Distributed Model Predictive Control for Heterogeneous Vehicle Platoon with Inter-Vehicular Spacing Constraints

Zhiwen Qiang, Li Dai*, Boli Chen, Yuanqing Xia

***Abstract*—This paper proposes a distributed control scheme for a platoon of heterogeneous vehicles based on the mechanism of model predictive control (MPC). The platoon composes of a group of vehicles interacting with each other via inter-vehicular spacing constraints, to avoid collision and reduce communication latency, and aims to make multiple vehicles driving on the same lane safely with a close range and the same velocity. Each vehicle is subject to both state constraints and input constraints, communicates only with neighboring vehicles, and may not know a priori desired setpoint. We divide the computation of control inputs into several local optimization problems based on each vehicle's local information. To compute the control input of each vehicle based on local information, a distributed computing method must be adopted and thus the coupled constraint is required to be decoupled. This is achieved by introducing the reference state trajectories from neighboring vehicles for each vehicle and by employing the interactive structure of computing local problems of vehicles with odd indices and even indices. It is shown that the feasibility of MPC optimization problems is achieved at all time steps based on tailored terminal inequality constraints, and the asymptotic stability of each vehicle to the desired trajectory is guaranteed even under a single iteration between vehicles at each time. Finally, a comparison simulation is conducted to demonstrate the effectiveness of the proposed distributed MPC method for heterogeneous vehicle control with respect to normal and extreme scenarios.**

***Index Terms*—Model predictive control, distributed control, heterogenous platoon control, coupled constraints**

I. INTRODUCTION

The platoon of vehicles has been developed throughout the world for several decades for its significant potential to benefit road transportation, including increasing the capacity of highways, enhancing road safety and improving energy efficiency, etc [1]–[6]. To achieve these benefits, appropriate platoon control algorithms are required to enable a group of vehicles to travel at the same speed while maintaining a pre-specified constant headway between any two consecutive vehicles [3], [7].

Early researches on vehicle control could go back to the Program on Advanced Technology for the Highway (PATH) in California of the USA, in the 1980s, where the basic principles and assumptions about sensors and actuators, control architecture, and string stability are summarized [7]. Since then, various vehicle control approaches are proposed. The near future will have to meet mixed traffic scenarios with the coexistence of human-driven vehicles and connected and autonomous vehicles [8]. After the transition period from human-driven vehicles to connected and autonomous vehicles, various proposed control approaches for autonomous vehicles can be adopted, e.g., Adaptive Cruise Control (ACC), and Cooperative Adaptive Cruise Control (CACC). The ACC functionality is capable of automatically adapting the cruise-control velocity of a vehicle via sensory devices for measuring the headway [9], [10], and is widespread and available in numerous commercial vehicles [3]. However, when considering traffic throughput, energy efficiency, safety of highway traffic while achieving a small inter-vehicular spacing, string-unstable driving behavior may occur if employing ACC. Hence CACC is proposed as an extension equipped with a wireless inter-vehicular communication link based on ACC [11]–[13]. Various aspects of vehicle control focusing on CACC are investigated, such as spacing policies [14], [15], communication topology [16]–[19], communication and dynamic delays [20], [21], vehicle dynamics heterogeneity [22]–[24], centralized and distributed control [25], [26], and so on. Despite a large volume of existing literature in CACC, state and input constraints that exist in practice are accounted for in rare cases. A lack of consideration of these constraints may lead to saturated control, low performance, instability or even catastrophic collisions.

Model predictive control (MPC), as one potential research direction in look ahead cruise control (a research direction of CACC), has been successfully applied in many areas over the last decades due to its ability to explicitly deal with diverse constraints and to account for optimization considerations [27], [28]. Traditionally, MPC is used for a single-agent system, and therefore is implemented in a centralized way. Nevertheless, a centralized MPC strategy, when applied to a vehicle platoon, may suffer from heavy communication and computational burden due to the spatially distributed vehicles. Recently, distributed model predictive control (DMPC) has been made a profound study [29]–[31]. In the context of standard DMPC, a positive definite cost function associated with the underlying optimization problem is often used to penalize the deviation between the state and input point of the actual system and the predefined setpoint or the reference signal. However, in a platoon control problem, the setpoint that embodies the position, velocity and acceleration of the leader vehicle may not be known to all following vehicles due to the communication restrictions. Each vehicle only communicates with its neigh-

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boring vehicles and may not be able to exchange information directly with the leader vehicle. Besides, vehicles in a platoon are not identical in most cases due to diversified powertrain, body types, etc. Given all of the above, it is difficult to apply the existing DMPC methods to platoon control. Existing works on heterogeneous platoon control can be categorized into two main groups. One focuses on the implementation of DMPC to vehicle platooning whereas the theoretical analyses are not taken into consideration [32], [33]. The other category presents DMPC algorithms for heterogeneous platoons with rigorous justification of the convergence and stability properties of the DMPC controllers, which relies on terminal equality constraints imposed on the state average of the neighboring vehicles [34]–[36]. This kind of terminal equality constraint is restrictive and makes it difficult to ensure the feasibility of optimization problem in theory for the first N steps, before all following vehicles indirectly access the position and uniform velocity information from the leader vehicle, where N is the number of vehicles in the platoon. The lack of feasibility implies that a valid control solution can not be guaranteed at each time step, which hinders the application in real-time. Moreover, only the local input constraints are considered in [35], [36], whereas the state-coupled inter-vehicular spacing constraints are omitted. Therefore, collision avoidance is not guaranteed. Although coupled state constraints are considered in [34], the target state (position, velocity and acceleration) of the leader vehicle has to be known for each follower, which demands extra communication burden.

With the aim of addressing the aforementioned challenges in terms of coupled state constraints and the feasibility for the first N steps, this paper proposes a novel DMPC scheme for controlling heterogeneous vehicles platoons subject to a bidirectional topology among the follower vehicles. The proposed method utilizes a sequential computational protocol, which consecutively solves local MPC problems for odd and even indexed vehicles within a sampling time interval. The contributions of this paper are summarized as follows:

1) The recursive feasibility, especially the satisfaction of the state constraints, of the DMPC algorithm can be completely guaranteed, including the initial N steps, under the condition that the control targets are unknown for a subset of the follower vehicles due to the lack of the leader vehicle's information. The key is to design a local feedback control law resorting to centrally derived tracking error invariant sets of each following vehicle. Such a DMPC framework also ensures the asymptotic stability of the platoon system.
2) The admissible range of velocity variability of the leader vehicle in terms of stability of the platoon is numerically investigated. Compared to the existing method, the proposed method permits more flexible leader velocity changes, which turns out to be useful in practice to accommodate traffic perturbations.

The rest of this paper is organized as follows. In Section II, the platoon of vehicles is modelled, and the control problem is presented. Then a DMPC algorithm is proposed in Section III, and it details the formulation of the decoupled MPC optimization problem for each vehicle. Section IV establishes system theoretic properties, including recursive feasibility, constraints satisfaction and closed-loop stability under the proposed algorithm. A numerical simulation is provided in Section V compared with an existing DMPC scheme for platoon control. Conclusions are drawn in Section VI.

Notation: Given two sets $\mathcal{X}, \mathcal{Y} \subseteq \mathbb{R}^n$, the Minkowski set addition is defined by $\mathcal{X} \oplus \mathcal{Y} = \{x + y \in \mathbb{R}^n \mid x \in \mathcal{X}, y \in \mathcal{Y}\}$ and the Minkowski set difference is defined by $\mathcal{X} \ominus \mathcal{Y} = \{x \in \mathbb{R}^n \mid x \oplus \mathcal{Y} \subseteq \mathcal{X}\}$. Given sets $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_M$, the set $\mathcal{A} \triangleq \Pi_{i=1}^{M} \mathcal{A}_i$ is defined as $\mathcal{A} \triangleq \{a = [a_1^\top, a_2^\top, \ldots, a_M^\top]^\top \mid a_i \in \mathcal{A}_i,\ i = 1, 2, \ldots, M\}$. Given a vector $x \in \mathbb{R}^n$ and a matrix $Q \in \mathbb{R}^{n \times n}$, denote $\|x\|_Q^2 \triangleq x^\top Q x$. Let $\mathrm{diag}(A_1, A_2, \ldots, A_N)$ denote a block-diagonal matrix with matrices $A_1, A_2, \ldots, A_N$ on the main diagonal and zeros everywhere else, and $(t_k + i|t_k)$ denote a prediction of a variable i steps ahead from time t_k. The modulus or remainder operation is defined as $a \ \mathbf{mod}\ b = c$ for all integers. Matrix inequality $A \preceq B$ means that $B - A$ is a positive semidefinite matrix, and a matrix A is Schur if all its eigenvalues lie in the interior of the unit circle.

II. PLATOON MODELING AND CONTROL OBJECTIVE

This paper considers a platoon of $N + 1$ heterogeneous vehicles forming a string, as shown in Fig. 1. The front vehicle indexed by 0 is regarded as the leader, and the downstream vehicles indexed from 1 to N are regarded as the followers. The leader vehicle 0 only transmits information to vehicle 1 and receives no information from all follower vehicles. The communication among all follower vehicles is enabled between consecutive vehicles and is assumed to be undirected.

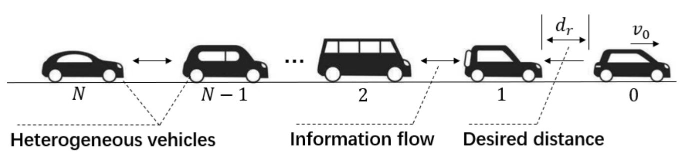

Fig. 1: Cooperative control architecture of a heterogeneous and connected vehicle platoon.

Denote $\mathcal{N} \triangleq \{1, 2, \ldots, N\}$ and $\mathcal{N}^+ \triangleq \{0, 1, \ldots, N\}$. The longitudinal dynamics of each vehicle $i, i \in \mathcal{N}^+$ in the platoon is modelled by a linear discrete-time system [37]

$$s_i(t_{k+1}) = s_i(t_k) + v_i(t_k)\Delta t + \frac{1}{2} a_i(t_k)\Delta t^2, \tag{1}$$

$$v_i(t_{k+1}) = v_i(t_k) + a_i(t_k)\Delta t, \tag{2}$$

$$a_i(t_{k+1}) = \left(1 - \frac{\Delta t}{\tau_i}\right) a_i(t_k) + \frac{\Delta t}{\tau_i} u_i(t_k), \tag{3}$$

where $s_i(t_k)$, $v_i(t_k)$ and $a_i(t_k)$ denote respectively the position, velocity and acceleration of vehicle i at time t_k. $u_i(t_k)$ is the control input acting on vehicle i at time t_k. Δt symbolizes the sampling interval. The vehicle dynamics heterogeneity lies in τ_i, which characterizes the inertial time-lag of vehicle longitudinal dynamics.

