Sliding mode predictive control: a survey

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***Abstract*— This paper reviews the development and application of sliding mode predictive control (SMPC) in a tutorial manner. Two core design paradigms are revealed in the combination of sliding mode control (SMC) and model predictive control (MPC). In the first case, MPC is used in the reaching phase to ensure a sliding mode is attained. In the second case, MPC is used to solve the existence problem and define the required performance in the sliding mode. The two approaches are discussed in detail from the perspectives of both theory and application. Finally, some future challenges and opportunities in the area of SMPC are summarized.**

***Index Terms*— sliding mode control, model predictive control, sliding mode predictive control, optimal control, control input constraint**

I. INTRODUCTION

The process industry has significant and increasing requirements with regard to control performance [1]–[5]. From the perspective of control design, optimisation and robustness are key requirements. In terms of robustness of the controller, any industrial process may be subject to external disturbances and uncertainties, which may cause mismatch between the model used for controller design and the plant. The degradation of product quality as well as equipment failure must also be accommodated. During operation, temperature, concentration, pressure, liquid level and so on have a limited range of operation, so the control input, system output and states will consequently be constrained to lie within certain ranges. The ability to design an optimal control with strong robustness which can accommodate constraints is of great importance in the theory and application of process control.

To address issues of robustness, many control strategies have been developed over the past few decades, such as robust control [6]–[9], adaptive control [10]–[12] and sliding mode control (SMC) [13], [14]. The robust control approach uses high control gains in general [15], [16] and an adaptive control requires a condition on the persistency of excitation and high modelling accuracy [17], which may limit the application scope particularly in the domain of process control. SMC has been extensively studied as a classic robust control [18], [19]. The main idea of SMC is to design a discontinuous control that drives the system state to a sliding surface and then ensure the states will approach zero along the chosen sliding surface. An appropriately designed SMC exhibits insensitivity to matched disturbances and uncertainties when the state moves on the sliding surface. These strong robustness properties, the ability to prescribe dynamic performance and the simple design philosophy make SMC a strong candidate to achieve robust control in many industrial applications.The design of the sliding surface can generally be divided into two types: (1) Linear sliding surface [20]–[22], which is a linear function of the system state $s(x) = Cx$; (2) Nonlinear sliding surfaces, which is nonlinear function of system state $s(x) = f(x)$. Within the broader approach to SMC, integral SMC (ISMC) [23]–[29] seeks to define a desired sliding mode dynamics without reducing the system order i.e. the order of the dynamics in the sliding mode are the same as the order of the system dynamics. The advantage of this approach is that the system state is on the sliding surface initially, which can reduce chattering effects without losing the

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robustness. Effectively the sliding mode reaching phase is eliminated. To improve performance and reducing chattering, SMC has been combined with other approaches, to develop adaptive SMC [30]–[34], neural network SMC [35]–[39], fuzzy SMC [40]–[45]. However, these control strategies have not considered the constraints and optimization.

MPC is well established in the process control community due to its capability to deliver an optimal control in the presence of constraints [46]–[50]. Though robust MPC can solve the constrained optimization online by using receding horizon methods and simultaneously gives consideration to robustness, there is still a need to enhance the ability of MPC to deal with external disturbances and uncertainties. It should be noted that the existing MPC approaches have not considered the case of unmatched disturbances and/or uncertainty explicitly in the control design. To achieve this goal, a new control has been derived by combining SMC and MPC. This approach is named sliding mode predictive control (SMPC) in the literature. SMPC seeks to preserve the advantages of MPC and SMC, such as strong robustness, straightforward implementation, the ability to consider both matched and unmatched disturbances and/or uncertainty, optimization and the capacity to deal explicitly with constraints. Significant work has been done in the area of SMPC [51]–[61]. There are two main approaches in the development of SMPC: (I) MPC is used in the reaching phase to make the system states reach the sliding surface with optimal performance [54]–[57]; (II) MPC is used in the sliding phase to ensure the system has optimal performance along the sliding surface and robustness is enhanced [58]–[61]. This survey focuses on the methods and developing trends of SMPC and presents some interesting/open issues as topics for future research. The methodology used to inform the initial literature search underpinning this article is shown in TABLE IX in Appendix B.

II. SMC : Brief Tutorial Background

The design of classical SMC includes the selection of a sliding surface to define the desired system performance and the use of a usually discontinuous control to render the desired system performance attractive. This control is usually determined using an appropriate reaching law, where numerous reaching laws are available. The time before the selected sliding surface is reached is termed the reaching phase. In the reaching phase, a discontinuous reaching law will ensure the system states reach the sliding surface in finite time. Once the sliding surface is reached, the system is said to be in a sliding mode and enters the sliding phase. In the sliding phase, the states will converge to an equilibrium point along the sliding surface in the case of a regulation problem. Similarly, if a tracking problem is considered, the corresponding tracking error state will converge to zero. Classical SMC defines the reaching mode (RM), sliding mode (SM) and steady-state mode (SS) to reflect this behaviour. Fig. 1 and Fig. 2 show the phase diagrams of a continuous time and discrete time SMC respectively for a second order system. From Fig. 1, for the continuous time system, the trajectory of the states move toward the sliding surface and arrive at the surface in finite time. They then converge to the equilibrium point along the sliding surface. However, from Fig. 2, for the discrete time system, the states reach the sliding surface but do not stay on it. They are seen to cross the sliding surface and move in a zigzag manner.

There are several types of reaching laws reported in the literature to drive the system states to the sliding surface [62]–[66], [69]–[72]. For continuous SMC, a classical reaching condition is given as $s_i \dot{s}_i < 0$. Improved forms of the reaching law are proposed in [62]–[66]. For discrete time SMC, an early reaching condition is given as $|s_i(k+1)| < |s_i(k)|$ [67], [68], which is equivalent to $[s_i(k+1) - s_i(k)]\, sign(s_i(k)) < 0$ and $[s_i(k+1) + s_i(k)]\, sign(s_i(k)) \geq 0$. The reaching condition is fundamentally different for continuous and discrete systems. In the continuous time case, the reaching law ensures the system state switches across the sliding surface with theoretically infinite frequency. The state of the system is thereby constrained to the sliding surface. A similar reaching law is not practical in the case of a discrete time system as the sampled nature of the system may result in the system state crossing and recrossing the sliding surface with increasing amplitude. Reaching laws for discrete time SMC are described in [69] and a reaching law-based quasi-SMC method has been proposed to improve both the steady-state accuracy and convergence rate [70]–[72].

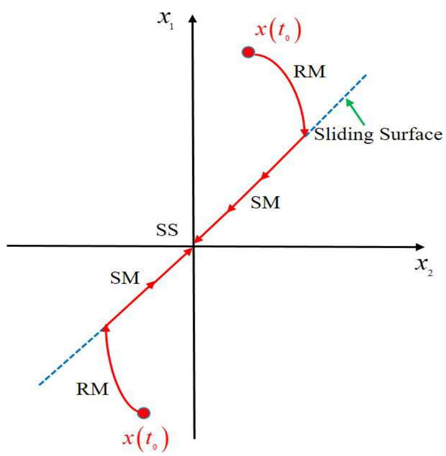

Fig. 1: Phase diagram of continuous SMC.

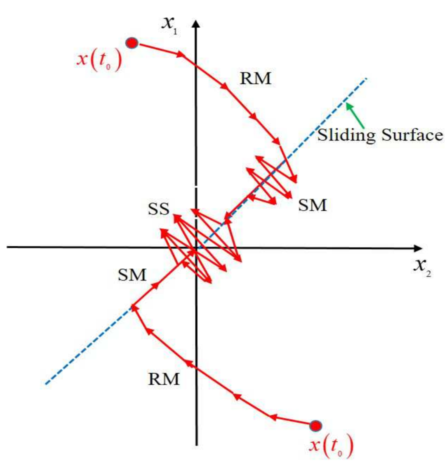

Fig. 2: Phase diagram of a discrete SMC.

A. *Design of continuous time SMC*

The following nominal linear continuous time system is considered for ease of exposition:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input and $A \in R^{n\times n}$ and $B \in R^{n\times m}$ are system matrices.

A commonly used switching function is given by:

$$s(t) = C_c x(t) \tag{2}$$

where $C_c \in \mathrm{R}^{m\times n}$ is a constant design matrix.

Popular reaching laws are listed in the following [62], [63]:

(1) The constant rate reaching law

$$\dot{s}(t) = -\varepsilon \mathrm{sgn}(s(t)), \quad \varepsilon > 0 \tag{3}$$

(2) The constant plus proportional rate reaching law

$$\dot{s}(t) = -\varepsilon \mathrm{sgn}(s(t)) - qs(t), \quad \varepsilon > 0, \, q > 0 \tag{4}$$

(3) The power rate reaching law

$$\dot{s}(t) = -q(s(t))^{\alpha} \mathrm{sgn}(s(t)), \;\; q > 0, \;\; 0 < \alpha < 1 \tag{5}$$

B. *Design of discrete time SMC*

The following equivalent discrete-time nominal linear system is considered:

$$x(k+1) = Fx(k) + Gu(k) \tag{6}$$

where $x(k) \in \mathrm{R}^n$ is the state vector, $u(k) \in \mathrm{R}^m$ is the control vector and $F \in \mathrm{R}^{n\times n}$ and $G \in \mathrm{R}^{n\times m}$ are the system matrices.

As for the continuous case, the sliding surface is frequently designed as

$$s(k) = C_d x(k) \tag{7}$$

where $C_d \in \mathrm{R}^{m\times n}$ is a constant design matrix.

Corresponding popular reaching laws are shown as follows [13], [69]:

(1) The practical reaching law

$$s(k+1) - s(k) = -qTs(k) - \varepsilon T \mathrm{sgn}(s(k)), \quad 1 - qT > 0 \tag{8}$$

where T, $\varepsilon > 0$ and $q > 0$ represent sampling period, reaching rate and approximate rate index, respectively.

(2) The reaching law is obtained by the one-step stabilization method [13]. For the system (6), the control input $u(k)$ can be designed by solving the following sliding mode function

$$s(k+1) = C_d\left(Fx(k) + Gu(k)\right) = 0 \qquad (9)$$

In other words, the control input $u(k)$ can be chosen as follows:

$$u(k) = -(C_d G)^{-1} C_d F x(k) \qquad (10)$$

when the matrix $C_d G$ is invertible. It should be noted that the choice of control in (10) is also referred to as the *equivalent control* in the domain of sliding mode control. This is the control effort which will ensure a known system attains a sliding mode; the equivalent control is usually denoted u_{eq} and is frequently used in the analysis of sliding mode control systems [18].

Due to inertia, time delay and other factors as well as the possible discontinuous nature of the control, high frequency chattering for SMC is sometimes inevitable in practice. Chattering not only affects the accuracy of the control and energy consumption, but also may excite high-frequency unmodelled dynamics in the system, which will have corresponding adverse effects on system performance. Many studies have considered the chattering problem [73]–[78]. Further, the optimal performance in the presence of constraints is not considered in many existing methods. These issues have motivated the integration of MPC into the sliding mode control design paradigm. [].

In the following section, the inclusion of MPC in the solution of the reachability problem is first considered.

III. MPC SOLUTIONS TO THE REACHABILITY PROBLEM

An attractive feature of MPC is its straightforward implementation to solve an online optimisation problem. This method has been incorporated within the reaching law to enhance the performance in the reaching phase. A dual-mode control method has been proposed in which MPC is used as the reaching law when the system state is outside a selected terminal horizon and SMC with off-line design is used when the system state is inside the chosen terminal horizon [54]–[57], [79], [80]. The continuous time case will first be considered.

A. Dual-mode control type reaching law for continuous systems

The following nonlinear continuous time system is considered:

$$\begin{aligned} &\dot{x}(t) = f\left(x(t), u(t)\right) \\ &x(0) = x_0 \\ &x(t) \in X,\ \ u(t) \in U \end{aligned} \qquad (11)$$

where $x(t) \in X \subset \mathrm{R}^n$ is the state vector, $u(t) \in U \subset \mathrm{R}^m$ is the control vector and $f : \mathrm{R}^n \times \mathrm{R}^m \to \mathrm{R}^n$ is a mapping with $f(0,0) = 0$. To optimize the control sequence at each sampling time by MPC, the nonlinear system (11) is discretized as follows [55]:

$$\begin{aligned} x(k+1) &= \tilde{f}\left(x(k), u(k)\right) \\ \tilde{f}\left(x(k), u(k)\right) &= x(k) \\ &\quad + \int_0^{\eta} f\left(x_u\left(\tau; x(k), k\eta\right), u(k)\right) d\tau \end{aligned} \qquad (12)$$

where $x_u\left(\tau; x(k), k\eta\right)$ is the state trajectory of system (11) with initial state $\left(x(k), k\eta\right)$ under control action $u(\tau) = u(k)$, $\tau \in [k\eta, (k+1)\eta]$, and $t = k\eta$.

For the discretized system (12), the following dual-mode control reaching law (DMCRL) is designed. Whether the SMC or MPC is adopted depends on whether the system state is inside a selected terminal set. To describe the DMCRL it is thus first necessary to define appropriate sets.

Definition 1: [55] The following sets are defined

$$S_i^{\Delta_{1i}} = \{x | x(k) \in X,\ |s_i(x)| \le \Delta_{1i},\ u_{smc}(x) \in U,$$
$$\Delta_{1i} = \varepsilon_i T > 0\};$$
$$S^{\Delta_1} = \bigcup_{i=1}^{m} S_i^{\Delta_{1i}};$$
$$S_i^{\Delta_{2i}} = \{x | x(k) \in X,\ |s_i(x)| \le \Delta_{2i},\ u_{smc}(x) \in U,$$
$$\Delta_{2i} > \Delta_{1i}\};$$
$$S^{\Delta_2} = \bigcup_{i=1}^{m} S_i^{\Delta_{2i}};$$
$$S_i^{\Delta} = S_i^{\Delta_{2i}} \Big/ S_i^{\Delta_{1i}},\ S^{\Delta} = \bigcup_{i=1}^{m} S_i^{\Delta}. \tag{13}$$

The control set U is a convex compact subset of space R^m, the state set X is a convex closed subset of space R^n, and $0_{m\times 1} \in U$, $0_{n\times 1} \in X$. $S \subset X$ and $x = 0 \in S$, $s(0_{n\times 1}) = 0_{m\times 1}$.

The DMCRL consists of two parts; one defines an optimal MPC sequence, denoted $u_{MPC}^D(k)$, which is to be used in the reaching phase, and the other uses an SMC in the sliding phase. Before the DMCRL is designed, the optimal MPC sequence can be obtained by solving the following optimization problem [55]:

$$M_N(x): \quad \min_u J_N(s,u) \tag{14}$$

$$s.t \begin{cases} x_k(i+1) = f_d(x_k(i), u_k(i)) \\ x_k(0) = x(k) \\ u_k(i) \in U, \quad i = 0,1,\cdots,N-1 \\ x_k(i) \in X, \quad i = 0,1,\cdots,N-1 \\ s_k(N) \in S^{\Delta} \end{cases} \tag{15}$$

where $J_N(s,u) = \sum_{i=0}^{N-1} l(s_k(i), u_k(i)) + F(s_k(N))$, $l(s_k(i), u_k(i)) = 1/2\left(\|s_k(i)\|_Q^2 + \|u_k(i)\|_R^2\right)$, $F(s_k(N)) = 1/2\|s_k(N)\|_{R_1}^2$. Q, R, $R_1 \in \mathrm{R}^{m\times m}$ are positive definite symmetric matrices. By solving the nonlinear programming problem (14), the optimal predictive control sequences $u_N^*(k) = \left\{u_k^*(0), u_k^*(1), \cdots, u_k^*(N-1)\right\}$ are obtained under the constraints (15). The first element of $u_N^*(k)$ defines the current control law, i.e. $u(k) = u_k^*(0)$.

