Efficiency versus Optimality in Procurement*

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Abstract

We study procurement procedures that simultaneously determine the specification and price of a good. Suppliers can offer and produce the good in either of two possible specifications, both of which are equally good for the buyer. Production costs are interdependent and unknown at the time of bidding. Each supplier receives two signals about production cost, one per specification. Our model is a special case of the interdependent-value settings with multi-dimensional types in Jehiel and Moldovanu (2001) where an efficient and incentive compatible mechanism exists. We characterize equilibrium bidding behavior if the winning supplier is selected purely on the basis of price, regardless of the specification offered. While there is a positive chance of obtaining an inefficient specification, this procurement mechanism involves lower information rents than efficient mechanisms, suggesting that there is a trade-off between minimizing expected expenditure for the good, and ensuring that the efficient specification is chosen.

Keywords: Procurement · interdependent valuations · multi-dimensional information · efficient mechanisms · optimal mechanisms.

JEL classification: C72 · D44 · D82.

1 Introduction

In the procurement of complex projects (such as construction work, or defence equipment) the exact quality or specification of the project is rarely fully determined by the buyer at the outset. Instead, the purpose of the procurement process is to determine the project specification, along with the price and the identity of the supplier. Such procurement processes (known as two-stage "design-bid-build" tenders) are commonly used by purchasers of construction work.[1] The first stage tender serves to elicit information about the available or technically feasible design specifications from expert contractors (e.g. architects, engineering consultancies, or defence companies). Thereafter, any contractor who has participated in the first stage has the opportunity to tender the second stage, where the exact specification and the price of the project will be determined alongside the identity of the supplier.[2]

Our focus in this paper is on the second stage of the procurement process. In particular, we consider situations where the buyer, following the first stage, has drawn up a list of available project specifications that are deemed close substitutes.[3] As an example, note that in the procurement of certain types of military equipment, the UK Ministry of Defence assesses available equipment specifications against minimum criteria, considering all that meet or exceed these criteria as "operationally equivalent".[4] While the buyer and the contractors are aware of the available project specifications prior to the second stage tender, there typically remain considerable cost uncertainties surrounding each of the design specifications in which contractors can deliver the project.[5] Contractors participating in the second stage tender therefore have to carefully account for the cost uncertainties when

[1]See "Procurement" factsheet published by "Constructing Excellence" available at www.constructingexcellence.co.uk

[2]Similar multi-stage procedures are used in the procurement of an innovation, as studied by Che and Gale (2003), and Schöttner (2008).

[3]We thereby abstract from the question whether participants in the first stage tender have an incentive to withhold or bias their information about feasible project specifications in order to influence the list of those that are considered substitutes by the buyer. The literature comprises some contributions where sellers choose strategically the information they provide to buyers, albeit in settings different from ours: Bergemann and Pesendorfer (2007) consider optimal revelation of relevant product information by a seller to bidders in a standard one-shot auction, while Ivanov (2011) does so in an oligopolistic market setting with competing sellers and a single buyer.

[4]See "Cost Effectiveness in UK Defence Procurement: The CO-EIA" available from NATO Research & Technology Organisation at http://ftp.rta.nato.int/public//PubFulltext/AGARD/CP/AGARD-CP-602///18CHAP15.pdf

[5]E.g. contractor cost uncertainties arise if the project is a custom-designed piece of military equipment, along with full contractor support and maintenance throughout the service life of the equipment. Similar cost uncertainties also arise in "TotalCare" contracts between UK power systems

preparing bids and committing to a design specification. Equally, a well-designed second-stage tender must take into account how participating firms deal with the cost uncertainties in terms of the design specification and the price that they choose to offer.

In this paper, we investigate how the buyer, who cares about the price and the design specification of the project, should structure the second stage procurement process when participating firms are subject to cost uncertainty at the time of tender. We model cost uncertainties by studying a common value environment where firms have only partial information about the cost of providing each of the available design specifications. While the *ex post* cost of producing a given specification are assumed to be identical across firms, they are unknown to the firms at the time of tender. The assumption of common values can be viewed as capturing supplier uncertainty about buyer characteristics, which affect the cost of customizing each design specification to the buyer's particular requirements.[6]

Our common value information structure represents a departure from what has been assumed in the procurement literature to date (Laffont and Tirole, 1987, Che, 1993, Branco, 1997, Asker and Cantillon 2008, 2010, and Rezende, 2009). There, the focus is on "private value" environments where firms have heterogeneous cost structures, but do not face uncertainty about their own cost of providing each of the available design specifications. While it is likely that firms in real-world procurement situations face cost uncertainties as well as being heterogeneous in their cost structure, we abstract here from the latter issue in order to concentrate on the question to which extent a "good" procurement procedure is able to select the "right" (or efficient) product specification.

We pursue this question in a stylized environment with two possible product specifications where each participating firm has multi-dimensional private information.[7] This information consists of a pair of cost-signals, one for each project specification. In line with our focus on the second stage of a two-stage tender, we assume here that the two specifications of the project generate the same benefit to the buyer. Therefore, the question of which specification is efficient reduces to the question of which specification has the lower *ex post* production cost. We ask if it is advisable for the buyer to insist on an efficient specification choice if he wants

company Rolls-Royce and commercial airlines for the provision of, and lifetime maintenance for, aircraft engines.

[6]The assumption that the cost of the suppliers depend on the buyer's characteristics are also made in Lauermann and Wolinsky (2009), albeit in a different setting.

[7]It is important to emphasize that with multi-dimensional private information, efficient mechanisms can be implemented only in private value environments, or in very special interdependent value environments (see Dasgupta and Maskin, 2000, and Jehiel and Moldovanu, 2001). The common value model we study here falls into the latter category.

to minimize his expenditure for the project. In other words, is the efficient specification also the cheapest from the buyer's point of view? Or should the buyer focus solely on price and disregard differences in specification? In order to address these questions, we start our investigation into procurement mechanisms with a study of a specific benchmark procedure. Under this procedure, each participating firm submits a two-dimensional bid that consists of a specification and a price for the project. The two-dimensional bids are then evaluated according to the net benefit that they generate for the buyer, and the winning bid is the one with the highest net benefit. As both specifications generate the same (gross) benefit, the prices alone determine the identity of the supplier and the chosen design specification. We refer to this procurement procedure as *minimum price mechanism* (henceforth MPM).

The main applied contribution of this paper is the equilibrium analysis of the MPM. An interesting result that emerges from this analysis is that the MPM - a very natural procurement procedure given that the available specifications are substitutes - will not always result in the efficient specification choice (i.e. the specification whose *ex post* production cost are lowest).[8] However, a rationale for using the MPM despite its inefficiency emerges when one compares the expected expenditure for the project under the MPM to a procurement procedure that guarantees delivery of the efficient specification. In this efficient procedure, all firms participating in the tender receive lump-sum payments in exchange for their expert information about the cost of the alternative specifications. By selecting at random a supplier of the efficient specification, this procedure aligns the firms' incentives with that of the buyer (because misrepresentation of a firm's information does not pay, as it may result in the firm being chosen to supply the more costly specification). The MPM delivers lower expected expenditure for the project than this efficient procedure in many settings. This suggests there is a trade-off between efficiency and expenditure-minimization. In order to explore this trade-off more formally, we subsequently adopt a mechanism design approach. One key insight that emerges is that, in symmetric settings, the MPM displays a feature that all expenditure-minimizing procurement processes necessarily share. The other key insight is that an optimal procurement process will not be efficient. It is interesting to note that this inefficiency property of expenditure-minimizing mechanisms arises in the absence of any a priori bias that the buyer may have in favor of a particular supplier (as in Rezende, 2009), and in the absence of any "price ceilings" that the buyer may wish to impose. This contrasts with optimal procurement pro-

[8] In this respect, we come to a similar conclusion as Ewerhart and Fieseler (2003). They study procurement auctions involving unit-price contracts (albeit in a private value setting with one-dimensional private information). They find that such auctions perform better in terms of ex ante expenditure than an auction which guarantees ex post efficient outcomes (even if a characterization of the optimal auction in their setting remains elusive).

cedures for a fully specified project when participating firms have one-dimensional private information (e.g. Myerson, 1981, or Che, 1993). There, in the absence of a "price ceiling", the contract is always allocated to the most efficient firm. The tension between efficiency and expenditure-minimization in our model arises from the multi-dimensional nature of the firms' information, and the added scope for manipulation that this offers: As the buyer will purchase just one unit of the good, and in only one specification, firms can boost the chances of being selected to supply their chosen specification by strategically misrepresenting their information about the other specification. In order to deter such behavior, it is important that any firm be selected only to supply the specification associated with its minimum signal. But by making a commitment to use only information about firms' minimum signals, the buyer foregoes the opportunity of securing an efficient specification.

The main technical contribution of this paper is to suggest a way for dealing with the considerable technical difficulties that are inherent in any study of optimal mechanisms in settings with multi-dimensional private information. With the exception of the single-agent monopoly screening models studied by Armstrong (1996) and Rochet and Choné (1998), there are, to date, no general characterizations of optimal mechanisms for such settings - be that with private or with interdependent values.[9] However, there exist in the literature characterizations of incentive compatible mechanisms for environments with multi-dimensional signals, extending the well-known "payoff equivalence" result for settings with single-dimensional private information due to Myerson (1981) (see, e.g., Jehiel et al., 1999, Krishna and Perry, 2000, Jehiel and Moldovanu, 2001, and Krishna and Maenner, 2001). In our setting, these payoff equivalence results establish a formal connection between the allocation rule by which a specification and a supplier are chosen, and the payments received by the firms participating in the mechanism. In particular, a firm's interim expected payment must be a path-independent path integral of a vector-valued function. This function, which is the interim expectation of the allocation rule, contains, for each specification, the probability that a given firm is chosen as supplier of that specification. Path-independence means that each firm's interim expected payment in an incentive compatible mechanism is the same regardless of the path of integration that has been chosen to compute the interim payment. In other words, the value of a firm's expected payment for a given signal-vector must be unaffected by the chosen path of integration. It is this incentive compatibility condition that distinguishes settings with multi-dimensional signals from those with one-dimensional

[9]Exceptions are Asker and Cantillon (2010) and Armstrong and Rochet (1999), who circumvent the technical difficulties by modeling the components of agents' multi-dimensional signals as discrete random variables, rather than continuous random variables as is typically the case in the literature.

signals. The requirement of path-independence as a condition for incentive compatibility is awkward to handle as a constraint in any characterization of optimal mechanisms. The main technical innovation of this paper is our approach for taking into account the path-independence requirement. To the best of our knowledge, this approach is novel in the mechanism design literature. It allows us to derive an expression for the ex ante expected payments received by firms participating in an incentive compatible mechanism. The great advantage of this expression is that it can be readily contrasted with the expected payments in a benchmark common value procurement model with a single product specification (where suppliers' private information is one-dimensional), thereby shedding light on the sources of firms' information rents.

Related Literature. Our model contributes to the literature on procurement when price and quality matter (Laffont and Tirole, 1987, Che, 1993, Branco, 1997, and Asker and Cantillon, 2008, 2010). In this literature, the "design specifications" are different quality levels, measured by a continuous variable. All papers apart from Asker and Cantillon (2008, 2010) study models in which firms have one-dimensional private information about their cost of providing the various quality-levels. Asker and Cantillon (2010) consider a private value model in which firms have two-dimensional private information about their fixed cost, as well as their marginal cost of producing different quality-levels. In order to overcome the technical difficulties associated with the characterization of optimal procurement mechanisms, they assume that the components of firms' private information (fixed and marginal cost) are discrete random variables, each with two possible realizations. This simplification allows them to fully characterize the optimal procurement mechanism for their setting. The optimal mechanism is not efficient, even in the absence of a reserve price (unlike the optimal procurement mechanism characterized by Laffont and Tirole, 1987 for the case of one-dimensional signals). This tension between ex post efficiency and ex ante expenditure minimization is a feature shared with our model.

This paper also makes a contribution to the literature on mechanism design for settings where agents have multi-dimensional private information (see Dasgupta and Maskin, 2000, Krishna and Perry, 2000, and Jehiel and Moldovanu, 2001). Within this literature, the main focus to date has been on the existence and characteristics of efficient mechanisms. For the case of private multi-dimensional values, Krishna and Perry (2000) show that within the class of efficient mechanisms, generalized Vickrey-Clarke-Groves mechanisms are optimal (in the sense of maximizing revenue), even if Bayes Nash implementation (rather than dominant strategy implementation) is used. We make a contribution to this literature by showing a way in which the path-independence property of incentive compatible mechanisms can be explicitly taken into account in the optimization problem that forms the basis for the characterization of optimal mechanisms.

The remainder of this paper is organized as follows: Section 2 contains the basics of our model. Section 3 describes the MPM and provides a characterization of firms' equilibrium strategies. In Section 4 we introduce direct revelation mechanism, as well as the notation and concepts required for our mechanism design analysis of procurement procedures. Section 5 contains our main characterization results regarding the ex ante expenditure of incentive compatible procurement mechanisms, as well as the derivation of necessary conditions that any expenditure-minimizing mechanism must display. In Section 6 we build on these results to argue that expenditure-minimizing mechanisms will not be efficient. Section 7 concludes, and the Appendix in Section 8 contains all proofs.

2 Model

2.1 Model basics

Setting. A buyer wishes to purchase a single unit of an indivisible good that can be produced in two different design specifications. Let $K \equiv \{A, B\}$ denote the set of available design specifications. We assume that the buyer considers the two specifications perfect substitutes, and derives the same benefit $b > 0$ from each of them. The buyer's von Neumann Morgenstern utility is $b - p$ if he pays a price of p for the good, regardless of its specification. There are n firms from whom this good can be sourced. Let $I \equiv \{1, \ldots, n\}$ be the set of firms. The production cost C_k of each specification $k \in K$ are common to the suppliers. Firm i's von Neumann Morgenstern utility is $p - C_k$ if the good is purchased from firm i in specification k at price p. If the good is not sourced from firm i, then firm i incurs no cost. The production cost C_k of each specification $k \in K$ are unknown to the buyer and to the firms at the time of competing for the buyer's custom. However, the firms have an informational advantage over the buyer, in that each firm has some private information about the cost C_k of each specification.

Information Structure. We assume that firm i's private information consists of two cost-signals: s^i_A and s^i_B. Signal s^i_k is firm i's private information about the production cost of specification k. We refer to the signal-vector $s^i \equiv (s^i_A, s^i_B)$ as firm i's *type*, and denote by $S \equiv [0,1]^2$ the set of possible types. We assume that s^i is a random variable which is only observed by firm i. The firms' types are stochastically independent, and they are identically distributed according to a continuous joint density f such that (s.t.) $f(s_A, s_B) > 0$ for all (s_A, s_B) in the interior of S. Regarding the connection between the firms' types and the production cost of each specification, we assume that the cost of specification k is given by the average of the firms' signals about specification k:

$$C_k \equiv \frac{1}{n}\sum_{i\in I} s_k^i \quad \forall k \in K \tag{1}$$

Notation. We denote by $\mathbf{s} \equiv (s^1, \ldots, s^n)$ the vector of all firms' types, with $\mathbf{s} \in S \times \ldots \times S \equiv S^n$. Let $\mathbf{s}^{-i} \equiv (s^1, \ldots, s^{i-1}, s^{i+1}, \ldots, s^n) \in S^{n-1}$ be the vector of all but firm i's types. Likewise, let $\mathbf{s}_k \equiv (s_k^1, \ldots, s_k^n)$ be the vector of all firms' k-signals, and let $\mathbf{s}_k^{-i} \equiv (s_k^1, \ldots, s_k^{i-1}, s_k^{i+1}, \ldots, s_k^n)$ be the vector of all but firm i's k-signals. We write $g(\mathbf{s}) \equiv \prod_{i\in I} f(s^i)$ and $g(\mathbf{s}^{-i}) \equiv \prod_{j\neq i} f(s^j)$. For ease of notation, let $m_i \equiv \arg\min_{k\in K} s_k^i$ be the specification associated with firm i's minimum signal, and $M_i \equiv \arg\max_{k\in K} s_k^i$ the specification associated with firm i's maximum signal.

2.2 Symmetry

Given the independent nature of the firms' information, and the symmetry of the cost functions C_k, we say that our setting is *symmetric with respect to (w.r.t.) firms*.[10] In addition to this form of symmetry, our model admits a second form of symmetry:

Definition 1. *We say the setting is* ***symmetric w.r.t. specifications*** *if the joint density f is symmetric around the 45°-line: $f(s_B, s_A) = f(s_A, s_B)$ for all $(s_A, s_B) \in S$.*[11]

In what follows, we shall focus on two types of settings, depending on whether the model displays symmetry w.r.t. specifications, or not:

Symmetric correlated settings. The setting displays symmetry w.r.t. firms *and* w.r.t. specifications.

Independent asymmetric settings. In such settings, the cost signals s_k are independently but non-identically distributed, with cumulative distribution function H_k on $[0,1]$ for all $k \in K$. We write H_k f.o.s.d. H_l ($l \in K$, $l \neq k$) if H_k first-order stochastically dominates H_l. Each H_k has a continuous derivative h_k, with $h_k(s_k) > 0$ for

[10] For each specification k, the cost function $C_k(\mathbf{s}_k)$ is symmetric in the following sense: $C_k(\mathbf{s}_k) = (1/n)(s_k^i + \mathbf{s}_k^{-i})\ \forall i \in I$.

[11] If f is symmetric around the 45°-line then $\Pr\{s_A < s_B\} = 1/2$.

all $s_k \in (0,1)$, so that $f(s_A, s_B) = h_A(s_A)h_B(s_B) > 0$ for all (s_A, s_B) in the interior of S.[12]

2.3 Efficiency

Given our assumption that the buyer derives the same benefit from each design specification, it follows that for all type-vectors $\mathbf{s} \in S^n$, the ex post efficient design specification is determined by the production cost alone:

Definition 2. *Specification A is **efficient** if $C_A(\mathbf{s}_A) < C_B(\mathbf{s}_B) \Leftrightarrow \sum_{i \in I} s_A^i < \sum_{i \in I} s_B^i$; otherwise, specification B is **efficient**.*

Because of the common value nature of the production cost, efficiency pertains solely to the specification of the object, not to the identity of the supplier. We adopt the particular common value cost structure in (1) so as to address the question whether an efficient procurement process can ever be expenditure-minimizing. For this purpose, we need an environment where efficient and implementable procurement procedures exist. As Jehiel and Moldovanu (2001) show, efficient procedures can be implemented only in special settings, which is why we assume the cost structure in (1).[13]

3 Minimum price mechanism

In this section, we start our investigation of procurement procedures available to the buyer with a simple and intuitive benchmark mechanism. In this mechanism, the buyer selects a supplier on the basis of price alone, disregarding any differences in the firms' design specifications. We refer to this procedure as "minimum price mechanism" (MPM).

[12] If the assumption of independent cost-signals appears restrictive, note that the same qualitative results obtain when s_A and s_B are non-independent random variables with a joint density of the form $f(s_A, s_B) = as_A + (2-a)s_B$, where $a \in (0,2)$ and $a \neq 1$.

[13] The existence of implementable efficient procedures in our setting follows from Theorem 4.3 in Jehiel and Moldovanu (2001). To see that the theorem applies, note that our setting is a special case of the environment studied by Jehiel and Moldovanu. Our setting can be thought of as having $2n$ different "social alternatives", with generic social alternative $(k,i) \in K \times I$. For every alternative (k,i), each firm $j \in I$ has a one-dimensional signal $s^j_{(k,i)} = s^j_k$ that affects (in different ways) the cost functions of the n firms: in alternative (k,i), the cost function $\mathscr{C}^i_{(k,i)}(s^1_{(k,i)}, \ldots, s^n_{(k,i)})$ of firm i takes the value $C_k(\mathbf{s}_k)$ given in (1), and the cost function $\mathscr{C}^j_{(k,i)}(s^1_{(k,i)}, \ldots, s^n_{(k,i)})$ of any firm j $(j \neq i)$ takes the value zero. It is straightforward to verify that the conditions of Theorem 4.3 in Jehiel and Moldovanu (2001) hold in this setting.

3.1 Mechanism basics

Rules. The buyer asks each firm to submit a two-dimensional bid consisting of a specification and a price. Let $k_i \in K$ be the specification chosen by firm i, and denote by $p^i_{k_i}$ the price that firm i demands in return for delivery of the good in specification k_i. Under the MPM, the buyer commits to sourcing the good at the lowest price, irrespective of the specifications proposed by the firms. I.e. given prices $(p^1_{k_1}, \dots, p^n_{k_n})$, the buyer sources the good from firm $j \in \arg\min_{i \in I} p^i_{k_i}$. Firm j is paid an amount equal to its price $p^j_{k_j}$ in return for producing the good in specification k_j. All firms other than the chosen supplier j receive no payment.

