An Extended Version of the ISRS GN model in Closed-Form Accounting for Short Span Lengths and Low Losses

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Abstract *A closed-form formula for the nonlinear interference (NLI) estimation of arbitrary modulation formats, supporting short span lengths and low losses in ultra-wideband optical transmission systems is presented. The formula is tested over 20 THz and accurately estimates the NLI at every point of the fibre span.*

Introduction

To address the current capacity limitations in the installed networking infrastructure, new technologies are being explored in optical communications. These include extending the optical transmission bandwidth beyond the conventionally-used C+L band[1]–[3], resulting in ultra-wideband (UWB) transmission. Associated with this, research in intelligent network planing tools is being carried out[4] and an effective approach is to bring physical layer awareness to the control plane level[5]. Therefore, an efficient, fast and accurate model to estimate nonlinear interference (NLI) at *every step* of the fibre span is essential.

For UWB systems, the inter-channel stimulated Raman scattering (ISRS) effect must be considered in the estimation of the NLI. To enable real-time prediction of the UWB system performance, formulations in closed-form are needed. These formulations must offer a fast, yet accurate, evaluation of the network characteristics, and can be widely used for network optimisation purposes[1],[2]. Numerous closed-form equations for the Gaussian Noise (GN) model[6] have been proposed to account for the ISRS effect[7] in[8]–[11]. A limitation of all these works is that the proposed formulas do not account for short span lengths and extremely low-losses, due to the approximations made to derive them. The closed-form formulas proposed in[12],[13] do account for short span lengths and extremely low-losses but do not include the ISRS effect, and hence are not suitable for UWB modeling.

In this paper, we propose a new closed-form formula, which is obtained by removing one of the main approximations used in deriving the formulas in[9],[10]. Enabled by this, we derive, for the first time, a closed-form expression capable of accurately estimating the NLI in the presence of ISRS for any fibre span length and for fibres with extremely low-losses (~ 0.02 dB/km). The proposed closed-form expression accounts for all modulation formats, wavelength-dependent attenuation and dispersion, and its accuracy is compared with the ISRS GN model in integral form[7],[10].

The closed-form formula

For any optical fibre link, the signal-to-noise ratio for the channel i (SNR_i) at the end of the span after amplification can be estimated as

$$\text{SNR}_i^{-1} \approx \text{SNR}_{\text{NLI},i}^{-1} + \text{SNR}_{\text{ASE},i}^{-1} + \text{SNR}_{\text{TRX},i}^{-1}, \quad (1)$$

where $\text{SNR}_{\text{NLI},i}$, $\text{SNR}_{\text{ASE},i}$ and $\text{SNR}_{\text{TRX},i}$ originate from fibre nonlinearity, amplifier noise and transceiver noise, respectively. This work is devoted to the calculation of $\text{SNR}_{\text{NLI},i}$.

Let $\tilde{T}_k = -\frac{P_{\text{tot}} C_r}{\tilde{\alpha}} f_k$, $T = 1 + \tilde{T}_k$, $\phi = -4\pi^2 (f_1 - f_i)(f_2 - f_i)[\beta_2 + \pi\beta_3(f_1 + f_2)]$ and $\alpha_l = \alpha + l\tilde{\alpha}$. By assuming the normalised power evolution along the fibre $\rho(z, f_i)$ as a semi-analytical solution of the Raman differential equations[14],[15], the so-called link function[6] of the ISRS GN model in closed-form is approximated as[9],[10]

$$\mu(f_1, f_2, f_i) \approx \left| -T \sum_{0 \le l \le 1} \left(\frac{-\tilde{T}_i}{T} \right)^l \left(\frac{1 - e^{-(\alpha_l - j\phi)L}}{-\alpha_l + j\phi} \right) \right|^2, \quad (2)$$

where β_2 and β_3 are, respectively, the group velocity dispersion (GVD) parameter and its linear slope, L is the span length, P_{tot} is the total launch power, f_i is the frequency of the channel of interest (COI), α is the fibre loss, C_r is the slope of the Raman gain spectrum and $\tilde{\alpha}$ models the gain/loss due to the ISRS effect along the fibre. Note that, the last three parameters (α, $\tilde{\alpha}$ and C_r)

