 2015). Furthermore, children's own use of number and spatial words may mediate the association between math language exposure and math skills at school entry (Levine et al., 2010; Pruden et al., 2011). Such research highlights the need to better understand young children's math language experiences-the number and spatial words that children hear and produce moment-to-moment-in the natural home setting. However, caregivers' spontaneous math language to infants and toddlers remains understudied, with the sparse research in this area targeting non-Hispanic, English-speaking households (e.g., Levine et al., 2010; Pruden et al., 2011).

We advance literature on the social context of early math cognition in two ways. First, we critically document the real-time temporal rhythms of math talk in the home, focusing on children's exposure to and production of math words during unscripted, everyday activities. Our work offers high resolution into the dynamics of mother and child math talk in the moment, and helps illuminate the learning mechanisms that underlie documented links between early math language input and children's later math
skills (e.g., Levine et al., 2010; Pruden et al., 2011). Second, we extend the study of naturalistic math language to Spanish-speaking, U.S. Hispanic mothers and their 1- to 2-year-old children in an effort to replicate mother-child math language associations in samples not typically represented in the literature (e.g., Nielsen et al., 2017). The Hispanic community comprises nearly $20 \%$ of the U.S. population, contains the largest immigrant population in the United States (Pew Research Center, 2019), and is a key community to target for understanding sources of emerging individual differences in math cognition.

## Types of Math Language

Math language falls into two domains, number and spatial language, both of which are foundational to skills ranging from basic addition to physics. Number language includes counting, assigning number-values to sets, and identifying written number symbols (Klibanoff et al., 2006). Spatial language provides information about the intrinsic properties of objects, such as shape names and object features; and the extrinsic relations between objects, such as location (e.g., "behind"), direction ("down"), and orientation ("backwards") (Cannon et al., 2007). Furthermore, both number and spatial language convey concepts of magnitude and comparison through quantifiers (e.g., "more"), spatial dimension words ("big"), and comparatives ("bigger"). Although researchers typically study number and spatial language separately, much of formal math learning requires manipulating numbers to describe spaces, as in geometry and calculus.

Furthermore, some math words offer greater precision in the information they convey than do others. For example, when speaking about location, words such as
under, on top of, and next to provide relatively precise information about where something is or belongs compared to deictics such as here and there, which could refer to any location, depending on context and other cues such as gesture. If a parent instructs a child to put a toy "here," the child must use parental cues such as eye gaze or a point to infer the implied location. Thus, deictics require the listener to draw inferences about the speaker's intention more so than do words that clearly specify quantities, magnitudes, and spatial relations.

## Toddler Math Language

Children use words to express number and spatial concepts well before they count cookies on their plate. Math permeates interactions as basic as requesting more food or asking to be picked up, and children grow in their math vocabularies across the second year as they begin to talk about numbers, quantities, and spatial relations. Already by 12 months, approximately $77 \%$ of children produce recognizable words (Reilly et al., 2009), and the production of math words soon thereafter indicates that math concepts appear among some children's first words. For example, at 16 months, approximately 45\% of Spanish-speaking children say "más" (more), and 25\% of children produce the word "ahí" (there) and count "uno, dos, tres" (one, two, three) (Frank et al., 2021). By 24 months, these proportions rise to $75 \%, 65 \%$, and $65 \%$, respectively. Likewise, even words that only $5 \%$ of children produce at 16 -months, such as "abajo" (down or under), appear in the vocabularies of over $50 \%$ of children by 24 months, highlighting the explosion of math language in the second year.

Nonetheless, averages mask substantial differences among young children in their understanding and production of math words and concepts (Von Hippel et al.,

2018; Xenidou-Dervou et al., 2013). Two- to three-year-old children vary widely in skills of counting, number knowledge, and shape-identification (e.g., Anders et al., 2012; Silver et al., 2021; Suh et al., 2021), suggesting that toddlers build math concepts at home under varying levels of input as early as the second year of life.

## Math Words in the Moment

Young children learn math words as they participate in everyday activities with caregivers (Son \& Hur, 2020; Susperreguy \& Davis-Kean, 2016). Parents introduce children to number and spatial language when counting grapes, naming shapes, and stacking cups. Although all parents communicate math concepts to children, they differ in the frequency of their math talk. For example, during structured play with toddlers, parents produced between 20 and 151 math utterances in fifteen minutes (Ramani et al., 2015). Similarly, parents of 16 -to-30-month-old children ranged widely in their spatial and number talk at home, exposing children to distinct math language environments (Levine et al., 2010; Pruden et al., 2011).