When the control horizon is finite [55], the DMCRL can be defined as follows:

$$u(k) = \begin{cases} u_k^*(0), & x(k) \in X \big/ S^{\Delta_2} \\ u_{SMC}(x(k)), & x(k) \in S^{\Delta_2} \end{cases} \tag{16}$$

To prove that (16) can stabilize the system (12), the following two lemmas are introduced. Lemma 1 shows that the optimal predictive control $u_{MPC}^D(k)$ ensures the system reaches the terminal horizon in finite time. Lemma 2 shows that the control $u_{SMC}(k)$ renders the system stable in the terminal horizon.

Lemma 1: [55] 1)The optimization problem has a solution at the initial time k;
2) For $\forall x(k) \in S^{\Delta_2}$, there exists $\gamma \ge 0$, such that $\left\|u_{SMC}(x(k)) - u_{eq}(x(k))\right\|_R^2 \le \gamma \|s(x(k))\|_R^2$;
3) Design ε_i and $q_i\,(i = 1,2,\cdots,m)$ so that $2\alpha_i \ge 1$;
4) Design positive definite symmetric matrices Q, R, R_1 and α, such that $Q + \gamma R + \alpha^T R_1 \alpha - R_1$ is negative definite or semi-negative definite;
5) The system state $x(k) \notin S^{\Delta_2}$, then $J_N^*(k+1) - J_N^*(k) \le -l(s_k(i), u_k(i))$, and the system state can enter the terminal horizon in finite time.

Remark 1: At time k, the optimal predictive control sequence $u_N^*(k)$ is obtained, the first element $u(k) = u_k^*(0)$ is implemented, and the corresponding optimal cost is denoted as $J_N^*(k)$. At the subsequent time $k+1$, the corresponding optimal predictive control sequence and optimal cost are denoted as $u_N^*(k+1)$ and $J_N^*(k+1)$ respectively. In $J_N^*(k+1) - J_N^*(k) \le -l(s_k(i), u_k(i))$, $s_k(i)$ and $u_k(i)$ denote the actual value of the switching function and control law at time k respectively. The system cost $J_N^*(\cdot)$ is decreasing while $x(k) \in X\big/S^{\Delta_2}$, so the system state can reach the terminal horizon in finite time.

Lemma 2: [55] When the system state $x(k_1) \in S^{\Delta_2}$ at time k_1, the off-line designed $u_{SMC}(x(k))$ is adopted to ensure $x(k_1+i) \in S^{\Delta_2}, i \ge 1$, and the SMC system after time k_1 is asymptotically stable.

Theorem 1: [55] Based on Lemma 1 and Lemma 2, (16) can guarantee asymptotic stability of the closed-loop system under the conditions of Lemma 1.

This strategy can also be applied in the case of a cost function of infinite horizon as described in

[54].

The main equations for computer implementation of the DMCRL for continuous time systems are given in TABLE I and the associated pseudocode is given below:

Algorithm 1 DMCRL for continuous time systems

Step 1 Select the sampling interval η, and then $t = k\eta$;
Step 2 Set $k = 0$ and give the initial state $x(0)$;
Step 3 At each time k, design a sliding surface $s(k) = C_c x(k)$;
Step 4 Set the SMC input as $u_{SMC}(k) = u_{eq}(k)$ and the MPC input as $u_{MPC}(k) = u_k^*(0)$;
Step 5 Implement the control law as $u(k)$ subject to (14) and (15);
Step 6 Set $k = k+1$ and go back to Step2.

B. A dual-mode control type reaching law for discrete time systems

The following discrete-time systems are considered:

$$\begin{array}{l} x(k+1) = Ax(k) + B(u(k) + d(k)) \\ x(0) = x_0 \\ x(k) \in X, \;\; u(k) \in U \end{array} \tag{17}$$

where $x(k) \in X \in \mathrm{R}^n$ is the state vector, $u(k) \in U \in \mathrm{R}^m$ is the control vector and $A \in \mathrm{R}^{n\times n}$, $B \in \mathrm{R}^{n\times m}$ are the state transition and input distribution matrices.The pair (A,B) is assumed to be controllable. The vector $d(k) \in \mathrm{R}^m$ is an unknown disturbance which is bounded by $0 < \|d(k)\|_\infty \le \rho$.

For discrete systems of the form (17), the DMCRL design idea follows as for the continuous case [56]. First, the optimal predictive control $u_{MPC}^D(k)$ is obtained by solving an optimization problem to ensure the state reaches the quasi-sliding band as soon as possible. Then, the sliding mode control $u_{SMC}(k)$ is obtained by prescribing the sliding function to maintain the state within the band. The MPC sequences are given as follows:

$$\begin{array}{l} U_N(k) = \{u'_k(0), u'_k(1), \cdots, u'_k(N-1)\} \\ u'_k(i) = u'(k+i), \;\; i = 0,1,\cdots N-1 \end{array} \tag{18}$$

The corresponding predictive values are:

$$\left\{ \begin{array}{l} X_N(k) = \{x'_k(1), x'_k(2), \cdots, x'_k(N)\} \\ S_N(k) = \{s'_k(1), \; s'_k(2), \cdots, s'_k(N)\} \\ s'_k(i) = s(x'_k(i)), \quad i = 1,2,\cdots,m \end{array} \right. \tag{19}$$

The cost function is defined as follows:

$$J(k) = \sum_{i=0}^{N-1} L\left(s'_k(i), u'_k(i)\right) \tag{20}$$

where $L(s'_k(i), u'_k(i)) = \|s'_k(i)\|_Q^2 + \|u'_k(i) - u_{eq}(i)\|_R^2$, $Q \in \mathrm{R}^m$ and $R \in \mathrm{R}^m$ are symmetric positive definite matrices with appropriate dimensions. Then, the optimization problem becomes [56]:

$$\min_{U_N(k)} J(k) \tag{21}$$

$$s.t \left\{ \begin{array}{l} x'_k(i+1) = Ax'_k(i) + Bu'_k(i) \\ x'_k(0) = x(k) \\ u'_k(i) \in U, \quad i = 0,1,\cdots,N-1 \\ x'_k(i) \in X, \quad i = 0,1,\cdots,N-1 \\ x'_k(N) \in S^\Delta \end{array} \right. \tag{22}$$

At time k, the optimal predictive control sequences $U_N^*(k) = \left\{u_k^*(0), u_k^*(1), \ldots, u_k^*(N-1)\right\}$ are obtained under constraints (22) by solving the nonlinear programming problem (21).

The sliding mode begins when the system trajectory converges to the terminal horizon S^Δ. The sliding condition is $s(x) = [s_1(x), s_2(x), \cdots, s_m(x)]^T = \mathbf{0}$. Set S and the terminal sliding mode set S^Δ are defined as follows:

$$S = \{x | s(x) = \mathbf{0}\}, \quad S^\Delta = \{x \in S | x(k) \in X\} \tag{23}$$

From the sliding condition $s(x) = \mathbf{0}$, the following equivalent control can be designed [56]:

$$u_{eq}(k) = \left\{ \begin{array}{ll} 0, & x(k) \notin S^\Delta \\ u_{eq}(k), & x(k) \in S^\Delta \end{array} \right. \tag{24}$$

Let $u_{MPC}^D(k) = u_k^*(0)$, then the DMCRL is defined as follows [56]:

$$u(k) = \left\{ \begin{array}{ll} u_{MPC}^D(k), & x(k) \notin S^{\Delta_2} \\ u_{SMC}(k), & x(k) \in S^{\Delta_2} \end{array} \right. \tag{25}$$

Remark 2: The optimal predictive control $u_{MPC}^D(k)$ optimizes the cost function through predictive variables, and it realizes the optimal reaching performance. After reaching the sliding

TABLE I: Main equations of DMCRL and ISMPC for continuous time systems

Continuous time systems		
	DMCRL [55]	ISMPC [58]
Sliding fuction	$s(k) = C_d x(k)$	$s(x(t),t) = D(x(t) - x(0)) - D\int_0^t (f(x(\tau)) + Bu_{MPC}(\tau))\,d\tau$
Equivalent control	$u_{eq}(k) = -(CB)^{-1}[CAx(k) - (1-qT)s(k) + \varepsilon T \mathrm{sgn}(s(k))]$	\
Sliding mode control	$u_{SMC}(k) = u_{eq}(k)$	$u_{ISM}(t) = -\rho \frac{(DB)^T s(x(t),t)}{\left\|(DB)^T s(x(t),t)\right\|}$
Model predictive control	$u_{MPC}(k) = u_k^*(0)$	$u_{MPC}^I(t) = \kappa(x(t_k)) = \bar{u}^o_{[t_k,t_{k+N-1}\|t_k]}(t_k), \quad t \in [t_k, t_{k+1})$
Sliding mode predictive control	$u(k) = \begin{cases} u_k^*(0), & x(k) \in X/S^{\Delta_2} \\ u_{SMC}(k), & x(k) \in S^{\Delta_2} \end{cases}$	$u(t) = u_{MPC}^I(t) + u_{ISM}(t)$
Cost function	$J_N(s,u) = \sum_{i=0}^{N-1} 1/2\left(\|s_k(i)\|_Q^2 + \|u_k(i)\|_R^2\right) + 1/2\|s_k(N)\|_{R_1}^2$	$J\left(\bar{x}, \bar{u}_{[t_k,t_{k+N-1}\|t_k]}, N\right) = \int_{t_k}^{t_{k+N}} \left(x^T(t)Qx(t) + u^T(t)Ru(t)\right)d\tau + x^T(t)\Xi x(t)$

surface, the quasi-sliding mode control $u_{SMC}(k)$ is adopted.

The system is closed-loop stable if and only if the sliding mode reaches the terminal horizon S^{Δ}. It can be seen from $J^*(k+1) - J^*(k) \leq -L(s'_k(i), u'_k(i))$ that the cost function is decreasing. From $L(s'_k(i), u'_k(i)) = 0 \Leftrightarrow s'(k) = 0$, $u'(k) = u_{SMC}(k)$, it follows that $\lim_{M\to\infty} s'(k+M) = 0$, $\lim_{M\to\infty} u'(k+M) = u_{SMC}(k+M)$.

The method of terminal equality constraints makes the prediction horizon and control horizon longer, but will increase computation for the case of multiple input multiple output systems. Therefore, the DMCRL may be designed with terminal inequality constraints and the terminal horizon Ω is defined as follows:

$$\Omega = \{x | x \in X, \ \ \|s(x)\| \leq \Delta, \ u(x) \in U, \ \Delta > 0\} \tag{26}$$

The DMCRL with terminal inequality constraints is described as follows [56]:

$$\min_{U_N(k)} J(k) \tag{27}$$

$$s.t \begin{cases} x'_k(i+1) = Ax'_k(i) + Bu'_k(i) \\ x'_k(0) = x(k) \\ u'_k(i) \in U, \quad i = 0,1,\cdots,N-1 \\ x'_k(i) \in X, \quad i = 0,1,\cdots,N-1 \\ x'_k(N) \in \Omega \end{cases} \tag{28}$$

where $J(k) = \sum_{i=0}^{N-1} L(s'_k(i), u'_k(i)) + F(s'_k(N))$, $F(s'_{k+1}(N)) = \|s'_{k+1}(N)\|_{Q_1}^2$. This method reduces computation when compared with the case of terminal equality constraints, as demonstrated in [56].

The main equations for computer implementation of the DMCRL for discrete time systems are given in TABLE II and the associated pseudocode is given in Algorithm 2.

C. Numerical examples

In order to evaluate the DMCRL method, the following system will be considered:

$$x(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} (u(k) + d(k)) \tag{29}$$

where $d(k) = 0.01\sin(10k)$ and $\|d(k)\|_\infty \leq \alpha = 0.01$.

TABLE II: Main equations of DMCRL, ISMPC and SMLPC for discrete time systems

Discrete time systems			
	DMCRL [56]	ISMPC [59]	SMLPC [97]
Sliding fuction	$s(k)=C_d x(k)$	$\begin{cases} s(k)=Gx(k)+\varepsilon(k) \\ \varepsilon(k+1)=\varepsilon(k)+G(x(k) \\ \quad -Ax(k)-Bu_{MPC}(k)) \\ \varepsilon(0)=-Gx(0) \end{cases}$	$s(k)=C_d x(k)$
Equivalent control	$u_{eq}(k)=\begin{cases} 0, & x(k)\notin S^{\Delta} \\ -(CB)^{-1}[CAx(k) & \\ -(1-qT)s(k) & \\ +\varepsilon T\mathrm{sgn}(s(k))], & x(k)\in S^{\Delta} \end{cases}$	\	\
Sliding mode control	$u_{SMC}(k)=u_{eq}(k)$	$u_{ISM}(t)=-M\mathrm{sgn}(s(k))$	\
Model predictive control	$u^{D}_{MPC}(k)=u^{*}_{k}(0)$	$u^{I}_{MPC}(k)=Fx(k)$	\
Sliding mode predictive control	$u(k)=\begin{cases} u^{D}_{MPC}(k), & x(k)\notin S^{\Delta_2} \\ u_{SMC}(k), & x(k)\in S^{\Delta_2} \end{cases}$	$u(k)=u^{I}_{MPC}(k)+u_{ISM}(k)$	$u(k)=-[1,0,\ldots,0]^T$ $(\Theta^T\Theta+G)^{-1}\Theta^T\,[\Xi x(k)$ $+O_pE(k)-S_r(k+1)]$
Cost function	$J(k)=\sum_{i=0}^{N-1}\left(\Vert s'_k(i)\Vert_Q^2+\Vert u'_k(i)-u_{eq}(i)\Vert_R^2\right)$ $+\Vert s'_{k+1}(N)\Vert_{Q_1}^2$	$J(k)=\sum_{i=1}^{N-1}(x^T(k+i)Qx(k+i)$ $+u^T(k+i)Ru(k+i))$ $+x^T(k+N)Px(k+N)$	$J_p(k)=\sum_{j=1}^{N}t_j[\tilde{s}_p(k+1)$ $-\ s_r(k+j)]^2$ $+\sum_{l=1}^{M}d_l[u(k+l-1)]^2$

Algorithm 2 DMCRL for discrete time systems

Step 1 Set $k=0$ and give the initial state $x(0)$;
Step 2 At each time k, design a sliding surface $s(k)=C_d x(k)$;
Step 3 Set the SMC input as $u_{SMC}(k)=u_{eq}(k)$ and the MPC input as $u^{D}_{MPC}(k)=u^{*}_{k}(0)$;
Step 4 Implement the control law as $u(k)$, subject to (21) and (22);
Step 5 Set $k=k+1$ and go back to Step1.

