# The Liquidity Channel of Fiscal Policy ${ }^{\star}$ 

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#### Abstract

Expansionary fiscal policy lowers the return difference between public debt and less liquid assets - the liquidity premium. We rationalize this finding in an estimated heterogeneousagent New-Keynesian model with incomplete markets and portfolio choice, in which public debt affects private liquidity. This liquidity channel stabilizes fixed-capital investment. We then quantify the long-run effects of higher public debt and find little crowding out of capital, but a sizable decline of the liquidity premium, which increases the fiscal burden of debt. The revenue-maximizing level of public debt is positive and has increased to 60 percent of US GDP post-2010.


Keywords: Fiscal Policy and Public Debt, Incomplete Markets, Liquidity Premium, HANK, Impulse Response Matching
JEL-Codes: C11, D31, E21, E32, E63

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## 1. Introduction

In response to the 2020 recession, governments have issued substantial public debt to finance large-scale transfers and government spending. With public debt climbing to levels unprecedented in peacetime, it has become a pressing issue to understand the effects of public debt on the economy and particularly on government bond yields-both in the short run and in the long run. In this, an essential aspect of public debt is its role as private liquidity (Woodford, 1990). Liquidity, throughout this paper, is understood as an asset's usefulness for self-insurance against idiosyncratic income shocks. Assets differ in this regard, for example, because of transaction costs, because of taxation, or because of thinner markets. Consequently, households require an expected return premium to hold less liquid assets relative to more liquid ones - a liquidity premium. In the present paper, we quantify how this liquidity premium responds to public debt supply: first, showing empirically that fiscal policy has a sizable impact on the return differences between public debt and less liquid assets, and, second, rationalizing and analyzing this finding using a monetary business cycle model with heterogeneous agents and incomplete markets (a "HANK model") in which public debt provides private liquidity and asset classes vary in their degree of liquidity.

Concretely, we first estimate the effects of an increase in public debt, induced by a spending shock, using local projections (Jordà, 2005). Importantly, this goes beyond the effects on aggregates and looks at the return premia of various assets. The estimation uses quarterly data from the US as well as annual international data. An increase in public debt via higher government spending decreases the excess return of less liquid assets over public debt. The effect is sizable. For a one percent increase in US public debt, it ranges from a two basis points (annualized) lower yield premium of AAA-corporate bonds to a 35 basis points lower premium on real estate - always relative to a long-term government-bond yield. We are, to our knowledge, the first to provide evidence for this differential effect of fiscal shocks on asset returns. International data corroborates the US evidence. It also allows us to exploit cross-country heterogeneity. Countries that rely more heavily on deficits to finance spending also see a larger decline of the liquidity premium to a government spending shock.

Next, we extend the heterogeneous-agent New-Keynesian model of Bayer et al. (2020), introducing financial intermediation and long-term bonds, and estimate it using Bayesian impulse response matching (Christiano et al., 2010). This model is well-suited to study fiscal policy featuring all frictions of the Smets and Wouters (2007) model, as well as self-insurance, the private creation of liquid assets through financial intermediation, and portfolio choice between assets of different liquidity. Therefore, fiscal policy operates through more than the traditional Keynesian channels because it additionally affects the liquidity premium.

When the government runs a larger deficit, it provides the economy with a greater supply
of liquid savings devices on top of the pre-existing public and private debt. Households hold these additional assets only when their return gets closer to the one of illiquid assets. Hence, equilibrium real interest rates on liquid and illiquid assets are a function of public debt in circulation. The model matches the local projections well and, hence, provides a good laboratory to study the importance of this liquidity channel of fiscal policy.

Looking at short-run changes in government spending shows that, in the model, the liquidity premium falls after an expansionary fiscal policy shock. The magnitude ( -7 basis points after a 1 percent increase in public debt) is broadly in line with our empirical evidence from the local projections. In addition, the decrease in the liquidity premium is stronger the less tax-financed the spending shock is - in line with the international evidence.

In the short run, this movement in the liquidity premium increases the economy's response to the fiscal stimulus. Fiscal multipliers are larger in the economy with an endogenous liquidity premium relative to the same economy with a constant one. There are two forces behind this result. First, the increase in liquidity improves the self-insurance of households overall, boosting consumption. Second, as liquid and illiquid assets are imperfect substitutes, an increase in public debt does not one-for-one substitute physical assets as savings devices. As a result, there is less crowding out of investment, making the response to stimulus stronger. This has persistent effects on the capital stock, and the cumulative fiscal multipliers of both models diverge as the time horizon increases.

Importantly for current debates, the model also allows us to study persistent changes in public debt, where the evidence from local projections does not allow us to make predictions. In particular, we ask how an increase in public debt affects interest rates in the long run and, in addition, what effects such a policy has on the capital stock and inequality. We consider a quasi-permanent increase in the debt target (debt-to-GDP ratio) by 10 percent, stretched over 20 years. The debt increase is paid out as non-distortionary transfers. This fiscal policy increases the real rate (permanently) by 25 basis points (annualized). ${ }^{1}$ The return on the illiquid asset, by contrast, moves very little. This affects the relative incentives to save for the rich (who mostly save illiquid) and the poor (who mostly save liquid) asymmetrically. As a result, the increase in debt persistently lowers wealth inequality.

Our model also allows us to study how changes in the US economy post-2010 might affect the response of the real bond rate to fiscal expansions. Higher inequality, be it because of higher markups or higher income risk, increases the interest rate elasticity slightly. Higher discount factors slightly lower it. Changes in the provision of private liquidity also have an effect and, specifically, the expansion of private credit lowers the elasticity. Overall, there is no strong evidence for a substantially different interest rate elasticity post-2010.

[^1]The fact that public debt and fixed capital are imperfect substitutes from the household's point of view is behind both, the pronounced interest-rate response and the limited capital crowding out. If all assets are equally liquid and hence perfect substitutes, as in the standard incomplete markets setup (see, e.g., Aiyagari and McGrattan, 1998), there is more crowding out and a smaller movement in the interest rate. If on top, there are complete markets, such that there is a representative agent and Ricardian equivalence, a debt increase has neither an effect on interest rates nor aggregates if financed by changes in non-distortionary taxes.

As the crowding out of capital by public debt is smaller compared to the standard incomplete-markets setup, the government can substantially increase the capital stock if it uses the receipts from issuing public debt to foster fixed-capital investment. This is modeled as a sovereign wealth fund. Such an extension of the government's balance sheet drives down the liquidity premium and increases output and capital in the long run. As wages increase and the return on capital falls, the economy becomes more equal. However, in the estimated model the necessary increase in bond yields on outstanding public debt dominates what the government can earn as return on the additional capital. Hence, taxes need to increase slightly in the long run to finance the sovereign wealth fund.

This statement depends crucially on the initial amount of outstanding public debt because bond rates are a function of the latter. This implies a Laffer-curve relationship. The government can earn a form of "liquidity tax", the difference between the bond rate and the economy's growth rate, if bonds are scarce (c.f. Bassetto and Cui, 2018; Blanchard, 2019; Reis, 2021). Lowering public debt decreases the "tax base" of this tax such that the revenue, the product of the two, falls once public debt becomes very scarce. Expressed conversely, the maximal amount the government can earn from rolling over debt is positive but finite. Using our approximation for the US, the revenue-maximizing public-debt level was around 60 percent of GDP for the last decade. Any target level below this number provides less liquidity to the private sector and fewer revenues to the government at the same time. Historically, however, this critical debt level has been, with 20 percent of GDP, much smaller. Our model predicts that in the last decade, a debt-to-GDP ratio of 160 percent would have achieved a zero interest-rate-growth differential.

With these results, we contribute to three literatures. First, our approach is closely related to the recent literature on HANK models that quantitatively studies the importance of heterogeneity for business cycles and policy. ${ }^{2}$ To our knowledge, our paper is the first to use a two-asset HANK model to investigate the liquidity channel of fiscal policy. Auclert

[^2]et al. (2018) and Hagedorn et al. (2019) also study fiscal multipliers but do so in models without portfolio choice. We show that the liquidity channel of public debt amplifies the multiplier obtained in models with perfectly liquid capital.

Second, the two-asset structure is also crucial for the long run as it significantly changes the extent to which public debt crowds out fixed capital. With perfectly liquid capital, such as in Aiyagari and McGrattan (1998), there is much stronger crowding out of capital through public debt. This key point has already been emphasized by Woodford (1990). However, much of this literature has focused on the optimal level of public debt with perfectly liquid capital. ${ }^{3}$ Our analysis is positive and adds to this literature by quantifying the importance of liquidity in the presence of illiquid capital in an estimated model that matches micro and macro moments of the data as well as the short-run response of the economy to a public debt injection. We share this focus on dynamics with Heathcote (2005) and Challe and Ragot (2011). The former looks at tax shocks in a calibrated Aiyagari (1994) model, and the latter at government spending shocks in a tractable model with incomplete markets.

Finally, our paper provides new empirical evidence on the effect of public debt on differential asset returns. Several papers have documented that higher debt tends to raise government bond rates (see, e.g., Brook, 2003; Engen and Hubbard, 2004; Kinoshita, 2006; Laubach, 2009). Our approach goes beyond what this literature has done, by showing that bond rates and returns on less liquid assets are affected differently. ${ }^{4}$ This focus is shared with Krishnamurthy and Vissing-Jorgensen (2012), who document the unconditional evolution of various asset returns relative to US debt. Our analysis complements theirs by conditioning on identified fiscal shocks, by comparing to international data, and by adding returns to fixed capital and housing, as well as interpreting the findings through the lens of a DSGE model.

The remainder of this paper is organized as follows: Section 2 provides evidence for the liquidity channel using identified fiscal policy shocks and a flexible local projection technique to identify their dynamic effects. Section 3 describes our model economy, its sources of fluctuations, and its frictions. Section 4 discusses the parameters calibrated to match steadystate targets and the parameter estimated to match the local projections. Section 5 discusses the short-run dynamics of the estimated model and how they fit with our local-projection estimates from Section 2. Section 6 then asks what the model implies for the fiscal burden of changes in public debt levels in the long run. Section 7 concludes.

[^3]
## 2. Evidence from Local Projections

We start by documenting that fiscal expansions affect aggregate quantities and the return differences between public debt and less liquid assets. Subsection 2.1 focuses on the US case, for which a variety of liquidity premia and identification approaches are available. International evidence in Subsection 2.2 corroborates the US findings.

In understanding the effects of an expansion of public debt, a difficulty arises from the fact that most changes in public debt are endogenous responses to other shocks. For example, public debt might increase in a recession when tax revenues decline. Therefore, it is necessary to look at exogenous changes in government spending or taxes-for which identification approaches are established in the literature - that increase public debt in their aftermath. As a baseline, government spending shocks are identified by the assumption, dating back to Blanchard and Perotti (2002), that government spending is predetermined within the quarter. This identification strategy allows us to run the same local projections for the US and other countries. This is robust to using narratively identified spending and tax shocks for the US.

Government expenditure shocks are standardized so that the peak increase in public debt is one percent. Our focus is to look at the return differences between public debt and alternative assets. Of course, these returns include various premia, and a government spending shock potentially affects these returns through other channels than just through the supply of more debt. For this reason, we will later use an estimated structural model to isolate the effects of a public debt increase on the liquidity premium.

### 2.1. US Evidence

As discussed by Blanchard and Perotti (2002), the rationale for assuming that government spending is predetermined within the quarter is that it can only be adjusted subject to decision lags. Also, there is no automatic response since government spending does not include transfers or other cyclical items. Appendix B shows that the results remain if using military spending news à la Ramey (2011) to identify exogenous variation in government spending, as well as when studying increases in debt induced by narratively-identified exogenous tax changes à la Romer and Romer (2010).

Our empirical estimates are based on local projections à la Jordà (2005) estimated on quarterly US time series from 1947Q1 to 2015Q4. ${ }^{5}$ Letting $x_{t+h}$ denote the variable of interest in period $t+h$, we estimate how it responds to fiscal shocks in period $t$ on the basis of the

[^4]following specification:
\[

$$
\begin{equation*}
x_{t+h}=\beta_{0}+\beta_{1} t+\beta_{2} t^{2}+\psi_{h} \log g_{t}+\Gamma(L) Z_{t-1}+u_{t+h} . \tag{1}
\end{equation*}
$$

\]

Here, $g_{t}$ is real per capita government spending in period $t$, and $Z_{t-1}$ is a vector of control variables that always includes four lags of government spending, output, and debt (all three in real per capita terms), plus the real interest rate on long-term bonds and lags of the respective dependent variable if not already included. Under the Blanchard and Perotti (2002)-predeterminedness assumption, the coefficient $\psi_{h}$ provides a direct estimate of the impulse response at horizon $h$ to the government spending shock in $t .{ }^{6}$ The regression also includes linear and quadratic time trends, $t$ and $t^{2}$, respectively. The error term $u_{t+h}$ is assumed to have zero mean and strictly positive variance. Confidence bands are based on Newey and West (1987)-standard errors that are robust with respect to heteroskedasticity and serial correlation.

First, Figure 1(a) shows the responses of a number of standard macroeconomic variables. It reconfirms that our fiscal policy shocks yield sensible aggregate results. Depicted are impulse response functions (IRFs) to a positive government spending shock that is scaled so that the maximum response of public debt is 1 percent. Government spending itself increases and follows a hump-shaped pattern, while public debt increases persistently. Output increasesat least in the short run-and investment falls, while private consumption increases with a delay. Overall, as in Ramey (2016), fiscal spending shocks have a muted effect on aggregate quantities when considering the whole post-war period. The bottom-right panel of Figure 1(a) shows that the real long-term government bond rate increases by 25 basis points after the fiscal expansion. ${ }^{7}$

The novel contribution is to estimate the response of a variety of proxies for the liquidity premium, i.e., the difference in returns of less liquid assets and long-term government bonds. The liquidity premium in the top-center panel of Figure 1(b) is based on the return to all capital computed by Gomme et al. (2011). ${ }^{8}$ Next, the return on housing from Jordà et al. (2019) provides an alternative measure of illiquid asset returns to compute the premium (top-right panel). Similarly, the liquidity premium can also be computed based on AAArated corporate bonds (the convenience yield as in Krishnamurthy and Vissing-Jorgensen, 2012). Next, we look at the federal-funds rate minus bond returns to capture the return

[^5]Figure 1: Empirical Responses to a Fiscal Expansion (US)


Notes: Impulse responses to a government spending shock. IRFs based on Blanchard and Perotti (2002)-style recursive identification; IRFs scaled so that the maximum debt response is 1 percent. Light (dark) gray areas are 90 percent ( 68 percent) confidence bounds based on Newey and West (1987)-standard errors.
Panel (a) from top left to bottom right: Government spending, federal debt held by the public, gross national expenditures, investment, consumption, real return on long-term government bonds.
Panel (b) from top left to bottom right: Liquid assets: deposits plus directly held stocks and debt of households; liq. premium capital: rate of return on capital minus long-term gov. bond rate; liq. premium housing: rate of return on housing minus long-term gov. bond rate; liq. premium corp. bonds: AAA corporate bond yield minus long-term gov. bond rate; liq. premium money: gov. bond rate minus (shadow) federal funds rate. Equity premium: Return on stocks minus long-term gov. bond rate.
premium over even more liquid assets as in Bredemeier et al. (2022). Finally, the figure includes Shiller (2015)'s equity premium.

The fiscal expansion increases total liquid assets-i.e., deposits, stocks, and debt-held directly by households by up to 0.4 percent, see top-left panel of Figure 1(b), and goes along with a significant fall in all liquidity premium measures. The premia on capital and housing fall by around $20-35$ basis points. The convenience yield falls by 2 basis points, which is the most conservative measure of the liquidity premium because it looks at the spread between very similar financial assets - government and corporate bonds - that are highly marketable. The equity premium also falls somewhat, however much less than the liquidity premia (except for the convenience yield). This is important because all return-on-capital measures, of course, include other premia besides the one on liquidity. Note that our results do not contradict the findings in Bredemeier et al. (2022), who look at the excess return of bonds over more liquid assets and find that this premium goes up in fiscal expansions. We can replicate their finding as is apparent from the positive response of the liquidity premium of money over government bonds shown in the lower left panel of Figure 1(b). In summary, the return premia of less liquid assets over bonds decrease and the return premium of bonds over more liquid assets increases after a deficit-financed spending shock.

Given the debate on the potential forecastability of Blanchard-Perotti shocks (see, e.g., Ramey, 2011, 2016), we also consider an alternative estimation replacing $\log g_{t}$ in Equation (1) by the military spending news series from Ramey (2011), deflated by the GDP deflator, in Appendix B.1. The IRFs are again scaled so that the maximum response of public debt is 1 percent. Results for aggregates and premia can be found there - see Figures B.1(a) and B.1(b). The IRFs look very similar, with the fall in the liquidity premia being somewhat more drawn out but even slightly larger quantitatively in this specification.

Overall, this novel evidence shows that fiscal policy has sizable effects on the liquidity premium. Fiscal expansions drive down the excess returns on assets that are less liquid than government bonds. As shown later, our estimated model can replicate the sign and size of the empirical responses.