But how might parent math language support child math language in the moment? Nothing is known about how parent-child math talk unfolds during everyday interactions, although it is likely that learning math words relies on general mechanisms of language learning such as contingent responding (Bloom et al., 1974; Goldstein \& Schwade, 2008; Hirsh-Pasek \& Golinkoff, 2012; Rovee-Collier, 1995; Speidel \& Nelson, 2012; Tamis-LeMonda et al., 2013; Tamis-LeMonda et al., 2014). Parents and children engage in responsive exchanges during book-reading and play (e.g., Kurchirko et al., 2018), and if patterns of reciprocal contingency extend to math talk, children may learn math words by having their own math words reinforced, imitating the math words that
adults produce, and responding to parents' math words with new math words. Furthermore, referential clarity in parent talk (i.e., the extent to which the referent of a spoken word is obvious) supports vocabulary development beyond the quantity of parent talk (Cartmill et al., 2013), with joint attention predicting referential clarity (Trueswell et al., 2016). Accordingly, children may identify the referents of math words more readily when those words are embedded in the context of other math talk. For instance, a child might be likely to infer that the unknown word "triangle" refers to an object's shape rather than its color when a mother uses the word in the context of a back-and-forth exchange about shape names. It follows, then, that the real-time coupling of parent and child math language may support math word acquisition at short and long timescales through bidirectional processes.

## Extending the Study of Math Language to a Hispanic Sample

Studies of young children's exposure to and use of math language largely target non-Hispanic White, highly educated, English-speaking samples. In a globalized world, research on child math language learning must extend to families from different cultures and economic strata, including children from U.S. Spanish-speaking immigrant households who may hold unique childrearing practices and beliefs around school readiness (e.g., Sawyer et al., 2021). In addition to culture, the ways that languages express mathematical concepts may influence the way individuals learn math (e.g., Dowker \& Nuerk, 2016), and the Spanish language provides unique features to encode math concepts. Spanish deictic words, for instance, are far more specific than their English counterparts of "here" and "there". Specifically, Spanish offers distance-related gradations of here and there: for instance, "ahi" and "allá" both mean there, but "ahi"
denotes a closer distance and "allá" a farther distance. Spanish also allows for less ambiguity in its use of deictics: Spanish spatial deictic words only refer to location, whereas in English, the words here and there may also be used colloquially to express non-spatial concepts (e.g., "there, there" used for comfort, "listen here", "there we go!", and so on). Therefore, certain categories of math talk may offer different information to Spanish-speaking children than they do to English-speaking children.

Moreover, the scarce research on Spanish-speaking children from low-income households typically compares group averages and identifies disparities relative to nonHispanic White and Asian children (e.g., Sonnenschein \& Sun, 2016). However, striking variation exists in the amounts and diversity of child and caregiver language in Hispanic samples (Escobar \& Tamis-LeMonda, 2017). Whether such variation—and associations between parent and child talk-extends to the domain of math remains unexamined. Investigation of different populations offers a critical test of the generalizability of learning principles across cultures and socioeconomic strata (Nielsen et al., 2017).

## The Current Study

We sought to describe moment-to-moment math talk during naturalistic home activities in Spanish-speaking mother-child dyads with relatively low maternal education, providing a snapshot of these children's earliest interactions around math talk. In doing so, we shed light on the mechanisms behind associations between mother and child math language. Our research extends to an understudied sample of children who on average show delays in STEM skills by school entry (Sonnenschein \& Sun, 2016), yet likely vary significantly in their early math language experiences. Moreover, we move beyond lab tasks that present materials intentionally designed to elicit math talk.

Although maternal speech to children during structured tasks correlates strongly with input during moments of peak language interactions at home, structured tasks reveal little about spontaneous language exchanges across time and routines (TamisLeMonda et al., 2017). Thus, we add to the sparse investigations of spontaneous math language in the home setting (Levine et al., 2010; Pruden et al., 2011; Susperreguey \& Davis-Kean, 2016). Specifically, we asked:
(1) How much and which types of math language do Spanish-speaking mothers spontaneously direct to their 1 - to 2 - year-old toddlers in the home setting? We expected mothers to use deictic words such as "aqui" (here) and "allá" (there) more than precise math language, but to vary considerably at an individual level, as observed in mothers from other samples (e.g., Levine et al, 2010; Pruden et al., 2011).
(2) How much and which types of math language do Spanish-speaking 1- to 2-year-olds use in their first forays into math talk? We expected children to vary in their math talk across age and individuals, but that at this young age, most math talk would consist of deictics.
(3) Does maternal math language relate to toddler math language? And do specific categories of mother math language relate more strongly than others? We expected mothers' total math language to correlate with children's math language during the visit and mothers' precise math language to yield stronger association than deictics (i.e., imprecise math talk). However, we were unsure whether associations would maintain when including mother non-math language in models (i.e., perhaps mothers' talkativeness per se rather than math talk predicts child math talk).
(4) How does math talk between mothers and children unfold in real time? We examined the temporal characteristics of mother-child math utterances (i.e., the latencies and ordering of math utterances in one partner and math utterances in the other). We expected math talk to be reciprocally responsive, with children largely producing math words shortly after mother math utterances and mothers reinforcing their children's math talk with contingent math talk.

## Methods

## Participants

Participants were 50 U.S. Hispanic mothers and their 12-to-26-month-old children ( $M=16.82, S D=3.72$ ), recruited from community agencies and clinics in a large urban city. The majority of mothers spoke only Spanish, with 18 using English to some degree. Mothers averaged $32.40(\mathrm{SD}=4.84)$ years of age; had lived an average of $12.63(S D=5.51)$ years in the United States; and completed $8.71(S D=5.14)$ years of education in their home countries on average. The majority of mothers were Mexican, with two from Guatemala, one from Ecuador, and one from Spain.