The cost function is defined by (20) and the corresponding positive definite weighting matrices are chosen as

$$Q=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, R=1 \tag{30}$$

The initial values and sliding function are given by: $x(0)=\begin{bmatrix} 3 & 5 \end{bmatrix}^T$, $s(k)=C_d x(k)=\begin{bmatrix} 3 & 1 \end{bmatrix}^T x(k)$. The selection of sliding function C_d renders the sliding dynamics stable where the design of C_d is shown in Chapter 3 of [22]. The performance of the DMCRL in (25) is compared with a classical discrete time sliding mode control where a reaching law based on (8) is used to design the control

$$u_{SMC}(k)=-(C_dB)^{-1}[C_dAx(k)-(1-qT)s(k) \\ +\varepsilon T\mathrm{sgn}(s(k))]-d(k)$$

where $T=0.01$, $q=10$ and $\varepsilon=0.5$.

The simulation results for both methods can be seen in Figs. 3-7.

TABLE III: ITAE for both DMCRL and SMC

ITAE	$x_1(k)$	$x_2(k)$
DMCRL	17.3700	19.6266
SMC	48.3290	24.0389

Note that the initial conditions, the sliding functions and the disturbance signal are identical for

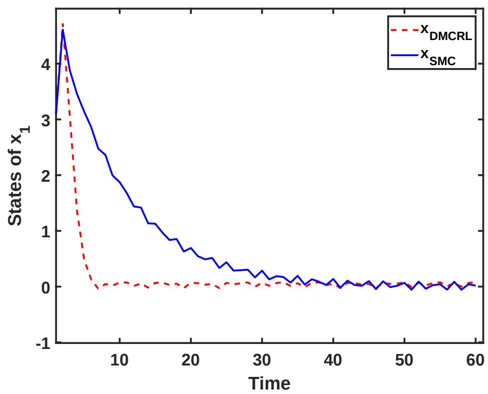

Fig. 3: The response of $x_1(k)$ in the simulation test.

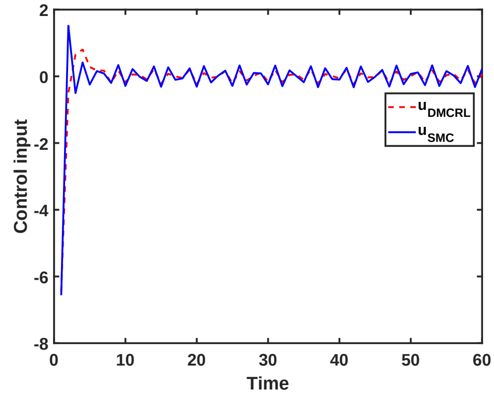

Fig. 5: The control input $u(k)$ in the simulation test.

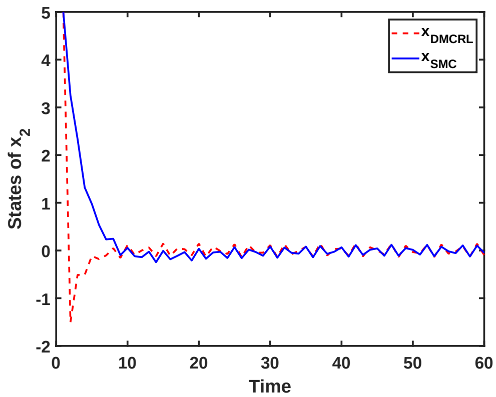

Fig. 4: The response of $x_2(k)$ in the simulation test.

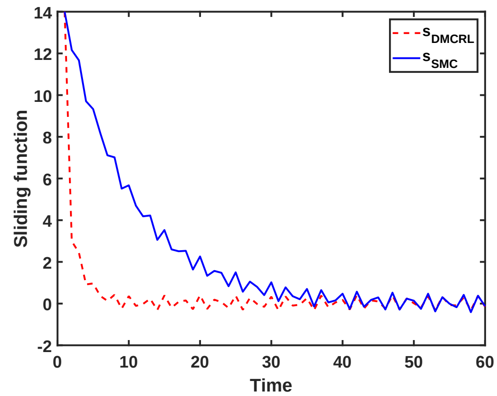

Fig. 6: The sliding function $s(k)$ in the simulation test.

both DMCRL and SMC simulations. The dotted line represents the DMCRL and the solid line represents SMC. It can be seen from Fig. 3, Fig. 4 and Fig. 6 that the DMCRL ensures the system state converges to the sliding surface faster than SMC in the reaching phase. Fig. 7 shows that the DMCRL can also improve system performance. The Integral of Time and Absolute Error (ITAE) for both methods is listed in Table III. It is clear that the ITAE of the DMCRL is smaller than that obtained for SMC. Meanwhile, the simulation results also clearly show that DMCRL is superior to SMC in the reaching phase.

D. Summary

Compared with the reaching law approach of classical SMC as described in section II, a DMCRL can reduce chattering, accelerate the reaching speed and improve the system performance. Further, compared with the quasi-infinite horizon nonlinear MPC method [81], the system does not need to be linearized at the origin and the terminal sliding mode horizon can be used in the final attraction horizon of the origin. However, in DMCRL the switching function $S(x)$ and the terminal horizon S^{Δ} may be difficult to design off-line while satisfying the stability and control constraints. The

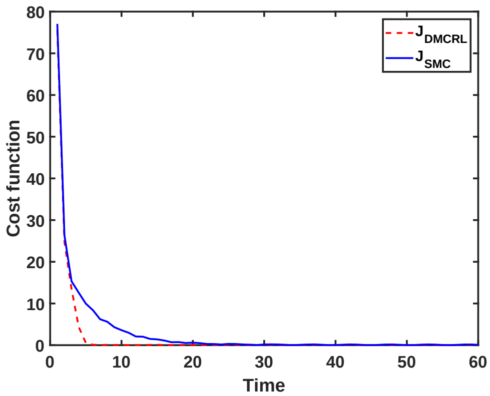

Fig. 7: The cost function $J(k)$ in the simulation test.

performance in the presence of disturbances of a DMCRL is not as robust as that of SMC. Note that the control is optimal only in the reaching phase but not in the sliding phase.

IV. MPC USED IN THE SLIDING PHASE

The robustness of SMC does not hold in the reaching phase for traditional SMC. For this reason ISMC has been proposed [82]–[85]. Since optimization and constraints are not accommodated in ISMC, integral sliding mode predictive control (ISMPC) has been recently proposed [58]–[61], [86]–[92]. In ISMPC, the control signal is composed of two parts: one is generated by the ISMC approach to deal with matched disturbances/uncertainties, the other is generated as an MPC to achieve optimal control with constraints whilst dealing with unmatched disturbances/uncertainties.

ISMPC is applied to a class of uncertain continuous and discrete systems in [58]–[61] to enhance the robustness, in which the closed-loop system in the presence of matched disturbances behaves exactly as the nominal system under nominal control, and the unmatched disturbances are not amplified by selecting an optimal projection matrix on the integral sliding surface. In [86]–[88], a state observer is used to accurately estimate the state and the matched disturbance is suppressed by using output information. The ISMPC is designed for uncertain multivariable linear systems with time delays present in the state and control in [89]. The self-triggered ISMPC method is investigated for networked nonlinear continuous-time systems subject to state and input constraints with additive disturbances and uncertainties in [90]. A tractable robust MPC scheme with adaptive SMC is designed for a class of nonlinear systems in normal form with two types of uncertainties in [91]. Although the systems are distinct in [89]–[92], ISMC can be used to reduce the influence of disturbances and uncertainties and enhance the system robustness.

A. Integral sliding mode control

a. ISMC of continuous systems

Since the integral sliding mode control has no reaching phase, it can effectively reduce system chattering. Consider the following nonlinear continuous time system

$$\dot{x}(t) = g(x(t)) + Bu(t) + h(t), t \geq 0 \tag{31}$$

where $x(t) \in \mathrm{R}^n$ is the state, $u(t) \in \mathrm{R}^m$ is the control and $h(t)$ is a perturbation due to model uncertainties or external disturbances. It should be noted that $h(t)$ can be decomposed into matched disturbances and unmatched disturbances.

The form of the control law is proposed as follows [82]:

$$u(t) = u_0(t) + u_1(t) \tag{32}$$

where the nominal control $u_0(t)$ is responsible for the performance of the nominal system and $u_1(t)$ is a discontinuous control that rejects the matched disturbances by ensuring the existence of a sliding motion.

The sliding manifold is defined by the set $\{x|\sigma(x,t) = 0\}$. The following nonlinear integral sliding surface is proposed [93]:

$$\begin{aligned}\sigma(x,t) &= Dx(t) - Dx(t_0) \\ &\quad - D\int_{t_0}^{t} [g(x(\tau)) + Bu_0(\tau)]\, d\tau\end{aligned} \tag{33}$$

where $D \in \mathrm{R}^{m\times n}$ is a projection matrix and it is assumed that the matrix DB is invertible. It can be seen that the additional integral term provides one more degree of freedom in design than the linear sliding surface. In addition, the term $-Dx(t_0)$ achieves the desirable property that $\sigma(x(t_0), t_0) = 0$, such that the reaching phase is eliminated.

Proposition 1: [84] For any matrix $B \in \mathrm{R}^{n\times m}$ satisfying $rank\ B = m$, the identity $I_n = BB^+ + B^\perp B^{\perp +}$ holds, where B^+ is understood as the left inverse of B, that is $B^+ = \left(B^T B\right)^{-1} B^T$ and the columns of $B^\perp \in \mathrm{R}^{n\times(n-m)}$ span the null space of B^T.

This allows the disturbances to be separated into matched and unmatched components:

$$\begin{aligned} &h = h_m + h_u \\ &h_m \triangleq BB^+ h \\ &h_u \triangleq B^\perp B^{\perp +} h \end{aligned} \tag{34}$$

where h_m and h_u are the components of the matched and unmatched disturbances, respectively. The equivalent control method [12] is adopted to determine the motion on the sliding manifold, so that the equivalent disturbances $h_{eq} \triangleq \left[I - B(DB)^{-1}D\right] h_u$ can be obtained.

Proposition 2: B^T is a matrix which minimizes the norm of h_{eq}, i.e.

$$D^* = B^T = \arg\min_{D\in R^{m\times n}} \left\| \left[I - B(DB)^{-1}D\right] h_u \right\|_2 \tag{35}$$

From Proposition 2, an optimal projection matrix D^* can be obtained, so that the equivalent disturbances h_{eq} are equal to the unmatched disturbances h_u, which means the effects of the unmatched disturbances are not amplified.

b. ISMC of discrete-time systems

Discrete time ISMC [83], [85] has been proposed to improve the control performance of sampled data systems. Like the continuous-time ISMC [82], [93], the closed-loop system can achieve the expected control performance while avoiding the generation of overly large control inputs.

Here the discrete-time system (17) is considered, where $d(k)$ is decomposed into matched and unmatched disturbances to enhance the robustness, as seen in [85].

The following form of the control law is proposed:

$$u(k) = u_0(k) + u_1(k) \tag{36}$$

where $u_0(k)$ is the ideal control which can stabilize the nominal system and $u_1(k)$ is an additional control input designed to achieve disturbance rejection.

The following discrete-time integral sliding surface is defined [85]:

$$\begin{aligned} &s(k) = Gx(k) - Gx(0) + \sigma(k) \\ &\sigma(k) = \sigma(k-1) - (GBu_0(k-1) + GAx(k-1)) \end{aligned} \tag{37}$$

where $s(k) \in \mathrm{R}^n$, $\sigma \in \mathrm{R}^m$, $\sigma(0) = 0$ and $G \in \mathrm{R}^{m\times n}$ is to be designed. The term $Gx(0)$ is used to eliminate the reaching phase. The ISMC forces the system state to move along the sliding surface from the initial time, which overcomes the shortcoming of reduced robustness in the reaching phase exhibited by standard SMC. However, optimization is not considered when the system has control and state constraints. The use of MPC in the sliding phase is proposed to address these issues.

B. ISMPC method

In recent decades, the ISMC method has become more mature and has been considered an effective tool for dealing with disturbances. MPC has been considered as an optimal control method to effectively dealing with constraints. Therefore, the ISMPC method cannot only effectively handle the system external disturbances, but also solve an optimization problem which includes system constraints. The ISMPC designed process is described as follows.

Definition 2: [58] Consider the continuous system with initial state $x_0 \in \mathrm{R}^n$. Given the positive integer N, the quadratic cost function $g(x,u) \triangleq x^T Qx + u^T Ru$ ($Q \in \mathrm{R}^n$ and $R \in \mathrm{R}^m$ are symmetric positive definite matrices), the quadratic terminal penalty $V_f(x) \triangleq x^T \Xi x$ ($\Xi \in \mathrm{R}^n$ is a symmetric positive definite matrix) and the terminal set χ_f, the Finite-Horizon Optimal Control Problem (FHOCP) problem with respect to $\bar{u}_{[t_k, t_{k+N-1}|t_k]}$ can be formulated as

$$\begin{aligned} J\left(\bar{x}, \bar{u}_{[t_k, t_{k+N-1}|t_k]}, N\right) = &\int_{t_k}^{t_{k+N}} g(x(\tau), u(\tau)) d\tau \\ &+ V_f(x(t_{k+N})) \end{aligned} \tag{38}$$

subject to the following constraints:

1) The state dynamics $\dot{x}(t) = g(x(t)) + Bu(t)$ with disturbances is zero, for all $t \in [t_k, t_{k+N})$;

2) The state constraint $x(t) \in \chi_{t-t_k}$, for all $t \in [t_k, t_{k+N})$;

3) The control constraint $u(t) \in U$;

4) The terminal state constraint $x(t_{k+N}) \in \chi_f$.

Definition 3: If there exists a KL-function β, K-function γ and a constant $c \geq 0$ such that

$$|x(t,\xi,u)| \leq \beta(|\xi|,t) + \gamma(\|u\|_\infty) + c \tag{39}$$

holds for each control u and each $\xi \in \mathrm{R}^n$, the system is said to be Input-to-State practically Stable (ISpS) [95], [96].

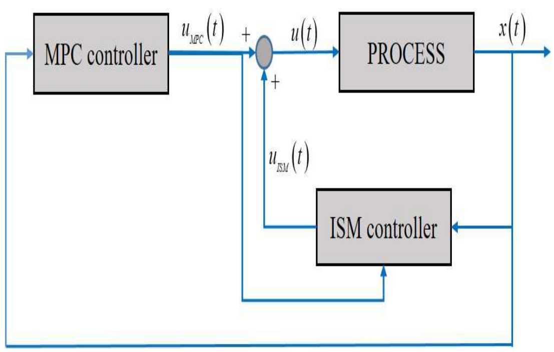

Fig. 8: Block diagram of ISMPC strategy

a. ISMPC of continuous system [58]

For the continuous system (31), the state and control variables are restricted to fulfill the following constraints

$$\begin{array}{l} x(t) \in X \\ u(t) \in U \end{array} \tag{40}$$

where X and U are compact sets containing the origin as the interior point.

According to the current state feedback, an integral sliding surface based on the MPC solution is designed:

$$\begin{aligned} s(x(t),t) &\triangleq D(x(t) - x(0)) \\ &- D\int_0^t \left(f(x(\tau)) + Bu_{MPC}^I(\tau)\right) d\tau \end{aligned} \tag{41}$$

where $D \in \mathrm{R}^{m \times n}$ is a projection matrix and DB is invertible [84].