### 2.2. International Evidence

International panel data from the Jordà-Schularick-Taylor Macrohistory Database (Jordà et al., 2017) allow us to show that the response of US liquidity premia is not exceptional. What is more, we can exploit heterogeneity across countries relating the response of the liquidity premium to the amount of debt issued to finance the fiscal expansion. This relationship, as later shown, is also present in our model.

Besides containing a consistent set of macroeconomic aggregates, the database also con-

Figure 2: Evidence from Country Panel
(a) Pooled Empirical Responses to Fiscal Expansion


Notes: Impulse responses to a government spending shock. IRFs based on Blanchard and Perotti (2002)style recursive identification; IRFs scaled so that the maximum debt response is 1 percent. Light (dark) gray areas are 90 percent ( 68 percent) confidence bounds based on Driscoll and Kraay (1998)-standard errors. Liq. premium housing: rate of return on housing minus long-term gov. bond rate.
(b) Heterogeneity in Debt and Liquidity Premium Responses


Notes: Dots represent, for each country, the debt and liquidity premium responses in years 3 to 5 (left panel) and average responses from years 3 to 5 (right panel) to a 1-percent government spending shock, based on country-by-country local projections. Standard errors for the regression line in parentheses.
tains annual housing returns for 16 advanced economies. The panel starts in 1947 to exclude direct effects of the second world war, and the last year available in the dataset is 2016. ${ }^{9}$ We again run the local projection, Equation (1), now at the annual level with $Z_{t-1}$ containing the first lag of the same set of controls. Intercepts, linear and quadratic trends are allowed to vary across countries. Given the panel dimension, confidence bands are now based on Driscoll and Kraay (1998)-standard errors that are robust with respect to heteroskedasticity, serial correlation, and cross-sectional correlation.

Figure 2(a) shows the responses to a fiscal expansion that increases real per-capita debt

[^6]by 1 percent, based on the Blanchard and Perotti (2002)-style recursive identification. ${ }^{10}$ Note that the x-axis now represents years, not quarters. The fiscal expansion leads to a persistent build-up in debt and an increase in the real interest rate on long-term bonds by about 10 basis points. Reassuringly, in this post-war country panel, the liquidity premium falls by about 20-30 basis points.

What is more, the panel regression masks an important heterogeneity. Not all countries finance the increase in government spending to the same extent by raising public debt. Some countries finance spending hikes in a more balanced-budget manner. This difference in financing behavior allows us to look at the question at hand, i.e., how does an increase in public debt change liquidity premia, through yet another angle. We run the local projections country-by-country and plot in Figure 2(b) the change in the liquidity premium against the change in public debt around four years after the spending shock. The left panel shows the pooled responses for years 3,4 , and 5 . The right panel, given the noise in the estimation, shows the average responses between years 3 and 5 for each country. The four-year horizon roughly coincides with the average peak response in public debt and ensures that more direct effects of the government spending surprises have faded out.

In those countries in which public debt increases more, the liquidity premium also declines significantly more. The size of the effect is with $17-19$ basis points for a 1 percent increase in debt consistent with the estimate for the US in the previous subsection. Compared to Summers and Rachel (2019)'s long-run estimates for the effect of public debt on government bond yields, the estimated short-run response of the liquidity premium is rather on the high side. However, we later show that theory predicts an overshooting of the liquidity premium response on impact. In the model, the short-run response of the liquidity premium to a fiscal spending shock can easily be three times stronger than the long-run response to an increase in debt itself.

## 3. Model

The economy features a firm sector, a household sector, and a government sector. ${ }^{11}$ Price setting for the final goods as well as wage-setting by unions is subject to a pricing friction à la Calvo (1983). Households earn income from supplying (raw) labor and capital and from owning the firm sector, absorbing all its rents that stem from the market power of unions and final goods producers, and capital goods production.

[^7]The government sector runs both a fiscal authority and a monetary authority. The fiscal authority levies taxes on labor income and profits, issues long-term government bonds, and adjusts expenditures to stabilize debt in the long run and aggregate demand in the short run. The monetary authority controls the nominal interest rate on deposits and sets it according to a Taylor rule.

### 3.1. Households

The household sector is subdivided into two types of agents: workers and entrepreneurs. The transition between both types is stochastic. Both rent out physical capital, but only workers supply labor. The efficiency of a worker's labor evolves randomly, exposing workerhouseholds to labor-income risk. Entrepreneurs do not work but earn all pure rents and banking profits in our economy, except for the rents of unions which are equally distributed across workers. All households self-insure against the income risks they face by saving in a liquid nominal asset (deposits) and a less liquid asset (capital). Trading illiquid assets is subject to random participation in the capital market.

To be specific, there is a continuum of ex-ante identical households of measure one, indexed by $i$. They are infinitely lived, have time-separable preferences with discount factor $\beta$, and derive felicity from consumption $c_{i t}$ and leisure. They obtain income from supplying labor, $n_{i t}$, renting out capital, $k_{i t}$, and earning interest on deposits, $d_{i t}$, and potentially from profits or union transfers. Households pay taxes on labor and profit income.

### 3.1.1. Productivity, Labor Supply, and Labor Income

A household's gross labor income $w_{t} n_{i t} h_{i t}$ is composed of the aggregate wage rate on raw labor, $w_{t}$, the household's hours worked, $n_{i t}$, and its idiosyncratic labor productivity, $h_{i t}$. We assume that productivity evolves according to a $\log -\mathrm{AR}(1)$ process and a fixed probability of transition between the worker and the entrepreneur state:

$$
h_{i t}= \begin{cases}\exp \left(\rho_{h} \log h_{i t-1}+\epsilon_{i t}^{h}\right) & \text { with probability } 1-\zeta \text { if } h_{i t-1} \neq 0  \tag{2}\\ 1 & \text { with probability } \iota \text { if } h_{i t-1}=0 \\ 0 & \text { else, }\end{cases}
$$

The shocks $\epsilon_{i t}^{h}$ to productivity are normally distributed with constant variance. $h$ is rescaled to obtain an average productivity of 1 .

With probability $\zeta$ households become entrepreneurs $(h=0)$. With probability $\iota$ an entrepreneur returns to the labor force with median productivity. An entrepreneur obtains a fixed share of the pure rents (aside from union rents), $\Pi_{t}^{F}$, in the economy (from monopolistic competition in the goods sector, banking, and the creation of capital). We assume that the
claim to the pure rent cannot be traded as an asset. Union rents, $\Pi_{t}^{U}$ are distributed lumpsum across workers, leading to labor-income compression.

This modeling strategy serves two purposes. First and foremost, it generally solves the problem of the allocation of pure rents without distorting factor returns and without introducing another tradable asset. ${ }^{12}$ Second, we use the entrepreneur state in particular-a transitory state in which incomes are very high - to match the income and wealth distribution following the idea by Castaneda et al. (1998). The entrepreneur state does not change the asset returns or investment opportunities available to households.

Concerning leisure and consumption, households have Greenwood et al. (1988) ${ }^{13}$ (GHH) preferences and maximize the discounted sum of felicity:

$$
\begin{equation*}
\mathbb{E}_{0} \max _{\left\{c_{i t}, n_{i t}\right\}} \sum_{t=0}^{\infty} \beta^{t} u\left[c_{i t}-L\left(h_{i t}, n_{i t}\right)\right] . \tag{3}
\end{equation*}
$$

The maximization is subject to the budget constraints described further below. The felicity function $u$ exhibits a constant relative risk aversion (CRRA) of degree $\xi>0$,

$$
u\left(x_{i t}\right)=\frac{1}{1-\xi} x_{i t}^{1-\xi}
$$

where $x_{i t}=c_{i t}-L\left(h_{i t}, n_{i t}\right)$ is household $i$ 's composite demand for goods consumption $c_{i t}$ and leisure and $L$ measures the disutility from work. Goods consumption bundles varieties

[^8]$j$ of differentiated goods according to a Dixit-Stiglitz aggregator:
$$
c_{i t}=\left(\int c_{i j t}^{\frac{\eta_{t}-1}{\eta_{t}}} d j\right)^{\frac{\eta_{t}}{\eta_{t}-1}}
$$

Each of these differentiated goods is offered at price $p_{j t}$, so that for the aggregate price level, $P_{t}=\left(\int p_{j t}^{1-\eta_{t}} d j\right)^{\frac{1}{1-\eta_{t}}}$, the demand for each of the varieties is given by

$$
c_{i j t}=\left(\frac{p_{j t}}{P_{t}}\right)^{-\eta_{t}} c_{i t}
$$

The disutility of work, $L\left(h_{i t}, n_{i t}\right)$, determines a household's labor supply given the aggregate wage rate, $w_{t}$, and a labor income tax, $\tau$, through the first-order condition:

$$
\begin{equation*}
\frac{\partial L\left(h_{i t}, n_{i t}\right)}{\partial n_{i t}}=(1-\tau) w_{t} h_{i t} . \tag{4}
\end{equation*}
$$

When the Frisch elasticity of labor supply is constant, $\frac{\partial L\left(h_{i t}, n_{i t}\right)}{\partial n_{i t}}=(1+\gamma) \frac{L\left(h_{i t}, n_{i t}\right)}{n_{i t}}$ with $\gamma>0$, the disutility of labor is a constant fraction of labor income, which simplifies the expression for the composite consumption good $x_{i t}$, making use of the first-order condition (4):

$$
\begin{equation*}
x_{i t}=c_{i t}-L\left(h_{i t}, n_{i t}\right)=c_{i t}-\frac{(1-\tau) w_{t} h_{i t} n_{i t}}{1+\gamma} . \tag{5}
\end{equation*}
$$

Therefore, in both the household's budget constraint and its felicity function, only after-tax income enters and neither hours worked nor productivity appears separately.

This implies that we can assume $L\left(h_{i t}, n_{i t}\right)=h_{i t} \frac{i_{i t}^{1+\gamma}}{1+\gamma}$ without further loss of generality as long as we treat the empirical distribution of income as a calibration target. ${ }^{14}$ This functional form simplifies the household problem as $h_{i t}$ drops out and all households supply $n_{i t}=N\left(w_{t}\right)$. Total effective labor input, $\int n_{i t} h_{i t} d i$, is also equal to $N\left(w_{t}\right)$ because $\mathbb{E} h=1$.

### 3.1.2. Consumption, Savings, and Portfolio Choice

Given labor income, households optimize intertemporally. They make savings and portfolio choices between liquid deposits and illiquid capital in light of a capital market friction that renders participation in the capital market random and i.i.d. in the sense that only a fraction, $\lambda$, of households is selected to be able to adjust their capital holdings in a given period.

What is more, we assume that there is a wasted intermediation cost that drives a wedge,

[^9]$\bar{R}$, between the policy rate of the central bank $R_{t}^{b}$ and the interest paid by/to households $R_{t}$ on deposits, when households resort to unsecured borrowing. This means, we specify:
\[

R\left(d_{i t}, R_{t}^{d}\right)= $$
\begin{cases}R_{t}^{d} & \text { if } d_{i t} \geq 0 \\ R_{t}^{d}+\bar{R} & \text { if } d_{i t}<0\end{cases}
$$
\]

Therefore, the household's budget constraint reads:

$$
\begin{align*}
c_{i t}+d_{i t+1}+q_{t} k_{i t+1}= & (1-\tau)\left(h_{i t} w_{t} N_{t}+\mathbb{I}_{h_{i t} \neq 0} \Pi_{t}^{U}+\mathbb{I}_{h_{i t}=0} \Pi_{t}^{F}\right)+\mathcal{T}_{t}\left(h_{i t}\right)  \tag{6}\\
& +d_{i t} \frac{R\left(d_{i t}, R_{t}^{d}\right)}{\pi_{t}}+\left(q_{t}+r_{t}\right) k_{i t}, \quad d_{i t+1} \geq \underline{d}, \quad k_{i t+1} \geq 0,
\end{align*}
$$

where $\mathcal{T}_{t}$ is non-distortionary transfers, $\Pi_{t}^{U}$ is union profits, $\Pi_{t}^{F}$ is firm profits, $d_{i t}$ is real deposit holdings, $k_{i t}$ is the amount of illiquid assets, $q_{t}$ is the price of these assets, $r_{t}$ is their dividend, $\pi_{t}=\frac{P_{t}-P_{t-1}}{P_{t-1}}$ is realized inflation, and $R(\cdot)$ is the nominal interest rate schedule on deposits. All households that do not participate in the capital market $\left(k_{i t+1}=k_{i t}\right)$ still obtain dividends and can adjust their deposits. Depreciated capital has to be replaced for maintenance, such that the dividend, $r_{t}$, is the net return on capital. Deposits have to be above an exogenous debt limit $\underline{d}$, and holdings of capital have to be non-negative.

For simplicity, we summarize all effects of all aggregate state variables, including the distribution of wealth and income, by writing the dynamic planning problem with timedependent continuation values. This leaves us with three functions that characterize the household's problem: value function $V^{a}$ for the case where the household adjusts its capital holdings, the function $V^{n}$ for the case in which it does not adjust, and the expected value, $\mathbb{W}$, over both:

$$
\begin{align*}
V_{t}^{a}(d, k, h) & =\max _{k^{\prime}, d_{a}^{\prime}} u\left[x\left(d, d_{a}^{\prime}, k, k^{\prime}, h\right)\right]+\beta \mathbb{E}_{t} \mathbb{W}_{t+1}\left(d_{a}^{\prime}, k^{\prime}, h^{\prime}\right) \\
V_{t}^{n}(d, k, h) & =\max _{d_{n}^{\prime}} u\left[x\left(d, d_{n}^{\prime}, k, k, h\right)\right]+\beta \mathbb{E}_{t} \mathbb{W}_{t+1}\left(d_{n}^{\prime}, k, h^{\prime}\right)  \tag{7}\\
\mathbb{W}_{t+1}\left(d^{\prime}, k^{\prime}, h^{\prime}\right) & =\lambda V_{t+1}^{a}\left(d^{\prime}, k^{\prime}, h^{\prime}\right)+(1-\lambda) V_{t+1}^{n}\left(d^{\prime}, k^{\prime}, h^{\prime}\right)
\end{align*}
$$

Expectations about the continuation value are taken with respect to all stochastic processes conditional on the current states. Maximization is subject to (6).

### 3.2. Firm Sector

The firm sector consists of five sub-sectors: (a) a labor sector composed of "unions" that differentiate raw labor and labor packers who buy differentiated labor and then sell labor services to intermediate goods producers, (b) intermediate goods producers who hire labor
services and rent out capital to produce goods, (c) final goods producers who differentiate intermediate goods and then sell them to goods bundlers, who finally sell them as consumption goods to households, and to (d) capital goods producers, who turn bundled final goods into capital goods. Finally, a banking sector (e) issues deposits and invests the receipts in government bonds. Through arbitrage, the interest rate on deposits has to be equal to the policy rate set by the central bank.

When profit-maximization decisions in the firm sector require intertemporal decisions (price and wage setting and producing capital goods), we assume for tractability that they are delegated to a mass-zero group of households (managers) that are risk neutral and compensated by a share in profits. ${ }^{15}$ They do not participate in any asset market and have the same discount factor as all other households. Since managers are a mass-zero group in the economy, their consumption does not show up in any resource constraint and all but the unions' profits go to the entrepreneur households (whose $h=0$ ). Union profits go lump sum to worker households.

### 3.2.1. Labor Packers and Unions

Worker households sell their labor services to a mass-one continuum of unions indexed by $j$, each of which offers a different variety of labor to labor packers who then provide labor services to intermediate goods producers. Labor packers produce final labor services according to the production function

$$
\begin{equation*}
N_{t}=\left(\int \hat{n}_{j t}^{\frac{\zeta-1}{\zeta}} d j\right)^{\frac{\zeta}{\zeta-1}}, \tag{8}
\end{equation*}
$$

out of labor varieties $\hat{n}_{j t}$. Cost minimization by labor packers implies that each variety of labor, each union $j$, faces a downward-sloping demand curve

$$
\hat{n}_{j t}=\left(\frac{W_{j t}}{W_{t}^{F}}\right)^{-\zeta_{t}} N_{t}
$$

where $W_{j t}$ is the nominal wage set by union $j$ and $W_{t}^{F}$ is the nominal wage at which labor packers sell labor services to final goods producers.

Since unions have market power, they pay the households a wage lower than the price at which they sell labor to labor packers. Given the nominal wage $W_{t}$ at which they buy labor from households and given the nominal wage index $W_{t}^{F}$, unions seek to maximize their

[^10]discounted stream of profits. However, they face a Calvo-type (1983) adjustment friction with indexation with the probability $\lambda_{w}$ to keep wages constant. They therefore maximize
\[

$$
\begin{equation*}
\mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t} \lambda_{w}^{t} \frac{W_{t}^{F}}{P_{t}} N_{t}\left\{\left(\frac{W_{j t} \bar{\pi}_{W}^{t}}{W_{t}^{F}}-\frac{W_{t}}{W_{t}^{F}}\right)\left(\frac{W_{j t} \bar{\pi}_{W}^{t}}{W_{t}^{F}}\right)^{-\zeta}\right\} \tag{9}
\end{equation*}
$$

\]

by setting $W_{j t}$ in period $t$ and keeping it constant except for indexation to $\bar{\pi}_{W}$, the steadystate wage inflation rate.