## Procedure

A female researcher video-recorded each dyad for one to two hours at home ( $M$ $=1.45, S D=0.27$ ) with a handheld digital camera ( 60 fps ). The camcorder's Wind Shield Zoom Microphone suppressed background noises, and aluminum electrolytic capacitors reduced sound distortion. Visits were scheduled between 8am and 6pm according to mothers' convenience, mostly on weekdays. Most visits lasted at least 1.5 hours; visits ended before the 1.5 -hour mark if the child fell asleep, the mother was
called into work, or the family was unable to continue due to other obligations.
Participants received $\$ 50$ gift cards for their time.
During visits, the researcher asked mothers to ignore her and go about their normal routines. The researcher then followed children with the camera and recorded child behaviors with minimal interference. Children and mothers engaged in activities such as mealtime, playtime, chores, grooming, TV-viewing, and so on. The researcher attempted to keep toddler and mother in the frame whenever possible, focusing the camera on the child if the two split up. Most visits only involved mother and child, but other family members were sometimes present during filming, though largely out of the way. With parent permission, videos of home observations are shared with authorized investigators of Databrary (https://nyu.databrary.org/volume/484).

## Coding of Math Language

Videos were transcribed and coded in Datavyu (datavyu.org), coding software that time-locks utterances to video frames. Doctoral students and senior staff members trained bilingual transcribers to parse language at the utterance level, following conventions of the Codes for the Human Analysis of Transcripts (CHAT) (CHILDES; MacWhinney, 2000). Speech from toys and media was not transcribed. Examination of a random sample of 4 transcripts ( $8 \%$ of the sample) confirmed that the vast majority of utterances were child-directed: out of 6,318 mother utterances, approximately 170 (or $2.7 \%$ ) were directed towards other people. Only 19 non-child-directed utterances contained any math language ( $0.3 \%$ of all utterances). To generate totals for all types and tokens of non-math words, Datavyu transcripts were exported to CLAN.

Researchers coded 7 categories of math language using codes modified from existing math language coding manuals (Cannon et al., 2007; Levine et al., 2010): (1) number, including integers up to a hundred, orders of magnitude such as "thousand" or "million", and number phrases such as "two cookies"; (2) shapes, names of shapes such as "circle", "square", or "triangle"; 3) spatial features and properties, words that describe the features or physical properties of an object or shape, such as "round", "point", or "side"; (4), magnitude and comparison, words that refer to amount, portion, or relative size or quantity, such as "all", "more", "big", and "bigger"; (5) location and direction, words that indicate the relative position of objects and people in space, such as "above", "underneath", and "next to"; (6) orientation, words that refer to the absolute orientation of objects or people in space, independent of other objects, such as "upside down" or "backwards"; and (7) deictics, the concepts of "here" and "there."

After defining math categories and accompanying words, bilingual researchers translated each word into Spanish equivalents. To ensure inclusion of Spanish math words without direct English equivalents, researchers reviewed 8 hours of Spanishlanguage home-visit videos and 74 structured task sessions from other studies in the lab. A computer Ruby script then searched mother and child language for words from the two lists: an English-language list that contained 346 math words and a Spanishlanguage list that contained 483 words. The wider range of word endings in Spanish (e.g., masculine, feminine, singular, and plural endings) resulted in a longer Spanishlanguage list.

Researchers produced a final Math Language Coding Manual (openly shared at https://nyu.databrary.org/volume/1403) that contained, for each category (English and

Spanish): 1) an exhaustive word list, 2) descriptions of inclusion and exclusion criteria, and 3) a comprehensive list of "false alarms", or words that are not math-related in a given context (e.g., "behind schedule"). Trained bilingual coders used the manual to classify script-identified math words into appropriate categories.

To examine inter-coder agreement, a second coder independently scored $25 \%$ of each visit, with reliability segments distributed across the beginning, middle, and end of the visit (e.g., 10 min drawn from each 30 min of a 2-hour session). Cohen's kappa ranged from 0.81 to 1.0 across categories. Disagreements were resolved through discussion, but were rare, with modifications to primary coder data made on only $1.6 \%$ of data. Because automatic computer scripts enabled coders to avoid categorizing utterances without math language, only utterances that contained math language factored into reliability calculations, thus preventing inflation from utterances without math words (such as coders agreeing that no math talk occurred).

## Data Analysis

All measures of mother and child language were pro-rated by visit length. Thus, descriptive results present utterances per hour, word types per hour, and word tokens per hour, and models were based on these per-hour units. A validity check confirmed that all children in the sample used at least one word (math or non-math) during the visit. The five categories of precise spatial math language were grouped into intrinsic and extrinsic words for analyses (see Figure 1). To obtain estimates of non-math language, math words were subtracted from word totals calculated in CLAN. Real-time correspondence between individual mother-child utterances was examined by computing time spans between the onsets of mother and child math utterances. Ten-
second thresholds determined if mother math talk prompted (i.e., came before) or reinforced (i.e., followed) child math talk. Analyses were conducted in R 3.6.3 and SPSS Statistics 27 (scripts openly shared at https://nyu.databrary.org/volume/1403).