$$
\begin{aligned}
\mathrm{SNR}_{\mathrm{NLI},i}^{-1} \approx T \sum_{\substack{0\le l\le 1\\ 0\le l'\le 1}} \left(\frac{-\tilde{T}_i}{T}\right)^{l+l'} \kappa_l\kappa_{l'} \Bigg(\Bigg\{ \frac{16}{27}\frac{\pi\gamma^2 P_i^2 n^{1+\epsilon}}{B_i^2 2C_l\phi_i} \\
\times \left[\frac{B_l+2C_l}{\sqrt{D_l+B_lC_l}}\mathrm{asinh}\left(\sqrt{\frac{D_l+B_lC_l}{2B_l^2C_l^2+A_l^2+2B_lC_lD_l}}\frac{3\phi_iB_i^2}{8\pi}\right) - \frac{B_l-2C_l}{\sqrt{D_l-B_lC_l}}\mathrm{asinh}\left(\sqrt{\frac{D_l-B_lC_l}{2B_l^2C_l^2+A_l^2-2B_lC_lD_l}}\frac{3\phi_iB_i^2}{8\pi}\right)\right]\Bigg\} \\
+\frac{32}{27}\sum_{k=1,k\neq i}^{N_{\mathrm{ch}}} \frac{\gamma^2P_k^2}{B_k}\Bigg\{\frac{n+\frac{5}{6}\Phi}{2C_l\phi_{i,k}}\left[\frac{(B_l+2C_l)\mathrm{atan}(\frac{\phi_{i,k}B_i}{2\sqrt{D_l+B_lC_l}})}{\sqrt{D_l+B_lC_l}} - \frac{(B_l-2C_l)\mathrm{atan}(\frac{\phi_{i,k}B_i}{2\sqrt{D_l-B_lC_l}})}{\sqrt{D_l-B_lC_l}}\right] \\
+\frac{5}{6}\frac{\Phi 2\pi\tilde{n}}{|\phi|\,B_k^2A_l}\left[(2\,|f_k-f_i|-B_k)\ln\left(\frac{2\,|f_k-f_i|-B_k}{2\,|f_k-f_i|+B_k}\right)+2B_k\right]\Bigg\}\Bigg).
\end{aligned}
\tag{4}
$$

are channel dependent and matched using nonlinear least-squares fitting to reproduce the true power profile, which is obtained by numerically solving the differential Raman equations[15].

If the approximation $e^{-\alpha_l L} \ll 1$ is assumed in Eq. (2), such that $1-e^{-(\alpha_l - j\phi)L} \approx 1$, the closed-form formulas published in[10],[16],[17] are obtained. This assumption is generally satisfied for relatively long span lengths and high losses. In order to remove the above-mentioned limitation and obtain a set of new closed-form formulas which accurately account for any spans lengths and any values of fibre loss in the presence of ISRS, we follow the approach in[12], and approximate the fraction presented in Eq. 2 as

$$
\frac{1-e^{-(\alpha_l-j\phi)L}}{-\alpha_l+j\phi} \approx \frac{\kappa_l}{-\tilde{a}_l+j\phi}, \tag{3}
$$

where κ_l and $\tilde{a}_l$ are chosen such that the first-order Taylor approximation of both the left and the right side of Eq. (3) around the variable $\phi = 0$ become equals. This yields

$$
\tilde{a}_l = \frac{\alpha_l(1-e^{-\alpha_l L})}{1-e^{-\alpha_l L}-\alpha_l L e^{-\alpha_l L}}
$$

and

$$
\kappa_l = \frac{\tilde{a}_l(1-e^{-\alpha_l L})}{\alpha_l}.
$$

The proposed approximation presented in Eq. (3) captures the effect of the attenuation in the oscillatory term $e^{-(\alpha_l-j\phi)L}$. This would also be important when modeling links employing backward Raman amplification, as a similar term arising in such cases must also be taken into account[18].

Inserting the approximation from Eq. (3) into Eq. (2), the $\mathrm{SNR}_{\mathrm{NLI},i}$ can be calculated as Eq. (4). In this equation, $\phi = -4\pi^2\left[\beta_2+\pi\beta_3(f_i+f_k)\right]L$, $\phi_i = -4\pi^2\left(\beta_2+2\pi\beta_3 f_i\right)$, $\phi_{i,k} = -4\pi^2\left(f_k-f_i\right)\left[\beta_2+\pi\beta_3\left(f_i+f_k\right)\right]$, Φ is the excess kurtosis of the modulation format, n is the number of spans, $\tilde{n} = \{0 \text{ for } n=1\ ,\ n \text{ for } n>1\}$, P_i is the channel launch power with bandwidth B_i, γ is the nonlinear coefficient, N_{ch} is the number of channels, ϵ is the coherent factor[6], $A_l = \tilde{a}_l\tilde{a}_{l'}$, $B_l = \tilde{a}_l - \tilde{a}_{l'}$, $C_l = \frac{1}{2}\sqrt{4A_l+B_l^2}$ and $D_l = \frac{1}{2}(2A_l+B_l^2)$. Note that, in the limit of $\alpha_l L \to \infty$, Eq. (4) converges to the closed-form formula reported in[17].

