Innovation Management on Online Platforms

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I, Sıdıka Tunç Candoğan, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the work.

To Güven for his unconditional love and support.

Abstract

In this thesis, I study operational decisions on online platforms (e.g., InnoCentive, Topcoder, Kickstarter, and Indiegogo) that facilitate crowd-based innovation and new product development, and I provide insights into how the economic value generated on these platforms can be enhanced. In the first two chapters, I focus on crowdsourcing platforms (e.g., InnoCentive and Topcoder) that create value by regularly organizing innovation contests to find solutions to their clients' innovation-related problems from the crowd. Although the value generated on crowdsourcing platforms is highly dependent on contest organizers' decisions on the contest rules such as award scheme, duration, and whether to allow team collaboration, these decisions and related trade-offs have received limited attention in the literature. To address this, in the first chapter, I analyze how an organizer should decide on the duration and award scheme for a contest; and in the second chapter, I study in what circumstances an organizer should encourage solvers to collaborate as teams or discourage team collaboration. In the third chapter, I focus on crowdfunding platforms (e.g., Kickstarter and Indiegogo), where entrepreneurs launch campaigns to crowdsource funding to bring their innovative ideas to life. Although crowdfunding is well recognized as an alternative means of funding, recent research and practice suggest that entrepreneurs often use crowdfunding as a mechanism for involving customers in product development. This motivates me to provide an understanding of crowdfunding as a product development mechanism and to study a key operational decision faced by an entrepreneur: whether to launch a crowdfunding campaign for a basic or enhanced version of a product.

Impact Statement

In recent years, online platforms have emerged as disruptive businesses that create value by enabling interactions between independent producers (sellers) and consumers (buyers). The operations management literature has looked at important operational decisions—such as those related to pricing and matching—on online platforms including Uber, Airbnb, TaskRabbit, among others (cf. Benjaafar and Hu 2020). Overall, my research contributes to this literature on online platforms by taking an innovation lens to platform operations with a focus on how they can be used to facilitate crowd-based innovation and new product development.

More specifically, the study in the first chapter contributes to the literature by identifying a novel trade-off that drives the optimal contest duration, and my interviews with practitioners at crowdsourcing platforms have confirmed the importance of this trade-off. This enables me to provide practically-consistent insights about the optimal contest duration and its relationship with the award scheme. The study in the second chapter contributes to the literature by identifying the gap between the theory and different policies adopted by crowdsourcing platforms about team collaboration and by generating practical insights into when the organizer can benefit from team collaboration. The study in the final chapter contributes to the literature by providing a nuanced understanding of crowdfunding as a product development mechanism and by generating managerial insights into entrepreneurs' product development and improvement decisions on crowdfunding platforms. It is also noteworthy that this study is the first in the crowdfunding literature to combine theoretical analysis, empirical analysis, and text analysis.

The impact of my thesis has been recognized by the operations management community and practitioner journals several times. Specifically, the study based on the first chapter was published in Manufacturing & Service Operations Management in 2021 and selected as the Institute for Operations Research and the Management Sciences (INFORMS) Technology, Innovation Management and Entrepreneurship Section (TIMES) Paper of the Month in June 2021; the study based on the second chapter won the 2020 INFORMS TIMES Best Working Paper Award; and the study based on the third chapter was selected as a runner-up of the 2021 INFORMS TIMES Best Working Paper Award and was featured by the Financial Times, the UCLA Anderson Review, and the Wall Street Journal.

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Last but not least, words cannot express my gratitude to my husband, Güven, who have taken care of us during this difficult journey. I feel extremely lucky to have you in my life. Thank you for being my family and best friend, and for helping me improve everyday in many ways.

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Chapter 1

Introduction

With the advancement in the technology and internet, online platforms such as Uber and Airbnb have emerged as disruptive business in recent years. These platforms create value by enabling interactions between external producers and consumers, and operational decisions such as pricing and matching significantly affect the economic value generated on these platforms (cf. Allon and Babich 2020, Benjaafar and Hu 2020). This thesis consists of three chapters that take an innovation lens to platform operations, focusing on how they can be used to facilitate crowd-based innovation and new product development. To this end, I leverage game-theoretical models, empirical analysis, and natural language processing techniques.

In Chapter 2 and Chapter 3, I focus on crowdsourcing platforms (e.g., InnoCentive, Topcoder, and 99designs) that create value by regularly organizing innovation contests to find solutions to their clients' innovation-related problems from the crowd (Erat and Krishnan 2012). Although the value generated on crowdsourcing platforms is highly dependent on contest organizers' decisions on the contest rules, some of these decisions and related trade-offs have received limited attention in the literature (e.g., Terwiesch and Xu 2008, Mihm and Schlapp 2019). To address this, in Chapter 2, I study the duration and the award scheme of an innovation contest organized on crowdsourcing platforms. I use a game-theoretical model where the organizer decides on the contest duration and the award scheme while each solver decides on her participation and determines her effort over the contest duration. The quality of a solver's solution improves with her effort, but it is also subject to an output uncertainty. I show that the optimal contest duration increases as the relative impact of the solver uncertainty on her output increases, and it decreases if the solver productivity increases over time. These results suggest that the optimal contest duration increases with the novelty or sophistication of solutions that the organizer seeks, and it decreases when the organizer can offer support tools that can increase the solver productivity over time. More interestingly, I characterize an optimal award scheme, and show that giving multiple (almost always) unequal awards is optimal when the organizer's urgency in obtaining solutions is below a certain threshold. This result helps explain why many contests on crowdsourcing platforms give multiple unequal awards. Finally, consistent with empirical findings, I show that there is a positive correlation between the optimal contest duration and the optimal total award.

In Chapter 3, I study team collaboration in innovation contests. Although solvers are capable of developing solutions individually and making *individual submissions*, if the organizer encourages collaboration, solvers may collaborate as teams and make *team submissions*. Motivated from different policies adopted by crowdsourcing platforms, I identify conditions under which the organizer can benefit from team submissions. By examining equilibrium outcomes of game-theoretical models, I show, interestingly, that when the organizer seeks high-novelty solutions to a *nondecomposable* problem (e.g., design challenges at InnoCentive), the organizer can benefit from team submissions despite the decrease in solvers' efforts. Yet, when the organizer seeks low-novelty solutions to a nondecomposable problem (e.g., logo design challenges at 99designs), the organizer may not benefit from team submissions unless teams are highly diverse. I further show that when the organizer seeks low-novelty or high-novelty solutions to a *decomposable* problem (e.g., software challenges at Topcoder), the organizer can benefit from team submissions, but interestingly, only under certain conditions. Finally, I identify conditions under which solvers can benefit from collaborating as teams because the organizer's benefit from team collaboration hinges upon solvers' decisions. I show that solvers can benefit from team collaboration in the absence of substantial synergistic gains, because without such gains, team collaboration decreases each solver's effort and hence cost in equilibrium.

In Chapter 4, I focus on crowdfunding platforms which are well recognized as an alternative way of raising funds for entrepreneurs (e.g., Hu et al. 2015, Belavina et al. 2020). Yet, crowdfunding goes beyond raising funds. Entrepreneurs often use crowdfunding to solicit feedback from customers to improve their products (e.g., Mollick 2016), and may therefore prefer to launch crowdfunding campaigns for a basic version of their products with few or no enhancements (i.e., limited features). Yet, customers may not be persuaded by a campaign if a product appears too basic. In view of this trade-off, a key question for an entrepreneur is how far a product should be enhanced before launching a crowdfunding campaign. Analyzing a game-theoretical model and testing its predictions empirically, I study how a product's level of enhancement at campaign launch influences both whether an entrepreneur continues to improve the product during the campaign and whether the campaign is successful. I show that as the product's level of enhancement at campaign launch increases, the likelihood of product improvement during a campaign at first increases (because customers are more likely to provide feedback) and then decreases (because of increased production cost for the entrepreneur). Furthermore, although the theoretical model intuitively predicts that the likelihood of campaign success will always increase when an entrepreneur launches a campaign for a more enhanced product, empirical analysis shows that the likelihood of campaign success first increases and then decreases. This counterintuitive result may be due to customers being overwhelmed with the complexity of highly enhanced products. Finally, while crowdfunding experts believe that products should be enhanced as much as possible before a campaign, I show that this is *not* always the best strategy.

To conclude, in this thesis, I aim to improve the understanding of operations on crowdsourcing and crowdfunding platforms and to generate practical insights into how the economic value generated on these platforms can be enhanced. I hope that my research contributes to the emerging literature on operations management on online platforms, motivates practitioners to make better decisions on crowdsourcing and crowdfunding platforms, and encourage other researchers to study crowd-based innovation and new product development and to combine theoretical and empirical analysis.

I would like to note that for the study in Chapter 2, I collaborated with Gizem Korpeoglu and Ersin Körpeoğlu; for the study in Chapter 3, I collaborated with Gizem Korpeoglu and Christopher S. Tang; and for the study in Chapter 4, I collaborated with Philipp Cornelius, Bilal Gokpinar, Ersin Körpeoğlu, and Christopher S. Tang. I performed all analyses and wrote all parts of the chapters myself.

Chapter 2

Optimal Duration of Innovation Contests

2.1 Introduction

In recent years, crowdsourcing has developed into a legitimate business tool, and online crowdsourcing platforms such as InnoCentive and Topcoder have enjoyed a significant growth, generating \$1 billion in revenue with an annual growth rate of 37.1% (Chen et al. 2020). These platforms create value for their customers such as Siemens, Pfizer, Unilever, and NASA by regularly organizing *innovation contests*. In an innovation contest, an organizer announces a problem along with a set of contest rules such as duration (i.e., how long the contest runs for) and award scheme (i.e., the set of awards).[1] Each participating solver generates a solution, and submits it to the organizer within the announced duration. At the end of the contest, the organizer evaluates all solutions, and gives award(s) based on the announced award scheme. In this study, we aim to generate insights into how an organizer should decide on the contest duration along with the award scheme.

At crowdsourcing platforms such as InnoCentive or Topcoder, we observe that the contest duration is determined based on contest characteristics. For instance, our analysis of contests (i.e., challenges) organized at InnoCentive in 2018 shows that the average duration of reduction-to-practice (in short, RTP) challenges that seek

[1] While there are other contest rules such as feedback policies, we focus on the organizer's decisions of the contest duration and the award scheme.

working prototypes is 81 days, while the average duration of theoretical challenges that seek theoretical solutions is 48 days. Similarly, at Topcoder, design challenges that seek innovative solutions (e.g., designing an app) often run longer than development challenges that seek low-novelty solutions (e.g., hunting bugs in a software). In addition to problem-related contest characteristics, the contest duration seems to be related to support tools provided for solvers. For instance, an organizer at Topcoder may offer support tools such as test cases, deployment guides, and documentation that can boost the solver productivity by reducing "non-functional decisions," and such tools can "shrink timelines" (Topcoder 2019).

In addition to the examples above, empirical studies and our interviews with practitioners at crowdsourcing platforms establish the managerial relevance of the contest duration, and they point to the following intrinsic drivers for it. Wang et al. (2015) empirically show that a longer contest duration may hinder solvers' participation but induces participating solvers to perform better. Dr. Kelly Higgins from InnoCentive explains this trade-off as follows: "we have found that increasing the length of posting at times has adverse effects ... Solvers [solvers] may think that if there is an extraordinary length of time for a challenge, it must be extremely difficult and therefore bypass the challenge." Thus, as the contest duration increases, solvers anticipate that they may have to exert more effort, and hence incur higher cost, so they may choose not to participate in the contest. In addition to these incentive effects, our interviews have also revealed that although the quality of solutions is the main concern, "clients [organizers] like to receive their solutions as early as possible." Thus, increasing the contest duration leads to discounting in the organizer's payoff (Seel 2018). Due to these opposing drivers, it is not obvious how an organizer should decide on the contest duration given different contest characteristics. The theoretical contest literature is of little help because it mostly overlooks the organizer's decision of the contest duration, and a few studies that consider the contest duration (Lang et al. 2014, Seel 2018) provide limited insights because they fail to capture all intrinsic drivers.

Our interviews indicate that practitioners factor in the contest duration when determining the award scheme. Yet, the prior literature on the award scheme overlooks the contest duration, and often suggests that an organizer should give a single award, i.e., adopt the winner-take-all (hereafter, WTA) award scheme. However, we observe that about three fourths of challenges organized at Topcoder have given multiple awards, and Stouras et al. (2017) also report that about two thirds of challenges organized at InnoCentive have given multiple awards. A few studies (e.g., Kalra and Shi 2001, Ales et al. 2017) show that giving multiple awards can rarely be optimal when solvers possess certain characteristics (e.g., risk aversion or specific beliefs about the solver uncertainty), but these studies neither explain why giving multiple awards is so common in practice nor can they account for why different contests on the same platform with a similar pool of solvers adopt different award schemes. Thus, it is important to investigate whether the organizer's decision of the contest duration can account for these policies in practice.

To address the gaps between theory and practice, we ask the following research questions. (Q1) How does the optimal contest duration change with contest characteristics? (Q2) What is the relationship between the contest duration and the award scheme?

As a first step towards answering these important research questions, we use a static game-theoretic model where the organizer determines the contest duration and the award scheme to maximize his profit. Then, each solver decides on her participation, and each participating solver decides on effort levels she will exert over the contest duration to maximize her utility.[2] The quality of a solver's solution increases with her effort, but it is also subject to an output uncertainty. Consistent with most challenges at InnoCentive and Topcoder, we assume that solvers do not receive feedback from the organizer.

To capture important contest characteristics and drivers about the contest duration in practice, our model contains the following key features. First, a solver

[2]Consistent with most papers in the innovation-contest literature (e.g., Terwiesch and Xu 2008, Ales et al. 2019a), our model assumes that solvers do not receive any information update throughout the contest, so they can statically determine effort levels they will exert over the contest duration. Alternatively, one can study a dynamic model where solvers dynamically determine their efforts based on information they receive over time. We provide more detailed discussion of such a dynamic model in §2.7, and defer this analysis to future research.

optimally allocates her (total) effort over the contest duration based on her per-time productivity. The solver productivity may decrease over time due to factors such as fatigue (e.g., Amabile et al. 2002) or may increase over time due to factors such as deeper understanding of concepts (e.g., Jain 2013). For instance, support tools such as test cases, deployment guides, and documentation at Topcoder can boost the solver productivity by reducing fatigue due to non-functional decisions and by facilitating deeper understanding of concepts.

Second, each solver endogenously determines whether to participate or not. Thus, the organizer ensures that a certain number of solvers chooses to participate in the contest by determining the contest rules accordingly. When modeling how the number of solvers is determined, we not only analyze the standard setting in the innovation-contest literature where the number of solvers is given exogenously, but also analyze a more novel setting where the organizer influences the number of solvers (i.e., the number of solvers is endogenous) while determining the contest rules. Third, we assume that (all else being equal) an organizer prefers obtaining solutions earlier rather than later. We capture this intuitive property by assuming that the organizer's payoff is discounted at a rate that depends on how urgently the organizer needs solutions. In addition to these features, our model helps us tease out the impact of the contest duration because we show that under a fixed duration, our model is equivalent to the standard modeling framework of the innovation-contest literature (e.g., Terwiesch and Xu 2008, Mihm and Schlapp 2019).

Using our model, we first analyze the optimal contest duration. We show that as the contest duration increases, each participating solver exerts more (total) effort, and hence generates a higher-quality solution. However, exerting more effort raises the solver's cost of effort, and hence reduces her utility from the contest. Thus, as the contest duration increases, it gets harder for the organizer to ensure solvers' participation. In addition, as the contest duration increases, the organizer's payoff is discounted more. We show that which of these three effects drives the optimal contest duration depends on how urgently the organizer needs solutions. For the organizer with high urgency (hereafter, impatient organizer), the trade-

Table 2.1: Summary of managerial insights.

<table>
<tr><th></th><th colspan="2">Patient organizer</th><th>Impatient organizer</th></tr>
<tr><th></th><th>Low urgency</th><th>Moderate urgency</th><th>High urgency</th></tr>
<tr><td>Optimal duration given award scheme</td><td colspan="2">Increases with the novelty or sophistication of solutions, and is shorter when the solver productivity increases over time.</td><td>Decreases with the novelty or sophistication of solutions, and is longer when the solver productivity increases over time.</td></tr>
<tr><td>Optimal distribution of awards</td><td>Giving multiple awards is optimal.</td><td colspan="2">The winner-take-all award scheme is optimal.</td></tr>
<tr><td>Optimal duration and total award</td><td colspan="3">Both increase with the novelty or sophistication of solutions.</td></tr>
</table>

off between increasing solvers' efforts and incurring more discounting (hereafter, effort-discounting trade-off) drives the optimal contest duration. However, for the organizer with low or moderate urgency (hereafter, patient organizer), the trade-off between increasing solvers' efforts and ensuring their participation (hereafter, effort-participation trade-off) drives the optimal contest duration. Interestingly, our interviews with practitioners at crowdsourcing platforms support our finding because an organizer at a crowdsourcing platform rarely has high urgency, and practitioners choose the contest duration considering the effort-participation trade-off. Because the patient-organizer case seems more consistent with practice, we focus on this case.

After characterizing the optimal contest duration, we analyze how it changes with contest characteristics. We show that the optimal contest duration increases with the novelty or sophistication of solutions that the organizer seeks. The intuition is as follows. As the novelty or sophistication increases, the impact of the solver's effort on her expected award decreases, so the solver reduces her effort, which reduces her cost of effort, and hence raises her utility. Thus, the organizer increases the contest duration to induce solvers to exert more effort while ensuring their participation. Our finding seems consistent with practice. For example, at Topcoder, design challenges that seek innovative solutions have longer durations than development challenges that seek low-novelty solutions. Similarly, at InnoCentive, RTP challenges have longer durations than theoretical challenges. We show that the optimal contest duration also depends on how the solver productivity changes over time. Although one may expect the organizer to set a longer contest duration when the solver productivity increases over time, we show that the opposite is true. This is because although an increase in productivity induces solvers to exert more effort, the organizer sets a shorter duration to ensure their participation. This result suggests that an organizer who provides support tools that increase the solver productivity over time (e.g., test cases, deployment guides, and documentation at Topcoder) can set a shorter duration without sacrificing the quality of solutions.[3]

We next analyze the award scheme under the optimal contest duration. We show, interestingly, that for an organizer with low urgency, it is always optimal to give multiple awards as the organizer can induce solvers to exert more effort by setting a longer contest duration. We further show that giving multiple awards is even more desirable when the solver productivity increases over time. This suggests that giving multiple awards goes hand in hand with offering support tools (e.g., test cases, deployment guides, and documentation at Topcoder) that increase the solver productivity over time. As another novel result, we explicitly characterize an optimal award scheme where it is almost always optimal to give unequal awards. These results help explain why many contests on platforms give multiple unequal awards because our interviews have revealed that many organizers on platforms have low urgency in obtaining solutions. Finally, we show that both the optimal contest duration and the optimal total award increase with the novelty or sophistication of solutions, which provides a plausible mechanism for an empirically-proven positive correlation between the contest duration and the total award (Yang et al. 2009, Shao et al. 2012).

[3] These support tools are unlikely to affect the solver uncertainty because these tools aim to reduce the cost of development and help with the implementation of solutions. Our insight does not encompass tools that may affect the solver uncertainty (e.g., by affecting solvers' creativity).

2.2 Related Literature

Our study is related to the new-product-development (hereafter, NPD) literature, the innovation-contest literature, and the scant literature on the contest duration.[4]

The traditional NPD literature (e.g., Chao and Kavadias 2008) focuses on an in-house development process of a new product where a product developer has full control over development efforts. Yet, with a shift in the landscape of classical research and development, a growing number of organizations have started to look beyond their boundaries towards outsourcing NPD activities (Eppinger and Chitkara 2006). A cost effective and time saving tool to outsource NPD activities is an innovation contest (e.g., Boudreau and Lakhani 2013). Different from a product developer, a contest organizer has to incentivize competing solvers to participate and exert costly efforts. Despite this contextual difference, some studies in the NPD literature show related results to ours. Specifically, Loch et al. (2001) analyze the optimal composition of parallel and sequential tests, and identify a trade-off between the cost and duration of testing because parallel testing is faster but costlier than sequential testing. This is in-line with our interim finding that the solver's cost of effort decreases with the contest duration. Dahan and Mendelson (2001) and Erat and Kavadias (2008) show that the number of parallel and sequential tests should increase with uncertainty. We show the opposite in an innovation contest, specifically, solvers' incentives to exert effort decrease with uncertainty. Indeed, we show that, to compensate for solvers' reduced incentives to exert effort, a patient organizer should increase the contest duration. In addition to the subtleties in our results arising from the contextual difference, we differ from the NPD literature by studying the relationship between the contest duration, a decision relevant to both innovation-contest and NPD settings, and the award scheme, a decision specific to

[4]As we factor in time as a model component, our study is also broadly related to the race literature (e.g., Loury 1979, Dasgupta and Stiglitz 1980, Lee and Wilde 1980), which analyzes competition among solvers where the first solver whose solution satisfies a certain quality requirement receives an award. In a race, the quality requirement is fixed, and the race duration is inherently unknown; while in a contest, the solution quality is variable, and the contest duration is known. In the race literature, to our knowledge, only Judd et al. (2012) show a result related to our study, and state that setting a higher quality requirement for a race leads to a longer race duration, which is consistent with our interim result that the quality of a solution increases with the contest duration.

an innovation-contest setting.

Our study contributes to the innovation-contest literature. Terwiesch and Xu (2008) pioneer a modeling framework of the innovation-contest literature, and show that a free-entry open-innovation contest is always optimal. Building on the modeling framework of Terwiesch and Xu (2008), Ales et al. (2020) show that a free-entry open-innovation contest is optimal only when the solver uncertainty is sufficiently large or the organizer is interested in many solutions. Building on the same modeling framework, Nittala and Krishnan (2016) analyze when and how a firm should organize an internal innovation contest; Hu and Wang (2020) compare a joint and a separate contest in the presence of multiple attributes; Körpeoğlu et al. (2017) study the impact of parallel innovation contests; and Mihm and Schlapp (2019) study the optimal feedback policy. We contribute to the innovation-contest literature by studying the organizer's decision of the contest duration and by analyzing the relationship between the contest duration and the award scheme. Indeed, we show that when the organizer considers a fixed contest duration, our model becomes equivalent to the standard modeling framework of the innovation-contest literature.

The closest study to our study is by Ales et al. (2017), who analyze the award scheme in an innovation contest by considering a fixed duration. They show that when the solver uncertainty has a log-concave density and her participation condition is satisfied, the WTA award scheme is optimal; and show that giving multiple awards is optimal in *rare* cases where one of these conditions is violated (e.g., when the solver uncertainty follows a log-convex and heavy tailed distribution). Our work differs from Ales et al. (2017) in the following key aspects. First, we show that even when the solver uncertainty has a log-concave density and the solver's participation condition is satisfied under the WTA award scheme, giving multiple awards is *always* optimal for the organizer with low urgency (as in most contests on crowdsourcing platforms). Second, we show the novel result that giving multiple awards is more desirable when the organizer provides support tools that increase the solver productivity over time (e.g., test cases, deployment guides, and documentation at Topcoder). Third, different from Ales et al. (2017), we show that these results hold when the organizer influences the number of solvers who participate in the contest, i.e., when the number of solvers is endogenous. Finally, we complement the results in this literature including Ales et al. (2017) by explicitly characterizing an optimal award scheme under the optimal contest duration.[5]

The innovation-contest literature also contains studies that use different modeling frameworks and assume a fixed contest duration. For example, Erat and Krishnan (2012) study design contests where each solver chooses from a set of design approaches. Bimpikis et al. (2019) analyze the role of intermediate awards and feedback in a two-stage contest where they assume that two solvers make continuous memoryless trials where the success rate of a solver is determined by her per-time effort. Because the tractability of a general model with both the solver uncertainty and heterogeneity is very limited (cf. Terwiesch and Xu 2008, Ales et al. 2019a), several papers in the innovation-contest literature focus on heterogeneity by abstracting away from uncertainty. For instance, Körpeoğlu and Cho (2018) analyze how the solver's equilibrium effort and the organizer's profit change with the number of solvers in the contest. Stouras et al. (2020) show that giving multiple *equal* awards may be optimal when the organizer aims to increase the number of solvers who participate in the contest. As opposed to their result, we show that when the WTA award scheme is not optimal, the organizer should almost always give *unequal* awards. For a detailed review of the contest literature, we refer the reader to Ales et al. (2019a) and Segev (2020).[6]

Despite its practical relevance, the contest duration has received little attention from the theoretical contest literature. Lang et al. (2014) characterize solvers' equilibria for an exogenously given duration without characterizing the optimal duration. Neglecting the solver uncertainty and participation decision, Seel (2018)

[5]As another related paper, Kalra and Shi (2001) study sales contests, and show that when solvers are risk averse, giving multiple awards can be optimal; whereas when solvers are risk neutral, the WTA award scheme is optimal. In contrast, we show that giving multiple awards to risk-neutral solvers is optimal when the organizer has low urgency in obtaining solutions. Also, different from Kalra and Shi (2001), we explicitly characterize an optimal distribution of awards when it is optimal to give multiple awards.

[6]Also, for recent studies in empirical research on crowdsourcing, we refer the reader to Hwang et al. (2019) and Aggarwal et al. (2020) and references therein.

suggests limiting the contest duration based on the effort-discounting trade-off. We contribute to this scant literature as follows. First, we identify the effort-participation trade-off, and show that this novel trade-off drives the optimal contest duration for an organizer without high urgency. As we discuss above, our interviews with practitioners at crowdsourcing platforms have revealed that an organizer at a crowdsourcing platform rarely has high urgency, so the effort-participation trade-off is more relevant to practice than the effort-discounting trade-off. Second, different from these papers, we provide practically-consistent insights about how the optimal contest duration changes with the solver productivity over time and the solver uncertainty, and the relationship between the contest duration and the award scheme.

2.3 Model

We consider an innovation contest where a contest organizer ("he") elicits solutions to an innovation-related problem from a set of N solvers ("she"), and solvers develop their solutions within a contest duration T. Given a population of $\overline{N}$ solvers, $N(\leq \overline{N})$ can be interpreted as the number of solvers that the organizer aims to attract to the contest. In our main analysis, we take N as given following the innovation-contest literature (e.g., Ales et al. 2017, Hu and Wang 2020, Mihm and Schlapp 2019). However, in §2.6, we extend our main results to the case where N is endogenous to the organizer's profit-maximization problem.

solvers. Each solver $i \in \{1,2,...,N\}$ generates an output y_i that represents the quality of her solution or solution's monetary value to the organizer. solver i's output y_i depends on her effort throughout the contest duration and an output shock.

First, to improve her output, each solver i exerts effort $\eta_i(t)$ (≥ 0) at time t over the contest duration T. For instance, solver i's per-time effort $\eta_i(t)$ may represent per-time resources that solver i allocates to the contest such as the full-time equivalent of labor hours or the amount of capital. solver i's per-time effort $\eta_i(t)$ leads to a deterministic improvement in her output y_i at the rate of $\theta(t)\eta_i(t)$ where the per-time productivity $\theta(t)$ (> 0) represents the marginal impact of the solver's per-time effort $\eta_i(t)$ on her output. Several factors may affect $\theta(t)$. On one hand, as t increases, solvers may rush or get exhausted, reducing the per-time productivity $\theta(t)$ (e.g., Amabile et al. 2002). On the other hand, as a solver spends more time on developing a solution, "activities can be sequenced in an efficient order. Consequently, unnecessary steps are eliminated, ... [and it] leads to a deeper understanding of concepts" (Jain 2013, page 1685). Thus, spending more time on the contest may lead to an increase in the per-time productivity $\theta(t)$ over time. If positive effects dominate, $\theta(t)$ can be increasing; if negative effects dominate, $\theta(t)$ can be decreasing; and if positive and negative effects offset each other, $\theta(t)$ can be constant. Throughout the study, whenever we need to capture how $\theta(t)$ changes over time, we consider the functional form $\theta(t) = \exp(\rho t)$, where the productivity exponent $\rho < 0$ captures decreasing productivity, $\rho = 0$ captures constant productivity, and $\rho > 0$ captures increasing productivity.

Second, each solver is exposed to an output shock $\widetilde{\xi}_i$.[7] For example, a chemist participating in an ideation challenge at InnoCentive faces an uncertainty about the value of her solution to the organizer. For each solver i, the output shock $\widetilde{\xi}_i (\in \Xi)$ is independent, and follows a cumulative distribution function H and a density function h with $E[\widetilde{\xi}_i] = 0$ over support $\Xi = [\underline{s}, \overline{s}]$, where $\underline{s} \in \mathbb{R} \cup \{-\infty\}$ and $\overline{s} \in \mathbb{R} \cup \{\infty\}$. We assume that h is log-concave (i.e., $\log(h)$ is concave), which is satisfied by most commonly used distributions such as Gumbel (e.g., Terwiesch and Xu 2008), uniform (e.g., Mihm and Schlapp 2019), normal, exponential, and logistic distributions. Let $\widetilde{\xi}^N_{(j)}$ be a random variable that represents the j-th largest output shock among $\{\widetilde{\xi}_1, \widetilde{\xi}_2, ..., \widetilde{\xi}_N\}$ with a cumulative distribution $H^N_{(j)}$ and a density $h^N_{(j)}(s) = \frac{N!}{(N-j)!(j-1)!}(1-H(s))^{j-1}H(s)^{N-j}h(s)$. To analyze the relative impact of the solver uncertainty on her output compared to her effort without imposing a

[7]Our interviews with practitioners at InnoCentive and Topcoder reveal that solvers receive email notifications right after a new contest is posted, they almost never receive feedback, and do not see other solvers" submissions. Thus, increasing the contest duration does not lead to a significant information update, and hence the solver uncertainty over time can be captured by a single output shock. Mihm and Schlapp (2019) point this out by stating that "[i]n the benchmark case of no feedback, the firm [i.e., organizer] does not provide any interim performance information to the solvers [i.e., solvers]. As a result, each solver's two-stage effort choice problem reduces to a simultaneous, single-stage utility maximization problem" (Mihm and Schlapp 2019, page 5). Note that a model that incorporates time does not need to be dynamic. For instance, time is an important component in the race literature (e.g., Loury 1979, Dasgupta and Stiglitz 1980, Lee and Wilde 1980), yet it is common in this literature to adopt a static model.

distribution assumption, we use the notion of a scale transformation. Two distribution functions H and $\widehat{H}$ differ by a scale transformation if there exists a parameter $\alpha > 0$ such that $\widehat{H}(s) = H(s/\alpha)$ for all $s \in \Xi$ (cf. Ales et al. 2019a). When $\widetilde{\xi}_i$ is transformed with the scale parameter $\alpha > 1$, the transformed output shock $\widehat{\xi}_i = \alpha\widetilde{\xi}_i$ has mean 0 and variance $\alpha^2 Var(\widetilde{\xi}_i)$, so the relative impact of the solver uncertainty increases and the relative impact of her effort decreases. Throughout the study, whenever we analyze the relative impact of the solver uncertainty or her effort, we use the scale parameter α, and whenever we do not, we normalize α to 1 for ease of illustration.

Given solver i's per-time effort $\eta_i(t)$, per-time productivity $\theta(t)$, and output shock $\widetilde{\xi}_i$, solver i's output takes the following form:

$$y_i = \int_0^T \theta(t)\eta_i(t)dt + \widetilde{\xi}_i. \tag{2.1}$$

We assume that solver i's cost of per-time effort takes the form $c\eta_i(t)^b$, where $c > 0$ and $b > 1$. Assuming convex cost of per-time effort is quite standard in the literature on multi-stage contests (e.g., Deng and Elmaghraby 2005, Mihm and Schlapp 2019), the product-development literature (e.g., Chao et al. 2009, Kouvelis et al. 2017), the race literature (e.g., Judd et al. 2012), and the project-management literature (e.g., Wu et al. 2014). For instance, Mihm and Schlapp (2019) study a two-period model where the per-period cost function is a special case of ours with $b = 2$. solver i's cost over the contest duration T is $\psi(\eta_i, T) = \int_0^T c\eta_i(t)^b dt$. Let $e_i \equiv \int_0^T \theta(t)\eta_i(t)dt$ be solver i's total deterministic improvement of her output over the contest duration T, and it can represent the total amount of tasks that the solver performs to improve her solution quality over the contest duration T. Throughout the study, we refer to e_i as the solver's effort.

The following lemma characterizes the cost function $\psi(e_i, T)$ of effort e_i by considering that solver i can optimally allocate her effort e_i over T. We present all proofs in Appendix A.

Lemma 1 *For any e_i and T, solver i's optimal per-time effort is* $e_i\theta(t)^{\frac{1}{b-1}}\tau(T)^{-1}$, *where* $\tau(T) = \int_0^T \theta(t)^{\frac{b}{b-1}} dt$. *Thus, for any* T, $\psi(e_i, T) = ce_i^b \tau(T)^{1-b}$. *Moreover,* $\psi(e_i, T)$ *is increasing and convex in* e_i*, and decreasing in* T.

Lemma 1 shows that the initial model that we present can be simplified to a model where each solver i can determine her (total) effort e_i, and then optimally allocate her effort over the contest duration T such that she exerts more per-time effort at times of higher per-time productivity. In this case, solver i's decision can be represented by e_i, where the solver's output function is $y_i = e_i + \widetilde{\xi}_i$ and her cost of effort can be simplified as $\psi(e_i, T) = ce_i^b \tau(T)^{1-b}$. This simplified model has two desirable properties. First, Lemma 1 shows that $\psi(e_i, T)$ is increasing and convex in effort e_i, and decreasing in the contest duration T. The cost function decreasing in the contest duration captures an intuitive property in practice that it is easier for a solver to allocate her effort over a longer period of time (e.g., Amabile et al. 1976, Ariely and Zakay 2001). Second, when T is fixed, our simplified model boils down to the standard modeling framework of the innovation-contest literature (e.g., Ales et al. 2017, Hu and Wang 2020, Mihm and Schlapp 2019).

Each solver maximizes her utility, which is a function of the award she receives from the contest and the cost she incurs. Following the economics and operations literature (e.g., Moscarini and Smith 2001, Kim and Lim 2015), we assume that the solver discounts her award with an interest rate β. Then, solver i's utility takes the form $U(e_i, T, z_i) = \exp(-\beta T) z_i - ce_i^b \tau(T)^{1-b} - F$, where z_i is the award solver i receives, and $F (> 0)$ is a fixed cost of participation in the contest.

Organizer. The organizer decides on the contest duration T and a vector of awards $(A_{(1)}, A_{(2)}, ..., A_{(N)})$ that we refer to as the award scheme. To isolate the impact of the award scheme from the impact of the contest duration, we assume that the organizer sets the present value of awards. (Using present values is also common in the race literature (e.g., Loury 1979, Dasgupta and Stiglitz 1980), where time is an important model component.) If solver i produces the j-th largest output $y_{(j)}$, then she receives an award $z_i = \exp(\beta T) A_{(j)}$. Consistent with practice and the literature (e.g., Terwiesch and Xu 2008), we assume $A_{(j)} \geq A_{(j+1)}$ for all $j \in \{1, 2, ..., N-1\}$. Let $A = \sum_{j=1}^N A_{(j)}$ be the present value of the total award, and $(\gamma_{(1)}, \gamma_{(2)}, ..., \gamma_{(N)})$

be the distribution of awards such that $A_{(j)} = \gamma_{(j)} A$ for all $j \in \{1, 2, ..., N\}$, and $\sum_{j=1}^{N} \gamma_{(j)} = 1$. We refer to the solver with the largest output as the winner, and refer to the award scheme that gives an award only to the winner (i.e., $\gamma_{(1)} = 1$) as the winner-take-all (hereafter, WTA) award scheme.