Since all unions are symmetric, we focus on a symmetric equilibrium and obtain the linearized wage Phillips curve from the corresponding first-order condition as follows, leaving out all terms irrelevant at a first-order approximation around the stationary equilibrium:

$$
\begin{equation*}
\log \left(\frac{\pi_{t}^{W}}{\bar{\pi}_{W}}\right)=\beta \mathbb{E}_{t} \log \left(\frac{\pi_{t+1}^{W}}{\bar{\pi}_{W}}\right)+\kappa_{w}\left(\frac{w_{t}}{w_{t}^{T}}-\frac{\zeta-1}{\zeta}\right) \tag{10}
\end{equation*}
$$

with $\pi_{t}^{W}=\frac{W_{t}^{F}}{W_{t-1}^{F}}=\frac{w_{t}^{F}}{w_{t-1}^{F}} \pi_{t}^{Y}$ being wage inflation, $w_{t}$ and $w_{t}^{F}$ being the respective real wages for households and firms, and $\frac{\zeta}{\zeta-1}$ being the target mark-down of wages the unions pay to households, $W_{t}$, relative to the wages charged to firms, $W_{t}^{F}$ and $\kappa_{w}=\frac{\left(1-\lambda_{w}\right)\left(1-\lambda_{w} \beta\right)}{\lambda_{w}}$.

### 3.2.2. Final Goods Producers

Similar to unions, final goods producers differentiate a homogeneous intermediate good and set prices. They face a downward-sloping demand curve, $y_{j t}=\left(p_{j t} / P_{t}\right)^{-\eta_{t}} Y_{t}$, for each good $j$ and buy the intermediate good at the nominal price $M C_{t}$. As for the unions, we assume price adjustment frictions à la Calvo (1983) with indexation.

Under this assumption, the firms' managers maximize the present value of real profits given this price adjustment friction, i.e., they maximize:

$$
\begin{equation*}
\mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t} \lambda_{Y}^{t}(1-\tau) Y_{t}\left\{\left(\frac{p_{j t} \bar{\pi}_{Y}^{t}}{P_{t}}-\frac{M C_{t}}{P_{t}}\right)\left(\frac{p_{j t} \bar{\pi}^{t}}{P_{t}}\right)^{-\eta}\right\} \tag{11}
\end{equation*}
$$

with a time constant discount factor.
The corresponding first-order condition for price setting implies a Phillips curve

$$
\begin{equation*}
\log \left(\frac{\pi_{t}}{\bar{\pi}}\right)=\beta \mathbb{E}_{t} \log \left(\frac{\pi_{t+1}}{\bar{\pi}}\right)+\kappa_{Y}\left(m c_{t}-\frac{\eta-1}{\eta}\right) \tag{12}
\end{equation*}
$$

where we again dropped all terms irrelevant for a first-order approximation and have $\kappa_{Y}=$ $\frac{\left(1-\lambda_{Y}\right)\left(1-\lambda_{Y} \beta\right)}{\lambda_{Y}}$. Here, $\pi_{t}$ is the gross inflation rate of final goods, $\pi_{t}=\frac{P_{t}}{P_{t-1}}, m c_{t}=\frac{M C_{t}}{P_{t}}$ is the real marginal costs, $\bar{\pi}$ is steady-state inflation and $\frac{\eta}{\eta-1}$ is the target markup.

### 3.2.3. Intermediate Goods Producers

Intermediate goods are produced with a constant returns to scale production function:

$$
Y_{t}=N_{t}^{\alpha}\left(u_{t} K_{t}\right)^{1-\alpha}
$$

where $u_{t} K_{t}$ is the effective capital stock taking into account utilization $u_{t}$, i.e., the intensity with which the existing capital stock is used. Using capital with an intensity higher than normal results in increased depreciation of capital according to $\delta\left(u_{t}\right)=\delta_{0}+\delta_{1}\left(u_{t}-1\right)+$ $\delta_{2} / 2\left(u_{t}-1\right)^{2}$, which, assuming $\delta_{1}, \delta_{2}>0$, is an increasing and convex function of utilization. Without loss of generality, capital utilization in the steady state is normalized to 1 , so that $\delta_{0}$ denotes the steady-state depreciation rate of capital goods.

Let $m c_{t}$ be the relative price at which the intermediate good is sold to final goods producers. The intermediate goods producer maximizes profits, $m c_{t} Y_{t}-w_{t}^{F} N_{t}-\left[r_{t}+q_{t} \delta\left(u_{t}\right)\right] K_{t}$, where $r_{t}$ and $q_{t}$ are the rental rate and the price of capital goods, respectively. The intermediate goods producer is a price-taker in the factor markets, such that the real wage and the user costs of capital are given by the marginal products of labor and effective capital:

$$
\begin{align*}
w_{t}^{F} & =\alpha m c_{t}\left(\frac{u_{t} K_{t}}{N_{t}}\right)^{1-\alpha}  \tag{13}\\
r_{t}+q_{t} \delta\left(u_{t}\right) & =u_{t}(1-\alpha) m c_{t}\left(\frac{N_{t}}{u_{t} K_{t}}\right)^{\alpha} \tag{14}
\end{align*}
$$

We assume that utilization is decided by the owners of the capital goods, taking the aggregate supply of capital services as given. The optimality condition for utilization is

$$
\begin{equation*}
q_{t}\left[\delta_{1}+\delta_{2}\left(u_{t}-1\right)\right]=(1-\alpha) m c_{t}\left(\frac{N_{t}}{u_{t} K_{t}}\right)^{\alpha} \tag{15}
\end{equation*}
$$

i.e., capital owners increase utilization until the marginal maintenance costs equal the marginal product of capital services.

### 3.2.4. Capital Goods Producers

Capital goods producers take the relative price of capital goods, $q_{t}$, as given in deciding about their output, $I_{t}$, i.e., they maximize

$$
\begin{equation*}
\mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t} I_{t}\left\{q_{t}\left[1-\frac{\phi}{2}\left(\log \frac{I_{t}}{I_{t-1}}\right)^{2}\right]-1\right\} \tag{16}
\end{equation*}
$$

Optimality requires (again dropping all terms irrelevant up to first order)

$$
\begin{equation*}
q_{t}\left[1-\phi \log \frac{I_{t}}{I_{t-1}}\right]=1-\beta \mathbb{E}_{t}\left[q_{t+1} \phi \log \left(\frac{I_{t+1}}{I_{t}}\right)\right], \tag{17}
\end{equation*}
$$

and each capital goods producer will adjust its production until (17) is fulfilled.
Since the producers are symmetric, we obtain as the law of motion for aggregate capital

$$
\begin{equation*}
K_{t}-\left(1-\delta\left(u_{t}\right)\right) K_{t-1}=\left[1-\frac{\phi}{2}\left(\log \frac{I_{t}}{I_{t-1}}\right)^{2}\right] I_{t} . \tag{18}
\end{equation*}
$$

The functional form assumption implies that investment adjustment costs are minimized and equal to 0 in steady state.

### 3.2.5. Banks

Finally, banks issue deposits, the total of which is $D_{t}$, that are held by the households. They invest the receipts into government bonds. ${ }^{16}$ The interest rate they pay on deposits, $R_{t}^{d}$, is effectively set by the central bank. Bonds are long-term and we model their maturity as a geometric decay. Each bond pays one nominal unit as interest and then each period a fraction $\delta_{B}$ of government bonds retire (without repaying the principal). Bonds are traded by banks at the nominal price $q^{B}$ and through perfect competition between banks (which we assume risk neutral), the equilibrium condition is given by

$$
\begin{equation*}
R_{t+1}^{d} q_{t}^{B}=\mathbb{E}_{t} q_{t+1}^{B}\left(1-\delta_{B}\right)+1 \tag{19}
\end{equation*}
$$

On the left-hand side is the expected nominal return on the deposits necessary to buy one government bond and on the right-hand side is the expected nominal return of that investment. The value of deposits $D_{t+1}$ issued at $t$ and redeemable (nominal) in period $t+1$ has to be equal to the market value of the banks investment $B_{t+1}$ at time $t$, the market value of government debt. The promised coupons on government debt payable next period are $\frac{B_{t}}{q_{t}^{b}}$.

Ex-post, banks make profits/losses when interest rates and therefore bond prices change. The per-period real profit of a bank is given by

$$
\begin{equation*}
\Pi_{t}^{B}=\frac{B_{t}}{\pi_{t}}\left[\left(1-\delta_{B}\right) \frac{q_{t}^{B}}{q_{t-1}^{B}}+\frac{1}{q_{t-1}^{B}}-R_{t}^{d}\right] . \tag{20}
\end{equation*}
$$

Finally, the expected yield on the government bond is given by $R_{t+1}^{b}=1-\delta_{B}+\frac{1}{q_{t}^{B}}$.

[^11]
### 3.3. Government

Monetary policy sets the nominal interest rate following a Taylor-type (1993) rule with interest rate smoothing:

$$
\begin{equation*}
\frac{R_{t+1}^{d}}{\bar{R}^{d}}=\left(\frac{R_{t}^{d}}{\bar{R}^{d}}\right)^{\rho_{R}}\left(\frac{\pi_{t}}{\bar{\pi}}\right)^{\left(1-\rho_{R}\right) \theta_{\pi}}\left(\frac{Y_{t}}{Y_{t-1}}\right)^{\left(1-\rho_{R}\right) \theta_{Y}}\left(\frac{B_{t}}{\bar{B}}\right)^{\left(1-\rho_{R}\right) \theta_{B}} . \tag{21}
\end{equation*}
$$

The coefficient $\bar{R}^{d} \geq 0$ determines the nominal interest rate in steady state. The coefficients $\theta_{\pi}, \theta_{Y} \geq 0$ govern the extent to which the central bank attempts to stabilize inflation and output growth, while $\rho_{R} \geq 0$ captures interest rate smoothing. The coefficient $\theta_{B}$ captures the possibility that the central bank takes into account an effect of government debt on the neutral rate-like the one we documented. If the neutral rate goes up after a government debt increase, this creates persistent inflationary pressure. When the central bank adjusts its interest target, such pressure is avoided.

The government follows an expenditure rule:

$$
\begin{equation*}
\frac{G_{t}}{\bar{G}}=\left(\frac{G_{t-1}}{\bar{G}}\right)^{\rho_{G_{1}}}\left(\frac{G_{t-2}}{\bar{G}}\right)^{\rho_{G_{2}}}\left(\frac{Y_{t}}{Y_{t-1}}\right)^{\left(1-\rho_{G_{1}}-\rho_{G_{2}}\right) \gamma_{Y}}\left(\frac{B_{t}}{\bar{B}}\right)^{\left(1-\rho_{G_{1}}-\rho_{G_{2}}\right) \gamma_{B}} \epsilon_{t}^{G} \tag{22}
\end{equation*}
$$

where we use an $\mathrm{AR}(2)$-process for government spending to capture the shape of expenditures in the local projections in Section 2.1 and $\epsilon_{t}^{G}$ is a log-normally distributed i.i.d. government spending shock with zero mean. The parameters $\gamma_{B}$ and $\gamma_{Y}$ measure, respectively, how the spending reacts to debt deviations from steady state and output growth.

The government uses tax revenues $T_{t}$ and bonds $B_{t+1}$ to finance expenditures, interest payments, and outstanding debt. Tax revenues are then $T_{t}=\tau\left(w_{t} N_{t}+\Pi_{t}^{U}+\Pi_{t}^{F}\right)-\mathcal{T}_{t}$, with constant tax rate $\tau$. Here we assume that transfers are linear in $h_{i t}$. The transfers are set to zero except for counterfactual experiments. The government budget constraint determines the real market value of government bonds residually:

$$
\begin{equation*}
B_{t+1}=G_{t}-T_{t}+\frac{B_{t}}{\pi_{t}}\left[\left(1-\delta_{B}\right) \frac{q_{t}^{B}}{q_{t-1}^{B}}+\frac{1}{q_{t-1}^{B}}\right]=G_{t}-T_{t}+\frac{R_{t}^{d}}{\pi_{t}} B_{t}-\Pi_{t}^{B} \tag{23}
\end{equation*}
$$

In words, because of long-term bonds, a persistent surprise change in the real interest rate redistributes, through banks profits, between the government and the private sector.

### 3.4. Goods, Bonds, Capital, and Labor Market Clearing

The labor market clears at the competitive wage given in (13). The bond market clears whenever the following equation holds:

$$
\begin{equation*}
B_{t+1}=B^{d}\left(R_{t}^{d}, r_{t}, q_{t}, q_{t}^{B}, \Pi_{t}^{F}, \Pi_{t}^{U}, w_{t}, \pi_{t}, \Theta_{t}, \mathbb{W}_{t+1}\right):=\mathbb{E}_{t}\left[\lambda d_{a, t}^{*}+(1-\lambda) d_{n, t}^{*}\right] \tag{24}
\end{equation*}
$$

where $d_{a, t}^{*}, d_{n, t}^{*}$ are functions of the states $(d, k, h)$, and depend on how households value asset holdings in the future, $\mathbb{W}_{t+1}(\cdot)$, and the current set of prices $\left(R_{t}^{d}, r_{t}, q_{t}, q_{t}^{B}, \Pi_{t}^{F}, \Pi_{t}^{U}, w_{t}, \pi_{t}\right)$. Future prices do not show up because we can express the value functions such that they summarize all relevant information on the expected future price paths. Expectations in the right-hand-side expression are taken w.r.t. the distribution $\Theta_{t}(d, k, h)$. Equilibrium requires the total net amount of deposits the household sector demands, $D^{d}$, to equal the supply of government bonds. In gross terms, there are more liquid assets in circulation as some households borrow up to $\underline{d}$. The aggregate amount of private liquidity is defined as $I O U_{t}=$ $\int_{\underline{d}}^{0} d d \Theta_{t}$, the sum over all private debt.

Last, the market for capital has to clear:

$$
\begin{equation*}
K_{t+1}=K^{d}\left(R_{t}^{d}, r_{t}, q_{t}, q_{t}^{B}, \Pi_{t}^{F}, \Pi_{t}^{U}, w_{t}, \pi_{t}, \Theta_{t}, \mathbb{W}_{t+1}\right):=\mathbb{E}_{t}\left[\lambda k_{t}^{*}+(1-\lambda) k\right] \tag{25}
\end{equation*}
$$

such that the aggregate supply of funds from households-both those that trade capital, $\lambda k_{t}^{*}$, and those that do not, $(1-\lambda) k$ - equals the capital used in production. Again $k_{t}^{*}$ is a function of the current prices and continuation values. The goods market then clears due to Walras's law, whenever labor, deposit, bond, and capital markets clear. For a formal definition of the equilibrium, see Appendix C.

## 4. Calibration and Estimation

The estimation of the model follows a two-step procedure: First, we calibrate all parameters that affect the steady state of the model. Second, we estimate by Bayesian limitedinformation methods (concretely, this is by IRF matching, see Christiano et al., 2010) all parameters that only matter for the dynamics of the model, i.e., the aggregate government spending shock, real and nominal frictions, and policy rules. Table 1 summarizes the calibrated parameters and the calibration targets, and Table 2 lists the estimated parameters. One period in the model refers to a quarter of a year and our sample covers the US from 1947 to 2019.

Table 1: Calibration (Quarterly Frequency)

| Parameter | Value | Description | Target |
| :--- | ---: | :--- | :--- |
| Households: | Income Process |  |  |
| $\rho_{h}$ | 0.98 | Persistence labor income | Storesletten et al. (2004) |
| $\sigma_{h}$ | 0.12 | STD labor income | Storesletten et al. (2004) |
| $\iota$ | $6.25 \%$ | Trans.prob. from E. to W. | Guvenen et al. (2014) |
| $\zeta$ | $0.05 \%$ | Trans.prob. from W. to E. | Top 10\% wealth share, $0.68 \%$ |
| Households: | Preferences |  |  |
| $\xi$ | 4.00 | Relative risk aversion | Kaplan et al. (2018) |
| $\gamma$ | 2.00 | Inverse of Frisch elasticity | Chetty et al. (2011) |
| $\beta$ | 0.983 | Discount factor | Capital to output, $K / Y=11.5$ |
| $\lambda$ | $6.40 \%$ | Portfolio adj. prob. | Public liquidity, $B / Y=2.36$ |
| $\bar{R}$ | $1.00 \%$ | Borrowing penalty | Private liquidity $I O U s / Y=0.56$ |
| Firms |  |  |  |
| $\alpha$ | 0.68 | Share of labor | $62 \%$ labor income |
| $\delta_{0}$ | $1.75 \%$ | Depreciation rate | $7.0 \%$ p.a. |
| $\bar{\eta}$ | 11 | Elasticity of substitution | Price markup $10 \%$ |
| $\bar{\zeta}$ | 11 | Elasticity of substitution | Wage markup $10 \%$ |
| Government |  |  |  |
| $\tau$ | 0.28 | Tax rate level | Gov.t expend. share, $G / Y=20 \%$ |
| $\delta_{B}$ | 0.20 | Bond Duration | Average time to maturity US debt |
| $\bar{R}$ | 1.00 | Nominal rate | Growth $\approx$ interest rate, see text |
| $\bar{\pi}$ | 1.00 | Inflation | Indexation, see text |

Notes: Capital stocks relative to GDP from NIPA, market value of public debt relative to GDP from FRED, private liquidity from the flow of funds, top $10 \%$ wealth share from WID, see Appendix A for details.