By letting $x_i(t_k) \triangleq [s_i(t_k), v_i(t_k), a_i(t_k)]^\top$, the longitudinal dynamics (1)-(3) can be rewritten in a compact form, for $i \in \mathcal{N}^+$, as

$$x_i(t_{k+1}) = A_i x_i(t_k) + B_i u_i(t_k), \tag{4}$$

where

$$A_i = \begin{bmatrix} 1 & \Delta t & \frac{\Delta t^2}{2} \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 - \frac{\Delta t}{\tau_i} \end{bmatrix},\ B_i = \begin{bmatrix} 0 \\ 0 \\ \frac{\Delta t}{\tau_i} \end{bmatrix}.$$

The platoon is dynamically decoupled and is constrained by the spatial formations and the operational limitations of all vehicles. Constraints imposed on each vehicle i are given by

$$s_i(t_k) - s_{i-1}(t_k) + \bar{d}_{i,i-1} \in \Delta\mathcal{S}_i, \tag{5a}$$
$$v_i(t_k) \in \mathcal{V}_i, \tag{5b}$$
$$a_i(t_k) \in \mathcal{A}_i, \tag{5c}$$
$$u_i(t_k) \in \mathcal{U}_i, \tag{5d}$$

where $\bar{d}_{i,i-1}$ is the desired constant position gap between vehicles i and $i-1$. The admissible set $\Delta\mathcal{S}_i \triangleq \{\Delta s \mid \Delta s_i^{\min} \le \Delta s \le \Delta s_i^{\max}\}$ forces vehicle i to be neither too close to (avoid collision) nor too far away from (avoid network latency or packet loss) the predecessor vehicle $i-1$. We denote $\mathcal{V}_i \triangleq \{v \mid v_i^{\min} \le v \le v_i^{\max}\}$, $\mathcal{A}_i \triangleq \{a \mid a_i^{\min} \le a \le a_i^{\max}\}$ and $\mathcal{U}_i \triangleq \{u \mid u_i^{\min} \le u \le u_i^{\max}\}$ the admissible sets of velocity, acceleration and control input of vehicle i, respectively. Sets $\Delta\mathcal{S}_i$, $\mathcal{A}_i$, $\mathcal{U}_i$ are assumed to contain the origin as an interior point.

This paper focuses on cooperative longitudinal platoon control for $N+1$ heterogeneous vehicles. The communication network topology among follower vehicles is assumed to be undirected between adjacent vehicles, that is, each vehicle i only communicates with vehicles $i-1$ and $i+1$. The leader vehicle travels at a constant velocity v_0, which also implies $a_0 = 0$ and $u_0 = 0$ in the steady-state condition. The control objective is to ensure that all the following vehicles can track the speed of the leader vehicle subject to constraints (5a)-(5d), while keeping a small constant inter-vehicular distance between any two neighboring vehicles to improve traffic throughput, i.e., for all $i \in \mathcal{N}$,

$$\lim_{t_k \to \infty} |s_i(t_k) - s_{i-1}(t_k) + \bar{d}_{i,i-1}| = 0, \tag{6a}$$
$$\lim_{t_k \to \infty} |v_i(t_k) - v_0| = 0, \tag{6b}$$
$$\lim_{t_k \to \infty} |a_i(t_k)| = 0. \tag{6c}$$

III. DMPC Algorithm for Heterogeneous Vehicle Platoon

This section develops a DMPC scheme for the heterogeneous platoon control problem formulated in Section II.

A. Problem reformulation

Considering the position $s_0(t_k)$, the uniform velocity $v_0(t_k) = v_0$ and the acceleration $a_0(t_k) = 0$ of the leader vehicle, the desired state and input for vehicle i can be defined as

$$x_{\mathrm{des},i}(t_k) = [s_{\mathrm{des},i}(t_k), v_{\mathrm{des},i}(t_k), a_{\mathrm{des},i}(t_k)]^\top, \tag{7}$$
$$u_{\mathrm{des},i}(t_k) = 0, \tag{8}$$

where $s_{\mathrm{des},i}(t_k) = s_0(t_k) - \bar{d}_{i,0}$ ($\bar{d}_{i,j} = \sum_{s=j+1}^{i} \bar{d}_{s,s-1}, j \le i-1$), $v_{\mathrm{des},i}(t_k) = v_0$ and $a_{\mathrm{des},i}(t_k) = 0$. In many previous works [34], [38], [39], a priori known set point or trajectory $x_{\mathrm{des},i}(\cdot)$ is assumed to be available for each vehicle i in order to ensure theoretical result. This assumption implies that all following vehicles are required to connect to the leader vehicle directly or indirectly via multi-hop V2X communication, which usually demands more communication resources and therefore is more restrictive. In this paper, the desired state and input are not necessary to be universally known to all followers and each vehicle can only use information from neighbors to make local control decisions.

Since the information from the leader vehicle is not accessible to all following vehicles, a general control objective such as $x_i(t_k) - x_{\mathrm{des},i}(t_k) \to 0$, as $t_k \to \infty$ cannot be adopted in each local MPC. Instead, the gap between adjacent vehicles is used locally. Consider $d_{i,j} = [\bar{d}_{i,j}, 0, 0]^\top$ the desired state gap and $e_i(t_k) = x_i(t_k) - x_{i-1}(t_k) + d_{i,i-1}$ the tracking error between vehicles i and $i-1$. The control objective for each vehicle i can then be described as $e_i(t_k) \to 0$, as $t_k \to \infty$.

To construct an invariant terminal set about all tracking errors e_i, $i \in \mathcal{N}$ for the design of DMPC in Section III-B, a terminal feedback control law for vehicle i is firstly given by $u_i(t_k) = K_{a,i} x_i(t_k) + K_{f,i} e_i(t_k)$. Under this control law, the dynamics of state deviation between vehicles i and $i-1$ can be described as

$$e_i(t_{k+1}) = A_d e_i(t_k) + B_i \bar{u}_i(t_k) - B_{i-1} \bar{u}_{i-1}(t_k), \tag{9}$$

where $\bar{u}_i(t_k) = K_{f,i} e_i(t_k)$ ($K_{f,i}^\top \in \mathbb{R}^3$ is a gain), and

$$A_d = \begin{bmatrix} 1 & \Delta t & \frac{\Delta t^2}{2} \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix}, K_{a,i} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

By taking all vehicles into consideration and letting $e(t_k) \triangleq [e_1^\top(t_k), e_2^\top(t_k), \ldots, e_N^\top(t_k)]^\top$ and $\bar{u}(t_k) \triangleq [\bar{u}_1(t_k), \bar{u}_2(t_k), \ldots, \bar{u}_N(t_k)]^\top$, the whole dynamics of tracking errors can be formulated as

$$e(t_{k+1}) = A_D e(t_k) + B_D \bar{u}(t_k), \tag{10}$$

where $A_D = \mathrm{diag}(A_d, A_d, \ldots, A_d) \in \mathbb{R}^{3N \times 3N}$ and

$$B_D = \begin{bmatrix} B_1 & & & & \\ -B_1 & B_2 & & & \\ & -B_2 & \ddots & & \\ & & \ddots & \ddots & \\ & & & -B_{N-1} & B_N \end{bmatrix}.$$

Concerning system (10), the following assumption is needed to design all ingredients associated with the terminal condition (i.e. terminal cost, terminal control law and terminal set) for the MPC optimal control problem (OCP), and these terminal ingredients are essential to guarantee theoretical results.

Denote $\Delta\mathcal{S} \triangleq \Pi_{i=1}^{N}\Delta\mathcal{S}_i$, $\mathcal{V} \triangleq \Pi_{i=1}^{N}\mathcal{V}_i$, $\mathcal{A} \triangleq \Pi_{i=1}^{N}\mathcal{A}_i$ and $\mathcal{U} \triangleq \Pi_{i=1}^{N}\mathcal{U}_i$.

Assumption 3.1: There exists a terminal invariant set $\mathcal{E} \triangleq \{e \in \mathbb{R}^{3N} \mid \sum_{i=1}^{N} e_i^\top P_i e_i \leq \gamma\}$ and a feedback control law $\bar{u}(t_k) = K_f e(t_k)$ with a block-diagonal matrix $K_f = \mathrm{diag}(K_{f,1}, K_{f,2}, \ldots, K_{f,N})$ such that:

(i) $A_D + B_D K_f$ is Schur;

(ii) $\mathcal{E}$ is an invariant set for system (10), i.e., $\forall e(t_k) \in \mathcal{E}$, $e(t_{k+1}) \in \mathcal{E}$ and $(I_N \otimes [1, 0, 0])e(t_k) \in \Delta\mathcal{S}$;

(iii) $\forall e(t_k) \in \mathcal{E}$, $\bar{u}(t_k) \in \mathcal{U} \ominus \mathcal{A}$;

(iv) $\forall e(t_k) \in \mathcal{E}$,

$$\sum_{i=1}^{N}\Big(||e_i(t_{k+1})||_{P_i}^2 - ||e_i(t_k)||_{P_i}^2 + ||e_i(t_k)||_{Q_i}^2\Big) \leq 0,$$

where positive parameter γ and symmetric positive definite matrices $P_i \in \mathbb{R}^{3\times3}$ and $Q_i \in \mathbb{R}^{3\times3}$ are prespecified by the designers.