Transmission system setup

The transmission system under consideration is similar to that in[17] and consists of a WDM transmission with N_{ch}=451 Nyquist spaced channels centered on 1540 nm. Each channel was modulated at the symbol rate of 40 GBd. This resulted in a total bandwidth of 20 THz (158 nm), ranging from 1470 nm to 1615 nm, corresponding to the transmission over the S- (1470 nm - 1530nm), C- (1530 nm - 1565nm) and L- (1565 nm - 1615nm) bands. Spectral gaps of 10 nm and 5 nm were considered between the S-/C- and C-/L- bands, respectively. The channels were transmitted over 5 x 20 km spans using a single-mode fibre (SMF). A spectrally uniform launch power profile was used, where each channel carries -2 dBm. The Raman gain spectrum and the wavelength-dependent attenuation were measured from a Corning© SMF-28© ULL fibre and are shown in[17]. Dispersion and nonlinearity parameters were $D = 18\ \frac{\mathrm{ps}}{\mathrm{nm\cdot km}}$, $S = 0.067\ \frac{\mathrm{ps}}{\mathrm{nm^2\cdot km}}$ and $\gamma = 1.2\frac{1}{\mathrm{W\cdot km}}$.

Results

The $\mathrm{SNR}_{\mathrm{NLI}}$ for each WDM channel is shown in Fig. 1 for the transmission setup described in the previous section. The transmission system performance estimation using the proposed closed-form formula, i.e, Eq. (4), are shown for two cases: Gaussian and 64-QAM constellations. The accuracy of Eq. (4) is compared with the ISRS GN model in integral form, for both Gaussian constellation[7] and arbitrary modulation formats[10]. The closed-form formula proposed in[17]

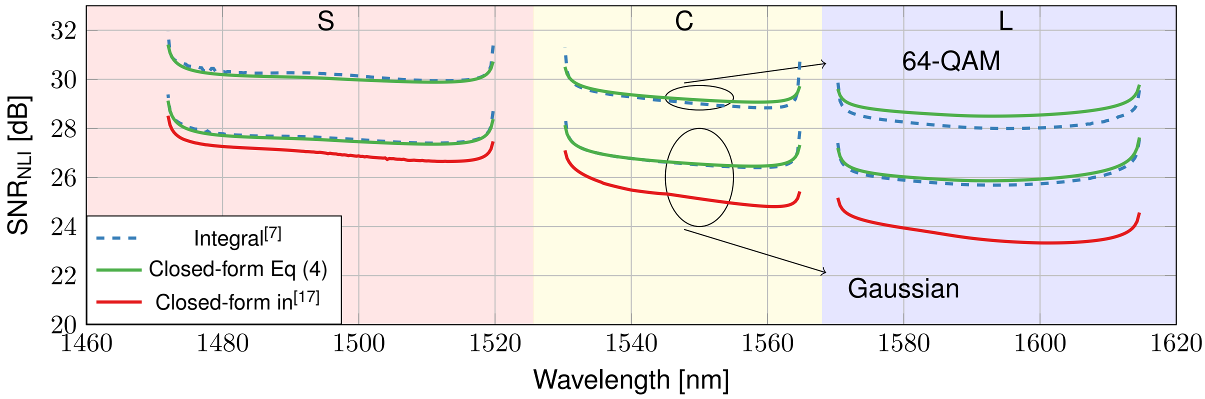

Fig. 1: Nonlinear performance after 5 x 20 km transmission.