### 4.1. Calibration

We fix a number of parameters either following the literature or targeting steady-state ratios; see Table 1. For the household side, the relative risk aversion is set to 4, which is common in the incomplete markets literature; see Kaplan et al. (2018). The Frisch elasticity is set to 0.5 ; see Chetty et al. (2011). We take estimates for idiosyncratic income risk from Storesletten et al. (2004), and set $\rho_{h}=0.98$ and $\sigma_{h}=0.12$. Guvenen et al. (2014) provide the probability that a household will fall out of the top 1 percent of the income distribution in a given year, which represents the transition probability from entrepreneur to worker, $\iota=6.25$ percent.

To calibrate the remaining household parameters, we match 4 targets (relative to annual output): (1) average illiquid assets $(K / Y=2.87$, annual), (2) public liquidity $(B / Y=0.59$, annual), (3) private liquidity $(I O U / Y=0.14$, annual), and (4) the average top 10 percent share of wealth, which is 68 percent. This yields a discount factor of 0.983 , a portfolio
adjustment probability of 6.4 percent, a borrowing penalty of 1.0 percent quarterly (given a borrowing limit of one-time average annual income), and a transition probability from worker to entrepreneur of 0.05 percent. ${ }^{17}$

The total supply of liquid assets, $I O U+B$, in our calibration is 25 percent larger than the supply of liquidity through government bonds alone. As in Huggett (1993), when some households borrow, they create liquid assets for others to save in. We match this private liquidity to the aggregate amount of unsecured consumer credit in the flow of funds.

For the firm side, the labor share in production, $\alpha$, is set to 68 percent to match a labor income share of 62 percent, which corresponds to the average BLS labor share. The depreciation rate is 1.75 percent per quarter. An elasticity of substitution between differentiated goods of 11 yields a markup of 10 percent. The elasticity of substitution between labor varieties is also set to 11 , yielding a wage markup of 10 percent. Both are standard values.

The tax rate, $\tau$, is set to clear the government budget constraint that corresponds to a government share of $G / Y=20$ percent. Steady-state inflation is set to zero as there is indexation to it in the Phillips curves. The steady-state net interest rate on bonds is set to 0.0 percent to capture the average federal funds rate relative to nominal output growth over 1947 - 2019. The average time to maturity for US government bonds has been roughly five years during that time period and $\delta_{B}=0.2$ is set accordingly.

### 4.2. Estimation

We follow Christiano et al. (2010) in employing a Bayesian variant of the Christiano et al. (2005)-type impulse response matching approach. The idea is to treat the empirical impulse responses $\hat{\psi}$ as "data" and to choose parameters $\vartheta$ to make the model impulse responses $\psi(\vartheta)$ as close as possible to $\hat{\psi}$. Christiano et al. (2010) refer to this strategy as a "limited information Bayesian approach". The Bayesian log posterior is then given by

$$
\begin{equation*}
\log f(\vartheta \mid \psi) \propto-\frac{1}{2}(\hat{\psi}-\psi(\vartheta))^{\prime} W(\hat{\psi}-\psi(\vartheta))^{\prime}+\log p(\vartheta) \tag{26}
\end{equation*}
$$

where $W$ is a weighting matrix and $p(\vartheta)$ denotes the priors on $\vartheta$.
The vector $\hat{\psi}$ stacks the empirical impulse responses at all horizons of the aggregates shown in Figure 1(a), i.e., those of government spending, public debt, output, investment, and the real yield on government bonds. Consumption is not included as output is constructed as the sum of its components. In addition to these variables, an impulse response of the liquidity premium is included. As there is more than one premium, we run a principal component analysis of the return premia of capital, housing, corporate bonds, money, and

[^12]Table 2: Prior and Posterior Distributions of Estimated Parameters

| Parameter | Distribution | Prior |  | Posterior |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Mean | Std. Dev. | Mean | Std. Dev. | $5 \%$ | $95 \%$ |
| Frictions |  |  |  |  |  |  |  |
| $\delta_{s}$ | Gamma | 5.00 | 2.00 | 2.296 | 1.738 | 0.380 | 5.774 |
| $\phi$ | Gamma | 4.00 | 2.00 | 3.929 | 1.278 | 2.183 | 6.276 |
| $\kappa$ | Gamma | 0.10 | 0.02 | 0.109 | 0.021 | 0.077 | 0.145 |
| $\kappa_{w}$ | Gamma | 0.10 | 0.02 | 0.108 | 0.021 | 0.077 | 0.144 |
| Monetary policy rule |  |  |  |  |  |  |  |
| $\rho_{R}$ | Beta | 0.75 | 0.20 | 0.977 | 0.011 | 0.956 | 0.991 |
| $\theta_{\pi}$ | Normal | 1.70 | 0.30 | 1.620 | 0.268 | 1.189 | 2.078 |
| $\theta_{Y}$ | Normal | 0.13 | 0.05 | 0.124 | 0.050 | 0.042 | 0.206 |
| $\theta_{B}$ | Gamma | 0.05 | 0.04 | 0.022 | 0.006 | 0.012 | 0.032 |
| Fiscal policy rule |  |  |  |  |  |  |  |
| $-\gamma_{B}$ | Gamma | 0.50 | 0.25 | 0.567 | 0.284 | 0.169 | 1.092 |
| $\gamma_{Y}$ | Normal | 0.00 | 1.00 | -0.188 | 0.987 | -1.804 | 1.460 |
| $\rho_{G *}$ | Beta | 0.50 | 0.20 | 0.962 | 0.007 | 0.951 | 0.973 |
| $\rho_{G 2}$ | Normal | 0.00 | 0.50 | -0.477 | 0.056 | -0.563 | -0.38 |
| $\sigma_{G}$ | Inv.-Gamma | 1.00 | 2.00 | 0.602 | 0.042 | 0.537 | 0.675 |

Notes: To estimate the $\operatorname{AR}(2)$-process for government spending, we estimate $\rho_{G *}=\rho_{G 1}+\rho_{G 2}$ in addition to $\rho_{G 2}$. The standard deviation of the government spending shock is expressed in percent.
equities over government bonds, controlling for a linear-quadratic time trend and treat the first principal component as an index of "the" liquidity premium. Scaling this component in order to predict the capital premium allows us to estimate an impulse response of "the" liquidity premium that takes into account information of all alternative measures. We estimate this impulse response by local projections and include this as our target in $\hat{\psi}$.

Following common practice in the impulse-response-matching literature (Christiano et al., 2005,2010 ), we use a diagonal weighting matrix $W$, where the diagonal entries are 1 divided by the squared standard error of the respective empirical impulse response.

Columns 1-4 of Table 2 present the estimated parameters and their assumed prior distributions. Priors follow standard values in the literature (Smets and Wouters, 2007; Justiniano et al., 2011) and are independent of the underlying data. Following Justiniano et al. (2011), a gamma distribution with prior mean of 5.0 and standard deviation of 2.0 is imposed on $\delta_{2} / \delta_{1}$, the elasticity of marginal depreciation with respect to capacity utilization, and a gamma prior with mean 4.0 and standard deviation of 2.0 for the parameter controlling investment adjustment costs, $\phi$. The slopes of the price and wage Phillips curves, $\kappa_{Y}$ and $\kappa_{w}$, follow gamma priors with mean 0.1 and standard deviation 0.02 , which corresponds to
contracts having an average length of one year.
Regarding monetary policy, the inflation and output feedback parameters in the Taylorrule, $\theta_{\pi}$ and $\theta_{Y}$, follow normal distributions with prior means of 1.7 and 0.13 , respectively, while the interest rate smoothing parameter $\rho_{R}$ follows a beta distribution with mean 0.75 and standard deviation 0.1 . For $\theta_{B}$, the parameter governing the adjustment of the interest rate target to public debt, we assume a gamma prior with mean 0.05 and standard deviation 0.04 , implying a prior mode of 0.021 which is in line with Summers and Rachel (2019)-long-run-elasticity of the interest rate to government debt.

For the fiscal policy rule, the parameter governing feedback to output $\gamma_{Y}$, follows a standard normal distribution, while for the parameter governing feedback to debt, $\gamma_{B}$, we assume that $-1 * \gamma_{B}$ follows a gamma prior with mean 0.5 and standard deviation 0.25 . This means, only parameter draws that yield determinacy $\left(\gamma_{B}>(R B-1) * B+d R B / d B * B>0\right)$ are allowed. To estimate the $\operatorname{AR}(2)$-process for government spending, we estimate $\rho_{G *}=$ $\rho_{G 1}+\rho_{G 2}$, which follows a beta distribution with mean 0.5 and standard deviation 0.2 , and $\rho_{G 2}$ which follows a normal distribution with zero mean and standard deviation 0.5 . The standard deviation of the government spending shock is assumed to follow an inverse-gamma prior distribution with prior mean of $1.00 \%$.

Columns 5-8 of Table 2 report the posterior distributions of the estimated parameters. They are based on a standard random walk Metropolis-Hastings algorithm using 450,000 draws including burn-in. ${ }^{18}$ Overall, the parameter estimates are in line with the representative-agent literature, both regarding the nominal and the real frictions.

The estimated Taylor rule coefficients on inflation and output growth are $\theta_{\pi}=1.6$ and $\theta_{Y}=0.12$, respectively, and there is substantial inertia of $\rho_{R}=0.98$. The fiscal rule that governs government spending exhibits a countercyclical response to output growth, $\gamma_{Y}=$ -0.2 , debt stabilization, $\gamma_{B}=-0.57$, and inertia, $\rho_{G 1}+\rho_{G 2}=0.96$. The AR-2 produces a mildly hump-shaped response to spending shocks as in the local projections of Section 2.

The data suggest that the Fed takes the effect of debt on the neutral rate into account; the estimated semi-elasticity is $\theta_{B}=0.02$ - an admittedly non-standard element of the Taylor rule. When the neutral rate is a function of public debt and the central bank does not take this into account, it would generate persistent inflation movements. Suppose the neutral rate goes up, but the central bank's target remains constant, then this causes persistently higher inflation. When the central bank's target adjusts (and market participants understand this), the positive co-movement of the long-term neutral rate and inflation breaks down. Real rates can increase persistently without an impact on inflation - in line with the data.

[^13]
## 5. The Short Run: Government Spending Shocks

This section looks at the aggregate effects of transitory government spending shocks that increase public debt in their aftermath. First, we show that the estimated model can match the target evidence from the local projections in Section 2. What is more, we discuss the role of the liquidity channel in the transmission of policy and how the response of the liquidity premium depends on the extent of debt financing and degree of illiquidity. Finally, the robustness of our findings is shown.

### 5.1. Model Dynamics

Figure 3 (blue solid lines) shows the impulse responses to a government spending shock in the estimated model together with the six local-projection IRFs the model is matched to (black dashed lines): output, government spending, investment, government debt, the ex-post real yield of government bonds, and the liquidity premium. The estimates follow reasonably closely the responses estimated via the Blanchard and Perotti (2002)-approach in Section 2.

Government spending persistently goes up, peaking at around 1 percent at quarter 3 and slowly returns to its steady-state level after 4 years. In response to higher government spending, output increases but investment falls. The increased public spending crowds out capital. This happens, first, because reducing investment smooths consumption by providing the resources absorbed by the government. This effect is also present in a model with complete markets. With incomplete markets, however, another channel arises. With increasing public debt, further savings devices become available to households. To make households willing to hold these assets, their yield needs to rise. In fact, we estimate that the central bank reacts accordingly and real bond yields go up by more than 10 basis points (annualized). In our model, this does not fully spill over to capital, because capital and bonds are imperfect substitutes. The expected return on capital does not follow the expected bond yield one-for-one and, in turn, the liquidity premium, the expected excess return of illiquid assets over bonds, falls by 7 basis points (annualized) after 12 quarters. This model-implied response of the liquidity premium lines up well with the local projections - except for the first year when the model-implied premium overshoots. This overshooting reflects a strong decline in the price of capital implied by our estimated investment adjustment costs. After 4 quarters, the price of capital recovers and the premium follows the differential response of capital dividend vs real rate on bonds.

In the data, the response of the liquidity premium is negative throughout because in reality some asset markets, e.g., housing, show inertia in prices. In the figure, the empirical liquidity premium response refers to the first principal component of the premia in capital,

Figure 3: IRFs to Government Spending Shock: Local Projection vs. Model


Notes: IRFs to the estimated government spending shock. Blue solid line: Baseline model estimated via IRF-matching, HANK-2. Black dashed line: Local-projection estimates. Gray shaded areas: 90 percent confidence bounds based on Newey and West (1987)-standard errors; blue shaded areas: 90 percent bands based on simulating 10,000 sets of IRFs using posterior draws. Y-axis: Percent deviation from steady state, except for $R_{t}^{b} / \pi_{t}$, and $L P_{t}$ that are in annualized percentage points. X-axis: Quarters.
housing, and corporate bonds. This strikes a compromise between the various measures of the liquidity premium presented in the empirical section. We scale this component back so as to predict the capital liquidity premium from Figure 1(b). The movement of the liquidity premium implies less crowding out of capital than in a single-asset Aiyagari and McGrattan (1998)-type of model.

Appendix F provides impulse responses for other, non-matched, variables. There, we show that the increase in public liquidity, while it crowds out capital only little, crowds out private liquidity substantially. At higher rates, households borrow less, IOUs go down by 0.25 percent. Consequently, total (gross) deposits increase less than the increase in government

Figure 4: Sensitivity of the Liquidity Premium Response
Liquidity premium response and ...


Notes: Change in the liquidity premium after 16 quarters relative to steady state. Left panel: Varying the illiqudity of capital (expected time to trade), recalibrating intermediation costs on liquid assets to keep the $\mathrm{B} / \mathrm{Y}$ ratio constant. Government debt increases by 1 percent at peak. Right panel: Varying the peak debt response through non-distortionary transfers. Dots: Solutions from simulation. Dashed lines: interpolated values. See main text for further details.
debt suggests-also in line with our empirical findings.
Furthermore, Appendix F also compares our baseline model to a variant of the model where all assets are equally liquid (HANK-1) and to a complete markets version (RANK). We show that the fiscal multiplier is larger in the HANK-2 model than in HANK-1. The weaker crowding out of capital is behind this result. Also compared to the complete markets model aggregate effects are stronger. In the two-asset economy, households are on average worse insured (see Kaplan et al., 2018), such that Keynesian effects are stronger while at the same time aggregate demand increases more. Expressed differently, public debt is private wealth when markets are incomplete and this wealth translates, through portfolio effects, into investment and thus goods demand.

Finally, we documented heterogeneity in the liquidity premium response across different asset classes and across countries in Section 2. Figure 4 shows that the model can successfully replicate this heterogeneity as well. The left panel shows how the liquidity premium (after 16 quarters) responds differently, when the liquidity of the illiquid asset is modified. A housing unit in the US is sold every 25 years on average, corporate bonds can be traded much more easily. Varying the trading probability from every two to every twelve years, keeping the bond to output ratio constant, varies the response in the liquidity premium from -1 to -15 basis points to a spending shock that increases government debt by 1 percent.

The model also successfully reproduces the cross-country evidence from Figure 2(b). For
this purpose, we vary the speed of repayment $\gamma_{B}$ and transfers, setting $\mathcal{T}_{t} \bar{Y}=-\gamma_{B}^{\tau}\left(\log B_{t+1}-\right.$ $\log B_{t}$ ); considering a range of values for $\gamma_{B}^{\tau}$ form -7.5 to 7.5. The higher the value for $\gamma_{B}^{\tau}$, the more the government finances spending shocks in a balanced-budget manner. In the baseline model, this response is absent. Here, it allows us to obtain variations in the size of the public debt response to a fiscal spending shock. The right panel in Figure 4 displays the results of this exercise. For an additional increase in debt of 1 percent, the liquidity premium falls by roughly 5 basis points after 4 years. The slope of this relationship in the model is well within the confidence bounds of the empirical exercise in Section 2.2.

### 5.2. Robustness

Appendix G shows that the model behavior is robust to re-estimating the model under the assumption of King et al. (1988) preferences, under the assumption of a degree of risk aversion of 2 instead of 4 , and under the assumption of the spending process being an ARMA(1,1) instead of an $\operatorname{AR}(2)$. In all versions, there is a significant decline in the liquidity premium after the spending shock. Compared to the evidence from the local projections, the liquidity premium falls too little in the medium run under KPR preferences. A lower degree of risk aversion/higher intertemporal substitution results in somewhat less investment crowding out and hence higher multipliers and a slightly stronger liquidity premium response. An ARMA process leads to virtually indistinguishable results compared to our baseline, except for a slightly worse fit of the government spending IRF.