Remark 3.1: Assumption 3.1 is a standard assumption in the field of MPC theory (see, for example, Assumption 2 in [29]). In Assumption 3.1, a terminal invariant set $\mathcal{E}$ and a feedback control law $\bar{u}(t_k) = K_f e(t_k)$ are required to be designed. The purpose is to limit the behavior of each vehicle at the end of the prediction horizon. Firstly, the platoon must be controllable according to Assumption 3.1 (i). Secondly, if the terminal tracking error $e(t_{k+H}|t_k)$ at time t_k is constrained in the terminal invariant set $\mathcal{E}$, then under the feedback control law $\bar{u}(t_{k+H}|t_k)$, the terminal tracking error $e(t_{k+1+H}|t_{k+1}) = e(t_{k+1+H}|t_k)$ at time t_{k+1} will still stay in $\mathcal{E}$ by Assumption 3.1 (ii) (it also implies the satisfaction of the state constraints (5a)-(5c)). Meanwhile, from Assumption 3.1 (iii), the feedback control law $\bar{u}(t_{k+H}|t_k)$ will always satisfy the input constraint (5d) for each follower vehicle i, $i \in \mathcal{N}$, and the platoon is asymptotically stable under the feedback control law $\bar{u}(t_{k+H}|t_k)$ when states $x_i(t_{k+H}|t_k)$, $i \in \mathcal{V}$ locate in the invariant set $\mathcal{E}$ according to Assumption 3.1 (iv). Note that although the terminal control law K_f is assumed on the whole dynamic system (10), it is decoupled for each vehicle i as the feedback control law $\bar{u}(t_k)$ can be divided into $\bar{u}_i(t_k) = K_{f,i}e_i(t_k)$, $i \in \mathcal{N}$.

B. Algorithm design

Since all vehicles are coupled through inter-vehicular spacing constraints (5a), in order to achieve a distributed control in the presence of coupling constraints, the N local MPCs are updated sequentially by following a two-step approach at each time step. More specifically, at an arbitrary sampling instant t_k, vehicles with odd indices solve in parallel their local control problems in the first instance and then the rest with even indices update their control inputs. Multiple iterations are permitted at a sampling instant and the number can be treated as a tunable parameter to balance coordinated control performance and computational efficiency. For the sake of further discussion, let us consider $\bar{p} \geq 1$ the desired number of iterations. When $\bar{p} = 1$, the sequential DMPC algorithm is executed in a non-iterative way, where the computational cost is minimized. Conversely, multiple iterations ($\bar{p} \geq 2$) tend to show a better convergence performance at a price of increased computational complexity. The influence of $\bar{p}$ on the control performance will be shown in Section V by simulations.

For each vehicle $i, i \in \mathcal{N}$, as shown in Fig. 1, its local OCP only requires the information from vehicles $i-1$ and $i+1$. Without loss of generality, the same length of prediction horizon H is used in all local problems. Define $J_i(\cdot)$ as the tracking error based cost function for vehicle i

$$J_i(t_k) \triangleq \sum_{j=0}^{H-1} l_i(x_i, x_{i-1}, x_{i+1}, t_{k+j}|t_k) + l_{f,i}(x_i, x_{i-1}, x_{i+1}, t_{k+H}|t_k),$$

which consists of the stage cost function l_i at each prediction time step defined, for $j = 0, 1, \ldots, H-1$, by

$$l_i(x_i, x_{i-1}, x_{i+1}, t_{k+j}|t_k) \triangleq h_i(x_i, x_{i-1}, t_{k+j}|t_k) + h_{i+1}(x_{i+1}, x_i, t_{k+j}|t_k),$$

and the terminal cost function $l_{f,i}$ at the end of the prediction horizon defined by

$$l_{f,i}(x_i, x_{i-1}, x_{i+1}, t_{k+H}|t_k) \triangleq h_{f,i}(x_i, x_{i-1}, t_{k+H}|t_k) + h_{f,i+1}(x_{i+1}, x_i, t_{k+H}|t_k),$$

with

$$h_i(x_i, x_{i-1}, t_{k+j}|t_k) \triangleq ||x_i(t_{k+j}|t_k) - x_{i-1}(t_{k+j}|t_k) + d_{i,i-1}||_{Q_i}^2,$$

and

$$h_{f,i}(x_i, x_{i-1}, t_{k+H}|t_k) \triangleq ||x_i(t_{k+H}|t_k) - x_{i-1}(t_{k+H}|t_k) + d_{i,i-1}||_{P_i}^2.$$

We formulate the local finite-horizon OCP $\mathcal{P}_i(t_k)$ for each vehicle $i,\ i \in \mathcal{N}$, at time t_k, by

$$\min_{u_i(t_k|t_k),\ldots,u_i(t_{k+H-1}|t_k)} J_i(t_k) \tag{11a}$$

s.t. for all $j = 0, 1, \ldots, H-1$

$$x_i(t_{k+j+1}|t_k) = A_i x_i(t_{k+j}|t_k) + B_i u_i(t_{k+j}|t_k), \tag{11b}$$
$$x_i(t_k|t_k) = x_i(t_k), \tag{11c}$$
$$s_i(t_{k+j+1}|t_k) - \hat{s}_{i-1}(t_{k+j+1}|t_k) + \bar{d}_{i,i-1} \in \Delta\mathcal{S}_i, \tag{11d}$$
$$\hat{s}_{i+1}(t_{k+j+1}|t_k) - s_i(t_{k+j+1}|t_k) + \bar{d}_{i+1,i} \in \Delta\mathcal{S}_{i+1}, \tag{11e}$$
$$v_i(t_{k+j+1}|t_k) \in \mathcal{V}_i, \tag{11f}$$
$$a_i(t_{k+j+1}|t_k) \in \mathcal{A}_i, \tag{11g}$$
$$u_i(t_{k+j}|t_k) \in \mathcal{U}_i, \tag{11h}$$
$$l_{f,i}(x_i, \hat{x}_{i-1}, \hat{x}_{i+1}, t_{k+H}|t_k) \leq l_{f,i}(\hat{x}_i, \hat{x}_{i-1}, \hat{x}_{i+1}, t_{k+H}|t_k), \tag{11i}$$

where $\hat{\mathbf{x}}_i(t_k) \triangleq [\hat{x}_i(t_k|t_k), \hat{x}_i(t_{k+1}|t_k), \ldots, \hat{x}_i(t_{k+H}|t_k)]$ with $\hat{x}_i = [\hat{s}_i, \hat{v}_i, \hat{a}_i]^\top$ is a reference state trajectory of vehicle i and it will be transmitted to vehicles $i-1$ and $i+1$ for reference at time t_k. Constraints (11d) and (11e) enforce vehicle i to maintain the proper distance gap against the predecessor vehicle $i-1$ and the following vehicle $i+1$. Constraints (11f)-(11h) respectively limit the velocity, acceleration and control input of vehicle i over the prediction horizon. Terminal inequality constraint (11i) enforces the sum of tracking errors between vehicles i and $i-1$ and between vehicles $i+1$ and i inside the invariant set $\mathcal{E}$ at the terminal horizon because the right term in (11i) is a constructed state point based on Assumption 3.1 which stays inside the invariant set $\mathcal{E}$.

After solving the local OCP $\mathcal{P}_i(t_k)$, $i \in \mathcal{N}$ for the p-th iteration at the sampling instant t_k with $p \in [1, \bar{p}]$, vehicle i determines the optimal state trajectory $\mathbf{x}_i^p(t_k) \triangleq [x_i^p(t_k|t_k), x_i^p(t_{k+1}|t_k), \ldots, x_i^p(t_{k+H}|t_k)]$ and the corresponding optimal input trajectory $\mathbf{u}_i^p(t_k) \triangleq [u_i^p(t_k|t_k), u_i^p(t_{k+1}|t_k), \ldots, u_i^p(t_{k+H-1}|t_k)]$, and then transmits its reference state trajectory $\hat{\mathbf{x}}_i(t_k) = \mathbf{x}_i^p(t_k)$ to its neighboring vehicles $i-1$ and $i+1$. After the maximum number of iterations $\bar{p}$, each vehicle i can construct a new reference state trajectory $\hat{\mathbf{x}}_i(t_{k+1})$ for the next sampling instant t_{k+1}, given by

$$\hat{x}_i(t_{k+1+j}|t_{k+1}) = \begin{cases} x_i^{\bar{p}}(t_{k+1+j}|t_k), \ j = 0, 1, \ldots, H-1, \\ \hat{x}_i(t_{k+1+H}|t_{k+1}), \ j = H, \end{cases} \tag{12}$$

and the corresponding input trajectory defined by

$$\hat{u}_i(t_{k+1+j}|t_{k+1}) = \begin{cases} u_i^{\bar{p}}(t_{k+1+j}|t_k), \ j = 0, 1, \ldots, H-2, \\ \hat{u}_i(t_{k+H}|t_{k+1}), \ j = H-1, \end{cases} \tag{13}$$

where the last elements in the reference state and input sequences are specified by

$$\begin{aligned} \hat{x}_i(t_{k+1+H}|t_{k+1}) &= A_i \hat{x}_i(t_{k+H}|t_{k+1}) + B_i \hat{u}_i(t_{k+H}|t_{k+1}), \\ \hat{u}_i(t_{k+H}|t_{k+1}) &= K_{f,i} \hat{e}_i(t_{k+H}|t_{k+1}) + \hat{a}_i(t_{k+H}|t_{k+1}), \end{aligned}$$

with $\hat{e}_i(t_{k+H}|t_{k+1}) = \hat{x}_i(t_{k+H}|t_{k+1}) - \hat{x}_{i-1}(t_{k+H}|t_{k+1}) + d_{i,i-1}$.

Remark 3.2: For the sake of narrative convenience, the formulation of OCP $\mathcal{P}_N(t_k)$ for vehicle N at time t_k is not rewritten though vehicle N is the last vehicle with no vehicle following. It can be regarded as a case where a phantom vehicle indexed by $N+1$ is perfectly tracking vehicle N such that parts of formulas l_i and $l_{f,i}$ are always zero, i.e., $x_N(t_{k+j}|t_k) - x_{N+1}(t_{k+j}|t_k) - d_{N+1,N} = 0, j = 0, 1, \ldots, H$. Similarly, the formulation of OCP $\mathcal{P}_1(t_k)$ for vehicle 1 at time t_k can also be easily accommodated in the presented setting. The state information of the leader vehicle is accessible for vehicle 1 since the leader vehicle runs with constant velocity v_0, that is, $\hat{x}_0(t_{k+j}|t_k) = A_0^s x_0(t_k)$ and $x_0(t_k) = [s_0(t_k), v_0, 0]^\top$ in OCP $\mathcal{P}_1(t_k)$.