Strategies. In the Bayesian game induced by the MPM, a strategy for any firm $i \in I$ consists of three functions: A specification choice rule $\delta_i : S \to K, s^i \mapsto \delta_i(s^i)$, and a pricing function $p^i_k : S \to \mathbb{R}_+, s^i \mapsto p^i_k(s^i)$ for each specification $k \in K$. The interpretation is as follows: each firm i commits to producing the good in specification $\delta_i(s^i) = k_i$ in return for a payment of $p^i_{k_i}(s^i)$. To facilitate the characterization of equilibrium strategies (δ_i, p^i_A, p^i_B), we restrict attention those that satisfy the following properties P1-P3:

P1 The specification choice δ_i rule takes the following form: Given an increasing and continuous function $X_i : \mathbb{R} \to \mathbb{R}$, $s^i_A \mapsto X_i(s^i_A)$, with inverse X_i^{-1} s.t. either $X_i^{-1}(0) \geq 0$, or $X_i^{-1}(1) \leq 1$, or neither:

$$\delta_i(s^i) = \begin{cases} A & \text{if } s^i_B > X_i(s^i_A) \\ B & \text{if } s^i_B < X_i(s^i_A) \end{cases}$$

P2 The price of any specification depends only on the signal pertaining to that specification. I.e. for all $i \in I$, all $s^i \in S$, and all $k \in K$: $p^i_k(s^i) = p^i_k(s^i_k)$.

P3 If a firm is indifferent between specification A and specification B, it quotes the same price regardless of the specification it chooses. I.e. for all $i \in I$, and all $s^i \in S$ s.t. $s^i_B = X_i(s^i_A)$: $p^i_A(s^i_A) = p^i_B(X_i(s^i_A))$.

Fig. 1 illustrates two specification choice rules of the form in P1. While there are four types of functions X_i that are compatible with P1, the two types shown in Fig. 1 are representative. This is because the remaining two types of X_i can be generated from those in Fig.1 by interchanging the specification labels A and B, and then defining a new function $\hat{X}_i$ s.t. $\hat{X}_i(s^i_A) \equiv X_i^{-1}(s^i_A)$ for all $s^i_A \in [0, \min\{1, X_i(1)\}]$. For a specification choice rule δ_i with $X_i^{-1}(0) \geq 0$ (as illustrated in both panels of Fig. 1), properties P2 and P3 together imply that a single pricing function p^i_A suffices to generate the prices of both specifications A and B:

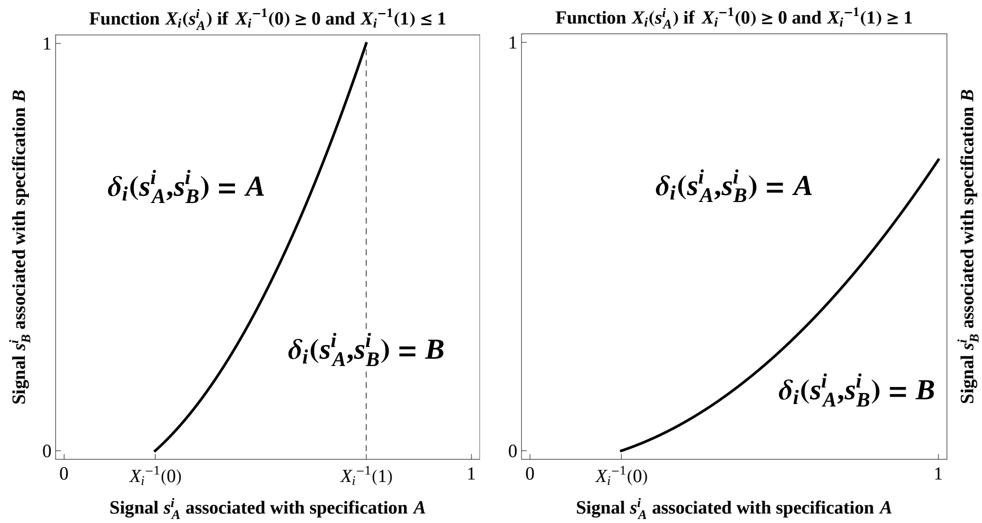

Figure 1: Two specification choice rules δ_i that satisfy property P1

Lemma 1. *If a strategy (δ_i, p_A^i, p_B^i) in the MPM satisfies properties P1-P3, then the pricing function p_B^i for specification B is the composition of $p_A^i : [0, \min\{X_i^{-1}(1), 1\}] \to \mathbb{R}_+$ and X_i. I.e. $p_B^i(x) = p_A^i(X_i^{-1}(x))$ for all $x \in [0, \min\{1, X_i(1)\}]$.*

In the following, we look for symmetric equilibria of the MPM where the strategy (δ, p_A, p_B) used by all firms satisfies properties P1-P3 above.[14] Building on Lemma 1 we can establish:

Lemma 2. *Any symmetric equilibrium (δ, p_A, p_B) of the MPM that satisfies properties P1-P3 features a pricing function p_A that is increasing and differentiable everywhere, with the exception of $x = X^{-1}(0)$.*

3.2 Equilibrium specification choice rule

Properties P1-P3 in Section 3.1 play an important role in the equilibrium characterization. They allow us to pin down, independently of the pricing functions p_A and p_B, the function X that determines the equilibrium specification choice rule δ. To see this, consider a firm with type s^i who is indifferent between offering specification A or B: $s_B^i = X(s_A^i)$. P3 requires firm i to quote the same price regardless of the specification it chooses: $p_A(s_A^i) = p_B(X(s_A^i)) \equiv p$. For this to be compatible

[14]There may of course be other equilibria.

with equilibrium behavior, the expected production cost (conditional on winning the contract with a price of p) must be identical for the two specifications. This observation gives rise to the following characterization of the equilibrium specification choice rule:

Lemma 3. *Any symmetric equilibrium (δ, p_A, p_B) of the MPM that satisfies properties P1-P3 features a specification choice rule δ where, for every $x \in [0,1]$, the function $X(x)$ is defined implicitly by:*

$$\Pr\{s_A > x, s_B > X(x)\}^{n-1} (x - X(x) + (n-1)\mathbb{E}[s_A - s_B \,|\, s_A > x, s_B > X(x)]) = 0$$

The next result shows which form the equilibrium function X takes if the model is symmetric w.r.t. firms and w.r.t. specifications:

Lemma 4. *In correlated symmetric settings, if the equilibrium strategy (δ, p_A, p_B) satisfies properties P1-P3, the unique specification choice rule δ is characterized by the function $X(x) = x$ for all $x \in [0,1]$.*

I.e. in fully symmetric settings, each firm chooses the specification associated with its minimum signal. This is no longer the case in settings that are not symmetric w.r.t. specifications.

Lemma 5. *In independent asymmetric settings where H_B f.o.s.d. H_A (resp. H_A f.o.s.d. H_B), if the equilibrium strategy (δ, p_A, p_B) satisfies properties P1-P3, the unique specification choice rule δ is characterized by an increasing and differentiable function X s.t. $X(x) \leq x$ (resp. $X(x) \geq x$) for all $x \in [0,1)$, and $X(1) = 1$.*

The left-hand (resp. right-hand) panel of Fig. 2 illustrates the specification choice rule δ by depicting the function X that arises if H_B f.o.s.d. H_A (resp. H_A f.o.s.d. H_B).[15] Types s^i above the solid black curve in both panels of Fig. 2 (which represents the graph of X) choose specification A, while types s^i below the solid black curve choose specification B.

3.3 Equilibrium pricing functions

In correlated symmetric settings, where the specification choice rule is characterized by the function $X(x) = x$, the pricing functions for the two specifications are

[15] Given a joint density $f(s_A, s_B) = as_A + (2-a)s_B$, a function X of the form in the left-hand panel of Fig. 2 arises if $a < 1$, while a function X of the form in the right-hand panel arises if $a > 1$. Our characterization of the equilibrium pricing functions p_A and p_B in Section 3.3 therefore also applies to this class of non-independent asymmetric settings.

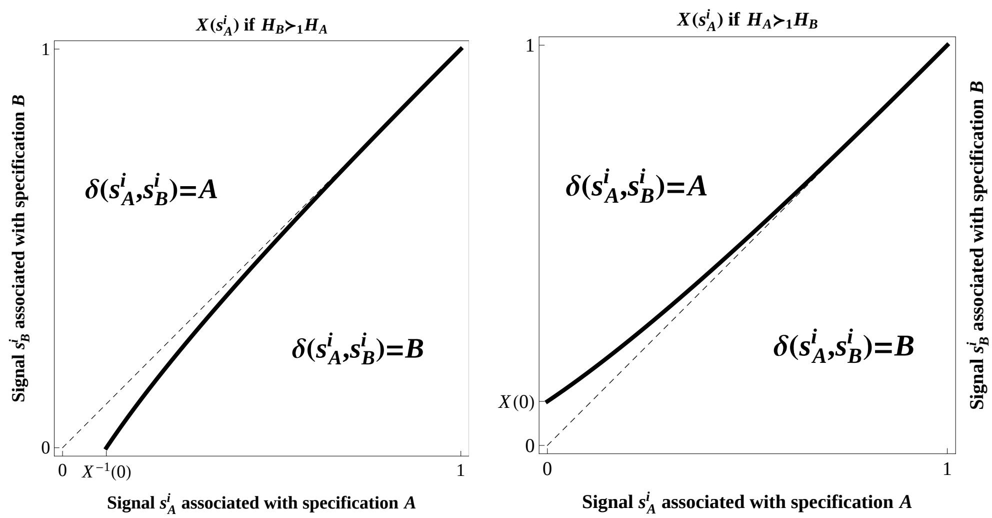

Figure 2: Representative equilibrium specification choice rules in independent asymmetric settings

identical: $p_A(x) = p_B(x) \equiv p(x)$ for all $x \in [0,1]$. This means that in a symmetric equilibrium (δ, p) each firm i offers the specification associated with its minimum signal at the price $p(s^i_{m_i})$. The following result completes our equilibrium characterization for correlated symmetric settings:

Proposition 1. *In correlated symmetric settings, the unique symmetric equilibrium (δ, p) that satisfies properties P1-P3 features the following pricing function:*

$$p(x) = \frac{n-1}{n\omega(x)^{n-1}} \int_x^1 \omega(r)^{n-2} \left(r + \mathbb{E}[s_k \,|\, s_k > r, s_l = r] + (n-2)\mathbb{E}[s_k \,|\, s_k, s_l > r]\right) dr$$

$$+ \frac{n-1}{n\omega(x)^{n-1}} (n-1) \int_x^1 \omega(r)^{n-2} \left(2r + (n-2)\mathbb{E}[s_k \,|\, s_k, s_l > r]\right) dr \qquad (2)$$

where $\omega(x) \equiv \Pr\{s_A, s_B > x\}$.

Appearances notwithstanding, the pricing function p in Proposition 1 has a neat interpretation. To see this, suppose that firm i's minimum signal takes the value r, and pertains to specification k. Suppose also that firm i is the chosen supplier. In the symmetric equilibrium, where all firms use the same increasing pricing function, this implies that firm i's minimum signal r is lower than the minimum signals of all other firms. Now consider the second term of (2), where the expression $2r + (n-2)\mathbb{E}[s_k|s_k, s_l > r]$ captures firm i's expected production cost of specification k if the minimum signal of one other firm is r and pertains to specification k, while the minimum signals of the remaining $n-2$ firms are all greater than r. Next, consider the first term of (2), where the expression $r + \mathbb{E}[s_k|s_k > r, s_l = r] + (n-2)\mathbb{E}[s_k|s_k, s_l > r]$ captures firm i's expected production cost of specification k if the minimum signal of one other firm is r and pertains to specification l, while the minimum signals of the remaining $n-2$ firms are all greater than r. This reveals that the equilibrium price of any firm i equals the expected production cost of its chosen specification when its own minimum signal is equal to the lowest minimum signal amongst its $n-1$ competitors. In computing these expected production cost, firm i must account for the fact that the lowest minimum signal among its competitors can pertain either to the same specification as its own minimum signal (see second term of (2)), or to the other specification (see first term of (2)).

We now turn to independent asymmetric settings, in which it is no longer the case that a single function p suffices to generate the price for both specifications.[16] The reason is that in any setting where the specification choice rule is as shown in the left-hand panel of Fig. 2, the pricing function for specification A must be defined piecewise: one function $\breve{p}$ for types s^i s.t. $s^i_A < X^{-1}(0)$, and another function $\bar{p}$ for types s^i s.t. $s^i_A \geq X^{-1}(0)$. The pricing functions p_A and p_B are then given as follows:

$$p_A(x) = \begin{cases} \breve{p}(x) & \text{if } 0 \leq x < X^{-1}(0) \\ \bar{p}(x) & \text{if } X^{-1}(0) \leq x \leq 1 \end{cases} \tag{3}$$

and

$$p_B(x) = \bar{p}(X^{-1}(x)), \text{ where } X^{-1}(0) < x \leq 1 \tag{4}$$

The next proposition completes the equilibrium characterization for independent asymmetric settings by stating the functions $\bar{p}$ and $\breve{p}$ if H_B f.o.s.d. H_A.[17]

[16]Recall that "asymmetry" here refers to the shape of the joint density f from which each firm's type is drawn, rather than different type-distributions across the n firms. We therefore do not encounter here the challenges associated with the analytical characterization of equilibrium strategies in the first-price auction when bidders' types are drawn from different distributions, as e.g. studied by Kaplan and Zamir (2010) for private-value settings, or Cheng and Tan (2010) for common-value settings.

[17]Observe that once we know the equilibrium pricing functions for this case, we automatically know the pricing functions for settings where H_A f.o.s.d. H_B. The reason is that for any joint density $f(s_A, s_B)$ s.t. X is as shown in the right-hand panel of Fig. 2, we can define a new density $\hat{f}(s_A, s_B) = f(s_B, s_A)$, for which the associated function $\hat{X}$ is as shown in the left-hand panel of Fig. 2, with $X(s_A) = \hat{X}^{-1}(s_A)$ for all $s_A \in [0,1]$.

Proposition 2. *In independent asymmetric settings where H_B f.o.s.d. H_A, the unique symmetric equilibrium (δ, p_A, p_B) that satisfies properties P1-P3 features the following pricing functions: for all $x \in [X^{-1}(0), 1]$,*

$$\bar{p}(x) = \frac{n-1}{n\omega(x)^{n-1}} \int_x^1 \omega(r)^{n-2} \left(r + (n-1)\mathbb{E}_A[s_A \,|s_A > r]\right) X'(r) \left(1 - H_A(r)\right) h_B(X(r)) dr$$
$$+ \frac{n-1}{n\omega(x)^{n-1}} \int_x^1 \omega(r)^{n-2} \left(2r + (n-2)\mathbb{E}_A[s_A \,|s_A > r]\right) \left(1 - H_B(X(r))\right) h_A(r) dr \tag{5}$$

where $\omega(x) \equiv \Pr\{s_A > x, s_B > X(x)\}$; and for all $x \in [0, X^{-1}(0)]$,

$$\breve{p}(x) = \frac{n-1}{n\left(1 - H_A(x)\right)^{n-1}} \int_x^{X^{-1}(0)} \left(1 - H_A(r)\right)^{n-2} \left(2r + (n-2)\mathbb{E}_A[s_A \,|s_A > r]\right) h_A(r) dr$$
$$+ \frac{\left(1 - H_A(X^{-1}(0))\right)^{n-1} \bar{p}(X^{-1}(0))}{\left(1 - H_A(x)\right)^{n-1}} \tag{6}$$

In the Appendix we provide a unified proof of Propositions 1 and 2 by assuming that X is an increasing and differentiable function with $X^{-1}(0) \geq 0$ and $X(1) = 1$. This assumption covers specification choice rules of the form in the left-hand panel of Fig. 2, as well as the specification choice rule for correlated symmetric settings in Lemma 3. Given the functions $\breve{p}$ and $\bar{p}$, the pricing function p for correlated symmetric settings (where $X^{-1}(0) = 0$) is given by $p(x) = \bar{p}(x)$ for all $x \in [0, 1]$.[18]

4 Direct revelation mechanisms

In this section, we adopt a mechanism design approach to the study of procurement procedures at the buyer's disposal. By the revelation principle, we can restrict our study of procurement procedures to the class of direct revelation mechanisms.

Definition 3. *A* ***social choice rule (SCR)*** *is a function $Q : S^n \to \Delta(K \times I)$, assigning to each $\mathbf{s} \in S^n$ probabilities $\{Q_k^i(\mathbf{s})\}_{(k,i) \in K \times I}$, where $\forall \mathbf{s} \in S^n$: $0 \leq Q_k^i(\mathbf{s}) \leq 1$ $\forall (k,i) \in K \times I$ and $\sum_{(k,i) \in K \times I} Q_k^i(\mathbf{s}) = 1$.*

[18] To account for the fact that the fully symmetric version of our model allows for correlated cost-signals, while the asymmetric case assumes signal independence, the proof derives the functions $\bar{p}$ and $\breve{p}$ in terms of general conditional distributions $F_{k|l}$ and marginal densities f_k (where $k, l \in K$, $l \neq k$).

Definition 4. *A* ***direct revelation mechanism (DRM)*** *is a pair* (Q,T)*, where* $T: S^n \to \mathbb{R}^{|I|}$ *is a payment scheme, with* $\mathbf{s} \mapsto T(\mathbf{s}) = (T_1(\mathbf{s}),\ldots,T_n(\mathbf{s}))$.

For a given DRM (Q,T), define for every $i \in I$ and every report $r^i \in S$ a conditional expected payment function $t_i(r^i) \equiv \mathbb{E}\left[T_i(r^i,\mathbf{s}^{-i})\right]$ and a conditional expected probability assignment function $q^i(r^i) \equiv (q^i_A(r^i), q^i_B(r^i))$, with $q^i_k(r^i) \equiv \mathbb{E}\left[Q^i_k(r^i,\mathbf{s}^{-i})\right]$ for all $k \in K$. If all other firms report truthfully their types $\mathbf{s}^{-i}$ and firm i reports a type r^i instead of its true type s^i, we write i's expected profit as $u_i(r^i,s^i) \equiv t_i(r^i) - \mathbb{E}\left[\sum_{k\in K} Q^i_k(r^i,\mathbf{s}^{-i})C_k(\mathbf{s}_k)\right]$. Denote by $\mu_i(s^i) \equiv u_i(s^i,s^i)$ firm i's expected profit from truthful revelation of its type, and let $c_i(s^i) \equiv \mathbb{E}\left[\sum_{k\in K} Q^i_k(\mathbf{s})C_k(\mathbf{s}_k)\right]$ be firm i's expected production cost when it reports truthfully its type. We can then express i's expected payment in a DRM as $t_i(s^i) = \mu_i(s^i) + c_i(s^i)$. As the firms' types are privately observed, in practice we can only implement incentive compatible DRMs.

Definition 5. (Q,T) *is* ***incentive compatible*** *if* $\forall i \in I$, $\forall s^i, r^i \in S$: $\mu_i(s^i) \geq u_i(r^i,s^i)$.

We assume that participation in any DRM is voluntary for the firms, and that each firm can guarantee itself a profit of zero by opting out of any proposed DRM.

Definition 6. (Q,T) *is* ***individually rational*** *if* $\forall i \in I$, $\forall s^i \in S$: $\mu_i(s^i) = u_i(s^i,s^i) \geq 0$.

We evaluate DRMs according to the level of expenditure that the buyer expects to commit ex ante to the purchase of the good.

Definition 7. ***Ex ante expenditure*** *associated with* (Q,T) *is* $\mathbb{E}\left[\sum_{i\in I} T_i(\mathbf{s})\right]$.

Definition 8. (Q,T) *is* ***optimal*** *if it minimizes ex ante expenditure among all incentive compatible and individually rational DRMs.*[19]

5 Expenditure minimization

Implementable DRMs. We now translate into our setting the standard characterization of incentive compatible DRMs when agents have multidimensional types. Because the proof of this result is familiar from the literature, we omit it.[20]

[19] We have in mind a setting in which the buyer must purchase the good, and therefore does not have the option of setting a price-ceiling for the good.

[20] For a proof see, e.g., Theorem 3.1 in Jehiel and Moldovanu (2001). Alternative sources include Jehiel et al. (1999), Krishna and Perry (2000) and Rochet (1987).

Proposition 3. *For given SCR Q and interim expected payments $\tau_i \in \mathbb{R}$ to the boundary types $s^i = (1,1) \equiv \mathbf{1}$, there exist payments T_i for all $i \in I$ s.t. (Q,T) is incentive compatible, individually rational, and $t_i(\mathbf{1}) = \tau_i$, if and only if:*

(i) q^i *is monotone and conservative for all* $i \in I$,
(ii) $\tau_i \geq c_i(\mathbf{1})$ *for all* $i \in I$.

Moreover, firm i's interim expected payment t_i associated with T_i is given by:

$$t_i(s^i) = t_i(\mathbf{1}) - c_i(\mathbf{1}) + c_i(s^i) + \int_{\Gamma(s^i,\mathbf{1})} q^i \cdot d\alpha \tag{7}$$

for all $i \in I$ and all $s^i \in S$, and for any continuous, piecewise smooth path Γ in S joining s^i and the boundary type $\mathbf{1}$.