## 6. The Long Run: Public Debt and Interest Rates

The previous section has shown that our estimated model is capable of explaining the short-run dynamics of the liquidity premium, matching our local-projection evidence. Next, we use it to investigate a permanent change in fiscal policy, for which empirical evidence is, almost by definition, very limited. In particular, we analyze the effects of a transition to a new steady state with higher public debt on real interest rates, the capital stock, and inequality. Analyzing the interest rate movements equips us with a simple approximation for the fiscal burden of public debt.

### 6.1. The Economic Consequences of Increasing the Debt Target

Consider an increase of the government debt target by 10 percent. This increase is, for all practical purposes, permanent and implemented over 20 years. ${ }^{19}$ The receipts (and long-

[^14]term costs) are distributed to the households through the non-distortionary transfer $\mathcal{T} .{ }^{20}$ To avoid any distortion of the picture by long-run inflation triggered by neutral rate changes, we recalibrate the central bank's response such as to match the long-run elasticity of bond yields to debt levels. Figure 5 shows the model responses to the change in the debt target over 100 years. Again, we display and compare results across the HANK-2, HANK-1, and RANK variants of our model. The RANK model is only displayed for completeness as it features Ricardian equivalence and, hence, the increase in public debt has no aggregate or price consequences whatsoever. ${ }^{21}$

Our main finding is that the higher public-debt target has a persistent and strong effect on the real interest rate of public debt in the HANK-2 model. A 10 percent increase (i.e., an increase in the initially targeted (annual) public-debt-to-output ratio of roughly 6 percentage points) increases the (annualized) real government bond yield by 25 basis points in the long run, i.e., the semi-elasticity of the real rate with respect to public debt is 0.025 . This number aligns with Summers and Rachel (2019), who summarize the literature with a semi-elasticity of $0.021 .{ }^{22}$ At the same time, the marginal product of capital hardly moves and capital declines only mildly by 0.4 percent.

After the 20 years of fiscal expansion, this leads to a pronounced difference to the standard incomplete markets version, in which all assets are liquid and the liquidity premium is constant. In the HANK-1 model, capital falls by 1 percent and output decreases twice as much as in HANK-2. At the same time government bond yields increase much less. The semi-elasticity in the HANK-1 model is, with 0.005 , a fifth of the elasticity in HANK-2. In terms of inequality measures, it takes substantial time to reach the new steady state. Wealth accumulation is slow. The initial disbursement of the transfers lowers wealth inequality. In HANK-2 also the higher interest rate on liquid assets incentivizes the relatively poor to accumulate more. Deposits are the first step in terms of accumulating wealth. Poor households, primarily saving in liquid form, profit from the higher returns on these liquid assets more than rich households do (see Bayer et al., 2019, who document in the SCF a decline by 23 percentage points of the ratio of liquid to illiquid assets when moving from the 25th to the 75 th percentile of the wealth distribution, see their Figure 6. In our calibration, this difference is 26 percentage points in the model). As a result, poor households increase their savings more than rich households when deposit rates go up, and wealth inequality

[^15]Figure 5: Response to an Increase in the Debt Target


Notes: Impulse responses to a 10 percent debt-target shock financed by non-distortionary transfers. Black solid line: Baseline model, HANK-2. Blue dash-dotted: Liquid-capital model, HANK1. Red dotted line: Complete-markets model, RANK. Y-axis: Percent deviation from steady state, except for $\mathcal{T}_{t} / \bar{Y}, R B_{t}$, and $L P_{t}$ that are in annualized percentage points. X-axis: Quarters.
persistently decreases. While wealth inequality falls, consumption inequality rises in the long run. Lower wages drive the increase in consumption inequality, but less so in HANK-1 compared to HANK-2 because of the smaller increase in taxes.

While the crowding out of capital is smaller in HANK-2 compared to HANK-1, the crowding out of private liquidity is stronger. With a permanent increase in debt, the crowding out of private liquidity is three times as strong as for the temporary increase studied in Section 5. IOUs fall by 8 percent in HANK-2 and below 2 percent in HANK-1. However, since IOUs make up only a fifth of total liquid assets (in HANK-2), the total supply of liquid assets (public plus private debt) still increases by 4.9 percentage points of annual output. Taking also into account the crowding out of capital, the total amount of assets still increases by 3.8 percentage points. In contrast, when capital is liquid (HANK-1), the total amount of assets in the economy increases by much less. The increase is only 2.7 percentage points of annual output because the 6 percentage points increase in public debt crowds out 3.0 percentage points of capital and 0.3 percentage points of private debt.

The increase in interest rates implies that the government needs to pay more on its outstanding debt and, therefore, the transfers in the future need to fall. In HANK-1, this channel is muted because interest rates increase less. There, however, the tax base becomes smaller as capital and, hence, output declines. For this reason, in both incomplete markets models, transfers become negative as debt increases in the long-run even though the steadystate return on bonds is zero.

### 6.2. The Fiscal Implications of Public Debt

As alluded to above, the substantial long-run elasticity of the real interest rate to public debt has important fiscal implications. Let $\mathcal{R}(B):=\frac{R_{t}^{b}}{\pi_{t}}-\log \left(\frac{Y_{t}}{Y_{t-1}}\right)$ be the real government bond yield net of output growth. The fiscal burden of rolling over public debt is then $B \cdot \mathcal{R}(B)$. We can approximate how $\mathcal{R}(B)$ responds to changes in debt, $B$, log-linearly with a constant semi-elasticity $\eta_{B}$ :

$$
\begin{equation*}
\mathcal{R}(B) \approx \mathcal{R}(\bar{B})+\eta_{B} \ln \left(\frac{B}{B}\right), \tag{27}
\end{equation*}
$$

where $\bar{B}$ is the steady-state debt level. Our estimate of this semi-elasticity $\eta_{B}$ is 2.5 percent. In our calibration, the interest-growth difference is zero, $\mathcal{R}(\bar{B})=0$, at a debt-to-output ratio of 59 percent - our steady state debt level.

This implies that at debt levels smaller than $\bar{B}$ the difference $\mathcal{R}$ is negative and the government constantly generates revenues when rolling over a debt level that is positive but smaller than $\bar{B}$. Of course, these revenues vanish again if the government would move to zero debt. The fiscal revenues $-B \cdot \mathcal{R}(B)$ are quasi-quadratic in $B$ and there is a positive maximal

Figure 6: Fiscal Implications of Public Debt

Fiscal revenues of debt, $-\frac{B \cdot \mathcal{R}(B)}{Y}$


Interest-growth differential, $\mathcal{R}(B)$


Notes: Left panel: Interest-burden-to-GDP ratio vs. public-debt-to-GDP ratio (both in annual percent). Right panel: Interest rate-growth differential vs. public-debt-to-GDP ratio (both in annual percent). Black solid line: US data from 1947 to 2019; red dashed line: US data from 2011 to 2020 .
revenue for some positive debt level $B^{*} \in(0, \bar{B})$. Expressed differently, for the government the return on bonds is not constant. There is limited competition for the provision of aggregate liquidity; in a sense, the government faces a Laffer curve for liquidity provision.

Figure 6 shows this graphically. It displays the fiscal revenues, $-\frac{B \cdot \mathcal{R}(B)}{Y}$, from maintaining a constant debt-to-GDP ratio (left panel) and the interest-growth differential, $\mathcal{R}(B)$, (right panel) both plotted against the debt-to-output ratio, $\frac{B}{Y}$, (all annualized). The black line corresponds to our baseline, which is calibrated to average US public debt and interest rates over the last 70 years. The two intercepts of the fiscal burden of debt with zero are, as explained, at $\frac{B}{Y}=0$ and at $\frac{B}{Y}=59 \%$. The red dashed line is based on 2011-2020 data.

A corollary of the dependence of $\mathcal{R}$ on the debt level is that the marginal fiscal burden of debt exceeds the current interest-growth differential because the government also has to pay a higher interest rate on all debt that is already outstanding when increasing its debt. Again, we can use the log-linear approximation with constant semi-elasticity to gauge the marginal fiscal burden of debt as:

$$
\begin{equation*}
\frac{\partial[\mathcal{R}(B) B]}{\partial B}=\mathcal{R}(B)+\underbrace{\frac{\partial \mathcal{R}(B)}{\partial B} B}_{\eta_{B}} \approx \mathcal{R}(\bar{B})+\eta_{B}\left[\ln \left(\frac{B}{\bar{B}}\right)+1\right] . \tag{28}
\end{equation*}
$$

The marginal cost of debt exceeds the interest-rate growth difference by $\eta_{B}$ and the level of debt, $B^{*}$, which maximizes revenues is given by

$$
\begin{equation*}
\frac{\partial\left[\mathcal{R}\left(B^{*}\right) B^{*}\right]}{\partial B}=0 \Longrightarrow \ln B^{*}=\ln \bar{B}-1-\frac{\mathcal{R}(\bar{B})}{\eta_{B}} \tag{29}
\end{equation*}
$$

Our estimates of the semi-elasticity and the zero interest-growth difference at a debt-to-output ratio of 59 percent yield that the revenue maximizing debt level has been at 21 percent of GDP on average (black solid line, in Figure 6) for the US over the last seven decades. Any target below this level provides less liquidity to the private sector and less revenues to the government.

If we apply the formula to the most recent decade (2011-2020, red dashed line), however, the results look very different even though it includes the worst recession after the second world war. For this period, the US interest-growth differential is minus 1 percent and the average debt-to-GDP ratio is 110 percent. ${ }^{23}$ This implies that any debt-to-GDP ratio below 60 percent leads to permanently lower fiscal revenues and less liquidity provision. Similarly, one can use the approximation to calculate the debt level needed to close the interest-growth gap as $\ln B^{0}=\ln \bar{B}-\frac{\mathcal{R}(\bar{B})}{\eta_{B}}$. This formula suggests that, to close the interest rate growth gap, US public debt would need to be roughly 160 percent of GDP today.

### 6.3. Post-2010 Scenarios for the Interest Rate Elasticity

The analysis so far assumes a constant semi-elasticity of the real interest rate to persistent public debt movements, $\eta_{B}$. Our baseline estimate, $\eta_{B}=2.5 \%$, corresponds to the time period of 1947 to 2019. This section asks how much this elasticity might have changed through the lens of our model post-2010. We do so by studying six scenarios that can explain the post-2010 increase in the debt-to-GDP ratio to 110 percent and the fall in the real interest rate growth difference to minus 1 percent. Table 3 shows the interest-rate semielasticity and the three non-targeted steady-state statistics for each scenario. If a higher discount factor or shift in the risk premium explains the post-2010 US experience, the model predicts a lower elasticity, $2.4 \%$. An increase in the price markup or income risk can also explain higher demand for public debt and a lower real rate. In this case, our model predicts a slightly higher elasticity between $2.9 \%$ and $3.8 \%$. The effects of a higher illiquidity of capital are similar. Finally, a tightening of the borrowing limit implies a substantially larger elasticity, $4.3 \%$ to $7.7 \%$. The latter scenario, however, predicts a strong decline in private liquidity that contradicts the US experience of increasing private liquidity post-2010. An increase in the price markup or the illiquidity of capital best match US private liquidity post-2010.

We also assess the robustness of our estimates of the semi-elasticity, $\eta_{B}$, to higher private liquidity coming from four sources: 1) loosening of the borrowing limit, 2) higher probability of trading capital, and extending the balance sheet of banks by allowing them to invest in

[^16]Table 3: Post-2010 Scenarios for the Interest Rate Elasticity

| Parameter | Capital <br> (K/Y) | Private liquidity <br> (IOUs/Y) | Top 10\% <br> wealth share | Interest Rate <br> semi-elasticity (\%) |
| :--- | :---: | :---: | :---: | :---: |
| Baseline | 2.87 | 0.14 | 0.68 | 2.50 |
| Data (1947-2019) | 2.87 | 0.14 | 0.68 |  |
| Data (2010-2019) | 2.95 | 0.18 | 0.72 |  |
| A) Post-2010 public-debt-to-GDP ratio of 110\% |  |  |  |  |
| Discount factor | 3.10 | 0.08 | 0.64 | 2.42 |
| Risk premium | 2.83 | 0.06 | 0.66 | 2.40 |
| Income risk | 3.00 | 0.08 | 0.57 | 3.78 |
| Price markup | 2.93 | 0.14 | 0.77 | 2.89 |
| Portfolio liquidity | 2.78 | 0.15 | 0.67 | 3.63 |
| Borrowing limit | 2.85 | 0.00 | 0.62 | 7.68 |
|  | B) Post-2010 real interest rate of $-1 \%$ |  |  |  |
| Discount factor | 3.04 | 0.14 | 0.68 | 2.56 |
| Risk premium | 2.87 | 0.14 | 0.69 | 2.42 |
| Income risk | 2.97 | 0.13 | 0.62 | 2.97 |
| Price markup | 2.92 | 0.19 | 0.76 | 2.63 |
| Portfolio liquidity | 2.84 | 0.19 | 0.69 | 2.71 |
| Borrowing limit | 2.88 | 0.07 | 0.67 | 4.27 |

Notes: We adjust the respective parameters to hit either the post-2010 public-debt-to-GDP ratio (keeping the real rate on public debt constant), Panel A, or the real interest rate on public debt (keeping the public-debt-to-GDP ratio constant), Panel B.
3) capital or in 4) tradable profits; See Appendix I for details. The first two counterfactuals correspond to different parameterizations of our baseline model while the latter two expand our baseline model to allow for a richer asset structure. The results reported in Table I. 3 show that in all four scenarios, the interest rate elasticity is between $1.5 \%$ and $2.3 \%$, i.e., within the range of estimates from our baseline-model scenarios in Table 3. However, the expansion of private liquidity in each scenario requires a higher return on liquid assets in contradiction with the decline of this return in the US post-2010.

### 6.4. Debt-Financed Investment Programs

Several governments, including the current US administration, have been discussing largescale investment programs both in terms of private and public capital over the last years. Some European commentators have suggested building up a well-diversified sovereign wealth fund (SWF) that buys private capital. ${ }^{24}$ At a first glance, the return difference of 1.5

[^17]Figure 7: Response to a debt-financed sovereign wealth fund build-up


Notes: IRFs to a sovereign wealth fund buying $1 \%$ of private capital financed by issuing debt. Non-distortionary transfers adjust to clear the government's budget if necessary. Black solid line: Baseline model, HANK-2. Blue dash-dotted: Liquid-capital model, HANK-1. Red-dotted line: Complete-markets model, RANK. Y-axis: Percent deviation from steady state. X-axis: Quarters.
percentage points (annualized) between public debt and capital that our model generates seems to be a strong fiscal argument in favor of such programs. ${ }^{25}$ However, the marginal fiscal burden is equal to the estimated semi-elasticity of 2.5 percent and thus larger than the liquidity premium of 1.5 percent, which suggests that the government, because of the outstanding legacy debt, would need to raise revenues to finance such a fund.

At the same time, a simple comparison of $\eta_{B}$ and the liquidity premium might be misleading because through its investments the SWF adds physical capital. This in turn improves government tax revenues by raising output. Figure 7 shows that an SWF that buys capital by issuing public debt is more than self-financing in our estimated model. We let the government buy capital equal to one percent of the steady-state capital stock, distributed again over 20 years. The government issues debt to finance the investments. It stabilizes the debt net of the value of the fund at the old steady-state level by slowly adjusting transfers. Since tax revenues together with investment returns increase more than the interest payments, the government can finance transfers in the long run with this plan. The capital stock of the economy goes up even in the long run, however by far less than the 1 percent acquired by the fund. The total increase in capital is only 0.2 percent. Employment and wages go up in line with that. Through the swap of capital for bonds, the fund increases bond yields and lowers the liquidity premium. Consumption inequality drops both because labor incomes are higher and because transfers become positive (even though they are linked to produc-

[^18]tivity). On top, the fund has some stimulating short-run effect. With liquid capital, i.e., in HANK-1, the debt-financed addition of public capital crowds out private capital one for one; it is irrelevant to the households whether they hold capital directly or indirectly through government bonds (see also the irrelevance in RANK).

## 7. Conclusion

The liquidity channel of public debt is important for understanding the effects of fiscal policy. We provide novel empirical evidence that fiscal expansions that result in higher public debt lower liquidity premia. An estimated monetary business cycle model with heterogeneous agents, incomplete markets, and portfolio choice can replicate this evidence. We then use this model as a framework to quantify the liquidity channel of fiscal policy, and find the liquidity channel to be important in the transmission of transitory and permanent changes in fiscal policy. In the short run, fiscal multipliers are larger because there is less crowding out of capital once the liquidity role of public debt is taken into account. In the long run, and in line with the theoretical argument in Woodford (1990), there is a limited impact on the private capital stock as well. However, as a fiscal expansion increases the interest rate on existing public debt, it has a strong impact on the government's budget.