Denote $\mathcal{N}_{odd} \triangleq \{i \in \mathcal{N} \mid i \ \mathbf{mod} \ 2 = 1\}$ and $\mathcal{N}_{even} \triangleq \{i \in \mathcal{N} \mid i \ \mathbf{mod} \ 2 = 0\}$. Now the DMPC algorithm for a platoon of heterogeneous vehicles with inter-vehicular spacing constraints is summerized in Algorithm 1, where the parameter tuning mechanism is detailed in Section III-C.

Algorithm 1 DMPC for a Heterogeneous Vehicle Platoon with Inter-Vehicular Spacing Constraints

Offline: Determine matrices K_f, P, Q, and set $\mathcal{E}$. Set the maximum number of iterations $\bar{p}$ and the running step number $\bar{k}$. Let $p = 1$ and $k = 0$. At the beginning time t_0, each vehicle $i \in \mathcal{N}^+$ measures its state $x_i(t_0)$. Choose prediction horizon H long enough such that there exists a reference state trajectory $\hat{\mathbf{x}}_i(t_0)$ for each follower vehicle i satisfying constraints (11b)-(11h) and the terminal condition $\sum_{i=1}^{N} ||\hat{x}_i(t_H|t_0) - \hat{x}_{i-1}(t_H|t_0) + d_{i,i-1}||_{P_i}^2 \leq \gamma$.

Online: For each vehicle $i \in \mathcal{N}$:

```
while k < k̄ do
    Measure state x_i(t_k).
    while p ≤ p̄ do
        if i ∈ N_odd then
            Receive x̂_{i−1}(t_k) and x̂_{i+1}(t_k).
            Solve P_i(t_k) for x_i^p(t_k) & u_i^p(t_k).
            Let x̂_i(t_k) = x_i^p(t_k).
            Send x̂_i(t_k) to vehicles i − 1 and i + 1.
            p ← p + 1.
        else i ∈ N_even
            Send x̂_i(t_k) to vehicles i − 1 and i + 1.
            Receive x̂_{i−1}(t_k) and x̂_{i+1}(t_k).
            Solve P_i(t_k) for x_i^p(t_k) & u_i^p(t_k).
            Let x̂_i(t_k) = x_i^p(t_k).
            p ← p + 1.
        end if
    end while
    Apply input u_i(t_k) = u_i^p̄(t_k|t_k).
    Construct x̂_i(t_{k+1}) via (12) and (13).
    k ← k + 1 and p ← 1.
end while
```

C. Guidelines on parameter design

In Algorithm 1, several parameters introduced in Assumption 3.1 have to be determined at the offline stage. In the following, we provide a viable method to obtain feasible matrices K_f, $P \triangleq \mathrm{diag}(P_1, P_2 \ldots, P_N) \in \mathbb{R}^{3N \times 3N}$, $Q \triangleq \mathrm{diag}(Q_1, Q_2 \ldots, Q_N) \in \mathbb{R}^{3N \times 3N}$ and the invariant set $\mathcal{E}$ in the practical implementation. First, by adjusting α, $0 < \alpha \leq 1$, find matrices K_f, P and Q satisfying the following inequalities

$$(A_D + B_D K_f)^\top P (A_D + B_D K_f) \preceq \alpha^2 P, \tag{14}$$

$$(A_D + B_D K_f)^\top P (A_D + B_D K_f) \preceq P - Q. \tag{15}$$

Then adjust the value of γ in the definition of terminal set $\mathcal{E}$ such that the condition (iii) in Assumption 3.1 is valid. Meanwhile, Assumption 3.1 only involves the relative information, i.e., the tracking errors, and the absolute information about the velocity and acceleration of each vehicle i is required to be taken into consideration. For each vehicle i, constraints (5b)-(5c) have to be satisfied in the terminal set $\mathcal{E}$ at the end of the prediction horizon. Since the state of the predecessor vehicle $i-1$ changes during the control process, it leads to difficulties in guaranteeing constraints (5b)-(5c) via $x_i = e_i + x_{i-1} - d_{i,i-1}$. For all $i \in \mathcal{N}$, there must exist a constant $\beta \geq 1$ satisfying $P_i \preceq \beta P_j, j = 1, 2, \ldots, i$ such that

$$\begin{aligned}||x_i - x_0 + d_{i,0}||^2_{P_i} \leq & \sum_{j=1}^{i} ||x_j - x_{j-1} + d_{j,j-1}||^2_{P_i} \\ \leq & \beta \sum_{j=1}^{i} ||x_j - x_{j-1} + d_{j,j-1}||^2_{P_j} \\ \leq & \beta \sum_{j=1}^{N} ||x_j - x_{j-1} + d_{j,j-1}||^2_{P_j} \\ \leq & \beta\gamma.\end{aligned}$$

The last inequality holds due to the definition of $\mathcal{E}$ and the condition (ii) in Assumption 3.1. Let $v_i - v_0 \in \Delta\mathcal{V}_{i,0} \triangleq \{[0,1,0]v \mid ||v||^2_{P_i} \leq \beta\gamma, v \in \mathbb{R}^3\}$ and $a_i - a_0 = a_i \in \Delta\mathcal{A}_{i,0} \triangleq \{[0,0,1]a \mid ||a||^2_{P_i} \leq \beta\gamma, a \in \mathbb{R}^3\}$. For all vehicles, constraints (5b)-(5c) can be satisfied via adjusting sets $\mathcal{V}_i$, $\mathcal{A}_i$, $i \in \mathcal{N}^+$ and the value of γ, such that for any $i \in \mathcal{N}$

$$\mathcal{V}_0 \oplus \Delta\mathcal{V}_{i,0} \subseteq \mathcal{V}_i, \quad (16)$$

$$\Delta\mathcal{A}_{i,0} \subseteq \mathcal{A}_i. \quad (17)$$

At the beginning of the actual control process, all vehicles are tasked with their initialization. The initialization step is performed by solving an optimization problem (11b)-(11i) (only once) for a collection of admissible control process $\hat{\mathbf{x}}_i(t_0)$ and $\hat{\mathbf{u}}_i(t_0)$ satisfying relations (11b)-(11i). The initialization term is another research topic about distributed optimization. In previous works on DMPC, the initialization was either implemented by a centralized way or by a trial-and-error method [29] when it was performed in the distributed fashion.

IV. CLOSED-LOOP THEORETIC PROPERTIES OF ALGORITHM 1

To analyze the closed-loop theoretic properties under the proposed distributed control framework in Algorithm 1, the standard way of proving recursive feasibility, closed-loop constraints satisfaction, and stability in MPC need to be adapted. Specifically, it is necessary to account adequately for the interaction between vehicles with odd indices and vehicles with even indices.

Theorem 4.1: (Recursive Feasibility) Suppose Assumption 3.1 holds. If OCP $\mathcal{P}_i(t_k)$ for each vehicle i, $i \in \mathcal{N}$, is feasible at the beginning time t_0 with the initial reference state $\hat{\mathbf{x}}_i(t_0)$ and input $\hat{\mathbf{u}}_i(t_0)$ and the terminal condition $\sum_{i=1}^{N} ||\hat{x}_i(t_H|t_0) - \hat{x}_{i-1}(t_H|t_0) + d_{i,i-1}||^2_{P_i} \leq \gamma$ is satisfied, then OCP $\mathcal{P}_i(t_k)$ in Algorithm 1 is recursively feasible at all subsequent time steps for all vehicles.

Proof: Assume that, at some time t_k, OCP $\mathcal{P}_i(t_k)$ for each vehicle i, $i \in \mathcal{N}$, is feasible with the feasible reference state $\hat{\mathbf{x}}_i(t_k)$ and input $\hat{\mathbf{u}}_i(t_k)$, and the terminal condition $\sum_{i=1}^{N} ||\hat{x}_i(t_{k+H}|t_k) - \hat{x}_{i-1}(t_{k+H}|t_k) + d_{i,i-1}||^2_{P_i} \leq \gamma$ is satisfied. The main content in the proof of this theorem is then broken down into two steps (or parts):

(1) prove that OCP $\mathcal{P}_i(t_k)$ at the p-th iteration of time t_k is always feasible no matter what the number of iteration p is;

(2) prove that the reference state trajectory $\hat{\mathbf{x}}_i(t_{k+1})$ together with the corresponding input trajectory $\hat{\mathbf{u}}_i(t_{k+1})$ constructed by (12)-(13) is a feasible solution of OCP $\mathcal{P}_i(t_{k+1})$, $i \in \mathcal{N}$, at time t_{k+1}.

Next, we justify **Step (1)** and **Step (2)** in detail.

Step (1): Suppose OCP $\mathcal{P}_i(t_k)$ for each vehicle i, $i \in \mathcal{N}$, is feasible at the p-th iteration of time t_k with the reference state $\mathbf{x}^p_i(t_k)$ and input $\mathbf{u}^p_i(t_k)$ and the terminal condition $\sum_{i=1}^{N} ||x^p_i(t_{k+H}|t_k) - x^p_{i-1}(t_{k+H}|t_k) + d_{i,i-1}||^2_{P_i} \leq \gamma$ is satisfied. At the $(p+1)$-th iteration, the vehicles with odd indices compute in parallel at first followed by vehicles with even indices compute in parallel, and thus the proof can be divided into two sub-steps.

(i) For each vehicle with odd index $i \in \mathcal{N}$ ($i \bmod 2 = 1$) at any $(p+1)$-th iteration of time t_k, information from coupled neighboring even vehicles $i \pm 1$ are fixed temporarily, that is, vehicles $i \pm 1$ temporarily do not optimize their OCPs $\mathcal{P}_{i\pm1}(t_k)$ and keep the reference trajectories $\mathbf{x}^p_{i\pm1}(t_k)$ unchanged (if $i = 1$, it can be regarded as the leader always keeping its solution unchanged). As a result, OCP $\mathcal{P}_i(t_k)$, $i \in \mathcal{N}$ at the $(p+1)$-th iteration of time t_k is always feasible because at least the previous solution $\mathbf{x}^p_i(t_k)$ and $\mathbf{u}^p_i(t_k)$ is a feasible solution for OCP $\mathcal{P}_i(t_k)$ satisfying constraints (11b)-(11i). Each vehicle with odd index $i \in \mathcal{N}$ can therefore solve its OCP $\mathcal{P}_i(t_k)$ and obtain the optimal solution $\mathbf{x}^{p+1}_i(t_k)$ and $\mathbf{u}^{p+1}_i(t_k)$. Besides, from the satisfaction of (11i), the optimal solution also satisfies $\sum_{i=1}^{N} ||x^{p+1}_i(t_{k+H}|t_k) - x^p_{i-1}(t_{k+H}|t_k) + d_{i,i-1}||^2_{P_i} \leq \gamma$.