According to Figure 8, the control is designed as follows:

$$u(t) = u_{MPC}^I(t) + u_{ISM}(t) \tag{42}$$

where $u_{MPC}^I(t) = \kappa(x(t_k)) \triangleq \overline{u}^o_{[t_k, t_{k+N-1}|t_k]}(t_k)$, $t \in [t_k, t_{k+1})$, where the optimal control sequence $\overline{u}^o_{[t_k, t_{k+N-1}|t_k]}$ is obtained by solving the FHOCP.

The control input to achieve disturbance rejection is designed as follows:

$$u_{ISM}(t) \triangleq -\rho \frac{(DB)^T s(x(t),t)}{\left|(DB)^T s(x(t),t)\right|} \tag{43}$$

where ρ is the control gain and $s(x(t),t) = 0$ for all time.

The main equations for computer implementation of the ISMPC for continuous time systems are given in TABLE I and the associated pseudocode is given in Algorithm 3.

Algorithm 3 ISMPC for continuous time systems

Step 1 Set $t = 0$ and give the initial state $x(0)$;

Step 2 At each time t, design a sliding surface $s(x(t),t)$ based on the solution of MPC $u_{MPC}^I(t)$, subject to (38);

Step 3 Set the SMC input as $u_{ISM}(t)$;

Step 4 Implement the control law as $u(t)$;

Step 5 Repeat the procedure 1) to 4), i.e. $t = t + 1$ at next time instance.

b. ISMPC for discrete time systems [59]

For the discrete time system (17), a set of constraints is given as follows:

$$\begin{array}{l} x(k) \in X \\ u(k) \in U \end{array} \tag{44}$$

The integral sliding surface is defined by

$$\begin{cases} s(k) = Gx(k) + \varepsilon(k) \\ \varepsilon(k+1) = \varepsilon(k) + G\left(x(k) - Ax(k) - Bu_{MPC}^I(k)\right) \\ \varepsilon(0) = -Gx(0) \end{cases} \tag{45}$$

where $G \in \mathrm{R}^{m \times n}$ is a projection matrix and GB is invertible.

The control input of the system is:

$$u(k) = u^{I}_{MPC}(k) + u_{ISM}(k) \tag{46}$$

where $u^{I}_{MPC}(k) = Fx(k)$ and the discontinuous control input is given by $u_{ISM}(t) = -M\text{sgn}(s(k))$. Then, the optimization problem as follows:

$$\begin{aligned} &\min_{u} J(k) = \min_{u} \Big[\sum_{i=1}^{N-1} \big(x^T(k+i) Qx(k+i) + \\ &\quad u^T(k+i) Ru(k+i) \big) + x^T(k+N) Px(k+N) \Big] \\ &s.t \qquad x(k+1) = Ax(k) + Bu(k) \\ &\qquad\qquad x(k) \in X \\ &\qquad\qquad u(k) \in U \end{aligned} \tag{47}$$

The stability of the ISMPC method for continuous systems [58] has been proved by application of Input-to-State practical Stability (ISpS). For the discrete time case [59], the system is Lyapunov stable, as long as the state trajectory is maintained within the quasi-sliding mode band $\Delta = \{|s(k_i)| \le \zeta,\ \zeta = 2\delta \|CB\|_\infty, i \in I\}$ The main equations for computer implementation of the ISMPC for discrete time systems are given in TABLE II and the associated pseudocode is given in Algorithm 4.

Algorithm 4 ISMPC for discrete time systems

Step 1 Set $k = 0$ and give the initial state $x(0)$;
Step 2 At each time k, design a sliding surface $s(k)$ based on the solution of MPC $u^{I}_{MPC}(k)$, subject to (47);
Step 3 Set the SMC input as $u_{ISM}(k)$;
Step 4 Implement the control law as $u(k)$;
Step 5 Repeat the procedure 1) to 4), i.e. $k = k+1$ at next time instance.

C. Numerical example

Consider the following discrete-time system

$$x(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} u(k) + d(k) \tag{48}$$

where $d(k) = \begin{bmatrix} 1 \\ -0.6 \end{bmatrix} \sin(10k)$ and the control input constraint is $u(k) \in U = \{u | -5 \le u(k) \le 5\}$.

For the ISMPC method, the disturbance is first decomposed as

$$d(k) = \underbrace{\begin{bmatrix} 0.2 & 0.4 \\ 0.4 & 0.8 \end{bmatrix}}_{BB^+} d(k) + \underbrace{\begin{bmatrix} 0.8 & -0.4 \\ -0.4 & 0.2 \end{bmatrix}}_{B^\perp B^{\perp +}} d(k)$$

The first component is matched and will be eliminated by the discontinuous control $u_{ISM}(k)$, the second is unmatched and will be compensated using the continuous control $u^{I}_{MPC}(k)$.

The cost function is defined by (38) and the corresponding positive definite weighting matrices are chosen as $Q = 1$, $R = 1$, $\Xi = 0.01$. The horizon length is $N = 20$. The system starts from $x(0) = \begin{bmatrix} 2 & 0 \end{bmatrix}$ and the projection matrix $G = \begin{bmatrix} 0.5 & 1 \end{bmatrix}$. The $u^{I}_{MPC}(k) = Fx(k)$ and $F = YQ^{-1}$ is obtained using LMI techniques. Then, the following ISMPC control law is implemented:

$$u(k) = Fx(k) + 0.1\text{sgn}(s(k)) \tag{49}$$

The performance when a comparator model predictive control is also shown where $u_{mpc}(k) = Fx(k)$. The simulation results for ISMPC and MPC are shown in Figs. 9-13.

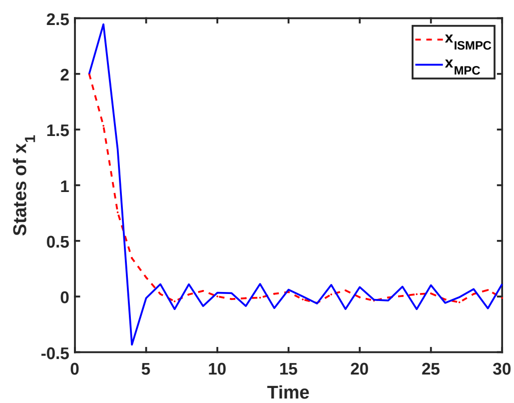

Fig. 9: The response of $x_1(k)$ in the simulation test.

TABLE IV: ITAE for both ISMPC and MPC

ITAE	$x_1(k)$	$x_2(k)$
ISMPC	7.3951	5.8292
MPC	13.3061	43.1737

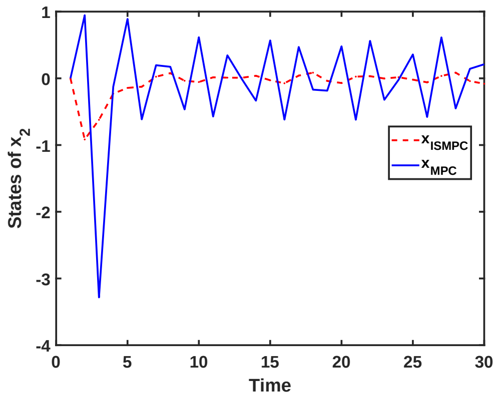

Fig. 10: The response of $x_2(k)$ in the simulation test.

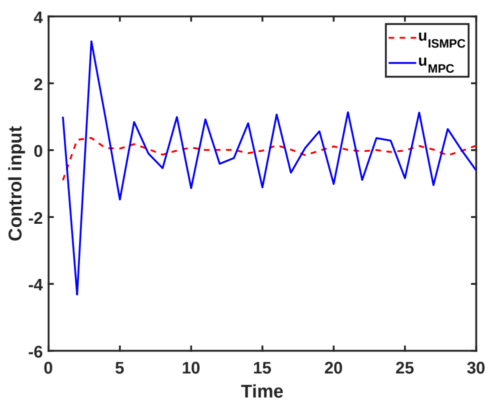

Fig. 11: The control input $u(k)$ in the simulation test.

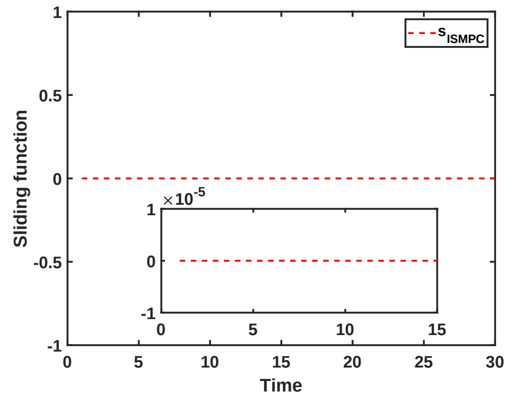

Fig. 12: The sliding function $s(k)$ in the simulation test.

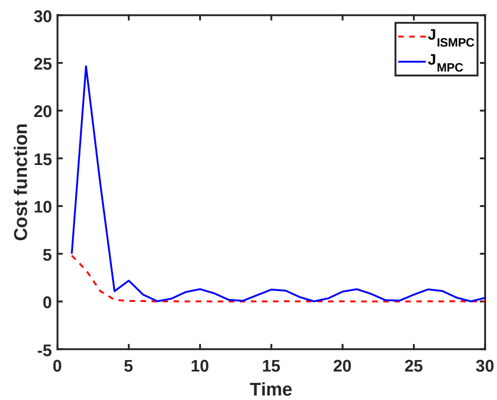

Fig. 13: The cost function $J(k)$ in the simulation test.

Figs. 9-13 show the system simulation results under the same disturbance conditions for both ISMPC and MPC respectively, where the dotted line represents ISMPC, and the solid line represents MPC. As can be seen from Fig. 9, Fig. 10 and Fig. 11, ISMPC has stronger robustness properties than MPC in dealing with external disturbances and uncertainties. Fig. 12 shows the convergence of the integral sliding variable $s(k)$, which means that the state $x(k)$ moves along the sliding surface from the initial time. Fig. 13 shows that ISMPC can maintain better system performance while demonstrating stronger robustness. The ITAE for both methods is listed in Table IV. It is clear that the ITAE of ISMPC is generally smaller than that of MPC. In summary, the simulation results and ITAE show that ISMPC is superior to MPC.

D. Summary

The ISMPC method not only has the properties of strong robustness and straightforward implementation of ISMC, but also has the advantages of MPC in being able to accommodate constrained optimization problems. In process control, ISMPC can overcome the effects of matched disturbances and uncertainties, effectively handle constraints and reduce chattering. Further, it can achieve global stability and optimization of the system. However, although the ISMPC algorithm is straightforward, the stability analysis is difficult [58], [59]. In addition, the use of discontinuous control can cause system chattering.

V. SLIDING MODE LIKE PREDICTIVE CONTROL

Besides the two mentioned SMPC, there is another sliding mode like predictive control where a predictive sliding mode approach and a corresponding control stabilize the system state to zero asymptotically. Because this approach has no obvious reaching phase and sliding phase, the authors define it as sliding mode like predictive control (SMLPC) in this review.

In [97]–[106], the SMLPC method is synthesized by applying a predictive sliding surface and a reference trajectory, combined with a state feedback correction and rolling optimization method aligned with the MPC strategy. This method not only reduces the chattering phenomenon, but also guarantees the robust stability of the closed-loop system. In [104]–[106], the SMLPC method is applied to uncertain discrete switching systems, uncertain networked control system with random time delay and networked control system with time delay and consecutive packet dropout, respectively. Research on the SMLPC method is centred on discrete-time system representations. The implementation process is described in the following.

A. SMLPC method

Assume a sliding mode function of the following form: $s(k) = C_d x(k)$. Taking the nominal model of the discrete system (17) as the prediction model, the value of the sliding function at time $k+p$ in the future is predicted as follows [97]:

$$s(k+p) = C_d A^p x(k) + \sum_{j=1}^{p} C_d A^{j-1} Bu(k+p-j) \tag{50}$$

The sliding mode prediction value of time k at time $(k-p)$ can be derived as follows:

$$s(k/k-p) = C_d A^p x(k-p) + \sum_{j=1}^{p} C_d A^{j-1} Bu(k-j) \tag{51}$$

Rewrite (51) in vector form:

$$S_p(k+1) = \Xi\, X(k) + \Theta U(k) \tag{52}$$

In practical applications, in order to correct the sliding mode prediction value $s(k+p)$, an error is defined between the actual value of the sliding variable $s(k)$ and the corresponding predicted value $s(k/k-p)$. The output value $\tilde{s}_p(k+p)$ of the sliding mode variable can be obtained as follows [97]:

$$\begin{aligned}\tilde{s}_p(k+p) &= s(k+p) + o_p e(k) \\ &= C_d A^p x(k-p) \\ &\quad + \sum_{j=1}^{p} C_d A^{j-1} Bu(k+p-j) + o_p e(k)\end{aligned} \tag{53}$$

where $e(k) = s(k) - s(k/k-p)$, $o_p = diag\left[o_p^1,\ o_p^2,\ \ldots,\ o_p^m\ \right]$ and $o_p^j > 0$ is a correction coefficient. Writing (53) in vector form:

$$\tilde{S}_p(k+1) = S_p(k+1) + \mathrm{O}_P E(k) \tag{54}$$

The corresponding optimization cost function is defined [97]:

$$\begin{aligned}J_p &= \sum_{j=1}^{N} t_j [\tilde{s}_p(k+1) - s_r(k+j)]^2 \\ &\quad + \sum_{l=1}^{M} d_l [u(k+l-1)]^2 \\ &\quad = \left\|\tilde{S}_p(k+1) - S_r(k+1)\right\| + \|U(k)\|_G^2\end{aligned} \tag{55}$$

where $s_r(k+1)$ is the sliding mode reference trajectory and t_j and d_l are weighting coefficients, respectively.

The sliding mode reference trajectory can be selected to be of many forms. One ideal case is that $s_r(k+p) = s_0(k+p) = \mathbf{0}^{m\times 1},\ S_r(k+1) = \mathbf{0}^{Nm\times 1}$. Since too rapid a convergence rate may cause overshoot, the following sliding mode reference trajectory can be selected to increase the design freedom [98]:

$$\begin{cases} s_r(k+p) = \rho s_r(k+p-1) + (I-\rho)s_0(k+p) \\ s_r(k) = s(k) \end{cases} \tag{56}$$

Letting $\frac{\partial J_p}{\partial U(k)} = 0$, the corresponding optimal control law is obtained as follows:

$$U(k) = -\left(\Theta^T\Theta + G\right)^{-1}\Theta^T\left[\Xi x(k) + O_p E(k) - S_r(k+1)\right] \tag{57}$$

Only the current control input signal is implemented by moving horizon optimization and the remaining elements in $U(k)$ are not implemented. The first element of $U(k)$ is described as follows:

$$u(k) = -[1,0,\ldots,0]^T\left(\Theta^T\Theta + G\right)^{-1}\Theta^T\left[\Xi x(k) + O_p E(k) - S_r(k+1)\right] \tag{58}$$

In the next round of optimization, (57) will serve as the initial value. When the system state reaches the sliding surface, the optimal control input $u(k)$ is provided at each step to minimize chattering by obtaining and analysing prediction errors.

The main equations for computer implementation of the SMLPC for discrete time systems are given in TABLE II and the associated pseudocode is given in Algorithm 5.