The organizer maximizes the present value of his expected profit, which consists of the organizer's payoff from the contest minus the total award given to solvers. We make the following assumptions about the organizer's profit. First, as is common in the innovation-contest literature (e.g., Mihm and Schlapp 2019), we assume that the organizer is interested in the quality of the best solution.[8] Second, all else being equal, the organizer prefers obtaining solutions earlier rather than later. We capture this by assuming that the organizer discounts his payoff with a discount factor δ (≥ 0) (e.g., Moscarini and Smith 2001, Seel 2018, Bimpikis et al. 2019). Hence, the organizer's profit $\Pi = \exp(-\delta T) y_{(1)} - \exp(-\beta T) \exp(\beta T) A = \exp(-\delta T) y_{(1)} - A$.[9] Note that the discount factor δ is different from the interest rate β because δ is also related to the value of solutions to the organizer and how urgently the organizer needs these solutions. For instance, if the organizer incurs a significant opportunity cost for not implementing a solution earlier, δ may be large.

The sequence of events is as follows. First, the organizer announces the contest duration and the award scheme. Then, each solver decides on whether to participate in the contest, and each participating solver decides on her effort, optimally allocates her effort over the contest duration, and generates a solution. Finally, the organizer collects and evaluates all solutions, and awards the best solution(s) based on the announced award scheme. solvers learn about the quality of their solutions only after the organizer evaluates all solutions.

Equilibrium among solvers. In our base model, following the innovation-contest

[8]Note that all our results extend to the case where the organizer is interested in multiple solutions. Also, following the innovation-contest literature (e.g., Terwiesch and Xu 2008, Mihm and Schlapp 2019), we assume that a solver can submit a solution of any quality. Thus, a solver who chooses to participate always submits a solution because the solver's probability of winning an award is positive when she submits a solution, whereas this probability is zero when she does not submit a solution.

[9]This profit function assumes the same interest rate for the organizer and solvers. However, our supplementary analysis shows that our main results extend to a case where solvers' cash flows are more sensitive than the organizer's.

literature (e.g., Terwiesch and Xu 2008, Hu and Wang 2020, Mihm and Schlapp 2019), we focus on a symmetric pure-strategy Nash equilibrium among participating solvers.[10] However, we show the robustness of our main findings by considering asymmetric pure-strategy Nash equilibria and mixed-strategy Nash equilibria in Appendix B.1 and Appendix B.2, respectively. We utilize the best-response argument to derive the symmetric pure-strategy Nash equilibrium, where each solver exerts the equilibrium effort e^*. Given that all other solvers exert the equilibrium effort e^*, solver i's probability of producing the j-th largest output (hence ranking the j-th) when she exerts effort e_i is as follows:

$$P^N_{(j)}[e_i, e^*] = \int_{s\in\Xi} \frac{(N-1)!}{(N-j)!(j-1)!} H(s+e_i-e^*)^{N-j}(1-H(s+e_i-e^*))^{j-1}h(s)ds.$$

Each solver i chooses her effort e_i to maximize her expected utility by solving

$$\max_{e_i\in\mathbb{R}_+} \sum_{j=1}^{N} P^N_{(j)}[e_i, e^*]A_{(j)} - ce_i^b\tau(T)^{1-b} - F. \tag{2.2}$$

Evaluating the first-order condition of (2.2) at $e_i = e^*$ yields

$$\sum_{j=1}^{N} I^N_{(j)}A_{(j)} = cb(e^*)^{b-1}\tau(T)^{1-b}, \tag{2.3}$$

where $I^N_{(j)} \equiv \left.\frac{\partial P^N_{(j)}[e_i,e^*]}{\partial e_i}\right|_{e_i=e^*}$, and it can be derived as follows:

$$I^N_{(j)} = \int_{s\in\Xi} \Bigg[\frac{(N-1)!}{(N-j)!(j-1)!} H(s)^{N-j-1}(1-H(s))^{j-2} \times [(N-j)(1-H(s))-(j-1)H(s)]\,h(s)^2 \Bigg] ds. \tag{2.4}$$

Let $x \equiv \sum_{j=1}^{N} I^N_{(j)}\gamma_{(j)}$. Noting that $I^N_{(j)}$ is independent of e^*, the solver's equilibrium

[10] Note that we allow for asymmetric participation behavior where N solvers participate and $\overline{N}-N$ solvers do not. Which N solvers participate in the contest is immaterial to our analysis because all solvers are identical.

effort

$$e^* = \left(\frac{Ax}{cb}\right)^{\frac{1}{b-1}} \tau(T). \tag{2.5}$$

As $e_i = e^*$ in equilibrium, a solver's probability of ranking the j-th is $1/N$. a solver participates in the contest if her expected utility is non-negative, i.e., $\frac{1}{N}\sum_{j=1}^{N} A_{(j)} - c(e^*)^b \tau(T)^{1-b} - F \geq 0$.

The organizer's problem. The organizer solves the following profit-maximization problem:

$$\max_{T,(A_{(1)},A_{(2)},\ldots,A_{(N)})} \exp(-\delta T)\left(e^* + E\left[\tilde{\xi}_{(1)}^N\right]\right) - \sum_{j=1}^{N} A_{(j)} \tag{2.6}$$

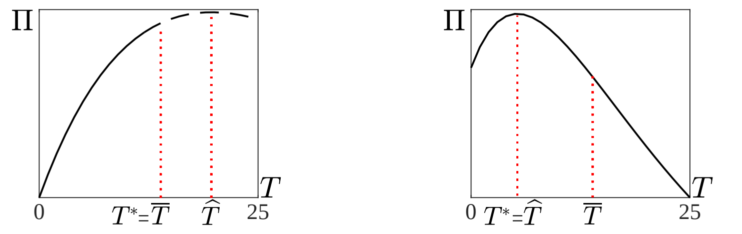
$$\text{s.t.} e^* = arg \max_{e_i \in \mathbb{R}_+} \sum_{j=1}^{N} P_{(j)}^N [e_i, e^*] A_{(j)} - c e_i^b \tau(T)^{1-b} - F, \tag{2.7}$$

$$\frac{1}{N}\sum_{j=1}^{N} A_{(j)} - c(e^*)^b \tau(T)^{1-b} - F \geq 0. \tag{2.8}$$

The organizer decides on the contest duration T and the award scheme $(A_{(1)}, A_{(2)}, \ldots, A_{(N)})$ to maximize his expected profit in (2.6) subject to the solver's incentive-compatibility constraint (2.7) and the solver's participation condition (2.8). We make the following mild assumptions to ensure that a solution to (2.6)-(2.8) exists and it is characterizable. First, we assume that $F < A/N$ because otherwise, the organizer cannot attract N solvers to the contest, so the organizer's problem (2.6)-(2.8) becomes infeasible. Second, we assume that the organizer's profit Π is unimodal in T and $\left.\frac{\partial \Pi}{\partial T}\right|_{T=0} > 0$ so that the Kuhn-Tucker conditions can characterize the optimal contest duration. Note that when the per-time productivity takes the form $\theta(t) = \exp(\rho t)$, Π is always unimodal in T and $\left.\frac{\partial \Pi}{\partial T}\right|_{T=0} > 0$ if the discount factor $\delta < \left(\frac{Ax}{cb}\right)^{\frac{1}{b-1}} / E\left[\tilde{\xi}_{(1)}^N\right]$.

2.4 Optimal Contest Duration

In this section, we analyze the optimal contest duration T^* by taking the award scheme $(A_{(1)}, A_{(2)}, \ldots, A_{(N)})$ as given. The following lemma characterizes the optimal contest duration T^*.



(a) Patient organizer ($\delta = 5\%$). (b) Impatient organizer ($\delta = 10\%$).

Figure 2.1: The organizer's profit Π as a function of the contest duration T, where T^* is the optimal contest duration and $\overline{T}$ and $\widehat{T}$ are as in Lemma 2. Setting: $\widetilde{\xi}_i \sim$ Gumbel with mean 0 and scale parameter 1; $\theta(t) = \exp(\rho t)$, $\rho = 0.01$, $b = 2$, $c = 1$, $F = 0.25$, $N = 2$, and $(A_{(1)}, A_{(2)}) = (1, 0)$.

Lemma 2 *Let $\overline{T}$ and $\widehat{T}$ solve the following equations, respectively:*

$$\int_0^{\overline{T}} \theta(t)^{\frac{b}{b-1}} dt = \frac{A - NF}{cN} \left(\frac{Ax}{cb}\right)^{\frac{-b}{b-1}} \quad \textit{and} \tag{2.9}$$

$$\theta(\widehat{T})^{\frac{b}{b-1}} - \delta \int_0^{\widehat{T}} \theta(t)^{\frac{b}{b-1}} dt = \delta E\left[\widetilde{\xi}^N_{(1)}\right] \left(\frac{Ax}{cb}\right)^{\frac{-1}{b-1}}. \tag{2.10}$$

When the organizer's profit Π is non-monotonic in the contest duration T, there exists δ_1 such that $T^* = \overline{T}$ for any $\delta < \delta_1$,[11] *and $T^* = \widehat{T}$ for any $\delta \geq \delta_1$. When Π is monotonic in T, $T^* = \overline{T}$.*

The intuition of Lemma 2 is as follows. Increasing the contest duration T has the following three effects on the organizer's profit-maximization problem (2.6)-(2.8). First, it increases the solver's equilibrium effort e^*, and hence improves the solver's solution quality. Second, because of the increase in e^*, the solver's cost of effort increases, her utility decreases, and hence the solver's participation condition (2.8) becomes tighter. Third, increasing T leads to more discounting of the organizer's payoff. When the organizer's profit is non-monotonic with respect to T (i.e., when the solver productivity does not increase very fast), which of these three

[11] A solution to (2.7) should exist under $T = \overline{T}$ so that the organizer is able to set the contest duration at $\overline{T}$. In Appendix B.3, we provide sufficient conditions (e.g., the fixed cost of participation F is sufficiently large) for e^* in (2.5) to be the unique solution of (2.7) under $T = \overline{T}$. Note that a necessary condition for a solution to (2.7) to exist under $T = \overline{T}$ is that $A_{(N)} \leq F$. Although the exact value of the optimal contest duration T^* depends on our focus on symmetric pure-strategy Nash equilibria, we show, in Appendix B.1 and Appendix B.2, that the intrinsic drivers of T^* are the same under asymmetric Nash equilibria and mixed-strategy Nash equilibria.

effects drives the optimal contest duration T^* depends on the discount factor δ. Throughout the study, we refer to an organizer with $\delta < \delta_1$ as a "patient organizer," and refer to an organizer with $\delta \geq \delta_1$ as an "impatient organizer." For a patient organizer, the impact of discounting is small, so the trade-off between increasing the equilibrium effort e^* and satisfying the participation condition (2.8) (hereafter, effort-participation trade-off) drives the optimal contest duration T^*. Thus, the organizer sets T such that the solver's participation condition is binding, i.e., $T^* = \overline{T}$. Hence, interestingly, even for a patient organizer (i.e., an organizer who does not worry about discounting much), it is optimal to limit the contest duration to guarantee solvers' participation; see Figure 2.1(a). For an impatient organizer, the impact of discounting is large, so the trade-off between increasing the solver's equilibrium effort e^* and incurring more discounting (hereafter, effort-discounting trade-off) drives the optimal contest duration T^*. Thus, the organizer sets T such that the impact of effort and the impact of discounting are balanced, i.e., $T^* = \widehat{T}\ (\leq \overline{T})$; see Figure 2.1(b).

Lemma 2 also shows that when the organizer's profit always increases with the contest duration T, the organizer always sets T according to the effort-participation trade-off, i.e., $T^* = \overline{T}$. This happens when the solver productivity increases very fast. Throughout the study, we focus our discussion on the case where the organizer's profit is non-monotonic so that we can also generate insights for the case of an impatient organizer, but all our results and their intuitions for the patient organizer apply to the case where the organizer's profit is monotonic. Note that as we discuss in §2.1, our interviews with practitioners have revealed that most organizers on platforms seem to be patient, so the case of a patient organizer is more relevant to practice. Thus, we focus on a patient organizer in §2.4.1, and we supplement our analysis by considering an impatient organizer in §2.4.2.

2.4.1 Analysis of Patient Organizer

The following theorem analyzes how the optimal contest duration T^* for a patient organizer changes with the relative impact of the solver uncertainty on her output compared to her effort. We measure this impact with a scale parameter α. Also, under the per-time productivity $\theta(t) = \exp(\rho t)$, we analyze the impact of the productivity exponent ρ on T^*.

Theorem 1 *(a) The optimal contest duration T^* is increasing in any scale parameter α such that $\delta < \delta_1$.*
(b) Suppose that the per-time productivity $\theta(t) = \exp(\rho t)$. Then, T^* is decreasing in any productivity exponent ρ such that $\delta < \delta_1$.

As discussed in §2.4, the effort-participation trade-off drives the optimal contest duration T^* for a patient organizer, so we explain the intuition of Theorem 1(a) by focusing on the effort-participation trade-off. As demonstrated in (2.3), the solver balances the marginal benefit of her effort (i.e., Ax) with the marginal cost of her effort $\big(\text{i.e., } cb(e^*)^{b-1}\tau(T)^{1-b}\big)$. When the relative impact of the solver uncertainty measured by α increases, the marginal benefit of the solver's effort decreases, so she reduces her equilibrium effort e^*. Because e^* decreases, the solver's cost of effort decreases, and hence her utility increases. This increase in the solver's utility allows the organizer to increase the contest duration T without violating the solver's participation condition, so T^* increases with α.

Theorem 1(a) has an important managerial implication. As the scale parameter α increases, the solver uncertainty becomes relatively more impactful and the solver's effort becomes relatively less impactful. The relative impact of the solver uncertainty can be associated with the novelty of solutions that the organizer seeks (e.g., Terwiesch and Xu 2008). The relative impact of the solver's effort can be associated with the sophistication of solutions that the organizer seeks because as the organizer seeks more sophisticated solutions, the solver's unit effort has relatively less impact on her solution. Therefore, Theorem 1(a) suggests that the optimal contest duration increases with the novelty or sophistication of solutions that a patient organizer seeks. This result seems consistent with practice. For example, at Topcoder, development challenges that seek low-novelty solutions (e.g., hunting bugs in software) have shorter contest durations than design challenges that seek innovative solutions (e.g., designing an app). Similarly, at InnoCentive, theoretical challenges that seek theoretical solutions have shorter contest durations than RTP challenges that seek working prototypes. Not only is our result consistent with practice but also the drivers of our result seem consistent with practice. Specifically, our interviews with practitioners at InnoCentive and Topcoder have revealed that most organizers at crowdsourcing platforms seem to be patient, so our results suggest that the optimal contest duration should be determined by the effort-participation trade-off. Indeed, our interviews corroborate our model prediction, and indicate that practitioners use the effort-participation trade-off while determining the optimal contest duration.

Theorem 1(b) shows that for a patient organizer, the optimal contest duration T^* decreases with the productivity exponent ρ. We discuss the intuition focusing on increasing productivity (i.e., $\rho > 0$), but the same intuition applies to decreasing productivity (i.e., $\rho < 0$). As the productivity exponent ρ increases, the solver productivity increases faster over time, so one may think that the organizer should increase the contest duration T to benefit from the increased productivity. However, Theorem 1(b) shows, somewhat counterintuitively, that the opposite is true. The intuition is as follows. As the productivity exponent $\rho (> 0)$ increases, the marginal benefit of the solver's per-time effort increases faster with T, and hence the solver increases her equilibrium effort e^* faster. Thus, the solver's cost of effort increases faster, and hence the solver's utility decreases faster. Hence, the solver's participation condition (2.8) binds under a smaller T, and hence the organizer sets a shorter contest duration to guarantee solvers' participation.[12] Our result indicates that the organizer may benefit from designing a contest with a shorter duration when the solver productivity increases over time. For instance, an organizer running an app development challenge at Topcoder can provide support tools such as test cases, deployment guides, and documentation that can boost the solver productivity by reducing "nonfunctional decisions" (Topcoder 2019). Our result shows that the organizer offering such support tools may set a shorter contest duration. Our prediction seems consistent with practice. For example, Topcoder promotes these support tools by stating that they can "shrink timelines," i.e., reduce contest durations (see

[12] Although Theorem 1(b) shows that T^* decreases with ρ, the organizer's profit Π intuitively increases with ρ.

Topcoder 2019, page 1).

2.4.2 Analysis of Impatient Organizer

In practice, an organizer may be impatient when his profit is significantly affected by how soon he obtains solutions. For example, Perpetual Motion, a startup that runs a logo-design contest at 99designs may require the logo quickly in order to launch its business (99designs 2018). The following proposition analyzes how the optimal contest duration T^* changes with the relative impact of the solver uncertainty measured by a scale parameter α. Under the per-time productivity $\theta(t) = \exp(\rho t)$, it also analyzes the impact of the productivity exponent ρ on T^*.

Proposition 1 *(a) The optimal contest duration T^* is decreasing in any scale parameter α such that $\delta > \delta_1$.*
(b) Suppose that the per-time productivity $\theta(t) = \exp(\rho t)$. Then, T^* is increasing in the productivity exponent ρ such that $\delta > \delta_1$.

Proposition 1(a) shows that the optimal contest duration T^* decreases with the relative impact of the solver uncertainty measured by α. The intuition is as follows. As we discuss after Lemma 2, the impatient organizer chooses T by balancing the marginal benefit of T that arises from a larger effort e^* with the marginal cost of T that arises from more discounting. As α increases, the marginal impact of the solver's effort on her expected award decreases, leading to a smaller effort e^* and a smaller impact of increasing T on e^* (i.e., smaller $\frac{\partial e^*}{\partial T}$). The former effect reduces the marginal cost of increasing T, whereas the latter effect reduces the marginal benefit of increasing T. Proposition 1(a) shows that the marginal benefit decreases more than the marginal cost, and hence the organizer reduces T with α. A managerial insight from Proposition 1(a) is that the optimal contest duration decreases with the novelty or sophistication of solutions that an impatient organizer seeks. This is primarily because the impatient organizer has so much urgency, and hence the impact of discounting is so large that the organizer sacrifices the solution quality for receiving solutions quickly. For example, a startup that urgently needs a logo to launch its business may prefer a satisfactory logo design quickly rather than waiting for the best-quality logo design.

Proposition 1(b) shows, somewhat intuitively, that the optimal contest duration T^* for the impatient organizer increases with the productivity exponent ρ. We discuss the intuition for increasing productivity (i.e., $\rho > 0$), but the same intuition applies to decreasing productivity (i.e., $\rho < 0$). As $\rho(> 0)$ increases, the marginal benefit of the solver's per-time effort increases faster with the contest duration T, and hence the solver increases her equilibrium effort e^* faster. Thus, the marginal benefit of T increases, leading to a longer T^*.

2.5 Contest Duration and Award Scheme

This section proceeds as follows. In §2.5.1, we consider the organizer's decisions of the contest duration T and the distribution of awards $(\gamma_{(1)}, \gamma_{(2)}, ..., \gamma_{(N)})$ by taking the total award A as given. In §2.5.2, we consider the organizer's decisions of T and A by taking $(\gamma_{(1)}, \gamma_{(2)}, ..., \gamma_{(N)})$ as given.

2.5.1 Contest Duration and Distribution of Awards

The following theorem analyzes when it is optimal for the organizer to adopt the WTA award scheme. As a preparation for the theorem, we let δ_1^{WTA} be the threshold on the discount factor δ below which $T^* = \overline{T}$ under the WTA award scheme (see Lemma 2 for the definition of δ_1).

Theorem 2 *(a) There exists $\overline{\delta}_0 (\leq \delta_1^{WTA})$ such that if $\delta > \overline{\delta}_0$, the WTA award scheme is optimal; and there exists $\underline{\delta}_0 (\leq \overline{\delta}_0)$ such that if $\delta < \underline{\delta}_0$, giving multiple awards is optimal. Also, there exists $M \geq 0$ such that if $\frac{\theta'(\overline{T})}{\theta(\overline{T})} \leq M$ for any $(\gamma_{(1)}, \gamma_{(2)}, ..., \gamma_{(N)})$, $\overline{\delta}_0 = \underline{\delta}_0 = \delta_0$.*
(b) Suppose that the per-time productivity $\theta(t) = \exp(\rho t)$. Then, $\underline{\delta}_0$ is increasing in the productivity exponent ρ.

Theorem 2(a) first shows that when the discount factor δ is above a threshold $\overline{\delta}_0$, the WTA award scheme is optimal. The intuition is as follows. For a fixed contest duration T, the equilibrium effort e^* is maximized under the WTA award scheme because the marginal impact of the solver's effort on her probability of becoming the winner is larger than that on her probability of attaining any other rank. When the organizer has high urgency in obtaining solutions (i.e., $\delta \geq \delta_1^{WTA} \geq \overline{\delta}_0$), the solver's participation condition (2.8) does not bind, and hence the effort-discounting trade-off drives T^*. In this case, since the WTA award scheme maximizes e^* for any T, the WTA award scheme maximizes the organizer's profit under T^*. When the organizer has moderate urgency (i.e., $\delta \in (\overline{\delta}_0, \delta_1^{WTA})$), (2.8) binds under the WTA award scheme. Because the WTA award scheme elicits a larger e^* for any fixed T, to satisfy (2.8), the organizer sets a shorter T under the WTA award scheme than T under other award schemes that make (2.8) binding. Thus, such an award scheme may yield a larger e^* at the expense of more discounting. When $\delta > \overline{\delta}_0$, the negative effect of more discounting outweighs the positive effect of a larger e^*, and hence the WTA award scheme is optimal. However, more interestingly, when the organizer has low urgency (i.e., $\delta < \underline{\delta}_0$), the positive effect of a larger e^* outweighs the negative effect of more discounting, so giving multiple awards is optimal. Although Theorem 2(a) presents two thresholds, $\underline{\delta}_0$ and $\overline{\delta}_0$, unless the solver productivity increases very fast over time (i.e., unless $\theta'(\overline{T})/\theta(\overline{T})$ is very large), these thresholds take a common value δ_0, above which the WTA award scheme is optimal and below which giving multiple awards is optimal.

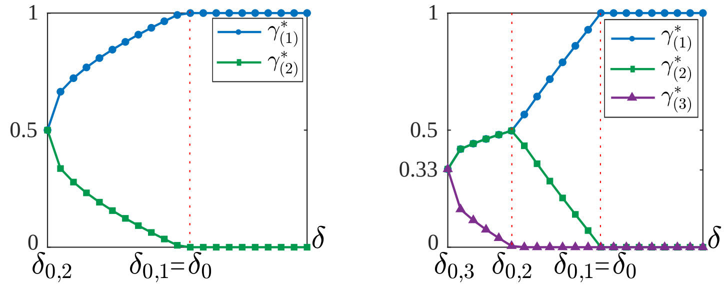
Theorem 2(a) has important implications for the contest theory and practice. By assuming a fixed contest duration, Ales et al. (2017) prove that the WTA award scheme is optimal when the solver's participation condition is satisfied and the density function h is log-concave as in our model. (Note that under a fixed T, our model becomes a special case of their model.) By factoring in the organizer's decision of T, our study complements their analysis on two fronts. First, we show that their result extends to the case when the organizer has moderate or high urgency in obtaining solutions. Second, and more interestingly, when the organizer has low urgency in obtaining solutions, the WTA award scheme is no longer optimal even if the conditions specified by Ales et al. (2017) are satisfied. Thus, our result helps explain why many contests on crowdsourcing platforms give multiple awards because as we discuss in §2.1, many organizers on crowdsourcing platforms have low urgency in obtaining solutions.

Theorem 2(b) shows that when the productivity exponent ρ increases, we have a larger threshold $\underline{\delta}_0$ under which giving multiple awards is optimal. The intuition is as follows. As discussed above, compared to the WTA award scheme, giving multiple awards may elicit a larger e^* under T^* at the expense of more discounting due to a longer T^*. Yet, as ρ increases, T^* decreases (see Theorem 1(b)), and hence the negative effect of more discounting due to giving multiple awards decreases. Thus, the organizer can benefit from giving multiple awards under a larger δ. An interesting managerial insight is that giving multiple awards is more desirable for an organizer that can provide support tools such as test cases, deployment guides, and documentation to increase the solver productivity over time. Thus, giving multiple awards goes hand in hand with offering such support tools.

As Theorem 2 shows, the WTA award scheme is not optimal when the organizer has low urgency. We next study an optimal distribution of awards and how it changes with the organizer's urgency. To ensure that there is a single threshold δ_0, we assume that $\frac{\theta'(\overline{T})}{\theta(\overline{T})} \leq M$, where M is as in Theorem 2. This assumption is satisfied in most settings including the constant per-time productivity assumed by the contest literature (e.g., Deng and Elmaghraby 2005, Mihm and Schlapp 2019).

Proposition 2 *Suppose that* $\frac{\theta'(\overline{T})}{\theta(\overline{T})} \leq M$ *for any* $(\gamma_{(1)}, \gamma_{(2)}, ..., \gamma_{(N)})$*, where* $M(\geq 0)$ *is defined as in Theorem 2. There exist an optimal distribution of awards* $(\gamma^*_{(1)}, \gamma^*_{(2)}, ..., \gamma^*_{(N)})$ *and thresholds* $(\delta_{0,1}, \delta_{0,2}, ..., \delta_{0,N})$ *such that* $0 \leq \delta_{0,N} < \delta_{0,N-1} < ... < \delta_{0,1} = \delta_0$*; and for any* $j \in \{2, 3, ..., N\}$*, when* $\delta \in (\delta_{0,j},\ \delta_{0,j-1})$*,* $\gamma^*_{(k)}$ *is increasing in* δ *for all* $k \in \{1, 2, ..., j-1\}$*,* $\gamma^*_{(j)}$ *is decreasing in* δ*, and* $\gamma^*_{(k)} = 0$ *for all* $k \in \{j+1, ..., N\}$.

Proposition 2 characterizes an optimal award scheme where the organizer gradually shifts awards from lower-ranked solvers towards higher-ranked solvers as the discount factor δ increases. For example, when the number of solvers $N = 2$, as δ increases, the optimal share of the winner award $\gamma^*_{(1)}$ increases, and the optimal share of the runner-up award $\gamma^*_{(2)}$ decreases; see Figure 2.2(a). For $N = 3$, as δ increases, both $\gamma^*_{(1)}$ and $\gamma^*_{(2)}$ increase as long as the optimal share of the third award



(a) $N=2$.

(b) $N=3$.

Figure 2.2: An optimal distribution of awards $(\gamma^*_{(1)}, \gamma^*_{(2)}, \ldots, \gamma^*_{(N)})$ as a function of the discount factor δ. The setting is the same as Figure 2.1.

$\gamma^*_{(3)}$ is positive; and when $\gamma^*_{(3)} = 0$, $\gamma^*_{(1)}$ increases and $\gamma^*_{(2)}$ decreases; see Figure 2.2(b). Thus, Proposition 2 shows that the more urgency an organizer has, the fewer awards he should give, and the larger share he should allocate to the winner. The intuition is similar to that of Theorem 2. The organizer can increase e^* by increasing the contest duration T and the number of awards. Yet, increasing T comes at the expense of more discounting. As δ increases, the negative effect of discounting increases, so the number of awards as well as the optimal contest duration T^* decreases.[13]

Proposition 2 has important implications for the contest theory and practice. Although the prior literature shows that multiple awards can be optimal in *rare* cases by assuming a fixed contest duration, these studies either do not explicitly characterize an optimal award scheme (e.g., Ales et al. 2017) or show that giving multiple *equal* awards is always optimal (e.g., Stouras et al. 2020). However, Proposition 2 characterizes an optimal award scheme where giving equal awards is almost never optimal when considering the organizer's simultaneous decisions on the contest duration and the award scheme. Our finding is indeed consistent with practice. For instance, among 52 challenges organized at Topcoder in 2019, 45 challenges give multiple awards, and only two of them give equal awards. Thus, our results

[13] We characterize an intuitive and easy to implement optimal award scheme. Although there may be other optimal award schemes that do not change with δ in the same manner as in Proposition 2, all optimal award schemes have the same intuition. As δ increases, the organizer benefits from reducing discounting, and achieves this by shifting his award scheme towards the WTA award scheme (hence increasing $\sum_{j=1}^{N} I^N_{(j)} \gamma^*_{(j)}$) and reducing the contest duration. Our supplementary analysis shows that any optimal award scheme features unequal awards for most δ values.

help explain not only why contests with multiple awards are common in practice, but also why these contests give unequal awards.

2.5.2 Contest Duration and Total Award

This section analyzes how the optimal contest duration T^* and the optimal total award A^* change with the scale parameter α and the productivity exponent ρ. For analytical tractability, we assume that the discount factor $\delta = 0$ (i.e., the organizer is patient as in §2.4.1), but the main result of this section (Proposition 3(a)) can also be shown for any δ when the fixed cost of participation $F = 0$ and the per-time productivity $\theta(t) = \theta$.

Proposition 3 *(a) The optimal contest duration T^* and the optimal total award A^* are increasing in the scale parameter α.*
(b) When $\theta(t) = \exp(\rho t)$, T^* is decreasing in the productivity exponent ρ and A^* does not change with ρ.

Proposition 3(a) shows that the optimal contest duration and the optimal total award increase with the scale parameter α. Thus, regarding the optimal contest duration, Proposition 3(a) yields the same result and has the same intuition as Theorem 1(a). A direct corollary of Proposition 3(a) is that under the WTA award scheme, as the scale parameter α increases, the optimal contest duration and the optimal winner award both increase, so they are positively correlated. Empirical studies by Yang et al. (2009) and and Shao et al. (2012) corroborate this positive correlation by using data from Taskcn.com and zhubajie.com (two largest crowdsourcing platforms in China). Although these papers do not suggest a mechanism for this correlation, Proposition 3(a) suggests that a plausible mechanism may be how the contest duration and the winner award change with the novelty or sophistication of solutions that the organizer seeks (see §2.4.1 for the discussion about how larger α implies larger novelty or sophistication of solutions).

Proposition 3(b) shows that the optimal contest duration decreases with the productivity exponent ρ, and hence Proposition 3(b) yields the same result and has the same intuition as Theorem 1(b). Proposition 3(b) further shows that the optimal total award does not change with the productivity exponent ρ. This is because as ρ changes, the organizer changes T^* such that the marginal benefit and the marginal cost of additional total award on the organizer's profit stay the same. Thus, the optimal total award A^* does not change with ρ.

2.6 Endogenous Number of Participating solvers

In our main model, as is common in the innovation-contest literature (e.g., Ales et al. 2017, Hu and Wang 2020, Mihm and Schlapp 2019), we consider the case where the organizer ensures that N solvers participate in the contest, where N is taken exogenously. In this section, we consider the case where the number of participating solvers (hereafter, participants) is endogenous to the organizer's profit-maximization problem (cf. Terwiesch and Xu 2008, Ales et al. 2020). Given a population of $\overline{N}(\geq 2)$ solvers, let $N(\leq \overline{N})$ be the number of participants. When N is endogenous, the organizer's profit-maximization problem becomes:

$$\max_{T,N\in\{2,3,\ldots,\overline{N}\},(A_{(1)},A_{(2)},\ldots,A_{(N)})} \exp(-\delta T)\left(e^*+E\left[\tilde{\xi}_{(1)}^N\right]\right)-\sum_{j=1}^{N}A_{(j)} \text{ s.t. } (2.7),(2.8).$$

The number of participants N is not a free decision of the organizer because it is subject to the solver's participation condition (2.8). Instead, the organizer can endogenously affect N by setting the contest duration T and the award scheme $(A_{(1)},A_{(2)},\ldots,A_{(N)})$ accordingly. For instance, the organizer may induce more solvers to participate by setting a shorter T or setting larger award(s).

We first extend Theorem 1 to the case where the organizer decides on the optimal contest duration T^* and the optimal number of participants N^* to maximize his profit. Note that as in our main analysis, which N^* solvers participate in the contest is immaterial to our analysis because all solvers are identical. To study the problem of a patient organizer (as in Theorem 1) while retaining analytical tractability, we assume that the discount factor $\delta=0$ (i.e., the organizer is patient as in §2.4.1). Although we cannot analytically characterize N^* when $\delta>0$, our numerical analysis shows that Theorem 1 extends to the case where the organizer decides on T^* and

N^* under $\delta > 0$.[14]

Proposition 4 *Suppose that $\delta = 0$. (a) T^* is increasing in the scale parameter α. (b) Suppose further that $\theta(t) = \exp(\rho t)$. Then, T^* is decreasing in the productivity exponent ρ.*

Proposition 4(a) shows that for a patient organizer, the optimal contest duration T^* increases with the scale parameter α, and Proposition 4(b) shows that the optimal contest duration T^* decreases with the productivity exponent ρ. Intuitions of Proposition 4(a) and 4(b) are the same as intuitions of Theorem 1(a) and 1(b), respectively, because N^* does not depend on α or ρ. Specifically, the organizer determines N^* by balancing the contribution of the equilibrium effort e^* on the organizer's profit Π with the contribution of the expected value of the maximum output shock $E\left[\widetilde{\xi}^{N^*}_{(1)}\right]$ on Π. Interestingly, both of these terms change at the same rate as α, and do not depend on ρ (see Π in (A.21) in Appendix A). Thus, N^* does not depend on α or ρ. We next extend our results about the award scheme.

Proposition 5 *(a) Let $\overline{\delta}_0[N]$ and $\underline{\delta}_0[N]$ be the thresholds in Theorem 2 when N solvers participate in the contest. Then, there exists $\overline{\delta}^*_0 \equiv \max_{N\in\{2,3,\ldots,\overline{N}\}} \overline{\delta}_0[N]$ such that if $\delta > \overline{\delta}^*_0$, the WTA award scheme is optimal; and there exists $\underline{\delta}^*_0 \equiv \min_{N\in\{2,3,\ldots,\overline{N}\}} \underline{\delta}_0[N]$ such that if $\delta < \underline{\delta}^*_0$, giving multiple awards is optimal.*
(b) Suppose that $\theta(t) = \exp(\rho t)$. Then, $\underline{\delta}^*_0$ is increasing in ρ.
(c) Giving multiple unequal awards is optimal for any $\delta \in (\delta_{0,N}, \underline{\delta}^*_0)$, where $\delta_{0,N} (\geq 0)$ is defined as in Proposition 2 and $\delta_{0,N}$ is independent of the number of participants N.

Proposition 5(a) shows that when the organizer has sufficiently high urgency (i.e., $\delta > \overline{\delta}^*_0$), the WTA award scheme maximizes the organizer's profit under the optimal number of participants N^*. The intuition is similar to that of Theorem 2(a). The organizer can increase e^* by simultaneously increasing T and giving multiple awards.