In turn, it is insufficient to only look at current bond yields to assess the fiscal consequences of increasing public debt. A simple formula can approximate the marginal fiscal burden of debt and calculate both a revenue maximizing level of public debt and the level of debt that equates the rates of interest and growth. We exemplify the fiscal cost of public debt in excess of the current interest rate by looking at an increase in public debt that either finances a transfer program or finances a sovereign wealth fund. The returns this fund makes on its investment in capital are higher than its financing cost, and the government's budget in total improves somewhat by the introduction of such a fund. There are long term fiscal cost with outright transfers because the capital stock decreases. What is more, an increase in public debt by compressing the liquidity premium lowers wealth inequality.

Our analysis restricts itself to the positive assessment of public debt expansions. The importance of the liquidity channel therein, as captured by the return differential between more and less liquid assets and the limited crowding out of capital, calls for a reassessment of the welfare consequences of public debt. Of course, one needs to take into account that our model economy is a closed economy. For many economies smaller than the US, the estimated elasticity of the interest rate on bonds is potentially too high. This open economy perspective and the more normative question of optimal public debt policy are important research areas for future work.

## References

Acikgöz, Ö., Hagedorn, M., Holter, H., and Wang, Y. (2018). The optimum quantity of capital and debt. CEPR Discussion Paper, No. DP12952.

Aiyagari, S. R. (1994). Uninsured idiosyncratic risk and aggregate saving. The Quarterly Journal of Economics, 109(3):659-684.

Aiyagari, S. R. and McGrattan, E. R. (1998). The optimum quantity of debt. Journal of Monetary Economics, 42(3):447-469.

Angeletos, G.-M., Collard, F., and Dellas, H. (2016). Public debt as private liquidity: Optimal policy. NBER Working Papers 22794.

Auclert, A., Bardóczy, B., and Rognlie, M. (2021). MPCs, MPEs, and multipliers: A trilemma for New Keynesian models. Review of Economics and Statistics, forthcoming.

Auclert, A., Rognlie, M., and Straub, L. (2018). The intertemporal Keynesian Cross. NBER Working Papers 25020.

Auclert, A., Rognlie, M., and Straub, L. (2020). Micro jumps, macro humps: Monetary policy and business cycles in an estimated HANK model. NBER Working Papers 26647.

Azzimonti, M. and Yared, P. (2019). The optimal public and private provision of safe assets. Journal of Monetary Economics, 102(C):126-144.

Bassetto, M. and Cui, W. (2018). The fiscal theory of the price level in a world of low interest rates. Journal of Economic Dynamics and Control, 89:5-22. Fed St. Louis-JEDC-SCG-SNB-UniBern Conference, titled: Fiscal and Monetary Policies.

Bayer, C., Born, B., and Luetticke, R. (2020). Shocks, frictions, and inequality in US business cycles. CEPR Discussion Paper No. DP14364.

Bayer, C. and Luetticke, R. (2020). Solving heterogeneous agent models in discrete time with many idiosyncratic states by perturbation methods. Quantitative Economics, 11:12531288.

Bayer, C., Luetticke, R., Pham-Dao, L., and Tjaden, V. (2019). Precautionary savings, illiquid assets, and the aggregate consequences of shocks to household income risk. Econometrica, 87(1):255-290.

Bhandari, A., Evans, D., Golosov, M., and Sargent, T. J. (2017). Public debt in economies with heterogeneous agents. Journal of Monetary Economics, 91:39-51.

Blanchard, O. (2019). Public debt and low interest rates. American Economic Review, 109(4):1197-1229.

Blanchard, O. J. and Perotti, R. (2002). An empirical characterization of the dynamic effects of changes in government spending and taxes on output. Quarterly Journal of Economics, 117(4):1329-1368.

Born, B. and Müller, G. J. (2012). Government spending shocks in quarterly and annual time series. Journal of Money, Credit and Banking, 44(2-3):507-517.

Born, B. and Pfeifer, J. (2014). Policy risk and the business cycle. Journal of Monetary Economics, 68:68-85.

Bredemeier, C., Juessen, F., and Schabert, A. (2022). Why are fiscal multipliers moderate even under monetary accommodation? European Economic Review, 141.

Broer, T., Hansen, N.-J. H., Krusell, P., and Öberg, E. (2019). The New Keynesian transmission mechanism: A heterogenous-agent perspective. The Review of Economic Studies, 87(1):77-101.

Brook, A.-M. (2003). Recent and prospective trends in real long-term interest rates: Fiscal policy and other drivers. Technical Report 367, OECD Economics Department Working Papers.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. Journal of Monetary Economics, 12(3):383-398.

Castaneda, A., Diaz-Gimenez, J., and Rios-Rull, J.-V. (1998). Exploring the income distribution business cycle dynamics. Journal of Monetary Economics, 42(1):93-130.

Challe, E. and Ragot, X. (2011). Fiscal policy in a tractable liquidity-constrained economy. The Economic Journal, 121(551):273-317.

Challe, E. and Ragot, X. (2015). Precautionary saving over the business cycle. The Economic Journal, 126(590):135-164.

Chetty, R., Guren, A., Manoli, D., and Weber, A. (2011). Are micro and macro labor supply elasticities consistent? A review of evidence on the intensive and extensive margins. The American Economic Review, 101(3):471-475.

Christiano, L. J., Eichenbaum, M., and Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. Journal of Political Economy, 113(1):1-45.

Christiano, L. J., Trabandt, M., and Walentin, K. (2010). DSGE models for monetary policy analysis. In Friedman, B. M. and Woodford, M., editors, Handbook of Monetary Economics, volume 3 of Handbook of Monetary Economics, chapter 7, pages 285-367. Elsevier.

Cui, W. (2016). Monetary-fiscal interactions with endogenous liquidity frictions. European Economic Review, 87:1-25.

Den Haan, W. J., Rendahl, P., and Riegler, M. (2017). Unemployment (fears) and deflationary spirals. Journal of the European Economic Association, 16(5):1281-1349.

Driscoll, J. C. and Kraay, A. C. (1998). Consistent covariance matrix estimation with spatially dependent panel data. Review of Economics and Statistics, 80(4):549-560.

Engen, E. M. and Hubbard, R. G. (2004). Federal government debt and interest rates. In NBER Macroeconomics Annual, volume 19, pages 83-160.

Floden, M. (2001). The effectiveness of government debt and transfers as insurance. Journal of Monetary Economics, 48(1):81-108.

Fratzscher, M. (2019). Ein Deutschlandfonds statt Aktien für Arme. Die ZEIT.
Geweke, J. (1992). Evaluating the accuracy of sampling-based approaches to the calculation of posterior moments. In Bernardo, J. M., Berger, J. O., Dawid, A. P., and Smith, A. F. M., editors, Bayesian Statistics, volume 4, pages 641-649. Clarendon Press, Oxford.

Gomme, P., Ravikumar, B., and Rupert, P. (2011). The return to capital and the business cycle. Review of Economic Dynamics, 14(2):262-278.

Gornemann, N., Kuester, K., and Nakajima, M. (2012). Monetary policy with heterogeneous agents. Philadelphia Fed Working Paper 12-21.

Gottardi, P., Kajii, A., and Nakajima, T. (2015). Optimal taxation and debt with uninsurable risks to human capital accumulation. American Economic Review, 105(11):3443-70.

Greenwood, J., Hercowitz, Z., and Huffman, G. W. (1988). Investment, capacity utilization, and the real business cycle. American Economic Review, 78(3):402-417.

Gros, D. and Mayer, T. (2012). A German sovereign wealth fund to save the euro. $C E P R$ VOXEU.

Guerrieri, V. and Lorenzoni, G. (2017). Credit crises, precautionary savings, and the liquidity trap. The Quarterly Journal of Economics, 132(3):1427-1467.

Guvenen, F., Kaplan, G., and Song, J. (2014). How risky are recessions for top earners? The American Economic Review, 104(5):148-153.

Hagedorn, M., Manovskii, I., and Mitman, K. (2019). The fiscal multiplier. NBER Working Paper, No. 25571.

Heathcote, J. (2005). Fiscal policy with heterogeneous agents and incomplete markets. The Review of Economic Studies, 72(1):161-188.

Huggett, M. (1993). The risk-free rate in heterogeneous-agent incomplete-insurance economies. Journal of Economic Dynamics and Control, 17(5-6):953-969.

Jaimovich, N. and Rebelo, S. (2009). Can news about the future drive the business cycle? American Economic Review, 99(4):1097-1118.

Jordà, Ò. (2005). Estimation and inference of impulse responses by local projections. American Economic Review, 95(1):161-182.

Jordà, Ò., Knoll, K., Kuvshinov, D., Schularick, M., and Taylor, A. M. (2019). The rate of return on everything, 1870-2015. The Quarterly Journal of Economics, 134(3):1225-1298.

Jordà, Ò., Schularick, M., and Taylor, A. M. (2017). Macrofinancial history and the new business cycle facts. NBER Macroeconomics Annual, 31(1):213-263.

Justiniano, A., Primiceri, G. E., and Tambalotti, A. (2011). Investment shocks and the relative price of investment. Review of Economic Dynamics, 14(1):101-121.

Kaplan, G., Moll, B., and Violante, G. L. (2018). Monetary policy according to HANK. American Economic Review, 108(3):697-743.

Kaplan, G. and Violante, G. L. (2010). How much consumption insurance beyond selfinsurance? American Economic Journal: Macroeconomics, 2(4):53-87.

King, R. G., Plosser, C. I., and Rebelo, S. T. (1988). Production, growth and business cycles: I. The basic neoclassical model. Journal of Monetary Economics, 21(2):195-232.

Kinoshita, N. (2006). Government debt and long-term interest rates. IMF Working Paper WP/06/63.

Krishnamurthy, A. and Vissing-Jorgensen, A. (2012). The aggregate demand for Treasury debt. Journal of Political Economy, 120(2):233-267.

Krueger, D. and Perri, F. (2006). Does Income Inequality Lead to Consumption Inequality? Evidence and Theory1. The Review of Economic Studies, 73(1):163-193.

Krueger, D., Perri, F., Pistaferri, L., and Violante, G. L. (2010). Cross-sectional facts for macroeconomists. Review of Economic Dynamics, 13(1):1-14.

Laubach, T. (2009). New evidence on the interest rate effects of budget deficits and debt. Journal of the European Economic Association, 7(4):858-885.

Luetticke, R. (2021). Transmission of monetary policy with heterogeneity in household portfolios. American Economic Journal: Macroeconomics, 13(2):1-25.

McKay, A., Nakamura, E., and Steinsson, J. (2016). The power of forward guidance revisited. The American Economic Review, 106(10):3133-3158.

Mertens, K. and Ravn, M. O. (2012). Empirical evidence on the aggregate effects of anticipated and unanticipated US tax policy shocks. American Economic Journal: Economic Policy, 4(2):145-81.

Newey, W. K. and West, K. D. (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. Econometrica, 55(3):703-708.

Ramey, V. A. (2011). Identifying government spending shocks: It's all in the timing. Quarterly Journal of Economics, 126(1):1-50.

Ramey, V. A. (2016). Macroeconomic shocks and their propagation. Handbook of Macroeconomics, 2:71-162.

Reis, R. (2021). The constraint on public debt when $\mathrm{r}<\mathrm{g}$ but $\mathrm{g}<\mathrm{m}$. CEPR Discussion Paper, No. DP15950.

Röhrs, S. and Winter, C. (2017). Reducing government debt in the presence of inequality. Journal of Economic Dynamics and Control, 82:1-20.

Romer, C. D. and Romer, D. H. (2010). The macroeconomic effects of tax changes: estimates based on a new measure of fiscal shocks. American Economic Review, 100(3):763-801.

Schmitt-Grohé, S. and Uribe, M. (2012). What's news in business cycles. Econometrica, 80(6):2733-2764.

Shiller, R. J. (2015). Irrational exuberance: Revised and expanded third edition. Princeton university press.

Smets, F. and Wouters, R. (2007). Shocks and frictions in US business cycles: A Bayesian DSGE approach. American Economic Review, 97(3):586-606.

Sterk, V. and Tenreyro, S. (2018). The transmission of monetary policy through redistributions and durable purchases. Journal of Monetary Economics, 99:124-137.

Storesletten, K., Telmer, C. I., and Yaron, A. (2004). Cyclical dynamics in idiosyncratic labor market risk. Journal of Political Economy, 112(3):695-717.

Summers, L. H. and Rachel, L. (2019). On falling neutral real rates, fiscal policy and the risk of secular stagnation. In Brookings Papers on Economic Activity, volume Spring 2019.

Taylor, J. B. (1993). Discretion versus policy rules in practice. In Carnegie-Rochester Conference Series on Public Policy, volume 39, pages 195-214. Elsevier.

Weiß, M. (2021). Stock price booms from technology news in a HANK model with portfolio choice. Mimeo, Bonn University.

Wong, A. (2019). Refinancing and the transmission of monetary policy to consumption. Mimeo, Princeton University.

Woodford, M. (1990). Public debt as private liquidity. American Economic Review, 80(2):382-388.

Wu, J. C. and Xia, F. D. (2016). Measuring the macroeconomic impact of monetary policy at the zero lower bound. Journal of Money, Credit and Banking, 48(2-3):253-291.

## Appendix A. Data

## Appendix A.1. Data for Local Projections

Unless otherwise noted, all series are available at quarterly frequency from 1947Q1 to 2019Q4 from the St.Louis FED - FRED database (mnemonics in parentheses). ${ }^{26}$ Corresponding series for the annual country panel (1947-2016) are taken from the Jordà-SchularickTaylor Macrohistory Database (Jordà et al., 2017). ${ }^{27}$

Output. Nominal GDP (GDP) divided by the GDP deflator (GDPDEF).
Investment. Gross private domestic investment (GPDI) divided by the GDP deflator (GDPDEF).

Consumption. Sum of personal consumption expenditures for nondurable goods (PCND), durable goods (PCDG) and services (PCESV) divided by the GDP deflator (GDPDEF).

Government spending. Government consumption expenditures and gross investment (GCE) divided by the GDP deflator (GDPDEF).

Public debt. Market value of gross federal debt (MVGFD027MNFRBDAL) divided by the GDP deflator (GDPDEF).

Nominal interest rate. Quarterly average of the effective federal funds rate (FEDFUNDS). From 2009Q1 till 2015Q4 we use the Wu and Xia (2016) shadow federal funds rate. Before 1954Q3, we use the 3-month t-bill rate (TB3MS).

Long-term rate on government bonds. Yield on long-term U.S. government securities (LTGOVTBD) until June 2000 and 20-Year Treasury Constant Maturity Rate (GS20) afterwards (see Krishnamurthy and Vissing-Jorgensen, 2012).

Real interest rate. Long-term rate on government bonds minus log-difference of GDP Deflator (GDPDEF).

[^19]Liquid assets. Sum of total currency and deposits including money market fund shares (FL154000025.Q), corporate equities (LM153064105.Q), and debt securities (LM154022005.Q) directly held by households from the Board of Governor's Flow of Funds tables; divided by the GDP deflator (GDPDEF).

Return to capital. After-tax returns to all capital taken from Gomme et al. (2011) and available till 2015Q4.

Return to housing. Annual return to housing from Jordà et al. (2019), available at annual frequency until 2016 and interpolated to quarterly frequency via cubic splines.

Liquidity premia. Difference between the respective return to capital or housing and the long-term rate on government bonds.

Liquidity premium on corporate bonds. Convenience yield: Spread between Moodys Aaa-rated corporate bond yield and the long-term rate on government bonds.

Liquidity premium on money. Spread between the long-term rate on government bonds and the (shadow) federal funds rate.

Equity premium. Computed from Bob Shiller's CAPE measure as 1/CAPE minus the long-term rate on government bonds.

Military spending shocks. Ramey (2011)-series of narratively-identified defense news shocks. Series available from 1947Q1 to 2015Q4 on Valerie Ramey's homepage (https : //econweb.ucsd.edu/~vramey/research.html).

Tax shocks. Romer and Romer (2010)-series of narratively-identified exogenous tax changes which are measured as total revenue impact as a ratio of GDP in the previous quarter. We focus on those classified as unanticipated by Mertens and Ravn (2012). Series available from 1948Q1 to 2007Q3 on Karel Mertens' homepage (https : //karelmertens.com/research/).

Tax revenues. Federal government current tax receipts (W006RC1Q027SBEA) divided by the GDP deflator (GDPDEF).

## Appendix A.2. Data for Calibration

We use the following four moments to calibrate the steady-state wealth distribution.
Mean illiquid assets. Private fixed assets (NIPA table 1.1) over quarterly GDP, averaged over 1947-2019.

Public liquidity. Gross federal debt (MVGFD027MNFRBDAL) over quarterly GDP, averaged over 1947-2019.

Private liquidity. Private unsecured credit (HCCSDODNS) from the flow of funds over quarterly GDP, averaged over 1947-2019.

Average top 10 percent share of wealth. Source is the World Inequality Database (1947-2019).