(ii) For each vehicle with even index $i \in \mathcal{N}$ ($i \bmod 2 = 0$) at any $(p+1)$-th iteration of time t_k, information from coupled neighboring odd vehicles $i \pm 1$ are fixed temporarily. Constraints (11b)-(11c) and (11f)-(11i) in OCP $\mathcal{P}_i(t_k)$ are obviously satisfied by the previous solution $\mathbf{x}^p_i(t_k)$ and $\mathbf{u}^p_i(t_k)$. Constraint (11d) in OCP $\mathcal{P}_i(t_k)$ is equivalent to constraint (11e) in OCP $\mathcal{P}_{i-1}(t_k)$ which has been taken into consideration for vehicle $i-1$ computing its OCP $\mathcal{P}_{i-1}(t_k)$. In other words, constraint (11d) is satisfied by $\mathbf{x}^p_i(t_k)$ and $\mathbf{x}^{p+1}_{i-1}(t_k)$, where $\mathbf{x}^{p+1}_{i-1}(t_k)$ is computed by odd vehicle $i-1$ before. Thus, (11d) is satisfied by the previous solution $\mathbf{x}^p_i(t_k)$ and $\mathbf{u}^p_i(t_k)$. Similarly, $\mathbf{x}^p_i(t_k)$ and $\mathbf{u}^p_i(t_k)$ can also ensure the satisfaction of (11e). Hence, problem $\mathcal{P}_i(t_k)$ for each vehicle with even index $i \in \mathcal{N}$ is feasible at the $(p+1)$-th iteration of time t_k and can be solved for the solution $\mathbf{x}^{p+1}_i(t_k)$ and $\mathbf{u}^{p+1}_i(t_k)$. The satisfaction of (11i) further ensures the satisfaction of $\sum_{i=1}^{N} ||x^{p+1}_i(t_{k+H}|t_k) - x^{p+1}_{i-1}(t_{k+H}|t_k) + d_{i,i-1}||^2_{P_i} \leq \gamma$.

Consider the assumption that OCP $\mathcal{P}_i(t_k)$ for each vehicle i, $i \in \mathcal{N}$, is feasible at time t_k with the feasible reference state $\hat{\mathbf{x}}_i(t_k)$ and input $\hat{\mathbf{u}}_i(t_k)$ and the terminal condition $\sum_{i=1}^{N} ||\hat{x}_i(t_{k+H}|t_k) - \hat{x}_{i-1}(t_{k+H}|t_k) + d_{i,i-1}||^2_{P_i} \leq \gamma$ is satisfied, where the feasible reference state $\hat{\mathbf{x}}_i(t_k)$ and input $\hat{\mathbf{u}}_i(t_k)$ can be regarded as a solution at the 0-th iteration of time t_k, i.e., $\mathbf{x}^0_i(t_k) = \hat{\mathbf{x}}_i(t_k)$ and $\mathbf{u}^0_i(t_k) = \hat{\mathbf{u}}_i(t_k)$. By using mathematical induction, there always exists a solution $\mathbf{x}^p_i(t_k)$ and $\mathbf{u}^p_i(t_k)$ for OCP $\mathcal{P}_i(t_k)$ and the terminal condition $\sum_{i=1}^{N} ||x^p_i(t_{k+H}|t_k) - x^p_{i-1}(t_{k+H}|t_k) + d_{i,i-1}||^2_{P_i} \leq \gamma$ is satisfied at all iteration steps p of time t_k.

Step (2): From the proof of **Step (1)**, OCP $\mathcal{P}_i(t_k)$ for each vehicle i, $i \in \mathcal{N}$, is feasible at the $\bar{p}$-th iteration of time t_k with the feasible reference state $\mathbf{x}_i^{\bar{p}}(t_k)$ and input $\mathbf{u}_i^{\bar{p}}(t_k)$ and the terminal condition $\sum_{i=1}^{N} ||x_i^{\bar{p}}(t_{k+H}|t_k) - x_{i-1}^{\bar{p}}(t_{k+H}|t_k) + d_{i,i-1}||_{P_i}^2 \leq \gamma$ is satisfied. A candidate solution at time t_{k+1} can then be constructed as $\hat{\mathbf{x}}_i(t_{k+1})$ and $\hat{\mathbf{u}}_i(t_{k+1})$ via (12) and (13). It is obvious that $\hat{x}_i(t_{k+1+j}|t_{k+1})$, $j = 0, 1, \ldots, H-1$ and $\hat{u}_i(t_{k+1+j}|t_{k+1})$, $j = 0, 1, \ldots, H-2$ satisfy constraints (11b)-(11i) in OCP $\mathcal{P}_i(t_{k+1})$, $i \in \mathcal{N}$, and constraint (11i) certainly holds with equality. In addition, since the terminal condition $\sum_{i=1}^{N} ||x_i^{\bar{p}}(t_{k+H}|t_k) - x_{i-1}^{\bar{p}}(t_{k+H}|t_k) + d_{i,i-1}||_{P_i}^2 \leq \gamma$ at the $\bar{p}$-th iteration of time t_k is satisfied, the terminal condition $\sum_{i=1}^{N} ||\hat{x}_i(t_{k+1+H}|t_{k+1}) - \hat{x}_{i-1}(t_{k+1+H}|t_{k+1}) + d_{i,i-1}||_{P_i}^2 \leq \gamma$ at time t_{k+1} is satisfied by Assumption 3.1 (ii). Assumption 3.1 (iii) also implies that $\hat{x}_i(t_{k+1+H}|t_{k+1})$ and $\hat{u}_i(t_{k+H}|t_{k+1})$ satisfy constraints (11b), (11d)-(11e) and (11h)-(11i). Constraints (16) and (17) guarantee the satisfaction of constraints (11f)-(11g) of $\hat{x}_i(t_{k+1+H}|t_{k+1})$. Hence the candidate solution $\hat{\mathbf{x}}_i(t_{k+1})$ and $\hat{\mathbf{u}}_i(t_{k+1})$ is a feasible solution for OCP $\mathcal{P}_i(t_{k+1})$, $i \in \mathcal{N}$ at time t_{k+1}, and satisfies the terminal condition $\sum_{i=1}^{N} ||\hat{x}_i(t_{k+1+H}|t_{k+1}) - \hat{x}_{i-1}(t_{k+1+H}|t_{k+1}) + d_{i,i-1}||_{P_i}^2 \leq \gamma$.

Based on **Step (1)** and **Step (2)**, it can be concluded, by induction, that if the conditions stated in the theorem hold, OCP $\mathcal{P}_i(t_k)$ in Algorithm 1 is recursively feasible for each vehicle $i, i \in \mathcal{N}$. ■

The following theorem shows that all constraints are satisfied in the closed-loop operation under Algorithm 1.

Theorem 4.2: (Closed-loop Constraint Satisfaction) Provided that the conditions in Theorem 4.1 hold, then the closed-loop trajectories $x_i(\cdot)$ and $u_i(\cdot)$ for each vehicle $i, i \in \mathcal{N}$ under Algorithm 1 satisfy the constraints (5a)-(5d) at all time steps.

Proof: From the online constraints (11d)-(11h) and the recursive feasibility established in Theorem 4.1, the closed-loop constraint satisfaction holds true trivially. ■

The remaining part of this section is to analyze the closed-loop stability by using Lyapunov method through showing that the positive definite cost function is decreasing as the time increases. In standard MPC, the cost functions at consecutive time instants t_k and t_{k+1} are compared and built a relation by introducing the cost associated with a constructed feasible solution at time t_{k+1}. However, this idea cannot be applied straightforwardly here because of the specific distributed mechanism in Algorithm 1.

Theorem 4.3: (Asymptotic Stability) Provided that the conditions in Theorem 4.1 hold, then each vehicle $i, i \in \mathcal{N}$ is asymptotically stable to the desired trajectory $x_{\text{des},i}(t_k)$ as $t_k \to \infty$ under the application of Algorithm 1.

Proof: For the ease of presentation, the state of vehicle 0 is denoted as $x_0^p(t_{k+1+j}|t_{k+1})$ for any p instead of $x_0(t_{k+1+j})$. First, define the optimal value function of $\mathcal{P}_i(t_{k+1})$ at the p-th iteration of time t_{k+1}, for $p \geq 1$ and $i \in \mathcal{N}_{odd}$, by

$$J_i^{odd}(t_{k+1}, p) \triangleq \sum_{j=0}^{H-1} l_i(x_i^p, x_{i-1}^{p-1}, x_{i+1}^{p-1}, t_{k+1+j}|t_{k+1}) + l_{f,i}(x_i^p, x_{i-1}^{p-1}, x_{i+1}^{p-1}, t_{k+1+H}|t_{k+1}), \quad (18)$$

and for $p \geq 1$ and $i \in \mathcal{N}_{even}$, by

$$J_i^{even}(t_{k+1}, p) \triangleq \sum_{j=0}^{H-1} l_i(x_i^p, x_{i-1}^p, x_{i+1}^p, t_{k+1+j}|t_{k+1}) + l_{f,i}(x_i^p, x_{i-1}^p, x_{i+1}^p, t_{k+1+H}|t_{k+1}). \quad (19)$$

Further, denote

$$h_{\sum,i}(x_i, x_{i-1}, t_k) \triangleq \sum_{j=0}^{H-1} h_i(x_i, x_{i-1}, t_{k+j}|t_k) + h_{f,i}(x_i, x_{i-1}, t_{k+H}|t_k).$$

For $p \geq 1$, define the sum of all local optimal value functions $J_i^{odd}(t_{k+1}, p)$ over $i \in \mathcal{N}_{odd}$ as an auxiliary function

$$\begin{aligned} J^{odd}(t_{k+1}, p) &\triangleq \sum_{i \in \mathcal{N}_{odd}} J_i^{odd}(t_{k+1}, p) \\ &= \sum_{i \in \mathcal{N}_{odd}} h_{\sum,i}(x_i^p, x_{i-1}^{p-1}, t_{k+1}) \\ &\quad + \sum_{i \in \mathcal{N}_{even}} h_{\sum,i}(x_i^{p-1}, x_{i-1}^p, t_{k+1}), \end{aligned}$$

and the sum of all local optimal value functions $J_i^{even}(t_{k+1}, p)$ over $i \in \mathcal{N}_{even}$ as another auxiliary function

$$\begin{aligned} J^{even}(t_{k+1}, p) &\triangleq \sum_{i \in \mathcal{N}_{even}} J_i^{even}(t_{k+1}, p) \\ &= \sum_{i=2}^{N} h_{\sum,i}(x_i^p, x_{i-1}^p, t_{k+1}). \end{aligned}$$

After that, define a Lyapunov candidate as

$$J(t_{k+1}, p) \triangleq J^{even}(t_{k+1}, p) + h_{\sum,1}(x_1^p, x_0^p, t_{k+1}).$$

We will prove the theorem in two steps.