[14]We take $\theta(t) = \exp(\rho t)$, and randomly generate 10,000 instances where $\delta < \delta_1$. We observe that in all instances, T^* increases with α and decreases with ρ. In each instance, we randomly select A from Uniform(0,10), F from Uniform(0,0.5A), ρ from Uniform(-0.01,0.01), α from Uniform(0,2), b from Uniform(2,10), and δ from Uniform(0,0.1); and assume $c = 1$, $\overline{N} = 100$, $\gamma_{(1)} = 1$, and $\widetilde{\xi}_i$ follows Gumbel distribution with mean 0 and scale parameter 1.

Yet, increasing T comes at the expense of more discounting, so giving multiple awards is optimal when the discount factor is sufficiently small. Proposition 5(b) extends Theorem 2(b) and has the same intuition. Finally, Proposition 5(c) shows that the main message of Proposition 2 is also preserved. Specifically, when it is optimal to give multiple awards, these awards should almost always be unequal.

2.7 Discussion and Conclusion

In this study, we have analyzed the duration and award scheme of an innovation contest intermediated by a crowdsourcing platform. Although practitioners consider the contest duration as a first-order decision, this decision has received only cursory attention from the theoretical contest literature. We take the first step toward filling this gap between the theory and practice.

We develop a normative model of an innovation contest, where an organizer determines the contest duration and the award scheme and each participating solver generates a solution by exerting effort. The quality of an solver's solution improves with her effort but is also subject to an output uncertainty. To capture intrinsic drivers in practice that we discuss in §2.1, our model contains the following key features. First, a solver optimally allocates her (total) effort over the contest duration according to her per-time productivity, which may decrease over time due to factors such as fatigue (e.g., Amabile et al. 2002) or may increase over time due to factors such as deeper understanding of concepts (e.g., Jain 2013). Second, each solver endogenously determines whether to participate or not. Thus, the organizer ensures that a certain (exogenous or endogenous) number of solvers participate by setting the contest rules accordingly. Third, (all else being equal) an organizer prefers obtaining solutions earlier rather than later, and hence the organizer's payoff is discounted at a rate that depends on his urgency. In addition to these features, our model helps us tease out the impact of the contest duration because we show that under a fixed duration, our model is equivalent to the standard modeling framework of the innovation-contest literature.

Our analysis yields the following novel insights. First, we show that the drivers for the optimal contest duration depends on how urgently the organizer needs solutions. Thus, although the contest literature (Seel 2018) posits that the contest duration should be determined by the effort-discounting trade-off, we identify the effort-participation trade-off, and show that this novel trade-off drives the optimal contest duration unless an organizer has high urgency. Our interviews with practitioners at crowdsourcing platforms indicate that an organizer rarely has high urgency, so we focus on the organizer without high urgency, and show that the optimal contest duration increases with the novelty or sophistication of solutions that the organizer seeks. This result seems consistent with the commonly adopted policy at crowdsourcing platforms. Perhaps more interestingly, our interviews with practitioners indicate that the underlying driver in practitioners' decision making seems to be the effort-participation trade-off as our model predicts. Furthermore, we analyze how the change in the solver productivity over time affects the optimal contest duration. Although one may expect the organizer to set a longer contest duration when the solver productivity increases over time, we show that the opposite is true. This is because although a longer contest duration increases the solver productivity and may help the organizer receive better solutions from solvers, it also hinders solvers' participation. This result suggests that an organizer who can provide support tools that increase the solver productivity over time (e.g., test cases, deployment guides, and documentation at Topcoder) should set a shorter contest duration.

Focusing on the contest duration and the distribution of awards, we first show that the organizer should give multiple awards when he has low urgency in obtaining solutions, as in many contests on crowdsourcing platforms. This result helps explain why many contests on these platforms give multiple awards. Although a few papers in the innovation-contest literature show that giving multiple awards can be optimal under *rare* cases discussed in §2.1, they either do not characterize an optimal award scheme (e.g., Kalra and Shi 2001, Ales et al. 2017) or show that the organizer should give multiple *equal* awards when the WTA award scheme is not optimal (e.g., Stouras et al. 2020). In contrast, we characterize an optimal award scheme where it is almost always optimal for the organizer to give *unequal* awards

– a result consistent with practice, and provide a possible explanation for this commonly applied strategy in practice. We further show that giving multiple awards is more desirable for an organizer who can provide support tools (e.g., test cases, deployment guides, and documentation at Topcoder) to increase the solver productivity over time. Thus, giving multiple awards goes hand in hand with offering such support tools.

Finally, focusing on the contest duration and the total award, we show that the optimal contest duration and the optimal total award increase with the novelty or sophistication of solutions that the organizer seeks. This result provides a plausible theory for recent empirical findings. Specifically, the positive correlation between the optimal contest duration and the optimal total award can be due to an increase in the novelty or sophistication of solutions that the organizer seeks.

Our study is the first step towards understanding the impact of the organizer's decision of the contest duration, and naturally has some limitations. Specifically, although we use a static model, one can imagine alternative settings where a dynamic model can be more appropriate. We use a static model because it captures the first-order effects and primary trade-offs in contests on crowdsourcing platforms such as InnoCentive and Topcoder. Also, we show that under a fixed contest duration, our model becomes equivalent to the standard modeling framework of the innovation-contest literature (which is also static), and this equivalence allows us to tease out the impact of the organizer's decision of the contest duration. However, a dynamic model may be useful to analyze alternative settings where the organizer provides feedback. An interesting research avenue is to study the relationship between the contest duration and feedback policies by employing a dynamic model. By analyzing a dynamic model, one can also study the impact of the contest duration on solvers' participation and effort decisions over time.[15] Furthermore, while we use a simple approach to capture the impact of a deeper understanding of concepts and

[15]It is worth noting that any dynamic model leads to an asymmetry among solvers, so it requires the analysis of a model with both solver heterogeneity and output uncertainty. It is well-established in the innovation-contest literature that such a model has a very limited tractability (e.g., Mihm and Schlapp 2019, Ales et al. 2019a, Hu and Wang 2020), and hence it requires many restrictive assumptions (e.g., the presence of only two solvers and a specific distribution for the solver uncertainty) that our model does not make.

fatigue on the per-time productivity, a more comprehensive dynamic model may be necessary to analyze the case where the per-time productivity of a solver at any point in time depends on her effort or her output uncertainty up to that point, and the analysis of such a dynamic model can be an interesting future research direction. Finally, instead of focusing on the case where the organizer runs only one contest, it can be interesting to examine the relationship between the contest duration and the scheduling of contests when the organizer runs multiple contests.

Chapter 3

Team Collaboration in Innovation Contests

3.1 Introduction

The internet has enabled organizations to look beyond their boundaries to solve their problems, and hence organizing *an innovation contest* has emerged as a viable tool to elicit high-quality solutions cost effectively without paying for "work, failure, or trial and error" (InnoCentive 2018). In an innovation contest, an organizer aims to obtain the best (i.e., highest-quality) solution for an innovation-related problem from a group of solvers, and solvers compete to develop this best solution and receive an award. Solvers can develop their solutions individually and make *individual submissions*,[1] or they may collaborate as teams and make *team submissions* if the organizer allows team collaboration. The prior studies on innovation management (e.g., Girotra et al. 2010) and innovation contests (e.g., Terwiesch and Xu 2008) suggest that quality of the best solution depends on the number of solutions generated, the average quality of solutions, and the variance in the quality of solutions. Team collaboration potentially affects all of these aspects, so it is unclear how team collaboration affects the quality of the best solution and hence the organizer's benefit from the innovation contest. Therefore, in this research, we study

[1] In some contests, solvers are incapable of making individual submissions due to a necessity of strong expertise in diverse topics (e.g., chemistry and biology). In such a contest, the organizer might always encourage team collaboration because receiving complete solutions requires collaboration. We naturally focus on contests where solvers are capable of making individual submissions.

when the organizer can benefit from team submissions.

Recent studies in the innovation literature suggest that the problem type plays a significant role in whether collaborating as a team leads to a better performance. Specifically, when the problem can be *decomposed* to different tasks (hereafter, *decomposable* problems), each of which can be performed by a team member, team collaboration can lead to a better performance (Chan et al. 2021). For example, software challenges can be classified as decomposable problems (e.g., MacCormack 2001, Chan et al. 2021) because a software system can be decomposed according to different functionalities, each of which can be performed independently (e.g., Apel and Kästner 2008). In this case, team members' efforts are substitutable. However, when the problem is very complex and hence requires a "holistic" solution, it may not be possible to decompose the problem to independently workable tasks, so the problem can be categorized as a *nondecomposable* problem (e.g., Orth and Malkewitz 2008, Chan et al. 2021). In this case, the prior literature suggests that a team may perform worse than a group of individuals because working as a team may require strong coordination and communication (e.g., Chan et al. 2021) and individuals' parallel working can be beneficial (e.g., Sommer and Loch 2004, Sommer et al. 2020). For example, design challenges are often classified as nondecomposable problems because of their holistic nature (e.g., Chan et al. 2021).

According to the above work, the inherent nature of the problem type — whether it is nondecomposable and decomposable — may help us understand why team collaboration is beneficial in certain settings and not in others. Indeed, when we observe popular crowdsourcing platforms such as InnoCentive, Topcoder, and 99designs, we realize that they have different problem types and different policies regarding team submissions. For example, InnoCentive mostly organizes (system) design challenges (such as design of affordable sanitation facility or inflation system for paddle boards), which require holistic solutions. If one applies the results of the innovation literature that we discuss above to the setting of InnoCentive, one might deduce that team submissions should be discouraged at InnoCentive, yet InnoCentive encourages solvers to make team submissions in about 70% of its challenges.[2] Similarly, applying the results of the innovation literature to the setting of Topcoder, one might expect that team submissions should be encouraged at Topcoder that organizes software challenges, yet Topcoder does not allow team submissions. These gaps between theory and practice may stem from the competitive nature in innovation contests, which is not relevant in traditional innovation/problem-solving settings, and hence not explicitly captured in the innovation literature.

Another factor that may play a part in how beneficial team collaboration is the "novelty" of solutions sought in an innovation contest. Indeed, the innovation-contest literature (e.g., Mihm and Schlapp 2019, Hu and Wang 2020) emphasizes that the novelty of solutions sought plays an important role in designing an innovation contest because for a low-novelty solution, the solution quality depends more on solvers' efforts, yet for a high-novelty solution, the solution quality depends more the uncertainty involved in the problem (e.g., Terwiesch and Xu 2008). For example, at Topcoder, development challenges such as Database Setup Challenge expect solvers to develop elementary solutions (Topcoder 2021a), so they can be considered as seeking low-novelty solutions (incremental innovations). In contrast, data science challenges such as Streamflow Forecast Challenge expect solvers to develop innovative algorithms (Topcoder 2021b), so they can be considered as seeking high-novelty solutions (breakthrough innovations). Despite the potential impact of the novelty of solutions sought on team collaboration, this factor has not been explored in the literature on teams.

To address these gaps in the prior theory and provide an explanation for the mixed policies in practice, we ask the following research question: (Q1) When should the organizer encourage or discourage team submissions?[3] Interestingly, our interviews with InnoCentive have revealed that, even when the organizer encourages team submissions, solvers can still make individual submissions in some

[2]We have collected the data from InnoCentive's website, and analyzed 60 contests whose announcements are available on August 30, 2019. Also, we have interviewed John Elliott (former vice president of sales at InnoCentive) and Greg Bell (former head of marketing and community at Topcoder) to gain insights into their operations.

[3]We do not distinguish a crowdsourcing platform from its clients because their incentives are aligned with regard to encouraging team submissions.

contests. As the organizer's benefit from team submissions hinges upon whether solvers benefit from collaborating as teams, we ask the following research question: (Q2) When do solvers benefit from team collaboration?

To answer our research questions, we incorporate a team's collective output into the standard modeling framework in the innovation-contest literature (e.g., Ales et al. 2017, Mihm and Schlapp 2019, Hu and Wang 2020). In our model, the organizer is interested in the quality of the best submission, and aims to determine whether to encourage or discourage team submissions. When solvers make individual submissions, our model boils down to the standard modeling framework of the innovation-contest literature, where each solver decides on her effort that improves her individual output. In this case, a solver's output depends on her *effort* and *output shock*, which is related to the novelty of solutions sought (e.g., Terwiesch and Xu 2008, Mihm and Schlapp 2019).

When solvers make team submissions, the quality of a team's *collective solution* depends on whether the organizer seeks solutions for a nondecomposable or decomposable problem. When the organizer has a nondecomposable problem, team members in each team can pursue alternative solution approaches in parallel (e.g., Amaldoss et al. 2000, Roels 2014). Here, the key motivation for solvers to collaborate is that solvers cannot be sure which solution approach is better before developing solutions (Amaldoss et al. 2000). Although team members pursue alternative solution approaches in parallel, they can share their ideas about different solution approaches (e.g., by brainstorming (Girotra et al. 2010) or providing feedback to each other (Gino 2019)). Such interactions of team members can lead to more diverse ideas about the problem (e.g., Taylor and Greve 2006) or better performance in identifying promising solution approaches (e.g., Singh and Fleming 2010).[4] We capture this effect by considering an *interaction shock* in addition to the team member's *effort* and *output shock*. After team members in each team de-

[4]We focus on solvers' diversity in background and experiences that help teams to generate more diverse set of ideas and solutions. Despite these differences, we assume that solvers are still comparable in terms of their skills and hence have identical output functions. It is well-established in the innovation contest literature that a model that captures both the solver's uncertainty (as in our study) and heterogeneity in skills has a very limited analytical tractability (cf. Ales et al. 2019b).

Table 3.1: Impact of team submissions on the organizer's profit.

	Low-novelty solutions (Effort-driven Solutions)	**High-novelty solutions (Shock-driven Solutions)**
Nondecomposable Problems	Team submissions are detrimental unless teams are highly diverse (e.g., logo design challenges at 99designs)	Team submissions are beneficial (e.g., design challenges at InnoCentive).
Decomposable Problems	Team submissions are beneficial when (a) teams are diverse or (b) teams are less diverse and increasing effort is difficult (e.g., development challenges at Topcoder).	Team submissions can be beneficial when teams are diverse (e.g., data science challenges at Topcoder).

velop their solutions in parallel, they select the most promising one and submit it as a team's solution.

When the organizer seeks solutions for a decomposable problem, team members can work on different tasks to develop the team's solution. In this case, team members can share their ideas about how to tackle their tasks, and hence a team's output depends on team members' *interaction shocks* in addition to their *efforts* and *output shocks*. Note that for any type of problems, as the organizer is interested in the quality of the best submission, the organizer's profit depends on solvers' efforts (hereafter, effort contribution) and the maximum of shocks (hereafter, shock contribution) under both individual submissions and team submissions.

As we summarize in Table 3.1, our equilibrium analysis yields the following results. Based on the innovation literature (e.g., Sommer et al. 2020, Chan et al. 2021), one might expect that the organizer should discourage team submissions when seeking solutions to a nondecomposable problem. Indeed, we show that this is the optimal strategy when the organizer seeks low-novelty solutions to a nondecomposable problem (e.g., logo design challenges at 99designs) and team diversity is sufficiently low. This is because solvers' incentive to exert effort is smaller when collaborating due to sharing the award and free-riding effect, and this negative effect can be outweighed by the benefit of team members' interactions on the shock contribution only when teams are highly diverse. However, we show that when the organizer seeks high-novelty solutions to a nondecomposable problem (e.g., design challenges at InnoCentive), the organizer can benefit from team submissions. The intuition is as follows. When the organizer seeks high-novelty solutions, the impact of team collaboration on the organizer's profit mostly depends on its impact on the shock contribution. When solvers make team submissions, the shock contribution increases because of team members' interactions, and hence the organizer's profit increases. This result may explain why team collaboration is encouraged in most InnoCentive challenges, despite the possible coordination losses in solvers' efforts.

We next analyze the case when the organizer seeks solutions to a decomposable problem, and find that the organizer should encourage team submissions, but interestingly, only under certain conditions. Specifically, when the organizer seeks high-novelty solutions (e.g., in Streamflow Forecast Challenge at Topcoder) or low-novelty solutions (e.g., in Database Setup Challenge at Topcoder), the organizer can benefit from team submissions through the increase in the shock contribution due to team members' interactions. Yet, in this case, the number of generated solutions decreases when solvers collaborate as teams. Thus, the benefit from team members' interactions should be sufficiently large, which can be achieved by more diverse teams (e.g., Taylor and Greve 2006, Singh and Fleming 2010), so that it can outweigh the negative impact of the decrease in the number of solutions on the shock contribution. Therefore, it can be better for the organizer to encourage (form, if possible) more diverse teams to benefit from the increase in the shock contribution.

We further show that when seeking low-novelty solutions to a decomposable problem, the organizer can benefit from team submissions also through the increase in the effort contribution if increasing effort is difficult for solvers. The intuition is as follows. When solvers collaborate as teams, although sharing the award and free-riding effect decrease each solver's incentive to exert effort, this effect can be outweighed by the benefit of accumulation of team members' efforts when increasing effort is difficult for solvers. It is worth noting that, in this case, diversity of ideas due to team members' interactions reduces team members' incentive to exert effort. Therefore, in this case, it can better for the organizer to encourage (form, if possible) less diverse teams to be able to benefit from the increase in the effort contribution. Our results for decomposable problems may help explain why Topcoder discourages team submissions because it may not be easy for Topcoder to control the diversity of teams, yet they also show that there is an opportunity to benefit from team submissions that Topcoder may consider in the future.

Finally, to answer our second research question, we compare the individual solver's utility and the team member's utility, and show that the team member's utility is larger. The intuition is as follows. When solvers collaborate as teams, their incentive to exert effort decreases due to sharing the award and free-riding effect. Thus, team members exert less effort than individual solvers, and hence incurs a smaller cost of effort. Because members in all teams end up exerting less effort, the expected award of solvers does not change when they collaborate, and hence the team member's utility is larger than the individual solver's utility. However, our additional analysis shows that when team collaboration results in synergistic gains, team members' incentive to exert effort increases, and hence each solver's utility can decrease with team collaboration. This result may explain why in some InnoCentive challenges, solvers make individual submissions although they are encouraged to make team submissions.

3.2 Related Literature

Our study is related to the innovation-contest literature, the literature on collaboration in other competitive settings (e.g., sales contests and competition between alliances), and the innovation and product-development literature on collaboration.

The innovation-contest literature analyzes the organizer's decisions in an innovation contest such as the number of participants (Terwiesch and Xu 2008, Ales et al. 2020), award scheme (Ales et al. 2017, Korpeoglu et al. 2020), feedback mechanism (Mihm and Schlapp 2019), duration of a contest (Korpeoglu et al. 2020), whether to run an internal contest (Nittala and Krishnan 2016), whether to run a simultaneous or sequential contest (Hu and Wang 2020), whether to run multiple parallel contests (Körpeoğlu et al. 2017), and whether to run multi-staged curated contests (Khorasani et al. 2020). We build on the modeling framework of these studies, which focus on the output uncertainty in an innovation contest.[5] We contribute to this literature by comparing the case of individual submissions with the case of team submissions in innovation contests, and by specifying conditions under which the organizer and solvers can benefit from team submissions.

Our study is also related to the literature on collaboration in other competitive settings such as sales contests and competition between alliances. In this stream of research, the closest study to ours is by Chen and Lim (2013), who show that team members exert larger efforts than individual solvers only when team members feel guilt aversion. Thus, in the absence of such a behavioral effect, solvers always reduce their efforts when making team submissions, and the organizer is always worse off. This is because the organizer maximizes the total output of solvers, and hence the organizer's profit only depends on solvers' efforts (not directly on their output shocks). Thus, whenever each solver's effort decreases, so does the organizer's profit. In an innovation contest, the organizer maximizes the best output, so our study has two fundamental differences from the setting of Chen and Lim (2013). First, the organizer's profit depends on solvers' efforts as well as the expected value of the maximum of output shocks. Second, the total effort exerted for the best output can be larger under team submissions even when each solver decreases her effort.

Other studies in this stream of research are as follows: Amaldoss et al. (2000) compare efforts of same-function alliance members with efforts of parallel-development alliance members; Amaldoss and Rapoport (2005) study the impact of the competition structure on alliance members' efforts for developing a product and a market; Amaldoss and Staelin (2010) compare efforts of same-function alliance members with efforts of cross-function alliance members; and Chen and Lim (2017) analyze the impact of the team's ability composition on team members' efforts. These papers do not consider an organizer who incentivizes alliance/team

[5]There are also studies (e.g., Stouras et al. 2020, Körpeoğlu and Cho 2018) focusing on the heterogeneity of solvers by suppressing the output uncertainty. (We refer the reader to Ales et al. (2019b) and Segev (2020) for detailed reviews.)

members to maximize profit, and do not compare alliance/team members' efforts with efforts that they would exert as individuals. Thus, we contribute to this literature by comparing the case of team submissions with the case of individual submissions and by characterizing when the organizer and solvers benefit from team submissions.

Our study is also related to the innovation and product-development literature on teams (e.g., Sosa et al. 2004, Bhaskaran and Krishnan 2009, Girotra et al. 2010, Sting et al. 2016, Taneri and Meyer 2017). The most relevant studies to ours are the theoretical study by Kavadias and Sommer (2009), the experimental study by Sommer et al. (2020), and empirical studies by Taylor and Greve (2006), Singh and Fleming (2010), and Chan et al. (2021). Kavadias and Sommer (2009) study the impact of collaboration on the quality of the best solution by comparing two extreme cases where all solvers work together to develop a collective solution and where each solver first develops a solution and then solvers together choose the best solution to implement, and show that for complex problems, individuals can perform better than a team. Sommer et al. (2020) experimentally validate this theoretical result, and suggest that this result is caused by the fact that working in parallel can be better for complex problems (Sommer and Loch 2004) and groupthink can harm the team's collective solution (Bendoly 2014). Similarly, Chan et al. (2021) suggest that individuals can perform better than a team when the problem is non-decomposable because in this case, collaboration may lead to coordination losses. These studies neither capture the competition among solvers nor do they consider an organizer who needs to incentivize competing solvers to exert costly efforts to maximize profit. As we discuss in §3.1, the gap between these results and crowdsourcing practice necessitates a study of team collaboration in innovation contests, and our study aims to fill this gap. Finally, to model the case of team submissions and interpret our theoretical results in this study, we build on the empirical findings that teams can generate more diverse solutions than individuals (e.g., Taylor and Greve 2006) and teams can perform better when identifying promising solution approaches (e.g., Singh and Fleming 2010). These results suggest that team collaboration may shift the average solution quality as well as increasing the variance of the quality of solutions.

3.3 Model

We consider an innovation contest in which an organizer elicits solutions to an innovation-related problem from a group of solvers ("she").[6] We study cases where each solver is capable of developing an individual solution, and hence the solver either develops her individual solution and makes an *individual submission;* or if the organizer encourages collaboration, she can collaborate with other solver(s) and make a *team submission*.

Motivated by practice and the literature (e.g., Chan et al. 2021) as we discuss in §3.1, we focus on two types of problems: (1) a nondecomposable problem, which is not amenable for team members to share the requisite tasks without significant coordination (e.g., Chan et al. 2021), and (2) a decomposable problem with (perfectly) substitutable tasks, where team members can divide up the tasks to be done. In §3.3.1 and §3.3.2, we present the models for the case of a nondecomposable problem and the case of a decomposable problem with (perfectly) substitutable tasks, respectively.[7]

3.3.1 Nondecomposable Problem

We analyze each solver's equilibrium effort and utility and the organizer's profit when solvers make individuals submissions and team submissions for a nondecomposable problem, respectively.

Individual Submissions. To model the case of individual submissions for a nondecomposable problem, we use the standard modeling framework in the innovation-contest literature (e.g., Ales et al. 2017, Mihm and Schlapp 2019). Let $\mathscr{N}$ be the

[6]In our model, the organizer can be an enterprise that organizes its own contest or a crowdsourcing platform that organizes a contest on behalf of its client, and we do not distinguish these two cases because incentives of a crowdsourcing platform and its client are aligned with regard to team submissions.

[7]In §3.5.3, we also consider the case when team collaboration may result in synergistic gains or coordination losses, and extend our results for the case of a problem decomposable to substitutable tasks. Also, in Appendix D.2, we consider a problem that can be decomposable to complementary tasks.

set of solvers who participate in the contest, and let $N \equiv |\mathscr{N}|$ denote the number of these solvers. Each solver $i \in \mathscr{N}$ generates an "output" y_i, which represents her solution quality and takes the following form:

$$y_i = e_i + \xi_i, \tag{3.1}$$

where e_i is solver i's effort and ξ_i is solver i's "output shock."[8] We assume that ξ_i's are independent, and follow normal distribution with mean zero and standard deviation σ (i.e., Normal$(0, \sigma^2)$), where $h(s)$ and $H(s)$ denote ξ_i's density and distribution functions, respectively. Output shock ξ_i represents the uncertainty in individual solution-generation and evaluation processes, and σ captures the novelty of solutions sought (e.g., Mihm and Schlapp 2019, Hu and Wang 2020). Specifically, σ is large when the organizer elicits high-novelty solutions (e.g., breakthrough innovations); and σ is small when the organizer elicits low-novelty solutions (e.g., incremental innovations).

Following the innovation-contest literature (e.g., Mihm and Schlapp 2019, Hu and Wang 2020), we restrict our attention to a symmetric pure-strategy Nash equilibrium. Let e^* be each solver's equilibrium effort when solvers make individual submissions, and let $P_N[e_i, e^*]$ be solver i's probability of producing the largest output when she exerts effort e_i given that all other solvers exert the equilibrium effort e^*. We derive $P_N[e_i, e^*]$ as follows:

$$P_N[e_i, e^*] = P\{y_i > y_j, \forall\, j \in \mathscr{N} \backslash \{i\}\} = \int_{-\infty}^{\infty} H\left(s + e_i - e^*\right)^{N-1} h(s) ds. \tag{3.2}$$

Consistent with the innovation-contest literature (e.g., Mihm and Schlapp 2019, Hu and Wang 2020), the organizer gives a winner award A (> 0) to the solver who produces the best output. If solver i produces the best output, she receives an award A; otherwise, she receives nothing.

When solver i exerts effort e_i, she incurs a cost of ce_i^b, where $c > 0$ and $b > 1$ (e.g., Chen and Lim 2013, Mihm and Schlapp 2019). Here, parameter b represents

[8] Solver i's output can be written as $y_i = e_i + (\xi_i^I + \xi_i^E)$, where ξ_i^I represents the uncertainty related to individual solution-generation process and ξ_i^E represents the uncertainty related to evaluation process. To ease our exposition, we represent both uncertainties with a single output shock ξ_i (e.g., Mihm and Schlapp 2019, Hu and Wang 2020).

the convexity of the cost function, which measures how quickly the marginal cost of additional effort increases, and hence "how difficult it is to increase effort" (e.g., Hu and Wang 2020). Accounting for ce_i^b and $P_N[e_i, e^*]$ in (3.2), solver i chooses her effort e_i by solving the following utility-maximization problem:

$$U^* \equiv \max_{e_i \in \mathbb{R}_+} \left(A \cdot P_N \left[e_i, e^* \right] - ce_i^b \right). \tag{3.3}$$

Evaluating the first-order condition of (3.3), we derive e^* and U^* in the following lemma.

Lemma 1 *Each solver's equilibrium effort e^* and utility U^* satisfy:*[9]

$$e^* = \left(\frac{A \cdot L_N}{cb} \right)^{\frac{1}{b-1}}, \text{ where } L_N \equiv (N-1) \int_{-\infty}^{\infty} H(s)^{N-2} h(s)^2 ds, \text{ and} \tag{3.4}$$

$$U^* = \frac{A}{N} - c \left(\frac{AL_N}{cb} \right)^{\frac{b}{b-1}}. \tag{3.5}$$

Note that e^* is decreasing in N because L_N in (3.4) is decreasing in N (cf. Ales et al. 2020).

The organizer is interested in the quality of the best solution, and hence the organizer's expected profit in equilibrium is

$$\Pi^* = E \left[\max_{i \in \mathcal{N}} \{y_i\} - A \right] = e^* + E \left[\max_{i \in \mathcal{N}} \{\xi_i\} \right] - A = \left(\frac{AL_N}{cb} \right)^{\frac{1}{b-1}} + m_N - A, \tag{3.6}$$

where $m_N = E\left[\max_{i \in \mathcal{N}} \{\xi_i\}\right]$. We refer to the first term e^* as the *solver-effort contribution* and the second term m_N as the *solver-shock contribution*, which is increasing in N by the definition of the expected value of the maximum of random variables.

Team Submissions. We now examine the case where solvers collaborate as teams and make team submissions for a nondecomposable problem. In preparation, we define a team t as a partition of the set of solvers $\mathcal{N}$, where we denote the set of solvers in team t by $\mathcal{N}_t^\tau$ and $\bigcup_t \mathcal{N}_t^\tau = \mathcal{N}$. Note that we use the superscript τ to distinguish notation related to team submissions from the notation related to individual submissions. Let n (≥ 2) be the number of solvers in each team (hereafter, team size). We assume that N is divisible by team size n so that $|\mathcal{N}_t^\tau| = n$ for all

[9] In Appendix D.3, we provide sufficient conditions for e^* in (3.4) to be the unique solution of (3.3), and we assume that at least one of these sufficient conditions holds (e.g., Mihm and Schlapp 2019, Ales et al. 2020).

$t \in \mathscr{T} \equiv \{1,2,...,N/n\}$ (e.g., Amaldoss et al. 2000, Chen and Lim 2013).

As we discuss in §3.1, when the organizer has a nondecomposable problem, the key motivation for solvers to collaborate is that they cannot be sure which solution approach is better before developing their solutions (Amaldoss et al. 2000). Thus, solvers collaborate as teams so that team members in each team t can pursue alternative solution approaches in parallel (e.g., Amaldoss et al. 2000, Roels 2014). Similar to the solution of each individual solver in (3.1), the solution of each team member $i(\in \mathscr{N}_t^\tau)$ depends on her effort e_{ti}^τ and "output shock" ξ_{ti} related to solution-generation and evaluation processes, which is analogous to ξ_i in (3.1): ξ_{ti}'s are independent and follows Normal$(0,\sigma^2)$. Because team members can share their ideas about different solution approaches (e.g., by brainstorming (Girotra et al. 2010), or providing feedback (Gino 2019)), the solution of each team member $i(\in \mathscr{N}_t^\tau)$ also depends on an "interaction shock" ξ_{ti}^B generated through team members' interactions. Thus, each team member i's output y_{ti}^τ takes the following form:

$$y_{ti}^\tau = e_{ti}^\tau + \xi_{ti}^B + \xi_{ti}.$$

We assume that ξ_{ti}^B follows Normal(μ_B,σ_B^2), where $\mu_B \geq 0$ and $\sigma_B > 0$; and ξ_{ti}^B's and ξ_{ti}'s are independent within each team t, considering that team members try alternative (mostly independent) solution approaches and team members come up with diverse (mostly independent) ideas. Also, ξ_{ti}^B's and ξ_{ti}'s are independent across teams, consistent with the common assumption in the literature that solvers' output shocks are independent (e.g., Hu and Wang 2020). Our model of the team member's output captures the idiosyncrasies in team members as well as the additional uncertainty and the performance enhancement generated through team members' interactions captured by parameters σ_B and μ_B, respectively.

To ease our exposition, let $\xi_{ti}^\tau = \xi_{ti}^B + \xi_{ti}$, which represents the team member's "overall output shock." Assuming that ξ_{ti}^B and ξ_{ti} are independent for each team member i in team t, since ξ_{ti}^B follows Normal(μ_B,σ_B^2) and ξ_{ti} follows Normal$(0,\sigma^2)$, each team member's overall output shock ξ_{ti}^τ follows Normal$(\mu_B,\sigma^2+\sigma_B^2)$, where $g(s)$ and $G(s)$ denote ξ_{ti}^τ's density function and distribution function, respectively.[10] The definitions of the team member's overall output shock ξ_{ti}^{τ} and the individual solver's output shock ξ_i enable us to capture two first-order effects of team collaboration identified in the innovation literature. First, the variance in team members' output shocks is larger than the variance in individual solvers' output shocks (i.e., $\sigma^2 + \sigma_B^2 > \sigma^2$) because team members bring diverse ideas about different solution approaches, and this effect increases as the team becomes more diverse (Taylor and Greve 2006). Second, the mean of team members' output shocks can be larger than the mean of individual solvers' output shocks (i.e., $\mu_B \geq 0$) because teams can be better at identifying promising solution approaches (Singh and Fleming 2010). This effect also increases as the team becomes more diverse (Singh and Fleming 2010).

After team members in team t develop their solutions in parallel, they select the best solution and submit it as team t's solution. Thus, team t's output y_t^{τ} can be calculated as follows:

$$y_t^{\tau} = \max_{i \in \mathcal{N}_t^{\tau}} y_{ti}^{\tau} = \max_{i \in \mathcal{N}_t^{\tau}} \{e_{ti}^{\tau} + \xi_{ti}^{\tau}\}. \tag{3.7}$$

Let $e^{*,\tau}$ be a team member's effort in a symmetric pure-strategy Nash equilibrium, and let $P_N^{\tau}[e_{ti}^{\tau}, e^{*,\tau}]$ be team t's probability of winning when team member $i \in \mathcal{N}_t^{\tau}$ exerts e_{ti}^{τ} given that other $(n-1)$ members of team t and members of other teams exert $e^{*,\tau}$. By using (3.7), we derive team t's probability of winning $P_N^{\tau}[e_{ti}^{\tau}, e^{*,\tau}]$ as follows:

$$\begin{aligned} P_N^{\tau}[e_{ti}^{\tau}, e^{*,\tau}] &= P\{y_t^{\tau} > y_k^{\tau}, \forall\, k \in \mathcal{T}\backslash\{t\}\} \\ &= P\left\{ \max_{m \in \mathcal{N}_t^{\tau}} \{e_{tm}^{\tau} + \xi_{tm}^{\tau}\} > \max_{j \in \mathcal{N}_k^{\tau}} \left\{e_{kj}^{\tau} + \xi_{kj}^{\tau}\right\}, \forall\, k \in \mathcal{T}\backslash\{t\} \right\} \\ &= P\left\{ \max\left\{ e_{ti}^{\tau} + \xi_{ti}^{\tau}, \max_{m \in \mathcal{N}_t^{\tau}\backslash\{i\}} \{e^{*,\tau} + \xi_{tm}^{\tau}\} \right\} > e^{*,\tau} + \max_{j \in \mathcal{N}_k^{\tau}} \left\{\xi_{kj}^{\tau}\right\}, \forall\, k \in \mathcal{T}\backslash\{t\} \right\} \\ &= \int_{-\infty}^{\infty} G(s + e_{ti}^{\tau} - e^{*,\tau})^{N-1} g(s) ds + (n-1) \int_{-\infty}^{\infty} G(s + e^{*,\tau} - e_{ti}^{\tau}) G(s)^{N-2} g(s) ds, \end{aligned} \tag{3.8}$$

where the first term is the probability that team member i in team t generates the

[10] When ξ_{ti}^{B} and ξ_{ti} are correlated for each team member i in team t, ξ_{ti}^{τ} follows Normal$(\mu_B, \sigma^2 + \sigma_B^2 + 2\rho\sigma\sigma_B)$, where ρ is the correlation. In Appendix D.1, we show that our main results continue to hold under such correlation.

best solution and the second term is the probability that one of other $(n-1)$ members in team t generates the best solution. The first derivative of (3.8) with respect to e_i^τ evaluated at $e_i^\tau = e^{*,\tau}$ yields:

$$\left.\frac{\partial P_N^\tau\left[e_{ti}^\tau, e^{*,\tau}\right]}{\partial e_{ti}^\tau}\right|_{e_{ti}^\tau = e^{*,\tau}} = (N-n)\int_{-\infty}^{\infty} G(s)^{N-2} g(s)^2 ds. \tag{3.9}$$

We assume that winning team members share the award A equally (e.g., Chen and Lim 2013).[11] Thus, team member i chooses her effort e_{ti}^τ by solving the following utility-maximization problem:

$$U^{*,\tau} \equiv \max_{e_{ti}^\tau \in \mathbb{R}_+} \left(\frac{A}{n} P_N^\tau\left[e_{ti}^\tau, e^{*,\tau}\right] - c(e_{ti}^\tau)^b\right). \tag{3.10}$$

Evaluating the first-order condition of (3.10), we derive $e^{*,\tau}$ and $U^{*,\tau}$ in the following lemma.