## Untargeted moments.

The model also does well in matching the household portfolio composition. As documented by Bayer et al. (2019) using the SCF, wealth-poor households hold more liquid portfolios than wealth-rich households in the US. This is true in the model because wealthpoor households are indebted and then first pay-off this debt and build a buffer stock of liquid savings before investing in illiquid capital. In the model, the ratio of liquid to illiquid assets falls by 26 percentage points for households in the 25 th vs. 75 th percentile of the wealth distribution. Bayer et al. (2019) find a decline by 23 percentage points of this ratio in the SCF, see their Figure 6. The model also features a higher Gini coefficient for liquid wealth than net wealth, as is the case in the US.

In addition, the model also matches the distribution of income and consumption in the US well. The Gini coefficient for consumption is 0.28 in the model and Krueger and Perri (2006) estimate this Gini coefficent to be around 0.25 in the US using data from the CEX. The variance of log net income is 0.39 in the model and Krueger et al. (2010) estimate this number to be 0.41 in the US using data from the PSID.

The calibration produces average annualized marginal propensities to consume of 8.5 and 47.3 percent for transitory and persistent income shocks, respectively. These are on the low end of what the literature usually calibrates to, but in line with the evidence by Kaplan and Violante (2010).

## Appendix B. Additional Empirical Results

## Appendix B.1. Aggregate Responses to Military News Shocks

Figure B.1(a) provides the impulse responses for narratively identified government spending shocks following Ramey (2011).

Figure B.1: Empirical Responses to Fiscal Expansion (US)


Notes: Impulse responses to a government spending shock. IRFs based on narrative identification via military news series from Ramey (2011); IRFs scaled so that the maximum debt response is 1 percent. Light (dark) gray areas are 90 percent ( 68 percent) confidence bounds based on Newey and West (1987)-standard errors. Panel (a) from top left to bottom right: Government spending, federal debt held by the public, gross national expenditures, investment, consumption, real return on long-term government bonds.
Panel (b) from top left to bottom right: Liquid assets: deposits plus stocks plus debt directly held by households; liq. premium capital: rate of return on capital minus long-term gov. bond rate; liq. premium housing: rate of return on housing minus long-term gov. bond rate; liq. premium corp. bonds: AAA corporate bond yield minus long-term gov. bond rate; liq. premium money: gov. bond rate minus (shadow) federal funds rate. Equity premium: Return on stocks minus long-term gov. bond rate.

Figure B.2: Empirical Responses to Romer-Romer Narrative Tax Shocks


Notes: Impulse responses to a tax shock. IRFs based on Romer and Romer (2010)-narrative tax changes; IRFs scaled so that the maximum debt response is 1 percent. Light (dark) gray areas are 90 percent (68 percent) confidence bounds based on Newey and West (1987)-standard errors.

## Appendix B.2. Tax Shocks

As discussed in Section 2, we are not interested in government spending shocks per se, but rather use them as a vehicle to study how an increase in public debt affects liquidity premia. Of course, increases in government spending are not the only causes for changes in the level of debt. Here, we study whether increases in debt induced by tax changes show a similar link between debt and liquidity premia.

To this end, we employ the Romer and Romer (2010)-series of narratively-identified exogenous tax changes, focusing on those classified as unanticipated by Mertens and Ravn (2012), which is available from 1948 till 2007. We replace $\log g_{t}$ in Equation (1) by the exogenous tax shock measure and include as additional control lags of real federal tax revenues. Results are shown in Figure B.2, where we again scale the IRFs so that the maximum response of public debt is 1 percent. Public debt takes some time to build up but once it
does, premia start to fall. So the negative link between public debt and liquidity premia also holds true if the increase in debt comes from the revenue and not the expenditure side of the government budget constraint. Focusing on the liquidity premium on housing, we see that the elasticity is quantitatively in the same ballpark. ${ }^{28}$

[^20]
## Appendix C. Equilibrium

A sequential equilibrium with recursive planning in our model is a sequence of policy functions $\left\{x_{a, t}^{*}, x_{n, t}^{*}, d_{a, t}^{*}, d_{n, t}^{*}, k_{t}^{*}\right\}$, of value functions $\left\{V_{t}^{a}, V_{t}^{n}\right\}$, of prices $\left\{w_{t}, w_{t}^{F}, \Pi_{t}^{F}, \Pi_{t}^{U}, q_{t}, q_{t}^{B}, r_{t}, R_{t}^{d}, \pi_{t}, \pi_{t}^{W}\right\}$, of shocks $\epsilon_{t}^{G}$, aggregate capital and labor supplies $\left\{K_{t}, N_{t}\right\}$, distributions $\Theta_{t}$ over individual asset holdings and productivity, and expectations such that

1. Given the functional $\mathbb{W}_{t+1}$ for the continuation value and period-t prices, policy functions $\left\{x_{a, t}^{*}, x_{n, t}^{*}, d_{a, t}^{*}, d_{n, t}^{*}, k_{t}^{*}\right\}$ solve the households' planning problem, and given the policy functions $\left\{x_{a, t}^{*}, x_{n, t}^{*}, d_{a, t}^{*}, d_{n, t}^{*}, k_{t}^{*}\right\}$ and prices, the value functions $\left\{V_{t}^{a}, V_{t}^{n}\right\}$ are a solution to the Bellman equation (7).
2. Distributions of wealth and income evolve according to households' policy functions.
3. The labor, the final goods, the bond, the capital, and the intermediate goods markets clear in every period, interest rates on deposits are set according to the central bank's Taylor rule, fiscal policy is set according to the fiscal rule, and stochastic processes evolve according to their laws of motion.
4. Expectations are model consistent.

## Appendix D. Numerical Solution and Estimation Technique

We solve the model by perturbation methods. We choose a first-order Taylor expansion around the stationary equilibrium following the method of Bayer and Luetticke (2020). This method replaces the value functions with linear interpolants and the distribution functions with histograms to calculate a stationary equilibrium. Then it performs dimensionality reduction before linearization but after calculation of the stationary equilibrium. The dimensionality reduction is achieved by using discrete cosine transformations (DCT) for the value functions and perturbing only the largest coefficients of this transformation and by approximating the joint distributions through distributions with an approximated copula and full marginals. We approximate changes in the Copula relative to the steady state in a similar way we approximate the value function with DCTs (plus additional constraints ensuring it remains a probability distribution). We solve the model originally on a grid of $80 x 80 x 11$ points for liquid assets, illiquid assets, and income, respectively. We apply a dimensionality reduction step after a first model solution based on the priors along the lines described in Bayer et al. (2020).

Approximating the sequential equilibrium in a linear state-space representation then boils down to the linearized solution of a non-linear difference equation

$$
\begin{equation*}
\mathbb{E}_{t} F\left(P x_{t}, X_{t}, P x_{t+1}, X_{t+1}, \sigma \Sigma \epsilon_{t+1}\right), \tag{D.1}
\end{equation*}
$$

where $x_{t}$ is "idiosyncratic" states and controls: the value and distribution functions, and $X_{t}$ is aggregate states and controls: prices, quantities, productivities, etc. The error term $\epsilon_{t}$ represents fundamental shocks. $P$ is a model reduction matrix.

## Appendix E. Estimation Diagnostics

## Appendix E.1. Convergence Checks

We estimate the parameters of the baseline and each of the alternative models using single RWMH chains after an extensive mode search. After burn-in, 430,000 draws from the posterior distribution are used to compute the posterior statistics. The acceptance rates are between 20 and 30 percent. Here, we provide Geweke (1992) convergence statistics (for all models) as well as traceplots (for the baseline model) of individual parameters. Geweke (1992) tests the equality of means of the first 10 percent of draws and the last 50 percent of draws (after burn-in). If the samples are drawn from the stationary distribution of the chain, the two means are equal and Geweke's statistic has an asymptotically standard normal distribution. Table E. 1 reports the Geweke z -score statistic and the p -value for the chains of each parameter. Taking the evidence from Geweke (1992) and traceplot graphs together, we conclude that our chains have converged.

Table E.1: Geweke (1992) Convergence Diagnostics

|  | HANK (base) |  | HANK |  | (ARMA) | HANK (RA2) |  | HANK (KPR) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Parameter | z-stat | p-value | z-stat | p-value | z-stat | p-value | z-stat | p-value |  |
| $\delta_{s}$ | 0.343 | 0.732 | -0.965 | 0.334 | -0.592 | 0.554 | -0.307 | 0.759 |  |
| $\phi$ | -1.256 | 0.209 | -0.266 | 0.791 | -2.398 | 0.016 | 1.471 | 0.141 |  |
| $\kappa$ | 0.576 | 0.565 | 0.252 | 0.801 | 0.793 | 0.428 | -0.217 | 0.828 |  |
| $\kappa_{w}$ | -0.912 | 0.362 | 0.472 | 0.637 | -0.267 | 0.789 | -0.303 | 0.762 |  |
| $\rho_{R}$ | 1.272 | 0.203 | -0.531 | 0.595 | -1.66 | 0.097 | 0.007 | 0.994 |  |
| $\theta_{\pi}$ | -0.531 | 0.596 | -0.837 | 0.403 | -1.469 | 0.142 | -0.236 | 0.814 |  |
| $\theta_{Y}$ | -0.643 | 0.520 | 0.148 | 0.882 | -0.121 | 0.904 | -1.394 | 0.163 |  |
| $\theta_{B}$ | -1.595 | 0.111 | -1.216 | 0.224 | 0.172 | 0.863 | 0.151 | 0.880 |  |
| $\gamma_{B}$ | -1.303 | 0.192 | 0.504 | 0.614 | 0.624 | 0.533 | -1.227 | 0.220 |  |
| $\gamma_{Y}$ | 0.587 | 0.557 | 0.048 | 0.962 | 0.197 | 0.844 | -0.783 | 0.434 |  |
| $\rho_{G *}$ | -1.252 | 0.210 | -0.157 | 0.875 | 1.020 | 0.308 | -0.337 | 0.736 |  |
| $\rho_{G 2}$ | -1.605 | 0.109 | -1.713 | 0.087 | -0.069 | 0.945 | -1.434 | 0.151 |  |
| $\sigma_{G}$ | -0.371 | 0.710 | 1.662 | 0.096 | -1.083 | 0.279 | -0.996 | 0.319 |  |

Notes: Geweke (1992) equality of means test of the first 10 percent vs. the last 50 percent of draws. Failure to reject the null of equal means indicates convergence. HANK (ARMA) denotes HANK model with ARMA $(1,1)$ process for government spending instead of AR(2), HANK (RA2) denotes HANK model with risk aversion 2 instead of 4, HANK (KPR) denotes HANK model with KPR instead of GHH preferences. To estimate the $\operatorname{AR}(2)$-process for government spending, we estimate $\rho_{G *}=\rho_{G 1}+\rho_{G 2}$ in addition to $\rho_{G 2}$. For HANK (ARMA), $\rho_{G *}=\rho_{G 1}$ and $\rho_{G 2}$ is the coefficient on the MA term.

Figure E.3: MCMC draws of baseline HANK-2 model


## Appendix F. Comparison to HANK-1 and RANK

To understand better how important the liquidity injection is in the short run; how important the portfolio choice is; and how our model compares to a complete markets setup, we run the same spending shock in two alternative specifications of the model under the baseline (HANK-2) parameterization. First, we look at the shock in an incomplete markets model in which all assets are liquid and thus, up to first order, the return difference between capital and bonds is constant (HANK-1). Second, we look at a version of the model with a representative agent, i.e., with complete markets (RANK). IRFs for all three model variants are displayed in Figure F.4. ${ }^{29}$

The IRFs across models are similar but diverge over time when the capital crowding out kicks in. When there is no movement in the liquidity premium, the decline of investment is much stronger compared to our baseline, in which capital is illiquid from the point of view of the household. Without portfolio choice and thus without an endogenous response of the liquidity premium, there is more crowding out of capital. Conversely, there is less crowding out of private bonds. The stronger decline of investment in RANK and HANK-1 also has consequences for the fiscal multiplier at longer horizons because capital falls less in HANK-2. After 3 years, when government spending is back at zero, the cumulative multipliers are 0.07 in HANK-2, -0.02 in HANK-1, and -0.02 in RANK. The impact fiscal multiplier, however, is quite similar - 0.54 (HANK-2) vs. 0.53 (HANK-1) vs. 0.48 (RANK).

[^21]Figure F.4: Impulse Response Functions to a Government Spending Shock


Notes: Impulse responses to the estimated government spending shock. Black solid line: Baseline model, HANK-2. Blue dash-dotted: Liquid-capital model, HANK-1. Red dotted line: Completemarkets model, RANK. Both alternative models under baseline parameters. Y-axis: Percent deviation from steady state, except for $R B_{t}, \pi_{t}$, and $L P_{t}$ that are in annualized percentage points. X-axis: Quarters.

## Appendix G. Robustness

In this appendix, we investigate robustness of our main results with respect to variations in the fiscal spending rule, risk aversion, and KPR preferences. For the latter two, we need to recalibrate the steady state to match the capital-to-output ratio, the public-debt-to-output ratio, the private-debt-to-output ratio, and the wealth held by the top 10 percent as reported in Table 1.

For GHH with risk aversion parameter of 2 , this yields a discount factor of $\beta=0.9905$, a portfolio adjustment probability of $\lambda=4.8$ percent, a borrowing penalty of $\bar{R}=1.0$ percent, and a probability of becoming an entrepreneur of $1 / 1700$.

For KPR preferences, this yields a discount factor of $\beta=0.9855$, a portfolio adjustment probability of $\lambda=9$ percent, a borrowing penalty of $\bar{R}=1.0$ percent, and a probability of becoming an entrepreneur of $1 / 1600$. The felicity function $u$ now reads:

$$
u\left(c_{i t}, n_{i t}\right)=\frac{c_{i t}^{1-\xi}-1}{1-\xi}-\Gamma \frac{n_{i t}^{1+\gamma}-1}{1+\gamma}
$$

with risk aversion parameter $\xi>0$ and inverse Frisch elasticity $\gamma>0$. The first-order condition for labor supply is:

$$
n_{i t}=\left[\frac{1}{\Gamma} u^{\prime}(c)\left(1-\tau_{t}\right)\left(w h_{i t}\right)\right]^{\left(\frac{1}{\gamma}\right)} .
$$

Table G. 2 shows the posterior distributions of the re-estimated parameters for all variants and Figure G. 5 presents the corresponding IRFs.

Table G.2: Prior and Posterior Distributions of Estimated Parameters: Alternative Models