We conducted linear and logistic regressions to assess associations between mother and child math talk. Because children's math language data were skewed, the data did not meet assumptions required for linear regressions. We thus based analyses on logistic regressions, which represented child math language as a dichotomous dependent variable ( 0 if a child used no math words during the observation, or 1 if a child used any math words). However, we tested linear regressions by implementing square root transformations of the independent (mother math language) and dependent (child math language) variables. Notably, both analytic approaches yielded similar findings: Results did not change when the child math language variable was treated as continuous (as in the case of linear regressions) or dichotomous (as in the case of logistic regressions). However, to avoid reporting model slopes and intercepts on the transformed scale (which are less interpretable), we mainly report results from logistic regressions, for which data met assumptions (see Table 3 for linear regression results). Because regressions produced similar results whether child math talk included deictics or strictly precise math language, all models include overall child math talk unless otherwise specified. Model interpretations used the $b$-coefficient to calculate the slope of each regression curve at its highest point. The maximum slope gives the maximum effect of the addition of one word on the probability of a child producing math language.

## Results

We first describe the amount, types, and categories of math language produced by mothers and toddlers, including exploratory analysis of differences between mothers who directed some English toward their child during the observation and mothers who did not. Next, we examine associations between mothers' math language input and children's production of math language at the visit level, exploring whether precise mother math talk related more strongly to children's math language production than did mother deictic input. Finally, we investigate the temporal distribution of mother and child math words from moment to moment.

Given the spread of child age, we expected older toddlers to use more math language than younger ones, and reciprocally, mothers' math language to increase with child age. Although child age related to total toddler math types and tokens (r's (48) = $.66, .65, p$ 's < .001), the association attenuated to non-significance ( $p=.88$ ) in logistic regressions that also included parent math talk. All regression models therefore adjust for child age. Contrary to expectations, child age did not relate to mother math types or tokens (r's $(48)=.13, .094, p$ 's $=.38, .51$ ). Mother education did not relate to any measure of mother math talk, as expected given the low and homogeneous education level across the sample (r's (48) $=-.042, .0062 ; p$ 's $=.78, .96$ ). Child sex did not relate to any measure of math talk and was therefore excluded from analyses.

## Mothers' and Children's Spontaneous Math Language

Mothers varied substantially in their use of math language, even within this relatively homogenous, Hispanic, low-educated sample (Figure 1). In aggregate, mothers produced 40,666 utterances, with $19 \%$ of utterances containing some type of
math language (i.e., number and/or spatial words and phrases). Within those utterances, mothers produced a total of 5,967 math words. Individual mothers expressed between 53 and 1,512 total utterances per hour ( $M=566.98, S D=347.92$ ); 4 and 342 math word tokens per hour ( $M=99.45, S D=80.21$ ); and 2 and 31 math word types per hour ( $M=15.55, S D=7.04$ ) (Figure 2a). Mothers' total word tokens per hour and total word types per hour related to mothers' math tokens per hour $(r(48)=0.81, p$ <.001) and math types per hour ( $r(48)=0.82, p<.001$ ), respectively. Thus, more talkative mothers were likely to provide their children with more math language, underscoring the need to test whether mothers' math language specifically or language input generally related to child math talk.
[Figure 1 near here]
[Figure 2 near here]

## Mothers' Math Words by Categories

As hypothesized, mothers used more deictic words per hour ( $M=42.51, S D=$ 36.62) than any single type of precise math language, $p<.001$ (Figure 3). However, collapsing across types of precise language, mothers' total precise language surpassed deictic language, $p<.05$, and also varied substantially, with mothers producing between 1 and 194 precise math words per hour ( $M=56.94, S D=51.23$ ). Notably, mothers who used more precise math words tended to also use more deictic words per hour $(r(48)=$ $0.66, p<.001$ ).
[Figure 3 near here]
Certain types of precise math talk were more prevalent than others (Figure 3), as confirmed by a within-subjects one-way ANOVA on the hourly frequencies of 6
categories of precise math language, $F(6,294)=42.52, p<.001$ (Table 1). Mothers tended to mostly use precise math words to express concepts of number ( $M=22.81$, $M d n=9.13, S D=27.78)$, magnitude and comparison $(M=16.47, M d n=13.18, S D=$ 14.53), and location and direction $(M=14.07, M d n=8.31, S D=14.19)$. Mothers rarely used words for intrinsic spatial concepts (shapes and features and properties) or orientation. Post-hoc comparisons yielded significant differences between location and direction words and each of the three rarest categories, with all greater differences also significant, p's < . 001 (Table 2).
[Table 1 near here]
[Table 2 near here]
In an exploratory analysis, we investigated whether mothers' use of math language differed between mothers who spoke exclusively Spanish or Spanish and English combined, given the unique characteristics for encoding certain math concepts in each language. Of the 50 mothers, $18(36 \%)$ used some English math language with their children. Counter to expectations, the two groups did not differ in education or years lived in the United States ( $p$ 's $=.42, .24$ ). A Wilcoxon rank sum exact test found that mothers who used at least some English math language with their children used more precise math tokens per hour $(M=73.86, M d n=70.45, S D=53.57)$ than did mothers who only used Spanish $(M=47.42, M d n=29.55, S D=48.10), W=389, p<$ .05 , with differences most pronounced for references to shapes and magnitudes and comparisons ( $p$ 's < .001). In contrast, mothers in the two groups did not differ in their use of deictic words per hour, $W=389, p=.83$ ( $M$ 's $=41.13,43.28$ ).