Lemma 2 *Each team member's equilibrium effort $e^{*,\tau}$ and utility $U^{*,\tau}$ satisfy:*[12]

$$e^{*,\tau} = \left(\frac{A \cdot L_N^\tau}{ncb}\right)^{\frac{1}{b-1}}, \text{ where } L_N^\tau \equiv (N-n)\int_{-\infty}^{\infty} G(s)^{N-2} g(s)^2 ds, \text{ and} \tag{3.11}$$

$$U^{*,\tau} = \left(\frac{A}{n}\right)\left(\frac{n}{N}\right) - c(e^{*,\tau})^b = \frac{A}{N} - c\left(\frac{AL_N^\tau}{ncb}\right)^{\frac{b}{b-1}}. \tag{3.12}$$

We next analyze the organizer's profit under team submissions. As in the case of individual submissions, the organizer is interested in the quality of best solution. Let $m_N^\tau \equiv E\left[\max_{t\in\mathscr{T}}\left\{\max_{i\in\mathscr{N}_t^\tau}\left\{\xi_{ti}^\tau\right\}\right\}\right]$. For all $i \in \mathscr{N}_t$ and $t \in \mathscr{T}$, $e_{ti}^\tau = e^{*,\tau}$ as in (3.11), so by applying (3.7), the organizer's expected profit in equilibrium can be calculated as follows:[13]

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[11] Our interviews with practitioners and solvers at InnoCentive have revealed that equal share of the award is common in practice. Also, in our main analysis, to isolate the impact of team submissions, we assume that the organizer gives the same award A under individual and team submissions. However, in §3.5.1, we extend our main results to the case where the organizer sets optimal awards under individual submissions and team submissions.

[12] In Appendix D.3, we provide sufficient conditions for $e^{*,\tau}$ in (3.11) to be the unique solution of (3.10), and we assume that at least one of these sufficient conditions holds (e.g., Mihm and Schlapp 2019, Ales et al. 2020).

[13] Note that this does not require that each team t can gauge the exact value of ξ_{ti}^τ's, and only implies that each team's ranking of its members' outputs is consistent with the organizer's ranking of them.

$$\Pi^{*,\tau} = E\left[\max_{t\in\mathscr{T}}\{y_t^\tau\} - A\right] = E\left[\max_{t\in\mathscr{T}}\left\{\max_{i\in\mathscr{N}_t^\tau}\{e^{*,\tau}+\xi_{ti}^\tau\}\right\}\right] -$$
$$= \left(\frac{AL_N^\tau}{ncb}\right)^{\frac{1}{b-1}} + m_N^\tau - A. \tag{3.13}$$

Analogous to the terms in (3.6), we refer to the first term $e^{*,\tau}$ as the *team-effort contribution*, and the second term m_N^τ as the *team-shock contribution*.

3.3.2 Decomposable Problem

We next discuss the case of a decomposable problem that can be divided to substitutable tasks, each of which can be performed by different solvers. For example, a software development project can be decomposed into substitutable tasks, which may represent different "features," i.e., units of functionality (e.g., Apel and Kästner 2008). In this case, the efforts exerted by different solvers for each task are also substitutable.

Individual Submissions. Suppose that $\mathscr{K}$ is the set of substitutable tasks associated with the decomposable problem and $K \equiv |\mathscr{K}|$ denotes the number of tasks. Let y_{ik} be the performance of each solver $i \in \mathscr{N}$ for each task $k \in \mathscr{K}$. Analogous to solver's output y_i in (3.1), for each task k, solver i's performance depends on her effort e_{ik} and output shock ξ_{ik} (Hu and Wang 2020). For ease of exposition, we assume that ξ_{ik}'s are independently and identically distributed with Normal$(0, \sigma^2/K)$. Thus, each solver i's output associated with the decomposable problem is

$$y_i = \sum_{k\in\mathscr{K}} y_{ik} = \sum_{k\in\mathscr{K}} (e_{ik} + \xi_{ik}) = \sum_{k\in\mathscr{K}} e_{ik} + \sum_{k\in\mathscr{K}} \xi_{ik} = e_i + \xi_i, \tag{3.14}$$

where $e_i \equiv \sum_{k\in\mathscr{K}} e_{ik}$ is solver i's "overall effort" across all tasks; and $\xi_i \equiv \sum_{k\in\mathscr{K}} \xi_{ik}$ is solver i's "overall output shock." Since ξ_{ik} follows Normal$(0, \sigma^2/K)$, the solver's overall output shock ξ_i follows Normal$(0, \sigma^2)$. (This is analogous to ξ_i in §3.3.1; see Table 3.2 (bottom row)). Thus, the solver i's output y_i in (3.14) for a decomposable problem is identical to the solver i's output y_i in (3.1) for a nondecomposable problem. Therefore, in the case of a decomposable problem with substitutable tasks, the solver's equilibrium effort e^*, the solver's equilibrium utility U^*, and the organizer's profit Π^* are as stated in (3.4), (3.5), and (3.6), respectively.

Team Submissions. We now examine the case where solvers make team submissions for a decomposable problem with K substitutable tasks. As in §3.3.1, n (≥ 2) is the team size, and the number of solvers N is divisible by n so that $|\mathscr{N}_t^{\tau}| = n$ for all $t \in \mathscr{T}$. We also assume that the number of tasks K is divisible by n so that team member i performs each task $k \in \mathscr{K}_i$, where $\mathscr{K}_i$ is the set of tasks that team member i performs, $|\mathscr{K}_i| = K/n$ for each team member i, and $\bigcup_{i \in \mathscr{N}_t^{\tau}} \mathscr{K}_i = \mathscr{K}$. Similar to the case of individual submissions, for each task $k \in \mathscr{K}_i$, team member $i \in \mathscr{N}_t^{\tau}$'s performance depends on her effort e_{tik}^{τ} and output shock ξ_{tik} (Hu and Wang 2020), yet team members can also share their ideas about how to tackle their tasks. Therefore, the performance of team member i for task k also depends on a random variable ξ_{tik}^B generated by these interactions. Thus, team member i's performance across her set of tasks $\mathscr{K}_i$ can be calculated as follows:

$$y_{ti}^{\tau} = \sum_{k \in \mathscr{K}_i} (e_{tik}^{\tau} + \xi_{tik}^B + \xi_{tik}) = e_{ti}^{\tau} + \sum_{k \in \mathscr{K}_i} (\xi_{tik}^B + \xi_{tik}),$$

where $e_{ti}^{\tau} \equiv \sum_{k \in \mathscr{K}_i} e_{tik}^{\tau}$ is the total effort of team member i exerted for all tasks in $\mathscr{K}_i$ that she performs. Thus, team t's output y_t^{τ} can be calculated as follows:

$$y_t^{\tau} = \sum_{i \in \mathscr{N}_t^{\tau}} y_{ti}^{\tau} = \sum_{i \in \mathscr{N}_t^{\tau}} e_{ti}^{\tau} + \sum_{i \in \mathscr{N}_t^{\tau}} \sum_{k \in \mathscr{K}_i} (\xi_{tik}^B + \xi_{tik}) = \sum_{i \in \mathscr{N}_t^{\tau}} e_{ti}^{\tau} + \xi_t^B + \xi_t, \quad (3.15)$$

where $\xi_t \equiv \sum_{i \in \mathscr{N}_t^{\tau}} \sum_{k \in \mathscr{K}_i} \xi_{tik}$ is team t's "output shock" related to solution-generation and evaluation processes and ξ_t follows Normal$(0, \sigma^2)$ (this is analogous to ξ_{ti} in (3.7); see Table 3.2 (bottom row)); and $\xi_t^B \equiv \sum_{i \in \mathscr{N}_t^{\tau}} \sum_{k \in \mathscr{K}_i} \xi_{tik}^B$ is the "interaction shock" generated through team members' interactions and ξ_t^B follows Normal(μ_B, σ_B^2).[14] (This is analogous to ξ_{ti}^B in §3.3.1; see Table 3.2 (bottom row).) As in §3.3.1, we assume that ξ_t^B's and ξ_t's are independent within each team t and across teams. Let $\xi_t^{\tau} = \xi_t^B + \xi_t$, which represents the team's "overall output shock." Then, since ξ_t^B follows Normal(μ_B, σ_B^2) and ξ_t follows Normal$(0, \sigma^2)$, ξ_t^{τ} follows Normal$(\mu_B, \sigma^2 + \sigma_B^2)$, where $g(s)$ and $G(s)$ are ξ_{ti}'s density function and

[14]Note that as ξ_t follows Normal$(0, \sigma^2)$ and ξ_t is the summation of $n \cdot \frac{K}{n}$ number of ξ_{tik}'s, we assume that ξ_{tik}'s are independently and identically distributed with Normal$(0, \sigma^2/K)$ for ease of illustration. Similarly, as ξ_t^B follows Normal(μ_B, σ_B^2) and ξ_t^B is the summation of $n \cdot \frac{K}{n}$ number of ξ_{tik}^B's, we assume that ξ_{tik}^B's are independently and identically distributed with Normal$(\mu_B/K, \sigma_B^2/K)$.

Table 3.2: Each Individual Solver's Output and Each Team's Output.

	Individual Submissions	**Team Submissions**
Nondecomposable Problems	$y_i = e_i + \xi_i$, where $\xi_i \backsim$Normal$(0, \sigma^2)$	$y_t^\tau = \max_{i\in\mathcal{N}_t^\tau} y_{ti}^\tau = \max_{i\in\mathcal{N}_t^\tau} \{e_{ti}^\tau + \xi_{ti}^B + \xi_{ti}\}$, where $\xi_{ti}^B \backsim$Normal(μ_B, σ_B^2) and $\xi_{ti} \backsim$Normal$(0, \sigma^2)$
Decomposable Problems	$y_i = e_i + \xi_i$, where $\xi_i \backsim$Normal$(0, \sigma^2)$	$y_t^\tau = \sum_{i\in\mathcal{N}_t^\tau} y_{ti}^\tau = \sum_{i\in\mathcal{N}_t^\tau} e_{ti}^\tau + \xi_t^B + \xi_t$, where $\xi_t^B \backsim$Normal(μ_B, σ_B^2) and $\xi_t \backsim$Normal$(0, \sigma^2)$

distribution function, respectively.[15]

As we summarize in Table 3.2, the definitions of the team's overall output shock ξ_t^τ and the individual solver's output shock ξ_i enable us to capture two first-order effects of team collaboration identified in the innovation literature. First, the variance in teams' output shocks is larger than the variance in individual solvers' output shocks (i.e., $\sigma^2 + \sigma_B^2 > \sigma^2$) because team members bring diverse ideas for each task (Taylor and Greve 2006). Second, the mean of teams' output shocks can be larger than the mean of individual solvers' output shocks (i.e., $\mu_B \geq 0$) because teams can be better at identifying a promising solution approach for each task (Singh and Fleming 2010).

As in §3.3.1, let $e^{*,\tau}$ be a team member's (total) effort in a symmetric pure-strategy Nash equilibrium, and let $P_{N/n}^\tau[e_{ti}^\tau, e^{*,\tau}]$ be team t's probability of winning when team member $i \in \mathcal{N}_t^\tau$ exerts e_{ti}^τ given that other $(n-1)$ members of team t and members of other teams exert $e^{*,\tau}$. By using (3.15) and noting that the number of teams is $|\mathcal{T}| = N/n$, we derive $P_{N/n}^\tau[e_{ti}^\tau, e^{*,\tau}]$ as follows:

$$\begin{aligned} P_{N/n}^\tau[e_{ti}^\tau, e^{*,\tau}] &= P\{y_t^\tau > y_k^\tau, \forall\, k \in \mathcal{T}\backslash\{t\}\} \\ &= P\{e_{ti}^\tau + (n-1)e^{*,\tau} + \xi_t^\tau > ne^{*,\tau} + \xi_k^\tau, \forall\, k \in \mathcal{T}\backslash\{t\}\} \\ &= \int_{-\infty}^{\infty} G(s + e_{ti}^\tau - e^{*,\tau})^{\frac{N}{n}-1} g(s)ds. \end{aligned} \tag{3.16}$$

[15]For ease of illustration, we consider the case where ξ_t^B and ξ_t are independent for each team t. When ξ_t^B and ξ_t are correlated for each team t, ξ_t^τ follows Normal$(\mu_B, \sigma^2 + \sigma_B^2 + 2\rho\sigma\sigma_B)$, where ρ is the correlation. We show in Appendix D.1 that our main results continue to hold under such correlation.

As in §3.3.1, members of the winning team share the award A equally, so team member i chooses her effort e^{τ}_{ti} by solving the following utility-maximization problem:

$$U^{*,\tau} \equiv \max_{e^{\tau}_{ti} \in \mathbb{R}_+} \left(\frac{A}{n} P^{\tau}_{N/n} [e^{\tau}_{ti}, e^{*,\tau}] - c(e^{\tau}_{i})^b \right). \tag{3.17}$$

Evaluating the first-order condition of (3.17), we derive $e^{*,\tau}$ and $U^{*,\tau}$ in the following lemma.

Lemma 3 *Each team member's equilibrium effort $e^{*,\tau}$ and utility $U^{*,\tau}$ satisfy:*[16]

$$e^{*,\tau} = \left(\frac{A \cdot L^{\tau}_{N/n}}{ncb} \right)^{\frac{1}{b-1}}, \text{ where } L^{\tau}_{N/n} \equiv \left(\frac{N}{n} - 1 \right) \int_{-\infty}^{\infty} G(s)^{\frac{N}{n}-2} g(s)^2 ds, \tag{3.18}$$

$$U^{*,\tau} = \left(\frac{A}{n} \right) \left(\frac{n}{N} \right) - c(e^{*,\tau})^b = \frac{A}{N} - c \left(\frac{AL^{\tau}_{N/n}}{ncb} \right)^{\frac{b}{b-1}}. \tag{3.19}$$

We next analyze the organizer's profit under team submissions. As in §3.3.1, the organizer is interested in the quality of best submission. Let $m^{\tau}_{N/n} \equiv E\left[\max_{t \in \mathscr{T}} \xi^{\tau}_t\right]$. For all $i \in \mathscr{N}_t$ and $t \in \mathscr{T}$, $e^{\tau}_{ti} = e^{*,\tau}$ as in (3.18), so by applying (3.15), the organizer's expected profit in equilibrium

$$\begin{aligned} \Pi^{*,\tau} &= E\left[\max_{t \in \mathscr{T}} \{y^{\tau}_t\} - A \right] = ne^{*,\tau} + E\left[\max_{t \in \mathscr{T}} \{\xi^{\tau}_t\} \right] - A \\ &= n \left(\frac{AL^{\tau}_{N/n}}{ncb} \right)^{\frac{1}{b-1}} + m^{\tau}_{N/n} - A. \end{aligned} \tag{3.20}$$

Analogous to the terms defined in (3.13), we refer to the first term $ne^{*,\tau}$ as the *team-effort contribution* and the second term $m^{\tau}_{N/n}$ as the *team-shock contribution*.

3.4 Impact of Team Collaboration

By using the results established in Lemmas 1, 2, and 3 and the organizer's profits in (3.6),

(3.13), and (3.20), we now analyze the impact of team collaboration on the organizer's profit and the solver's utility when the organizer seeks solutions to a

[16] In Appendix D.3, we provide sufficient conditions for $e^{*,\tau}$ in (3.18) to be the unique solution of (3.17), and we assume that at least one of these sufficient conditions holds (e.g., Mihm and Schlapp 2019, Ales et al. 2020).

nondecomposable problem and a decomposable problem in §3.4.1 and §3.4.2, respectively.

3.4.1 Nondecomposable Problem

Analysis of Organizer's Profit. We compare the organizer's profit Π^* given in (3.6) for the case when solvers make individual submissions and the organizer's profit $\Pi^{*,\tau}$ given in (3.13) for the case when solvers make team submissions. This comparison yields:

$$\Pi^{*,\tau} - \Pi^* = (e^{*,\tau} - e^*) + (m_N^\tau - m_N). \tag{3.21}$$

In (3.21), the first term is the difference between the team-effort contribution and the solver-effort contribution, and the second term in (3.21) is the difference between the team-shock contribution and the solver-shock contribution. Recall from §3.3.1 that the individual solver's output shock follows Normal$(0, \sigma^2)$ and the team member's overall output shock follows Normal$(\mu_B, \sigma^2 + \sigma_B^2)$. In preparation, we first analyze the impact of team members' interactions (measured by σ_B and μ_B) on $(e^{*,\tau} - e^*)$ and $\left(m_N^\tau - m_N\right)$, and obtain the following proposition. (All proofs are presented in Appendix C.)

Proposition 1 *(Nondecomposable problem) (a) $e^{*,\tau} < e^*$, and $(e^{*,\tau} - e^*)$ is decreasing in σ_B and constant in μ_B. (b) $m_N^\tau > m_N$, and $(m_N^\tau - m_N)$ is increasing in σ_B and μ_B.*

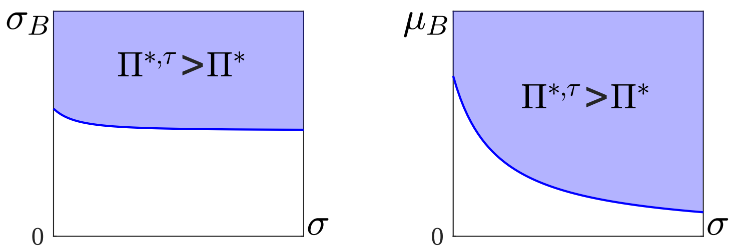
Proposition 1(a) first shows that when the organizer elicits solutions to a nondecomposable problem, the team-effort contribution $e^{*,\tau}$ is always smaller than the solver-effort contribution e^*. The intuition is as follows. When solvers collaborate as teams to solve a nondecomposable problem, as members of each team work in parallel, it can be observed from (3.7) that each team's submission can be generated by one of its members. Due to this free-riding effect, a team member has less incentive to exert effort than an individual solver who develops her own solution. Also, when solvers collaborate as teams, members of the winning team share the award, which further reduces the solver's incentive to exert effort. Thus, when solvers collaborate as teams, the effort contribution decreases (i.e., $e^{*,\tau} < e^*$).

Proposition 1(a) further shows that the difference of effort contributions (i.e., $e^{*,\tau} - e^*$) decreases with the uncertainty (i.e., σ_B) generated through team members' interactions. This is because with this additional uncertainty, the quality of each team's solution depends less on team members' efforts, and hence each team member's incentive to exert effort decreases. Finally, Proposition 1(a) shows that the difference of effort contributions does not change with the performance enhancement (i.e., μ_B) generated through team members' interactions. This is because this performance enhancement does not affect the relative ranking of teams' submissions, and hence does not have any impact on the team member's incentive to exert effort.

Proposition 1(b) shows that when the organizer elicits solutions to a nondecomposable problem, the team-shock contribution m_N^τ is larger than the solver-shock contribution m_N, and the difference of shock contributions (i.e., $m_N^\tau - m_N$) increases with both the additional uncertainty (i.e., σ_B) and the performance enhancement (i.e., μ_B) generated through team members' interactions. The intuition is as follows. When the organizer seeks solutions to a nondecomposable problem, even if solvers collaborate as teams, each solver develops one solution. Thus, under both individual and team submissions, the best solution is selected among N solutions. Yet, the additional uncertainty (i.e., σ_B) and the performance enhancement (i.e., μ_B) generated through team members' interactions both increase the team-shock contribution m_N^τ. Therefore, when solvers collaborate as teams, the shock contribution increases, and this benefit (i.e., $m_N^\tau - m_N$) increases with σ_B and μ_B.

In summary, Proposition 1 reveals the following tradeoff for a nondecomposable problem: team collaboration results in a lower effort contribution but results in a higher shock contribution. To examine which effect dominates the other, we now use (3.21) and characterize the conditions under which team collaboration results in a higher organizer's profit in the following theorem.

Theorem 1 *(Nondecomposable problem) (a) Suppose* $\mu_B = 0$. *Then, for any* σ, *there exist two thresholds* $\overline{\sigma_B}$ *and* $\underline{\sigma_B}$ *such that* $\Pi^{*,\tau} > \Pi^*$ *if* $\sigma_B > \overline{\sigma_B}$, *and* $\Pi^{*,\tau} < \Pi^*$



(a) Impact of σ and σ_B when $\mu_B = 0$. (b) Impact of σ and μ_B when $\sigma_B \to 0$.

Figure 3.1: For a nondecomposable problem, when is the organizer's profit $\Pi^{*,\tau}$ under team submissions larger than the organizer profit Π^* under individual submissions? Setting: $\xi_i \sim \text{Normal}(0, \sigma^2)$, $\xi_{ti} \sim \text{Normal}(0, \sigma^2)$, $\xi_{ti}^B \sim \text{Normal}(\mu_B, \sigma_B^2)$, $A = 1$, $b = 2$, $c = 1$, $n = 2$, and $N = 10$.

if $\sigma_B < \underline{\sigma_B}$.

(b) Suppose $\sigma_B \to 0$. *Then, there exist thresholds* $\overline{\sigma}$ *and* $\overline{\mu_B}$ *such that* $\Pi^{*,\tau} \geq \Pi^*$ *if* $\sigma \geq \overline{\sigma}$ *or* $\mu_B \geq \overline{\mu_B}$, *and* $\Pi^{*,\tau} < \Pi^*$ *if* $\sigma < \overline{\sigma}$ *and* $\mu_B < \overline{\mu_B}$.

When the organizer seeks solutions to a nondecomposable problem, one might expect that the organizer cannot benefit from team submissions due to the decrease in solvers' incentive to exert effort. Yet, Theorem 1 shows that the organizer can benefit from team submissions under some conditions. Specifically, Theorem 1(a) shows that when there is no performance enhancement generated through team members' interactions (i.e., $\mu_B = 0$), the organizer can benefit from team submissions if the uncertainty generated through team members' interactions is sufficiently large (i.e., $\sigma_B > \overline{\sigma_B}$). The intuition is as follows. As Proposition 1 shows, when solvers collaborate as teams, although the effort contribution decreases (i.e., $e^{*,\tau} < e^*$), the shock contribution increases (i.e., $m_N^\tau > m_N$). Proposition 1 also shows that this positive effect increases as the uncertainty (i.e., σ_B) generated through team members' interactions increases, and hence can dominate the decrease in the effort contribution. Therefore, as Figure 3.1(a) illustrates, the organizer can benefit from team submissions through the increase in the shock contribution when σ_B is above a threshold. However, when σ_B is below a threshold, the organizer cannot benefit from team submissions.

Theorem 1(b) shows that when there is no additional uncertainty generated through team members' interactions (i.e., $\sigma_B \to 0$), the organizer can benefit from team submissions if the performance enhancement generated through team members' interactions or the novelty of solutions sought is sufficiently large (i.e., $\sigma \geq \overline{\sigma}$ or $\mu_B \geq \overline{\mu_B}$). The intuition is as follows. As Proposition 1 shows, the increase in the shock contribution increases as the performance enhancement (i.e., μ_B) increases. Thus, when μ_B is above a threshold, the increase in the shock contribution dominates the decrease in the effort contribution, and hence the organizer's profit is larger under team submissions. Furthermore, as the solution novelty measured by σ increases, the decrease in the effort contribution becomes insignificant because each solver's incentive to exert effort decreases (e.g., Terwiesch and Xu 2008), whereas the increase in the shock contribution increases. Thus, when the solution novelty measured by σ is above a threshold, the organizer can also benefit from team submissions through the increase in the shock contribution. Therefore, as Figure 3.1(b) illustrates, the organizer can benefit from team submissions when $\sigma \geq \overline{\sigma}$ or $\mu_B \geq \overline{\mu_B}$. Otherwise, the organizer's profit decreases when solvers collaborate as teams. Overall, as we summarize in Table 3.3 (top row), Theorem 1 shows that for a large σ, the organizer can benefit from team submissions; and for a small σ, the organizer can benefit from team submissions only when $\sigma_B > \overline{\sigma_B}$ or $\mu_B > \overline{\mu_B}$.

Theorem 1 has important managerial implications when combined with prior empirical results that the additional uncertainty and performance enhancement generated through team members' interactions increases with the diversity of teams (e.g., Taylor and Greve 2006, Singh and Fleming 2010). Specifically, consistent with the innovation literature (e.g., Sommer et al. 2020, Chan et al. 2021), we show that it may be better for the organizer to discourage team submissions when eliciting low-novelty solutions (small σ) to a nondecomposable problem such as logo design challenges at 99designs unless teams are highly diverse. Yet, in contrast to the results in the literature, we show that when the nondecomposable problem requires high-novelty solutions (large σ), as in design challenges at InnoCentive for example, it may be beneficial for the organizer to encourage team submissions to obtain more diverse solutions. These results may explain why team collaboration is encouraged in most InnoCentive challenges, but discouraged at 99designs.

Table 3.3: Impact of team submissions on the organizer's profit.

	Small σ (Low-novelty solutions)	**Large σ (High-novelty solutions)**
Nondecomposable problem	Team submissions are detrimental unless σ_B or μ_B is very large (e.g., logo design challenges at 99designs)	Team submissions are beneficial (e.g., design challenges at InnoCentive).
Decomposable problem	Team submissions are beneficial when (a) σ_B or μ_B is large or (b) σ_B is small and b is large (e.g., development challenges at Topcoder).	Team submissions are beneficial when σ_B or μ_B is large (e.g., data science challenges at Topcoder).

In addition to these managerial implications, Theorem 1 has important implications for the contest theory. The prior literature focuses on the case where the organizer maximizes the total or average output of solvers, and shows that as the equilibrium effort is smaller under team submissions, the organizer cannot benefit from team submissions unless there exist some behavioral effects (e.g., guilt aversion) that increase the equilibrium effort (cf. Chen and Lim 2013). Different from this literature, we consider an innovation contest where the organizer maximizes the best output, so the organizer can benefit from team collaboration via the increase in the shock contribution. This positive effect of team collaboration disappears when the organizer maximizes the total or average output because, in that case, the organizer's profit depends only on solvers' efforts. Therefore, in an innovation contest, the organizer can benefit from team collaboration more than what the contest theory suggests for other types of contests (e.g., sales contests). Our prediction also seems to be consistent with the empirical finding of Girotra et al. (2010) that analyzing the average output may underestimate the benefits of teams.

Analysis of Solver's Utility. We now examine when solvers benefit from collaboration while generating solutions to a nondecomposable problem. To do that, we compare an individual solver's utility U^* given in (3.5) and a team member's utility $U^{*,\tau}$ given in (3.12) as stated in Lemmas 1 and 2, respectively. This comparison yields:

$$U^{*,\tau} - U^{*} = c\,(e^{*})^{b} - c\,(e^{*,\tau})^{b}. \tag{3.22}$$

As seen from (3.22), the difference between $U^{*,\tau}$ and U^{*} boils down to the difference between the solver's cost of effort and the team member's cost of effort. Thus, from Proposition 1(a), we obtain the following corollary, whose intuition is the same as the intuition of Proposition 1(a).

Corollary 1 *Team member's utility $U^{*,\tau}$ is always larger than individual solver's utility U^{*}.*

Corollary 1 suggests that when the organizer seeks solutions to a nondecomposable problem, the organizer can collect team submissions whenever the organizer encourages team submissions.

3.4.2 Decomposable Problem

Analysis of Organizer's Profit. We compare the organizer's profit Π^{*} given in (3.6) for the case when solvers make individual submissions and the organizer's profit $\Pi^{*,\tau}$ given in (3.20) for the case when solvers make team submissions. This comparison yields:

$$\Pi^{*,\tau} - \Pi^{*} = (ne^{*,\tau} - e^{*}) + \left(m^{\tau}_{N/n} - m_{N}\right). \tag{3.23}$$

In (3.23), the first term is the difference between the team-effort contribution (based on n members of the winning team) and the solver-effort contribution, and the second term is the difference between the team-shock contribution and the solver-shock contribution. Recall from §3.3.2 that the individual solver's output shock follows Normal$(0, \sigma^2)$ and the team's overall output shock follows Normal$(\mu_B, \sigma^2 + \sigma_B^2)$. In preparation, we first analyze the impact of teams' interactions (measured by σ_B and μ_B) on $(e^{*,\tau} - e^{*})$ and $\left(m^{\tau}_{N} - m_{N}\right)$, and obtain the following proposition.

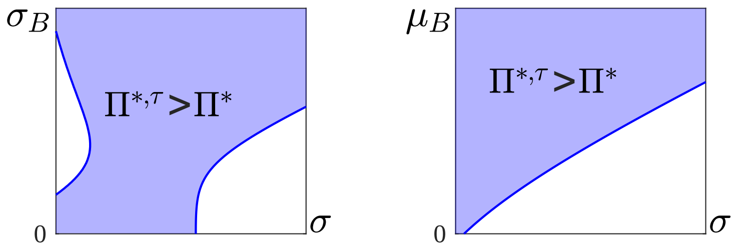
Proposition 2 *(Decomposable problem) (a) There exist thresholds σ_B' and $\underline{b}$ such that when $b > \underline{b}$, $ne^{*,\tau} > e^{*}$ if and only if $\sigma_B < \sigma_B'$. Also, $(ne^{*,\tau} - e^{*})$ is decreasing in σ_B and constant in μ_B.*

(b) There exist σ_B'' and μ_B' such that $m_N^\tau > m_N$ if $\sigma_B > \sigma_B''$ or $\mu_B > \mu_B'$, and $m_{N/n}^\tau \leq m_N$ if $\sigma_B \leq \sigma_B''$ and $\mu_B \leq \mu_B'$. Also, $(m_{N/n}^\tau - m_N)$ is increasing in σ_B and μ_B.

Proposition 2(a) shows that when increasing the effort is sufficiently difficult (i.e., $b > \underline{b}$), the team-effort contribution is larger than the solver-effort contribution (i.e., $ne^{*,\tau} > e^*$) if and only if the additional uncertainty (i.e., σ_B) generated through team members' interactions is sufficiently small. The intuition is as follows. When solvers collaborate as teams, although sharing the award and free-riding effect decrease their incentive to exert effort, as n team members exert effort to develop a team's solution, the team-effort contribution can be larger than the solver-effort contribution. Yet, this is possible under two conditions. First, increasing the effort should be sufficiently difficult such that the benefit of n team members' contribution can outweigh each team member's smaller incentive to exert effort. Second, the additional uncertainty generated through team members' interactions should be sufficiently small because this effect reduces each team member's incentive to exert effort. Proposition 2(a) further shows that the difference of effort contributions (i.e., $ne^{*,\tau} - e^*$) decreases with σ_B, yet it does not change with μ_B. The intuition is same as the intuition of Proposition 1(a).

Proposition 2(b) shows that the team-shock contribution m_N^τ is larger than the solver-shock contribution m_N when the additional uncertainty (i.e., σ_B) or the performance enhancement (i.e., μ_B) generated through team members' interactions is sufficiently large, and the difference of shock contributions (i.e., $m_{N/n}^\tau - m_N$) increases with both σ_B and μ_B. The intuition is similar to the intuition of Proposition 1(b), but in this case, when solvers collaborate as teams of size n, the number of solutions generated decreases from N to N/n. Thus, the benefit of team members' interactions should be sufficiently large to outweigh the negative effect of the decrease in the number of solutions, and hence the shock contribution increases when solvers collaborate as teams.

In summary, Proposition 2 reveals the following tradeoff for a decomposable problem: Although team members' interactions improve the shock contribution, they reduce the effort contribution. We now use (3.23) to characterize when $\Pi^{*,\tau} >$



(a) Impact of σ and σ_B when $\mu_B = 0$.

(b) Impact of σ and μ_B when $\sigma_B \to 0$.

Figure 3.2: For a decomposable problem, when is the organizer's profit $\Pi^{*,\tau}$ under team submissions larger than the organizer profit Π^* under individual submissions? Setting: $\xi_i \sim \text{Normal}(0,\sigma^2)$, $\xi_t \sim \text{Normal}(0,\sigma^2)$, $\xi_t^B \sim \text{Normal}(\mu_B,\sigma_B^2)$, $A=1$, $b=2$, $c=1$, $n=2$, and $N=10$.

Π^* in the following theorem.

Theorem 2 *(Decomposable problem) $\underline{b}$ is as defined in Proposition 2.*

(a) Suppose $\mu_B = 0$. Then, for any σ, there exists $\overline{\sigma_B}$ such that $\Pi^{*,\tau} > \Pi^*$ if $\sigma_B > \overline{\sigma_B}$. Furthermore, there exist thresholds $\underline{\sigma_B}$, $\underline{\sigma}$, and $\overline{\sigma}$ such that when $\sigma_B < \underline{\sigma_B}$, $\sigma < \underline{\sigma}$, and $b > \underline{b}$, $\Pi^{*,\tau} > \Pi^*$; and when $\sigma_B < \underline{\sigma_B}$ and $\sigma > \overline{\sigma}$, $\Pi^{*,\tau} < \Pi^*$.

(b) Suppose $\sigma_B \to 0$. Then, there exist thresholds $\overline{\mu_B}$, $\underline{\underline{\sigma}}$, and $\overline{\overline{\sigma}}$ such that $\Pi^{*,\tau} > \Pi^*$ if $\mu_B > \overline{\mu_B}$ or $\sigma < \underline{\underline{\sigma}}$ and $b > \underline{b}$; and $\Pi^{*,\tau} < \Pi^*$ if $\mu_B < \overline{\mu_B}$ and $\sigma > \overline{\overline{\sigma}}$.

Although one might expect that an organizer who seeks solutions to a decomposable problem always benefits from team submissions because team members can share the tasks, Theorem 2 shows that this expectation holds *only* under certain conditions. Specifically, Theorem 2(a) first shows that when there is no performance enhancement generated through team members' interactions (i.e., $\mu_B = 0$), the organizer can benefit from team submissions if the uncertainty generated through team members' interactions is sufficiently large (i.e., $\sigma_B > \overline{\sigma_B}$). The intuition is as follows. As Proposition 2 shows, when solvers collaborate as teams and σ_B is large, although the effort contribution decreases (i.e., $ne^{*,\tau} < e^*$), the shock contribution increases (i.e., $m^{\tau}_{N/n} > m_N$). Proposition 2 also shows that as σ_B increases, this positive effect on shock contribution increases, and hence outweighs the decrease in the effort contribution. Therefore, as Figure 3.2(a) illustrates, the organizer can benefit from team submissions through the increase in the shock contribution when the additional uncertainty generated through team members' interactions is sufficiently large.