Algorithm 5 SMLPC for discrete time systems

Step 1 Set $k=0$ and give the initial state $x(0)$;
Step 2 At each time k,design a sliding surface $s(k) = C_d x(k)$;
Step 3 At time $k+p$, the value of the sliding function $s(k+p)$ is predicted;
Step 4 Set the sliding mode reference trajectory $s_r(k+p)$;
Step 5 Implement the optimal control law $u(k)$, subject to (55);
Step 6 Set $k=k+1$ and go back to Step 1.

B. Numerical example

To illustrate the performance of the proposed SMLPC, an SMC based on the reaching law (8) is introduced as a comparison. This is selected so that the sliding surface is the same as the SMLPC.

Consider the following discrete time system:

$$x(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} u(k) + d(k) \tag{59}$$

where $d(k) = \begin{bmatrix} 1 \\ -0.6 \end{bmatrix} \sin(10k)$ and is decomposed into matched and unmatched components as follows

$$d(k) = \underbrace{\begin{bmatrix} 0.2 & 0.4 \\ 0.4 & 0.8 \end{bmatrix}}_{BB^+} d(k) + \underbrace{\begin{bmatrix} 0.8 & -0.4 \\ -0.4 & 0.2 \end{bmatrix}}_{B^\perp B^{\perp +}} d(k)$$

The initial states and sliding function are given by: $x(0) = \begin{bmatrix} -3 & 1 \end{bmatrix}^T$, $s(k) = \begin{bmatrix} 2 & 3 \end{bmatrix} x(k)$. The prediction horizon and the control horizon are selected as $N_p = 10$, $N_c = 4$.

The cost function is defined by (55) and the sliding mode reference trajectory is chosen as $s_r(k+p) = \mathbf{0}$. Then, the SMLPC law (58) is implemented and the correction coefficient is chosen as $O_p = diag\begin{pmatrix} 1 & 0.8 & 0.7 & 0.6 & 0.5 & 0.4 & 0.3 & 0.2 & 0.1 & 0.05 \end{pmatrix}$.

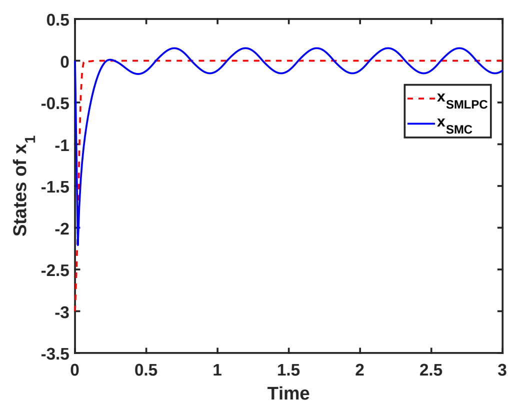

Fig. 14: The response of $x_1(k)$ in the simulation test.

The simulation results of both the SMLPC and an SMC are shown in Figs. 14-18. The same disturbance is applied in both cases. The dotted line represents SMLPC and the solid line represents SMC. It can be seen that the SMLPC can effectively

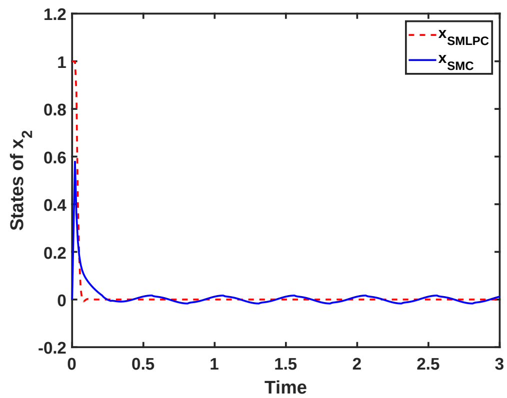

Fig. 15: The response of $x_2(k)$ in the simulation test.

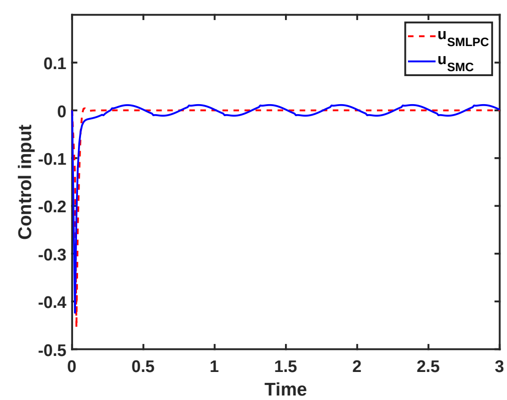

Fig. 16: The control input $u(k)$ in the simulation test.

TABLE V: ITAE for both SMLPC and SMC

ITAE	$x_1(k)$	$x_2(k)$
SMLPC	6.7208	3.5600
SMC	106.3972	11.7510

reduce system chattering and has stronger robustness than the SMC. Fig. 18 shows that SMLPC can improve system performance. The ITAE for both methods is listed in Table V. It is clear that the ITAE of SMLPC is much smaller than that of SMC. To sum up, the simulation results and ITAE show that SMLPC is superior to SMC.

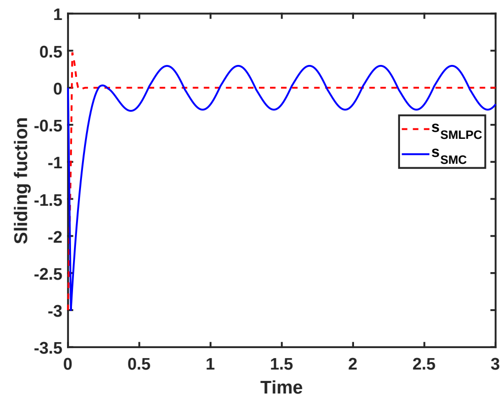

Fig. 17: The sliding function $s(k)$ in the simulation test.

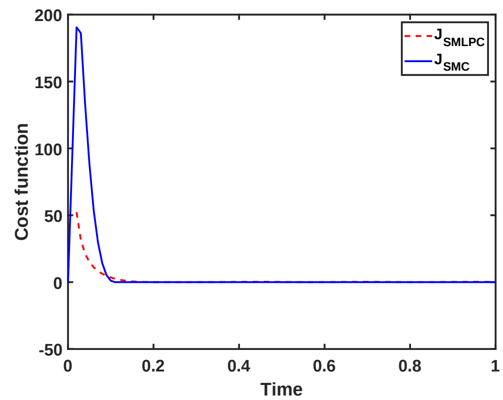

Fig. 18: The cost function $J(k)$ in the simulation test.

C. Summary

The advantages of SMLPC include: 1) The sliding mode prediction model is designed so that the future information on the sliding variable can be used. 2) The system uncertainties can be compensated by feedback correction and rolling optimization. 3) A non-switching SMC is obtained by solving a quadratic problem, so the chattering phenomenon does not exist. However, the structure of the multi-step sliding mode predictive control solved by this method is complicated and will typically increase computational load. Further, there is no reaching phase and sliding phase. Most importantly, the robustness will be reduced compared with a more standard sliding mode control approach.

VI. COMPARISON BETWEEN DMCRL, ISMPC AND SMLPC

Based on the review of DMCRL, ISMPC and SMLPC, comparative robustness testing will now be undertaken. The main equations for the computer implementation are as shown in TABLE I and TABLE II and the pseudo-codes for each implementation are as given in Algorithms 1-5. The following two comparative simulation tests are presented to compare these methods.

Example 1. Consider the following discrete time system

$$x(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} u(k) + d(k) \tag{60}$$

where $d(k) = \begin{bmatrix} 1 \\ -0.6 \end{bmatrix} \sin(10k)$ and is divided into matched and unmatched parts. DMCRL and SMLPC have the same sliding mode function $s(k) = \begin{bmatrix} 2 & 3 \end{bmatrix} x(k)$. The initial system state is $x(0) = \begin{bmatrix} 3 & 5 \end{bmatrix}^T$ and the positive definite weighting matrices are chosen as $Q = 1$, $R = 1$. The simulation results of the DMCRL, ISMPC and SMLPC are shown in Figs. 19-23, and their average computational times are $t^a_{DMCRL} = 2.1335$, $t^a_{ISMPC} = 0.1285$ and $t^a_{SMLPC} = 11.2396$, respectively. It can be seen that the computational time of SMLPC is greater than those of DMCRL and ISMPC, which shows that SMLPC is more complex with regard to implementation when compared with DMCRL and ISMPC.

TABLE VI: ITAE for DMCRL, ISMPC and SMLPC

ITAE	$x_1(k)$	$x_2(k)$
DMCRL	99.7369	260.2759
ISMPC	36.6220	51.0295
SMLPC	28.6914	20.5375

The system state responses are shown in Figs. 19 and. 20. It can be seen from Figs. 19 and 20 that

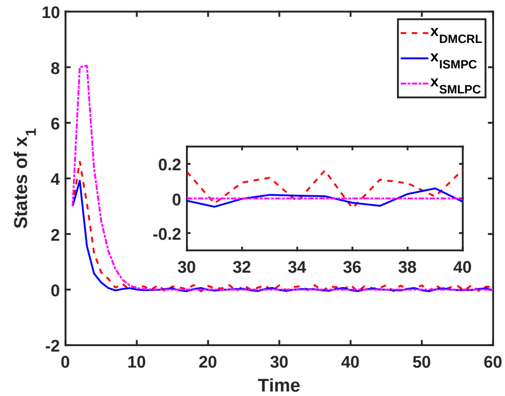

Fig. 19: The response of $x_1(k)$ in the simulation test.

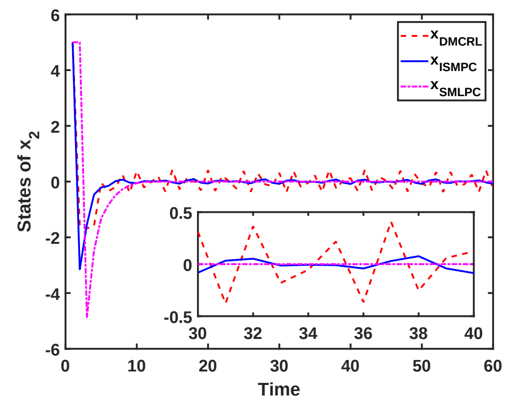

Fig. 20: The response of $x_2(k)$ in the simulation test.

all three control methods prescribe convergence of the system states under the given disturbance conditions. By contrast, when DMCRL and ISMPC are applied, the system state will chatter after reaching a neighbourhood of the origin. This phenomenon does not exist when SMLPC is applied. The evolution of the control input and sliding surface are shown in Figs. 21 and 22, respectively. It can be seen that DMCRL and ISMPC have similar dynamic performance. Compared with SMLPC, they are less able to reduce chattering in both the sliding surface and control input. Fig. 23 shows

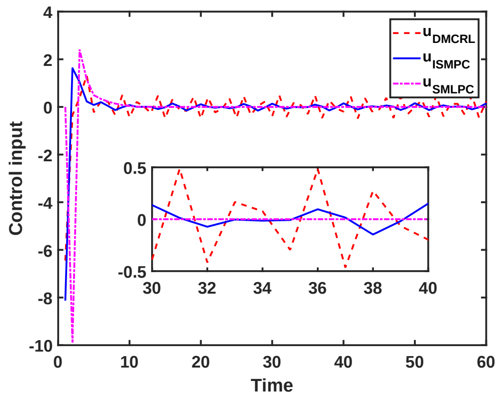

Fig. 21: The control input $u(k)$ in the simulation test.

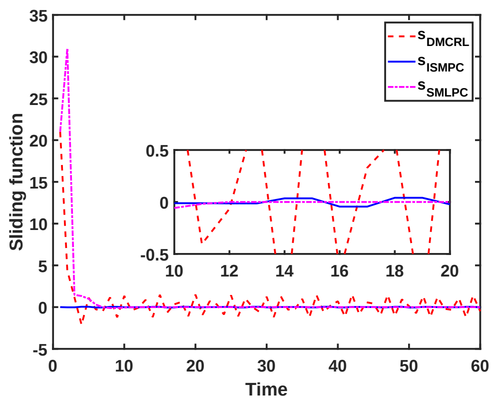

Fig. 22: The sliding function $s(k)$ in the simulation test.

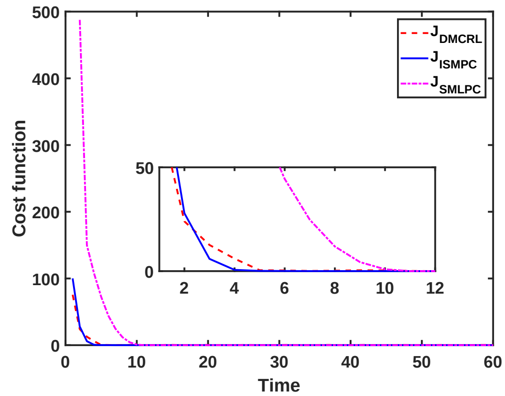

Fig. 23: The cost function $J(k)$ in the simulation test.

that the cost function value of SMLPC is higher than that of DMCRL and ISMPC. The reason for this result can be explained as follows: SMLPC sacrifices part of the energy to reduce chattering, which produced a higher cost function value and less obvious chattering than the other two methods. The ITAE for the three control methods is listed in Table VI. It can be seen from Table VI that the ITAE of SMLPC is smaller than those of DMCRL and ISMPC, which shows that SMLPC is superior to DMCRL and ISMPC.

Example 2. To further verify the methods analysed in this paper, classical SMC, MPC and proportion integral derivative (PID) control schemes are employed to provide comparative simulation results for the satellite system from [107]. The following discrete form is derived based on the forward-Euler discretization (FED) [108] method:

$$x(k+1)=\begin{bmatrix} 1 & Ts & 0 & 0 \\ 3*Ts & 1 & 0 & 2*Ts \\ 0 & 0 & 1 & Ts \\ 0 & -2*Ts & -3*Ts & 1 \end{bmatrix} x(k)+\begin{bmatrix} 0 \\ Ts \\ 0 \\ 0 \end{bmatrix} u(k)+d(k) \tag{61}$$

with the disturbance taken as $d(k) = 0.01(0.4\sin(0.8\pi k))$ and $Ts = 0.1$. The initial conditions are $x_1(0) = -0.99$, $x_2(0) = 0.5$, $x_3(0) = 0$, $x_4(0) = 0$. The design of the sliding surface is the same for SMC, DMCRL, SMLPC, which is $s(k) = \begin{bmatrix} 2.6667 & 1.0000 & -5.5000 & -2.1667 \end{bmatrix} x(k)$. The projection matrix of ISMPC is designed as $G = B^T$, and the parameter values for the PID controller are selected as $P = \begin{bmatrix} 13.0000 & 14.5000 & -12.2500 & -17.2500 \end{bmatrix}$, $D = \begin{bmatrix} -1 & -3 & -1 & -1 \end{bmatrix}$. The simulation results are shown as Figs. 24-30.