| Parameter | Distribution | Prior |  | Posterior |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Mean | Std. Dev. | HANK (base) | HANK (ARMA) | HANK (RA2) | HANK (KPR) |
| Frictions |  |  |  |  |  |  |  |
| $\delta_{s}$ | Gamma | 5.00 | 2.00 | $\begin{gathered} 2.296 \\ (0.380,5.774) \end{gathered}$ | $\begin{gathered} 1.916 \\ (0.334,6.345) \end{gathered}$ | $\begin{gathered} \hline 3.460 \\ (0.889,7.177) \end{gathered}$ | $\begin{gathered} \hline 4.867 \\ (2.167,8.584) \end{gathered}$ |
| $\phi$ | Gamma | 4.00 | 2.00 | $\begin{gathered} 3.929 \\ (2.183,6.276) \end{gathered}$ | $\begin{gathered} 4.082 \\ (2.088,6.738) \end{gathered}$ | $\begin{gathered} 4.406 \\ (2.441,7.000) \end{gathered}$ | $\begin{gathered} 1.232 \\ (0.627,2.026) \end{gathered}$ |
| $\kappa$ | Gamma | 0.10 | 0.02 | $\begin{gathered} 0.109 \\ (0.077,0.145) \end{gathered}$ | $\begin{gathered} 0.114 \\ (0.082,0.152) \end{gathered}$ | $\begin{gathered} 0.102 \\ (0.085,0.119) \end{gathered}$ | $\begin{gathered} 0.034 \\ (0.022,0.054) \end{gathered}$ |
| $\kappa_{w}$ | Gamma | 0.10 | 0.02 | $\begin{gathered} 0.108 \\ (0.077,0.144) \end{gathered}$ | $\begin{gathered} 0.113 \\ (0.081,0.149) \end{gathered}$ | $\begin{gathered} 0.101 \\ (0.085,0.118) \end{gathered}$ | $\begin{gathered} 0.050 \\ (0.028,0.079) \end{gathered}$ |
| Monetary policy rule |  |  |  |  |  |  |  |
| $\rho_{R}$ | Beta | 0.75 | 0.20 | $\begin{gathered} 0.977 \\ (0.956,0.991) \end{gathered}$ | $\begin{gathered} 0.979 \\ (0.958,0.992) \end{gathered}$ | $\begin{gathered} 0.973 \\ (0.946,0.990) \end{gathered}$ | $\begin{gathered} 0.718 \\ (0.567,0.848) \end{gathered}$ |
| $\theta_{\pi}$ | Normal | 1.70 | 0.30 | $\begin{gathered} 1.620 \\ (1.189,2.078) \end{gathered}$ | $\begin{gathered} 1.588 \\ (1.170,2.049) \end{gathered}$ | $\begin{gathered} 1.590 \\ (1.154,2.057) \end{gathered}$ | $\begin{gathered} 1.139 \\ (1.006,1.373) \end{gathered}$ |
| $\theta_{Y}$ | Normal | 0.13 | 0.05 | $\begin{gathered} 0.124 \\ (0.042,0.206) \end{gathered}$ | $\begin{gathered} 0.123 \\ (0.040,0.206) \end{gathered}$ | $\begin{gathered} 0.123 \\ (0.041,0.206) \end{gathered}$ | $\begin{gathered} 0.132 \\ (0.049,0.214) \end{gathered}$ |
| $\theta_{B}$ | Gamma | 0.05 | 0.04 | $\begin{gathered} 0.022 \\ (0.012,0.032) \end{gathered}$ | $\begin{gathered} 0.023 \\ (0.014,0.033) \end{gathered}$ | $\begin{gathered} 0.021 \\ (0.012,0.031) \end{gathered}$ | $\begin{gathered} 0.042 \\ (0.031,0.056) \end{gathered}$ |
| Fiscal policy rule |  |  |  |  |  |  |  |
| $-\gamma_{B}$ | Gamma | 0.50 | 0.25 | $\begin{gathered} 0.567 \\ (0.169,1.092) \end{gathered}$ | $\begin{gathered} 0.934 \\ (0.408,1.535) \end{gathered}$ | $\begin{gathered} 0.695 \\ (0.272,1.275) \end{gathered}$ | $\begin{gathered} 0.809 \\ (0.401,1.331) \end{gathered}$ |
| $\gamma_{Y}$ | Normal | 0.00 | 1.00 | $\begin{gathered} -0.188 \\ (-1.804,1.460) \end{gathered}$ | $\begin{gathered} -0.495 \\ (-2.12,1.195) \end{gathered}$ | $\begin{gathered} -0.161 \\ (-1.771,1.492) \end{gathered}$ | $\begin{gathered} -0.229 \\ (-1.855,1.402) \end{gathered}$ |
| $\rho_{G *}$ | Beta | 0.50 | 0.20 | $\begin{gathered} 0.962 \\ (0.951,0.973) \end{gathered}$ | $\begin{gathered} 0.958 \\ (0.943,0.969) \end{gathered}$ | $\begin{gathered} 0.964 \\ (0.953,0.974) \end{gathered}$ | $\begin{gathered} 0.968 \\ (0.958,0.976) \end{gathered}$ |
| $\rho_{G 2}$ | Normal | 0.00 | 0.50 | $\begin{gathered} -0.477 \\ (-0.563,-0.38) \end{gathered}$ | $\begin{gathered} 0.394 \\ (0.201,0.608) \end{gathered}$ | $\begin{gathered} -0.439 \\ (-0.532,-0.332) \end{gathered}$ | $\begin{gathered} -0.512 \\ (-0.587,-0.431) \end{gathered}$ |
| $\sigma_{G}$ | Inv.-Gamma | 1.00 | 2.00 | $\begin{gathered} 0.602 \\ (0.537,0.675) \\ \hline \end{gathered}$ | $\begin{gathered} 0.714 \\ (0.612,0.826) \end{gathered}$ | $\begin{gathered} 0.633 \\ (0.564,0.715) \end{gathered}$ | $\begin{gathered} 0.537 \\ (0.482,0.598) \\ \hline \end{gathered}$ |

Notes: Columns $5-8$ report posterior means with ( $0.05,0.95$ )-percentiles in parentheses. HANK (ARMA) denotes HANK model with ARMA(1,1) process for government spending instead of AR(2), HANK (RA2) denotes HANK model with risk aversion 2 instead of 4, HANK (KPR) denotes HANK model with KPR instead of GHH preferences. To estimate the AR(2)-process for government spending, we estimate $\rho_{G *}=$ $\rho_{G 1}+\rho_{G 2}$ in addition to $\rho_{G 2}$. For HANK (ARMA), $\rho_{G *}=\rho_{G 1}$ and $\rho_{G 2}$ is the coefficient on the MA term. The standard deviation of the government spending shock is expressed in percent.

Figure G.5: Impulse Response Functions (robustness)


Notes: Impulse responses to the estimated government spending shocks in the baseline model (black - solid line), model with $G_{t}$ following an ARMA process (blue dash-dotted line), model with risk aversion of 2 (red dotted line), and model with KPR preferences (green dot-dot-dashed line.) Y-axis: Percent deviation from steady state for $Y_{t}, C_{t}, I_{t}, G_{t}, B_{t}$, and $I O U_{t}$, and annualized percentage points for $R B_{t}, \pi_{t}$ and $L P_{t}$. X-axis: Quarters.

## Appendix H. Increasing the Debt Target to Finance Expenditures

Figure H. 6 shows the impulse responses for a 10 percent increase in the debt target used for government spending. In response, as in the baseline in which adjustment is done via non-distortionary transfers, the liquidity premium falls by around 25 basis points. In the long run, capital falls by around 0.3 percent-slightly less than in the baseline experiment. Similarly, wealth inequality falls in the long run because of the decline in the liquidity premium. The capital stock falls somewhat less than in the baseline, because the long-term cut in government spending increases resources available for consumption and investment. Both in HANK-1 and to a lesser extent HANK-2 there is some crowding out through higher debt. In RANK this is absent, and the long-run capital stock falls the least there. This resembles the baseline, where the debt-financed transfers in the RANK model had no effect on any variable and on capital in particular.

Figure H.6: Response to an Increase in the Debt Target (G adjusts)


Notes: Impulse responses to a 10 percent debt target shock used for government spending. Black solid line: Baseline model, HANK-2. Blue dash-dotted: Liquid-capital model, HANK-1. Red dotted line: Complete markets model, RANK. Y-axis: Percent deviation from steady state, except for $R B_{t}$ and $L P_{t}$ that are in annualized percentage points. X-axis: Quarters.

## Appendix I. Broader Definitions of Private Liquidity

Table I. 3 shows that our estimate of the semi-elasticity of the real rate to public debt is robust to including more private liquidity in the model. We increase private liquidity in the model in four ways: 1) loosening of the borrowing limit and, thus, allowing for $10 \%$ more unsecured borrowing, 2) a $10 \%$ higher probability of trading capital, 3) introducing secured borrowing in a model with leveraged capital, where we securitize $10 \%$ of the capital stock, and 4) introducing tradable profits that increase liquid assets by $10 \%$.

Table I.3: Robustness to Broader Private Liquidity

| Parameter | Capital <br> $(\mathrm{K} / \mathrm{Y})$ | Total liquidity <br> $((\mathrm{B}+\mathrm{IOUs}+\ldots) / \mathrm{Y})$ | Top $10 \%$ wealth <br> share | Interest rate <br> semi-elasticity $(\%)$ |
| :--- | :---: | :---: | :---: | :---: |
| Borrowing limit | 2.87 | 0.73 | 0.68 | 2.28 |
| Illiquidity of capital | 2.88 | 0.68 | 0.69 | 2.17 |
| Securitized capital | 2.92 | 0.79 | 0.70 | 1.50 |
| Tradable profits | 2.87 | 0.73 | 0.68 | 1.72 |

Notes: Row 1 allows for $10 \%$ more unsecured borrowing. Row 2 increases the liquidity of the capital stock by $10 \%$. Row 3 securitizes $10 \%$ of the capital stock. Row 4 allows for tradable profits making up $10 \%$ of liquid assets. In each case we calibrate to a public-debt-to-GDP ratio of $59.5 \%$ by adjusting the return to liquid assets.

We introduce leveraged capital by assuming that the banks invest some of the deposits in capital. Concretely a fraction $\Xi$ of the steady-state capital stock $\bar{K}$ is owned by banks. This implies more liquid saving vehicles for households and affects the clearing of asset markets in the following way:

$$
\begin{aligned}
B_{t+1}+\Xi \bar{K} & =\mathbb{E}_{t}\left[\lambda d_{a, t}^{*}+(1-\lambda) d_{n, t}^{*}\right] \\
K_{t+1} & =\mathbb{E}_{t}\left[\lambda k_{t}^{*}+(1-\lambda) k\right]+\Xi \bar{K} .
\end{aligned}
$$

Profits of the financial intermediary accrue to the entrepreneurs but we assume an intermediation cost equal to the steady-state liquidity premium to leave steady-state profits unaltered. We choose $\Xi=0.1$, which expands the supply of liquid assets by almost $50 \%$.

We introduce tradable profits by assuming that claims to a fraction $\omega^{\Pi}$ of profits can be traded at price $q_{t}^{\Pi}$ following the work of Weiß (2021). Again, we assume that banks hold these profit-stocks and the return on these claims has to fulfill the no-arbitrage condition:

$$
\begin{equation*}
\mathbb{E} \frac{R_{t+1}^{d}}{\pi_{t+1}}=\mathbb{E} \frac{q_{t+1}^{\Pi}\left(1-\iota^{\Pi}\right)+\omega^{\Pi} \Pi_{t+1}^{F}}{q_{t}^{\Pi}} \tag{I.1}
\end{equation*}
$$

where a fraction of $\iota^{\Pi}$ of those claims retire every period and lose value, while new claims are emitted by the entrepreneurs. Asset markets clearing now reads:

$$
\begin{aligned}
B_{t+1}+q_{t}^{\Pi} & =\mathbb{E}_{t}\left[\lambda d_{a, t}^{*}+(1-\lambda) d_{n, t}^{*}\right] \\
K_{t+1} & =\mathbb{E}_{t}\left[\lambda k_{t}^{*}+(1-\lambda) k\right]
\end{aligned}
$$

In words, banks invest deposits now either in stocks or government bonds. The real payout to entrepreneurs (bank profits aside) then becomes $\left(1-\omega^{\Pi}\right) \Pi_{t}^{F}+\iota^{\Pi} q_{t}^{\Pi}$. The fractions $\omega^{\Pi}$ and $\iota^{\Pi}$ are calibrated to yield a share of liquid assets held in stocks of $10 \%$, which implies $\omega^{\Pi}=0.05$ and $\iota^{\Pi}=0.016$. Since the net interest rate on deposits is zero in our steady-state calibration, the total profits including bank profits remain the same as in our baseline.


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[^1]:    ${ }^{1}$ This is in line with the summary of estimates in the literature in Summers and Rachel (2019), Table 2.

[^2]:    ${ }^{2}$ See, for example, Auclert et al. (2020); Bayer et al. (2019); Broer et al. (2019); Challe and Ragot (2015); Den Haan et al. (2017); Gornemann et al. (2012); Guerrieri and Lorenzoni (2017); Kaplan et al. (2018); Luetticke (2021); McKay et al. (2016); Sterk and Tenreyro (2018); Wong (2019).

[^3]:    ${ }^{3}$ See, for example, Floden (2001), Gottardi et al. (2015), Bhandari et al. (2017), Röhrs and Winter (2017), Acikgöz et al. (2018), Azzimonti and Yared (2019). There are exceptions that assess the importance of liquidity frictions, for example, Angeletos et al. (2016); Cui (2016).
    ${ }^{4}$ Complementary to our paper, Bredemeier et al. (2022) report that a fiscal expansion increases the return spread between treasury bonds and even more liquid assets like cash deposits.

[^4]:    ${ }^{5}$ The constraining factor is the availability of some of the liquidity premia after 2015. See Appendix A for more details on the data.

[^5]:    ${ }^{6}$ This is equivalent to a two-step approach, where $g_{t}$ is first regressed on lags of itself and additional covariates and the residual is then included in step 2 as the shock measure.
    ${ }^{7}$ We use the long-term government bond rate from Krishnamurthy and Vissing-Jorgensen (2012) with maturity of 10 years or more, see Appendix A.
    ${ }^{8}$ This combines business and housing capital. Looking at both returns separately yields similar results.

[^6]:    ${ }^{9}$ See Appendix A for more details on the data and the country coverage.

[^7]:    ${ }^{10}$ Of course, the Blanchard and Perotti (2002)-predeterminedness assumption is more restrictive at the annual than at the quarterly level. However, Born and Müller (2012) provide evidence for Australia, Canada, the UK, and the US that this assumption may not be too restrictive even for annual timeseries data.
    ${ }^{11}$ The model extends Bayer et al. (2020) and the exposition follows that paper where there is overlap.

[^8]:    ${ }^{12}$ There are basically three possibilities for dealing with the pure rents. One attributes them to capital and labor, but this affects their factor prices; one introduces a third asset that pays out rents as dividends and is priced competitively; or one distributes the rents in the economy to an exogenously determined group of households. The latter has the advantage that factor supply decisions remain the same as in any standard New-Keynesian framework and still avoids the numerical complexity of dealing with three assets.
    ${ }^{13}$ The assumption of GHH preferences is mainly motivated by the fact that many estimated DSGE models of business cycles find small aggregate wealth effects in labor supply; see, e.g., Schmitt-Grohé and Uribe (2012); Born and Pfeifer (2014). It is not feasible to estimate the flexible form of preference of Jaimovich and Rebelo (2009), which also encompasses King et al. (1988) (KPR) preferences. This would require solving the stationary equilibrium in every likelihood evaluation, which is substantially more time consuming than solving for the dynamics around this equilibrium. We provide a robustness check of our main results to assuming KPR preferences instead in Appendix G. The GHH assumption has been criticized by Auclert et al. (2021) on the basis of producing "too high" multipliers. We show that fiscal multipliers in our estimated model are of reasonable size both in the short and in the long run. The reason for this lies in the combination of model elements only briefly discussed or even absent in the stylized Auclert et al. (2021) economy: sticky wages, distortionary taxes, capacity utilization, and a Taylor rule. Capacity utilization allows for output adjustment without adjusting hours; additional wage stickiness translates increasing labor demand into higher wage markups instead of hours and consumption; distortionary taxes absorb an additional fraction of income; and the Taylor rule translates the fiscal shock into to a real interest rate increase. The back-of-the envelope calculation of the multiplier based on formula (15) in Auclert et al. (2021), counter-factually assuming fixed real rates and ignoring capacity utilization, would be: $(1-(1-\tau)(\eta-1) / \eta(\zeta-1) / \zeta)^{-1} \approx 2.5$. The estimated multiplier is, in line with the data, much smaller.

[^9]:    ${ }^{14}$ Hence, productivity risk can be read off from estimated income risk and both treated interchangeably.

[^10]:    ${ }^{15}$ Since we solve the model by a first-order perturbation in aggregate shocks, fluctuations in stochastic discount factors are irrelevant.

[^11]:    ${ }^{16}$ Appendix I extends the banks' balance sheet and hence private liquidity by allowing banks to invest in capital and profits as well.

[^12]:    ${ }^{17}$ Detailed data sources and a discussion of untargeted moments of the distributions of wealth, income, and consumption can be found in Appendix A.

[^13]:    ${ }^{18}$ Appendix E. 1 provides more details and convergence statistics.

[^14]:    ${ }^{19}$ The speed of this transition is not important for the long-run results. The debt target shock has a persistence of 0.9999. Long-term effects are similar to steady-state comparisons.

[^15]:    ${ }^{20}$ The transfers are proportional to a household's productivity in order to keep income risk constant in this exercise. No response of government spending to the debt target and cycle is allowed, $\gamma_{B}=\gamma_{Y}=0$.
    ${ }^{21}$ We also consider adjustment through government spending, such that there is no Ricardian equivalence in RANK. The IRFs are displayed in Appendix H. The findings are similar.
    ${ }^{22}$ See their Table 2, which reports a 3.5 basis point increase for a 1 percentage point increase in the debt-to-output ratio.

[^16]:    ${ }^{23}$ We take the 10-year bond yield minus nominal GDP growth. In terms of the model, a risk premium shock, $A_{t}$, for example, might have lowered the interest rate. See next section for a discussion.

[^17]:    ${ }^{24}$ These proposals have been around since the euro area hit the ZLB; see for example Gros and Mayer (2012) or Fratzscher (2019).

[^18]:    ${ }^{25}$ This sets aside political economy arguments or the government being less efficient when holding capital.

[^19]:    ${ }^{26}$ In the quarterly regressions, we use only data until 2015Q4 to have a consistent sample across all dependent variables. The constraining factor is the availability of some of the liquidity premia after 2015.
    ${ }^{27}$ Countries covered are Australia, Belgium, Switzerland, Germany, Denmark, Spain, Finland, France, United Kingdom, Italy, Japan, Netherlands, Norway, Portugal, Sweden, and the USA.

[^20]:    ${ }^{28}$ Interestingly, a tax cut in our sample is self-financing through an increase in economic activity and leads to a fall in debt. Given that we are only interested in the link between debt and liquidity premia, we flip all IRFs to facilitate comparison with the other experiments.

[^21]:    ${ }^{29}$ We recalibrate the Taylor-rule response to government debt, such that the interest rate response coincides in all models at $\mathrm{t}=16$. We do this, because the neutral-rate in the three models differs. When the central bank imposes its neutral rate estimate from HANK-2 as target in the other models, then this will create substantial deflation and much higher real rates, leading to even negative multipliers then.