Theorem 2(a) further shows that when the overall uncertainty is sufficiently small (i.e., $\sigma_B < \underline{\sigma_B}$ and $\sigma < \underline{\sigma}$) and increasing the effort is sufficiently difficult (i.e., $b > \underline{b}$), the organizer can benefit from team submissions. The intuition is as follows. As Proposition 2 shows, when solvers collaborate as teams under small σ_B, although the shock contribution decreases (i.e., $m^{\tau}_{N/n} < m_N$), the effort contribution increases (i.e., $ne^{*,\tau} > e^*$) under large b. Also, as the solution novelty measured by σ decreases, the increase in the effort contribution increases because each solver's incentive to exert effort increases, yet the decrease in the shock contribution decreases (e.g., Mihm and Schlapp 2019, Hu and Wang 2020). Therefore, when $\sigma_B < \underline{\sigma_B}$, $\sigma < \underline{\sigma}$, and $b > \underline{b}$, the increase in the effort contribution dominates the decrease in the shock contribution, and hence as Figure 3.2(a) illustrates, the organizer can benefit from team submissions through the increase in the effort contribution. On the other hand, when $\sigma_B < \underline{\sigma_B}$ and $\sigma > \underline{\sigma}$, the decrease in the shock contribution outweighs the (possible) increase in the effort contribution, and hence $\Pi^{*,\tau} < \Pi^*$.

Theorem 2(b) shows that when there is no additional uncertainty generated through team members' interactions (i.e., $\sigma_B \to 0$), the organizer can benefit from team submissions if the performance enhancement generated through team members' interactions is sufficiently large (i.e., $\mu_B > \overline{\mu_B}$), or the solution novelty is sufficiently small (i.e., $\sigma < \underline{\underline{\sigma}}$) and increasing the effort is sufficiently difficult (i.e., $b > \underline{b}$). The intuition is as follows. As Proposition 2 shows, the team-shock contribution increases with μ_B. Thus, when μ_B is above a threshold, the increase in the shock contribution dominates the (possible) decrease in the effort contribution, and hence the organizer can benefit from team submissions through the increase in the shock contribution. Furthermore, as Proposition 2 shows, the difference of effort contributions is positive when b is sufficiently large; and this difference increases as the solution novelty measured by σ decreases, and hence dominates the decrease in the shock contributions. Thus, when the solution novelty measured by σ is below a threshold and b is above a threshold, the organizer can benefit from team submissions through the increase in the effort contribution. Therefore, as Figure 3.1(b) illustrates, the organizer can benefit from team submissions when $\mu_B > \overline{\mu_B}$, or $\sigma < \underline{\underline{\sigma}}$ and $b > \underline{b}$. On the other hand, when $\mu_B < \overline{\mu_B}$ and $\sigma > \overline{\overline{\sigma}}$, the organizer's profit decreases when solvers collaborate as teams. Overall, as we summarize in Table 3.3 (bottom row), Theorem 2 shows that for any σ, the organizer can benefit from team submissions when $\sigma_B > \overline{\sigma_B}$ or $\mu_B > \overline{\mu_B}$. Furthermore, for a small σ, the organizer can also benefit from team submissions when σ_B is sufficiently small and b is sufficiently large.

Theorem 2 has important managerial implications. Specifically, consistent with the innovation literature (e.g., Sommer et al. 2020, Chan et al. 2021), we show that the organizer can benefit from team submissions when seeking solutions to a decomposable problem such as software challenges at Topcoder, but only under certain conditions. Specifically, when the organizer elicits high-novelty solutions (e.g., data science challenges at Topcoder) or low-novelty solutions (e.g., development challenges at Topcoder), the organizer can benefit from team submissions by encouraging (forming, if possible) more diverse teams. Moreover, when the organizer elicits low-novelty solutions to a decomposable problem and increasing the effort is difficult for solvers, the organizer can also benefit from team submissions by encouraging (forming, if possible) less diverse teams.

Although these results are in line with the literature, we find a contrasting result to the literature that team collaboration can also be harmful for the organizer who seeks solutions to a decomposable problem. For example, when eliciting high-novelty solutions from less diverse teams or eliciting low-novelty solutions from solvers who can easily increase their efforts, the organizer may not benefit from team submissions. Given these findings, it is understandable why Topcoder discourages team submissions, yet our findings above suggests that Topcoder may also find an opportunity to benefit from team submissions in some of its challenges.

Theorem 2 replicates the implications of Theorem 1, and also has another important implication for the contest theory. Specifically, since the organizer maximizes the best output in an innovation contest, even when $e^{*,\tau} < e^*$, the team-effort contribution $ne^{*,\tau}$ can be larger than the solver-effort contribution e^*. Thus, the organizer can benefit from team submissions through the increase in the effort contribution despite team members' smaller incentive to exert effort. This result is in contrast to other contest settings (e.g., sales contests), where team collaboration is always harmful when it leads to lower incentives to exert effort.

Analysis of Solver's Utility. We now examine when solvers benefit from collaboration while developing solutions to a decomposable problem. To do that, we compare an individual solver's utility U^* given in (3.5) and a team member's utility $U^{*,\tau}$ given in (3.19) as stated in Lemmas 1 and 3, respectively. This comparison yields:

$$U^{*,\tau} - U^* = c\left(\frac{AL_N}{cb}\right)^{\frac{b}{b-1}} - c\left(\frac{AL^{\tau}_{N/n}}{ncb}\right)^{\frac{b}{b-1}}. \tag{3.24}$$

As seen from (3.24), the difference between $U^{*,\tau}$ and U^* boils down to the difference between the solver's cost of effort and the team member's cost of effort, and by analyzing this difference, we obtain the following proposition.

Proposition 3 *Team member's utility $U^{*,\tau}$ is always larger than individual solver's utility U^*.*

Proposition 3 shows that when the organizer seeks solutions to a decomposable problem, solvers benefit from collaborating as teams. The intuition is as follows. When solvers collaborate as teams, on one hand, the number of competing solutions decreases from N to N/n, which motivates team members to exert a larger effort, on the other hand, the solver's expected award decreases from A to A/n. As the latter negative effect outweighs the former positive one, the solver's equilibrium effort decreases with team collaboration, and hence $U^{*,\tau} > U^*$. Therefore, Proposition 3 suggests that the organizer can benefit from team submissions whenever the organizer encourages it.

To conclude, Corollary 1 and Proposition 3 reveal that solvers can benefit from team collaboration. However, Theorems 1 and 2 suggest that whether the organizer should encourage team submissions would depend on: (1) whether the problem is nondecomposable or decomposable, (2) the novelty of solutions sought (i.e., σ), and (3) team members' interactions (i.e., σ_B and μ_B).

3.5 Additional Analysis

This section is organized as follows. In §3.5.1, we extend our main results to the case when the organizer sets optimal awards under individual and team submissions. In §3.5.2, we discuss the impact of the team size on the outcomes of team collaboration. In §3.5.3, we extend our results in §3.4.2 to the case when team collaboration may result in synergistic gains or coordination losses.

3.5.1 Optimal Awards under Individual Submissions and Team Submissions

In this section, we consider the case when the organizer sets the optimal award under both individual submissions and team submissions. In preparation, observe from (3.6), (3.13), and (3.20) that the organizer's profit under different settings can be expressed in the following generic form:

$$\Pi(F,M) = A^{\frac{1}{b-1}}F + M - A \tag{3.25}$$

First, observe from (3.6) that for the case of individual submissions, $F = \left(\frac{L_N}{cb}\right)^{\frac{1}{b-1}}$ and $M = m_N$. Second, observe from (3.13) that for the case of team submissions for a nondecomposable problem, $F = \left(\frac{L_N^\tau}{ncb}\right)^{\frac{1}{b-1}}$ and $M = m_N^\tau$. Finally, observe from (3.20) that for the case of team submissions for a decomposable problem, $F = n\left(\frac{L_{N/n}^\tau}{ncb}\right)^{\frac{1}{b-1}}$ and $M = m_{N/n}^\tau$. In all cases, F and M are independent of A. Therefore, the organizer maximizes $\Pi(F,M)$ over the award A to decide on the optimal award A^* under both individual submissions and team submissions. (We assume $b > 2$ such that $\Pi(F,M)$ is concave in A, and hence A^* is finite.) We evaluate the first derivative of $\Pi(F,M)$ in (3.25) with respect to A, and obtain A^* for any generic form of F and M as follows:

$$A^* = \left(\frac{F}{b-1}\right)^{\frac{b-1}{b-2}}. \tag{3.26}$$

Under the optimal award A^* in (3.26), the organizer's profit is

$$\Pi^*(F,M) = (A^*)^{\frac{1}{b-1}} F + M - A^* = \left(\frac{F}{b-1}\right)^{\frac{1}{b-2}} F + M - \left(\frac{F}{b-1}\right)^{\frac{b-1}{b-2}}$$
$$= (b-2)\left(\frac{F}{b-1}\right)^{\frac{b-1}{b-2}} + M. \tag{3.27}$$

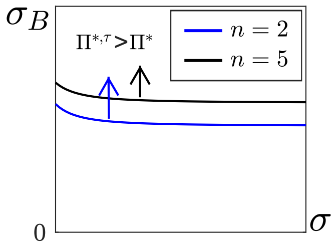
Using $\Pi^*(F,M)$ in (3.27), we make the following observations about the impact of team collaboration under A^*. First, when the organizer sets the optimal awards under individual submissions and team submissions, the organizer's benefit from the shock contribution M in (3.25) does not change. More interestingly, the impact of team collaboration on both the effort contribution under any A (i.e., $A^{\frac{1}{b-1}}F$ in (3.25)) and the net effort contribution under A^* (i.e., $(b-2)\left(\frac{F}{b-1}\right)^{\frac{b-1}{b-2}}$ in (3.27)) depends only on the impact of team collaboration on F. Therefore, Theorems 1 and 2 extend to the case where the organizer sets A^* under both individual submissions and team submissions. Also, notice from (3.26) that it is optimal for the organizer to set a larger award under the case where the effort contribution under any A is larger. This means that the organizer sets a larger A^* for solvers whose incentive to exert effort is already larger. Finally, even when the organizer sets the optimal award A^*, each solver compares her utility when making an individual submission and her utility when making a team submission under the same A^*. Thus, Corollary 1 and Proposition 3 for any A directly extend to the case when the organizer sets A^*.

3.5.2 Impact of Team Size

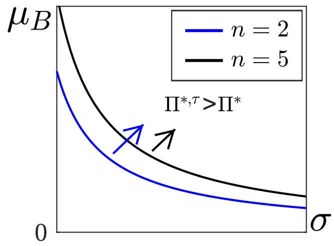
In this section, we analyze the impact of team size n on when the organizer and solvers benefit from team collaboration. Since the equilibrium outcomes under the case of individual submissions do not depend on team size n, we analyze the impact of team size n on the equilibrium outcomes under the case of team submissions.

Nondecomposable Problem. In the following proposition, we analyze the impact of team size n in the case of a nondecomposable problem.

Proposition 4 *(a) The team-effort contribution $e^{*,\tau}$ in (3.13) decreases with team size n. (b) The team-shock contribution m_N^τ in (3.13) does not change with n. (c)*



(a) Impact of σ and σ_B when $\mu_B = 0$.



(b) Impact of σ and μ_B when $\sigma_B \to 0$.

Figure 3.3: For a nondecomposable problem, when is the organizer's profit $\Pi^{*,\tau}$ under team submissions larger than the organizer profit Π^* under individual submissions? The setting is the same as Figure 3.1. Arrows depict regions where $\Pi^{*,\tau} > \Pi^*$.

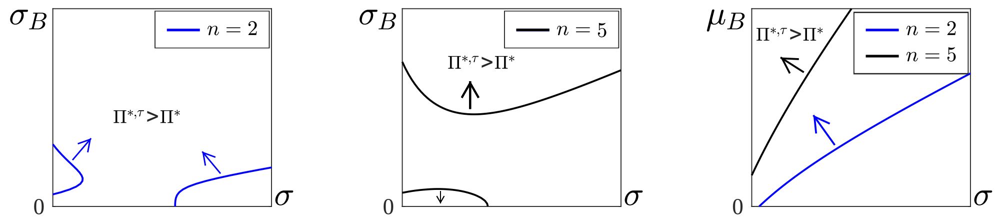
The team member's utility $U^{*,\tau}$ in (3.12) increases with n.

Proposition 4(a) shows that the team-effort contribution $e^{*,\tau}$ decreases with team size n because the award A for the winning team is divided among more team members. Proposition 4(b) shows that the team-shock contribution m_N^τ does not change with n. The reason is that when the organizer has a nondecomposable problem, team members work in parallel to develop solutions, and hence the total number of solutions generated in the contest depends only on the number of solvers N. Thus, Propositions 4(a) and 4(b) along with Theorem 1 show that as n increases, the organizer benefits less from team submissions, see Figures 3.3(a)-(b). Yet, Proposition 4(c) shows that as n increases, solvers benefit more from team collaboration as their incentive to exert effort decreases. Thus, Proposition 4 suggests that it can be better for the organizer to limit the team size when encouraging team submissions, if possible.

Decomposable Problem. In the following proposition, we analyze the impact of team size n on the team-effort contribution $ne^{*,\tau}$ and the team-shock contribution $m_{N/n}^\tau$ in (3.20), and the team member's utility $U^{*,\tau}$ in (3.19).

Proposition 5 *(a) The team-effort contribution $ne^{*,\tau}$ in (3.20) increases with team size n when b is sufficiently large. (b) The team-shock contribution $m_{N/n}^\tau$ in (3.20) decreases with n. (c) The team member's utility $U^{*,\tau}$ in (3.19) increases with n.*

Proposition 5(a) shows that $ne^{*,\tau}$ increases with team size n when increasing the



(a) Impact of σ and σ_B when $\mu_B = 0$. (b) Impact of σ and σ_B when $\mu_B = 0$. (c) Impact of σ and μ_B when $\sigma_B \to 0$.

Figure 3.4: For a decomposable problem, when is the organizer's profit $\Pi^{*,\tau}$ under team submissions larger than the organizer profit Π^* under individual submissions? The setting is the same as Figure 3.2. Arrows depict regions where $\Pi^{*,\tau} > \Pi^*$.

effort is sufficiently difficult (i.e., b is sufficiently large). The intuition is follows. As n increases, although more team members share the award A, which decreases their incentive to exert effort, more team members contribute to the team's solution. Because the former negative effect decreases with b, when b is large, the latter positive effect dominates the former negative effect, and hence $ne^{*,\tau}$ increases with n. Proposition 5(b) shows that $m^{\tau}_{N/n}$ decreases with n because the number of team submissions N/n decreases with n. As Figures 3.4(a)-(c) illustrate, Proposition 5 along with Theorem 2 show that as n increases, unless b is large, the organizer benefits less from team submissions. Yet, Proposition 5(c) shows that as n increases, solvers benefit more from making team submissions because their incentive to exert effort decreases with n. Therefore, Proposition 5 suggests that it can be better for the organizer to limit the team size when encouraging team submissions, if possible.

3.5.3 Synergistic Gains and Coordination Losses Arising from Team Collaboration

In this section, we extend our results in §3.4.2 to the case when team collaboration may result in synergistic gains or coordination losses. First, when solvers make individual submissions, the solver's equilibrium effort e^* and utility U^*, and the organizer's profit Π^* are as stated in (3.4), (3.5), and (3.6), respectively. We next discuss the case of team submissions. In this case, team member i in each team t exerts effort e^{τ}_{ti} to contribute to team t's output y^{τ}_t as in §3.3.2. To capture both synergistic gains and coordination losses within a team, we use the constant elasticity of substitution

(CES) function (cf. Arrow et al. 1961, Roels 2014). For analytical tractability, we assume a single output shock ξ_t (follows Normal$(0,\sigma^2)$ as in §3.3.2) related to the problem and a single interaction shock ξ_t^B (follows Normal(μ_B,σ_B^2) as in §3.3.2) generated through team members' interactions. Recall from §3.3.2 that the team's overall output shock $\xi_t^\tau = \xi_t^B + \xi_t$, where ξ_t^τ follows Normal$(\mu_B,\sigma^2+\sigma_B^2)$ and $g(s)$ and $G(s)$ are ξ_t^τ's density function and distribution function, respectively. Then, team t's output can be modeled as follows:

$$y_t^\tau = \left(\sum_{i\in\mathcal{N}_t^\tau}(e_{ti}^\tau)^\gamma\right)^{\frac{1}{\gamma}} + \xi_t + \xi_t^B = \left(\sum_{i\in\mathcal{N}_t^\tau}(e_{ti}^\tau)^\gamma\right)^{\frac{1}{\gamma}} + \xi_t^\tau, \tag{3.28}$$

where $\gamma = 1$ represents the case where team members' efforts are perfectly substitutable as in (3.15) (Amaldoss and Rapoport 2005). As γ increases from 1, it captures coordination losses (i.e., loss in efficiency; Roels 2014) within a team because $\left(\sum_{i\in\mathcal{N}_t^\tau}(e_{ti}^\tau)^\gamma\right)^{1/\gamma} < \sum_{i\in\mathcal{N}_t^\tau} e_{ti}^\tau$. However, as γ (>0) decreases from 1, it captures synergistic gains within a team because $\left(\sum_{i\in\mathcal{N}_t^\tau}(e_{ti}^\tau)^\gamma\right)^{1/\gamma} > \sum_{i\in\mathcal{N}_t^\tau} e_{ti}^\tau$.

By using (3.28), we derive team t's probability of winning $P_{N/n}^\tau[e_{ti}^\tau, e^{*,\tau}]$ for any γ as follows:

$$P_{N/n}^\tau[e_{ti}^\tau, e^{*,\tau}] = \int_{-\infty}^{\infty} G\left(s + ((e_{ti}^\tau)^\gamma + (n-1)(e^{*,\tau})^\gamma)^{\frac{1}{\gamma}} - n^{\frac{1}{\gamma}} e^{*,\tau}\right)^{\frac{N}{n}-1} g(s)ds. \tag{3.29}$$

The first derivative of (3.29) with respect to e_{ti}^τ evaluated at $e_{ti}^\tau = e^{*,\tau}$ yields:

$$\left.\frac{\partial P_{N/n}^\tau[e_{ti}^\tau, e^{*,\tau}]}{\partial e_{ti}^\tau}\right|_{e_{ti}^\tau = e^{*,\tau}} = n^{\frac{1}{\gamma}-1}\left(\frac{N}{n}-1\right)\int_{-\infty}^{\infty} G(s)^{\frac{N}{n}-2} g(s)^2 ds = n^{\frac{1}{\gamma}-1} L_{N/n}^\tau, \tag{3.30}$$

where $L_{N/n}^\tau$ is as defined in (3.18). We characterize the equilibrium in the following lemma.

Lemma 4 *Each team member's equilibrium effort $e^{*,\tau}$ and utility $U^{*,\tau}$ satisfy:*

$$e^{*,\tau} = \left(\frac{An^{\frac{1}{\gamma}-2} L_{N/n}^\tau}{cb}\right)^{\frac{1}{b-1}} \text{ and} \tag{3.31}$$

$$U^{*,\tau} = \frac{A}{N} - c\left(\frac{An^{\frac{1}{\gamma}-2} L_{N/n}^\tau}{cb}\right)^{\frac{b}{b-1}}. \tag{3.32}$$

Lemma 4 extends Lemma 3 to any value of γ, and shows that as γ increases, $e^{*,\tau}$ decreases and $U^{*,\tau}$ increases. In this case, the organizer's expected profit in equilibrium is

$$\Pi^{*,\tau} = \; n^{\frac{1}{\gamma}} e^{*,\tau} + E\left[\max_{t\in\mathscr{T}}\{\xi_t^\tau\}\right] - A = n^{\frac{1}{\gamma}}\left(\frac{An^{\frac{1}{\gamma}-2}L^\tau_{N/n}}{cb}\right)^{\frac{1}{b-1}} + m^\tau_{N/n} - A. \quad (3.33)$$

Notice that the team-effort contribution $n^{\frac{1}{\gamma}} e^{*,\tau}$ decreases with γ. By comparing Π^* given in (3.6) under individual submissions and $\Pi^{*,\tau}$ given in (3.33) under team submissions, we extend Theorem 2 in the following proposition.

Proposition 6 *(a) Suppose $\mu_B = 0$. Then, for any σ, there exists $\overline{\sigma_B}$ such that $\Pi^{*,\tau} > \Pi^*$ if $\sigma_B > \overline{\sigma_B}$. Furthermore, there exist thresholds $\underline{\sigma_B}$, $\underline{\sigma}$, and $\overline{\sigma}$ such that when $\sigma_B < \underline{\sigma_B}$, $\sigma < \underline{\sigma}$, and $b > \underline{b}$, $\Pi^{*,\tau} > \Pi^*$; and when $\sigma_B < \underline{\sigma_B}$ and $\sigma > \overline{\sigma}$, $\Pi^{*,\tau} < \Pi^*$.*
(b) Suppose $\sigma_B \to 0$. Then, there exist thresholds $\overline{\mu_B}$, $\underline{\underline{\sigma}}$, and $\overline{\overline{\sigma}}$ such that $\Pi^{*,\tau} > \Pi^*$ if $\mu_B > \overline{\mu_B}$ or $\sigma < \underline{\underline{\sigma}}$ and $b > \underline{b}$; and $\Pi^{*,\tau} < \Pi^*$ if $\mu_B < \overline{\mu_B}$ and $\sigma > \overline{\overline{\sigma}}$.
(c) As γ decreases, $\Pi^{*,\tau} - \Pi^*$ increases.

Propositions 6(a) and 6(b) extend Theorems 2(a) and 2(b) to the case when synergistic gains or coordination losses can arise from team collaboration. Furthermore, Proposition 6(c) shows that as γ decreases, team members increase their efforts due to synergistic gains, so the organizer benefits more from team submissions. This result is also consistent with the literature because when $\gamma \geq 1$, $1/\gamma$ can be interpreted as the "degree of decomposability" of the problem, and Chan et al. (2021) suggest that decomposability of an invention (i.e., decrease in γ) favors teams over individuals.

We next analyze when solvers benefit from team submissions. The following proposition compares an individual solver's utility U^* given in (3.5) and a team member's utility $U^{*,\tau}$ given in (3.32).

Proposition 7 *There exists $\overline{\gamma}$ such that $U^{*,\tau} > U^*$ if and only if $\gamma > \overline{\gamma}$.*

Proposition 7 extends Proposition 3, and shows that the team member's utility is smaller than the individual solver's utility when γ is below a threshold. This is because as γ decreases, synergistic gains create incentive for team members to exert more effort, leading to an increase in their cost of effort. This result may explain why solvers make individual submissions in some InnoCentive challenges even when team submissions are encouraged.

3.6 Discussion and Conclusion

In recent years, crowdsourcing platforms (e.g., InnoCentive, Topcoder, and 99designs) have enjoyed significant growth because innovation contests have emerged as a viable tool to outsource innovation. In an innovation contest, solvers can make individual submissions, or if the organizer encourages team submissions (as in about 70% of InnoCentive challenges), they can make team submissions. In both cases, the organizer's profit depends on the quality of the best submission, and the innovation literature (e.g., Girotra et al. 2010) show that the quality of the best solution depends on the number of solutions generated, the average quality of solutions, and the variance in the quality of solutions. Yet, it is unclear how team collaboration affects all these aspects in an innovation contest because the prior studies in the innovation literature on collaboration (e.g., Kavadias and Sommer 2009, Sommer et al. 2020, Chan et al. 2021) neither consider the competition among solvers nor do they consider the presence of an organizer. Motivated from this gap between the theory and practice and different policies adopted by crowdsourcing platforms, we identify conditions under which the organizer and solvers can benefit from team collaboration.

Inspired by the literature on coproduction (e.g., Amaldoss et al. 2000, Roels 2014) and the innovation literature (e.g., Taylor and Greve 2006, Singh and Fleming 2010), we incorporate a team's collective output into the standard modeling framework in the innovation-contest literature (e.g., Mihm and Schlapp 2019, Hu and Wang 2020), and generate the following insights. First, based on established results in the innovation literature, one might expect that the organizer should discourage team submissions when seeking solutions to a nondecomposable problem (e.g., Chan et al. 2021). Indeed, we show that this is the optimal strategy when the organizer seeks low-novelty solutions to a nondecomposable problem (e.g., logo design challenges at 99designs) and team diversity is sufficiently low, because solvers' incentive to exert effort decreases when they collaborate as teams. However, we establish a new result that cannot be derived from the ones in the prior literature: When the organizer seeks high-novelty solutions to a nondecomposable problem (e.g., design challenges at InnoCentive), the organizer can benefit from team submissions due to the additional uncertainty and the performance enhancement generated through team interactions (e.g., Taylor and Greve 2006, Singh and Fleming 2010). This result may explain why team submissions are encouraged in most InnoCentive challenges, despite possible coordination losses in solvers' efforts. Although practitioners may allow team collaboration just to provide an option for solvers, our results show that this strategy can lead to better contest outcome only for specific types of problems.

Second, when the organizer seeks solutions to a decomposable problem, one might expect that the organizer always benefits from team submissions (e.g., Sommer et al. 2020, Chan et al. 2021) because team members can share tasks. However, we show that this expectation holds only under certain conditions. Specifically, when the organizer seeks high-novelty solutions (e.g., in Streamflow Forecast Challenge at Topcoder) or low-novelty solutions (e.g., in Database Setup Challenge at Topcoder), the organizer can benefit from team submissions when the benefit from team members' interactions is sufficiently large, which can be achieved by more diverse teams (e.g., Taylor and Greve 2006, Singh and Fleming 2010). We further show that when seeking low-novelty solutions to a decomposable problem, the organizer can benefit from team submissions also through the increase in the effort contribution if increasing effort is difficult for solvers. As the diversity of ideas due to team members" interactions reduces team members" incentive to exert effort, in this case, it can better for the organizer to encourage (form, if possible) teams with limited diversity (e.g., by prompting solvers with similar backgrounds to form teams). Given these findings, it is understandable why Topcoder discourages team submissions, yet our findings above suggests that Topcoder may also find an opportunity to benefit from team submissions in some of its challenges.

Our results have also two important implications for the theory on teams in contests (e.g., Chen and Lim 2013). First, since the organizer maximizes the best output in an innovation contest, even when each team member's equilibrium effort is smaller than each individual solver's equilibrium effort, the team-effort contribution can be larger than the solver-effort contribution in the case of a decomposable problem. Thus, the organizer can benefit from team submissions due to the increase in the effort contribution although team members' incentive to exert effort is smaller. Second, different from traditional contests (e.g., sales contests) where the average or total output of solvers is maximized, in innovation contests, the organizer can benefit from team submissions due to the additional uncertainty and the performance enhancement generated through team members' interactions. These implications are consistent with the empirical finding of Girotra et al. (2010) that analyzing the average output may underestimate the benefits of collaboration.

Finally, we show that the team member's utility is larger than the individual solver's utility unless team collaboration results in substantial synergistic gains. This is because when solvers collaborate as teams, all solvers (in all teams) end up exerting less effort, leading to a similar expected award but a significantly lower cost of effort for each solver. Yet, if synergistic gains create sufficient incentive for team members to exert more effort than they would individually, solvers' utilities decrease when they collaborate as teams. This result may explain why in some InnoCentive challenges, solvers make individual submissions although they are encouraged to collaborate.

Our study is an initial attempt to analyze the impact of team submissions in an innovation contest, which provides opportunities for future research. First, consistent with the innovation-contest literature (e.g., Mihm and Schlapp 2019), we assume that solvers are identical to ensure tractability, yet it can be interesting to empirically or experimentally examine the impact of team submissions when solvers are heterogenous. Second, we focus on the organizer's decision on whether to encourage or discourage team submissions, and abstract away from dynamics associated with solvers' team formation. However, it would be interesting to conduct an experimental study to understand how solvers form teams to generate solutions to nondecomposable and decomposable problems. Finally, consistent with the innovation-contest literature (e.g., Hu and Wang 2020), we assume that the organizer seeks solutions from a fixed number of solvers regardless of the decision to encourage team submissions. Yet, it can be interesting to empirically analyze whether/how the number of solvers changes when the organizer encourages team submissions.

Chapter 4

Product Development in Crowdfunding: Theoretical and Empirical Analysis

4.1 Introduction

The internet has enabled entrepreneurs to use crowdfunding to raise funds from a large number of people for projects ranging from social entrepreneurship to for-profit enterprises.[1] In addition to crowdfunding being an important financial instrument (e.g., Hu et al. 2015, Belavina et al. 2020, Chakraborty and Swinney 2021), recent research and practice suggest that it can be used by entrepreneurs (hereafter *creators*) as a mechanism to involve customers in product development and thus to improve their products during their crowdfunding campaigns (e.g., Mollick 2016, Cornelius and Gokpinar 2020). Customer involvement in product development can lead to significant cost savings (e.g., Thomke and Bell 2001, Loch and Kavadias 2008), so a creator may consider launching a crowdfunding campaign for a basic version of a product (as opposed to a more enhanced version with more features; Althuizen and Chen 2021) to leave room for improvements to it. However, if a product appears too basic, customers may be discouraged from contributing to the campaign, making it less likely that the campaign will be successful (i.e., reach the

[1] The first recorded crowdfunding on the internet occurred in 1997, when the British band Marillion raised $60,000 from fans to fund its North American tour (Masters 2013).

funding goal). Thus, we study how a creator should choose the product's level of enhancement at campaign launch (hereafter *initial enhancement level*), taking into account the impact this may have on product improvement and campaign success.

To understand how customers can contribute to product improvement during a crowdfunding campaign, consider the following example from Kickstarter—a global crowdfunding platform that has raised $5 billion for entrepreneurs over the last decade (Kickstarter 2021g). In October 2015, the "onomo" team launched a campaign for an innovative bike navigation device, HAIZE (see Figure 4.1(a); Kickstarter 2021a). During the campaign, customers suggested that HAIZE could include a wristband so that they could also use the product when not on a bike (see Figure 4.1(b)). In response to suggestions from customers, the creator added a wristband to the product and revised the campaign description accordingly (see Figure 4.1(c)). In our interview, the creator of HAIZE explained this process as follows: "It's definitely very efficient for that kind of [market] research... the idea, for example, of adding a wristband to the device, it was always like floating... we're not sure if we should do this [or not]... But then, we began to receive very relevant testimonies of people who were having amazing ideas [about a wristband]..."

As this example aptly shows, a creator can make a strategic choice to launch a crowdfunding campaign for a basic product and improve it during the campaign. Since the initial enhancement level of a product can affect whether a creator improves it during the campaign, we start by investigating the following research question: (Q1) How does the initial enhancement level affect the likelihood that the product will be improved during the campaign?

Besides affecting product improvement, the initial enhancement level also influences whether customers pledge money to the campaign, and ultimately whether the campaign is successful. Indeed, experts on Kickstarter campaigns assert that a creator should "have as much done as possible" and "launch [a campaign] as close to production as possible" for a successful campaign (Kickstarter 2021e). The implicit assumption here is that a campaign for a more enhanced product is more likely to be successful, so we investigate this by asking: (Q2) How does the product's initial

(a) Initial product. (b) Customer comment. (c) Product improvement.

Figure 4.1: Example of initial product, customer comment, and product improvement.

enhancement level affect the success of the campaign?

Building on the descriptive analyses presented above, and with the aim of generating actionable insights, we study a case in which a creator chooses the optimal initial enhancement level to maximize the profit from the campaign. This allows us to test whether Kickstarter experts are correct in their assertions by investigating the following research question: (Q3) Should a creator enhance the product as much as possible before launching a campaign?

To answer these questions and inspired by the crowdfunding practice and literature (e.g., Hu et al. 2015, Belavina et al. 2020, Chakraborty and Swinney 2021), we build a parsimonious game-theoretical model of a “reward-based, all-or-nothing” crowdfunding campaign that takes into account the creator’s product improvement decision.[2] In such a campaign, a creator solicits funds from customers to finance the launch of a product. To this end, the creator specifies the initial enhancement level of the product (the more enhanced the product is, the greater the number of features it offers), and announces a funding goal and a pledge price. Having considered these, customers then decide whether to pledge money in return for the product, and after pledging, they can make comments to induce the creator to improve the product further. (Customers cannot make comments before pledging; see Kickstarter 2021h.) If the creator sees comments, the creator decides whether to improve the product, and if improvements are made, other potential customers will be seeing the improved product before making pledging decisions. By the end of

[2]There are also other forms of crowdfunding with respect to the type of reward and type of funding. We refer the reader to Chen et al. (2020) for a review of other forms of crowdfunding.

the campaign, if the total amount pledged reaches the funding goal, the campaign is successful; the creator receives the funds raised, and products are then produced and delivered to customers. If the total amount pledged fails to reach the funding goal, the campaign fails; the creator receives and delivers nothing, and customers are fully refunded.

We first analyze how the initial enhancement level of a product affects the likelihood of a product being improved during the campaign. Because making an improvement is costly, one might expect that the more enhanced a product is, the less likely the creator will be to make any further improvement to it during the campaign.[3] Yet, our analysis reveals an opposing effect at play. Specifically, when a product is more enhanced, customers are more likely to pledge and to comment, and hence the creator is more likely to receive a comment that can be used to improve the product. This increases the likelihood of a product being improved during the campaign. However, above a certain level of initial enhancement, the likelihood of product improvements decreases because of the additional costs the creator would incur if implementing them.

To address our second research question, we analyze how the initial enhancement level affects campaign success. Our theoretical model predicts that the likelihood of campaign success increases with the initial enhancement level, albeit at a decreasing rate when the initial enhancement level is high. The reason is that when the product is highly enhanced, customers are already very likely to pledge, so their incentive to pledge is less responsive to a higher initial enhancement level.

We test our theoretical results empirically using a unique large-scale data set from Kickstarter. Our data set contains detailed information about campaign characteristics as well as product descriptions from the beginning and end of the campaigns. To create a measure of enhancement levels of products (i.e., number of features that products offer; Ulrich and Eppinger 2016, Althuizen and Chen 2021), we take advantage of these detailed product descriptions, and use an unsupervised

[3]For instance, while responding to a customer comment asking for an extra USB port, one creator on Kickstarter explained why it was not feasible to improve the product in this way by saying: “Yes, I would love to have USB3, or USB type C... What you might not know fully is that it is a serious additional cost...” (Kickstarter 2021f).

natural language processing technique, latent Dirichlet allocation (LDA; Blei et al. 2003). LDA has been used by marketing and operations scholars for purposes such as extracting product features (Toubia et al. 2019) or generating a measure of a firm's innovation (Bellstam et al. 2020) from textual data. We address possible endogeneity concerns in our empirical analysis by exploiting a policy change on Kickstarter that reduced minimum campaign requirements and thereby provided an exogenous shock to initial enhancement levels.

Our empirical analysis supports the predicted relationship between the initial enhancement level and the likelihood of product improvement. More interestingly, although our theoretical model intuitively predicts that the likelihood of campaign success will increase with the initial enhancement level, we do not observe this empirically. Instead, the likelihood of campaign success first increases with the initial enhancement level but then, counterintuitively, decreases. This discrepancy between our normative theory and the empirical finding hints at the presence of possible behavioral effects. Specifically, when enhancing a product, a creator may end up with a too complex or too advanced product, which can be overwhelming for customers (e.g., Mick and Fournier 1998) or can lead to customer anxiety (e.g., Castaño et al. 2008, Goodman and Irmak 2013). Indeed, when we revise our theoretical model to incorporate such behavioral effects, our theoretical predictions become consistent with the empirical results.