Ex ante expenditure. In the spirit of Myerson (1981)'s approach to optimal auction design, we now derive a manageable expression for ex ante expenditure by incorporating binding feasibility constraints into the buyer's objective function. In classic mechanism design settings with one-dimensional private information, the only binding constraint is that interim payments t_i be determined by the SCR, up to an additive constant. For settings with multi-dimensional private information such as ours, Jehiel and Moldovanu (2001) show that an additional constraint is binding, namely that the conditional expected probability assignment functions q^i be conservative (see item (i) of Proposition 3). In order to mimic Myerson (1981)'s approach, we therefore have to find a way of explicitly incorporating this awkward constraint into the buyer's objective function. To do this, we exploit an equivalent property: The statement that q^i is conservative is equivalent to saying that, for any type s^i, the value of the path integral of q^i in (7) is the same for *any* continuous, piecewise smooth path joining types s^i and $\mathbf{1}$. This property is called *path independence*. It implies, in particular, that we obtain the *same* value for firm i's interim expected payment $t_i(s^i)$ regardless of whether we calculate the path integral in (7) along path Γ^1 or along path Γ^2, both shown in Fig. 3. Using this implication of path independence, we obtain as a corollary to Proposition 3 the following result:

Corollary 1. *For every $s^i \in S$ the interim expected payment t_i associated with an incentive compatible DRM (Q,T) can be expressed equivalently by evaluating the*

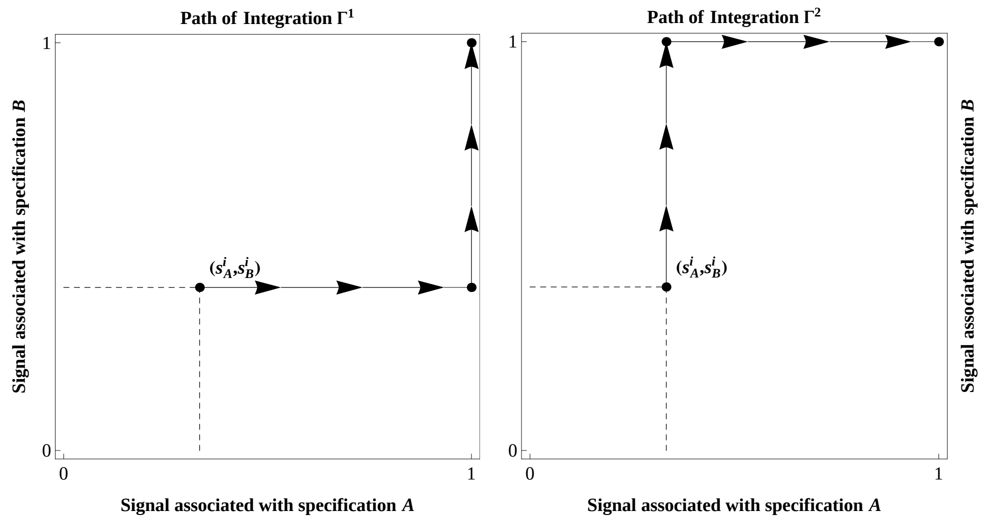

Figure 3: Two paths of integration

path integral of q^i between s^i and $\mathbf{1}$ along path Γ^1 (cf. (8)) or along path Γ^2 (cf. (9)):

$$t_i(s^i) = t_i(\mathbf{1}) - c_i(\mathbf{1}) + c_i(s^i) + \int_{s^i_A}^{1} q^i_A(x, s^i_B)dx + \int_{s^i_B}^{1} q^i_B(1,x)dx \tag{8}$$

$$= t_i(\mathbf{1}) - c_i(\mathbf{1}) + c_i(s^i) + \int_{s^i_B}^{1} q^i_B(s^i_A, x)dx + \int_{s^i_A}^{1} q^i_A(x,1)dx. \tag{9}$$

We can now state a manageable expression for ex ante expenditure when the buyer uses a DRM that generates interim payments t_i that satisfy the equality in Corollary 1. To obtain this expression, we have computed the ex ante expected payment $\mathbb{E}[t_i(s^i)]$ to any firm i by making use of *both* equations (8) and (9) in Corollary 1.[21] In particular, for types s^i s.t. $s^i_B < s^i_A$, we have used the expression for $t_i(s^i)$ in (8), and for types s^i s.t. $s^i_A < s^i_B$, we have used the expression for $t_i(s^i)$ in (9). By adding up the individual firms' ex ante payments, we can then write ex ante expenditure solely in terms of the probabilities Q^i_k that constitute the SCR Q.

[21] Note that our approach here differs form the one taken by Armstrong (1996) in a monopoly screening setting. When translated to our setting, his approach means computing ex ante expenditure under an incentive compatible DRM along a *single* path (the straight line from type s^i to the boundary type $\mathbf{1}$). However, in our setting it is not clear how to then derive conditions on the signal-density f s.t. a DRM which minimizes pointwise ex ante expenditure satisfies the binding requirement that the conditional expected probability assignment functions q^i are conservative.

Proposition 4. *For any incentive compatible DRM (Q,T) the buyer's ex ante expenditure $\sum_{i\in I}\mathbb{E}[t_i(s^i)]$ is:*[22]

$$\int_{S^n}\sum_{i\in I}\sum_{\substack{k,l\in K\\ l\neq k}}\left(Q_k^i(\mathbf{s})\left[C_k(\mathbf{s}_k)+\frac{F(s_k^i\,|s_l^i)}{f(s_k^i\,|s_l^i)}\right]+\left[Q_k^i((s_k^i,1),\mathbf{s}^{-i})-Q_k^i(\mathbf{s})\right]\frac{F(s_{m_i}^i\,|s_l^i)}{f(s_k^i\,|s_l^i)}\right)g(\mathbf{s})d\mathbf{s}$$
$$+\sum_{i\in I}(t_i(\mathbf{1})-c_i(\mathbf{1}))$$

With exception of the constants $t_i(\mathbf{1})-c_i(\mathbf{1})$, all terms in the expression for ex ante expenditure in Proposition 4 are fully determined by the SCR Q. The key advantage of expressing ex ante expenditure in this way is that we can gain a better understanding for the sources of firms' information rents in our setting. For this purpose, it is instructive to compare ex ante expenditure in Proposition 4 with ex ante expenditure in a benchmark model with one-dimensional private information: Consider a procurement setting with n firms and a *single* product specification. The firms' production cost are common and given by $C(\sigma_1,\dots,\sigma_n)\equiv(1/n)\sum_{i\in I}\sigma_i$, where the cost-signals $\sigma_i\in[0,1]$ are i.i.d. random variables with distribution H, observed privately by each firm i ($i\in I$). In this setting, a SCR assigns to each signal-vector $\sigma\equiv(\sigma_1,\dots,\sigma_n)$ probabilities $Q_i(\sigma)$ (one for each firm i), where $Q_i(\sigma)$ is the probability that the good is purchased from firm i when the firms' signals are σ. Using standard steps from the mechanism design literature with one-dimensional private information, we obtain the following expression for the buyer's ex ante expenditure in this benchmark setting:

$$\int_{[0,1]^n}\sum_{i\in I}\left(Q_i(\sigma)\left[C(\sigma)+\frac{H(\sigma_i)}{h(\sigma_i)}\right]\right)h(\sigma_1)\cdot\ldots\cdot h(\sigma_n)d\sigma+\sum_{i\in I}(t_i(1)-c_i(1))\qquad(10)$$

Now compare this benchmark expression for ex ante expenditure with the one in Proposition 4 (with i.i.d. cost signals s_k^i for comparability). There, if a firm is chosen to produce specification k, it receives information rents of $H(s_k^i)/h(s_k^i)$ which are needed to ensure truthful revelation of the signal pertaining to specification k. This source of information rents is present also in the benchmark setting (see

[22] We write $Q_k^i((s_k^i,1),\mathbf{s}^{-i})$ for the probability that supplier i is chosen to supply specification k when his k-signal is s_k^i, and the cost-signal for the *other* specification is 1. I.e. for $k=A$ we have $Q_A^i((s_A^i,1),\mathbf{s}^{-i})$ and for $k=B$ we have $Q_B^i((1,s_B^i),\mathbf{s}^{-i})$. Also, when there is no risk of confusion, we ease notation by writing $F(s_k^i\,|s_l^i)$ for the conditional distribution $F_{k|l}(s_k^i\,|s_l^i)$, $f(s_k^i\,|s_l^i)$ for the conditional density $f_{k|l}(s_k^i\,|s_l^i)$, and $f(s_k^i)$ for the marginal density $f_k(s_k^i)$.

equation (10)). However, ex ante expenditure in Proposition 4 features an additional term that influences a firm's level of information rents:

$$\left[Q_k^i((s_k^i,1),\mathbf{s}^{-i}) - Q_k^i(\mathbf{s})\right] H(s_{m_i}^i)/h(s_k^i)$$

This term awards additional rents to firm i if the probability of being chosen to produce specification k increases in firm i's l-signal. Intuitively, this term recognizes that any firm can always make any one of its two signals look relatively more attractive by exaggerating the realization of the *other* signal. Thereby, the firm can reveal truthfully one of its signals, while manipulating the relative cost of the two specifications. In order to deter manipulations of this kind, extra information rents may be necessary.

Properties of optimal DRMs. We now investigate properties of optimal DRMs in our setting by looking for SCRs that minimize ex ante expenditure in Proposition 4. For this purpose, we propose the following optimization problem of the buyer:

Choose a SCR Q and interim payments $t_i(\mathbf{1})$ to the boundary type $\mathbf{1}$ of each firm i so as to minimize ex ante expenditure:

$$\begin{aligned}
&\int_{S^n} \sum_{i\in I} \sum_{\substack{k,l\in K\\ l\neq k}} Q_k^i((s_k^i,1),\mathbf{s}^{-i}) \frac{F(s_{m_i}^i \,|s_l^i)}{f(s_k^i\,|s_l^i)} g(\mathbf{s})d\mathbf{s} \\
&+\int_{S^n} \sum_{i\in I} Q_{M_i}^i(\mathbf{s}) \left(C_{M_i}(\mathbf{s}_{M_i}) + \frac{F(s_{M_i}^i\,|s_{m_i}^i) - F(s_{m_i}^i\,|s_{m_i}^i)}{f(s_{M_i}^i\,|s_{m_i}^i)} \right) g(\mathbf{s})d\mathbf{s} \\
&+\int_{S^n} \sum_{i\in I} Q_{m_i}^i(\mathbf{s}) C_{m_i}(\mathbf{s}_{m_i}) g(\mathbf{s})d\mathbf{s} + \sum_{i\in I}(t_i(\mathbf{1}) - c_i(\mathbf{1}))
\end{aligned} \tag{11}$$

subject to the constraints that for every firm $i \in I$:

(i) the function q^i be monotone,

(ii) $t_i(\mathbf{1}) \geq c_i(\mathbf{1})$.

In this optimization problem, the objective function is given by ex ante expenditure in Proposition 4, however expressed equivalently in terms of the firms' minimum and maximum signals $s_{m_i}^i$ and $s_{M_i}^i$, resp. Recall that this objective function explicitly incorporates the constraint that for every firm i, the expressions for interim payments t_i in equations (8) and (9) must be equal for all $s^i \in S$. While this is only *one* manifestation of the path independence requirement of incentive compatible DRMs, we show below that it is *sufficient* to ensure that the functions q^i are conservative, as required by item (i) of Proposition 3. To see this, note first that any DRM that solves the buyer's optimization problem above must involve $t_i(\mathbf{1}) = c_i(\mathbf{1})$. This implies that the individual rationality constraint is binding only for the boundary type $\mathbf{1}$. We can therefore characterize optimal DRMs solely in terms of the SCR Q. While we are unable to fully characterize SCRs that minimize the objective function above, we can derive the following *necessary* feature that any solution to the above optimization problem must display: A firm is only ever selected to produce the specification associated with its minimum signal. This result is established by looking for SCRs that contribute to pointwise minimization of the components of ex ante expenditure in lines two and three of (11) when the monotonicity constraints are disregarded. Note that pointwise minimization of the full objective function in (11) appears intractable because, in addition to determining probabilities Q^i_k at any given type-vector $\mathbf{s}$ in the interior of S^n, one has to simultaneously determine the probabilities Q^i_k for types $((s^i_k, 1), \mathbf{s}^{-i})$ on the boundary of S^n.

Proposition 5. *If Q minimizes ex ante expenditure, then $Q^i_{M_i}(\mathbf{s}) = 0$ for all $\mathbf{s}$ in the interior of S^n and all $i \in I$.*

It is important to note that monotonicity of the functions q^i (as required by item (i) of Proposition 3) is *not* compromised by the use of SCRs that satisfy the property in Proposition 5. In other words, any SCR that solves the buyer's optimization problem inclusive of monotonicity constraint (i) *must* have the property in Proposition 5.[23] We can show furthermore:

Proposition 6. *If a SCR Q satisfies the necessary condition for optimality in Proposition 5 and features conditional expected probability assignment functions q^i s.t. (8) and (9) in Corollary 1 are equal, there exists $\rho_i : [0,1] \to [0,1]$ s.t. $\forall i \in I$, $\forall s^i \in S$:*

$$q^i_A(s^i) = \begin{cases} \rho_i(s^i_A) & \text{if } s^i_A < s^i_B \\ 0 & \text{if } s^i_A > s^i_B \end{cases} \quad \text{and} \quad q^i_B(s^i) = \begin{cases} 0 & \text{if } s^i_A < s^i_B \\ \rho_i(s^i_B) & \text{if } s^i_A > s^i_B. \end{cases} \tag{12}$$

Recall from item (i) of Proposition 3 that incentive compatible DRMs feature monotone conditional expected probability assignment functions. By Proposition 6, any optimal incentive compatible DRM therefore features monotonically

[23] Intuitively, monotonicity does not conflict with the property in Proposition 5, because for any type-vector $\mathbf{s} \in S^n$ s.t. specification $k \in K$ is chosen, there is at least one firm s.t. $m_i = k$. I.e. when minimizing information rents required for the purchase of specification k, there exists always a firm i which, if chosen as supplier of specification k, will *not* need to be paid the rent component $(F(s^i_{M_i}|s^i_{m_i}) - F(s^i_{m_i}|s^i_{m_i}))/f(s^i_{M_i}|s^i_{m_i})$ in (11). This component is positive for all s^i regardless of the distribution F, and it is therefore immaterial how it behaves as a function of s^i.

decreasing functions ρ_i. I.e. we can restrict our search for expenditure-minimizing DRMs to SCRs that display these characteristics:

Proposition 7. *Any SCR Q with conditional expected probability assignment functions q^i in Proposition 6 and decreasing functions ρ_i can be part of an incentive compatible DRM.*

Proposition 7 confirms that our approach of taking into account a *single* implication of the path-independence property of incentive compatible DRMs (in the form of Corollary 1) sufficiently restricts the class of admissible DRMs so that the associated conditional expected probability assignment functions q^i are conservative, as required by item (i) of Proposition 3. The MPM of Section 3 is an example of a SCR that satisfies the necessary condition for optimality in Proposition 5. The conditional expected probability assignment functions q^i associated with the MPM are given by (12), with $\rho_i(x) = \int_x^1 (1 - F(x|r)) f(r) dr$ for all $x \in [0,1]$. Fig. 4 shows the SCR associated with the MPM when there are $n = 2$ firms. The left-hand panel displays the chosen specification-firm-pair for a given type s^1 s.t. $s_A^1 < s_B^1$ and all possible types s^2 of firm 2. The right-hand panel shows the chosen specification-firm-pair for given s^1 s.t. $s_A^1 > s_B^1$ and all $s^2 \in S$. The shaded grey areas in both panels of Fig. 4 highlight all types s^2 s.t. the MPM, for given s^1, fails to select the efficient specification. This raises the question if there exist implementable DRMs that select the efficient specification for *all* type-vectors $\mathbf{s} \in S^n$, and if so, whether such a DRM may, in fact, be optimal. We address this question in the next section.

6 Optimality vs efficiency

While a full characterization of optimal (i.e. expenditure-minimal) DRMs has proved difficult to obtain, we can show that no optimal DRM will deliver an efficient specification choice for every type-vectors $\mathbf{s} \in S^n$. This means that the buyer will have to decide at the outset whether to forego efficiency in order to reduce his expected payment for the good, or to accept the inevitability of higher expenditure in return for a guaranteed efficient specification choice. Before exploring more formally this trade-off between efficiency and optimality, we start by defining efficient SCRs:

Definition 9. *A SCR Q is* ***efficient*** *if* $\forall \mathbf{s} \in S^n$*:* $\sum_{i \in I} s_A^i < \sum_{i \in I} s_B^i \Rightarrow \sum_{i \in I} Q_A^i(\mathbf{s}) = 1$, *and* $\sum_{i \in I} s_A^i > \sum_{i \in I} s_B^i \Rightarrow \sum_{i \in I} Q_B^i(\mathbf{s}) = 1$.

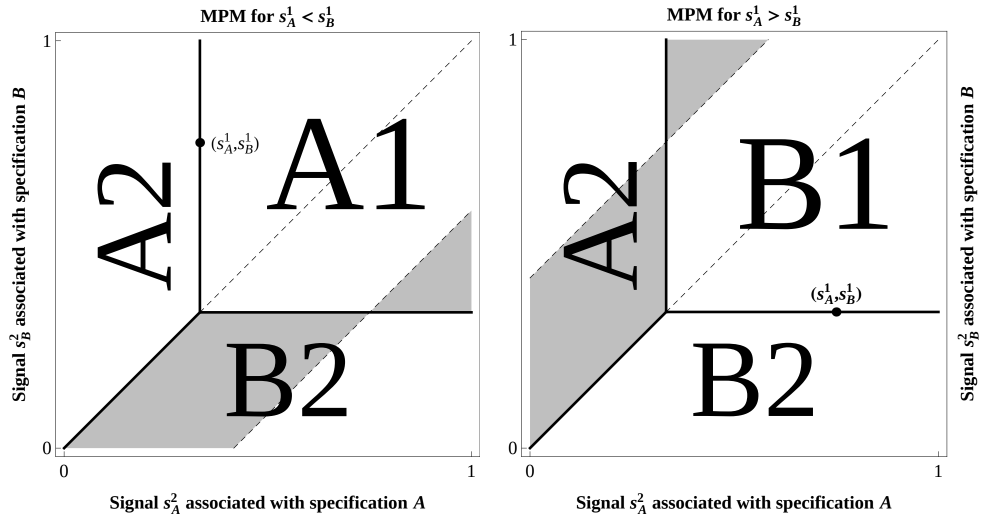

Figure 4: SCR associated with MPM in correlated symmetric setting with $n = 2$ firms

In order to show that efficient and implementable DRMs exist in our setup, consider the following SCR $\hat{Q}$ that selects by means of a random device a supplier to produce the efficient specification. I.e. for all $i \in I$ and all $k, l \in K$, $l \neq k$:

$$\hat{Q}_k^i(\mathbf{s}) \equiv \begin{cases} \lambda_i & \text{if } \sum_{i \in I} s_k^i < \sum_{i \in I} s_l^i \\ 0 & \text{if } \sum_{i \in I} s_k^i > \sum_{i \in I} s_l^i \end{cases}$$

where λ_i denotes the probability that firm i is chosen to produce the efficient specification, with $0 \leq \lambda_i \leq 1$, $\sum_{i \in I} \lambda_i = 1$. Now consider the payment scheme $\hat{T}$ under which each firm i receives a constant amount $\hat{T}_i$ equal to the expected production cost of the boundary type $s^i = \mathbf{1}$ under SCR $\hat{Q}$: $\hat{T}_i(\mathbf{s}) \equiv \lambda_i \hat{c}_i(\mathbf{1})$ for all $\mathbf{s} \in S^n$, where $c_i(\mathbf{1}) = \mathbb{E}\left[\sum_{k \in K} Q_k^i(\mathbf{1}, \mathbf{s}^{-i}) C_k(1, \mathbf{s}_k^{-i})\right]$. We can show:

Proposition 8. *The efficient DRM $(\hat{Q}, \hat{T})$ is incentive compatible and individually rational.*

The proof is omitted as the result follows immediately from the fact that if all other firms report truthfully their types $\mathbf{s}^{-i}$, then firm i minimizes expected production cost by reporting truthfully its own type. This is because each firm has a positive chance of being chosen to produce the efficient specification. By misrepresenting its type, firm i faces a positive chance of having to produce the more costly specification. This is not profitable as each firm's payment is constant.