The greater use of precise math words in mothers who spoke Spanish and English to their children came from both languages. That is, the average 26 -math-word advantage by mothers who mixed languages comprised 10 more Spanish math words per hour on average ( $M=57.21, M d n=49.00, S D=50.73$ ), supplemented by 16 English math words per hour ( $M=16.16, M d n=3.28, S D=26.21$ ).

## Toddlers' Math Language

Twenty-three children (46\%) used at least one math word during the visit, collapsing across precise and deictic categories. Figure $2 b$ shows the distribution of children's math types and tokens: children spoke between 0 and 8 math word types per hour ( $M=0.95, S D=1.63$ ) and between 0 and 21 math tokens per hour ( $M=2.21, S D$ $=4.38$ ). Collapsing categories of precise math language, children spoke between 0 and 9 precise math words per hour ( $M=1.26, S D=2.39$ ). Only two children used location and direction words, and children did not use words to refer to orientation or spatial features and properties.

## Associations Between Mother and Child Math Language

## Aggregated Mother Math Language Predicts Child Math Language

As hypothesized, mothers' use of math language related to children's use of math language. Mothers who used above the median level of math language were over three times as likely to have children who used math language than were mothers who fell below the median, $O R=3.60$. In fact, only one of 17 mothers in the bottom third of math language frequency had a child who used math language during the visit, compared to 22 of 33 mothers in the top two-thirds. Logistic regression with child math
language as a dichotomous variable revealed that a one-word increase in a mother's math language tokens per hour increased the chance of a child using math language during the visit by up to $0.55 \%, b=0.022, p<.01$ (Figure 4a). This seemingly small bump to child math language is meaningful considering the range of mother math tokens, from as few as 4 per hour to as many as 342 per hour ( $M=99.45, S D=80.21$ ). For instance, an increase of one standard deviation of mothers' math tokens per hour increased the chance that her child would use math language by $33.91 \%, b=1.36, p<$ .01. Similarly, aggregated measures of mothers' hourly math language types also predicted child math language use during the visit. A one-type increase in a mother's math language per hour increased the chances that a child used math language during the visit by up to $3.5 \%, b=0.14, p<.01$.

To rule out the possibility that general mother talk per se (rather than math talk specifically) predicted child math language use, we examined the association between mother non-math talk per hour and child math language. Results show that mother nonmath language did not predict child math language, $b=0.00053, p=.10$ (Figure 4b). The probability plots $a$ and $b$ in Figure 4 contrast the predictive value of mothers' math tokens per hour versus non-math tokens per hour on child dichotomous use of math language. This contrast suggests that exposure to math language specifically, as opposed to non-math language, supports children's developing vocabularies around mathematical concepts. Furthermore, within dyads, $95 \%$ of math words spoken by children had also been spoken by their mothers, suggesting a direct path from mother to child math vocabularies.

## [Figure 4 near here]

## Contrasting Precise and Deictic Math Talk

Counter to our hypothesis, mothers' precise math talk did not relate more strongly to child math language than did deictic math talk, even when child math language included strictly precise words. Specifically, mothers' precise and deictic talk predicted all types of child math language equally well, as indicated by the $95 \%$ confidence intervals (CI) around the differences between the models' AIC and BIC, Cls [-9.16, 12.74].

The predictive power of mother math language maintained for nearly all subcategories of precise math language. Although maternal extrinsic language increased a child's probability of using math words by up to $1.50 \%$ per additional extrinsic word, $b=0.060, p<.01$, making it a particularly strong predictor of child math language, $95 \%$ confidence intervals around AIC and BIC differences with other models did not improve model fit. Logistic regressions confirmed that nearly all categories of mothers' math language (precise math language, deictic math language, number language, and extrinsic math language) predicted equally well the odds of a child using math language (see model summaries in Table 3). Intrinsic language alone did not predict child math language, likely because of its low frequency.
[Table 3 near here]

## Real-Time Coupling Between Mother and Child Math Utterances

Aggregating across the subset of children who used math language yielded 127 child math utterances for analysis. Consistent with the hypothesis that mother-toddler
math talk would be reciprocally responsive, a significant percentage of child math utterances (65\%) were framed on one or both sides by mother math utterances within a 10 -second window, with $51 \%$ of those child utterances framed on both sides (Figure 5). In contrast, 35\% of child math utterances occurred outside the context of mother math talk. A binomial test indicated that the proportion of child math utterances positioned within 10 seconds of a mother math utterance ( $95 \% \mathrm{Cl}$ [ $0.56-0.74]$ ) was significantly greater than $0.5, p<.001$ (two-tailed). A chi-square test did not find any significant differences in the types of math language used by children in these different contexts ( $p$ $=0.28$ ).