Building on these results and to generate prescriptive insights, we investigate whether the creator should enhance the product as much as possible before the campaign. To this end, we analyze a model in which the creator chooses the optimal initial enhancement level to maximize profit. In contrast to what experts on Kickstarter campaigns suggest, we show that a creator should *not* always enhance a product as much as possible because there is a trade-off between cost savings and chances of success. Specifically, when customer involvement in product development can lead to substantial cost savings in product development activities, it can be better for the creator to launch the campaign for a basic version of a product and improve it during the campaign in response to customer feedback, even though this may decrease the chances of the campaign being successful.

4.2 Related Literature

As a phenomenon that has emerged quickly, crowdfunding has caught the attention of entrepreneurs, managers, and business scholars. Accordingly, there is a relatively new but growing literature on crowdfunding. We first discuss theoretical and empirical studies of crowdfunding and then summarize our contributions to this literature.

In the crowdfunding literature, Hu et al. (2015) have pioneered theoretical studies of reward-based, all-or-nothing crowdfunding by analyzing whether the creator should offer a single reward or multiple rewards. Follow-up studies (e.g., Du et al. 2017, Chakraborty and Swinney 2019, 2021, Burtch et al. 2020, Li et al. 2020) analyze the creator's other design decisions, including the funding goal, the pledge price, limited rewards, and the timing of referrals and contingent stimulus policies (e.g., limited time offer). There are also theoretical studies that analyze how crowdfunding platforms can be designed to prevent misconduct (Strausz 2017, Belavina et al. 2020).[4] Empirical studies focus mainly on factors that influence customers' pledging decisions and campaign success such as altruism, geographic proximity to creators, and creators' pre-campaign information sharing (e.g., Burtch et al. 2013, Mollick 2014, Agrawal et al. 2015, Lin and Viswanathan 2016, Kuppuswamy and Bayus 2017, Wei et al. 2020). Other empirical papers study broader aspects of crowdfunding such as the similarity between the evaluations of crowdfunding customers and those of experts (Mollick and Nanda 2016), the impact of crowdfunding on a creator's ability to reach venture capital investors (Sorenson et al. 2016), and differences between the pledge price and the post-campaign retail price (Blaseg et al. 2020). Recently, Cornelius and Gokpinar (2020) show that crowdfunding campaigns are more likely to be successful with greater customer involvement. For a detailed review of this literature, we refer the reader to Allon and Babich (2020)

[4]There are other theoretical studies that ask broader questions about crowdfunding such as when to use different forms of crowdfunding (e.g., Belleflamme et al. 2014, Bi et al. 2019) or how crowdfunding interacts with traditional financing sources (e.g., Roma et al. 2018, Babich et al. 2021). Recently, Chemla and Tinn (2020) analyze the value of crowdfunding as a tool to test the potential market. For a detailed review, we refer the reader to Chen et al. (2020).

and Chen et al. (2020).

While existing research has significantly improved our understanding of crowdfunding as a new financing (e.g., Hu et al. 2015, Belavina et al. 2020) and customer interaction mechanism (e.g., Mollick 2016, Cornelius and Gokpinar 2020), the product development decisions made by creators in crowdfunding have not received attention in the literature either theoretically or empirically. Our study fills this gap by providing a nuanced understanding of crowdfunding as a product development mechanism. Specifically, inspired by practice and theoretical models in the crowdfunding literature, we first construct a theoretical model that includes the creator's decision on product improvement during the campaign. This model helps us build a theory about the impact of the initial enhancement level of a product on the likelihood of product improvement and campaign success. We then test this theory empirically with a unique large-scale data set. Our theory helps us generate practically grounded insights into whether the creator should improve a product as much as possible before the campaign. To our knowledge, our study is the first in the crowdfunding literature to combine theoretical and empirical analyses. Thus, one of its key strengths is that our theoretical and empirical analyses inform each other. Specifically, our theory helps us generate testable hypotheses for our empirical study, and our empirical results hint at the presence of behavioral effects and help us improve our theoretical model.

Our work is also related to the NPD literature that studies when to launch a product by considering different trade-offs.[5] Specifically, assuming that a more enhanced product always increases the customer's utility, the majority of studies in this literature (e.g., Cohen et al. 1996, Özer and Uncu 2013, Gao et al. 2021) investigate how much to delay the launch to enhance the product by considering the risk of losing the first-mover advantage. Recently, Bhaskaran et al. (2020) con-

[5]Our study is also related to the broader NPD literature (e.g., Thomke and Bell 2001, Loch et al. 2001, Erat and Kavadias 2008, Sommer et al. 2009) that has mainly focused on operational decisions related to experimentation and testing (e.g., whether to test sequentially or in parallel) mostly to resolve technical uncertainty *before* product launch. This literature suggests that cost of making changes and redesign increases over time in a product development process. We build our theoretical model based on this result, and study an innovative setting where product development continues based on customer feedback *after* product launch (i.e., launch of a crowdfunding campaign).

sider a different trade-off: Launching a basic and immediately available version of a product brings earlier revenues (as opposed to delaying the launch to develop a more enhanced version), yet this may negatively affect the perception of future versions of the product. Different than the setting of these papers, we analyze a setting where launching a crowdfunding campaign for a basic version of the product neither brings any early revenue nor establishes any first-mover advantage. Therefore, we identify a unique trade-off for product enhancement before launch that is relevant in crowdfunding settings and contribute to this literature on several fronts. First, we empirically show that enhancing the product too much before a crowdfunding campaign may reduce the likelihood of campaign success, which implies that the customer's utility does not always increase with the initial enhancement level of the product. Thus, the common assumption in product launch literature does not seem to hold in crowdfunding settings. Second, we consider a setting, where customers receive the final version of the product even though they pledge before the creator improves the product during the campaign. This unique feature of crowdfunding eliminates the risk of losing future value unlike settings where customers stick with the product they purchased even though the product is later improved. Despite this advantage, there are additional challenges for the creator because the creator receives pledges only when the campaign is successful, and improvements hinge on customers' voluntary feedback. Therefore, launching a more enhanced product may solicit more feedback and increase the chances of campaign success in crowdfunding, which may not be concerns in settings analyzed in the NPD literature. In sum, our study expands the NPD literature on product launch decisions by focusing on crowdfunding, a novel product development and financing setting.

The remainder of the study is organized as follows. In §4.3, we discuss our theoretical model and its predictions; in §4.4, we discuss our empirical models and their results; in §4.5, we revise our theoretical model based on empirical results; in §4.6, we analyze the optimal initial enhancement level; and in §4.7, we discuss the results and limitations of our study.

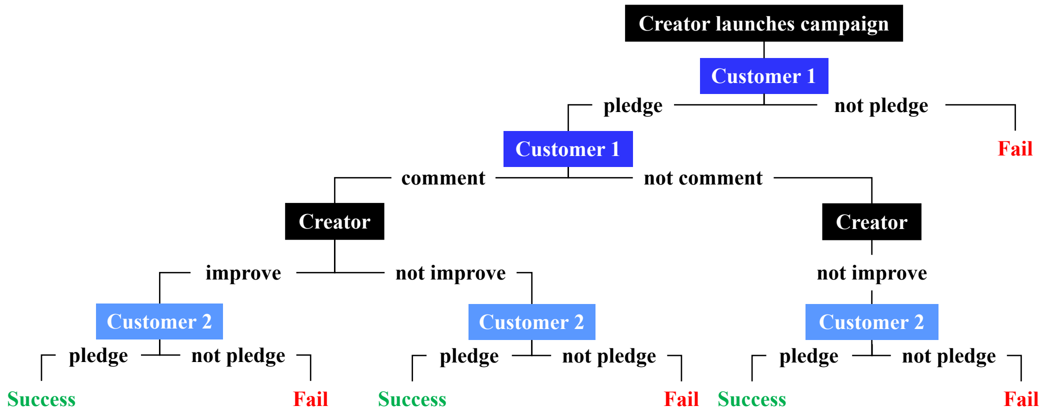


Figure 4.2: The sequence of decisions and events in a crowdfunding campaign where the product may potentially be improved during the campaign.

4.3 Theoretical Model and Analysis

We consider a reward-based crowdfunding campaign where a creator elicits funds from customers ("she") to finance the launch of a physical product,[6] and each customer pledges to receive the product as a reward. We focus on an all-or-nothing setting where the creator sets a funding goal, and if the total amount pledged exceeds the funding goal by the end of the campaign, the campaign is successful. In this case, the creator collects money pledged and delivers products to those who have pledged. When the total amount pledged does not meet the funding goal, the campaign fails, and hence the creator does not receive any funds or deliver any products and customers are refunded (e.g., Hu et al. 2015, Belavina et al. 2020).

Because crowdfunding is a nascent research area and our aim is to use our theoretical results to develop testable hypotheses, we develop a parsimonious model. Specifically, we build on the model of Hu et al. (2015) by incorporating the creator's product improvement decision and construct a four-stage game-theoretical model that involves one creator and two customers, as illustrated in Figure 4.2. Table 4.1 summarizes the main assumptions in our model. We will describe our model according to the sequence of events.

Stage 0: The creator launches a crowdfunding campaign. The creator specifies a funding goal G (> 0) and an initial enhancement level q_i (> 0), which represents

[6]Note that we focus on a campaign for a physical product such as camera equipment because a campaign for a non-physical product such as software may require a different model.

how enhanced the product is. For example, as we discuss in §4.1, HAIZE with wristband (i.e., a navigation device with more features) is a more enhanced version of the initially introduced navigation device. We assume that the creator incurs an investment cost of $C_i \cdot q_i$, where $C_i \geq 0$, to launch a campaign for a product whose initial enhancement level is q_i (e.g., Chakraborty and Swinney 2021). Although the creator can also incur some investment cost when improving the product during the campaign, we normalize this cost to zero for ease of illustration. (Note that our results are qualitatively similar when we incorporate this cost into our model.) Therefore, the initial investment cost C_i corresponds to the (opportunity) cost of enhancing the product before the campaign (e.g., the cost of market research, concept generation, design rework, or engineering changes) rather than involving customers in an earlier stage of product development. The creator sets the pledge price $p = G/2$ so that the campaign is successful if and only if both customers pledge (Hu et al. 2015).[7] To study our first two research questions (Q1) and (Q2) analytically and empirically, we consider the case when q_i is exogenously given, so the initial investment cost $C_i \cdot q_i$ is sunk. However, when we examine our third research question (Q3), q_i is endogenously determined.

Stage 1: Customer 1's pledging decision. Customer 1 with valuation v_1 arrives at the campaign. For $j = \{1,2\}$, valuation v_j measures customer j's marginal willingness to pay for the enhancement level of the product, and v_j's are independent across customers and drawn from a Uniform distribution with parameters 0 and 1 (e.g., Krishnan and Ramachandran 2011, Belleflamme et al. 2014).[8] Once arrived at the campaign, customer 1 not only observes the pledge price p and the initial enhancement level q_i of the product from its detailed description but also anticipates the final enhancement level q_f of the product. Thus, to decide whether to pledge or not, customer 1 compares her effective valuation $v_1 \cdot q_f$ of the product and the

[7] As a supplementary analysis, we also consider a model where two customers (instead of just customer 2) arrive after customer 1 yet the funding goal G is still $2p$. In this case, it is still possible that the campaign is successful if two out of three customers pledge, so no single customer is pivotal in whether the campaign will be successful. Our supplementary analysis of this case yields qualitatively similar results to our main results.

[8] In the crowdfunding literature (e.g., Hu et al. 2015), it is common to assume such a distribution on valuations.

Table 4.1: Summary of the main assumptions in the theoretical model.

Assumptions	References
Two customers arrive at the campaign sequentially, the funding goal $G = 2p$, and the per-unit production cost is cq_f^2, where $c > 0$.	Hu et al. (2015)
v_j's are i.i.d. with Uniform$(0,1)$.	Krishnan and Ramachandran (2011), Belleflamme et al. (2014)
Customers are fully refunded if the campaign fails.	Hu et al. (2015), Chakraborty and Swinney (2021)

pledge price p (e.g., Belleflamme et al. 2014). If customer 1 does not pledge, the campaign fails, and both customers receive a reservation value of 0. If customer 1 pledges, the next stage commences.

Stage 2: Customer 1's commenting decision. Customer 1 decides whether to make a comment to entice the creator to improve the product during the campaign, which can enhance the product by q_u (> 0). For instance, q_u may represent the degree of improvement achieved by adding a wristband to the HAIZE navigation device, as discussed in §4.1.[9] If customer 1 does not make any comment, then the final enhancement level of the product is $q_f = q_i$.

Stage 3: The creator's product improvement decision. If customer 1 makes a comment with a enhancement level of q_u, then the creator decides whether to improve the product further or not. If the creator improves the product, then the final enhancement level of the product is $q_f = q_i + q_u$ (e.g., Krishnan and Ramachandran 2011); otherwise, $q_f = q_i$. Note that $q_f = q_i + q_u$ can be achieved even before the campaign if the creator is willing to incur the related cost (e.g., extra market research).

Stage 4: Customer 2's pledging decision. Customer 2 with valuation v_2 arrives at the campaign. Upon observing the final enhancement level q_f and the pledge price

[9]For ease of illustration, we assume that the customer's cost of commenting is negligible compared to the utility she can obtain from a potential improvement in the product. In Appendix F.2, we extend our analysis to the case where the customer incurs some non-negligible cost when she makes a comment.

p, she decides whether to pledge or not. If customer 2 pledges along with customer 1, the campaign is successful, and the creator receives pledges and delivers products to customers by incurring a per-unit production cost of $c \cdot q_f^2$, where $c > 0$ (e.g., materials and labor; Guo and Zhang 2012, Hu et al. 2015). If customer 2 does not pledge, then the campaign fails. (Our results are qualitatively similar when the per-unit production cost is $c \cdot q_f$.)

4.3.1 Analysis of Sub-game Perfect Equilibrium

For any given pledge price p and initial enhancement level q_i, we determine the sub-game perfect equilibrium via backward induction. First, in Stage 4, customer 2 with valuation $v_2 \sim \text{Uniform}(0,1)$ pledges if and only if her expected utility $U_2 = v_2 q_f - p \geq 0$. If the creator improved the product in Stage 3, the final enhancement level $q_f = q_i + q_u$, and hence customer 2 pledges with probability $\frac{q_i+q_u-p}{q_i+q_u}$. Otherwise, the final enhancement level $q_f = q_i$, and hence customer 2 pledges with probability $\frac{q_i-p}{q_i}$. We assume that $q_i > p$ to avoid trivial cases where even a customer with maximum valuation does not pledge when there is no improvement.

In Stage 3, if customer 1 pledged and made a comment in Stages 1 and 2, the creator decides whether to improve the product during the campaign by comparing the creator's expected profit Π^I with improvement and expected profit Π^{NI} with no improvement. By taking customer 2's pledging probability $\frac{q_i+q_u-p}{q_i}$ and the per-unit production cost $c(q_i+q_u)^2$ into consideration, the creator's expected profit with an improvement is $\Pi^I = \left(\frac{q_i+q_u-p}{q_i+q_u}\right)\left(2p - 2c(q_i+q_u)^2\right) - C_i q_i$.[10] Similarly, the creator's expected profit without improvement is $\Pi^{NI} = \left(\frac{q_i-p}{q_i}\right)\left(2p - 2cq_i^2\right) - C_i q_i$. Thus, the creator improves the product during the campaign if and only if $\Pi^I \geq \Pi^{NI}$, i.e.,

$$\frac{p^2}{q_i(q_i+q_u)} - c(2q_i + q_u - p) \geq 0. \tag{4.1}$$

In Stage 2, if customer 1 pledged in Stage 1, she decides whether to make

[10] Although we normalize the unit investment cost C_u of improving the product to zero, our results are qualitatively similar when $C_u > 0$; i.e., $\Pi^I = \left(\frac{q_i+q_u-p}{q_i+q_u}\right)\left(2p - 2c(q_i+q_u)^2\right) - C_i q_i - C_u q_u$. Note that in this case, $C_i > C_u$ represents the case where the investment cost is mainly related to market research, while $C_u > C_i$ represents the case where the investment cost is mainly related to design rework.

a comment or not. First, suppose that condition (4.1) is violated. Then, even if customer 1 makes a comment, she anticipates that the creator will not improve the product in Stage 3 and customer 2 will pledge in Stage 4 with probability $\frac{q_i-p}{q_i}$. Thus, in this case, customer 1's expected utility is $U_1^C = \left(\frac{q_i-p}{q_i}\right)(v_1 q_i - p)$. Similarly, if customer 1 does not make a comment, she anticipates that customer 2 will pledge in Stage 4 with probability $\frac{q_i-p}{q_i}$, and hence customer 1's expected utility is $U_1^{NC} = U_1^C$. Since $U_1^{NC} = U_1^C$, the cases where customer 1 makes a comment and does not make a comment are both in equilibria.

Now, suppose that condition (4.1) holds. Then, if customer 1 makes a comment, she anticipates that the creator will improve the product in Stage 3 and customer 2 will pledge in Stage 4 with probability $\frac{q_i+q_u-p}{q_i+q_u}$. Thus, in this case, customer 1's expected utility is $U_1^C = \left(\frac{q_i+q_u-p}{q_i+q_u}\right)(v_1(q_i+q_u)-p)$. However, if customer 1 does not make a comment, she anticipates that customer 2 will pledge in Stage 4 with probability $\frac{q_i-p}{q_i}$, and hence customer 1's expected utility is $U_1^{NC} = \left(\frac{q_i-p}{q_i}\right)(v_1 q_i - p)$. Therefore, customer 1 makes a comment in Stage 2 if and only if $U_1^C \geq U_1^{NC}$, i.e., $v_1 \geq \frac{p^2}{q_i(q_i+q_u)}$.

Finally, in Stage 1, customer 1's pledging decision depends on whether she anticipates an improvement in the product or not. First, suppose that condition (4.1) holds so that customer 1 anticipates an improvement and makes a comment when $v_1 \geq \frac{p^2}{q_i(q_i+q_u)}$. Then, customer 1 decides whether to pledge or not by comparing her expected utility U_1^P when she pledges, where $U_1^P = U_1^C = \left(\frac{q_i+q_u-p}{q_i+q_u}\right)(v_1(q_i+q_u)-p)$, and her expected utility U_1^{NP} when she does not pledge, where $U_1^{NP} = 0$. Thus, customer 1 pledges if and only if $U_1^P \geq U_1^{NP}$, i.e., $v_1 \geq \frac{p}{q_i+q_u}$. Therefore, in a setting where customer 1 anticipates an improvement, she pledges *and* makes a comment if

$$v_1 \geq \max\left\{\frac{p^2}{q_i(q_i+q_u)}, \frac{p}{q_i+q_u}\right\} = \frac{p}{q_i+q_u}.$$

This means that if condition (4.1) holds, customer 1 makes a comment *whenever* she pledges. Second, suppose that condition (4.1) is violated. Then, regardless of customer 1's commenting decision in Stage 3, customer 2 pledges in Stage 4 with probability $\frac{q_i-p}{q_i}$. In this case, customer 1's expected utility when she pledges is $U_1^P = \left(\frac{q_i-p}{q_i}\right)(v_1 q_i - p)$, and her expected utility when she does not pledge is $U_1^{NP} = 0$. Thus, customer 1 pledges if and only if $v_1 \geq \frac{p}{q_i}$.

By using the creator's and customers' rational strategies explained above, we next characterize the equilibrium outcomes. We aim to understand how the initial enhancement level affects whether the product is improved during a campaign or not. Thus, we calculate the ex-ante probability that there is an improvement in the product during the campaign, and we denote this probability by $\mathbb{P}(improve)$. Also, let $\mathbb{P}(success)$ be the ex-ante probability that the campaign is successful. The following lemma characterizes these equilibrium outcomes. We present all proofs in Appendix E.

Lemma 1 *If* $\frac{p^2}{q_i(q_i+q_u)} - c(2q_i + q_u - p) \geq 0$, *then* $\mathbb{P}(improve) = \frac{q_i+q_u-p}{q_i+q_u}$ *and* $\mathbb{P}(success) = \left(\frac{q_i+q_u-p}{q_i+q_u}\right)^2$*; otherwise,* $\mathbb{P}(improve) = 0$ *and* $\mathbb{P}(success) = \left(\frac{q_i-p}{q_i}\right)^2$.

Lemma 1 first characterizes the case where condition (4.1) holds so that the creator is willing to improve the product during the campaign and customer 1 makes a comment whenever she pledges. In this case, customer 1 pledges and makes a comment with probability $\frac{q_i+q_u-p}{q_i+q_u}$, and hence $\mathbb{P}(improve) = \frac{q_i+q_u-p}{q_i+q_u}$. Also, customer 2 pledges with probability $\frac{q_i+q_u-p}{q_i+q_u}$, and hence $\mathbb{P}(success) = \left(\frac{q_i+q_u-p}{q_i+q_u}\right)^2$. Lemma 1 also characterizes the case where condition (4.1) is violated. In this case, regardless of customer 1 making a comment or not, $\mathbb{P}(improve) = 0$ and $\mathbb{P}(success) = \left(\frac{q_i-p}{q_i}\right)^2$. Note that, *ceteris paribus*, $\mathbb{P}(success)$ is greater when the creator improves the product during the campaign, which is consistent with the empirical finding of Cornelius and Gokpinar (2020).

4.3.2 Probability of Product Improvement

Using Lemma 1, we next answer our first research question (Q1). The following proposition characterizes the impact of the initial enhancement level q_i on $\mathbb{P}(improve)$.

Proposition 1 *There exists a threshold* $\overline{q_i}$ (≥ 0) *such that when the initial enhancement level* $q_i \leq \overline{q_i}$, $\mathbb{P}(improve)$ *increases with* q_i*; and when* $q_i > \overline{q_i}$, $\mathbb{P}(improve) = 0$.

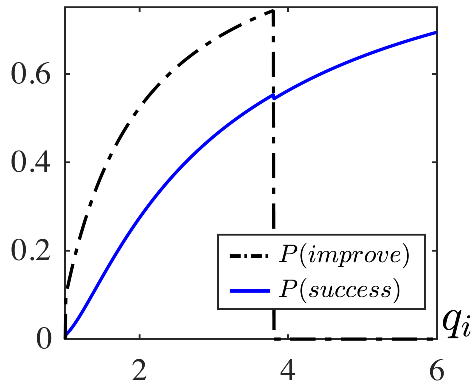


Figure 4.3: The impact of the initial enhancement level q_i on $\mathbb{P}(improve)$ and $\mathbb{P}(success)$. The setting: $p = 1$, $c = 0.01$, and $q_u = 0.1$.

One might expect that the higher the initial enhancement level q_i, the less likely the product is to be improved during the campaign. However, Proposition 1 shows that the probability of product improvement during the campaign increases with the initial enhancement level q_i up to a threshold $\overline{q_i}$ but above this threshold, the creator does not improve the product during the campaign, as illustrated in Figure 4.3. The intuition is as follows. $\mathbb{P}(improve)$ depends on the decisions of both the creator and customer 1. First, consider the creator's improvement decision. When the creator improves the product during the campaign, the product becomes more enhanced, and hence each customer is more likely to pledge, yet the creator expects to incur a higher production cost (i.e., $c(q_i + q_u)^2$ instead of cq_i^2). When the initial enhancement level q_i is higher, improving the product leads to a smaller increase in the customer's likelihood of pledging and a larger increase in the cost of production. Because of these opposing forces, the creator improves the product during the campaign only when q_i is below a certain threshold (i.e., $q_i \leq \overline{q_i}$).[11]

Next, consider the decisions of customer 1. As q_i increases, customer 1's expected utility increases, and customer 1 is thus more likely to pledge and make a comment, which increases $\mathbb{P}(improve)$. However, this is possible only when the creator improves the product further (i.e., $q_i \leq \overline{q_i}$). Thus, $\mathbb{P}(improve)$ increases with $q_i \leq \overline{q_i}$, but $\mathbb{P}(improve) = 0$ when $q_i > \overline{q_i}$.

[11]Even when the marginal cost of production is constant (i.e., cost of production is $c \cdot q_f$), this result continues to hold because as q_i increases, improving the product leads to a smaller increase in the customer's likelihood of pledging.

Based on Proposition 1, we establish the following hypothesis (hereafter H1).

Hypothesis 1 *As the initial enhancement level of a product increases, the likelihood that the product will be improved during the campaign first increases and then decreases.*

4.3.3 Probability of Campaign Success

We next use Lemma 1 to answer our second research question (Q2). The following proposition characterizes the impact of the initial enhancement level q_i on $\mathbb{P}(success)$.

Proposition 2 *(i) When* $\frac{p^2}{q_i(q_i+q_u)} - c(2q_i + q_u - p) \neq 0$, *$\mathbb{P}(success)$ is increasing in q_i. (ii) Also, there exists a threshold $\underline{q_i}$ (> 0) such that for $q_i > \underline{q_i}$, $\mathbb{P}(success)$ is concave in q_i.*

Proposition 2 shows that, regardless of whether condition (4.1) holds with strict inequality or it is violated (i.e., whether the creator is willing to improve the product during the campaign or not), the probability of campaign success increases with the initial enhancement level q_i, but at a decreasing rate when q_i is above a threshold; see Figure 4.3(a).[12] The intuition behind Proposition 2(i) is that as the initial enhancement level q_i increases, each customer's expected utility increases, and the customer is thus more likely to pledge. Thus, $\mathbb{P}(success)$ increases with q_i. Proposition 2(ii) stems from two opposing effects. First, as q_i increases, the marginal benefit of increasing q_i on each customer's probability of pledging decreases. Second, there is also an indirect effect of one customer's pledging on the other one's decision, and the marginal benefit of increasing q_i on this indirect effect increases with q_i. When q_i is above a certain threshold (i.e., $q_i > \underline{q_i}$), the former effect is more dominant than the latter, so the marginal benefit of increasing q_i on the probability of campaign success decreases.

Based on Proposition 2, we establish the following hypothesis (hereafter H2).

---

[12]Note that when $\frac{p^2}{q_i(q_i+q_u)} - c(2q_i + q_u - p) = 0$, the creator does not improve the product during the campaign as q_i increases, and hence $\mathbb{P}(success)$ decreases from $\left(\frac{q_i+q_u-p}{q_i+q_u}\right)^2$ to $\left(\frac{q_i-p}{q_i}\right)^2$.

Hypothesis 2 *As the initial enhancement level of a product increases, the likelihood of campaign success increases, but at a decreasing rate when the initial enhancement level is high.*

4.4 Empirical Models and Analysis

We test H1 and H2 by using a unique data set collected from Kickstarter, a crowdfunding platform that enables creators to launch reward-based, all-or-nothing campaigns. To launch a campaign on Kickstarter, each creator first prepares a campaign page that includes a textual description of the product along with supporting materials such as pictures. On the campaign page, the creator also specifies a funding goal and the pledge price required to receive the product as a reward. Once the campaign has been launched, a customer arriving on the campaign page reads the product description to decide whether or not to pledge. As we discuss in §4.1, if a customer pledges, she can make a comment on the campaign page to induce the creator to improve the product further (customers cannot make comments before pledging; see Kickstarter 2021h); and if the creator improves the product, the creator revises the product description accordingly. After the campaign ends, customers cannot pledge or the creator cannot revise the product description. As we discuss in §4.3, if the total amount pledged at the end exceeds the funding goal, the campaign is successful; otherwise, the campaign fails and customers are refunded. In the remainder of this section, we describe the sample, variables, empirical models, and empirical results.

4.4.1 Sample

Consistent with our theoretical model, we focus on 21,768 campaigns for physical products in the Technology and Design categories launched on Kickstarter between July 2013 and February 2016.[13] The sample contains 6,488 successful campaigns, 12,111 failed campaigns, and 3,169 cancelled campaigns.[14] We exclude 388 cam-

[13]These campaigns constitute the majority of the Technology and Design campaigns, and include product subcategories such as camera equipment, hardware, and product design, but not software, web, and graphic design. Our additional analysis shows that the empirical results reported in §4.4.4 continue to hold when we include campaigns for non-physical products.

[14]Campaigns can be cancelled due to intellectual property disputes or at the discretion of creators (Kickstarter 2021b). Although we exclude cancelled campaigns in our main empirical analyses, our

paigns that are not suitable for textual analysis (e.g., non-English campaigns, see §4.4.2), so the final sample contains 21,380 campaigns.

4.4.2 Dependent, Explanatory, and Control Variables

Dependent Variables: Product Improvement and Campaign Success. To test H1, we need a variable that measures whether a creator improves the product during a campaign or not. Hence, we generate a binary variable, product improvement I_k, for each campaign k that represents whether the final enhancement level q_{fk} of the product in campaign k is higher than its initial enhancement level q_{ik}. (We explain how we construct q_{ik} and q_{fk} when we discuss level of enhancement below.) Specifically, $I_k = 1$ if $q_{fk} > q_{ik}$; otherwise, $I_k = 0$. In our sample, 26% of all products and 43% of products in successful campaigns were improved during their campaigns.

To test H2, we create a binary variable, campaign success S_k, for each campaign k that represents whether the total amount pledged P_k at the end of the campaign is greater than or equal to the funding goal G_k (e.g., Mollick 2014, Wei et al. 2020). Specifically, $S_k = 1$ if $P_k \geq G_k$; otherwise, $S_k = 0$. This measure is important for creators to evaluate their success, and it is also consistent with how Kickstarter evaluates campaigns to analyze the performance of the platform (Kickstarter 2021g). In our sample, 35% of campaigns were successful (excluding cancelled campaigns).

Level of Enhancement. The NPD literature identifies that product developers typically design their products by first outlining the core features and then adding more features (e.g., Ulrich and Eppinger 2016). Products that are more enhanced therefore tend to have more features (e.g., Althuizen and Chen 2021). Utilizing this relationship, we measure the initial and final enhancement levels of products through the number of features by leveraging products' textual descriptions at the beginning and end of their campaigns.

Following the prior literature (e.g., Tirunillai and Tellis 2014, Toubia et al. 2019), we use latent Dirichlet allocation (LDA; Blei et al. 2003) to create a proxy

empirical results continue to hold when we treat them as failed campaigns (see §4.4.4).

for the number of features by extracting topics and their weights in each description. Our approach is as follows. We train the LDA model on 42,564 initial and final descriptions of 21,380 campaigns in our sample. (We discuss the details of the LDA model in Appendix F.1.) For each description, we multiply the description's topic weights by the description length (in words) to get the number of words associated with a particular topic. In line with the prior literature (e.g., Blei et al. 2003, Griffiths and Steyvers 2004), we consider a topic to be present in a description if it is associated with at least ten words. (Our empirical results continue to hold when we use different thresholds; see §4.4.4.) Then, in each campaign k, we measure the product's initial enhancement level q_{ik} as the number of topics in its initial description and the product's final enhancement level q_{fk} as the number of topics in its final description. For example, the initial enhancement level of the HAIZE navigation device discussed in §4.1 is calculated as 14, whereas its final enhancement level is calculated as 19, consistent with the fact that it was improved during its campaign. (See Figure F.1 in Appendix F.1 for the initial and final descriptions of this product.) In our sample, the average initial enhancement level is 9.02 and the average final enhancement level is 9.59. As we need both q_{ik} and q_{fk} to calculate I_k, we exclude 196 campaigns which only have either an initial or a final description after pre-processing (see Appendix F.1).

Control Variables. In our empirical models, we include several controls for campaign and creator characteristics. Specifically, we control for the *category* of each campaign (i.e., technology or design; we set design as the base category in empirical models) because the initial enhancement level of a product can differ across categories. Also, we control for each campaign's funding *goal* (natural logarithm of goal in US dollars) and *duration* (in days) (e.g., Mollick 2014, Blaseg et al. 2020). These variables enable us to control for the scale of a project because, for example, we can expect the goal to be higher and/or the duration of the campaign to be longer for a larger scale project. Additionally, we control for the median *pledge price* (in US dollars) of each campaign and the *delivery time* (the number of months between the last delivery date and the end of the campaign), which are also linked to the scale of the project.[15]

In our analysis, we derive the enhancement level of products from their textual descriptions. Because these descriptions can also include videos and pictures as well as a section in which creators discuss various risks associated with their campaigns, we control for the number of *videos*, the number of *pictures*, and *risk-section length* (in words) (e.g., Blaseg et al. 2020). We include additional control variables to take into account various creator-related factors that may affect the likelihood of product improvement or campaign success. First, we control for *creator experience* in terms of the number of previous campaigns launched by a creator. Second, we control for whether the creator is an *individual* or not, which we define as follows. If the majority of personal pronouns in the product description are singular and the creator is not an organization with legal name (e.g., Ltd, Inc), the creator is an individual; if not, the creator is not an individual. Finally, we control for the average level of *competition* during each campaign as follows. For each category and each day, we calculate the number of concurrent campaigns and the number of new customers,[16] and we then divide these two variables to obtain "campaign–customer" ratio. Then, for each campaign, we control for the average campaign–customer ratio during the campaign.

4.4.3 Model Specification

Our empirical strategy relies on probit models and an instrumental variable (IV) approach to address potential endogeneity concerns.

Probit Models. Because our dependent variables, product improvement I_k and campaign success S_k, are binary, we use probit models to test H1 and H2. Let X_k be the vector of control variables for campaign k. First, to test the nonlinear (first increasing and then decreasing) relationship between initial enhancement level q_{ik} on product improvement I_k in H1, we include both q_{ik} and $(q_{ik})^2$, and obtain the following

[15]Our additional analysis indicates that the median pledge price is a good proxy for the pledge price of the product in each campaign, but our empirical results hold when we control for the mean pledge price in each campaign.

[16]On December 19, 2013, the number of new customers is zero in both categories due to a server error, so we replace the number of new customers on this day with the average number of customers in each category.