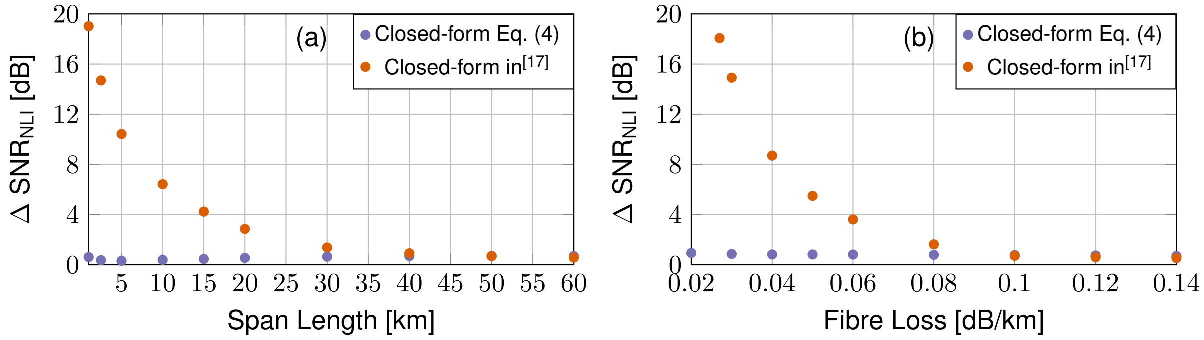

Fig. 2: Maximum per-channel $\mathrm{SNR_{NLI}}$ difference ($\Delta\mathrm{SNR_{NLI}}$) between the integral ISRS GN model and the proposed closed-form formula in Eq. (4) (purple points) for different (a) span lengths and (b) fibre losses. The $\Delta\mathrm{SNR_{NLI}}$ using the formula in[17] are also shown for comparison (orange points).

is shown for Gaussian constellations for comparison (the 64-QAM curve obtained using this expression was even less accurate).

The interaction between fibre attenuation, dispersion and normalised ISRS-power evolution profile, leads to the $\mathrm{SNR_{NLI}}$ profile as shown in Fig. 1. The high dispersion towards the L-band reduces the NLI for the long-wavelength channels. This reduction however is counterbalanced by the ISRS-transferred power, increasing the NLI for these channels, reducing the $\mathrm{SNR_{NLI}}$. For the Gaussian and 64-QAM constellations respectively, maximum errors of 0.55 dB and 1 dB between the proposed closed-form formula and the integral ISRS GN model were found showing good accuracy in estimating the NLI. For the closed-form formula in[17] a maximum error of 3 dB was found for Gaussian constellations; this error is larger towards long-wavelength channels because the wavelength-depend attenuation for these channels is lower, due to the ISRS-transferred power, breaking the assumption $e^{-\alpha_l L} \ll 1$.

To validate the accuracy of the proposed closed-form formula, the previous simulation scenario has been varied in two different ways: (a) the span length was swept from 1km to 60 km and (b) the span length was fixed at 80 km and a spectrally uniform loss profile ranging from 0.02 dB/km to 0.14 dB/km was considered. Fig. 2 shows the maximum per-channel $\mathrm{SNR_{NLI}}$ difference, i.e, the maximum per-channel error in terms of $\mathrm{SNR_{NLI}}$ between the integral ISRS-GN model and the proposed closed-form formula. The same analysis using the closed-form expression reported in[17] is also shown for comparison. The results were obtained considering Gaussian constellations. As shown in Fig. 2, the new closed-form formula proposed in this paper can accurately account for any span lengths and low losses; among all the scenarios considered in Fig. 2, maximum errors of 0.7 dB and 0.94 dB were found respectively when considering different span lengths and losses.

Conclusions

A closed-form formula that can accurately evaluate the NLI in the presence of ISRS at any step of the fibre span and in extremely low loss regimes (~ 0.02 dB/km) is proposed. The formula was applied in modeling an S+C+L band (20 THz) transmission system and validations were carried out using integral model simulations; the proposed closed-form formula estimates the NLI in a few microseconds, and is thus suitable for effective and intelligent UWB network planning tools and rapid performance evaluations.

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References

[1] H. Buglia, E. Sillekens, A. Vasylchenkova, *et al.*, "Challenges in extending optical fibre transmission bandwidth beyond c+l band and how to get there", in *2021 International Conference on Optical Network Design and Modeling (ONDM)*, 2021, pp. 1–4. DOI: 10.23919/ONDM51796.2021.9492434.

[2] H. Buglia, E. Sillekens, A. Vasylchenkova, P. Bayvel, and L. Galdino, "On the impact of launch power optimization and transceiver noise on the performance of ultra-wideband transmission systems", *J. Opt. Commun. Netw.*, vol. 14, no. 5, B11–B21, May 2022. DOI: 10.1364/JOCN.450726.

[3] A. Vasylchenkova, E. Sillekens, R. I. Killey, and P. Bayvel, "Mutual shaping and pre-emphasis gain magnification in the throughput maximisation for ultrawideband transmission", in *Optical Fiber Communication Conference (OFC) 2022*, Optica Publishing Group, 2022, Th1H.4. DOI: 10.1364/OFC.2022.Th1H.4.