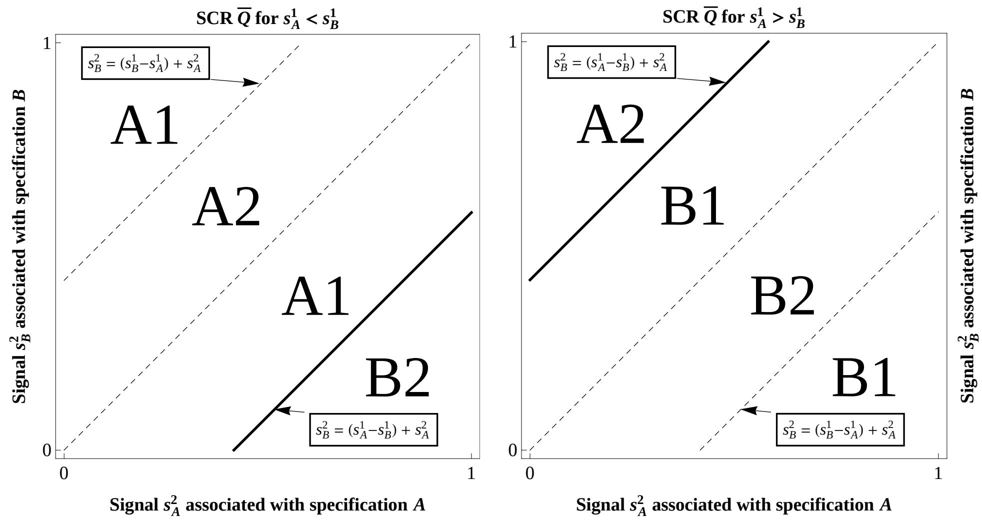

Figure 5: Efficient SCR $\bar{Q}$ in correlated symmetric setting with $n = 2$ firms

Observe that the efficient DRM $(\hat{Q}, \hat{T})$ does not satisfy the necessary condition for expenditure-minimization in Proposition 5, as each firm has a positive chance of being chosen to supply the efficient specification regardless of whether its minimum signal pertains to the efficient specification or not.

We now address the question whether an optimal DRM can ever be efficient in the sense of Definition 9. From Section 5 on expenditure minimization we know that, within the class of *all* implementable DRMs, an expenditure-minimal DRM must display the property in Proposition 5, by which no firm is chosen to supply the specification associated with its *maximum* signal. Therefore, if an optimal DRM is efficient, the SCR associated with it must be both efficient *and* display the necessary property for expenditure-minimization in Proposition 5. However, as we show now, such SCRs exist only in correlated symmetric settings with $n = 2$ firms. It follows immediately that in all other settings, no optimal DRM will be efficient. To see that efficiency and the necessary condition in Proposition 5 are compatible only in very special cases, consider the SCR $\bar{Q}$ in Fig. 5. It is obvious that $\bar{Q}$ is efficient and satisfies the condition in Proposition 5. To see that $\bar{Q}$ is also incentive compatible, note that it gives rise to expected probability assignment functions $\bar{q}^i$ of the form in Proposition 6, with $\bar{\rho}_i(x) = 1/2$ for all $x \in [0, 1]$. We can show furthermore:

Proposition 9. *In correlated symmetric settings with $n = 2$ firms, the only efficient SCR that satisfies the necessary condition for expenditure-minimization in Proposition 5 is $\bar{Q}$ in Fig. 5.*

Observe that incentive compatibility of the efficient SCR $\bar{Q}$ relies crucially on symmetry w.r.t. specifications. If the type-distribution is not symmetric around the 45°-line, the two events in the left-hand panel of Fig. 5 where firm 1 is chosen to produce specification A (labeled by "A1") will no longer generate an expected probability assignment function $q^1_A(s^1)$ that is constant for all s^1 s.t. $s^1_A < s^1_B$. In particular, it will hold that $\partial \bar{q}^1_A(s^1)/\partial s^1_B \neq \partial \bar{q}^1_B(s^1)/\partial s^i_A = 0$, which constitutes a violation of incentive compatibility.[24] Note furthermore that the construction in Fig. 5 does *not* generalize to settings with three or more firms, even if the type-distribution is symmetric. To see this, let $n = 3$. Suppose that $s^1_A < s^1_B$ and define $\bar{s}_A \equiv s^2_A + s^3_A$ and $\bar{s}_B \equiv s^2_B + s^3_B$. Specification A is efficient if $\bar{s}_B > (s^1_A - s^1_B) + \bar{s}_A$. Otherwise, specification B is efficient. In this case, the necessary condition for expenditure-minimization in Proposition 5 requires that firm 1 not be chosen as $s^1_A < s^1_B$. Now adapt as follows the construction of $\bar{Q}$ in Fig. 5: Apart from events $\Sigma \equiv \left\{(\bar{s}_A, \bar{s}_B) : (s^1_A - s^1_B) + \bar{s}_A < \bar{s}_B < \bar{s}_A\right\}$ and $\hat{\Sigma} \equiv \left\{(\bar{s}_A, \bar{s}_B) : \bar{s}_B > (s^1_B - s^1_A) + \bar{s}_A\right\}$, firm 1's probability of being chosen is zero. In events Σ and $\hat{\Sigma}$, choose with equal probability among all firms whose minimum signals pertains to specification A. This means that firm 1 is chosen with positive probability, but the precise magnitude of this probability depends on the realizations of types s^2 and s^3 of the other firms. In particular, in event Σ there can be at most one other firm whose minimum signal pertains to A, so that firm 1 is chosen to supply specification A either with probability 1 or with probability $1/2$. In event $\hat{\Sigma}$ there is at least one other firm whose minimum signal pertains to A, so that firm 1 is chosen to supply specification A either with probability $1/2$ or with probability $1/3$. Consequently, firm 1's expected probability of being chosen differs across the events Σ and $\hat{\Sigma}$. It is not hard to verify that $q^1_A(s^1)$, given by $\Pr\{(\bar{s}_A, \bar{s}_B) \in \Sigma \cup \hat{\Sigma}\}$, violates the necessary condition for incentive compatibility in footnote 24. In summary, apart from correlated symmetric settings with two firms, there exists no incentive compatible SCR that is both efficient and satisfies the necessary condition for expenditure-minimization in Proposition 5.

We now turn to correlated symmetric settings with two firms. Here, we need to address the question if the efficient SCR $\bar{Q}$ in Fig. 5 can ever be optimal overall, i.e. minimize expenditure within the class of *all* implementable SCRs. To show that the answer to this question is No, observe that *if* $\bar{Q}$ is expenditure-minimal overall, it also has to be expenditure-minimal within the restricted class of *efficient* SCRs. However, it is not hard to show that $\bar{Q}$ is *not* expenditure-minimal in this restricted class, because the efficient DRM $(\hat{Q}, \hat{T})$ (with $\lambda_1 = \lambda_2 = 1/2$) generates lower ex

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[24] By item (i) of Proposition 3, incentive compatibility requires $\bar{q}^i$ to be conservative. A necessary condition for q^i to be conservative is that $\partial \bar{q}^i_A(s^i)/\partial s^i_B = \partial \bar{q}^i_B(s^i)/\partial s^i_A = 0$ $\forall i \in I$ and $\forall s^i \in S$ at which $\bar{q}^i$ is differentiable (see Theorem 1.2 in chapter VII,§1, of Lang, 1987).

ante expenditure than the efficient DRM featuring SCR $\bar{Q}$ in Fig. 5. This can be seen from a comparison of ex ante expenditure under these two efficient DRMs, which amounts to a comparison of the firms' information rents under $\hat{Q}$ and $\bar{Q}$.[25] Under $\hat{Q}$ (with $\lambda_1 = \lambda_2 = 1/2$), the conditional expected probability assignment functions $\hat{q}^i$ are:

$$\hat{q}^i_A(s^i) = \begin{cases} \frac{1}{2}(1-\pi_B(s^i)) & \text{if } s^i_A < s^i_B \\ \frac{1}{2}\pi_A(s^i) & \text{if } s^i_A > s^i_B \end{cases} \text{ and } \hat{q}^i_B(s^i) = \begin{cases} \frac{1}{2}\pi_B(s^i) & \text{if } s^i_A < s^i_B \\ \frac{1}{2}(1-\pi_A(s^i)) & \text{if } s^i_A > s^i_B \end{cases}$$

where $\pi_A(s^i)$ denotes the probability that specification A is efficient when firm i's type is s.t. $s^i_A > s^i_B$, and $\pi_B(s^i)$ denotes the probability that specification B is efficient when firm i's type is s.t. $s^i_A < s^i_B$. Now fix any type s^i with $s^i_k < s^i_l$ ($k,l \in K$, $l \neq k$) and compute firm i's information rents under $\bar{Q}$ and $\hat{Q}$, resp:[26]

$$\int_{\Gamma_{(s^i,\mathbf{1})}} \bar{q}^i d\alpha = \frac{1}{2}(1-s^i_k) > \int_{\Gamma_{(s^i,\mathbf{1})}} \hat{q}^i d\alpha = \frac{1}{2}(1-s^i_k) - \frac{1}{2}\pi_l(s^i)(s^i_l - s^i_k)$$

This comparison shows that for any given type s^i, SCR $\bar{Q}$ awards each firm higher information rents than $\hat{Q}$. Consequently, ex ante expenditure is lower under $(\hat{Q},\hat{T})$ (with $\lambda_1 = \lambda_2 = 1/2$) than under an efficient DRM featuring SCR $\bar{Q}$. We can therefore conclude that any DRM that is optimal among all implementable DRMs (and therefore generates ex ante expenditure at least as low as an expenditure-minimal efficient DRM) cannot involve SCR $\bar{Q}$ in Fig. 5. But since $\bar{Q}$, as stated in Proposition 9, is the only efficient SCR to satisfy the necessary condition for expenditure-minimization in Proposition 5, it follows that even in correlated symmetric settings with two firms an optimal DRM will not be efficient.

At this point, it may be helpful to offer an intuition for why the condition in Proposition 5 is necessary for expenditure-minimization among all implementable DRMs, but is *not* necessary when attention is restricted to the class of efficient DRMs. Recall that Proposition 5 was established by partially minimizing pointwise ex ante expenditure in (11) in the absence of any constraints on the SCRs considered. However, when looking for an expenditure-minimal efficient SCR, ex ante expenditure in (11) must be minimized subject to the ex post constraint of

[25] Recall that in an implementable DRM, the buyer's expenditure consists of two components. The first is a reimbursement of (expected) production cost, and the second is the firms' information rents. The information rents under $\hat{Q}$ and $\bar{Q}$ can be computed as path-integrals of the conditional expected probability assignment functions $\hat{q}^i$ and $\bar{q}^i$, resp. The reimbursement of production cost is, of course, the same under any two efficient DRMs, so that any difference in ex ante expenditure between two efficient DRMs is driven solely by the different levels of information rents.

[26] If $k = A$ and $l = B$, we use integration-path Γ_2 in Fig. 3 to compute firm i's information rents under $\hat{Q}$ and $\bar{Q}$, while for $k = B$ and $l = A$ we use Γ_1.

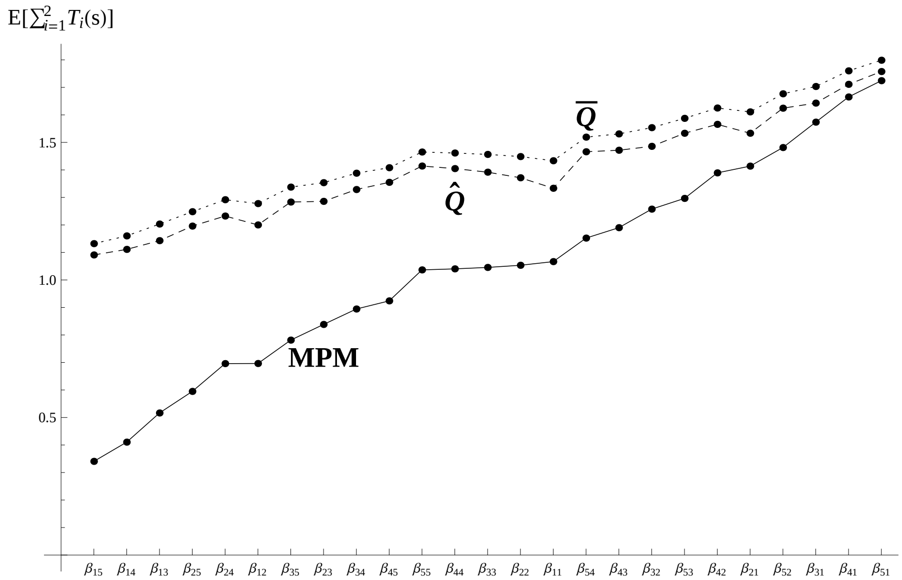

Figure 6: Ex ante expenditure under MPM and the SCRs $\hat{Q}$ and $\bar{Q}$ for 25 Beta-distributions

efficient specification choice. Fig. 5 shows that within this restricted class, the decision to select at every type-pair (s^1, s^2) in the interior of S^2 the supplier whose minimum signal pertains to the efficient specification (as required by the condition in Proposition 5) determines fully the choice of supplier at all associated type-pairs $((s_k^i, 1), s^{-i})$ on the boundary of S^2 (for all $k \in K$ and $i = 1, 2$). Therefore, when minimizing (11) subject to efficiency constraints, the values of the SCR Q at interior type-pairs cannot be chosen independently of the values of Q at boundary type-pairs. This contrasts with the partial pointwise minimization of (11) in the absence of a restriction to efficient SCRs (as in the proof of Proposition 5). There, any decision as to which firm supplies each specification at interior type-pairs does *not* restrict the choice of supplier for type-pairs on the boundary. Consequently, while the condition in Proposition 5 is necessary for pointwise minimization of (11) in the absence of ex post constraints on Q, it is not necessary for expenditure-minimization within the class of efficient SCRs.

To conclude this section, we provide a numerical comparison of ex ante expenditure under the MPM (which satisfies the necessary condition for expenditure minimization in Proposition 5) and the efficient SCRs $\hat{Q}$ (with $\lambda_1 = \lambda_2 = 1/2$) and $\bar{Q}$. For our simulations, the results of which are displayed in Fig. 6, we restrict ourselves to the independent symmetric case (i.e. $f(s_A, s_B) = h(s_A)h(s_B)$) where the distribution H is given by 25 different Beta-distributions.[27] While we do not know if the MPM is an optimal DRM, our numerical comparisons reveal that the MPM performs better than the efficient DRM $(\hat{Q}, \hat{T})$ for all 25 Beta-distributions, and does particularly well for those distributions that concentrate probability mass on low cost signals (such as the Beta-distribution β_{15} with parameters $a = 1$ and $b = 5$), where the MPM generates a level of ex ante expenditure that is less than 50% of expenditure associated with efficient DRM $(\hat{Q}, \hat{T})$. The advantage of the MPM over $(\hat{Q}, \hat{T})$ becomes less pronounced for Beta-distributions that concentrate probability mass on high cost-signals (e.g. under the Beta-distribution β_{51}, where ex ante expenditure in the MPM reaches approx. 98% of ex ante expenditure under $\hat{Q}$).

7 Conclusion

We have introduced a simple common-value procurement model in which both the specification and the price of the good matter. We have shown that the minimum price mechanism, which is a very natural procurement procedure, is both inefficient and involves lower (expected) expenditure than efficient mechanisms in many settings. The main technical contribution of this paper was to suggest a pragmatic approach for taking account of incentive compatibility constraints in mechanism design settings with multi-dimensional private information. In future work, we plan to explore if the techniques developed here can also be useful in other mechanism design settings with multi-dimensional private information.

8 Appendix

8.1 Proof of Lemma 1.

We have to show that if a strategy (δ_i, p^i_A, p^i_B) in the MPM satisfies properties P1-P3, then the pricing function p^i_A fully determines the pricing function p^i_B. Assume (w.l.o.g.) that the specification choice rule δ_i is as shown in either panel of Fig. 1. Now consider any type $\hat{s}^i$ s.t. $\delta_i(\hat{s}^i_A, \hat{s}^i_B) = B$. By P2, this type quotes the price $p^i_B(\hat{s}^i_B)$. Now note that there exists a type $\tilde{s}^i$, with $\tilde{s}^i_A = X_i^{-1}(\hat{s}^i_B)$ and $\tilde{s}^i_B = \hat{s}^i_B$, who is indifferent between offering specification A or B. Property P2 implies that if specification A is offered by type $\tilde{s}^i$, the quoted price will be $p^i_A(\tilde{s}^i_A) = p_A(X_i^{-1}(\hat{s}^i_B))$.

[27] We have generated these 25 distributions by letting each of the two parameters that characterize the Beta-distribution take all integer values between 1 and 5.

If, instead, specification B is offered, the quoted price will be $p_B^i(\hat{s}_B^i)$. By P3, these two prices must be the same. Therefore, $p_B^i(\hat{s}_B^i) = p_A^i(X^{-1}(\hat{s}_B^i))$. $\square$

8.2 Proof of Lemma 2.

The proof consists of three steps. In Step 1, we show that there exist symmetric equilibria that feature a nondecreasing pricing function p_A. In Step 2, we show that a symmetric equilibrium pricing function p_A must, in fact, be increasing. In Step 3, we finally show that $p_A(x)$ is differentiable at every $x \in [0, \min\{X^{-1}(1), 1\}]$, with the exception of $x = X^{-1}(0)$.

Step 1. We show here that the game induced by the MPM in Section 3.1 satisfies the single crossing condition for games of incomplete information (SCC). The SCC (see Definition 3 in Athey, 2001) ensures that if each firm j uses a nondecreasing function to generate the price for its chosen specification, then there exists a best response where firm i ($i \neq j$) also uses a nondecreasing function to generate its price quote. We may then look for a symmetric equilibrium of the MPM in which all firms use the same nondecreasing pricing functions. To show that SCC is satisfied, we have to prove that firm i's expected profit $\Pi_i(p^i; s_k^i)$ from offering the good in specification $\delta_i(s^i) = k$ at price p^i satisfies the single crossing differences property (SCD) when the strategy (δ, p_A, p_B) used by all other firms features nondecreasing pricing functions p_A and p_B.[28] Assuming that (δ, p_A, p_B) satisfies properties P1-P3, and (w.l.o.g.) that δ is based on a function X s.t. $X^{-1}(0) \geq 0$ (as illustrated in Fig. 1), each firm j uses a nondecreasing function p_A to generate its price if $\delta(s^j) = A$, and the nondecreasing function $p_A(X^{-1}(\cdot))$ to generate its price if $\delta(s^j) = B$. To obtain an expression for firm i's expected profit $\Pi_i(p^i; s_k^i)$ we need to identify, for each competitor j, the events in which the price quoted by j is no lower than p^i:

(i) Firm j offers the good in specification A: $\delta(s_A^j, s_B^j) = A \Leftrightarrow s_B^j > X(s_A^j)$. The price $p_A(s_A^j)$ quoted by firm j is no lower than firm i's price p^i: $p_A(s_A^j) \geq p^i$.

(ii) Firm j offers the good in specification B: $\delta(s_A^j, s_B^j) = B \Leftrightarrow s_A^j > X^{-1}(s_B^j)$. The price $p_B(s_B^j)$ quoted by firm j is no lower than firm i's price p^i: $p_B(s_B^j) \geq p^i$.