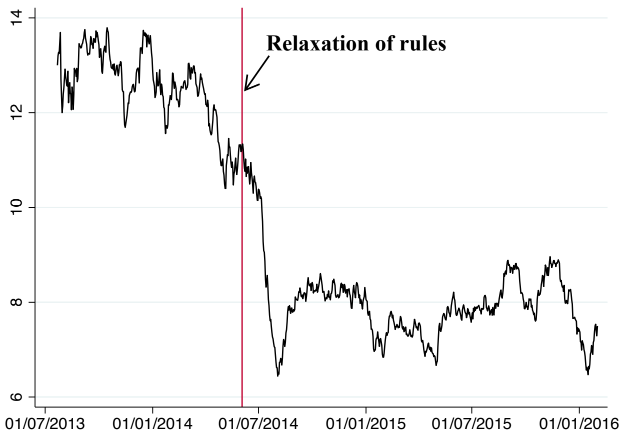


Figure 4.4: 30-day moving average of products' initial enhancement levels by campaign start dates.

probit model (Probit Model 1):

$$I_k = \beta_0 + \beta_1 q_{ik} + \beta_2 (q_{ik})^2 + \beta_X X_k + v_k.$$

Next, to test the nonlinear (increasing with diminishing returns after some initial enhancement level) relationship between initial enhancement level q_{ik} on campaign success S_k in H2, we include both q_{ik} and $(q_{ik})^2$, and obtain the following probit model (Probit Model 2):

$$S_k = \beta_0 + \beta_1 q_{ik} + \beta_2 (q_{ik})^2 + \beta_X X_k + v_k.$$

Instrumental Variable. Although we control for campaign and creator characteristics, there may still be unobserved factors that can simultaneously affect initial enhancement level q_{ik} in campaign k and product improvement I_k as well as campaign success S_k. To address this problem and any potential measurement errors, we use an exogenous shock to the level of product enhancement that is required before a Kickstarter campaign can be launched. Specifically, on June 3, 2014, Kickstarter introduced a new policy in which the campaign requirements were relaxed (Kickstarter 2021d). As Figure 4.4 illustrates, this exogenous shock leads to a substantial decrease in the average initial enhancement level of products. Using this exogenous shock as an instrument allows us to isolate the impact of the initial enhancement level on our outcome variables.[17]

[17] A concern may be that the relaxation of rules simultaneously leads to an increase in the number

To use this instrument in our IV models, we create a binary variable, *before relaxation of rules* B_k, where $B_k = 1$ if campaign k is launched before June 3, 2014, and $B_k = 0$ otherwise. Since we focus on campaigns launched between July 2013 and February 2016, we have comparable time periods before and after the instrument. This also enables us to avoid a confounding event on February 2, 2016, when Kickstarter started selecting curated campaigns (Kickstarter 2021c). Note that the F–statistic in the regression of the initial enhancement level on B_k is 900.10, which indicates that our IV satisfies the relevance condition.

IV Models. In our empirical analysis, we aim to analyze the nonlinear relationship between an endogenous explanatory variable and a binary dependent variable, so using a standard two-stage least squares (2SLS) approach would be problematic. Thus, we implement the instrumental variable using a control function approach (Wooldridge 2010, 2015), a two-step approach that allows us to condition out the variation in unobserved variables that depends on the endogenous variable, and hence the remaining variation in the endogenous variable is independent of the error (Petrin and Train 2010).

To test H1 with the IV model, we use the following procedure as explained in Wooldridge (2015) (for an example in the operations management literature, see Chan et al. 2021). In the first stage, we regress the initial enhancement level q_{ik} on the instrumental variable B_k and control variables in an ordinary least-squares model. We then use the predicted residuals $\hat{u}_k$ of the first-stage regression in the second-stage probit model whose dependent variable is product improvement I_k (Wooldridge 2015). Because $\hat{u}_k$ is an estimate from the first stage, which adds extra variation in the second stage (Petrin and Train 2010), we also use a nonparametric bootstrap to obtain valid standard errors in the second stage (Wooldridge 2010, 2015). For H1, we obtain the IV model (IV Model 1) with the following first- and

of campaigns and thereby reduces each campaign's likelihood of success. This would violate the exclusion restriction, by which an instrument cannot affect an outcome variable directly but only through the instrumented explanatory variable. To satisfy the exclusion restriction, we include the level of competition as an additional control variable. The instrument is then independent of campaign success, conditional on the explanatory variable and the level of competition.

Table 4.2: Descriptive statistics of variables in the empirical models ($n = 18,173$).

Variables	Mean	Standard deviation	Minimum	Maximum
Product improvement	0.26	0.44	0	1
Campaign success	0.35	0.48	0	1
Initial enhancement level	9.02	8.11	0	48
Goal (ln)	9.48	1.64	-0.28	18.52
Duration	34.63	10.5	1	61
Pledge price	174.95	535.09	0	10000
Delivery time	4.27	5.03	0	70
Videos	0.26	0.83	0	26
Pictures	11.08	12.11	0	119
Risk-section length	141.64	119.95	8	4981
Creator experience	0.18	0.78	0	21
Individual	0.31	0.46	0	1
Competition	0.16	0.04	0.05	0.54
Before relaxation of rules	0.22	0.41	0	1

Note. The minimum value of the natural logarithm of the goal (in US dollars) is negative because the minimum value of the goal is one Canadian dollar. Our empirical results continue to hold when we exclude 324 campaigns where the goal (in US dollars) is smaller than the 1st percentile ($168) or larger than the 99th percentile ($500,000).

second-stage regressions:

$$q_{ik} = \alpha_0 + \alpha_1 B_k + \alpha_X X_k + u_k, \text{ and}$$

$$I_k = \beta_0 + \beta_1 q_{ik} + \beta_2 (q_{ik})^2 + \beta_3 \hat{u}_k + \beta_4 (\hat{u}_k)^2 + \beta_X X_k + v_k.$$

To test the nonlinear relationship between q_{ik} and I_k, we include both $(q_{ik})^2$ and $(\hat{u}_k)^2$ in the second-stage regression (Wooldridge 2015, page 437).[18] Similarly, to test H2, we obtain the IV model (IV Model 2) with the following first- and second-stage regressions:

$$q_{ik} = \alpha_0 + \alpha_1 B_k + \alpha_X X_k + u_k, \text{ and}$$

$$S_k = \beta_0 + \beta_1 q_{ik} + \beta_2 (q_{ik})^2 + \beta_3 \hat{u}_k + \beta_4 (\hat{u}_k)^2 + \beta_X X_k + v_k.$$

[18] Note that there is no forbidden regression problem in our model because we do not directly plug predicted values of q_{ik} from the first stage in the nonlinear second-stage regression (cf. Angrist and Pischke 2009). Instead, we implement control function approach, which was developed as a solution to the forbidden regression problem (Wooldridge 2010, 2015, Petrin and Train 2010), and hence we use predicted residuals $\hat{u}_k$.

Table 4.3: Correlation matrix for variables in the empirical models ($n = 18,173$).

Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(1) *Product improvement*	1.00												
(2) *Campaign success*	0.28***	1.00											
(3) *Initial enhancement level*	0.18***	0.21***	1.00										
(4) *Goal* (ln)	0.07***	-0.22***	0.25***	1.00									
(5) *Duration*	0.04***	-0.05***	0.05***	0.22***	1.00								
(6) *Pledge price*	0.01	-0.04***	0.04***	0.18***	0.02***	1.00							
(7) *Delivery time*	0.01*	-0.08***	0.05***	0.26***	0.10***	0.10***	1.00						
(8) *Videos*	0.05***	0.04***	0.18***	0.11***	0.04***	0.01	0.01*	1.00					
(9) *Pictures*	0.19***	0.32***	0.53***	0.17***	0.08***	0.00	0.00	0.18***	1.00				
(10) *Risk-section length*	0.09***	0.06***	0.36***	0.20***	0.03***	0.03***	0.07***	0.09***	0.21***	1.00			
(11) *Creator experience*	0.01*	0.17***	-0.01	-0.18***	-0.03***	-0.03***	-0.06***	0.00	0.04***	-0.03***	1.00		
(12) *Individual*	-0.09***	-0.12***	-0.14***	-0.18***	-0.06***	-0.02***	0.01	-0.06***	-0.21***	-0.08***	0.00	1.00	
(13) *Competition*	-0.06***	-0.09***	-0.14***	0.01	0.06***	0.04***	0.05***	0.02***	-0.09***	-0.04***	0.02**	0.01*	1.00

**** p<0.01, ** p<0.05, * p<0.1*

4.4.4 Results

Tables 4.2 and 4.3 show the descriptive statistics and correlations (we exclude 3,011 cancelled campaigns). We report no issue of multicollinearity. All the results are presented in Table 4.4. The effects of control variables on our outcome variables in both models are as expected. For example, compared to teams, individual creators are less likely to improve their products and their campaigns are less likely to be successful.

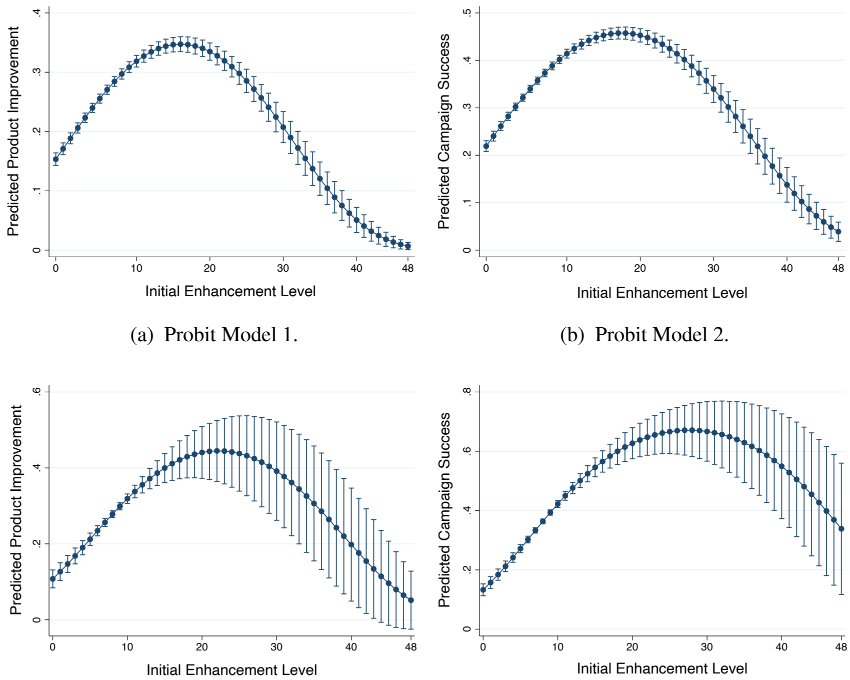
Results of Probit Models. In Probit Model 1, the coefficient of initial enhancement level q_{ik} is positive and significant ($\gamma_1 = 0.072$, $p < 0.01$) and the coefficient of $(q_{ik})^2$ is negative and significant ($\gamma_2 = -0.002$, $p < 0.01$). As Figure 4.5(a) illustrates, this result supports H1. In Probit Model 2, the coefficient of initial enhancement level q_{ik} is positive and significant ($\gamma_1 = 0.088$, $p < 0.01$) and the coefficient of $(q_{ik})^2$ is negative and significant ($\gamma_2 = -0.002$, $p < 0.01$). As Figure 4.5(b) illustrates, this result shows that as the initial enhancement level increases, the likelihood of campaign success first increases but then decreases. Hence, this result does not support H2.

Results of IV Models. We next discuss the results of the IV models (see Table 4.4). The second stage of IV Model 1 shows the results for H1. While the coefficient of initial enhancement level q_{ik} is positive and significant ($\gamma_1 = 0.103$, $p < 0.01$), the coefficient of $(q_{ik})^2$ is negative and significant ($\gamma_2 = -0.002$, $p < 0.01$). We also calculate the turning point as $q_{ik} = 23.12$ and its 95% confidence interval as $(19.46, 27.33)$, and both the turning point and its confidence interval are within the data range (e.g., Haans et al. 2016, Tan and Netessine 2019). As Figure 4.5(c) illustrates, this result indicates that as the initial enhancement level increases, the likelihood of product improvement first increases and then decreases, supporting H1.

Table 4.4: Results of probit and IV models.

	Probit Model 1	Probit Model 2	First Stage of IV Models 1 and 2	Second Stage of IV Model 1	Second Stage of IV Model 2
	Product improvement	Campaign success	Initial enhancement level	Product improvement	Campaign success
Initial enhancement level	.072***	.088***		.103***	.129***
	(.004)	(.004)		(.009)	(.01)
Initial enhancement level × Initial enhancement level	-.002***	-.002***		-.002***	-.002***
	(.000)	(.000)		(.000)	(.000)
Category: Technology	.059***	.025	1.004***	.026	-.016
	(.023)	(.021)	(.104)	(.025)	(.022)
Goal (ln)	-.006	-.331***	.579***	-.023***	-.354***
	(.008)	(.008)	(.034)	(.009)	(.009)
Duration	.003***	-.002*	-.019***	.003***	-.001
	(.001)	(.001)	(.005)	(.001)	(.001)
Pledge price	0	0***	0	0	0***
	(.000)	(.000)	(.000)	(.000)	(.000)
Delivery time	.002	-.005**	.011	.002	-.005**
	(.002)	(.002)	(.01)	(.002)	(.002)
Videos	.015	-.005	.696***	-.006	-.032**
	(.012)	(.012)	(.089)	(.014)	(.013)
Pictures	.013***	.033***	.296***	.004	.022***
	(.001)	(.001)	(.006)	(.003)	(.003)
Risk-section length	0***	0***	.015***	0	0**
	(.000)	(.000)	(.001)	(.000)	(.000)
Creator experience	.02	.257***	.203***	.018	.257***
	(.015)	(.023)	(.062)	(.015)	(.023)
Individual	-.14***	-.357***	-.04	-.14***	-.36***
	(.024)	(.02)	(.107)	(.024)	(.02)
Competition	-1.274***	-2.056***	-2.543*	-.595**	-1.163***
	(.250)	(.283)	(1.405)	(.303)	(.327)
Before relaxation of rules			3.513***		
			(.153)		
Residuals				-.031***	-.04***
				(.009)	(.009)
Residuals×Residuals				0	0*
				(.000)	(.000)
Constant	-1.04***	2.261***	-2.292***	-1.095***	2.192***
	(.084)	(.074)	(.366)	(.087)	(.078)
Wald χ^2	1490.12	4034.94	13279.55	1495.50	4001.20
pseudo R^2	.056	.204	.399	.057	.206
Observations	18173	18173	18173	18173	18173

*Nonparametric bootstrap standard errors (100 replications) in parentheses. *** p<.01, ** p<.05, * p<.1*



(a) Probit Model 1. (b) Probit Model 2.

(c) IV Model 1. (d) IV Model 2.

Figure 4.5: Predicted likelihood of product improvement and campaign success.

The second stage of IV Model 2 shows the results for H2. While the coefficient of initial enhancement level q_{ik} is positive and significant ($\gamma_1 = 0.129$, $p < 0.01$), the coefficient of $(q_{ik})^2$ is negative and significant ($\gamma_2 = -0.002$, $p < 0.01$). We also calculate the turning point as $q_{ik} = 26.01$ and its 95% confidence interval as $(22.15, 29.36)$, and both the turning point and its confidence interval are within the data range. As Figure 4.5(d) illustrates, this result shows that as the initial enhancement level increases, the likelihood of campaign success first increases but then decreases. Thus, like Probit Model 2, IV Model 2 does not support H2. In §4.5, we discuss a possible explanation for this relationship between the initial enhancement level and campaign success. Table 4.4 also shows that in the second stage of both IV models, the coefficients of residuals obtained from the first-stage model are negative and significant. Significant residuals confirm a possible endogeneity problem and support our use of an IV (Wooldridge 2010).

Robustness Check. To check the robustness of our empirical results, we also run spline regressions, which use knots to capture the different impact of an explanatory variable for different intervals (e.g., Kesavan et al. 2014). We try various spline regressions for the second-stage estimations in both IV models, and we find that the coefficient of the first spline is positive and significant and the coefficient of the second spline is negative and significant (see Table F.1 in Appendix F.3), supporting our results above.

Additionally, we run our empirical models for the following cases (see Appendix F.3 for details). First, to have equal time periods before and after the instrument, we exclude campaigns launched after April 28, 2015, and we show the robustness of our empirical findings by analyzing 11,764 campaigns. Second, we observe that in some campaigns, the number of topics in the final description of a product is slightly lower than in the initial description, often because of the decrease in the length of the description. When we exclude these campaigns and analyze 17,005 campaigns, our results continue to hold. Third, we treat cancelled campaigns as failed campaigns, and show the robustness of our findings by analyzing 21,184 campaigns. Fourth, as we explain in Appendix F.1, we train the LDA model with 50 topics, in line with the literature. Our results continue to hold when we set the number of topics to 20% above or below 50 in the LDA model and when we set the threshold to 20% above or below 10 words when counting the number of topics in each description. Finally, when testing H1, we control for the average competition in the first week of each campaign instead of the average competition during the campaign to avoid timing problem, and we show that our results hold.

4.5 Revised Theoretical Model and Analysis

Our empirical results do not support H2, and suggest that $\mathbb{P}(success)$ first increases and then decreases with the initial enhancement level. This discrepancy between the normative theory and empirical findings may suggest some behavioral effects. Specifically, when enhancing the product, a creator may end up with a too complex or too advanced product, which can overwhelm customers (e.g., Mick and Fournier 1998) or can lead to customer anxiety (e.g., Castaño et al. 2008, Goodman and Irmak 2013). To factor in such behavioral effects, we incorporate a cost of $b \cdot q_f^2$ into each customer's utility, where $b > 0$, and this cost discounts the value that each

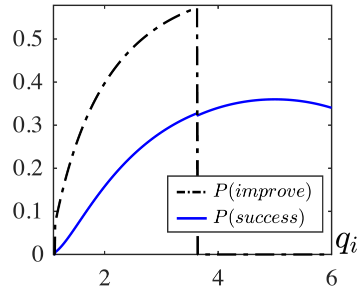


Figure 4.6: The impact of q_i on $\mathbb{P}(improve)$ and $\mathbb{P}(success)$. The setting is the same as in Figure 4.3 where $b = 0.1$.

customer receives from the product enhancement.[19] Specifically, we modify the utility function of customer j ($\in \{1,2\}$) in §4.3 as follows:

$$U_j = v_j(q_f - b \cdot q_f^2) - p, \text{ where } q_f = q_i \text{ or } q_f = q_i + q_u. \tag{4.2}$$

Note that when we omit behavioral effects, i.e., the behavioral cost parameter $b = 0$, this model boils down to our original model in §4.3. Considering U_j in (4.2), we use the same approach in §4.3 to analyze the impact of q_i on $\mathbb{P}(improve)$ and $\mathbb{P}(success)$, and obtain the following proposition.[20]

Proposition 3 *(a) There exists a threshold $\overline{q_i}$ (≥ 0) such that when the initial enhancement level $q_i \leq \overline{q_i}$, $\mathbb{P}(improve) = \frac{(q_i+q_u)-b(q_i+q_u)^2-p}{(q_i+q_u)-b(q_i+q_u)^2}$, which is increasing in q_i if and only if $b(q_i+q_u) < 0.5$. Furthermore, there exists $\underline{q_i}$ $(\in \mathbb{R}_+ \cup \{+\infty\})$ such that when $q_i > \underline{q_i}$, $\mathbb{P}(improve) = 0$.*
(b) There exist thresholds q_i' (≥ 0) and q_i'' $(\in \mathbb{R}_+ \cup \{+\infty\})$ such that $\mathbb{P}(success)$ is increasing in $q_i < q_i'$ and $\mathbb{P}(success)$ is decreasing in $q_i > q_i''$.

As Figure 4.6 illustrates, Proposition 3(a) extends Proposition 1 to the case where U_j is as in (4.2). In this case, as q_i increases, $\mathbb{P}(improve)$ first increases and may then decrease before $\mathbb{P}(improve) = 0$. More importantly, Proposition 3(b) shows that $\mathbb{P}(success)$ increases with the initial enhancement level q_i when q_i is below a certain threshold, and that $\mathbb{P}(success)$ decreases with q_i when q_i is above a certain

[19] Although we incorporate this cost for the final enhancement level q_f to have the impact of q_i and q_u on the customer's utility consistent, notice that this approach leads to an inverted U-shaped relationship between q_i and the customer's utility, as predicted in empirical analysis.

[20] In §4.5 and §4.6, we assume that $q_f - bq_f^2 - p > 0$, where $q_f = q_i$ (respectively, $q_f = q_i + q_u$), to avoid trivial cases where even a customer with maximum valuation does not pledge when there is no improvement (respectively, there is an improvement).

threshold; see Figure 4.6. (Note that in Figure 4.6, thresholds q_i' and q_i'' in Proposition 3(b) overlap.) The reason is that, although each customer's utility increases with q_i up to some point, too large an increase in q_i reduces each customer's utility, and it thus becomes less likely that customers will pledge. Therefore, when we consider the customer's utility in (4.2), our theoretical predictions and empirical observations become consistent.

Proposition 3(b) has the following interesting implication for crowdfunding practice. As we discuss in §4.1, Kickstarter experts suggest enhancing the product as much as possible before campaign launch (Kickstarter 2021e) by assuming that a more enhanced product is more likely to be successful. However, we show that this assumption does not necessarily hold. Instead, to run a successful campaign, a creator should avoid enhancing the product too much before the campaign as it can overwhelm customers or can lead to customer anxiety.

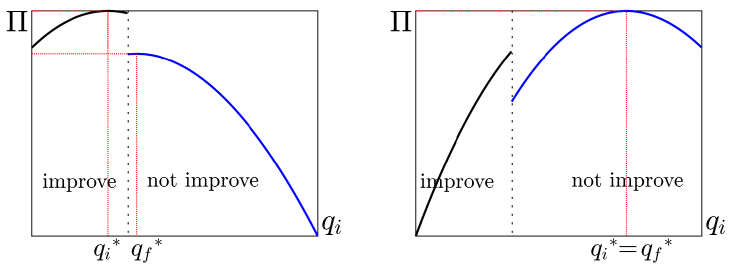
4.6 Optimal Level of Enhancement at Campaign Launch

So far, to answer our first two research questions (Q1) and (Q2), we have considered the case in which the initial enhancement level q_i is exogenously given. We now consider the optimal initial enhancement level to answer our last research question (Q3). For this analysis, we use our revised model in §4.5 as it yields predictions that are consistent with our empirical observations in §4.4.4.

We build on the backward induction argument presented in §4.3.1 to derive the sub-game perfect equilibrium. When customer j's utility U_j is as in (4.2), the creator improves the product during the campaign for any q_i if and only if

$$\begin{aligned} I \equiv & \frac{p^2(1-b(2q_i+q_u))}{q_i(q_i+q_u)(1-b(q_i+q_u))(1-bq_i)} \\ & - c\left(2q_i+q_u-\frac{p}{(1-b(q_i+q_u))(1-bq_i)}\right) \geq 0. \end{aligned} \tag{4.3}$$

This condition is analogous to the one in (4.1). Given this condition, in Stage 0 of



(a) $C_i = 0.05$. (b) $C_i = 0.01$.

Figure 4.7: The creator's ex-ante expected profit Π in Stage 0 as a function of q_i. The left-hand side of the figures represents q_i's which lead to improvement (i.e., $I \geq 0$) and the right-hand side represents q_i's which lead to no improvement (i.e., $I < 0$). The setting is the same as in Figure 4.6(a).

our revised model, the creator decides on q_i that maximizes the following profit:

$$\Pi = \begin{cases} \left(1 - \frac{p}{(q_i+q_u)-b(q_i+q_u)^2}\right)^2 \left(2p - 2c(q_i+q_u)^2\right) - C_i q_i, & \text{if } I \geq 0, \\ \left(1 - \frac{p}{q_i - bq_i^2}\right)^2 \left(2p - 2cq_i^2\right) - C_i q_i, & \text{otherwise.} \end{cases} \tag{4.4}$$

Let q_i^* and q_f^* be the optimal initial and final enhancement levels of the product respectively, where $q_f^* = q_i^* + q_u$ or $q_f^* = q_i^*$, depending whether $I \geq 0$ or $I < 0$ under q_i^*. To analyze whether it is always optimal for the creator to enhance the product as much as possible before the campaign (i.e., $q_i^* = q_f^*$), we make the following mild assumption.

Assumption 1 *$q_u - bq_u^2 - p > 0$ and $p - cq_u^2 > 0$.*

This assumption ensures that the cost of an improvement during a campaign can be recovered at least under zero initial enhancement so that making an improvement is an option for the creator. Under Assumption 1, we analyze the relationship between q_i^* and q_f^* in the following proposition.

Proposition 4 *For any b (≥ 0), there exists a threshold $\underline{C_i}$ (≥ 0) such that if $C_i > \underline{C_i}$, then $q_i^* = q_f^* - q_u$.*

As we discuss in §4.1, experts on crowdfunding campaigns suggest that products should be enhanced as much as possible before campaigns (i.e., $q_i^* = q_f^*$), yet Proposition 4 shows that this is *not* the optimal strategy for a creator when C_i is above a threshold.[21] The intuition is as follows. As C_i increases, it becomes more costly for the creator to increase q_i. Thus, to save on the initial investment cost, the creator prefers setting $q_i^* = q_f^* - q_u$ and improves the product during the campaign, as Figure 4.7(a) illustrates. Note that this result continues to hold even when there is no behavioral effect that leads the customer's utility to decrease with q_i, i.e., when $b = 0$. This result suggests that if enhancing the product fully before launching a campaign would entail significant additional costs for the creator, it may be better to go ahead with a basic version of a product and improve it during the campaign, even though this may decrease the chances of the campaign being successful. These additional costs might relate to market research, concept generation, design rework, or engineering changes. We also illustrate in Figure 4.7(b) that when C_i is small, the creator enhances the product fully (i.e., $q_i^* = q_f^*$) without leaving any room for further improvements during the campaign.

4.7 Discussion and Conclusion

Crowdfunding is more than just an effective financing instrument for entrepreneurs. One of its key advantages, which has received cursory attention from the crowdfunding literature, is that it enables creators to improve their products in response to customer feedback (Mollick 2014, Cornelius and Gokpinar 2020). To take advantage of this, a creator may launch a crowdfunding campaign for a basic version of a product (i.e., product with fewer features), leaving room for it to be improved. However, if the product appears too basic, customers may not pledge and the campaign may thus fail. Keen to investigate this key trade-off, we study how the initial enhancement level of a product affects the likelihood of it being improved during a campaign and the chances of the campaign being successful, and we ultimately analyze whether a creator should enhance a product as much as possible before launching a crowdfunding campaign.

Inspired by both crowdfunding practice and literature (e.g., Hu et al. 2015, Belavina et al. 2020, Chakraborty and Swinney 2021), we construct a parsimo-

[21] Our additional analysis shows that when $b = 0$, $q_i^* = q_f^* - q_u$ if and only if $C_i > \underline{C_i}$. Otherwise, $q_i^* = q_f^*$.

nious game-theoretical model of reward-based, all-or-nothing crowdfunding that takes into account the creator's product improvement decision. We obtain the following results. Although one might expect that more enhanced products are less likely to be improved since this will increase the creator's production cost, we show that this is only true when there is extensive initial enhancement. In contrast, when there is relatively little initial enhancement, the likelihood of product improvement increases with the initial enhancement level, because customers are then more likely to pledge and to leave comments. We also show the intuitive result that as the initial enhancement level increases, the likelihood of campaign success increases, but at a decreasing rate when the initial enhancement level is high.

We test these theoretical predictions using a unique data set from Kickstarter. Our empirical findings support the predicted relationship between the initial enhancement level (i.e., number of features) and the likelihood of product improvement. More interestingly, although our theoretical model intuitively predicts that the likelihood of campaign success will increase with the initial enhancement level, our empirical finding does not support this prediction. Instead, we show that the likelihood of campaign success at first increases but then, counterintuitively, decreases with the initial enhancement level. This can be because a highly enhanced product can be too complex or too advanced, which can overwhelm customers (e.g., Mick and Fournier 1998) or may lead to customer anxiety (e.g., Castaño et al. 2008). This result suggests that to run a successful campaign, the creator should avoid enhancing the product too much before the campaign.

Building on these results, we test the experts' recommended strategy of enhancing the product as much as possible before the campaign, and we show that this strategy is not always optimal. Instead, when customer involvement can reduce the product development costs—such as for market research, concept generation, design rework, or engineering changes—it can be better for the creator to go ahead with a simple version of a product and make improvements to it during the campaign, even though this may decrease the chances of the campaign being successful.

Our results not only contribute to the crowdfunding literature, but also add to the NPD literature (cf. Krishnan and Ulrich 2001, Loch and Kavadias 2008), as we revisit the debate around flexible approaches in product development and the role of customer feedback. Specifically, we point out that unlike traditional product development approaches (e.g., Bhattacharya et al. 1998, Thomke and Reinertsen 1998), crowdfunding enables a creator to improve a product based on customer feedback before committing to any production. Although this approach comes with a risk of campaign failure, our results suggest that it can still be better for the creator to leave scope to refine the product based on customer feedback during the campaign.

Our study is a first step towards analyzing creators' product development and improvement decisions in crowdfunding campaigns, so it naturally has some limitations that provide opportunities for future research. First, when analyzing how the initial enhancement level of a product affects a campaign's chances of success, we measure this by comparing the funds raised at the end of the campaign to the funding goal. Although this measure is consistent with our theoretical model and appropriate for the purpose of our study, it would also be interesting to analyze the impact of the initial enhancement level of a product on the likelihood of it being delivered to customers (i.e., a successful product launch). This analysis would require a more comprehensive data set that includes product launch information. Second, inspired by practice, in our theoretical model, we consider customer comments as providing a motivation for the creator to improve the product during the campaign, and we focus on the creator's decisions on the initial enhancement level of a product and whether to improve it during the campaign. However, an interesting direction for future research would be to focus in detail on customers' pledging and commenting decisions. Finally, our study focuses on reward-based crowdfunding, but it would also be interesting to analyze the impact of the initial enhancement level of a product in equity-based crowdfunding where customer and creator dynamics can be quite different.

Chapter 5

Conclusions

Online crowdsourcing and crowdfunding platforms have changed the way how firms, organizations, and entrepreneurs manage innovation. While crowdsourcing platforms enable established firms and organizations to look beyond their boundaries for innovative solutions, crowdfunding platforms enable entrepreneurs to raise funds and collect feedback for their innovative ideas from the crowd. Although the economic value generated on these platforms is highly dependent on operational decisions of firms, organizations, and entrepreneurs, some of these decisions have received limited attention from the literature on platform operations. This thesis contributes to the literature and practice by addressing this gap between the theory and practice.

Specifically, in Chapter 2, I study the contest organizer's decisions on the contest duration and award scheme. Analyzing a game-theoretical model, I show how the organizer should decide on the contest duration given different contest characteristics and provide an explanation for why giving multiple awards is so common in practice. In Chapter 3, I study the impact of team collaboration in innovation contests, motivated by the mixed policies adopted by crowdsourcing platforms. Analyzing game-theoretical models of innovation contests under both individual submissions and team submissions, I identify conditions under which the organizer and solvers can benefit from team collaboration and show that on crowdsourcing platforms, team collaboration is more likely to lead to better outcomes than as in traditional innovation settings. In Chapter 4, I study entrepreneurs' product development and improvement decisions in crowdfunding campaigns. Analyzing a game-theoretical model and testing its predictions empirically, I show that both the chances of campaign success and the likelihood of product improvement during a campaign first increase but then decrease with the product's initial level of enhancement, and it is not always best strategy to enhance products as much as possible before a campaign.

This thesis provides many opportunities for future research about innovation management on online platforms. In Chapter 2, I use a static model o capture the first-order effects and primary trade-offs in contests organized on crowdsourcing platforms such as InnoCentive and Topcoder. An interesting research avenue can be to study the relationship between the contest duration and feedback policies by using a dynamic model. By analyzing a dynamic model, one can also study the impact of the contest duration on solvers' participation and effort decisions over time. In Chapter 3, I focus on the organizer's decision on whether to encourage or discourage team collaboration and do not study solvers' team formation process. For future work, it would be interesting to conduct an experimental study to understand how solvers form teams to generate solutions to different types of problems on crowdsourcing platforms. In Chapter 4, campaign success is defined as whether the total amount pledged reaches the funding goal by the end of the campaign. Although this measure is consistent with the theoretical model and how Kickstarter defines campaign success, an interesting research direction would be to use a broader definition of success, such as whether the product is delivered to customers or product sales on platforms like Amazon after the campaign. Finally, I hope that this research encourages researchers to combine theoretical analysis, empirical analysis, and large-scale textual data analysis.

Appendix A

Proofs of Chapter 2

Proof of Lemma 1. For any $e_i \equiv \int_0^T \theta(t)\eta_i(t)dt$ and T, agent i can optimally allocate her effort over the contest duration by solving the following cost-minimization problem:

$$\min_{\eta_i} \int_0^T c\eta_i(t)^b dt, \text{ s.t. } e_i - \int_0^T \theta(t)\eta_i(t)dt = 0. \tag{A.1}$$

Let μ_e be the Lagrange multiplier of the constraint in (A.1). Then, the equilibrium per-time effort $\eta_i^*(t)$ and the optimal Lagrange multiplier $\mu_e^*(\geq 0)$ satisfy the Kuhn-Tucker conditions, which are $cb\eta_i^*(t)^{b-1} - \mu_e^*\theta(t) = 0$ and $\mu_e^*\left(e_i - \int_0^T \theta(t)\eta_i^*(t)dt\right) = 0$. Thus, $\eta_i^*(t) = \left(\frac{\mu_e^*\theta(t)}{cb}\right)^{\frac{1}{b-1}}$ and $\mu_e^* = cb\left(\frac{e_i}{\tau(T)}\right)^{\frac{1}{b-1}}$, and hence $\eta_i^*(t) = e_i\theta(t)^{\frac{1}{b-1}}\tau(T)^{-1}$ and agent i's cost $\psi(e_i,T) = ce_i^b\tau(T)^{1-b}$. Therefore, since $b>1$, $\frac{\partial\psi(e_i,T)}{\partial e_i} = cbe_i^{b-1}\tau(T)^{1-b} > 0$ and $\frac{\partial^2\psi(e_i,T)}{\partial e_i^2} = cb(b-1)e_i^{b-2}\tau(T)^{1-b} > 0$. Also, $\frac{\partial\psi(e_i,T)}{\partial T} = ce_i^b(1-b)\tau(T)^{-b}\tau'(T) < 0$ since $b>1$ and $\tau'(T) = \theta(T)^{\frac{b}{b-1}} > 0$. ■

Proof of Lemma 2. From (2.5), $e^* = \left(\frac{xA}{cb}\right)^{\frac{1}{b-1}}\tau(T)$. Substituting e^* into (2.6)-(2.8) yields

$$\max_T \exp(-\delta T)\left(\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}}\tau(T) + E\left[\widetilde{\xi}_{(1)}^N\right]\right) - A, \tag{A.2}$$

$$\text{s.t. } -\frac{A}{N} + c\left(\frac{xA}{cb}\right)^{\frac{b}{b-1}}\tau(T) + F \leq 0. \tag{A.3}$$

Suppose that Π is non-monotonic. Let μ be the Lagrange multiplier of the constraint in (A.3). T^* and the optimal Lagrange multiplier $\mu^*(\geq 0)$ satisfy the following

Kuhn-Tucker conditions:

$$\exp(-\delta T^*)\left[\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}}\left(-\delta\tau(T^*)+\tau'(T^*)\right)-\delta E\left[\widetilde{\xi}^N_{(1)}\right]\right]-\mu^* c\left(\frac{xA}{cb}\right)^{\frac{b}{b-1}}\tau'(T^*)=0, \tag{A.4}$$

$$\mu^*\left(-\frac{A}{N}+c\left(\frac{xA}{cb}\right)^{\frac{b}{b-1}}\tau(T)+F\right)=0. \tag{A.5}$$

Suppose that $\mu^*>0$. From (A.5), the optimal contest duration

$$T^*=\overline{T}=\tau^{-1}\left(\frac{A-NF}{cN}\left(\frac{xA}{cb}\right)^{\frac{-b}{b-1}}\right). \tag{A.6}$$

Using this equation to simplify (A.4) yields

$$\mu^*=\exp(-\delta T^*)\left[\left(\frac{-\delta b(A-NF)}{NxA}+\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}}\tau'(T^*)\right)-\delta E\left[\widetilde{\xi}^N_{(1)}\right]\right]\times$$
$$c^{-1}\left(\frac{xA}{cb}\right)^{\frac{-b}{b-1}}(\tau'(T^*))^{-1}.$$

$\mu^*>0$, so T^* satisfies (A.6) if $\delta<\delta_1\equiv\frac{\tau'\left(\tau^{-1}\left(\frac{A-NF}{cN}\left(\frac{xA}{cb}\right)^{\frac{-b}{b-1}}\right)\right)\left(\frac{Ax}{cb}\right)^{\frac{1}{b-1}}}{\frac{b(A-NF)}{NxA}+E\left[\widetilde{\xi}^N_{(1)}\right]}$. Note that $\delta_1>0$ since $\tau'(T)=\theta(T)^{\frac{b}{b-1}}>0$ for any T, $E\left[\widetilde{\xi}^N_{(1)}\right]>0$, $F<A/N$, and $x>0$ by Lemma 6 of Appendix B.