[4] R. Matzner, D. Semrau, R. Luo, G. Zervas, and P. Bayvel, "Making intelligent topology design choices: Understanding structural and physical property performance implications in optical networks", *J. Opt. Commun. Netw.*, vol. 13, no. 8, pp. D53–D67, Aug. 2021.

[5] D. Ives, S. Yan, L. Galdino, *et al.*, "Distributed abstraction and verification of an installed optical fibre network", *Scientific Reports*, vol. 11, p. 10 750, May 2021. DOI: 10.1038/s41598-021-89976-w.

[6] P. Poggiolini, "The GN model of non-linear propagation in uncompensated coherent optical systems", *Journal of Lightwave Technology*, vol. 30, no. 24, pp. 3857–3879, 2012. DOI: 10.1109/JLT.2012.2217729.

[7] D. Semrau, R. I. Killey, and P. Bayvel, "The Gaussian noise model in the presence of inter-channel stimulated Raman scattering", *Journal of Lightwave Technology*, vol. 36, no. 14, pp. 3046–3055, 2018. DOI: 10.1109/JLT.2018.2830973.

[8] P. Poggiolini, M. R. Zefreh, G. Bosco, F. Forghieri, and S. Piciaccia, "Accurate non-linearity fully-closed-form formula based on the GN/EGN model and large-data-set fitting", in *2019 Optical Fiber Communications Conference and Exhibition (OFC)*, 2019, pp. 1–3.

[9] D. Semrau, R. I. Killey, and P. Bayvel, "A closed-form approximation of the Gaussian noise model in the presence of inter-channel stimulated raman scattering", *Journal of Lightwave Technology*, vol. 37, no. 9, pp. 1924–1936, 2019. DOI: 10.1109/JLT.2019.2895237.

[10] D. Semrau, E. Sillekens, R. I. Killey, and P. Bayvel, "A modulation format correction formula for the Gaussian noise model in the presence of inter-channel stimulated Raman scattering", *Journal of Lightwave Technology*, vol. 37, no. 19, pp. 5122–5131, 2019. DOI: 10.1109/JLT.2019.2929461.

[11] M. R. Zefreh, F. Forghieri, S. Piciaccia, and P. Poggiolini, "A closed-form nonlinearity model for forward-Raman-amplified WDM optical links", in *Optical Fiber Communication Conference (OFC) 2021*, Optica Publishing Group, 2021, p. M5C.1. DOI: 10.1364/OFC.2021.M5C.1.

[12] M. R. Zefreh and P. Poggiolini, "A GN-model closed-form formula supporting ultra-low fiber loss and short fiber spans", *arXiv:2111.04584 [eess.SP]*, 2021.

[13] W. Klaus and P. J. Winzer, "Hollow-core fiber capacities with receiver noise limitations", in *2022 Optical Fiber Communications Conference and Exhibition (OFC)*, 2022, pp. 1–3.

[14] D. Christodoulides and R. Jander, "Evolution of stimulated raman crosstalk in wavelength division multiplexed systems", *IEEE Photonics Technology Letters*, vol. 8, no. 12, pp. 1722–1724, 1996. DOI: 10.1109/68.544731.

[15] M. Zirngibl, "Analytical model of Raman gain effects in massive wavelength division multiplexed transmission systems", *Electronics Letters*, vol. 34, no. 8, pp. 789–790, 1998.

[16] D. Semrau, R. I. Killey, and P. Bayvel, "A closed-form approximation of the Gaussian noise model in the presence of inter-channel stimulated Raman scattering", *Journal of Lightwave Technology*, vol. 37, no. 9, pp. 1924–1936, 2019. DOI: 10.1109/JLT.2019.2895237.

[17] D. Semrau, L. Galdino, E. Sillekens, D. Lavery, R. I. Killey, and P. Bayvel, "Modulation format dependent, closed-form formula for estimating nonlinear interference in S+C+L band systems", in *45th European Conference on Optical Communication (ECOC 2019)*, 2019, pp. 1–4. DOI: 10.1049/cp.2019.0892.

[18] D. Semrau, G. Saavedra, D. Lavery, R. I. Killey, and P. Bayvel, "A closed-form expression to evaluate nonlinear interference in Raman-amplified links", *Journal of Lightwave Technology*, vol. 35, no. 19, pp. 4316–4328, 2017. DOI: 10.1109/JLT.2017.2741439.