(1) prove that the Lyapunov candidate for any iteration step p at time t_{k+1} is not larger than the Lyapunov candidate at the 0-th iteration of time t_{k+1};
(2) prove that the Lyapunov candidate at the 0-th iteration of time t_{k+1} is not larger than the Lyapunov candidate at the $\bar{p}$-th iteration of time t_k.

Step (1): Denote the optimal solution at time t_k by $\mathbf{x}_i^{\bar{p}}(t_k)$ and $\mathbf{u}_i^{\bar{p}}(t_k)$ satisfying constraints (11b)-(11i) and $\sum_{i=1}^{N} ||x_i^{\bar{p}}(t_{k+H}|t_k) - x_{i-1}^{\bar{p}}(t_{k+H}|t_k) + d_{i,i-1}||_{P_i}^2 \leq \gamma$. At time t_{k+1}, a feasible solution is first constructed as $\mathbf{x}_i^0(t_{k+1}) = \hat{\mathbf{x}}_i(t_{k+1})$ and $\mathbf{u}_i^0(t_{k+1}) = \hat{\mathbf{u}}_i(t_{k+1})$ corresponding to $p = 0$, and $J(t_{k+1}, 0)$ can then be given by

$$\begin{aligned} J(t_{k+1}, 0) &= J^{even}(t_{k+1}, 0) + h_{\sum,1}(x_1^0, x_0^0, t_{k+1}) \\ &= \sum_{i=1}^{N} h_{\sum,i}(x_i^0, x_{i-1}^0, t_{k+1}), \end{aligned}$$

where

$$J^{even}(t_{k+1}, 0) \triangleq \sum_{i=2}^{N} h_{\sum,i}(x_i^0, x_{i-1}^0, t_{k+1}).$$

From the definitions of $J^{even}(t_k,p)$, $J^{odd}(t_k,p)$ and $J(t_k,p)$, it can be derived that

$$\begin{aligned}J(t_{k+1},p) =& J^{even}(t_{k+1},p) + h_{\sum,1}(x_1^p, x_0^p, t_{k+1})\\ \le& h_{\sum,1}(x_1^p, x_0^p, t_{k+1})\\ &+ \sum_{i\in\mathcal{N}_{odd}, i\neq 1} h_{\sum,i}(x_i^p, x_{i-1}^{p-1}, t_{k+1})\\ &+ \sum_{i\in\mathcal{N}_{even}} h_{\sum,i}(x_i^{p-1}, x_{i-1}^{p}, t_{k+1})\\ =& \sum_{i\in\mathcal{N}_{odd}} h_{\sum,i}(x_i^p, x_{i-1}^{p-1}, t_{k+1})\\ &+ \sum_{i\in\mathcal{N}_{even}} h_{\sum,i}(x_i^{p-1}, x_{i-1}^{p}, t_{k+1})\\ =& J^{odd}(t_{k+1},p),\end{aligned} \tag{20}$$

where the inequality comes from the optimality as the states x_i^p, $i \in \mathcal{N}_{even}$ are the optimal solutions of OCP $\mathcal{P}_i(t_{k+1})$, $i \in \mathcal{N}_{even}$ at the p-th iteration. By the similar argument, it yields that

$$\begin{aligned}J^{odd}(t_{k+1},p) =& \sum_{i\in\mathcal{N}_{odd}} h_{\sum,i}(x_i^p, x_{i-1}^{p-1}, t_{k+1})\\ &+ \sum_{i\in\mathcal{N}_{even}} h_{\sum,i}(x_i^{p-1}, x_{i-1}^{p}, t_{k+1})\\ \le& \sum_{i\in\mathcal{N}_{odd}} h_{\sum,i}(x_i^{p-1}, x_{i-1}^{p-1}, t_{k+1})\\ &+ \sum_{i\in\mathcal{N}_{even}} h_{\sum,i}(x_i^{p-1}, x_{i-1}^{p-1}, t_{k+1})\\ =& J^{even}(t_{k+1},p-1) + h_{\sum,1}(x_1^{p-1}, x_0^{p-1}, t_{k+1})\\ =& J(t_{k+1},p-1).\end{aligned} \tag{21}$$

Inequalities (20)-(21) together imply that

$$J(t_{k+1},p) \le J(t_{k+1},p-1). \tag{22}$$

and further

$$J(t_{k+1},\bar{p}) \le J(t_{k+1},0). \tag{23}$$

Step (2): Let $\Delta J(t_k,t_{k+1})$ as the difference between the Lyapunov candidate at the 0-th iteration of time t_{k+1} and the Lyapunov candidate at the $\bar{p}$-th iteration of time t_k.

$$\begin{aligned}&\Delta J(t_k,t_{k+1})\\ &\triangleq J(t_{k+1},0) - J(t_k,\bar{p})\\ &= \sum_{i=1}^{N} h_{\sum,i}(x_i^0, x_{i-1}^0, t_{k+1}) - \sum_{i=1}^{N} h_{\sum,i}(x_i^{\bar{p}}, x_{i-1}^{\bar{p}}, t_k)\end{aligned}$$

Based on the definition of $\hat{\mathbf{x}}_i(t_{k+1})$ in (12), $\Delta J(t_k,t_{k+1})$ can be rewirren as

$$\begin{aligned}&\Delta J(t_k,t_{k+1})\\ =& \sum_{i=1}^{N}\Big(h_i(\hat{x}_i, \hat{x}_{i-1}, t_{k+H}|t_{k+1}) - h_i(x_i^{\bar{p}}, x_{i-1}^{\bar{p}}, t_k|t_k)\\ &+ h_{f,i}(\hat{x}_i, \hat{x}_{i-1}, t_{k+1+H}|t_{k+1}) - h_{f,i}(x_i^{\bar{p}}, x_{i-1}^{\bar{p}}, t_{k+H}|t_k)\Big)\\ =& -\sum_{i=1}^{N} h_i(x_i^{\bar{p}}, x_{i-1}^{\bar{p}}, t_k|t_k) + \delta_h,\end{aligned} \tag{24}$$

where

$$\begin{aligned}\delta_h =& \sum_{i=1}^{N}\Big(h_i(\hat{x}_i, \hat{x}_{i-1}, t_{k+H}|t_{k+1}) - h_{f,i}(x_i^{\bar{p}}, x_{i-1}^{\bar{p}}, t_{k+H}|t_k)\\ &+ h_{f,i}(\hat{x}_i, \hat{x}_{i-1}, t_{k+1+H}|t_{k+1})\Big).\end{aligned}$$

From the condition (iv) in Assumption 3.1, δ_h in (24) is non-positive and it holds that

$$\Delta J(t_k,t_{k+1}) \le -\sum_{i=1}^{N} h_i(x_i^{\bar{p}}, x_{i-1}^{\bar{p}}, t_k|t_k) \le 0,$$

thereby having

$$J(t_{k+1},0) \le J(t_k,\bar{p}). \tag{25}$$

Taking (25) together with (23) implies the Lyapunov candidate function is decreasing, that is

$$J(t_{k+1},\bar{p}) \le J(t_k,\bar{p}). \tag{26}$$

From the definition of $J(t_k,p)$, it is obvious that $J(t_k,p) \ge 0$ for any sampling instant t_k and any iterations p due to the positive definiteness of symmetric matrices Q_i and P_i, $i \in \mathcal{N}$. The relation (26) therefore suffices to conclude the asymptotic stability of tracking error of each vehicle to 0 in the closed-loop operation, that is, the closed-loop platoon control for each vehicle $i, i \in \mathcal{N}$ is asymptotically stable under the application of Algorithm 1, such that

$$x_i(t_k) - x_{i-1}(t_k) + d_{i,i-1} \to 0,\ t_k \to \infty. \tag{27}$$

Furthermore, since the desired trajectory $x_{\text{des},i}(t_k)$ for vehicle i is $x_0(t_k) - d_{i,0}$, it can be derived that

$$\begin{aligned}x_i(t_k) =& x_i(t_k) - x_0(t_k) + d_{i,0} + x_{\text{des},i}(t_k)\\ =& \sum_{j=1}^{i}\Big(x_j(t_k) - x_{j-1}(t_k) + d_{j,j-1}\Big) + x_{\text{des},i}(t_k)\\ \to& x_{\text{des},i}(t_k),\ t_k \to \infty.\end{aligned}$$

Hence, under Algorithm 1, the state trajectory of vehicle $i, i \in \mathcal{N}$, asymptotically converges to the desired trajectory $x_{\text{des},i}(t_k)$. ■

From (27), it is immediate to obtain that each individual control objective of (6a)-(6c) is achieved.