Figs. 24-27 illustrate the state trajectories for SMC, MPC, ISMPC, SMLPC, DMCRL and PID,

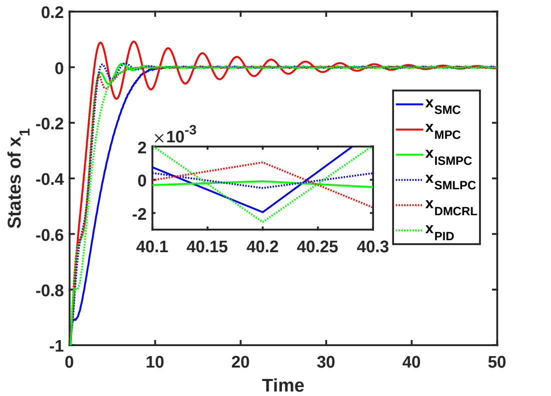

Fig. 24: The response of $x_1(k)$ in the simulation test.

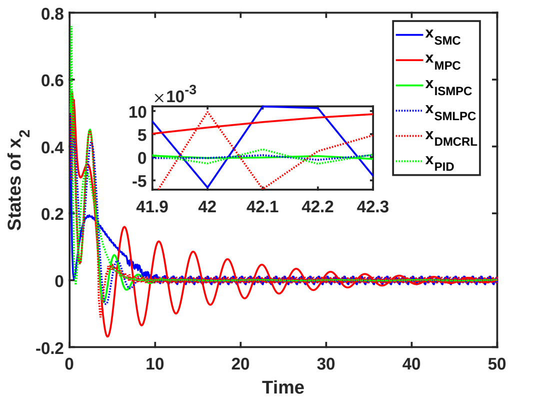

Fig. 25: The response of $x_2(k)$ in the simulation test.

TABLE VII: ITAE for the six methods

ITAE	$x_1(k)$	$x_2(k)$	$x_3(k)$	$x_4(k)$
SMC	36.3704	12.8559	8.2340	12.6626
MPC	23.8041	26.7204	46.6985	70.6670
PID	22.6911	13.5905	7.9869	12.6753
ISMPC	18.1763	11.5745	7.5902	6.1799
SMLPC	17.7352	10.3997	7.1108	3.5092
DMCRL	18.2969	11.7614	7.4969	11.1215

respectively. Under the same disturbance conditions, it is clear that MPC has obvious overshoot in contrast to the other methods. As can be seen from the magnified area, ISMPC and SMLPC are

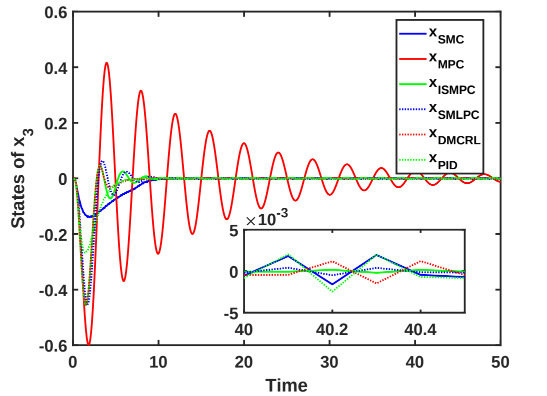

Fig. 26: The response of $x_3(k)$ in the simulation test.

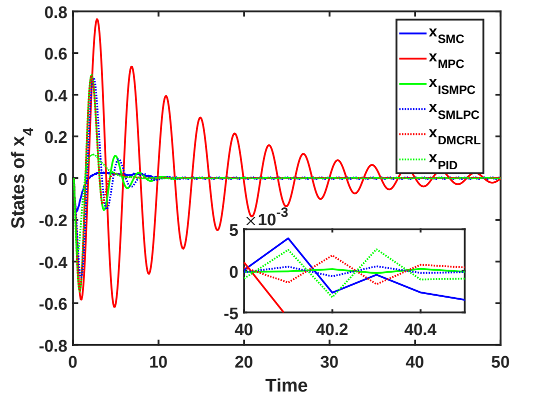

Fig. 27: The response of $x_4(k)$ in the simulation test.

superior to DMCRL, SMC and PID in reducing chattering. Figs. 28 and 29 depict the control inputs and sliding function trajectories, respectively. It can be seen from the magnified area that the trajectories of the SMLPC and ISMPC are flatter, which shows that SMLPC has stronger robustness compared with the other methods. Fig. 30 shows plots of the cost function. Overall, the control cost of PID and SMLPC is higher than the other methods. The ITAE for the six control methods is listed in TABLE VII. It is clear that the ITAE of ISMPC, SMLPC and DMCRL are generally smaller than that of SMC,

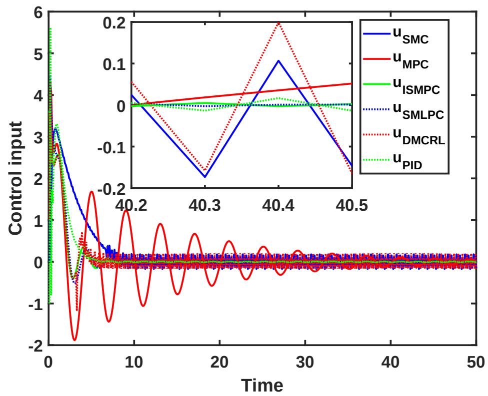

Fig. 28: The control input $u(k)$ in the simulation test.

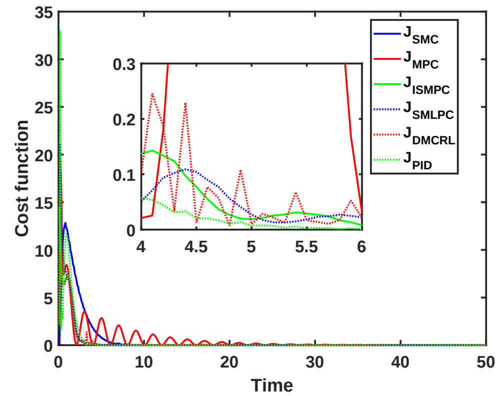

Fig. 30: The cost function $J(k)$ in the simulation test.

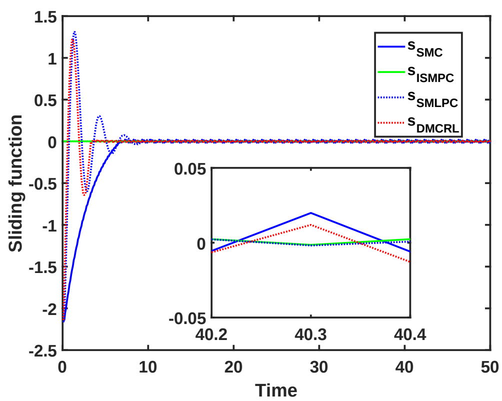

Fig. 29: The sliding function $s(k)$ in the simulation test.

MPC, PID. In summary, the simulation results and ITAE indicates that ISMPC, SMLPC and DMCRL are superior to SMC, MPC, PID.

VII. APPLICATIONS

SMPC is powerful in dealing with external disturbances and uncertainties and can improve the system performance. It has thus been widely used in various fields.

A. Application in process systems

SMPC is developed to alleviate delays and packet loss induced by network overload, and guarantee robust stability and fulfillment of constraints in [109]–[111]. Since SMPC can make the system state reach the sliding surface in an optimal manner, it is used to reduce the hysteresis effects of the piezoelectric actuator to obtain an approximate linear relationship between the input voltage and the output displacement [112], [113]. A sliding mode multi-model predictive control strategy is proposed for temperature regulation in a circulating fluidized bed (CFB) boiler. The control seeks to obtain better adaption to wide load variations [114] and SMC is used to ameliorate issues with variation in fuel quality and other unknown disturbances to enhance the safety and durability of CFB boilers. The application of the SMPC and a Volterra-MPC is compared and analyzed in a benchmark CSTR non-minimum phase reactor [115]. Because the switching frequency of the SMC is reduced in the SMPC method, it is used to reduce the vibration experienced by a cantilevered aluminum beam in [116]. The SMPC is applied to improve greenhouse inside air temperature control in [117]. The SMPC is also applied to heat exchangers and other temperature systems [118], [119].

B. Application in electromechanical systems

The SMPC method has been applied for motion control of robot manipulators [120]–[122]. Here the SMC component is employed to compensate unmodeled system dynamics and disturbances. The SMPC approach is verified for a COMAU Smart3-S2 industrial robot manipulator and 7-DOF prototype ABB YuMi robot arm [120], [121]. It has also been applied to driving, obstacle avoidance, steering and idling of autonomous vehicles [123]–[126]. SMPC has been used in flight control to eliminate the multi-frequency helicopter vibrations [127]–[129], and it is also applied to motor drives [130], [131]. Recently, SMPC strategies have been employed in space tethered satellite systems [132].

C. Application in energy systems

In this application domain, the SMPC method has been used to improve the utilization of solar energy by reducing the impact of disturbances caused by changes of solar irradiation and solar collector inlet temperature [133], [134]. It is also applied to frequency regulation in power grids [135], [136]. The SMPC method is applied in energy management system of microgrids, in which grid-connected and islanded operation modes are controlled by SMC, and MPC generates power reference for the energy storage systems [137]. The method has also been used to reduce the effect of unexpected disturbances such as the terminal DC-link voltage step and the load variation [138], [139], and it is also employed for synchronous DC-DC buck converter to further reduce the chattering phenomenon, steady-state error, overshoot and undershoot [140].

D. Application in traffic systems

The SMPC is applied to solve the mainstream inflow, the traffic demands on the on-ramps and the flows exiting the off-ramps problems in freeway traffic systems. Here, via the SMC component, the approach provides valuable robustness to uncertainties and provides fast response while MPC provides optimality [141], [142].

In general, for the DMCRL and SMLPC methods, due to the need for accurate system model information and the complexity of the calculations, there are few industrial applications. However, the application of the ISMPC method has achieved great success, especially in process control systems. It is considered an effective method to improve performance and enhance robustness. Wider application will promote new developments in the future. A summary of the application of the three methods is shown in the following table:

TABLE VIII: The application of the three methods

Control methods	DMCRL	ISMPC	SMLPC
Process systems	[112], [113], [116]	[109]–[111], [114] [115], [117]–[119]	\
Electromechanical systems	\	[120]–[132]	\
Energy systems	\	[133]–[139]	[140]
Traffic systems	\	[141], [142]	\

VIII. SOME CHALLENGING PROBLEMS AND FUTURE DIRECTIONS

Some methods, contributions and applications have been reviewed in the area of SMPC. However, there are still many challenging issues to be addressed. Below, several issues and further directions are discussed.

I. It is clear that the SMPC strategy continues to attract increasing attention. Most of the existing SMPC methods are used under the condition that the system states are available. However, system states may not all be available in practice, This has motivated the study of output feedback control. An observer may be used when the system state is unmeasurable, but this greatly increases the complexity of the system. It is desirable to develop an SMPC strategy that uses only measured output information.

II. SMPC is designed based on a mathematical model of the system assuming known information. However a mathematical model may be difficult to establish in many practical applications. Research which considers how to relax the dependence of an SMPC on the model used for design is valuable [143], [144].

III. Chattering may be difficult to eliminate because of the discontinuous nature of SMC, particularly in sampled situations. Though the chattering can be reduced by SMPC, it still exists. The larger the switching range of the control variables, the more obvious the chattering. In high-precision fields such as aerospace engineering and satellite systems, chattering may not be tolerable. More effort is required to find a way to eliminate chattering in the design of SMPC.

IV. The design of SMPC based on fuzzy models is a challenging problem. It is well known that T-S fuzzy systems can be used to approximate complex systems, and the key feature is to decompose nonlinear systems into several linear subsystems. Although this approach can greatly simplify modelling of the system, uncertainties inevitably exist. Considering the strong robustness of SMPC, the combination of fuzzy methods and SMPC is a novel direction [145]–[147].

V. SMPC applied in large-scale distributed systems is a worthy research topic. Currently, information exchange is cheaper and faster with the rapid development of network technology, which facilitates the control of large-scale systems. Distributed control has become a hot topic in the control of large-scale systems. In distributed systems, there exist problems such as computational complexity, uncertainties and coupling. SMPC is a good choice to address such problems because of its strong robustness and ability to incorporate optimization. As a result, applying SMPC to distributed systems can be a focus of future research.

VI. The combination of SMC with economic model predictive control (EMPC) is a further interesting topic for future research. In recent years, the research on EMPC has become quite mature. On the one hand, the requirements are becoming more and more stringent for economic performance indicators in industrial production processes. On the other hand, strong robust control is required due to the uncertainties and variabilities of the production environment. The addition of SMC may enhance EMPC.

VII. The design of event-triggered SMPC is also worthy of future study. Since event-triggered control is an effective method to reduce computational cost, its combination with SMPC has the potential to enhance robustness while at the same time reducing computational cost [109], [110].

IX. CONCLUSIONS

In this paper, the past and recent research results concerning SMPC have been reviewed. It has been seen that MPC can be used in the reaching phase and/or the sliding phase when designing an SMC. The advantages and disadvantages have been summarized and demonstrated using examples. The SMPC strategy not only overcomes the influence of matched disturbances and uncertainties to increase system robustness, but also effectively reduces system chattering to realize global optimization of the closed-system. It should be pointed out that SMPC has been studied for many years and seen many applications. However, it is necessary to conduct further study to eliminate the gap between theory and practical application. In the future, this type of control can be further developed for classes of systems including large-scale systems which may be distributed.

REFERENCES

[1] J L Ding, C E Yang, Y D Chen, T Y Chai. Research Progress and Prospects of Intelligent Optimization Decision Making in Complex Industrial Process [J]. Acta Automatica Sinica, 44(11), pp. 1931-1943, 2018.

[2] F Qian. Smart and optimal manufacturing: the key for the transformation and development of the process industry [J]. Engineering, 3(2), pp. 151-151, 2017.

[3] H Ding, R X Gao, A J Isaksson, et al. State of AI-Based Monitoring in Smart Manufacturing and Introduction to Focused Section [J]. IEEE-ASME Transactions on Mechatronics, 25(5), pp. 2143-2154, 2020.

[4] F Qian, W L Du, W M Zhong, Y Tang. Problems and Challenges of Smart Optimization Manufacturing in Petrochemical Industries [J]. Acta Automatica Sinica, 43(6), pp. 893-901, 2017.

[5] X Li, C Zhang. Some new trends of intelligent simulation optimization and scheduling in intelligent manufacturing [J]. Service Oriented Computing and Applications, 14(3), pp. 149-151, 2020.

[6] C Ishii, T Shen, K Tamura. Robust model following control for a robot manipulator [J]. Control Theory and Applications, IEE Proceedings-Control Theory and Applications, 144(1), pp. 53-60, 1997.

[7] W Lin, T Shen. Robust passivity and feedback design for minimum-phase nonlinear systems with structural uncertainty [J]. Automatica, 35(1), pp. 35-47, 1999.

[8] C D Mu, S W Mei, T Shen. New Developments in Robust Nonlinear Control Theory [J]. Control Theory and Applications, 18(1), pp. 1-6, 2001.

[9] I Postlethwaite, M C Turner, G Herrmann. Robust control applications [J]. Annual Reviews in Control, 31(1), pp. 27-39, 2007.

[10] Miroslav Kristic, Ioannis Kanellakopoulis, Petar Kokotovic. Nonlinear and Adaptive Control Design [M]. New York: Wiley, a Wiley InterScience Publication, 1995.

[11] K S Narendra, J Balakrishnan. Adaptive control using multiple models [J]. IEEE Transactions on Automatic Control, 42(2), pp. 171-187, 1997.