## [Figure 5 near here]

Moreover, latencies between child and mother math language were brief. Of child math utterances that followed mother math language within 10 seconds, $47 \%$ occurred less than 3 seconds after the prior mother utterance; $25 \%$ between three and five seconds afterwards; and 28\% between five and ten seconds afterwards (Figure 6a). The majority of these child utterances ( $80 \%$ ) involved some form of imitation, whether exact ( $60 \%$ ) or conceptual ( $20 \%$; e.g., the child said "circle" after the mother said "square"). Likewise, when mothers reinforced children's math talk, they responded quickly. Of mother math utterances that followed children's math language within 10 seconds, $76 \%$ fell within 3 seconds; $9 \%$ within 3 to 5 seconds; and $15 \%$ within 5 to 10 seconds (Figure 6b).
[Figure 6 near here]
Notably, however, children also used math language outside the context of mother math language, with $35 \%$ of child math talk occurring outside a 10 second
window of mother language. Indeed, most children (65\%) who used math language did so spontaneously at least once (i.e., distant from mother math language or prior to mother math language).

## Discussion

Already in the second year, children experience dramatically different mathlanguage environments that play out in their math language production. Children from Spanish-speaking families of relatively low education are no exception. Mothers' math talk ranged widely and uniquely predicted children's math talk, which was largely "sandwiched" between mothers' math utterances, suggesting processes of imitation and reinforcement in math language learning. Nonetheless, children's spontaneous use of math language suggests that toddlers' productions did not entirely depend on mothers' input. Real-time analysis of math language in a Hispanic sample adds to our general understanding of math language input and its social contexts, while extending that knowledge to an understudied group of families.

## No Two Mothers Are Alike

Variability in the math language of Spanish-speaking mothers mirrored that of English-speaking mothers (Levine et al., 2010). Some Spanish-speaking mothers used as few as 5 math words per hour, whereas others used well over 300 , which cautions against homogenizing the language experiences of children from U.S. Hispanic households. Mothers also differed by language characteristics, with mothers who used some English directing more math words to their children than mothers who used strictly Spanish. However, English likely served as an indicator and not a cause of higher math
language production in mothers, since mothers who used English produced far more of their math language in Spanish. Findings thus spur questions on the sources of variation, which in this case, cannot be attributed to mothers' education, ethnic background, or even geographic region.

Mothers' beliefs about math and parenting, and the materials present in the home environment, may contribute to variation in math talk. Parents' beliefs (e.g., feeling positively about math, viewing themselves as role models) and expectations about children's math skills relate to preschoolers' math abilities through effects on engagement in math activities (e.g., Sonnenschein et al., 2012; Missall et al., 2015; Silver et al., 2021; Vasilyeva et al., 2018). Accordingly, the presence of English in Spanish-speaking households may indicate values around education: Perhaps mothers who make an effort to use English in the home also intentionally engage in behaviors aimed at preparing their children for school, such as by using math language. Furthermore, the unique ways that mothers leverage object affordances for math talk may contribute to individual differences in math language. Caregivers can exploit opportunities to engage in math talk during block and puzzle play (Ramani et al., 2015), but also during everyday activities such as counting socks and remarking that objects are "under" the table or "inside" pots.

## Mother Math Talk Specifically Supports Child Math Talk

Mother math language input, but not non-math talk, related to child math language production even at this early age in a diverse sample, validating findings from the field's seminal investigation of parent-toddler math talk in a naturalistic setting (Pruden et al., 2011; Levine et al., 2010). The unique importance of math words echoes
associations for other specific types of language (e.g., Huttenlocher, 1998; Huttenlocher et al., 1991). Spanish-speaking children who were frequently exposed to math language often produced math words themselves, suggesting that children who hear math words build math concepts at very young ages. In fact, parent math language overshadowed child age when analyzed in models predicting child math talk. Although this may seem surprising, lab interactions that comprise most existing math talk studies likely amplify language-and therefore the amount of math language-produced by parents and children, which may magnify age differences. In everyday contexts at home, the realtime expression of math language by children at these young ages may depend more on the activity context in which they're embedded.

The connection between mother and child math language has implications for policies around early learning. Attention to the language experiences of toddlers may help identify children at risk of falling behind in math cognition, and offer opportunities to educate and intervene with caregivers. Caregivers should be encouraged to leverage everyday opportunities to engage in math-related exchanges, particularly as parents may be unaware that certain types of talk (e.g., location words) support math cognition already in toddlerhood.