Remark 4.1: The stability established in Theorem 4.3 will remain true if some vehicles decide not to optimize control inputs at a certain iteration of time t_k instead using the solution of the last iteration. For example, at time t_k, when some vehicles solve their OCPs $\mathcal{P}_i(t_k)$, $i \in \mathcal{N}$ at the p-th iteration, the other vehicles choose not to update control inputs. This may be due to the temporary communication interruption or limited computing resources. In this scenario, each vehicle which updates its control input obtains its latest trajectories $\mathbf{x}_i^p(t_k)$ and $\mathbf{u}_i^p(t_k)$ by solving its OCP $\mathcal{P}_i(t_k)$, and each vehicle which does not optimize its control input keeps its last calculated trajectories $\mathbf{x}_i^{p-1}(t_k)$ and $\mathbf{u}_i^{p-1}(t_k)$, i.e., $\mathbf{x}_i^p(t_k) = \mathbf{x}_i^{p-1}(t_k)$ and $\mathbf{u}_i^p(t_k) = \mathbf{u}_i^{p-1}(t_k)$. Choose Lyapunov candidate function as $J(t_k,p)$. It is obvious that inequalities (20), (21) and (25) are still valid, and thus (26), which implies the Lyapunov function $J(t_k,p)$ is decreasing as

t_k increases. Thus, the same conclusion about the asymptotic stability of the closed-loop state trajectory can be obtained. Hence, the number of iterations can be arbitrary and different for all vehicles without any effect on stability.

V. Numerical Examples

To illustrate the performance of Algorithm 1, numerical simulations are implemented on a heterogeneous vehicle platoon with a leader vehicle and 7 following vehicles. The heterogeneous engine time constants τ_i of all vehicles $i \in \mathcal{N}^+$ are listed in TABLE I [35], and the sampling interval is set as $\Delta t = 0.1s$. Based on the given platoon system parameters,

TABLE I: VEHICLE PARAMETERS IN THE PLATOON

Label	0	1	2	3	4	5	6	7
$\tau_i(s)$	0.65	0.51	0.75	0.78	0.70	0.73	0.72	0.62

to satisfy Assumption 3.1, inequality (14) can be transformed into a linear matrix inequality (LMI) problem

$$\begin{bmatrix} -X & A_D X + B_D L \\ (A_D X + B_D L)^\top & -\alpha^2 X \end{bmatrix} \preceq 0,$$

where $X \triangleq P^{-1}$, $L \triangleq K_f X$ and $\alpha = 1$. Once we obtain appropriate P and K_f, it is easy to compute matrix Q by (15). The invariant set $\mathcal{E}$ is then designed as $\mathcal{E} \triangleq \{e \in \mathbb{R}^{3N} \mid \sum_{i=1}^{N} e_i^\top P_i e_i \leq \gamma\}$ with $\gamma = 252$, and the constant β is set as $\beta = 1.082$ according to design rules presented in subsection III-C. Sets in constraints (5a)-(5d) are specified by

$$\begin{aligned} &\Delta\mathcal{S}_i = \{\Delta s \mid -8 \leq \Delta s \leq 8\}, \ \forall i \in \mathcal{N}, \\ &\mathcal{V}_0 = \{v \mid 2.4 \leq v \leq 29.6\}, \\ &\mathcal{V}_i = \{v \mid 0 \leq v \leq 32\}, \ \forall i \in \mathcal{N}, \\ &\mathcal{A}_i = \{a \mid -6 \leq a \leq 6\}, \ \forall i \in \mathcal{N}, \\ &\mathcal{U}_i = \{u \mid -20 \leq u \leq 20\}, \ \forall i \in \mathcal{N}. \end{aligned}$$

The prediction horizon H is set to 20 and the running step number is set to 100, i.e., $10s$. The initial state of the leader vehicle is $x_0(t_0) = [0, 20, 0]^\top$, which means the leader vehicle drives at the origin $s_0(t_0) = 0m$ with uniform velocity $v_0(t_0) = 20m/s$ and zero acceleration $a_0(t_0) = 0m/s^2$ at time t_0. The desired distance headway is given by $\bar{d}_{i,i-1} = 20m$ for all $i \in \mathcal{N}$. It is assumed a consensus condition where the initial state of the platoon is set as the desired state with zero tracking error, that is

$$x_i(t_0) - x_{i-1}(t_0) + d_{i,i-1} = 0, \ \forall i \in \mathcal{N}.$$

A case as shown in Fig. 2 is adopted where the leader alters its uniform velocity from $20m/s$ to $23m/s$ while satisfying constraints (5a)-(5d) within $2s$ starting at time t_0, that is, $v_0(t_0) = 20m/s$ and $v_0(t_{H+k}) = 23m/s$, $k = 0, 1, \ldots$.

The following computations are all done using Yalmip toolbox [40] and Gurobi 9.5.0 solver [41] in Matlab.

A. Platoon Control Performance

Fig. 3 demonstrates the tracking errors between vehicle i and the leader vehicle under the control of Algorithm 1 in non-iterative mode (i.e., the maximum iterations $\bar{p} = 1$).

Fig. 2: Velocity change of the leader vehicle from $20m/s$ to $23m/s$ while satisfying constraints (5a)-(5d) starting at time t_0.

The position error $s_i(t_k) - s_0(t_k) + \bar{d}_{i,0}$, the velocity error $v_i(t_k) - v_0(t_k)$, and the acceleration error $a_i(t_k) - a_0(t_k)$ (plotted respectively in 3 subfigures from the top to the bottom) all converge to zero. Meanwhile, it is indicated that constraints (11b)-(11h) in problem $\mathcal{P}_i(t_k)$ are satisfied for all vehicles. Hence, the control objective (6a)-(6c) is achieved. In Fig. 4, the tracking errors in iterative mode (the maximum iterations $\bar{p} = 3$) are plotted. The platoon transient performance in the control process is obviously improved in iterative mode referring to the significantly reduced dynamic variation of the tracking errors on position, velocity and acceleration compared with the simulation result with $\bar{p} = 1$.

The tracking errors under the algorithm in [35] executed with the predecessor following topology and the same vehicle model parameters, coefficient matrices and initial conditions are shown in Fig. 5. The convergence speed of the tracking errors is slightly faster than the result under Algorithm 1 in non-iterative mode $\bar{p} = 1$. It is natural because a strict equality terminal constraint is utilized in [35] to ensure some theoretical results, rather than an inequality terminal constraint used in the proposed method, which enables the feasibility properties at the price of slightly degraded performance. Nevertheless, compared with the result shown in Fig. 4, the dynamic variation of the tracking errors in Fig. 5 changes more.

Furthermore, the first N steps of the DMPC algorithm in [35] can not be proved feasible, which means that the optimization problem may have no solution and worse still the platoon may be uncontrollable. Consider a scenario where the leader vehicle, within $2s$, speeds up to $25m/s$ (instead of $23m/s$) from $20m/s$. Fig. 6 demonstrates the tracking errors under the control of Algorithm 1 in non-iterative mode $\bar{p} = 1$, however the optimization problem in [35] more often than not has no solution, which could lead to instability and even collision of the platoon control of heterogenous vehicles.

B. Verification of recursive feasibility

Since spacing constraint (5a) is not taken into account and handled in [35], it cannot always ensure vehicle i following the predecessor vehicle $i-1$ with a suitable distance. Consider

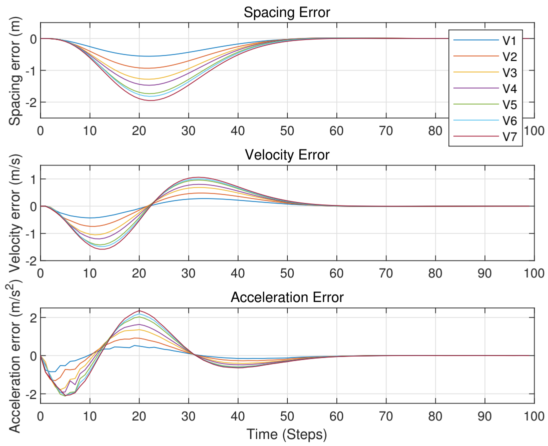

Fig. 3: State tracking error trajectories of all following vehicles tracking the leader vehicle under Algorithm 1 in non-iterative mode $\bar{p} = 1$, where the leader vehicle alters its uniform velocity from $20m/s$ to $23m/s$ within $2s$. Top: Spacing error; Middle: Velocity error; Bottom: Acceleration error.

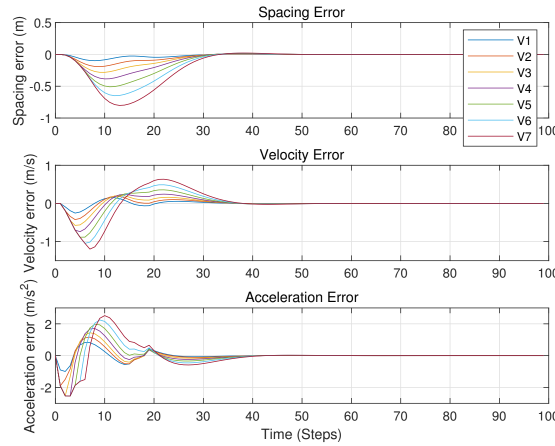

Fig. 5: State tracking error trajectories of all following vehicles tracking the leader vehicle under Algorithm in [35], where the leader vehicle alters its uniform velocity from $20m/s$ to $23m/s$ within $2s$. Top: Spacing error; Middle: Velocity error; Bottom: Acceleration error.

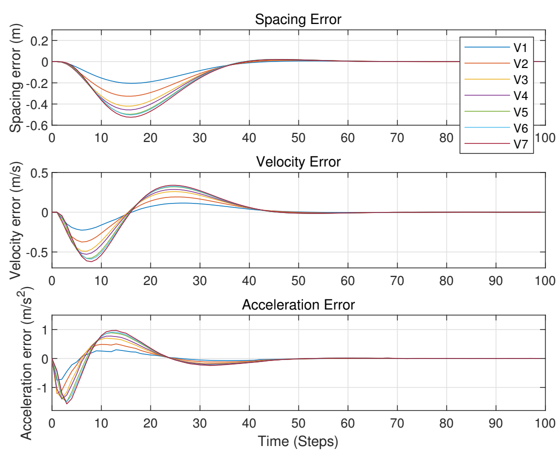

Fig. 4: State tracking error trajectories of all following vehicles tracking the leader vehicle under Algorithm 1 in iterative mode $\bar{p} = 3$, where the leader vehicle alters its uniform velocity from $20m/s$ to $23m/s$ within $2s$. Top: Spacing error; Middle: Velocity error; Bottom: Acceleration error.