Now define for each specification $k \in K$ a function $\check{s}_k : \mathbb{R}_+ \to \mathbb{R}_+$, $p \mapsto \check{s}_k(p)$, with $\check{s}_k(p) \equiv \inf\{s_k^i \in [0,1] : p_k(s_k^i) \geq p\}$. The value $\check{s}_k$ represents the lowest k-signal s.t. firm j's price for specification k is no lower than some given price p. Note that each $\check{s}_k$ is nondecreasing, with $\check{s}_k(p) = 0$ for all $p \leq p_k(0)$. Assuming

[28] For a definition of SCD, see chapter 4.1 of Milgrom (2004).

that each pricing function p_k is bounded, we also have $\check{s}_k(p) = \infty$ for all p above the upper bound of p_k.[29] Formally, the event that firm j's price is no lower than firm i's price p^i is: $\Omega(p^i) \equiv \{s^j \in S : s^j_A > \check{s}_A(p^i), s^j_B > X(s^j_A)\} \cup \{s^j \in S : s^j_B > \check{s}_B(p^i), s^j_A > X^{-1}(s^j_B)\}$. Firm i has positive probability of winning the contract iff all competitors $j \neq i$ quote prices no lower than p^i. This event can be described by the Cartesian product $\Omega(p^i) \times \ldots \times \Omega(p^i)$ across the $n-1$ competitors. To ease notation, let $\Omega(p^i)^{n-1} \equiv \Omega(p^i) \times \ldots \times \Omega(p^i)$. Having identified the events where all competitors charge a price no lower than p^i, we need to account for the possibility that, rather than winning outright, firm i's price p^i ties with the prices of one or more competitors. For this purpose, define for every $i \in I$ a tie-breaking function $\Theta_i : \mathbb{R}^n_+ \to [0,1]$ s.t. $\Theta_i(p^1, \ldots, p^n) = 1$ if $p^i < \min_{j \neq i} p^j$, and $\Theta_i(p^1, \ldots, p^n) \in (0,1)$ if p^i ties with the price of at least one competitor. Otherwise, $\Theta_i(p^1, \ldots, p^n) = 0$. We can now write as follows firm i's expected profit from offering specification k at price p^i:[30]

$$\Pi_i(p^i; s^i_k) \equiv \int_{\Omega(p^i)^{n-1}} \Theta_i(p^i, \mathbf{p}^{-i}(\mathbf{s}^{-i}))(p^i - C_k(s^i_k, \mathbf{s}^{-i}_k))g(\mathbf{s}^{-i})d\mathbf{s}^{-i} \tag{A.1}$$

Now define $\Psi(s^i_k) \equiv \Pi_i(\hat{p}^i; s^i_k) - \Pi_i(\tilde{p}^i; s^i_k)$ for any two prices $\hat{p}^i > \tilde{p}^i$ quoted by firm i for its chosen specification k. We say that $\Pi_i(p^i; s^i_k)$ satisfies SCD if for all $\hat{s}^i_k > \tilde{s}^i_k$: $\Psi(\tilde{s}^i_k) > 0 \Rightarrow \Psi(\hat{s}^i_k) > 0$ and $\Psi(\tilde{s}^i_k) \geq 0 \Rightarrow \Psi(\hat{s}^i_k) \geq 0$. Suppose that $\Psi(\tilde{s}^i_k) \geq 0$. Then, if $\Pi_i(p^i; s^i_k)$ satisfies SCD, it must hold that $\Psi(\hat{s}^i_k) - \Psi(\tilde{s}^i_k) \geq 0$ for any $\hat{s}^i_k > \tilde{s}^i_k$. We can express the difference $\Psi(\hat{s}^i_k) - \Psi(\tilde{s}^i_k)$ as follows:

$$\int_{\Omega(\tilde{p}^i)^{n-1}} \Theta_i(\tilde{p}^i, \mathbf{p}^{-i}(\mathbf{s}^{-i}))(\hat{s}^i_k - \tilde{s}^i_k)g(\mathbf{s}^{-i})d\mathbf{s}^{-i}$$
$$- \int_{\Omega(\hat{p}^i)^{n-1}} \Theta_i(\hat{p}^i, \mathbf{p}^{-i}(\mathbf{s}^{-i}))(\hat{s}^i_k - \tilde{s}^i_k)g(\mathbf{s}^{-i})d\mathbf{s}^{-i}$$

As $\check{s}_k(\tilde{p}^i) \leq \check{s}(\hat{p}^i)$ for all $k \in K$, it holds that $\Omega(\hat{p}_i) \subseteq \Omega(\tilde{p}^i)$. We can therefore partition the set $\Omega(\tilde{p}^i)$ into the disjoint subsets $\Omega(\hat{p}^i)$ and $\Omega(\tilde{p}^i) \backslash \Omega(\hat{p}^i)$.[31] We can

[29] This follows from the fact that $\inf \varnothing = \infty$.

[30] We write $\mathbf{p}^{-i}(\mathbf{s}^{-i}) \equiv (p^1_{k_1}, \ldots, p^{i-1}_{k_{i-1}}, p^{i+1}_{k_{i+1}}, \ldots, p^n_{k_n})$ for the vector of prices quoted by firms $j \neq i$ for their respective specifications, chosen according to the specification choice rule δ. I.e. $k_j = \delta(s^j)$.

[31] This set is $\{s^j \in S : \check{s}_A(\tilde{p}^i) < s^j_A < \check{s}_A(\hat{p}^i), s^j_B > X(s^j_A)\} \cup \{s^j \in S : \check{s}_B(\tilde{p}^i) < s^j_B < \check{s}_B(\hat{p}^i), s^j_A > X^{-1}(s^j_B)\}$.

now write the difference $\Psi(\hat{s}^i_k) - \Psi(\check{s}^i_k)$ as:

$$\int_{\Omega(\tilde{p}^i)^{n-1} \setminus \Omega(\hat{p}^i)^{n-1}} \Theta_i(\tilde{p}^i, \mathbf{p}^{-i}(\mathbf{s}^{-i}))(\hat{s}^i_k - \check{s}^i_k) g(\mathbf{s}^{-i}) d\mathbf{s}^{-i}$$
$$+ \int_{\Omega(\hat{p}^i)^{n-1}} \left[\Theta_i(\tilde{p}^i, \mathbf{p}^{-i}(\mathbf{s}^{-i})) - \Theta_i(\hat{p}^i, \mathbf{p}^{-i}(\mathbf{s}^{-i}))\right] (\hat{s}^i_k - \check{s}^i_k) g(\mathbf{s}^{-i}) d\mathbf{s}^{-i} \qquad \text{(A.2)}$$

It is easy to see that the first term in (A.2) is nonnegative: while the integrand is always positive, the set $\Omega(\tilde{p}^i)^{n-1} \setminus \Omega(\hat{p}^i)^{n-1}$ may be empty.[32] To see that the second term in (A.2) is nonnegative, observe that we are integrating over all type-vectors $\mathbf{s}^{-i}$ s.t. each competitor's price is no lower than $\hat{p}^i$. For $\tilde{p}^i < \hat{p}^i$ we have $\Theta_i(\tilde{p}^i, \mathbf{p}^{-i}(\mathbf{s}^{-i})) = 1$, and therefore $\Theta_i(\tilde{p}^i, \mathbf{p}^{-i}(\mathbf{s}^{-i})) - \Theta_i(\hat{p}^i, \mathbf{p}^{-i}(\mathbf{s}^{-i})) \geq 0$ for all $\mathbf{s}^{-i} \in \Omega(\hat{p}^i)^{n-1}$. We can conclude that $\Psi(\check{s}^i_k) \geq 0 \Rightarrow \Psi(\hat{s}^i_k) \geq 0$, which implies that $\Pi_i(p^i; s^i_k)$ satisfies SCD, as required.

Step 2. We show by contradiction that a symmetric equilibrium (δ, p_A, p_B) in the space of nondecreasing strategies must feature an increasing pricing function p_A. Suppose, instead, that there exist $\tilde{x}$, $\hat{x}$ with $0 \leq \tilde{x} < \hat{x} \leq \min\{X^{-1}(1), 1\}$ s.t. $p_A(x) = \hat{p}_A$ for all $x \in (\tilde{x}, \hat{x})$, $p_A(x) < \hat{p}_A$ if $x < \tilde{x}$, and $p_A(x) > \hat{p}_A$ if $x > \hat{x}$. Note that $\check{s}_A(\hat{p}_A) = \tilde{x}$, that $\lim_{\varepsilon \downarrow 0} \check{s}_A(\hat{p}_A - \varepsilon) = \tilde{x}$, and that $\lim_{\varepsilon \downarrow 0} \check{s}_A(\hat{p}_A + \varepsilon) = \hat{x}$. Now consider firm i with A-signal $s^i_A = x \in (\tilde{x}, \hat{x})$. Denoting by $\hat{p}^+_A$ the price for which $\check{s}(\hat{p}^+_A) = \hat{x}$, we can write firm i's expected profit $\Pi_i(\hat{p}_A; x)$ as follows:

$$\int_{\Omega(\hat{p}_A)^{n-1} \setminus \Omega(\hat{p}^+_A)^{n-1}} \Theta_i(\hat{p}_A, \mathbf{p}^{-i}(\mathbf{s}^{-i}))(\hat{p}_A - C_A(x, \mathbf{s}^{-i}_A)) g(\mathbf{s}^{-i}) d\mathbf{s}^{-i}$$
$$+ \int_{\Omega(\hat{p}^+_A)^{n-1}} (\hat{p}_A - C_A(x, \mathbf{s}^{-i}_A)) g(\mathbf{s}^{-i}) d\mathbf{s}^{-i} \quad \text{(A.3)}$$

The first integral in (A.3) is over type-vectors $\mathbf{s}^{-i}$ s.t. at least one firm $j \neq i$ has a type s^j in $\Omega(\hat{p}_A) \setminus \Omega(\hat{p}^+_A)$.[33] This means that at least one competitor quotes the same price $\hat{p}_A$ as firm i, and therefore $\Theta_i(\hat{p}_A, \mathbf{p}^{-i}(\mathbf{s}^{-i})) < 1$ for all $\mathbf{s}^{-i} \in \Omega(\hat{p}_A)^{n-1} \setminus \Omega(\hat{p}^+_A)^{n-1}$. The second integral in (A.3) is over type-vectors $\mathbf{s}^{-i}$ s.t. every competitor's price exceeds $\hat{p}^+_A$, which means that firm i wins outright. Now suppose that firm i deviates to some price $\hat{p}_A - \varepsilon$, which yields expected profit:

$$\Pi_i(\hat{p}_A - \varepsilon; x) = \int_{\Omega(\check{s}_A(\hat{p}_A - \varepsilon))^{n-1}} (\hat{p}_A - \varepsilon - C_A(x, \mathbf{s}^{-i}_A)) g(\mathbf{s}^{-i}) d\mathbf{s}^{-i}$$

[32] This happens if the pricing function p_A displays a jump discontinuity, in which case $\check{s}_A(\tilde{p}^i) = \check{s}_A(\hat{p}^i)$.

[33] This set is given by $\{s^j \in S : \tilde{x} < s^j_A < \hat{x}, s^j_B > X(s^j_A)\} \cup \{s^j \in S : \check{s}_B(\hat{p}_A) < s^j_B < \check{s}_B(\hat{p}^+_A), s^j_A > X^{-1}(s^j_B)\}$.

In the limit as $\varepsilon \downarrow 0$:

$$\lim_{\varepsilon\downarrow 0}\Pi_i(\hat{p}_A-\varepsilon;x) = \int_{\Omega(\hat{p}_A)^{n-1}\setminus\Omega(\hat{p}_A^+)^{n-1}}(\hat{p}_A - C_A(x,\mathbf{s}_A^{-i}))g(\mathbf{s}^{-i})d\mathbf{s}^{-i} \qquad (13)$$
$$+\int_{\Omega(\hat{p}_A^+)^{n-1}}(\hat{p}_A - C_A(x,\mathbf{s}_A^{-i}))g(\mathbf{s}^{-i})d\mathbf{s}^{-i}$$

The profit-gain $\lim_{\varepsilon\downarrow 0}\Pi_i(\hat{p}_A-\varepsilon;x)-\Pi_i(\hat{p}_A;x)$ from this deviation is:

$$\int_{\Omega(\hat{p}_A)^{n-1}\setminus\Omega(\hat{p}_A^+)^{n-1}}[1-\Theta_i(\hat{p}_A,\mathbf{p}^{-i}(\mathbf{s}^{-i}))](\hat{p}_A - C_A(x,\mathbf{s}_A^{-i}))g(\mathbf{s}^{-i})d\mathbf{s}^{-i}$$

which is positive as $\Theta_i(\hat{p}_A,\mathbf{p}^{-i}(\mathbf{s}^{-i}))<1$ for all $\mathbf{s}^{-i}\in\Omega(\hat{p}_A)^{n-1}\setminus\Omega(\hat{p}_A^+)^{n-1}$. This establishes the desired contradiction. We can therefore conclude that the equilibrium pricing function p_A must be increasing.

We can show furthermore that the equilibrium pricing function p_A must be continuous everywhere in its domain $[0,\min\{X^{-1}(1),1\}]$. To see this, note first that an increasing function p_A can only display jump discontinuities. Suppose now that p_A is continuous everywhere, with the exception of a point $\bar{s}_A\in(X^{-1}(0),\min\{X^{-1}(1),1\})$. At $\bar{s}_A$ let $p_A^-(\bar{s}_A)<p_A^+(\bar{s}_A)$, where $p_A^-(\bar{s}_A)$ and $p_A^+(\bar{s}_A)$ denote the right-hand and left-hand limit of p_A as $s_A\uparrow\bar{s}_A$ and $s_A\downarrow\bar{s}_A$, resp. We now ask: at which value $\bar{s}_B$ does the pricing function $p_B(\cdot)=p_A(X^{-1}(\cdot))$ feature a jump discontinuity, and what size is the jump? To answer this question, note that p_B is defined on the interval $[0,\min\{1,X(1)\}]$. At $s_B=0$ it holds that $p_B(0)=p_A(X^{-1}(0))$, while at $s_B=\min\{1,X(1)\}$ it holds that $p_B(\min\{1,X(1)\})=p_A(\min\{X^{-1}(1),1\})$. Finally, observe that in the limit as $s_B\uparrow X(\bar{s}_A)$, we have $p_B^-(X(\bar{s}_A))=p_A^-(\bar{s}_A)$, and in the limit as $s_B\downarrow X(\bar{s}_A)$, we have $p_B^+(X(\bar{s}_A))=p_A^+(\bar{s}_A)$. We can therefore conclude that p_B is continuous everywhere in $[0,\min\{1,X(1)\}]$, with the exception of the point $\bar{s}_B=X(\bar{s}_A)$, and that the size of the jump at this point is $p_B^+(X(\bar{s}_A))-p_B^-(X(\bar{s}_A))=p_A^+(\bar{s}_A)-p_A^-(\bar{s}_A)$. Therefore, the size in the jump of p_B at $X(\bar{s}_A)$ is equal to the size of the jump in p_A at $\bar{s}_A$.

We now show by contradiction that the equilibrium pricing function p_A cannot display a jump discontinuity at $\bar{s}_A$. To see this, consider firm i who offers specification A, and suppose its A-signal is $s_A^i=\bar{s}_A-\varepsilon$. In the limit as $\varepsilon\downarrow 0$, the pricing function p_A prescribes a price of $p_A^-(\bar{s}_A)$. With this price, we obtain thresholds $\check{s}_A(p_A^-(\bar{s}_A))=\bar{s}_A$ and $\check{s}_B(p_A^-(\bar{s}_A))=X(\bar{s}_A)$. The set $\Omega(p_A^-(\bar{s}_A))^{n-1}$ captures all type-vectors $\mathbf{s}^{-i}$ s.t. firm i wins the contract. Now suppose firm i deviates to the price $p_A^+(\bar{s}_A)$. Due to the identical right-hand and left-hand limits of p_A and p_B at their respective points of discontinuity, it follows that $\check{s}_A(p_A^+(\bar{s}_A))=\bar{s}_A$ and $\check{s}_B(p_A^+(\bar{s}_A))=X(\bar{s}_A)$, and therefore $\Omega(p_A^+(\bar{s}_A))^{n-1}=\Omega(p_A^-(\bar{s}_A))^{n-1}$. Consequently, when quoting $p_A^+(\bar{s}_A)$, firm i still wins against the same competitor-types $\mathbf{s}^{-i}$ as with the lower price $p_A^-(\bar{s}_A)$. However, with price $p_A^+(\bar{s}_A)$, firm i has strictly higher expected profit than with $p_A^-(\bar{s}_A)$, which makes this a profitable deviation. Therefore, p_A is continuous everywhere in $[X^{-1}(0), \min\{X^{-1}(1), 1\}]$. It is easy to argue that p_A must also be continuous in the remainder of its domain.

Step 3. In this final step of the proof of Lemma 2, we show that an increasing equilibrium pricing function p_A is a.e. differentiable on $[0,1)$. Suppose firm i's offers specification is A, and that its A-signal s_A^i is in $(X^{-1}(0), 1)$. Due to the additive nature of the cost function C_A in (1), we can write $\Pi_i(p_A(s_A^i); s_A^i)$ as

$$\int_{\Omega(s_A^i)^{n-1}} \left(p_A(s_A^i) - \frac{s_A^i}{n} - \sum_{j \neq i} \frac{s_k^j}{n} \right) g(\mathbf{s}^{-i}) d\mathbf{s}^{-i}$$
$$= \omega(s_A^i)^{n-1} \left(p_A(s_A^i) - \frac{s_A^i}{n} + \frac{n-1}{n} \mathbb{E}[s_A \,|s_A > s_A^i, s_B > X(s_A^i)] \right) \tag{A.4}$$

where $\Omega(s_A^i) \equiv \{(s_A, s_B) \in S : s_A > s_A^i, s_B > X(s_A^i)\}$ and $\omega(s_A^i) = \int_{s_A^i}^1 \int_{X(s_A^i)}^1 f(s_A, s_B) ds_A ds_B$. The term $\omega(s_A^i)$ represents the probability that any given competitor charges a price higher than $p_A(s_A^i)$. Now take x, y s.t. $X^{-1}(0) < x < y < 1$. In a symmetric equilibrium, type y must prefer the price $p_A(y)$ to the price $p_A(x)$:

$$\Pi_i(p(y); y) - \Pi_i(p(x); y) \geq 0 \tag{A.5}$$

Similarly, type x must prefer the price $p(x)$ to the price $p(y)$:

$$\Pi_i(p(x); x) - \Pi_i(p(y); x) \geq 0 \tag{A.6}$$

Setting $P(s_A^i) \equiv p_A(s_A^i)\omega(s_A^i)^{n-1}$, the profit difference in (A.5) can be written as:

$$\begin{aligned} \frac{n(P(y) - P(x))}{n-1} \geq \; & \frac{y}{n-1} \left(\omega(y)^{n-1} - \omega(x)^{n-1} \right) \\ & + \omega(y)^{n-1} \mathbb{E}[s_A \,|s_A > y, s_B > X(y)] \\ & - \omega(x)^{n-1} \mathbb{E}[s_A \,|s_A > x, s_B > X(x)] \end{aligned} \tag{A.7}$$

The right-hand side of (A.7) is a lower bound on $\frac{n}{n-1}(P(y) - P(x))$. Now note that the terms in the second and third line of (A.7) can be written as:

$$\begin{aligned} & \mathbb{E}[s_A \,|s_A > y, s_B > X(y)] \left(\omega(y)^{n-1} - \omega(x)^{n-1} \right) \\ & + \omega(x)^{n-1} \left(\mathbb{E}[s_A \,|s_A > y, s_B > X(y)] - \mathbb{E}[s_A \,|s_A > x, s_B > X(x)] \right) \end{aligned}$$

After dividing both sides of (A.7) by $y-x$, the lower bound on the right-hand side of (A.7) becomes:

$$\left(\frac{y}{n-1}+\mathbb{E}[s_A\,|s_A>y,s_B>X(y)]\right)\frac{\omega(y)^{n-1}-\omega(x)^{n-1}}{y-x} + \omega(x)^{n-1}\frac{\mathbb{E}[s_A\,|s_A>y,s_B>X(y)]-\mathbb{E}[s_A\,|s_A>x,s_B>X(x)]}{y-x} \quad \text{(A.8)}$$

As $\omega(s_A^i)^{n-1}$ and $\mathbb{E}[s_A\,|s_A>s_A^i,s_B>X(s_A^i)]$ are integrals, they are both differentiable. Therefore, in the limit as $y\downarrow x$ the expression in (A.8) converges to:

$$\frac{d}{dy}\left(\frac{x}{n-1}\omega(y)^{n-1}+\mathbb{E}[s_A\,|s_A>y,s_B>X(y)]\omega(y)^{n-1}\right)\Bigg|_{y=x} \quad \text{(A.9)}$$

Using similar steps, we can show that the upper bound on $\frac{n}{n-1}(P(y)-P(x))$ implied by (A.6) also converges to the limit in (A.9) as $y\downarrow x$. Therefore, the term $\frac{n}{n-1}(P(y)-P(x))$ converges to $\frac{n}{n-1}P(x)'$, which implies that the derivative $p_A'(x)$ exists. While the above argument pertains to $x\in(X^{-1}(0),1)$, it is straightforward to show that $p_A(x)$ is differentiable also for all $x\in[0,X^{-1}(0))$. However, for such values x the probability that any given competitor charges a price higher than $p_A(x)$ is given by $\omega(x)=1-F_A(x)$, which will result in a derivative $p_A'(x)$ for $x<X^{-1}(0)$ that is different from the derivative $p_A'(x)$ derived above for $x>X^{-1}(0)$. Therefore, while continuous everywhere, the function p_A is not differentiable at $x=X^{-1}(0)$. □

8.3 Proof of Lemma 3.

Suppose all n firms use a strategy (δ,p_A,p_B) that satisfies properties P1-P3. Now consider a firm i with type s^i s.t. $s_A^i=x<1$ and $s_B^i=X(x)$. By P3, firm i's equilibrium price quote is $p_A(x)=p_B(X(x))$, regardless of the specification it chooses. Therefore, the event that any given competitor charges a price higher than the price of firm i is $\Omega(x)\equiv\{(s_A,s_B)\in S: s_A>x,s_B>X(x)\}$. The probability that firm i wins the contract is then given by $\omega(x)=\int_{\Omega(x)}f(s_A,s_B)d(s_A,s_B)$, regardless of the specification it chooses. Given the additive nature of the cost functions C_k in (1), and the fact that the firms' types are independent random vectors, firm i's expected cost of producing any specification $k\in K$ are:

$$\frac{s_k^i}{n}\omega(x)^{n-1}+(n-1)\omega(x)^{n-1}\int_{\Omega(x)}\frac{s_k}{n}\frac{f(s_A,s_B)}{\omega(x)}d(s_A,s_B)$$