## Precise and Deictic Math Talk

We expected precise math talk to be especially predictive of child math language. However, deictic words predicted children's math language (even when child math talk included strictly precise language) as well as did precise math words. Children may benefit from mothers alternating between precise language and deictic language, with deictic language aiding toddlers' learning of more complex math terms. For
example, mothers may bracket specific instructions ("put it inside the circle hole") with general directions ("put it here."). We speculate that as long as deictic language does not replace instances of precise language, imprecise locatives such as here and there may foster familiarity with basic math concepts and bootstrap (Carey, 2004) children's learning of more complex and precise math words. Indeed, the relatively strong association between mothers' use of deictics and precise math language in this sample suggests that they were mixing the two, perhaps in complementary ways.

Deictic math language may also be particularly informative for Spanish-speaking children because of the greater variety of concepts such words encode compared to English deictic language. For example, words such as "ahí," "alli," and "allá" indicate gradations of distance, with "ahi" being the closest and "allá" the furthest. Similarly, Spanish divides the concept of here into two words, "aqui" and "acá", the former denoting stationary locations ("the book is here") and the latter indicating movement and nearly always attached to verbs ("come here"). Indeed, mothers in the sample used all of the above variations in their interactions with toddlers. Therefore, deictic words may provide more spatial information in Spanish than in English, supporting Spanishspeaking children's early math learning, particularly when accompanied by specific location words.

## The Social Setting of Child Math Talk

Children used math language in varied social situations, sometimes capitalizing on math language input and sometimes eliciting mother responses. Real-time analyses of mother and toddler math talk focused on children who produced math language, and
future research could increase power by targeting a larger sample of children who produce math language. However, the patterns identified here are informative as each child contributed multiple instances allowing for real-time analyses on 127 child math utterances. Findings show that dyads engaged in a significant amount of sandwiching-two-way coupling in which child responses to mother math talk prompted further mother math talk. Children's imitation of math words offers them opportunities to practice and model math language, and mothers' reinforcement of children's math language provides additional math input. The bidirectionality of math language exchanges suggests contingency in interactions around math concepts and highlights toddlers' active role in responding to the math language of parents while also eliciting responses from them (Begus \& Southgate, 2018; Kuchirko et al., 2018; Trautman \& Rollins, 2006).

Notably, however, children also produced math words outside the context of mother math input, emphasizing toddlers' independence in the learning process (e.g., Lockhart, 2011; Begus \& Southgate, 2018; Tamis-LeMonda et al., 2018). Learning mechanisms based on modeling, imitation, reinforcement, and independent practice play out moment-to-moment and may illuminate how children acquire and use words to express math concepts during everyday interactions with caregivers.

## Conclusions

Galileo famously said that, "The Book of Nature... is written in mathematical language, and its characters are triangles, circles, and other geometric figures... without these, one is wandering in a dark labyrinth." Navigating the labyrinth of math cognition begins at birth. Infants stand up, outgrow clothes, and retrieve balls lost under couches. But children do not embark on this journey alone: Caregivers accompany toddlers
through the early years of math learning by offering language to frame children's experiences and map words to concepts. Math learning is a dance between partners, and ultimately, toddlers inhabit vastly different math language environments that shape the course of their learning. Even within a relatively homogenous immigrant Hispanic community, parents and children bring unique tools for math learning to the table.

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## Declaration of Interest Statement

The authors whose names are listed below certify that they have NO affiliations or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers' bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript.

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## Data Availability Statement

The data that support the findings of this study are openly available on Databrary at https://nyu.databrary.org/volume/1403.

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Table 1
Mother Math Language by Category

|  | Mean (SD) | Range |
| :--- | :--- | :--- |
| Number | $22.81(27.78)$ | $0-106.80$ |
| Shapes | $2.37(5.49)$ | $0-26.39$ |
| Magnitude \& Comparison | $16.47(14.53)$ | $0-59.85$ |
| Location and Direction | $14.07(14.19)$ | $0.65-68.62$ |
| Orientation | $0.34(0.74)$ | $0-3.61$ |
| Features \& Properties | $0.68(1.56)$ | $0-7.09$ |
| Deictics | $42.51(36.62)$ | $2.86-182.86$ |

Note. Mean, standard deviation, and range of word tokens among mothers in each math language category.

## Table 2

## Mothers' Most Frequently Produced Math Words

Location \& Direction
Magnitude \& Comparison
Orientation

|  | \# | \% |  | \# | \% |  | \# | \% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| En (in) | 377 | 88 | Más (more)* | 485 | 86 | Vuelta (around) | 12 | 12 |
| Arriba (up/above) | 132 | 48 | Todo (all) | 166 | 70 | Revés (backwards) | 12 | 12 |
| Abajo (down) | 85 | 42 | Mucho (lots) | 119 | 60 | Backwards | 1 | 2 |
| Adentro (inside) | 70 | 38 | Poco/chiquito (small) | 97 | 54 |  |  |  |
| Afuera (outside) | 49 | 38 | Grande (big) | 95 | 44 |  |  |  |

Shapes
Features \& Properties
Number

|  | \# | \% |  | \# | \% |  | \# | \% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Estrella (star)* | 42 | 22 | Lado (side) | 33 | 22 | Uno (one)* | 768 | 90 |
| Círculo (circle) | 27 | 14 | Punta (point) | 12 | 10 | Dos (two)* | 306 | 66 |
| Cuadrado (square)* | 22 | 10 | Redondo (round) | 6 | 6 | Tres (three)* | 186 | 58 |
| Triángulo (triangle) | 22 | 8 | Esquina (corner) | 2 | 4 | Cinco (five) | 58 | 40 |
| Bola (sphere/ball) | 17 | 20 | Ovalado (oval) | 1 | 2 | Cuatro (four) | 53 | 36 |