[12] Z Qu, R A Hull, J Wang. Globally Stabilizing Adaptive Control Design for Nonlinearly Parameterized Systems [J]. IEEE Transactions on Automatic Control, 51(6), pp. 1073-1079, 2006.

[13] V Utkin, J Guldner, J Shi. Sliding Modes in Electro-Mechanical Systems [M]. Crc Press, Taylor and Francis, London, 1999.

[14] J K Liu. Sliding mode control design and MATLAB simulation: the basic theory and design method [M]. 2015.

[15] Wang Z, Ding D, Dong H, et al. H-infinity Consensus Control for Multi-agent Systems with Missing Measurements: The Finite-horizon Case [J]. Systems and Control Letters, 62(10), pp. 827-836, 2013.

[16] K. Oh, K. L. Moore and H. Ahn. Disturbance Attenuation in a Consensus Network of Identical Linear Systems: An H-infinity Approach [J]. IEEE Transactions on Automatic Control, 59(8), pp. 2164-2169, 2014.

[17] Mu X , Xiao X , Liu K , et al. Leader-following Consensus of Multi-agent Systems with Jointly Connected Topology Using Distributed Adaptive Protocols[J]. Journal of the Franklin Institute, 351(12), pp. 5399-5410, 2014.

[18] C Edwards, S K Spurgeon. Sliding Mode Control: Theory and Applications [M]. Crc Press, Taylor and Francis, London, 2009.

[19] G P Incremona, M Rubagotti, A Ferrara. Sliding Mode Control of Constrained Nonlinear Systems [J]. IEEE Transactions on Automatic Control, 62(6), pp. 2965-2972, 2016.

[20] V I Utkin. Variable Structure Systems with Sliding Modes [J]. IEEE Transactions Automatic Control, 22(2), pp. 212-222, 1977.

[21] K D Young, V I Utkin, U Ozguner. A Control Engineer's Guide to Sliding Mode Control [J]. IEEE Transactions on Control Systems Technology, 7(3), pp. 328-342, 1999.

[22] C Edwards, S K Spurgeon. Sliding Mode Control: Theory and Applications [M]. Crc Press Taylor and Francis Group, 1998.

[23] Z Chen, J Zhang, J Zeng. Integral Sliding Mode Variable Structure Control Based on Fuzzy Logic[C]. Hefei, China: 3th World Congress on Intelligent Control and Automation, 2000.

[24] J H Lee, E Paul, Gang T. Integral Sliding Mode Control of a Magnetically Suspended Balance Beam: Analysis, Simulation, and Experiment [J]. IEEE Transaction on Mechanical, 6(3), pp. 338-346, 2001.

[25] T L Chern, J S Wong. DSP Based Integral Variable Structure Control for Motor Servo Drives. IEE Proceeding Control Theory Applications, 142(5), pp. 444-450, 1995.

[26] J D Wang, T L Lee, Y T Juang. New Methods to Design an Integral Variable Structure Controller. IEEE Transactions on Automatic Control, 41(1), pp. 140-143, 1996.

[27] J H Lee, P E Allaire, G Tao. Integral Sliding Mode Control of a Magnetically Suspended Balance Beam: Analysis, Simulation and Experiment [J]. IEEE/ASME Transactions on Mechatronics, 6(3), pp. 338-346, 2001.

[28] A Poznyak, L Fridman, F J Bejarano. Min-max Integral Sliding-Mode Control for Multi-model Linear Uncertain Systems [J]. Transactions on Automatic Control, 49(1), pp. 97-102, 2004.

[29] M Rubagotti, A Estrada, F Castanos, et al. Integral Sliding Mode Control for Nonlinear Systems With Matched and Unmatched Perturbations [J]. IEEE Transactions on Automatic Control, 56(11), pp. 2699-2704, 2011.

[30] R H Sira, S O Llanes. Adaptive Dynamical Sliding Mode Control via Backstepping [C]. Texas, USA: 32nd IEEE Conference on Decision and Control, 1993, 2: 1422-1427.

[31] M R Bolivar, ASI Zinober, H Sira-Ramirez. Dynamic Adaptive Sliding Mode Output Tracking Control of a Class of Nonlinear Systems [J]. International Journal of Robust and Nonlinear Control, 7(4), pp. 387-405, 1997.

[32] G Wheeler, C H Su, Y Stepanenko. A Sliding Mode Controller with Improved Adaptation Laws for the Upper Bounds on the Norm of Uncertainties [J]. Automatica, 34(12), pp. 1657-1661, 1998.

[33] Y J Huang, T C Kuo, S H Chang. Adaptive Sliding-Mode Control for Nonlinear Systems with Uncertain Parameters [J]. IEEE Transactions on Systems Man and Cybernetics-Part B: Cybernetics, 38(2), pp. 534-539, 2008.

[34] X Chen. Adaptive Sliding Mode Control for Discrete-time Multi-Input Multi-Output Systems [J]. Automatica, 42(3), pp. 427-435, 2006.

[35] S C Lin, Y Y Chen. RBF Network Based Sliding Mode Control[C]. IEEE International Conference on Systems Man and Cybernetics, pp. 1957-1961, 1994.

[36] G Parma, B R Menezes, A P Braga, M A Costa. Sliding Mode Neural Network Control of an Induction Motor Drive [J]. International Journal of Adaptive Control and Signal Processing, 17(6), pp. 501-508, 2003.

[37] M Ertugrual, O Kaynak. Neuro Sliding Mode Control of Robotic Manipulators [J]. Mechatronics, 10(1-2), pp. 239-263, 2000.

[38] T Sun, H Pei, Y Pan, et al. Neural Network-based Sliding Mode Adaptive Control for Robot Manipulators [J]. Neurocomputing, 74(14-15), pp. 2377-2384, 2011.

[39] J Fei, H Ding. Adaptive Sliding Mode Control of Dynamic System Using RBF Neural Network [J]. Nonlinear Dynamics, 70(2), pp. 1563-1573, 2012.

[40] S W Kim, J J Lee. Design of a Fuzzy Controller with Fuzzy Sliding Surface [J]. Fuzzy Sets and Systems, 71(3), pp. 359-367, 1995.

[41] X Yu, Z Man, B Wu. Design of Fuzzy Sliding-Mode Control Systems [J]. Fuzzy Sets and Systems, 95(3), pp. 295-306, 1998.

[42] S Tong, X Li H. Fuzzy Adaptive Sliding-mode Control for MIMO Nonlinear Systems [J]. IEEE Transactions on Fuzzy Systems, 11(3), pp. 354-360, 2003.

[43] R J Wai, C M Lin, C F Hsu. Adaptive Fuzzy Sliding-mode Control for Electrical Servo Drive [J]. Fuzzy Sets and Systems, 143(2), pp. 295-310, 2004.

[44] Y Wang, H Shen, H R Karimi, et al. Dissipativity-based Fuzzy Integral Sliding Mode Control of Continuous-Time T-S Fuzzy Systems [J]. IEEE Transactions on Fuzzy Systems, 26(3), pp. 1164-1176, 2018.

[45] C Z mei, Z J gang, Z J chao. Integral Sliding Mode Variable Structure Control Based on Fuzzy Logic [J]. Pump Industry Analyst, 4(4), pp. 3009-3012, 2000.

[46] M Morari, J H Lee. Model predictive control: Past, Present and Future [J]. Computers and Chemical Engineering, 23(4), pp. 667-682, 1999.
[47] D Q Mayne, J B Rawlings, C V Rao, P O M Scokaert. Constrained Model Predictive Control: Stability and Optimality [J], Automatica, 36(6), pp. 789-814, 2000.
[48] Fontes, A C Fernando, C Magni. Min-Max Model Predictive Control of Nonlinear Systems Using Discontinuous Feedbacks [J]. IEEE Transactions on Automatic Control, 48(10), pp. 1750-1755, 2003.
[49] A Richards, J How. Robust stable model predictive control with constraint tightening [C]. American Control Conference, pp. 1557-1562, 2006.
[50] W Langson, I Chryssochoos, S V Rakovi, et al. Robust model predictive control using tubes [J]. Automatica, 40(1), pp. 125-133, 2004.
[51] M L Corradini, G Orlando. A VSC Algorithm Based on Generalized Predictive Control [J]. Automatica, 33(5), pp. 927-932, 1997.
[52] M Perez de la Parte, O Cammacho, E F Comacho. Development of a GPC-based Sliding Mode Controller [J]. Isa Transactions, 41(1), pp. 19-30, 2002.
[53] D M Raimondo, M Rubagotti, C N Jones, L Magni, A Ferrara, M Morari. Multirate Sliding Mode Disturbance Compensation for Model Predictive Control [J]. International Journal of Robust and Nonlinear Control, 25(6), pp. 2984-3003, 2015.
[54] J S Zhou, Z Y Liu, R Pei. Nonlinear Model Predictive Control Scheme with Terminal Sliding Mode Constraint [J]. Control and Decision, 16(4), pp. 473-475, 2001.
[55] J S Zhou, Z Y Liu, R Pei. A New Nonlinear Mode Predictive Control Scheme for Discrete-time System Based on Sliding Mode Control [C]. American Control Conference, 4, pp. 3079-3084, 2001.
[56] J S Zhou, Z Y Liu, R Pei. Sliding Mode Model Predictive Control with Terminal Constraints[C]. 3rd World Congress on Intelligent Control and Automation, 4, pp. 2791-2795, 2000.
[57] W Garcia-Gabin, E F Camacho, Sliding Mode Model Based Predictive Control for Non-minimum Phase Systems [C]. European Control Conference, pp. 904-909, 2003.
[58] M Rubagotti, D M Raimondo, A Ferrara. Robust Model Predictive Control with Integral Sliding Mode in Continuous time Sampled data Nonlinear Systems [J]. IEEE Transactions on Automatic Control, 56(3), pp. 556-570, 2011.
[59] Y W Liao, J K Hedrick. Robust Model Predictive Control with Discrete-time Integral Sliding Surface [C]. American Control Conference, pp. 1641-1646, 2015.
[60] R Errouissi, J Yang, W H Chen. Robust Nonlinear Generalized Predictive Control for a Class of Uncertain Nonlinear Systems via an Integral Sliding Mode Approach [J]. International Journal of Control, 89(8), pp. 1698-1710, 2016.
[61] M Rubagotti, G P Incremona, D M Raimondo, A Ferrara. Constrained Nonlinear Discrete-Time Sliding Mode Control Based on a Receding Horizon Approach [J]. IEEE Transactions on Automatic Control, 66(8), pp. 3802-3809, 2021.
[62] J Y Hung, W Gao. Variable Structure Control: A Survey [J]. IEEE Transactions on Industrial Electronics, 40(1), pp. 2-22, 1993.
[63] W Gao, J C Hung. Variable Structure Control of Nonlinear Systems: A New Approach [J]. IEEE Transactions on Industrial Electronics, 40(1), pp. 45-55, 1993.
[64] J Jun, Q W Chen, J Guo, W H Fan. Sliding Mode Stable Tracking Control for Mobile Satellite Communication System Based on a New Reaching Law [J].Control and Decision, 26(12), pp. 1904-1908, 2011.
[65] Z H Yao, Y Sun, C S Tang. An improved sliding mode reaching law for continuous-time system [J]. Journal of Chongqing University, 36(4), pp. 51-55, 2013.
[66] A Bartoszewicz, A new reaching law for sliding mode control of continuous time systems with constraints [J]. Transactions of the Institute of Measurement and Control, 37(4), pp. 515-521, 2015.
[67] D. Milosavljevic, General Conditions for the Existence of a Quasi-sliding Mode on the Switching Hyperplane in Discrete Variable Structure Systems [J]. Automation and Remote Control, 46 pp. 307-314, 1985.
[68] S Z Sarpturk, Y Istefanopulos, O Kaynak. On the Stability of Discrete-time Sliding Mode Control Systems [J]. IEEE Transactions on Automatic Control, 32(10), pp. 930-932, 1987.
[69] G W bing, W Y fu, Abdollah Homaifa, Discrete-time Variable Structure Control Systems [J]. IEEE Transactions on Industrial Electronics, 42(2), pp. 117-122, 1995.
[70] A Bartoszewicz. Discrete-time Quasi-sliding Mode Control Strategies [J]. IEEE Transactions on Industrial Electronics, 45(4), pp. 633-637, 1998.
[71] C Vivekanandan, R Prabhakar, M Gnanambigai. A Redefined Quasi-Sliding Mode Control Strategy [C]. Processing of World Academy of Science Engineering and Technology, 29, pp. 292-295, 2008.
[72] Y Niu, Z Wang. Improved Sliding Mode Control for Discrete-time Systems via Reaching Law [J]. IET Control Theory and Applications, 4(11), pp. 2245-2251, 2010.
[73] S H Yu, W Y Qiang, P C Fu. Chattering-free Discrete Quasi-sliding Mode Controller [J]. Control and Decision, 16(3), pp. 380-382, 2001.
[74] L Z Song, H S Li, Q H Yao. Discrete-time Variable-structure Control Based on Reaching Law Approach [J]. Control Theory and Applications, 25(3), pp. 525-528, 2008.
[75] B P Kang, J L Ju. Sliding Mode Controller with Filtered Signal for Robot Manipulators Using Virtual Plant Controller [J]. Mechatronics, 7(3), pp. 277-286, 1997.
[76] M Hamerlain, T Youssef, M Belhocine. Switching on the Derivative of Control to Reduce Chatter [J]. IEE Proceeding on Control Theory and Application, 148(1), pp. 81-96, 2001.
[77] G Bartolini, A Ferrara, E Usai. Chattering Avoidance by Second-order Sliding Mode Control [J]. IEEE Transactions on Automatic Control, 43(2), pp. 241-246, 1998.
[78] A Kawamura, H Itoh, K Sakamoto. Chattering Reduction of Disturbance Observer Base Sliding Mode Control [J]. IEEE Transaction on Industry Application, 30(2), pp. 456-461, 1994.
[79] W K Kwon, S Jeong, S Won. Sliding Mode Control for Uncertain Discrete-Time System Using Robust Model Predictive Control in Reaching Mode [C]. International Control Conference Automation and Systems, pp. 39-43, 2012.

[80] Mayne D Q, Rawlings J B, Rao C V, et al. Constrained Model Predictive Control: Stability and optimality [J]. Automatica, 2000, 36(6):789-814.

[81] C Hong, A ower Frank. A Quasi-Infinite Horizon Predictive Control Scheme for Constrained Nonlinear Systems [J]. Control Theory and Applications, 16(3), pp. 313-319, 1999.

[82] V I Utkin, J X Shi. Integral Sliding Mode in Systems Operating Under Uncertainty Conditions[C]. Kobe, Japan: 35th IEEE Conference on Decision and Control, 1996.

[83] K Abidi, X J Xu, Y X Huo. On the Discrete-time Integral Sliding-Mode Control [J]. IEEE Transactions on Automatic Control, 52(4), pp. 709-715, 2007.

[84] F Castanos, L Fridman. Analysis and Design of Integral Sliding Manifolds for Systems with Unmatched Perturbations [J]. IEEE Transactions on Automatic Control, 51(5), pp. 853-858, 2006.

[85] Z Xi, T Hesketh. Brief Paper: Discrete Time Integral Sliding Mode Control for Systems with Matched and Unmatched Uncertainties [J]. IET Control Theory and Applications, 4(5), pp. 889-896, 2010.