As we are looking to characterize the function X s.t. the expected production cost of the two specifications are the same, we now subtract the expected cost of B from the expected cost of A. This yields the following equation, which defines implicitly the function X:

$$\begin{aligned}&(x-X(x))\,\omega(x)^{n-1}\\&+(n-1)\omega(x)^{n-2}\left(\int_{\Omega(x)} s_A f(s_A,s_B)d(s_A,s_B)-\int_{\Omega(x)} s_B f(s_A,s_B)d(s_A,s_B)\right)=0\end{aligned} \tag{A.10}$$

It is easy to rewrite this equation in the form given in Lemma 3. □

8.4 Proof of Lemma 4.

Suppose the density f is symmetric around the 45°-line: $f(s_B,s_A)=f(s_A,s_B)$ for all $(s_A,s_B)\in S$. We first show that $X(x)=x$ is a *sufficient* condition for equation (A.10) in the proof of Lemma 3 to hold. Now fix some $x<1$. If $X(x)=x$, then (A.10) reduces to:

$$\int_x^1\int_x^1 s_A f(s_A,s_B)ds_B ds_A-\int_x^1\int_x^1 s_B f(s_A,s_B)ds_B ds_A=0$$

First consider the first double integral on the left-hand side. Note that we can change the order of integration. Next, consider the second double integral, and note that we can arbitrarily re-label the integration indices. In particular, re-label s_A as s_B, and vice versa. We can therefore equivalently write the left-hand side as:

$$\int_x^1\int_x^1 s_A[f(s_A,s_B)-f(s_B,s_A)]ds_A ds_B$$

Symmetry of f ensures that this expression is zero for every (s_A,s_B). The next step is to show that the condition $X(x)=x$ is also *necessary* for equation (A.10) to hold. We show this by contradiction. Fix some $x<1$, and suppose that (A.10) holds. Suppose also that $x<X(x)$. In this case, the first term of (A.10) is negative. Now consider the second term of (A.10). In particular, note that, due to the additivity of the double integral, we can write:

$$\begin{aligned}&\int_{\Omega(x)} s_A f(s_A,s_B)d(s_A,s_B)-\int_{\Omega(x)} s_B f(s_A,s_B)d(s_A,s_B)\\&=\int_x^{X(x)}\int_{X(x)}^1 (s_A-s_B)f(s_A,s_B)ds_B ds_A\\&+\int_{X(x)}^1\int_{X(x)}^1 s_A f(s_A,s_B)ds_B ds_A-\int_{X(x)}^1\int_{X(x)}^1 s_B f(s_A,s_B)ds_B ds_A\end{aligned}$$

We can change the order of integration in the second double integral. Also, in the third double integral, we can re-label s_A as s_B, and vice versa. This yields:

$$\int_x^{X(x)} \int_{X(x)}^1 (s_A - s_B) f(s_A, s_B) ds_B ds_A$$
$$+ \int_{X(x)}^1 \int_{X(x)}^1 s_A [f(s_A, s_B) - f(s_B, s_A)] ds_A ds_B$$

By symmetry of f this expression reduces to:[34]

$$\int_x^{X(x)} \int_{X(x)}^1 (s_A - s_B) f(s_A, s_B) ds_B ds_A < 0$$

We can conclude that if $x < X(x)$, then the expected cost difference in (A.10) is negative, which yields the desired contradiction. An analogous argument establishes that the expected cost difference in (A.10) is positive if $x > X(x)$. Therefore, it must be the case that $X(x) = x$ for all $x \in [0,1)$. By continuity, $X(1) = 1$. □

8.5 Proof of Lemma 5.

The proof proceeds in three steps. In Step 1 we show that $X(x) \leq x$. In Step 2, we prove that $X(1) = 1$, and in Step 3 we show that the derivative of X is positive.

Step 1. Assume that H_B f.o.s.d. H_A, which implies that $\mathbb{E}_B[s_B | s_B \geq x] \geq \mathbb{E}_A[s_A | s_A \geq x]$ for all $x \in [0,1)$. We now show by contradiction that $X(x) \leq x$ for all $x \in [0,1)$. We start from the premise that equation (A.10) in the proof of Lemma 3 is satisfied for $s_A^i = x < 1$. Suppose now that $X(x) > x$, which implies that the first term on the left-hand side of (A.10) is negative. Now consider the second term, noting that:

$$\begin{aligned}
&\int_{\Omega(x)} s_A f(s_A, s_B) d(s_A, s_B) - \int_{\Omega(x)} s_B f(s_A, s_B) d(s_A, s_B) \\
&= \int_{X(x)}^x \left(\int_x^1 s_A h_A(s_A) ds_A \right) h_B(s_B) ds_B + \int_x^1 \left(\int_x^1 s_A h_A(s_A) ds_A \right) h_B(s_B) ds_B \\
&- \int_x^1 \left(\int_x^1 s_B h_B(s_B) ds_B \right) h_A(s_A) ds_A - \int_x^1 \left(\int_{X(x)}^x s_B h_B(s_B) ds_B \right) h_A(s_A) ds_A
\end{aligned}$$

This term can be expressed equivalently as:

$$\begin{aligned}
&(1 - H_A(x)) (H_B(x) - H_B(X(x))) (\mathbb{E}_A[s_A | s_A \geq x] - \mathbb{E}_B[s_B | X(x) \leq s_B \leq x]) \\
&+ (1 - H_A(x)) (1 - H_B(x)) (\mathbb{E}_A[s_A | s_A \geq x] - \mathbb{E}_B[s_B | s_B \geq x]) \qquad \text{(A.11)}
\end{aligned}$$

[34] It is straightforward to see that this expression is negative: With $s_A \in (s, X(s))$ and $s_B \in (X(s), 1)$, it is obvious that $s_A - s_B < 0$ for every (s_A, s_B).

As H_B f.o.s.d. H_A, it follows that the second line of (A.11) is non-positive. In the first line of (A.11), the term $\mathbb{E}_A[s_A\,|s_A \geq x] - \mathbb{E}_B[s_B\,|X(x) \leq s_B \leq x]$ is positive given the assumption that $X(x) > x$, and therefore the first line of (A.11) is negative. This, however, implies that *both* terms on the left-hand side of equation (A.10) are negative, which establishes a contradiction to our premise that equation (A.10) holds. An analogue argument establishes that if H_A f.o.s.d. H_B, then $X(x) \geq x$ for all $x \in [0,1)$.

Step 2. We now show that $X(1) = 1$. To see this, re-write equation (A.10) using the fact that in independent asymmetric settings: $\omega(x) = (1-H_A(x))(1-H_B(X(x))$. Then substitute in the expression given in (A.11) above, and divide both sides of the resulting equation by $(1-H_A(x))^{n-1}$. Easing notation by writing X instead of $X(x)$, we obtain:

$$
\begin{aligned}
&(x-X)\,(1-H_B(X))^{n-1} \\
&+(n-1)\,(1-H_B(X))^{n-2}\,(1-H_B(x))\,(\mathbb{E}_A[s_A\,|s_A \geq x] - \mathbb{E}_B[s_B\,|s_B \geq x]) \\
&+(n-1)\,(1-H_B(X))^{n-2}\,(H_B(x)-H_B(X))\,(\mathbb{E}_A[s_A\,|s_A \geq x] - \mathbb{E}_B[s_B\,|X \leq s_B \leq x]) = 0
\end{aligned}
$$

When evaluated at $x = 1$, this expression reduces to:

$$(1-X)\,(1-H_B(X))^{n-1} + (n-1)\,(1-H_B(X))^{n-1}\,(1-\mathbb{E}_B[s_B\,|s_B \geq X\,]) = 0$$

It is obvious that this equation holds iff $X = 1$, which establishes that $X(1) = 1$.

Step 3. We show that $X'(x) > 0$. To see this, note that equation (A.10) can also be expressed as follows (again writing X instead of $X(x)$ to ease notation):

$$x - X + (n-1)\int_x^1 s_A \frac{h_A(s_A)}{1-H_A(s_A)} ds_A - (n-1)\int_X^1 s_B \frac{h_B(s_B)}{1-H_B(s_B)} ds_B = 0$$

Using Leibniz's rule to differentiate both sides of this equation w.r.t x, we obtain:

$$
\begin{aligned}
&1 + \frac{(n-1)h_A(s_A)}{1-H_A(s_A)}\,(\mathbb{E}_A[s_A\,|s_A \geq x] - x) \\
&\quad - X'(x)\left(1 + \frac{(n-1)h_B(s_B)}{1-H_B(s_B)}\,(\mathbb{E}_B[s_B\,|s_B \geq X\,] - X)\right) = 0
\end{aligned}
$$

It is straightforward to solve for $X'(x)$, which shows that $X'(x) > 0\;\forall x \in [0,1]$. □

8.6 Proof of Propositions 1 and 2.

The proof proceeds in three Steps. In Steps 1 and 2, we derive the equilibrium pricing functions in Prop. 2 from the first-order condition (f.o.c.) for a maximum of the expected profit of an arbitrary firm i. In Step 3, we then establish formally that the pricing functions obtained from the f.o.c. (and appropriate boundary conditions) together constitute a symmetric equilibrium of the MPM.

Step 1. Suppose all n firms use the equilibrium specification choice rule δ. Consider firm i with type s^i s.t. $s^i_A = x \geq X^{-1}(0)$ and $s^i_B > X(x)$. I.e. firm i offers the good in specification A. Now consider firm i's problem of which price to quote for specification A, when all *other* firms use the equilibrium pricing functions p_A and p_B in (3) and (4), resp. Suppose that firm i, instead of submitting the price $p_A(x) = \bar{p}(x)$, quotes some other price $\hat{p} \geq \bar{p}(X^{-1}(0))$ for specification A. Note that it does not pay for firm i to quote a price $\hat{p} > \bar{p}(1)$, because this means that firm i will never be chosen, resulting in a profit of zero. The same outcome can be achieved by setting $\hat{p} = \bar{p}(1)$. Therefore, we only need to consider prices $\hat{p} \in [\bar{p}(X^{-1}(0)), \bar{p}(1)]$, which is equivalent to choosing a signal-value $\hat{x} \in [X^{-1}(0)), 1]$ (where $\hat{x}$ need not be equal to firm i's A-signal x) and quoting the corresponding candidate equilibrium price $\bar{p}(\hat{x})$ for specification A. The expected profit of firm i when its A-signal is x and it quotes the price $\bar{p}(\hat{x})$ for specification A is:

$$\bar{\Pi}_i(\hat{x};x) \equiv \omega(\hat{x})^{n-1} \left(\bar{p}(\hat{x}) - \frac{x}{n} - (n-1) \int_{X(\hat{x})}^{1} \int_{\hat{x}}^{1} \frac{s_A}{n} \frac{f(s_A, s_B)}{\omega(\hat{x})} ds_A ds_B \right) \quad \text{(A.12)}$$

where $\omega(\hat{x}) = \Pr\{(s_A, s_B) \in \Omega(\hat{x})\}$, and $\Omega(\hat{x}) = \{(s_A, s_B) \in S : s_A > \hat{x}, s_B > X(\hat{x})\}$. Firm i solves $\max_{\hat{x}} \bar{\Pi}_i(\hat{x};x)$, which yields the following f.o.c.:

$$\begin{aligned} 0 = &\ \bar{p}'(\hat{x})\omega(\hat{x})^{n-1} + (n-1)\bar{p}(\hat{x})\left(\omega(\hat{x})\right)^{n-2}\omega'(\hat{x}) \\ &+ \frac{n-1}{n}\omega(\hat{x})^{n-2}\left(x + \hat{x} + (n-2)\mathbb{E}[s_A \,|(s_A, s_B) \in \Omega(\hat{x})]\right)\left(1 - F_{B|A}\left(X(\hat{x})\,|\hat{x}\right)\right) f_A(\hat{x}) + \\ &+ \frac{n-1}{n}\omega(\hat{x})^{n-2}\left(x + \mathbb{E}[s_A \,|s_A > \hat{x}, s_B = X(\hat{x})]\right) X'(\hat{x})\left(1 - F_{A|B}\left(\hat{x}|X(\hat{x})\right)\right) f_B(X(\hat{x})) \\ &+ \frac{n-1}{n}\omega(\hat{x})^{n-2}(n-2)\mathbb{E}[s_A \,|(s_A, s_B) \in \Omega(\hat{x})] X'(\hat{x})\left(1 - F_{A|B}\left(\hat{x}|X(\hat{x})\right)\right) f_B(X(\hat{x})) \end{aligned} \quad \text{(A.13)}$$

In a symmetric equilibrium, the optimal value of $\hat{x} = x$, so setting $\hat{x} = x$ in the f.o.c., we obtain the differential equation:

$$
\begin{aligned}
&-\frac{d}{ds}\bar{p}(x)\omega(x)^{n-1}\\
&=\frac{n-1}{n}\omega(x)^{n-2}\left(2x+(n-2)\mathbb{E}[s_A\,|(s_A,s_B)\in\Omega(x)]\right)\left(1-F_{B|A}\left(X(x)\,|x\right)\right)f_A(x)+\\
&+\frac{n-1}{n}\omega(x)^{n-2}\left(x+\mathbb{E}[s_A\,|s_A>x,s_B=X(x)]\right)X'(x)\left(1-F_{A|B}\left(x\,|X(x)\right)\right)f_B(X(x))\\
&+\frac{n-1}{n}\omega(x)^{n-2}(n-2)\mathbb{E}[s_A\,|(s_A,s_B)\in\Omega(x)]X'(x)\left(1-F_{A|B}\left(x\,|X(x)\right)\right)f_B(X(x))
\end{aligned}
\tag{A.14}
$$

We now derive a boundary condition associated with this differential equation. Suppose firm i's type s^i is s.t. $s^i_A=1$ and $s^i_B=X(1)$. Conditional on winning (which occurs in the event that $s^j_A=s^j_B=1$ for all $j\neq i$), the production cost of each specification is 1. Suppose first that the equilibrium pricing function $\bar{p}$ is s.t. $\bar{p}(1)>1$. In the event that firm i is awarded the contract, it has positive profit. However, the probability that firm i is chosen with a price of $\bar{p}(1)$ is zero, and so firm i's expected profit is zero. Now consider the following deviation: firm i offers specification A at a price $\hat{p}$, with $1<\hat{p}<\bar{p}(1)$, which corresponds to the equilibrium price submitted by a type with A-signal $\bar{p}^{-1}(\hat{p}))<1$. Therefore, firm i's expected profit from quoting the price $\hat{p}$ is:

$$\omega(\bar{p}^{-1}(\hat{p}))^{n-1}\left(\hat{p}-\frac{1}{n}-\frac{(n-1)}{n}\mathbb{E}[s_A\left|(s_A,s_B)\in\Omega(\bar{p}^{-1}(\hat{p}))\right]\right)$$

As $\hat{p}>1>((1/n)+((n-1)/n)\mathbb{E}[s_A\left|(s_A,s_B)\in\Omega(\bar{p}^{-1}(\hat{p}))\right])$, this constitutes a profitable deviation, and therefore establishes a contradiction to the premise that $\bar{p}$ is the equilibrium pricing function for all types s.t. $s^i_A>X^{-1}(0)$ and $s^i_B>X(s^i_A)$.

Next, suppose that the equilibrium pricing function $\bar{p}$ is such that $\bar{p}(1)<1$. Consider firm i and suppose $s^i_A=1-\varepsilon$, with $\varepsilon>0$ but small. In the event that firm i wins the contract (which occurs with probability $(\omega(1-\varepsilon))^{n-1}$), its profit is negative. Therefore, it is profitable for firm i to deviate to $\hat{p}=1$. In the limit as $\varepsilon\downarrow 0$, firm i's expected profit from quoting the price $\hat{p}=1$ is zero. We can therefore conclude that the boundary condition associated with the above differential equation is $\bar{p}(1)=1$. Given this boundary condition, the unique solution to the differential equation (A.14) is obtained by integrating both sides of (A.14) from x to 1. However, as $\omega(1)=0$, the function $\bar{p}$ is not defined at $x=1$. To verify that $\bar{p}$ nevertheless satisfies the boundary condition $p(1)=1$, we can show that $\lim_{x\uparrow 1}\bar{p}(x)=1$.[35]

[35]To show this, repeated use of L'Hôpital's rule is required. For the sake of brevity, we omit the details.

Step 2. Derivation of pricing function $\breve{p}$. As before, suppose all n firms use the equilibrium specification choice rule δ. Consider firm i with type s^i s.t. $s^i_A = x \leq X^{-1}(0)$. I.e. firm i offers the good in specification A, irrespective of the value of its B-signal. We again consider firm i's problem of which price to quote for specification A, given that *other* firms use the equilibrium pricing functions p_A and p_B in (3) and (4), resp. Suppose that firm i, instead of submitting the price $p_A(x) = \breve{p}(x)$, quotes some other price $\hat{p} \leq \overline{p}(X^{-1}(0))$ for specification A. Note that it does not pay for firm i to quote a price $\hat{p} < \breve{p}(0)$, because this means that firm i will be chosen with probability 1, but can do better by increasing its price slightly so that it is still chosen but has a higher profit. Therefore, we only need to consider prices $\hat{p} \in [\breve{p}(0), \breve{p}(X^{-1}(0))]$, which is equivalent to choosing a signal-value $\hat{x} \in [0, X^{-1}(0))]$ and quoting the corresponding candidate equilibrium price $\breve{p}(\hat{x})$ for specification A. This yields expected profit:

$$\breve{\Pi}_i(\hat{x};x) \equiv (1-F_A(\hat{x}))^{n-1}\left(\breve{p}(\hat{x}) - \frac{x}{n} - (n-1)\int_0^1\int_{\hat{x}}^1 \frac{s_A}{n}\frac{f(s_A,s_B)}{(1-F_A(\hat{x}))}ds_A ds_B\right) \tag{A.15}$$

where $F_A(\hat{x}) = \int_{\hat{x}}^1\int_0^1 f(s_A,s_B)ds_B ds_A$. Firm i solves $\max_{\hat{x}}\breve{\Pi}_i(\hat{x};x)$, which yields the following f.o.c.:

$$\begin{aligned} 0 &= \breve{p}'(\hat{x})\,(1-F_A(\hat{x}))^{n-1} - (n-1)\breve{p}(\hat{x})\,(1-F_A(\hat{x}))^{n-2} f_A(\hat{x}) \\ &+ \frac{n-1}{n}(1-F_A(\hat{x}))^{n-2}(x+\hat{x}+(n-2)\mathbb{E}[s_A\,|s_A \geq \hat{x}])\,f_A(\hat{x}) \end{aligned} \tag{A.16}$$

In a symmetric equilibrium, the optimal value of $\hat{x} = x$, so setting $\hat{x} = x$ in the f.o.c., we obtain the differential equation:

$$\begin{aligned} &-\frac{d}{ds}\breve{p}(x)\,(1-F_A(x))^{n-1} \\ &= \frac{n-1}{n}(1-F_A(x))^{n-2}(2x+(n-2)\mathbb{E}[s_A\,|s_A \geq x])\,f_A(x) \end{aligned} \tag{A.17}$$

To obtain a boundary condition for the differential equation (A.17), recall from Lemma 2 that the equilibrium pricing function p_A is continuous. This implies that $\breve{p}(X^{-1}(0)) = \overline{p}(X^{-1}(0))$. Given this boundary condition, the unique solution to the differential equation (A.17), given by the function $\breve{p}(x)$ in Prop. 2, is obtained by integrating both sides of (A.17) from x to $X^{-1}(0)$.

Step 3. We now prove sufficiency by verifying that the solutions to the differential equations in (A.14) and (A.17), together with the associated boundary conditions

$\bar{p}(1) = 1$ and $\breve{p}(X^{-1}(0)) = \bar{p}(X^{-1}(0))$, constitute an equilibrium. I.e. we need to show that if the $n-1$ competitors of firm i use the pricing functions p_A and p_B in (3) and (4), then it is optimal for firm i to do so. To show this, we derive in Steps 3.1 and 3.2 below properties of the expected profit functions $\bar{\Pi}_i(\hat{x};x)$ and $\breve{\Pi}_i(\hat{x};x)$, given by (A.12) and (A.15), resp. These properties are then used in Step 3.3 to conclude that the functions $\breve{p}$ and $\bar{p}$ constitute an equilibrium.

Step 3.1. We begin by considering a firm i whose chosen specification is A, and whose A-signal is x. If firm i submits the price $\bar{p}(X^{-1}(0))$, its expected profit is $\bar{\Pi}_i(X^{-1}(0);x)$. If, instead, firm i submits the price $\breve{p}(X^{-1}(0))$, its expected profit is $\breve{\Pi}_i(X^{-1}(0);x)$. While we omit the details here, it is not difficult to verify that:

$$\bar{\Pi}_i(X^{-1}(0);x) - \breve{\Pi}_i(X^{-1}(0);x) = \left[\bar{p}(X^{-1}(0)) - \breve{p}(X^{-1}(0))\right]\left[1 - F_A(X^{-1}(0))\right] = 0$$

This shows that for any given x, the two profit functions $\bar{\Pi}_i(\hat{x};x)$ and $\breve{\Pi}_i(\hat{x};x)$ intersect at $\hat{x} = X^{-1}(0)$.

