Multipath Routing for Multipartite State Distribution in Quantum Networks

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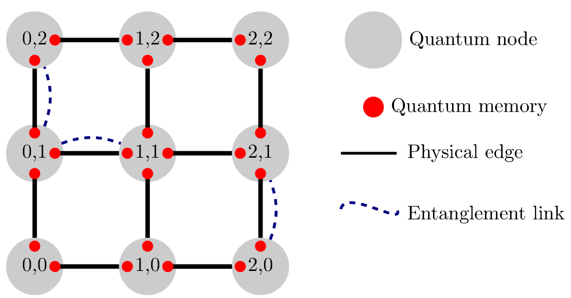
Abstract: Multipath routing for multipartite state distribution is proposed. Compared to shortest path routing, multipath routing achieved exponential rate improvement and an observed 6000× speedup on error-prone grid networks.

1. Introduction

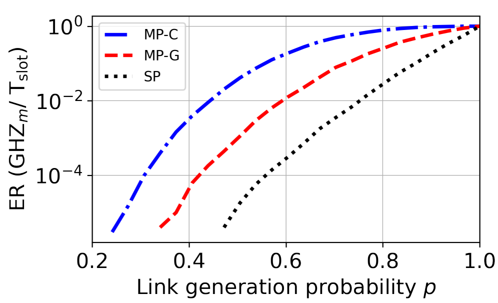
Many quantum applications utilise shared multipartite states, such as distributed quantum computation [1]. However, distributing entanglement over a network is challenging, due to the difficulty of entangling distant qubits, and short memory decoherence time [2]. Previous work on entanglement distribution between two users, has shown that multipath routing increases long distance entanglement rate (ER), by utilising more of the network capacity [3]. In this paper we present for the first time, multipath routing for multipartite state distribution. Results - obtained for different bipartite ER, quantum memory decoherence time, and network sizes - show that multipath routing exponentially increases multipartite distribution rate on grid topologies, compared to single path routing. The improvement of multipath routing over single path was highest, with a simulated 6000× ER speedup, for low entanglement success probability and short decoherence times. Results also show that the protocols improved by attempting entanglement over multiple timesteps, which was not implemented in previous work.

2. Quantum Network Model

Consider a quantum network defined by a graph $G(V,E)$, of nodes V and edges E. Nodes hold a quantum memory for each connected edge, as shown in Figure 1a where a 3×3 grid topology is depicted. The networks temporal evolution is modelled using discrete time slots of duration T_{slot}. Memory decoherence is modelled by a cut-off time T_c, expressed in the ratio $Q_c = T_c/T_{\text{slot}}$ after which the qubit in the memory is discarded [2]. Edges are fiber optic channels over which bipartite entanglement or *links* can be generated. Further links cannot be attempted on an edge while a link is present, as the quantum memories are already in use. Each node can perform entanglement swapping and multipartite state fusion [4]. Entanglement swapping allows long distant entanglement distribution, by combining entanglements at each node along a route. Multipartite state fusion for this report refers to a projective measurement onto the GHZ_m basis. The target multipartite state is assumed to be the m-qubit Greenberger–Horne–Zeilinger state $\text{GHZ}_m = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes m} + |1\rangle^{\otimes m})$, but the results are valid for general multipartite states. The fusion operation can combine multiple entangled bipartite pairs into the target GHZ_m states.



(a)



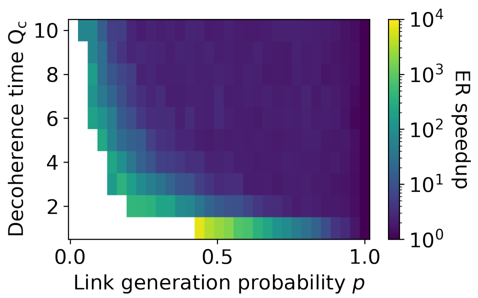
(b)

Fig. 1: a) Example 3×3 grid topology. b) ER with p, 5×5 grid, $Q_c = 1$ for a GHZ_5 state.