[86] M Steinberger, I Castillo, M Horn, L Fridman. Model Predictive Output Integral Sliding Mode Control[C]. International Workshop on Variable Structure Systems, pp. 228-233, 2016.

[87] M Steinberger, I Castillo, M Horn, L Fridman. Robust Output Tracking of Constrained Perturbed Linear Systems Via Model Predictive Sliding Mode Control [J]. International Journal of Robust and Nonlinear Control, 30(3), pp. 1258-1274, 2019.

[88] H Xiao, D Zhao, Discrete-Time Dynamic Output Feedback Integral Sliding Model Predictive Control [C]. 39th Chinese Control Conference, pp. 2357-2362, 2020.

[89] H Caballero-Barragan, LP Osuna-Ibarra, AG Loukianov, F Plestan. Sliding Mode Predictive Control of Linear Uncertain Systems with Delays [J]. Automatica, 94, pp. 409-415, 2018.

[90] Q Zhang, Y Shi, K Wu. Self-Triggered Robust MPC with ISM for Constrained Nonlinear Input-Affine Systems [C]. International Symposium on Industrial Electronics, pp. 2175-2180, 2019.

[91] S J Fesharaki, F Sheikholeslam, M Kamali. Tractable Robust Model Predictive Control with Adaptive Sliding Mode for Uncertain Nonlinear systems [J]. International Journal of Systems science, 51(12), pp. 2204-2216, 2020.

[92] Y W Liao, J K Hedrick. Discrete-time integral sliding model predictive control for unmatched disturbance attenuation[C]. American Control Conference (ACC), pp. 2675-2680, 2016.

[93] W J Cao, J X Xu. Nonlinear Integral-Type Sliding Surface for both Matched and Unmatched Uncertain Systems [J]. IEEE Transactions on Automatic Control, 49(8), pp. 1355-1360, 2004.

[94] E D Sontag, Y Wang. New Characterizations of Input-to-State Stability [J]. IEEE Transactions on Automatic Control, 41(9), pp. 1283-1294, 1996.

[95] Z P Jiang, A R Teel, L Praly. Small-Gain Theorem or ISS Systems and Applications [J]. Mathematics of Control, Signals and Systems, 7(2), pp. 95-120, 1994.

[96] Sontag, D Eduardo. On the Input-to-State Stability Property [J]. European Journal of Control, 1(1), pp. 24-36, 1995.

[97] L Z Song, S C Chen, Q H Yao. Discrete Variable Structure Control Algorithm Based on Sliding Mode Prediction [J]. Control Theory and Applications, 21(5), pp. 826-829, 2004.

[98] L Z Song, S C Chen, Q H Yao. Sliding Mode Predictive Control for Multi-Input Discrete-time Systems [J]. Electric Machines and Control, 9(2), pp. 128-132, 2005.

[99] L Z Song, S M Yan, L Q Yang. Robustness of Discrete-time Sliding Mode Prediction Control [J]. Journal of Naval University of Engineering, 22(5), pp. 40-44, 2010.

[100] L F Xiao. Sliding Mode Control for Discrete-time System Based on Predictive Control Strategy [D]. Zhejiang University, 2008.

[101] B M Houda, A Nizar, N A Said, Predictive Sliding Mode Control for Perturbed Discrete Delay Time Systems: Robustness Analysis [C]. International Conference on Electrical Engineering and Software Applications, pp. 1-6, 2013.

[102] H B Mansour. Discrete Predictive Sliding Mode Control for Multivariable Systems [C]. International Conference on Sciences and Techniques of Automatic Control and Computer Engineering, 2015.

[103] B M Houda, N A Said, Discrete Predictive Sliding Mode Control of Uncertain Systems [C]. International Multi-Conference on Systems, Signals and Devices, Chemnitz, pp. 1-6, 2012.

[104] Z L HE, M Wang, S H Liu. Discrete Sliding Mode Prediction Control of Uncertain Switched Systems [J]. Journal of Systems Engineering and Electronics, 20(5), pp. 1065-1071, 2009.

[105] Y Zhang, S Xie, L Zhang. Robust Sliding Mode Predictive Control of Uncertain Networked Control System with Random Time Delay [J]. Discrete Dynamics in Nature and Society, 2018, 2018.

[106] Y Zhang, S Xie, L Ren, et al. A new Predictive Sliding Mode Control Approach for Networked Control Systems with Time Delay and Packet Dropout [J]. IEEE Access, 7, pp. 134280-134292, 2019.

[107] S Pradeep. A New Tension Control Law for Deployment of Tethered Satellites [J]. Mechanics Research Communications, 24(3), pp. 247-254, 1997.

[108] X Li, G Sun, X Shao. Discrete-time Pure-tension Sliding Mode Predictive Control for the Deployment of Space Tethered Satellite with Input Saturation [J]. Acta Astronautica, 170, pp. 521-529, 2020.

[109] A Ferrara, G P Incremona, L Magni. Model-based Event-triggered Robust MPC/ISM [C]. European Control Conference, pp. 2931-2936, 2014.

[110] G P Incremona, A Ferrara, L Magni. Asynchronous Networked MPC with ISM for Uncertain Nonlinear Systems [J]. IEEE Transactions on Automatic Control, 62(9), pp. 4305-4317, 2017.

[111] H Li, H Yang, F Sun, et al. Sliding-Mode Predictive Control of Networked Control Systems under a Multiple-Packet Transmission Policy [J]. IEEE Transactions on Industrial Electronics, 61(11), pp. 6234-6243, 2014.

[112] B Kim, G Washington, H S Yoon. Hysteresis-reduced Dynamic Displacement Control of Piezoceramic Stack

Actuators Using Model Predictive Sliding Mode Control [J]. Smart Materials and Structures, 21(5), 2012.
[113] B Kim, G Washington. Nonlinear Position Control of Smart Actuators Using Model Predictive Sliding Mode Control [C]. Smart Materials, Adaptive Structures and Intelligent Systems, 2008.
[114] H Zhu, J Shen, K Y Lee. Multi-model Based Predictive Sliding Mode Control for Bed Temperature Regulation in Circulating Fluidized Bed Boiler [J]. Control Engineering Practice, 101, 2020.
[115] W Garcia-Gabin, F Dorado, C Bordons. Comparative Analysis of Sliding Mode and Volterra Model Predictive Controllers Applied to a Benchmark CSTR Non-minimum Phase Reactor [C]. European Control Conference, pp. 1662-1667, 2007.
[116] B Kim, G Washington. Active Vibration Control of a Cantilevered Beam Using Model Predictive Sliding Mode Control[C]. Structural Dynamics, and Materials Conference 16th AIAA/ASME/AHS Adaptive Structures Conference 10t. 2008.
[117] J B Oliveira, J Boaventura-Cunha, PB M Oliveira. A Feasibility Study of Sliding Mode Predictive Control for Greenhouses [J]. Optimal Control Applications and Methods, 37(4), pp.730-748, 2016.
[118] M Parte, E F Camacho. Application of a predictive Sliding Mode Controller to a Heat Exchanger [J]. IEEE International Conference on Control Applications, pp. 1219-1224, 2002.
[119] C Chao, L Yum, D Su. Compound Sliding-mode Predictive Control for a Temperature System of High-speed Heat-airflow Wind Tunnel [J]. Proceedings of the Institution of Mechanical Engineers, Part C-Journal of Mechanical Engineering Science, 228(11), pp. 1869-1879, 2014.
[120] G P Incremona, Ferrara A, Magni L. MPC for Robot Manipulators with Integral Sliding Modes Generation [J]. IEEE/ASME Transactions on Mechatronics, 22(3), pp. 1299-1307, 2017.
[121] D Nicolis, F Allevi, P Rocco. Operational Space Model Predictive Sliding Mode Control for Redundant Manipulators. IEEE Transactions on Robotics, 36(4). pp. 1348-1355, 2020.
[122] K R Muske, H Ashrafiuon, M Nikkhah. A Predictive and Sliding Mode Cascade Controller [C]. American Control Conference, pp. 44540-4545, 2007.
[123] D J Kim, C M Kang, S Lee, C C Chung. Discrete-Time Integral Sliding Model Predictive Control for Dynamic Lateral Motion of Autonomous Driving Vehicles [C]. Annual American Control Conference, Milwaukee, pp. 4757-4763, 2018.
[124] L Yi Wen, J Karl Hedrick. A Discrete-Time Integral Sliding Model Predictive Control for Obstacle Avoidance of Ground Vehicles [C]. ASME 2015 Dynamic Systems and Control Conference, 2015.
[125] C Huang, F Naghdy, H Du. Fault Tolerant Sliding Mode Predictive Control for Uncertain Steer-by-Wire System [J]. IEEE Transactions on Cybernetics, 49(1), pp. 261-272, 2019.
[126] S H Lee, C C Chung. Predictive Control with Sliding Mode for Autonomous Driving Vehicle Lateral Maneuvering [C]. American Control Conference. pp. 2998-3003, 2017.
[127] M Xun, L Yang, W Feng. Experimental Investigations on Active Control of Multi-frequency Helicopter Vibrations Using Discrete Model Predictive Sliding Mode Control [J]. Proceedings of the Institution of Mechanical Engineers Part G-Journal of Aerospace Engineering, 232(15), pp. 2898-2909, 2018.
[128] Y Lu, X J Ma. Active Control of Multi-frequency Helicopter Vibrations Using Discrete Model Predictive Sliding Mode Control [J]. Proceedings of the Institution of Mechanical Engineers Part G-Journal of Aerospace Engineering, 230(4), pp. 668-680, 2016.
[129] A Razzaghian, M Yaghoobi. Predictive Sliding Mode Control for Small-scale Unmanned Helicopter [C]. 4th International Conference on Applied Research in Electrical, Mechanical and Mechatronics Engineering. 2017.
[130] Y Zhong, Z Yang, T Xiang, Y Ru. Model Predictive Control Using Globe Exponential Reaching Law Sliding Mode Design Method for Induction Motor Drives [C]. Applied Power Electronics Conference and Exposition (APEC), pp. 2559-2563, 2019.
[131] Y F Chen, Y Q Wang, K Lu. Model Predictive Control Based on Sliding-Mode Observer for Speed Sensorless Asynchronous Motor [J]. Control and Application Technology, 46(11), pp. 12-16, 2016.
[132] X Li, G Sun, X Shao. Discrete-time Pure-tension Sliding Mode Predictive Control for the Deployment of Space Tethered Satellite with Input Saturation [J]. Acta Astronautica, 170, pp. 521-529, 2020.
[133] W Garcia-Gabin, D Zambrano, E F Camacho. Sliding Mode Predictive Control of a Solar Air Conditioning Plant [J]. Control Engineering Practice, 17(6), pp. 652-663, 2009.
[134] Parte de la. Application of Predictive Sliding Mode Controllers to a Solar Plant [J]. IEEE Transactions Control Systems Technology, 16(4), pp. 819-825, 2008.
[135] G Rinaldi, A Ferrara. Decentralized Integral Sliding Mode Approach for Frequency Control and Unknown Demand Reconstruction in Power Systems [C]. International Workshop on Variable Structure Systems, pp. 455-460, 2018.
[136] G Rinaldi, A Ferrara. Decentralized Model Predictive Control with Integral Sliding Mode for Frequency Regulation in Power Grids [C]. European Control Conference, Saint Petersburg, Russia, pp. 1885-1890, 2020.
[137] G P Incremona, M Cucuzzella, L Magni. MPC with Sliding Mode Control for the Energy Management System of Microgrids [C]. Ifac-Papersonline, pp. 7397-7402, 2017.
[138] A Musa, L R, Sabug, A Monti. Robust Predictive Sliding Mode Control for Multi-terminal HVDC Grids [J]. IEEE Transactions on Power Delivery, 33(4), pp. 1545-1555, 2018.
[139] M Wang, H Wang, Y Shi, et al. A Modified Sliding-mode Controller-based Mode Predictive Control Strategy for Three-phase Rectifier [J]. International Journal of Circuit Theory and Applications, 48(10), pp. 1564-1582, 2020.
[140] M J Deylamani, P Amiri, M H Refan. Design a Novel Digital Sliding Mode Predictive Control for Buck Converter [J].International Journal of Electronics, 109(2), pp. 246-270, 2021.
[141] A Ferrara, G P Incremona, G Piacentini. A Hierarchical MPC and Sliding Mode Based Two-Level Control for

Freeway Traffic Systems with Partial Demand Information [J]. European Journal of Control, 59, pp. 152-164, 2021.
[142] S Aval, N Eghbal. Feedback-based Cooperative Ramp Metering for Highway Traffic Flow Control: A Model Predictive Sliding Mode Control Approach [J]. International Journal of Robust and Nonlinear Control, 30(18), 2020.
[143] Y Wang, M Hou. Model-free Adaptive Integral Terminal Sliding Mode Predictive Control for a Class of Discrete-time Nonlinear Systems [J]. ISA Transactions, 93, pp. 209-217, 2019.
[144] J Zhang, S C Chai, B H Zhang. Distributed Model-free Sliding Mode Predictive Control of Discrete time Second Order Nonlinear Multiagent Systems with Delays [J]. IEEE Transactions on Cybernetics, 99, pp. 1-11, 2021.
[145] M Farbood, M Veysi, M Shasadeghi, A Izadian, T Niknam, J Aghaei. Robustness Improvement of Computationally Efficient Cooperative Fuzzy Model Predictive-Integral Sliding Mode Control of Nonlinear Systems [J]. IEEE Access, 9, pp. 147874-147887, 2021.
[146] C H Song. Optimal Control Algorithm of Constrained Fuzzy System Integrating Sliding Mode Control and Model Predictive Control [J]. Mathematical Problems in Engineering, 2015, 2015.
[147] A Mousavi, A H D Markazi. A Predictive Approach to Adaptive Fuzzy Sliding-mode Control of Under-actuated Nonlinear Systems with Input Saturation [J]. International Journal of Systems Science, 52(8), pp. 1599-1617, 2021.

APPENDIX A

TABLE IX: The methodology used to inform the initial literature search

Sources	The main sources of references in this paper are Institute of Electrical and Electronics Engineers (IEEE), The International Federation of Automatic Control (IFAC) and Society for Industry and Applied Mathematics (SIAM), etc.
Repositories	This paper is mainly based on the references obtained in Web of Science, IEEE Xplore Digital Library, Elsevier, Springer and other repositories.
Time range	The references are retrieved from 1970 to 2022.
Keywords	The keywords retrieved include sliding mode control, model predictive control and sliding mode predictive control.
Inclusion and Exclusion criteria	Document title: sliding mode control (and) model predictive control; sliding mode control (and) predictive control; sliding mode (and) predictive control; sliding mode (and) model predictive control; sliding mode control (and) receding horizon control; SMC (and) MPC; sliding mode control (and) MPC; SMC (and) receding horizon control (not) environment. Abstract: sliding mode control (and) model predictive control; sliding mode control (and) predictive control; sliding mode (and) predictive control; sliding mode (and) model predictive control; sliding mode control (and) receding horizon control; SMC (and) MPC; sliding mode control (and) MPC; sliding mode (and) MPC; SMC (and) receding horizon control; SMC (and) predictive control; SMC (and) model predictive control. **The inclusions of "Index terms" and "Full text" are the same as "Abstract"**.