Step 3.2. Next, we study the behavior of the functions $\bar{\Pi}_i(\hat{x};x)$ and $\breve{\Pi}_i(\hat{x};x)$, resp., as we vary $\hat{x}$, while treating x as a fixed "location parameter". To this end, we compute the first and second derivatives of the functions $\bar{\Pi}_i(\hat{x};x)$ and $\breve{\Pi}_i(\hat{x};x)$ w.r.t. $\hat{x}$. Observe first that $\partial\breve{\Pi}_i(\hat{x};x)/\partial\hat{x}$ is given by the right-hand side of the f.o.c. in (A.16). Now add and subtract $\hat{x}(1-F_A(\hat{x}))^{n-2}f_A(\hat{x})(n-1)/n$ from the expression for $\partial\breve{\Pi}_i(\hat{x};x)/\partial\hat{x}$. This yields:

$$\begin{aligned}\frac{\partial\breve{\Pi}_i(\hat{x};x)}{\partial\hat{x}} &= \frac{\partial\breve{\Pi}_i(\hat{x};\hat{x})}{\partial\hat{x}} - (\hat{x}-x)\frac{n-1}{n}(1-F_A(\hat{x}))^{n-2}f_A(\hat{x}) \\ &= -(\hat{x}-x)\frac{n-1}{n}(1-F_A(\hat{x}))^{n-2}f_A(\hat{x})\end{aligned}$$

where the second line follows from the fact that $\partial\breve{\Pi}_i(\hat{x};\hat{x})/\partial\hat{x} = \partial\breve{\Pi}_i(\hat{x};x)/\partial\hat{x}\big|_{\hat{x}=x} = 0$. This shows that $\partial\breve{\Pi}_i(\hat{x};x)/\partial\hat{x} > 0$ if $\hat{x} < x$, that $\partial\breve{\Pi}_i(\hat{x};x)/\partial\hat{x} < 0$ if $\hat{x} > x$, and that $\partial\breve{\Pi}_i(\hat{x};x)/\partial\hat{x} = 0$ if $\hat{x} = x$. It is straightforward to verify that:

$$\left.\frac{\partial^2\breve{\Pi}_i(\hat{x};x)}{\partial\hat{x}^2}\right|_{\hat{x}=x} = -\frac{n-1}{n}(1-F_A(\hat{x}))^{n-2}f_A(\hat{x}) < 0$$

which establishes that $\breve{\Pi}_i(\hat{x};x)$ has an interior global maximum at $\hat{x} = x$ if $x < X^{-1}(0)$, and that $\breve{\Pi}_i(\hat{x};x)$ reaches its global maximum at the upper boundary of its domain (i.e. at $\hat{x} = X^{-1}(0)$) if $x > X^{-1}(0)$.

Now observe that $\partial\bar{\Pi}_i(\hat{x};x)/\partial\hat{x}$ is given by the right-hand side of the f.o.c. in (A.13). Now add and subtract $\hat{x}\omega(\hat{x})^{n-2}\left(1-F_{B|A}\left(X(\hat{x})\,|\hat{x}\right)\right)f_A(\hat{x})(n-1)/n$ from the expression for $\partial\bar{\Pi}_i(\hat{x};x)/\partial\hat{x}$. This yields:

$$\begin{aligned}\frac{\partial\bar{\Pi}_i(\hat{x};x)}{\partial\hat{x}} &= \frac{\partial\bar{\Pi}_i(\hat{x};\hat{x})}{\partial\hat{x}} - (\hat{x}-x)\frac{(n-1)}{n}\omega(\hat{x})^{n-2}\left(1-F_{B|A}\left(X(\hat{x})\,|\hat{x}\right)\right)f_A(\hat{x}) \\ &= -(\hat{x}-x)\frac{(n-1)}{n}\omega(\hat{x})^{n-2}\left(1-F_{B|A}\left(X(\hat{x})\,|\hat{x}\right)\right)f_A(\hat{x})\end{aligned}$$

This establishes that $\partial\bar{\Pi}_i(\hat{x};x)/\partial\hat{x} > 0$ if $\hat{x} < x$, that $\partial\bar{\Pi}_i(\hat{x};x)/\partial\hat{x} < 0$ if $\hat{x} > x$, and that $\partial\bar{\Pi}_i(\hat{x};x)/\partial\hat{x} = 0$ if $\hat{x} = x$. As before, it is easy to verify that:

$$\left.\frac{\partial^2\bar{\Pi}_i(\hat{x};\hat{x})}{\partial\hat{x}^2}\right|_{\hat{x}=x} = -\frac{(n-1)}{n}\omega(\hat{x})^{n-2}\left(1-F_{B|A}\left(X(\hat{x})\,|\hat{x}\right)\right)f_A(\hat{x}) < 0$$

We can therefore conclude that $\bar{\Pi}_i(\hat{x};x)$ has an interior global maximum at $\hat{x} = x$ if $x > X^{-1}(0)$, and that $\bar{\Pi}_i(\hat{x};x)$ reaches its global maximum at the lower boundary of its domain (i.e. at $\hat{x} = X^{-1}(0)$) if $x < X^{-1}(0)$.

Step 3.3. To conclude the sufficiency argument, consider first a firm i with type s^i s.t. $s^i_A = \bar{x} > X^{-1}(0)$ and $s^i_B > X(\bar{x})$. If the functions $\breve{p}$ and $\bar{p}$ constitute an equilibrium, then firm i must prefer the price $\bar{p}(\bar{x})$ to any price $\bar{p}(\hat{x})$, where $\hat{x}$ is any other A-signal in $[X^{-1}(0),1]$. Likewise, firm i must prefer $\bar{p}(\bar{x})$ to any price $\breve{p}(\tilde{x})$, where $\tilde{x} \in [0,X^{-1}(0)]$. Our results in Steps 3.1 and 3.2 regarding the behavior of the functions $\bar{\Pi}_i(\hat{x};x)$ and $\breve{\Pi}_i(\hat{x};x)$ imply that for any $\bar{x} \in (X^{-1}(0),1]$: $\bar{\Pi}_i(\bar{x};\bar{x}) > \bar{\Pi}_i(\hat{x};\bar{x})$ for all $\hat{x} \in [X^{-1}(0),1]$ s.t. $\hat{x} \neq \bar{x}$; and $\bar{\Pi}_i(\bar{x};\bar{x}) > \bar{\Pi}_i(X^{-1}(0);\bar{x}) = \breve{\Pi}_i(X^{-1}(0);\bar{x}) > \breve{\Pi}_i(\tilde{x};\bar{x})$ for all $\tilde{x} \in [0,X^{-1}(0)]$. I.e. it is optimal for firm i to submit the price $\bar{p}(\bar{x})$. Now consider a firm i with type s^i s.t. $s^i_A = \breve{x} < X^{-1}(0)$. If the functions $\breve{p}$ and $\bar{p}$ constitute an equilibrium, firm i must prefer the price $\breve{p}(\breve{x})$ to any price $\breve{p}(\tilde{x})$, where $\tilde{x}$ is any other A-signal in $\in [0,X^{-1}(0)]$. Likewise, firm i must prefer $\breve{p}(\breve{x})$ to any price $\bar{p}(\hat{x})$, where $\hat{x} \in [X^{-1}(0),1]$. Our results regarding the behavior of the functions $\bar{\Pi}_i(\hat{x};x)$ and $\breve{\Pi}_i(\hat{x};x)$ imply that for any $\breve{x} \in [0,X^{-1}(0))$: $\breve{\Pi}_i(\breve{x};\breve{x}) > \breve{\Pi}_i(\tilde{x};\breve{x})$ for all $\tilde{x}$ in $[0,X^{-1}(0)]$ s.t. $\tilde{x} \neq \breve{x}$; and $\breve{\Pi}_i(\breve{x};\breve{x}) > \breve{\Pi}_i(X^{-1}(0);\breve{x}) = \bar{\Pi}_i(X^{-1}(0);\breve{x}) > \bar{\Pi}_i(\hat{x};\breve{x})$ for all $\hat{x} \in [X^{-1}(0),1]$. I.e. it is optimal for firm i to submit the price $\breve{p}(\breve{x})$. $\square$

8.7 Proof of Corollary 1

We can compute the path integral along the piecewise smooth path Γ^1 (see left-hand diagram in Fig. 3) as follows:

$$\int_{\Gamma^1(s^i,1)} q^i \cdot d\alpha^1 = \int_0^1 q_A^i(y+(1-y)s_A^i, s_B^i)(1-s_A^i)dy + \int_1^2 q_B^i(1, y-1+(2-y)s_B^i)(1-s_B^i)dy. \quad \text{(A.18)}$$

This expression can be simplified by appropriate integration by substitution. Consider the first integral term on the right-hand side of (A.18) and let $x = y+(1-y)s_A^i$. I.e. $dx/dy = 1-s_A^i$. For $y=0$ we have $x = {s^i}_A$, and for $y=1$ we have $x=1$. We can therefore rewrite the first integral term in (A.18) as follows:

$$\int_0^1 q_A^i(y+(1-y)s_A^i, s_B^i)(1-s_A^i)dy = \int_{s_A^i}^1 q_A^i(x, {s^i}_B)dx$$

Now consider the second integral term on the right-hand side of (A.18) and define $x = y-1+(2-y)s_B^i$. I.e. $dx/dy = 1-s_B^i$. For $y=1$ we have $x = s_B^i$, and for $y=2$ we have $x=1$. We can therefore rewrite the second integral term in (A.18) as:

$$\int_1^2 q_B^i(1, y-1+(2-y)s_B^i)(1-s_B^i)dy = \int_{s_B^i}^1 q_B^i(1,x)dx$$

The sum of these two integrals yields the expression for the path integral along Γ^1 featured in expression (8) in Corollary 1. Steps similar to those used above for computing the path integral along Γ^1 show that the path integral along Γ^2 is given by the expression in (9) in Corollary 1. $\square$

8.8 Proof of Proposition 4.

The proof requires us to compute the expected profit $\mathbb{E}[\mu_i(s^i)]$ of any firm i, using the expression for profit $\mu_i(s^i)$ implicit in (7) in Prop. 4:

$$\mu_i(s^i) - \mu_i(\mathbf{1}) = \int_{\Gamma(s^i,\mathbf{1})} q^i \cdot d\alpha \quad \text{(A.19)}$$

Due to path-independence, the path integral in (A.19) can be evaluated along any piecewise smooth path Γ. Here, we compute the expectation in (A.19) by using the equivalent expressions for the path integral in equations (8) and (9) of Corollary 1:

$$\int_{\Gamma^1(s^i,\mathbf{1})} q^i \cdot d\alpha = \int_{s_A^i}^1 q_A^i(x, s_B^i)dx + \int_{s_B^i}^1 q_B^i(1,x)dx \quad \text{(A.20)}$$

and

$$\int_{\Gamma^2(s^i,\mathbf{1})} q^i \cdot d\alpha = \int_{s_B^i}^1 q_B^i(s_A^i, x)dx + \int_{s_A^i}^1 q_A^i(x,1)dx \tag{A.21}$$

We can write:[36]

$$\int_S \left(\int_{\Gamma(s^i,\mathbf{1})} q^i \cdot d\alpha\right) f(s^i)ds^i = \int_0^1 \int_0^{s_A^i} \left(\int_{\Gamma^1(s^i,\mathbf{1})} q^i \cdot d\alpha\right) f(s^i)ds_B^i ds_A^i + \int_0^1 \int_{s_A^i}^1 \left(\int_{\Gamma^2(s^i,\mathbf{1})} q^i \cdot d\alpha\right) f(s^i)ds_B^i ds_A^i \tag{A.22}$$

Label the first term on the right-hand side of equation (A.22) as $\mathscr{T}_1$, and label the second term (in the second line) as $\mathscr{T}_2$. Replacing the path integral with the expression in (A.20), we can split $\mathscr{T}_1$ into two additive terms:

$$\mathscr{T}_1 = \int_0^1 \int_0^{s_A^i} \left(\int_{s_A^i}^1 q_A^i(x, s_B^i)dx\right) f(s^i)ds_B^i ds_A^i + \int_0^1 \int_0^{s_A^i} \left(\int_{s_B^i}^1 q_B^i(1,x)dx\right) f(s^i)ds_B^i ds_A^i \tag{A.23}$$

Similarly, by replacing the path integral in $\mathscr{T}_2$ with the expression in (A.21), we can split $\mathscr{T}_2$ into two additive terms.

$$\mathscr{T}_2 = \int_0^1 \int_{s_A^i}^1 \left(\int_{s_B^i}^1 q_B^i(s_A^i, x)dx\right) f(s^i)ds_B^i ds_A^i + \int_0^1 \int_{s_A^i}^1 \left(\int_{s_A^i}^1 q_A^i(x,1)dx\right) f(s^i)ds_B^i ds_A^i \tag{A.24}$$

Label the first term in (A.23) as $\mathscr{T}_{11}$, and label the second term in (A.24) as $\mathscr{T}_{22}$. Note that $\mathscr{T}_{11}$ and $\mathscr{T}_{22}$ can be expressed equivalently by changing the order of integration:

$$\begin{aligned}
\mathscr{T}_{11} &= \int_0^1 \int_{s_B^i}^1 \left(\int_{s_A^i}^1 q_A^i(x, s_B^i)dx\right) f(s^i)ds_A^i ds_B^i \\
\mathscr{T}_{22} &= \int_0^1 \int_0^{s_B^i} \left(\int_{s_A^i}^1 q_A^i(x,1)dx\right) f(s^i)ds_A^i ds_B^i
\end{aligned}$$

It is then easy to see that $\mathscr{T}_2$ can be obtained from $\mathscr{T}_1$ by interchanging the specification-subscripts A and B, and vice versa.[37] In the remainder of the proof, we therefore focus on $\mathscr{T}_1$ as the representative expression. We start by simplifying $\mathscr{T}_{11}$ using integration by parts on the inner double integral:

[36] See Theorem 10-25 on p. 267 in Apostol (1957), which shows that the multiple integral is additive.

[37] Note that when the specification-subscripts A and B are interchanged, the expression $q_A^i(x, s_B^i)$, for instance, becomes $q_B^i(s_A^i, x)$, and vice versa.

$$\begin{aligned}
&\int_{s_B^i}^1 \left(\int_{s_A^i}^1 q_A^i(x,s_B^i)dx\right) f(s_A^i,s_B^i)ds_A^i \\
= \quad & -\left(\int_{s_B^i}^1 q_A^i(x,s_B^i)dx\right)\left(\int_0^{s_B^i} f(y,s_B^i)dy\right) + \int_{s_B^i}^1 q_A^i(s^i)\left(\int_0^{s_A^i} f(y,s_B^i)dy\right)ds_A^i
\end{aligned}$$

Relabeling the integration index x as s_A^i simplifies this expression further:

$$\begin{aligned}
&-\int_{s_B^i}^1 q_A^i(s^i)\left(\int_0^{s_B^i} f(y,s_B^i)dy\right)ds_A^i + \int_{s_B^i}^1 q_A^i(s^i)\left(\int_0^{s_A^i} f(y,s_B^i)dy\right)ds_A^i \\
&= \int_{s_B^i}^1 q_A^i(s^i)\frac{F(s_A^i\,|s_B^i) - F(s_B^i\,|s_B^i)}{f(s_A^i\,|s_B^i)} f(s^i)ds_A^i
\end{aligned}$$

We can therefore write:

$$\mathscr{T}_{11} = \int_0^1 \int_{s_B^i}^1 q_A^i(s^i)\frac{F(s_A^i\,|s_B^i) - F(s_B^i\,|s_B^i)}{f(s_A^i\,|s_B^i)} f(s^i)ds_A^i ds_B^i \tag{A.25}$$

Now turn to term $\mathscr{T}_{12}$, which can be simplified using integration by parts on the inner double integral:

$$\begin{aligned}
&\int_0^{s_A^i} \left(\int_{s_B^i}^1 q_B^i(1,x)dx\right) g(s_B^i)ds_B^i \\
= \quad & \left(\int_{s_A^i}^1 q_B^i(1,x)dx\right)\left(\int_0^{s_A^i} f(s_A^i,y)dy\right) + \int_0^{s_A^i} q_B^i(1,s_B^i)\left(\int_0^{s_B^i} f(s_A^i,y)dy\right)ds_B^i
\end{aligned}$$

Relabeling the integration index x as s_B^i simplifies this expression further:

$$\begin{aligned}
&\int_{s_A^i}^1 q_B^i(1,s_B^i)\left(\int_0^{s_A^i} f(s_A^i,y)dy\right)ds_B^i + \int_0^{s_A^i} q_B^i(1,s_B^i)\left(\int_0^{s_B^i} f(s_A^i,y)dy\right)ds_B^i \\
= \quad & \int_0^1 q_B^i(1,s_B^i)\frac{F(s_{m_i}^i\,|s_A^i)}{f(s_B^i\,|s_A^i)} f(s^i)ds_B^i
\end{aligned}$$

We can therefore write:

$$\mathscr{T}_{12} = \int_0^1 \int_0^1 q_B^i(1,s_B^i)\frac{F(s_{m_i}^i\,|s_A^i)}{f(s_B^i\,|s_A^i)} f(s^i)ds_B^i ds_A^i \tag{A.26}$$

The full expression for $\mathscr{T}_1$ is the sum of $\mathscr{T}_{11}$ in (A.25) and $\mathscr{T}_{12}$ (A.26). The expressions $\mathscr{T}_{21}$ and $\mathscr{T}_{22}$ that constitute $\mathscr{T}_2$ can now be obtained by interchanging the specification-subscripts A and B in (A.25) and (A.26), resp. The full expression for $\mathscr{T}_2$ is then the sum of $\mathscr{T}_{21}$ and $\mathscr{T}_{22}$, and the expectation of the path integral in (A.19) is the sum of $\mathscr{T}_1$ and $\mathscr{T}_2$:

$$\int_S \left[q_A^i(s_A^i,1)\frac{F(s_{m_i}^i\,|s_B^i)}{f(s_A^i\,|s_B^i)} + q_B^i(1,s_B^i)\frac{F(s_{m_i}^i\,|s_A^i)}{f(s_B^i\,|s_A^i)}\right] f(s^i)ds^i$$
$$+\int_0^1\int_{s_B^i}^1 q_A^i(s^i)\frac{F(s_A^i\,|s_B^i)-F(s_B^i\,|s_B^i)}{f(s_A^i\,|s_B^i)}f(s^i)ds_A^i ds_B^i$$
$$+\int_0^1\int_{s_A^i}^1 q_B^i(s^i)\frac{F(s_B^i\,|s_A^i)-F(s_A^i\,|s_A^i)}{f(s_B^i\,|s_A^i)}f(s^i)ds_B^i ds_A^i$$

In the integrand in the second line, add and subtract $q_B^i(s^i)\left(F(s_B^i\,|s_A^i)f(s^i)/f(s_B^i\,|s_A^i)\right)$, and in the integrand in the third line, add and subtract $q_A^i(s^i)\left(F(s_A^i\,|s_B^i)f(s^i)/f(s_A^i\,|s_B^i)\right)$. Then, using the notation $s_{m_i}^i$ for firm i's minimum signal, we can collect terms and write $\mathbb{E}[\mu_i(s^i)]-\mu_i(\mathbf{1})$ as:

$$\int_S \sum_{\substack{k,l\in K\\ l\neq k}} \left(q_k^i(s^i)\frac{F(s_k^i\,|s_l^i)}{f(s_k^i\,|s_l^i)} + \left[q_k^i(s_k^i,1)-q^i{}_k(s^i)\right]\frac{F(s_{m_i}^i\,|s_l^i)}{f(s_k^i\,|s_l^i)}\right) f(s^i)ds^i$$

We can now write $\mathbb{E}[\mu_i(s^i)]-\mu_i(\mathbf{1})$ in terms of Q_k^i:

$$\int_{S^n}\sum_{\substack{k,l\in K\\ l\neq k}} Q_k^i(\mathbf{s})\frac{F(s_k^i\,|s_l^i)}{f(s_k^i\,|s_l^i)}g(\mathbf{s})d\mathbf{s} + \int_{S^n}\sum_{\substack{k,l\in K\\ l\neq k}} Q_k^i((s_k^i,1),\mathbf{s}^{-i})\frac{F(s_{m_i}^i\,|s_l^i)}{f(s_k^i\,|s_l^i)}g(\mathbf{s})d\mathbf{s}$$
$$-\int_{S^n}\sum_{\substack{k,l\in K\\ l\neq k}} Q_k^i(\mathbf{s})\frac{F(s_{m_i}^i\,|s_l^i)}{f(s_k^i\,|s_l^i)}g(\mathbf{s})d\mathbf{s} \quad \text{(A.27)}$$