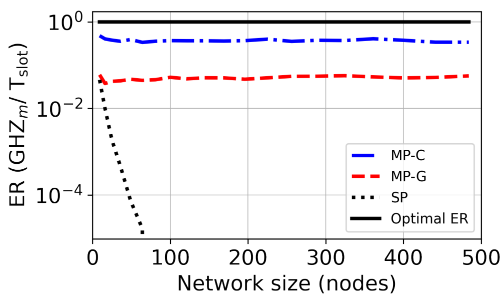
Bipartite protocols share a two-qubit entangled state, between two distant nodes through a network. This is done by entanglement swapping, which transforms a route of point-to-point links between the nodes into a single long distance entanglement [2]. In shortest path (SP) routing links are only attempted along a single shortest path. For multipath routing (MP) links are attempted on multiple or all edges in the network and once a valid route is generated, the entanglement swapping is performed [3, 5]. For multipartite protocols the aim is to distribute a m-qubit multipartite state between m nodes from a set of n nodes $\{V_1, V_2, \ldots, V_n\}$, where $n \geq m$. Two protocol variants - cooperative (MP-C) and greedy (MP-G) - of multipath routing for multipartite state distribution are proposed. For both, entanglement links are attempted on all edges in the network in each timestep. Links are successful with probability p and any current links older than Q_c are discarded. In the MP-C variant, links are attempted until there exists a connecting route of links between all destination nodes, when the target multipartite state is then created. For the MP-G protocol, as soon as a route is feasible between a source node V_1 and destination V_i, a long-distance bipartite entanglement is created between V_1-V_i. When all destinations share entanglement with V_1, a fusion operation combines the individually (entangled) states into the multipartite state. Both variants are compared to multipartite SP routing, which creates entanglement along edge-disjointed routes from a source node to each destination, e.g. as in [4].

3. Results

The performance of the protocols were evaluated, using a Monte Carlo simulation, in terms of the rate GHZ_m/T_{slot} of GHZ_m states generated per time-slot. In a 5×5 grid topology a 5-qubit multipartite state was shared between the node $V_{2,2}$ with the corner nodes $\{V_{0,0}, V_{0,4}, V_{4,0}, V_{4,4}\}$, where $V_{i,j}$ is node on row i and column j. Figure 1b shows the rate against link generation probability p for $Q_c = 1$. This value of Q_c means links must be consumed on the round they are generated. Both multipath variants provide significant improvement compared to SP. The MP-C protocol achieved the highest rate for all values of p. To study the impact of different values of Q_c, a parameter sweep of p and Q_c was performed for the same topology. Utilising multiple timeslots achieves a higher average rate (not shown due to length constraint) compared to single timeslot protocols, when $Q_c = 1$, for all protocols. Figure 2a shows the speedup of the MP-C protocol over the SP protocol. The MP-G protocol achieved speedup up to 120× lower than the MP-C protocol. Multipath routing increased entanglement for all values of p and Q_c, but the largest improvement, of up to a 6000× speedup, was observed for small p and Q_c values. As these are the likely hardware constraints of early quantum networks, multipath routing will be essential for multipartite state distribution.



(a)



(b)

Fig. 2: a) ER for MP-C over SP against p and Q_c, on a 5×5 grid. b) ER for grid topologies, $p = 0.675$, $Q_c = 1$.

The protocols were also tested on grid topologies of increasing size, sharing the state between the center and corner nodes. Figure 2b shows that the multipath protocols were able to achieve a rate independent of the network size for $p = 0.675$. This can be described in terms of percolation theory, where the majority of nodes form a single connected cluster above a critical link probability, which for grid lattices is $p \geq 0.5$ [6]. In contrast, the rate achieved by shortest path routing decreased exponentially, due to the increasing route length for larger networks.

References

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