Deictics

|  | \# | \% |
| :---: | :---: | :---: |
| Aquí (here)* | 1436 | 96 |
| Ahí (there)* | 653 | 82 |
| Allá (there)* | 326 | 90 |
| Allí (there)* | 291 | 48 |
| Acá (here)* | 281 | 82 |

Note. Table 2 lists the five most common math words in each category of math language, with English translations in parentheses when applicable. For each word, the table records the raw number of times the word was used across the sample, and the percent of mothers who used the word at least once. Asterisks indicate words also used by children.

## Table 3

Logistic and Linear Regression Models: Predicting Child Math Language

| IV | DV | b-value | p-value | AIC |
| :--- | :---: | :---: | :---: | :---: |
| Mother non-math tokens | Child Math Lang | 0.00053 | n.s. | 70.11 |
| Mother math tokens | Child Math Lang | 0.022 | $<.01$ | 58.21 |
| Mother math tokens, standardized | Child Math Lang | 1.36 | $<.01$ | 58.21 |
| Mother math types | Child Math Lang | 0.14 | $<.01$ | 63.44 |
| Mother precise tokens | Child Math Lang | 0.022 | $<.01$ | 61.58 |
| Mother deictic tokens | Child Math Lang | 0.041 | $<.01$ | 58.63 |
| Mother extrinsic tokens | Child Math Lang | 0.060 | $<.01$ | 57.75 |
| Mother intrinsic tokens | Child Math Lang | 0.023 | n.s. | 72.75 |
| Mother number tokens | Child Math Lang | 0.030 | $<.05$ | 66.61 |
| Child age | Child Math Lang | -0.031 | n.s. | 72.83 |
| Mother math tokens (linear reg) | Child Math Lang | 0.15 | $<.001$ | 156.32 |

Note. Regression models (i.e., independent and dependent variables in each model), results, and model statistics. The inclusion or exclusion of deictic language in the dependent variable of child math language did not change results; therefore, results are reported from models in which the dependent variable included all child math language.

## Figures

Figure 1
Individual Differences in Mothers' Math Language


Figure 2
Math Types and Tokens by Mothers and Toddlers


Figure 3
Frequencies of Mother Math Language Categories


## Figure 4

## Predicting Child Math Talk: Mother Math Language vs. Non-Math Language



Figure 5
Social Contexts of Child Math Language


Figure 6
Time Between Child Math Utterances and the Nearest Mother Math Utterances



## Figure Captions

## Figure 1

## Individual Differences in Mothers' Math Language

Note. Number of math words (tokens) produced by mothers per hour. Each bar represents a mother, with categories of math language denoted by bar colors. Black stars atop bars indicate dyads in which children also used math language during the visit.

## Figure 2

## Math Types and Tokens by Mothers and Toddlers

Note. Violin plots display distributions of mother (Plot A, left) and child (Plot B, right) math word types and tokens. The area of each plot visualizes the changing density of values across the range, with the widest point of the plot signifying the highest density of points. Horizontal lines denote mean values. All Y-axes show frequencies per hour.

## Figure 3

## Frequencies of Mother Math Language Categories

Note. Differences in math category usage among mothers quantified as number of math tokens by category per hour. Boxes denote medians and first and third quartiles, while lines indicate the range of values from minimum to maximum. Black dots signify outliers.

## Figure 4

Predicting Child Math Talk: Mother Math Language vs. Non-Math Language

Note. Estimated probability, ranging from 0-1, of child producing math language as a function of mother math ( A ) and non-math tokens ( B and C ) per hour. Panel A shows a strong association between mother math language and the likelihood of child producing math language. Panels $B$ and $C$ show a lack of association between mothers' non-math language and the likelihood of child producing math language. Blue lines represent the best estimate of the association; gray areas around the line indicate $95 \%$ confidence intervals. Panel C zooms into panel $B$ to equalize the range of words represented in panel A , accurately comparing associations from mothers' math and non-math language.

Figure 5

## Social Contexts of Child Math Language

Note. Timelines illustrate the distribution of child (red) and mother (green) math utterances across time, featuring the seven participants with the highest child math language. The blue box features a 5 -minute excerpt from one participant's timeline, zooming in on the three contexts of child math talk in higher resolution: two-way reciprocity, isolation, and one-way reciprocity.

## Figure 6

## Time Between Child Math Utterances and the Nearest Mother Math Utterances

Note. Time spans (in seconds) between each child math utterance and its closest prior mother math utterance (panel A). Time spans between each child math utterance and its closest subsequent mother math utterance (panel B). In both graphs, the few outlier utterances with latencies over five hundred seconds are grouped into $500+$ bins. The
right-skewed distribution shows that most pairings of child and mother math language fell within a few seconds of one another.