Adding firm i's ex ante expected production cost $\mathbb{E}\left[\sum_{k\in K} Q_k^i(\mathbf{s})C_k(\mathbf{s}_k)\right]$ to $\mathbb{E}[\mu_i(s^i)]-\mu_i(\mathbf{1})$ in (A.27), we obtain the following expression for $\mathbb{E}[t_i(s^i)]-(t_i(\mathbf{1})-c_i(\mathbf{1}))$:

$$\int_{S^n}\sum_{\substack{k,l\in K\\ l\neq k}}\left(Q_k^i(\mathbf{s})\left(C_k(\mathbf{s}_k)+\frac{F(s_k^i\,|s_l^i)}{f(s_k^i\,|s_l^i)}\right) + \left[Q_k^i((s_k^i,1),\mathbf{s}^{-i})-Q_k^i(\mathbf{s})\right]\frac{F(s_{m_i}^i\,|s_l^i)}{f(s_k^i\,|s_l^i)}\right)g(\mathbf{s})d\mathbf{s} \quad \text{(A.28)}$$

Summing (A.28) over all firms we finally obtain the buyer's expected expenditure in Prop. 4. □

8.9 Proof of Proposition 5.

We have to establish that $Q^i_{M_i}(\mathbf{s}) = 0$ for all $\mathbf{s}$ in the interior of S^n and all $i \in I$. To see this, consider the sum of the integrands in the second and third lines of the expression for ex ante expenditure in (11):

$$\sum_{i\in I}\left(Q^i_{m_i}(\mathbf{s})C_{m_i}(\mathbf{s}_{m_i}) + Q^i_{M_i}(\mathbf{s})\left(C_{M_i}(\mathbf{s}_{M_i}) + \frac{F(s^i_{M_i}|s^i_{m_i}) - F(s^i_{m_i}|s^i_{m_i})}{f(s^i_{M_i}|s^i_{m_i})}\right)\right) \quad \text{(A.29)}$$

A SCR Q that minimizes (11) must minimize (A.29) for every $\mathbf{s} \in S^n$, subject to $\sum_{i\in I}(Q^i_{m_i}(\mathbf{s}) + Q^i_{M_i}(\mathbf{s})) = 1$. Now consider a type-vector $\mathbf{s}$ in the interior of S^n s.t. $m_i = k\ \forall i \in I$. In this case, the coefficient associated with any $Q^i_k(\mathbf{s})$ in (A.29) is $C_k(\mathbf{s}_k)$, while that associated with any $Q^i_l(\mathbf{s})$ is $C_l(\mathbf{s}_l) + (F(s^i_l|s^i_k) - F(s^i_k|s^i_k)/f(s^i_l|s^i_k)) > C_l(\mathbf{s}_l) > C_k(\mathbf{s}_k)$. It is therefore optimal to set $Q^i_l(\mathbf{s}) = 0\ \forall i$. Next consider a type-vector $\mathbf{s}$ in the interior of S^n s.t. $m_i = l$ and $\exists j \in I, j \neq i$, s.t. $m_j = k$. In this case, the coefficient associated with $Q^j_k(\mathbf{s})$ in (A.29) is $C_k(\mathbf{s}_k)$, while the coefficient associated with $Q^i_k(\mathbf{s})$ is $C_k(\mathbf{s}_k) + (F(s^i_k|s^i_l) - F(s^i_l|s^i_l))/f(s^i_k|s^i_l) > C_k(\mathbf{s}_k)$. It is therefore optimal to set $Q^j_k(\mathbf{s}) = 0$. By the same logic, $Q^j_l(\mathbf{s}) = 0$. □

8.10 Proof of Proposition 6.

We have to establish that any Q that minimizes ex ante expenditure in (11), and which generates functions q^i s.t. (8) and (9) in Corollary 1 are equal, must be s.t. q^i_A and q^i_B are of the form shown in (12) in Prop. 6. By Corollary 1, $\forall i \in I$ and $\forall s^i \in S$: $\int_{s^i_A}^1 q^i_A(x, s^i_B)dx + \int_{s^i_B}^1 q^i_B(1,x)dx = \int_{s^i_B}^1 q^i_B(s^i_A, x)dx + \int_{s^i_A}^1 q^i_A(x,1)dx$. As Q minimizes expenditure in (11) (which, by Prop. 5, implies $Q^i_{M_i}(\mathbf{s}) = 0\ \forall \mathbf{s} \in S^n$), we can write:

(i) $\int_{s^i_B}^{s^i_A}\left[q^i_B(1,x) - q^i_B(s^i_A,x)\right]dx = \int_{s^i_A}^1\left[q^i_A(x,1) - q^i_B(1,x)\right]dx$ if $s^i_A > s^i_B$

(ii) $\int_{s^i_A}^{s^i_B}\left[q^i_A(x,1) - q^i_A(x,s^i_B)\right]dx = \int_{s^i_B}^1\left[q^i_B(1,x) - q^i_A(x,1)\right]dx$ if $s^i_A < s^i_B$.

For any s^i s.t. $s^i_A = s^i_B = s \in [0,1)$, (i) and (ii) hold iff $q^i_A(s,1) = q^i_B(1,s)\ \forall s \in [0,1)$. Thus:

(i) $\int_{s^i_B}^{s^i_A}\left[q^i_B(1,x) - q^i_B(s^i_A,x)\right]dx = 0 \Leftrightarrow q^i_B(s^i_A, s^i_B) = q^i_B(1, s^i_B)\ \forall s^i$ s.t. $s^i_A > s^i_B$

(ii) $\int_{s^i_A}^{s^i_B}\left[q^i_A(x,1) - q^i_A(x,s^i_B)\right]dx = 0 \Leftrightarrow q^i_A(s^i_A, s^i_B) = q^i_A(s^i_A, 1)\ \forall s^i$ s.t. $s^i_A < s^i_B$.

Setting $q^i_B(1,s) = q^i_A(s,1) \equiv \rho_i(s)$, we obtain $q^i_A(s^i)$ and $q^i_B(s^i)$ in Prop. 6. □

8.11 Proof of Proposition 7.

We need to show that any q^i in (12) with decreasing function ρ_i is monotone and conservative. It is straightforward to verify that if ρ_i is decreasing, then q^i in (12) is monotone. We now show in two steps that q^i is conservative. For this purpose, we draw on a result in Jehiel et al. (1999) that provides necessary and sufficient conditions for any piecewise continuous function q^i to be conservative. For convenience, we re-state this result in the notation of our paper:

Proposition 10. *(Jehiel et al., 1999) Assume $q^i : S \to [0,1]^{|K|}$ is piecewise continuous. That is, assume there exists a partition $\{M_1, ..., M_\vartheta\}$ of the unit square S such that q^i restricted to the interior of M_ζ is continuous for each $\zeta = 1, \ldots, \vartheta$. Suppose each M_ζ has a piecewise smooth boundary. Then, q^i is conservative iff (i) q^i restricted to M_ζ is conservative for each $\zeta = 1, \ldots, \vartheta$; and (ii) whenever M_ζ and M_η are two adjacent regions, the jump in $q^i(s^i)$ as s^i crosses from M_ζ to M_η is perpendicular to the common boundary between M_ζ and M_η. That is, if s^i is in the common boundary between M_ζ and M_η, and n is the unitary normal vector of the boundary between M_ζ and M_η at s^i, then the vector $\Delta q^i(s^i) \equiv \lim_{\varepsilon\to 0^+} q^i(s^i - \varepsilon n) - \lim_{\varepsilon\to 0^+} q^i(s^i + \varepsilon n)$ is parallel to n.*

Step 1. We verify that any q^i in (12) with monotonically decreasing function ρ_i satisfies item (ii) of Prop. 10. As ρ_i is decreasing on $[0,1]$, the left-hand limit $\rho_i^-(s) \equiv \lim_{x\to s^-} \rho_i(x)$ and the right-hand limit $\rho_i^+(s) \equiv \lim_{x\to s^+} \rho_i(x)$ are both finite at every $s \in (0,1)$, with $\rho_i^-(s) \geq \rho_i^+(s)$, and ρ_i can only display countably many jump discontinuities, if any.

Step 1 (a). Suppose first that ρ_i is continuous everywhere in $(0,1)$. Partition S into two convex subsets $M_k^0 \equiv \{s^i : s_k^i \leq s_l^i\}$ $(k, l \in K,\, l \neq k)$. Fix a type $s^i = (s,s)$ in the common boundary between M_A^0 and M_B^0, which is the 45°-line in the unit square S. A normal vector of the boundary between M_A^0 and M_B^0 at s^i is $n = (-1,1)$. The jump in $q^i(s^i)$ as s^i crosses from M_B^0 to M_A^0 is:

$$\begin{aligned}\Delta q^i(s,s) &= \lim_{\varepsilon\to 0^+} \begin{pmatrix} q_A^i(s+\varepsilon, s-\varepsilon) \\ q_B^i(s+\varepsilon, s-\varepsilon) \end{pmatrix} - \lim_{\varepsilon\to 0^+} \begin{pmatrix} q_A^i(s-\varepsilon, s+\varepsilon) \\ q_B^i(s-\varepsilon, s+\varepsilon) \end{pmatrix} \\ &= \begin{pmatrix} -\rho_i^-(s) \\ \rho_i^-(s) \end{pmatrix}\end{aligned}$$

It is easy to see that n and $\Delta q^i(s,s)$ are parallel vectors because their cross product is zero:[38] $(-1)\cdot\rho_i^-(s) - 1\cdot(-\rho_i^-(s)) = 0$.[39]

Step 1 (b). We now consider the case where ρ_i is not continuous at every $s \in (0,1)$. Let $\{s_1, s_2, \ldots, s_\vartheta\}$ be the set of discontinuities of ρ_i, where $\vartheta \in \mathbb{N}$ and $0 < s_1 < \ldots < s_\vartheta < 1$.[40] At each point s_η ($\eta = 1, ..., \vartheta$) we have $\rho_i^-(s_\eta) > \rho_i^+(s_\eta)$ and $\rho_i(s_\eta) \in [\rho_i^+(s_\eta), \rho_i^-(s_\eta)]$. Now define the following subsets of S: $\forall k, l \in K$, $l \neq k$, let $M_k^1 \equiv \{s^i : 0 \leq s_k^i \leq s_1, s_l^i \geq s_k^i\}$, $M_k^\zeta \equiv \{s^i : s_{\zeta-1} \leq s_k^i \leq s_\zeta, s_l^i \geq s_k^i\}$ for $\zeta = 2, ..., \vartheta$, and $M_k^{\vartheta+1} \equiv \{s^i : s_\vartheta \leq s_k^i \leq 1, s_l^i \geq s_k^i\}$. If the number of discontinuities of ρ_i is $\vartheta = 1$ we shall partition S into $\{M_k^1, M_k^{\vartheta+1}\}_{k\in K}$. If, instead, $\vartheta \geq 2$ we shall partition S into $\{M_k^1, M_k^\zeta, M_k^{\vartheta+1}\}_{k\in K, \zeta\in\{2,...,\vartheta\}}$. For given $\vartheta \geq 1$, fix $s^i = (s_\eta, s_B^i)$ (with $s_B^i > s_\eta$) in the common boundary between M_A^η and $M_A^{\eta+1}$ ($\eta = 1, \ldots, \vartheta$). This boundary is a vertical line. The unitary normal vector of the boundary between M_A^η and $M_A^{\eta+1}$ at s^i is $\tilde{n} = (-1, 0)$. The jump in $q^i(s^i)$ as s^i crosses from $M_A^{\eta+1}$ to M_A^η is:

$$\begin{aligned}
\Delta q^i(s_\eta, s_B^i) &= \lim_{\varepsilon\to 0^+}\begin{pmatrix} q_A^i(s_\eta+\varepsilon, s_B^i) \\ q_B^i(s_\eta+\varepsilon, s_B^i)\end{pmatrix} - \lim_{\varepsilon\to 0^+}\begin{pmatrix} q_A^i(s_\eta-\varepsilon, s_B^i) \\ q_B^i(s_\eta+\varepsilon, s_B^i)\end{pmatrix} \\
&= \begin{pmatrix} -[\rho_i^-(s_\eta) - \rho_i^+(s_\eta)] \\ 0 \end{pmatrix}
\end{aligned}$$

It is easy to see that the vectors $\tilde{n}$ and $\Delta q^i(s_\eta, s_B^i)$ are parallel because their cross product is zero: $(-1)\cdot 0 - 0\cdot(-[\rho_i^-(s_\eta) - \rho_i^+(s_\eta)]) = 0$. An analogous argument can be made for any $s^i = (s_A^i, s_\eta)$ (with $s_A^i > s_\eta$) in the common boundary between M_B^η and $M_B^{\eta+1}$ (where $\eta = 1, \ldots, \vartheta$). Furthermore, the argument in Step 1(a) can be replicated to establish that if $s^i = (s,s)$ is a point in the common boundary between M_A^η and M_B^η for any $\eta = 1, \ldots, \vartheta$, then the jump in $q^i(s^i)$ as s^i crosses from M_B^η to M_A^η is perpendicular to the common boundary between M_A^η and M_B^η. Therefore, any q^i given by (12), with decreasing ρ_i, satisfies item (ii) of Prop. 10.

Step 2. We now verify that any function q^i in (12) with decreasing ρ_i satisfies item (i) of Prop. 10. Observe that q^i is continuous when restricted to the interior

[38] In two dimensions, the cross product of two vectors $v_1 = (v_{11}, v_{12})$ and $v_2 = (v_{21}, v_{22})$ is $v_1 \times v_2 = \det(v_1, v_2) = v_{11}v_{22} - v_{12}v_{21}$.

[39] The unitary normal vector of the boundary between M_A^0 and M_B^0 at s^i is $(1/\sqrt{2})n$, with corresponding jump $\Delta q^i(s,s)$ given in the text. As $\Delta q^i(s,s)$ is parallel to n, it is obvious that $\Delta q^i(s,s)$ is also parallel to the unitary normal vector $(1/\sqrt{2})n$, as required by Prop. 10.

[40] Note that there could, in fact, be a countable infinity of discontinuities. While necessitating minor changes in notation, the proof would be otherwise unaffected.

$int(M_k^\eta)$ of any M_k^η in the partition of S ($k \in K$, $\eta = 1, \ldots, \vartheta + 1$). Each $int(M_k^\eta)$ is an open set in $\mathbb{R}^2$, and any two points in $int(M_k^\eta)$ can be connected by a path in $int(M_k^\eta)$. Given these properties of q^i and $int(M_k^\eta)$, we can appeal to Theorem 6 in chapter V, §5, of Lang (1973), which establishes that the existence of a potential function ϕ_i for q^i is equivalent to path-independence of the integral of q^i from one point in $int(M_k^\eta)$ to another. Therefore, all we have to show is that the continuous function q^i on any $int(M_k^\eta)$ has a potential function. It is easy to verify that, for all $k \in K$ and all $\eta = 1, \ldots, \vartheta + 1$, the function $\phi_k^\eta(s^i) = \int_{s_k^i}^{s_{\eta+1}} \rho_i(x)dx$ is a potential function for (12). □

8.12 Proof of Proposition 9.

Take an efficient and incentive compatible SCR Q that satisfies the necessary condition for expenditure-minimization in Proposition 5. Fix a type-pair $(\bar{s}^1, \bar{s}^2)$ with $\bar{s}_A^i < \bar{s}_B^i$ for all $i = 1, 2$. For fixed $\bar{s}^i$, we can depict Q in a (s_A^{-i}, s_B^{-i})-diagram (see either panel of Fig. 7). Efficiency and optimality fully determine which specification-firm-pair is chosen for types s^{-i} below the 45°-line in the (s_A^{-i}, s_B^{-i})-diagram. Now consider the left-hand panel of Fig. 7. As Q is efficient and satisfies the condition in Proposition 5, it must choose firm 1 to supply specification A for all types s^2 below the 45°-line for which specification A is efficient. This event is highlighted in the left-hand panel of Fig. 7 by the dark grey area below the 45°-line. To ensure incentive compatibility, the efficient SCR Q must generate conditional expected probability assignment functions q^i that satisfy the necessary condition in footnote 24. This implies that each q_A^i must be independent of firm i's maximum signal s_B^i. In particular, to ensure that this requirement holds for firm 1, Q must also choose firm 1 to supply specification A for all types s^2 in the dark grey area *above* the 45°-line in the left-hand panel of Fig. 7. It is easy to verify (given the fact that f is symmetric around the 45°-line) that the two dark grey areas together give rise to a conditional expected probability assignment function $q_A^1(\bar{s}^1)$ that is independent of $\bar{s}_B^1$:

$$q_A^1(\bar{s}^1) = \frac{1}{2} - \int_0^{\bar{s}_A^1} \int_{s_A^2 + (1 - \bar{s}_A^1)}^1 f(s_A^2, s_B^2) ds_B^2 s_A^2 \tag{A.30}$$

We now argue by contradiction that firm 1 with type $\bar{s}^1$ s.t. $\bar{s}_A^1 < \bar{s}_B^1$ must also be chosen to supply specification A for all types s^2 in the light grey area in the top left corner of the left-hand panel of Fig. 7. Suppose instead that firm 2 is chosen to produce specification A in the light grey area in the left-hand panel of Fig. 7. In particular, consider the type $(\bar{s}_A^2, \bar{s}_B^2)$ shown in the left-hand panel of Fig. 7, and suppose that firm 2 is chosen to supply specification A. Note that for each type s^2 in

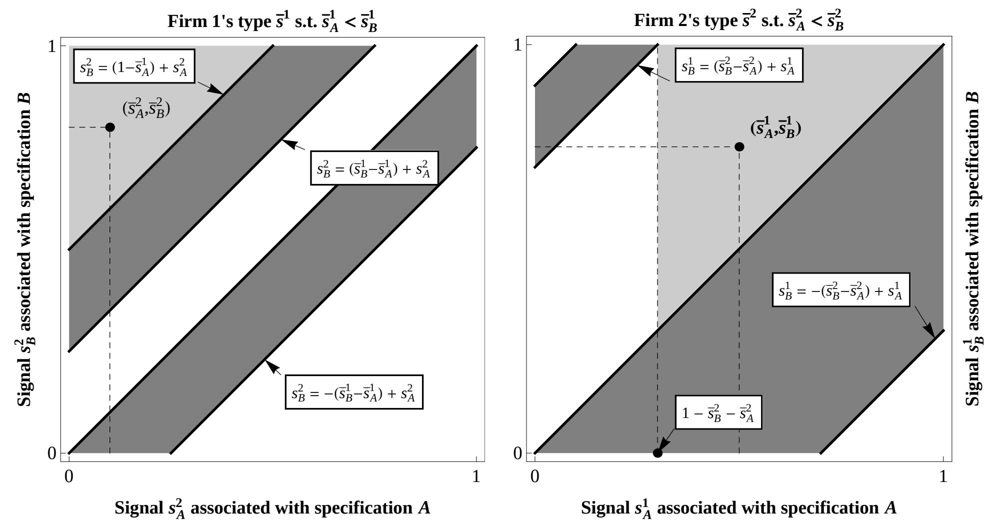

Figure 7: Elements of efficient SCR that satisfies the property in Proposition 5

the light grey area, it holds that $s^2_B > (1-\bar{s}^1_A) + s^2_A$, or equivalently $\bar{s}^1_A > 1-(s^2_B - s^2_A)$. Now turn to the right-hand panel of Fig. 7, where the type $\bar{s}^1$ highlighted there corresponds to the same type-pair $(\bar{s}^1, \bar{s}^2)$ as the type $\bar{s}^2$ highlighted in the left-hand panel of Fig. 7. Therefore, if firm 2 is chosen to supply specification A at type $\bar{s}^2$ in the left-hand panel, firm 2 is obviously also chosen to produce specification A at the type $\bar{s}^1$ in the right-hand panel. This implies that if firm 2 is chosen for *all* types s^2 in the light grey area of the left-hand panel, then firm 2 is chosen to supply specification A for *all* types s^1 in the light grey area of the right-hand panel (i.e. for all types s^1 above the 45°-line s.t. $s^1_A > 1-(\bar{s}^2_B - \bar{s}^2_A)$. This, however, implies that firm 2's conditional expected probability assignment function $q^2_A(\bar{s}^2)$ depends explicitly on $\bar{s}^2_B$, in violation of the necessary condition for incentive compatibility in footnote 24. We can therefore conclude that firm 1 must be chosen to supply specification A for all types s^2 s.t. $s^2_B > (\bar{s}^1_B - \bar{s}^1_A) + s^2_A$ in the left-hand panel of Fig. 7. This, however, means that the only efficient and incentive compatible SCR Q that satisfies the necessary condition in Proposition 5 is $\bar{Q}$ in Fig. 5. $\square$

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