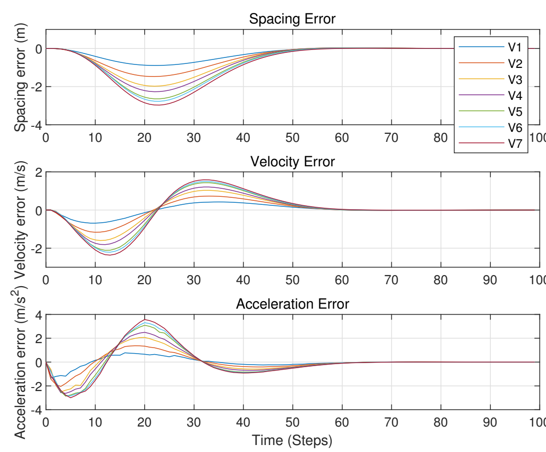

Fig. 6: State tracking error trajectories of all following vehicles tracking the leader vehicle under Algorithm 1 in non-iterative mode $\bar{p} = 1$, where the leader vehicle alters its uniform velocity from $20m/s$ to $25m/s$ within $2s$, and no control solution can be found when the algorithm in [35] is applied. Top: Spacing error; Middle: Velocity error; Bottom: Acceleration error.

an unconventional case, the initial state of the platoon is set as

$$s_i(t_0) - s_{i-1}(t_0) + d_{i,i-1} = \Delta s_i^{\max},\ \forall i \in \mathcal{N}.$$

For brevity, only the first 2 following vehicles are considered. Fig. 7 demonstrates the tracking errors under the control of the algorithm in [35], where the black dotted line in the top subfigure is the maximum admissible spacing error between vehicles 1 and 2. Hence at the previous 20 time steps, constraint (5a) is violated by vehicle 2 as the red line is higher than the black dotted line. However, under the control of Algorithm 1, constraint (5a) can be always satisfied, and simulation example in Fig. 8 verifies the result.

C. Maximum allowable velocity change of leader vehicle

This case study investigates the maximum velocity change of the leader vehicle that can be addressed by the proposed method. Suppose in a consensus platoon case where all tracking errors are zero at time t_k, then the leader vehicle alters its uniform velocity from v_0 to $\bar{v}_0$ in a horizon H. Considering an extreme case where the leader vehicle drives with the maximum input $u_0^{\max}$ until the acceleration increases up to the maximum value $a_0^{\max}$, then the leader vehicle drives with acceleration $a_0^{\max}$, and finally drives with the minimum input $u_0^{\min}$ until the acceleration drops back to zero, a curve of the

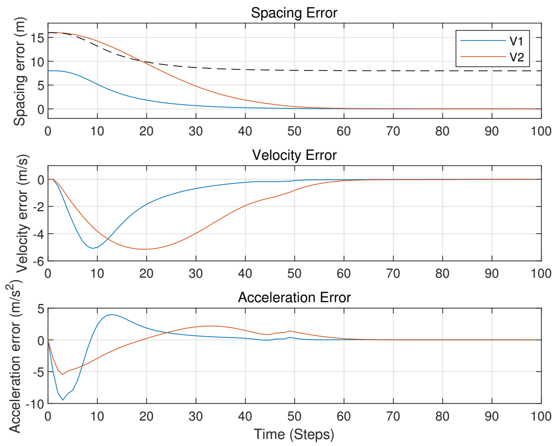

Fig. 7: State tracking error trajectories of two following vehicles tracking the leader vehicle under the algorithm in [35], where the leader vehicle drives with uniform velocity $20m/s$ and an unconventional case, in which the distance between all adjacent vehicles is set much closer at the beginning, is considered. Black dotted line is the maximum admissible spacing error between vehicle 1 and 2, and thus at the first 20 time steps, constraint (5a) is violated as the red line is higher than the black dotted line. Top: Spacing error; Middle: Velocity error; Bottom: Acceleration error.

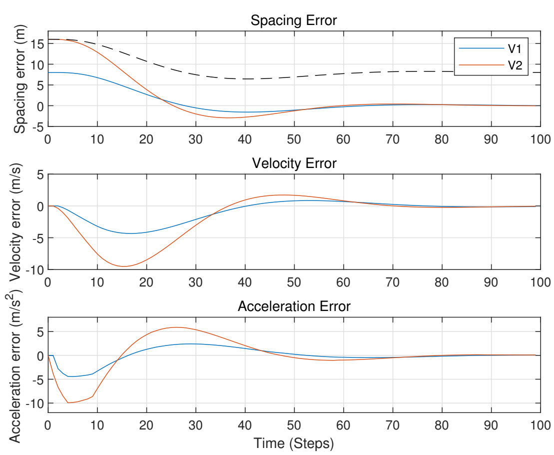

Fig. 8: State tracking error trajectories of two following vehicles tracking the leader vehicle under Algorithm 1 in non-iterative mode $\bar{p} = 1$, where the leader vehicle drives with uniform velocity $20m/s$ and the same unconventional case in Fig. 7 is considered. Constraint (5a) is always satisfied. Top: Spacing error; Middle: Velocity error; Bottom: Acceleration error.

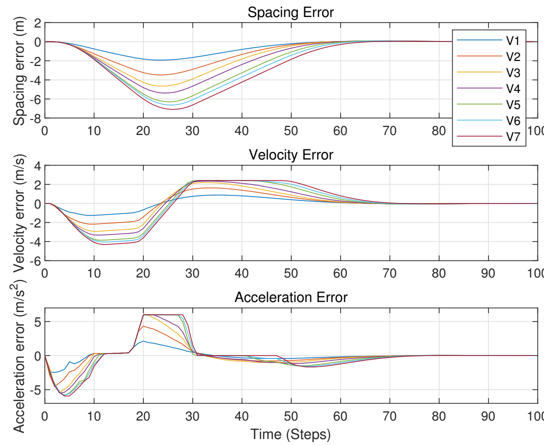

Fig. 9: State tracking error trajectories of all following vehicles tracking the leader vehicle under Algorithm 1 in non-iterative mode $\bar{p} = 1$, where the leader vehicle alters its uniform velocity from $20m/s$ to $29.6m/s$ within $2s$, and no control solution can be found when the algorithm in [35] is applied. Top: Spacing error; Middle: Velocity error; Bottom: Acceleration error.

leader vehicle's acceleration against time steps can be plotted. The maximum accelerated velocity $\Delta v_0^{\max} = \bar{v}_0^{\max} - v_0$ can be obtained by the integral of the plotted curve, given by

$$\begin{aligned}\Delta v_0^{\max} =& \frac{(N^+ + 1)N^+ u_0^{\max} - (N^- + 1)N^- u_0^{\min}}{2\tau_0}\Delta t^2 \\ &+ (H - N^+ - N^- - 1)a_0^{\max}\Delta t,\end{aligned}$$

where $H \geq N^+ + N^- + 2$, and

$$a_0^{\max} - \frac{u_0^{\max}}{\tau_0}\Delta t < \frac{u_0^{\max}}{\tau_0}N^+\Delta t \leq a_0^{\max},$$
$$a_0^{\max} + \frac{u_0^{\min}}{\tau_0}\Delta t < -\frac{u_0^{\min}}{\tau_0}N^-\Delta t \leq a_0^{\max}.$$

By the similar argument, the maximum decelerated velocity $\Delta v_0^{\min} = \bar{v}_0^{\min} - v_0$ is

$$\begin{aligned}\Delta v_0^{\min} =& \frac{(M^+ + 1)M^+ u_0^{\max} - (M^- + 1)M^- u_0^{\min}}{2\tau_0}\Delta t^2 \\ &+ (H - M^+ - M^- - 1)a_0^{\min}\Delta t,\end{aligned}$$

where $H \geq M^+ + M^- + 2$, and

$$a_0^{\min} \leq \frac{u_0^{\min}}{\tau_0}M^-\Delta t < a_0^{\min} - \frac{u_0^{\min}}{\tau_0}\Delta t$$
$$a_0^{\min} \leq -\frac{u_0^{\max}}{\tau_0}M^+\Delta t < a_0^{\min} + \frac{u_0^{\max}}{\tau_0}\Delta t.$$

Denote $\bar{\mathcal{V}}_0 \triangleq \{\bar{v}_0 \mid \Delta v_0^{\min} \leq \bar{v}_0 - v_0 \leq \Delta v_0^{\max}, \ \|[0, \bar{v}_0 - v_0, 0]^\top\|_{P_1} \leq \gamma\}$. The velocity v_0 of leader vehicle at time t_k can be accelerated (or decelerated) to any velocity $\bar{v}_0$ in set $\mathcal{V}_0 \cap \bar{\mathcal{V}}_0$. The velocity of the leader vehicle is accelerated to $29.6m/s$ in Fig. 9, and simulation example verifies the result. However no control solution can be found when the algorithm in [35] is applied.

VI. CONCLUSION

We have proposed a distributed MPC approach oriented toward heterogeneous platoon control with bidirectional topology and parallel computing tasks under the background of CACC. The platoon is dynamically decoupled, but constrained by both local state and input constraints and coupled inter-vehicular spacing constraints. To achieve distributed implementation, we design a two-level iterative control architecture. All vehicles with odd indices are located in the first level, which are simultaneously calculated in a parallel manner and sent the latest information to their neighboring vehicles with even indices in the second level. Vehicles with even indices then solve their local optimization problems in parallel by constructing and utilizing the neighbor's reference state trajectories and return their solutions to the first level. All intermediate results can be stored in memory to speed up the data acquisition at next iteration/time step. The recursive feasibility at all time steps including the first N steps is ensured by employing a tailored terminal set for the tracking error dynamics. The asymptotic stability of the proposed algorithm is also proved to ensure all vehicles can drive with a desired distance and the same velocity. Numerical simulation results demonstrate the efficacy of the proposed distributed control scheme and the improvement in terms of the guarantee of recursive feasibility and the flexibility of the leader vehicle's velocity changes.

Further research is extending the algorithm to a distributed robust MPC scheme for the platoon of heterogeneous vehicles with nonlinear dynamics and uncertainties. Other potential issues include dealing with time delay and packet loss in the communication between vehicles.

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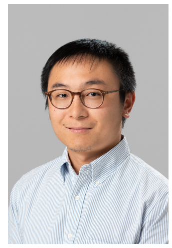

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