Suppose that $\mu^*=0$. $\exp(-\delta T^*)>0$, so from (A.4), the optimal contest duration $T^*=\widehat{T}$ satisfies

$$\tau'(\widehat{T})-\delta\tau(\widehat{T})=\delta E\left[\widetilde{\xi}^N_{(1)}\right]\left(\frac{xA}{cb}\right)^{\frac{-1}{b-1}}. \tag{A.7}$$

$T^*=\widehat{T}$ should satisfy (A.3). Plugging $\tau(T^*)=\frac{\tau'(T^*)}{\delta}-E\left[\widetilde{\xi}^N_{(1)}\right]\left(\frac{xA}{cb}\right)^{\frac{-1}{b-1}}$ into (A.3) gives $\delta\geq\delta_1$. Thus, $T^*=\widehat{T}$ if $\delta\geq\delta_1$. Let $\overrightarrow{\gamma}\equiv(\gamma_{(1)},\gamma_{(2)},...,\gamma_{(N)})$ and

$$\Phi(\overrightarrow{\gamma})\equiv\frac{\tau'\left(\tau^{-1}\left(\frac{A-NF}{cN}\left(\frac{xA}{cb}\right)^{\frac{-b}{b-1}}\right)\right)\left(\frac{Ax}{cb}\right)^{\frac{1}{b-1}}}{\frac{b(A-NF)}{NxA}+E\left[\widetilde{\xi}^N_{(1)}\right]}-\delta. \tag{A.8}$$

Then, for any distribution of awards $\overrightarrow{\gamma}$, if $\Phi(\overrightarrow{\gamma})>0$, then T^* is characterized by (A.6); and if $\Phi(\overrightarrow{\gamma})\leq 0$, then T^* is characterized by (A.7) when Π is non-monotonic in T. Also, when Π is monotonic in T, T^* is always characterized by (A.6) because $\frac{\partial\Pi}{\partial T}>0$. ■

Proof of Theorem 1. **(a)** Given a scale parameter $\alpha(>0)$, we have

$$E\left[\widehat{\xi}^N_{(1)}\right] = E\left[\alpha\widetilde{\xi}^N_{(1)}\right] = \alpha E\left[\widetilde{\xi}^N_{(1)}\right], \tag{A.9}$$

$$\sum_{j=1}^{N} \widehat{I}^N_{(j)}\gamma_{(j)} = \frac{1}{\alpha}\sum_{j=1}^{N} I^N_{(j)}\gamma_{(j)} = \frac{x}{\alpha}. \tag{A.10}$$

Suppose that $\delta < \delta_1$ under a given $\overrightarrow{\gamma}$ and for some α. Then, $\Phi(\overrightarrow{\gamma}) > 0$ from (A.8). By the continuity of Φ, in a sufficiently small neighborhood of α, we still have $\Phi(\overrightarrow{\gamma}) > 0$. Thus, from (A.6), under the scale parameter α, $T^*[\alpha] = \tau^{-1}\left(\frac{A-NF}{cN}\left(\frac{xA}{cb}\right)^{\frac{-b}{b-1}}\alpha^{\frac{b}{b-1}}\right)$. Then, since $\tau'(T^*[\alpha]) = \theta(T^*[\alpha])^{\frac{b}{b-1}} > 0$, for any $\delta < \delta_1$, $\frac{\partial T^*[\alpha]}{\partial\alpha} = \frac{\frac{A-NF}{cN}\left(\frac{xA}{cb}\right)^{\frac{-b}{b-1}}\frac{b}{b-1}\alpha^{\frac{1}{b-1}}}{\tau'(T^*[\alpha])} > 0$.

(b) Suppose that $\theta(t) = \exp(\rho t)$ and $\delta < \delta_1$ under a given $\overrightarrow{\gamma}$ for some ρ. Then, the optimal contest duration $T^* = \overline{T} = \frac{b-1}{b\rho}\log\left(\frac{b\rho}{b-1}\frac{A-NF}{cN}\left(\frac{xA}{cb}\right)^{\frac{-b}{b-1}}+1\right)$ and $\frac{\partial T^*}{\partial\rho} = \frac{b-1}{b\rho^2}\left[1-\exp\left(\frac{-b\rho T^*}{b-1}\right)-\frac{b\rho T^*}{b-1}\right]$. By L'Hopital's rule, $\lim_{\rho\to 0}\frac{\partial T^*}{\partial\rho} < 0$. Let $P = \frac{b\rho}{b-1}$. Then, for any $\rho \neq 0$, $\frac{\partial T^*}{\partial\rho} < 0$ if and only if

$$\frac{b\rho T^*}{b-1} + \exp\left(\frac{-b\rho T^*}{b-1}\right) = PT^* + \exp(-PT^*) > 1. \tag{A.11}$$

For any $\rho > 0$ (i.e., $P > 0$), $\frac{\partial(PT^*+\exp(-PT^*))}{\partial P} = T^* - T^*\exp(-PT^*) > 0$. Also, when $\rho = 0$ (i.e., $P = 0$), we have $0+\exp(0) = 1$. Thus, for any $\rho > 0$ (i.e., $P > 0$), (A.11) is satisfied, so $\frac{\partial T^*}{\partial\rho} < 0$. Finally, for any $\rho < 0$ (i.e., $P < 0$), $\frac{\partial(PT^*+\exp(-PT^*))}{\partial P} = T^* - T^*\exp(-PT^*) < 0$. Since $0+\exp(0) = 1$, for any $\rho < 0$ (i.e., $P < 0$), (A.11) is satisfied, so $\frac{\partial T^*}{\partial\rho} < 0$. Thus, T^* is decreasing in ρ. ■

Proof of Proposition 1. **(a)** Suppose that for some α, $\delta > \delta_1$ under a given distribution of awards $\overrightarrow{\gamma}$. Then, $\Phi(\overrightarrow{\gamma}) < 0$ from (A.8). By the continuity of Φ, in a sufficiently small neighborhood of α, we still have $\Phi(\overrightarrow{\gamma}) < 0$. From (A.9), $E\left[\widehat{\xi}^N_{(1)}\right] = \alpha E\left[\widetilde{\xi}^N_{(1)}\right]$; and from (A.10), $\sum_{j=1}^N \widehat{I}^N_{(j)}\gamma_{(j)} = \frac{x}{\alpha}$. Then, from (A.7), T^* under α is characterized by $\theta(T^*[\alpha])^{\frac{b}{b-1}} - \delta\int_0^{T^*[\alpha]}\theta(t)^{\frac{b}{b-1}}dt = \delta\alpha E\left[\widetilde{\xi}^N_{(1)}\right]\left(\frac{xA}{\alpha cb}\right)^{\frac{-1}{b-1}}$. Let $\Omega = \theta(T^*[\alpha])^{\frac{b}{b-1}} - \delta\int_0^{T^*[\alpha]}\theta(t)^{\frac{b}{b-1}}dt - \delta E\left[\widetilde{\xi}^N_{(1)}\right]\left(\frac{xA}{\alpha cb}\right)^{\frac{-1}{b-1}} = 0$. Then, for any $\delta > \delta_1$,

$$\frac{\partial T^*[\alpha]}{\partial\alpha} = -\frac{\frac{\partial\Omega}{\partial\alpha}}{\frac{\partial\Omega}{\partial T^*[\alpha]}} = \frac{\delta\frac{b}{b-1}\alpha^{\frac{1}{b-1}}E\left[\widetilde{\xi}^N_{(1)}\right]\left(\frac{xA}{cb}\right)^{\frac{-1}{b-1}}}{\frac{b}{b-1}\theta(T^*[\alpha])^{\frac{1}{b-1}}\theta'(T^*[\alpha]) - \delta\theta(T^*[\alpha])^{\frac{b}{b-1}}} < 0$$

if and only if $\frac{\theta'(T^*[\alpha])}{\theta(T^*[\alpha])} < \frac{\delta(b-1)}{b}$. Note that this condition is satisfied when Π is non-monotonic in T and unimodal as Lemma 7 in Appendix B shows. Thus, $T^* = \widehat{T}$ decreases with α.

(b) Suppose that $\theta(t) = \exp(\rho t)$ and $\delta > \delta_1$ under a given $\overrightarrow{\gamma}$ for some ρ. Let $P = \frac{b\rho}{b-1}$. Then, $\frac{\partial T^*}{\partial \rho} > 0$ if and only if $\frac{\partial T^*}{\partial P} > 0$. From (A.7), $T^* = \widehat{T}$ can be calculated as

$$\theta(T^*)^{\frac{b}{b-1}} - \delta \int_0^{T^*} \theta(t)^{\frac{b}{b-1}} dt = \frac{\delta}{P} + \exp(PT^*)\left[1 - \frac{\delta}{P}\right] = \delta E\left[\widetilde{\xi}_{(1)}^N\right]\left(\frac{xA}{cb}\right)^{\frac{-1}{b-1}}. \tag{A.12}$$

Using implicit function theorem, $\frac{\partial T^*}{\partial P} = \frac{\frac{\delta}{[P-\delta]}(1-\exp(PT^*))-PT^*\exp(PT^*)}{P^2\exp(PT^*)}$. $\lim_{P\to 0} \frac{\partial T^*}{\partial P} > 0$ by L'Hopital's rule. Note that $\rho < \frac{\delta(b-1)}{b}$, i.e., $P - \delta < 0$, when Π is non-monotonic in T and unimodal as Lemma 7 in Appendix B shows. Thus, $\frac{\partial T^*}{\partial P} > 0$ if and only if

$$\delta(1 - \exp(PT^*)) - PT^* \exp(PT^*)(P - \delta) < 0. \tag{A.13}$$

Suppose that $\rho < 0$ (i.e., $P < 0$). From (A.12), since $\delta E\left[\widetilde{\xi}_{(1)}^N\right]\left(\frac{xA}{cb}\right)^{\frac{-1}{b-1}} > 0$, $\frac{\delta}{P} + \exp(PT^*)\left[1 - \frac{\delta}{P}\right] > 0$, which means $\exp(PT^*)(P - \delta) < -\delta$ when $P < 0$. Thus, we have

$$\begin{aligned}\delta(1 - \exp(PT^*)) - PT^* \exp(PT^*)(P - \delta) &< \delta(1 - \exp(PT^*)) + \delta PT^* \\ &= \delta(1 - \exp(PT^*) + PT^*).\end{aligned}$$

Because $1 - \exp(0) + 0 = 0$ and $\frac{\partial(1-\exp(PT^*)+PT^*)}{\partial P} = -T^*\exp(PT^*) + T^* > 0$ when $P < 0$, $\delta(1 - \exp(PT^*) + PT^*) < 0$ when $P < 0$. Thus, when $\rho < 0$ (i.e., $P < 0$), the condition in (A.13) is satisfied, and hence $\frac{\partial T^*}{\partial \rho} > 0$. Next, suppose that $\rho > 0$ (i.e., $P > 0$). From (A.12), $\frac{\delta}{P} + \exp(PT^*)\left[1 - \frac{\delta}{P}\right] > 0$, which means $\exp(PT^*)(P - \delta) > -\delta$ when $P > 0$. Thus, we have

$$\begin{aligned}\delta(1 - \exp(PT^*)) - PT^* \exp(PT^*)(P - \delta) &< \delta(1 - \exp(PT^*)) + \delta PT^* \\ &= \delta(1 - \exp(PT^*) + PT^*).\end{aligned}$$

Because $1 - \exp(0) + 0 = 0$ and $\frac{\partial(1-\exp(PT^*)+PT^*)}{\partial P} = -T^*\exp(PT^*) + T^* < 0$ when $P > 0$, $\delta(1 - \exp(PT^*) + PT^*) < 0$ when $P < 0$. Thus, when $\rho > 0$ (i.e., $P > 0$), the condition in (A.13) is satisfied, and hence $\frac{\partial T^*}{\partial \rho} > 0$. Thus, $T^* = \widehat{T}$ increases with ρ. ■

Proof of Theorem 2. **(a)** Let $\Phi(\overrightarrow{\gamma})$ be defined as in (A.8). We have two cases.

Case 1: Suppose that $\Phi(1,0,0,...,0) \leq 0$ (i.e., $\delta \geq \delta_1^{WTA}$) and Π is non-monotonic in T under the WTA award scheme. Then, by Lemma 8 of Appendix B, Π is non-monotonic in T for any $\overrightarrow{\gamma}$, and hence for any $\overrightarrow{\gamma}$ such that $\Phi \leq 0$, $T^* = \widehat{T}$, which satisfies (A.7). By Lemma 8, the WTA award scheme yields a larger Π than any $\overrightarrow{\gamma}$ such that $\Phi \leq 0$. Also, for any $\overrightarrow{\gamma}$ such that $\Phi > 0$, $T^* = \overline{T}$ as in (A.6); and for any $\overrightarrow{\gamma}$, Π under $\overline{T}$ is always less than and equal to Π under $\widehat{T}$. Also, since the WTA award scheme yields a larger Π than any $\overrightarrow{\gamma}$ under $\widehat{T}$, it yields a larger Π than any $\overrightarrow{\gamma}$ such that $T^* = \overline{T}$. Thus, the WTA award scheme is optimal.

Case 2: Suppose that $\Phi(1,0,0,...,0) \leq 0$ (i.e., $\delta \geq \delta_1^{WTA}$) and Π is monotonic under the WTA award scheme or suppose that $\Phi(1,0,0,...,0) > 0$ (i.e., $\delta < \delta_1^{WTA}$). Let $\overrightarrow{\gamma_m} \equiv (\gamma^m_{(1)}, \gamma^m_{(2)}, ..., \gamma^m_{(N)})$. Any $\overrightarrow{\gamma_1} \neq (1,0,0,...,0)$ such that $\Phi(\overrightarrow{\gamma_1}) < 0$ and Π is non-monotonic cannot be optimal because by the continuity of Φ and the continuity of $\frac{\partial \Pi}{\partial T}$, we can find $\overrightarrow{\gamma_2}$ such that $\Phi(\overrightarrow{\gamma_2}) < 0$, Π is non-monotonic, and $\sum_{j=1}^N I^N_{(j)} \gamma^2_{(j)} > \sum_{j=1}^N I^N_{(j)} \gamma^1_{(j)}$; and $\overrightarrow{\gamma_2}$ yields a larger Π by Lemma 8 of Appendix B. Thus, without loss of optimality, we restrict attention to $\overrightarrow{\gamma}$ such that $\Phi(\overrightarrow{\gamma}) < 0$ and Π is monotonic or any $\overrightarrow{\gamma}$ such that $\Phi(\overrightarrow{\gamma}) \geq 0$. Then, $T^* = \overline{T} = \tau^{-1}\left(\frac{A-NF}{cN}\left(\frac{xA}{cb}\right)^{\frac{-b}{b-1}}\right)$ from (A.6), and Π under T^* is

$$\Pi = \exp\left(-\delta\tau^{-1}\left(\frac{A-NF}{cN}\left(\frac{xA}{cb}\right)^{\frac{-b}{b-1}}\right)\right)\left(\frac{b(A-NF)}{ANx} + E\left[\widetilde{\xi}^N_{(1)}\right]\right) - A. \quad \text{(A.14)}$$

As $\overrightarrow{\gamma} = (\gamma_{(1)}, \gamma_{(2)}, ..., \gamma_{(N)})$ affects Π only through x, the first derivative of Π with respect to x

$$\frac{\partial \Pi}{\partial x} = \exp\left(-\delta\tau^{-1}\left(\frac{A-NF}{cN}\left(\frac{xA}{cb}\right)^{\frac{-b}{b-1}}\right)\right)\frac{b(A-NF)}{ANx^2}\times$$
$$\left[\delta(\tau^{-1})'\left(\frac{A-NF}{cN}\left(\frac{xA}{cb}\right)^{\frac{-b}{b-1}}\right)\frac{b}{(b-1)}\left(\frac{xA}{cb}\right)^{\frac{-1}{b-1}}\left(\frac{b(A-NF)}{ANx} + E\left[\widetilde{\xi}^N_{(1)}\right]\right) - 1\right].$$
(A.15)

Let

$$\Delta(x) \equiv \frac{b-1}{b} \frac{\tau'\left(\tau^{-1}\left(\frac{A-NF}{cN}\left(\frac{xA}{cb}\right)^{\frac{-b}{b-1}}\right)\right)\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}}}{\frac{b(A-NF)}{ANx} + E\left[\widetilde{\xi}^N_{(1)}\right]}, \tag{A.16}$$

and $\widehat{\delta}_0 \equiv \max_x \Delta(x)$. Then, for any $\delta > \widehat{\delta}_0$, $\frac{\partial \Pi}{\partial x} > 0$ for any x. Also, let $\underline{\delta}_0 \equiv \Delta(I^N_{(1)})$. Then, for any $\delta < \underline{\delta}_0$, $\left.\frac{\partial \Pi}{\partial x}\right|_{x=I^N_{(1)}} < 0$. Note that $\widehat{\delta}_0 > 0$ and $\underline{\delta}_0 > 0$ because $\tau'(T) = \theta(T)^{\frac{b}{b-1}} > 0$ for any T, $E\left[\widetilde{\xi}^N_{(1)}\right] > 0$, $F < A/N$, and $x > 0$ by Lemma 6 of Appendix B.

Suppose that $\delta > \widehat{\delta}_0$, but the WTA award scheme is not optimal. Then, under an optimal distribution of awards $(\gamma^*_{(1)},\gamma^*_{(2)},...,\gamma^*_{(N)})$, there exists $l(>1)$ such that $\gamma^*_{(l)} > 0$ and $\gamma^*_{(j)} \geq \gamma^*_{(j+1)}$ for all $j \in \{1,2,...,N-1\}$. Let $k = \max\{l|\gamma^*_{(l)} > 0\}$. Consider a perturbation where the k-th award is shifted to the winner award by keeping other awards the same, i.e., $\widehat{\gamma}_{(k)} = \gamma^*_{(k)} - \gamma^*_{(k)} = 0$ and $\widehat{\gamma}_{(1)} = \gamma^*_{(1)} + \gamma^*_{(k)}$. After the perturbation, we still have $\widehat{\gamma}_{(j)} \geq \widehat{\gamma}_{(j+1)}$ for all $j \in \{1,2,...,N-1\}$, and x increases as $I^N_{(1)}(\gamma^*_{(1)} + \gamma^*_{(k)}) + \sum_{j=2}^{k-1} I^N_{(j)}\gamma^*_{(j)} > \sum_{j=1}^{k} I^N_{(j)}\gamma^*_{(j)}$ by Lemma 6 of Appendix B. Thus, the organizer's profit Π increases after the perturbation since $\frac{\partial \Pi}{\partial x} > 0$ when $\delta > \widehat{\delta}_0$. This contradicts the optimality of $(\gamma^*_{(1)},\gamma^*_{(2)},...,\gamma^*_{(N)})$, so the WTA award scheme is optimal when $\delta > \widehat{\delta}_0$.

Suppose that $\delta < \underline{\delta}_0$, but the WTA award scheme is optimal. Consider a perturbation where $\widehat{\gamma}_{(1)} = 1-\varepsilon$, $\widehat{\gamma}_{(2)} = \varepsilon$, and $\varepsilon(>0)$ is small. As $\Phi(1,0,0,...,0) > 0$ and Φ is continuous, we can find a sufficiently small ε such that $\Phi(\widehat{\gamma}_{(1)},\widehat{\gamma}_{(2)},0,0,...,0) > 0$. After the perturbation, the change in Π is

$$\Pi^{\Delta} = \exp\left(-\delta\tau^{-1}\left(\frac{A-NF}{cN}\left(\frac{\left(I^N_{(1)}(1-\varepsilon) + I^N_{(2)}\varepsilon\right)A}{cb}\right)^{\frac{-b}{b-1}}\right)\right) \times$$
$$\left(\frac{b(A-NF)}{AN\left(I^N_{(1)}(1-\varepsilon) + I^N_{(2)}\varepsilon\right)} + E\left[\widetilde{\xi}^N_{(1)}\right]\right)$$
$$-\exp\left(-\delta\tau^{-1}\left(\frac{A-NF}{cN}\left(\frac{I^N_{(1)}A}{cb}\right)^{\frac{-b}{b-1}}\right)\right)\left(\frac{b(A-NF)}{ANI^N_{(1)}} + E\left[\widetilde{\xi}^N_{(1)}\right]\right).$$

Since $I^N_{(2)} - I^N_{(1)} < 0$ by Lemma 6 of Appendix B, $\lim_{\varepsilon\to 0} \frac{\Pi^{\Delta}}{\varepsilon} > 0$ when $\delta < \underline{\delta}_0$.

Thus, the perturbation improves Π under T^*, which contradicts the optimality of the WTA award scheme.

Combining cases 1 and 2, let $\overline{\delta}_0 \equiv \min\{\widehat{\delta}_0, \delta_1^{WTA}\}$ when Π is non-monotonic in T under the WTA award scheme, and let $\overline{\delta}_0 \equiv \widehat{\delta}_0$ when Π is monotonic in T under the WTA award scheme. If $\delta > \overline{\delta}_0$, the WTA award scheme is optimal; and if $\delta < \underline{\delta}_0$, giving multiple awards is optimal. Also, noting that the WTA award scheme maximizes x by Lemma 6, $\Delta(x)$ in (A.16) is increasing in x, and hence $\widehat{\delta}_0 = \max_x \Delta(x) = \Delta(I_{(1)}^N) = \underline{\delta}_0$ when

$$\frac{\partial \tau'\left(\tau^{-1}\left(\frac{A-NF}{cN}\left(\frac{xA}{cb}\right)^{\frac{-b}{b-1}}\right)\right)}{\partial x} = \frac{NF-A}{cNx}\left(\frac{xA}{cb}\right)^{\frac{-b}{b-1}}\frac{\theta'\left(\overline{T}\right)}{\theta\left(\overline{T}\right)} \tag{A.17}$$

is sufficiently large because $\frac{\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}}}{\frac{b(A-NF)}{ANx}+E\left[\widetilde{\xi}_{(1)}^N\right]}$ is increasing in x. Thus, there exists $M \geq 0$ such that if $\frac{\theta'(\overline{T})}{\theta(\overline{T})} \leq M$ for any x, $\overline{\delta}_0 = \frac{b-1}{b}\delta_1 = \underline{\delta}_0$, and hence $\overline{\delta}_0 = \underline{\delta}_0 = \delta_0$.

(b) Suppose that $\theta(t) = \exp(\rho t)$. Then, $\frac{\partial \delta_0}{\partial \rho} = \frac{b-1}{b}\frac{\frac{b(A-NF)}{ANI_{(1)}^N}+\left(\frac{I_{(1)}^N A}{cb}\right)^{\frac{1}{b-1}}}{\frac{b(A-NF)}{ANI_{(1)}^N}+E\left[\widetilde{\xi}_{(1)}^N\right]} > 0$, so the result follows. ■

Proof of Proposition 2. We construct an optimal distribution of awards $(\gamma_{(1)}^*, \gamma_{(2)}^*, ..., \gamma_{(N)}^*)$ that satisfies the conditions in the proposition. Suppose that $\frac{\theta'(\overline{T})}{\theta(\overline{T})} \leq M$ for any $\overrightarrow{\gamma} = (\gamma_{(1)}, \gamma_{(2)}, ..., \gamma_{(N)})$, where $M(\geq 0)$ is defined as in Theorem 2. In this case, as discussed in the proof of Theorem 2, $\Delta(x)$ in (A.16) is increasing in x, and hence $\underline{\delta}_0 = \overline{\delta}_0 = \delta_0$. For any A, the organizer chooses T^* and $x^* \equiv \sum_{j=1}^{L} I_{(j)}^N \gamma_{(j)}^*$ by solving his profit-maximization problem in (2.6)-(2.8).

Let $x(1/K) \equiv \sum_{j=1}^{K} I_{(j)}^N / K$ for any $K \in \{1, 2, ..., N\}$, let $\delta_{0,K} \equiv \Delta(x(1/K))$ for any $K \in \{1, 2, ..., N-1\}$, and let $\delta_{0,N} \equiv \max\{0, \Delta(x(1/N))\}$. Because $x(1/K)$ is decreasing in K by Lemma 6 of Appendix B and $\Delta(x)$ is increasing in x, we have $\delta_{0,K} < \delta_{0,K-1}$ for any $K \in \{2, 3, ..., N\}$. Note that $\delta_{0,1} = \delta_0 = \Delta(I_{(1)}^N)$. From (2.7), $e^* = \left(\frac{xA}{cb}\right)^{\frac{1}{b-1}}\tau(T)$, where $\tau(T)$ is increasing in T, so $\Pi = \exp(-\delta T)\left(\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}}\tau(T) + E\left[\widetilde{\xi}_{(1)}^N\right]\right) - A$. From Theorem 1, when $\delta \leq \delta_1$, $T^* = \overline{T}$ for any $\overrightarrow{\gamma}$, and hence under $(\gamma_{(1)}^*, \gamma_{(2)}^*, ..., \gamma_{(N)}^*)$, we also have $T^* = \overline{T}$. Also, from

(A.15) and (A.16), when $\delta > \delta_{0,N} = \Delta(0)$, $\frac{\partial \Pi}{\partial x}\Big|_{T=\overline{T},x=0} > 0$, and hence x^* is interior under $T = \overline{T}$. Thus, the Kuhn-Tucker conditions are necessary for optimality. Let μ be the Lagrange multiplier of (2.8). Then, the Kuhn-Tucker conditions are given by (A.4)-(A.5) evaluated at $x = x^*$ and by $\frac{A\tau(T)}{(b-1)}\left(\frac{x^*A}{cb}\right)^{\frac{1}{b-1}}\left[\frac{\exp(-\delta T^*)}{x^*A} - \mu^*\right] = 0$. Thus, we have $\mu^* = \frac{\exp(-\delta T^*)}{x^*A} > 0$, so $T^* = \overline{T} = \tau^{-1}\left(\frac{A-NF}{cN}\left(\frac{xA}{cb}\right)^{\frac{-b}{b-1}}\right)$ from (A.5). Plugging μ^* and T^* into (A.4) yields $-\delta\left(\frac{b(A-NF)}{NAx^*} + E\left[\widetilde{\xi}^N_{(1)}\right]\right) + \frac{b-1}{b}\left(\frac{x^*A}{cb}\right)^{\frac{1}{b-1}}\tau'\left(\tau^{-1}\left(\frac{A-NF}{cN}\left(\frac{x^*A}{cb}\right)^{\frac{-b}{b-1}}\right)\right) = 0$, i.e., $-\delta + \Delta(x^*) = 0$ from (A.16). When $\delta = \delta_{0,N}$, $-\delta_{0,N} + \Delta(0) = 0$, so $(\gamma^*_{(1)}, \gamma^*_{(2)}, ..., \gamma^*_{(N)}) = (\frac{1}{N}, \frac{1}{N}, ..., \frac{1}{N})$.

Suppose $\delta \in (\delta_{0,N}, \delta_0)$. Consider a perturbation where δ is increased by a sufficiently small ε_δ. Then, x^* increases by $\varepsilon_{x^*} = \Delta^{-1}(\delta + \varepsilon_\delta) - \Delta^{-1}(\delta)$. Thus, any perturbation of $\gamma^*_{(j)}$ that increases x^* by ε_{x^*} leads to an optimal distribution of awards under $\delta + \varepsilon_\delta$. Consider the perturbation where $\gamma^*_{(N)}$ changes by $\varepsilon_{(N)} = \frac{\varepsilon_{x^*}}{-\frac{1}{N-1}\sum_{j=1}^{N-1} I^N_{(j)} + I^N_{(N)}}$ and $\gamma^*_{(j)}$ changes by $\varepsilon_{(j)} = -\varepsilon_{(N)}/(N-1)$ for $j \in \{1, 2, ..., N-1\}$. This perturbation increases x^* by ε_{x^*}. Note that $-\frac{1}{L-1}\sum_{j=1}^{L-1} I^N_{(j)} + I^N_{(L)} < 0$ for any $L \in \{2, ..., N\}$ since $I^N_{(j)} \geq I^N_{(j+1)}$ for any $j \in \{1, 2, ..., N-1\}$ by Lemma 6. Thus, this perturbation reduces $\gamma^*_{(N)}$ while increasing $\gamma^*_{(j)}$ for all $j \in \{1, 2, ..., N-1\}$. When δ increases up to $\delta_{0,N-1}$, since $-\delta_{0,N-1} + \Delta(x(1/(N-1))) = 0$, $\gamma^*_{(j)} = \frac{1}{N-1}$ for all $j \in \{1, 2, ..., N-1\}$ and $\gamma^*_{(N)} = 0$. As δ increases up to δ_0, we can repeat the same process by setting $\varepsilon_{(L)} = \frac{\varepsilon_{x^*}}{-\frac{1}{L-1}\sum_{j=1}^{L-1} I^N_{(j)} + I^N_{(L)}}$ and $\varepsilon_{(j)} = -\varepsilon_{(L)}/(L-1)$ for $j \in \{1, 2, ..., L-1\}$ when there are L non-zero awards. Therefore, for any $K \in \{2, 3, ..., N-1\}$, when $\delta \in (\delta_{0,K}, \delta_{0,K-1})$, $\gamma^*_{(j)} = 0$ for all $j \in \{K+1, K+2, ..., N\}$, $\gamma^*_{(j)}$ is increasing in δ for all $j \in \{1, 2, ..., K-1\}$, and $\gamma^*_{(K)}$ is decreasing in δ. As a side note, when $M = 0$, $\frac{\theta'(\overline{T})}{\theta(\overline{T})} \leq 0$, and hence from (A.17), $\Delta(0) \leq 0$. Thus, $\delta_{0,N} = \max\{0, \Delta(0)\} = 0$. ■

Proof of Proposition 3. Suppose that $\delta = 0$. Then, $\Pi = \left(\frac{xA}{cb}\right)^{\frac{1}{b-1}}\tau(T) + E\left[\widetilde{\xi}^N_{(1)}\right] - A$. Thus, the Kuhn-Tucker conditions (where $\mu(\geq 0)$ is the Lagrange multiplier of

(A.3)) are as follows:

$$\left(\frac{xA^*}{cb}\right)^{\frac{1}{b-1}}\tau'(T^*)-\mu^* c\left(\frac{xA^*}{cb}\right)^{\frac{b}{b-1}}\tau'(T^*)=0. \tag{A.18}$$

$$\frac{1}{A^*(b-1)}\left(\frac{xA^*}{cb}\right)^{\frac{1}{b-1}}\tau(T^*)-1-\mu^*\left[-\frac{1}{N}+\frac{b}{A^*(b-1)}c\left(\frac{xA^*}{cb}\right)^{\frac{b}{b-1}}\tau(T^*)\right]=0. \tag{A.19}$$

$$\mu\left[-\frac{A^*}{N}+c\left(\frac{xA^*}{cb}\right)^{\frac{b}{b-1}}\tau(T^*)+F\right]=0. \tag{A.20}$$

From (A.18), $\mu^*=\frac{b}{xA^*}>0$, and hence from (A.20), $\tau(T^*)=\frac{A^*-NF}{Nc}\left(\frac{xA^*}{cb}\right)^{\frac{-b}{b-1}}$. Plugging μ^* and $\tau(T^*)$ into (A.19) yields $A^*=\sqrt{\frac{bF}{x}}$, and hence $T^*=\tau^{-1}\left(\frac{\sqrt{\frac{bF}{x}}-NF}{Nc}\left(\frac{1}{c}\sqrt{\frac{xF}{b}}\right)^{\frac{-b}{b-1}}\right)$. Note that $\lim_{T\to\infty}\Pi=\infty$, but T^* is bounded by the participation condition (A.3). Thus, under T^*, $\Pi=\frac{b(A-NF)}{ANx}+E\left[\widetilde{\xi}^N_{(1)}\right]-A$, and hence $\lim_{A\to\infty}\Pi=-\infty$. Therefore, Kuhn-Tucker conditions above are necessary for optimality.

(a) Under the scale parameter α, $T^*=\tau^{-1}\left(\frac{\sqrt{\frac{\alpha bF}{x}}-NF}{Nc}\left(\frac{1}{c}\sqrt{\frac{xF}{\alpha b}}\right)^{\frac{-b}{b-1}}\right)$ and $A^*=\sqrt{\frac{\alpha bF}{x}}$. Since τ^{-1} is increasing and the terms inside τ^{-1} are increasing in α, $\frac{\partial T^*}{\partial\alpha}>0$. Also, $\frac{\partial A^*}{\partial\alpha}=\frac{1}{2}\sqrt{\frac{bF}{\alpha x}}>0$.

(b) Suppose that $\theta(t)=\exp(\rho t)$. A^* is independent of ρ, so Theorem 1(b) directly follows. ■

Proof of Proposition 4. Suppose that $\delta=0$. **(a)** $\Pi=\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}}\tau(T)+E\left[\widetilde{\xi}^N_{(1)}\right]-A$. Since $\frac{\partial\tau(T)}{\partial T}>0$, we have $\frac{\partial\Pi}{\partial T}>0$ for any N, and hence $T^*=\overline{T}$ as in Lemma 2. Under $T^*=\overline{T}$, the organizer's profit

$$\Pi=\frac{b(A-NF)}{ANx}+E\left[\widetilde{\xi}^N_{(1)}\right]-A. \tag{A.21}$$

Thus, the organizer decides on N^* to maximize Π in (A.21). Let $\Pi[\alpha]$ be the organizer's profit under α. Then, N^* that maximizes $\Pi[\alpha]=\frac{\alpha b(A-NF)}{ANx}+\alpha E\left[\widetilde{\xi}^N_{(1)}\right]-A$ maximizes $\frac{\Pi[\alpha]+A}{\alpha}$, which is independent of α. Thus, N^* does not depend on α, and hence by Theorem 1, T^* increases with α.

(b) Suppose that $\theta(t)=\exp(\rho t)$. Then, Π in (A.21) does not depend on ρ. Thus,

N^* does not depend on ρ, and hence by Theorem 1, T^* decreases with ρ. ■

Proof of Proposition 5. **(a)** Let $\overline{\delta}_0^* = \max_{N\in\{2,3,\ldots,\overline{N}\}} \overline{\delta}_0[N]$, where $\overline{\delta}_0[N]$ is the threshold defined in Theorem 2(a) when there are N participants. Let $\overrightarrow{\gamma_1} \equiv (\gamma^1_{(1)}, \gamma^1_{(2)}, \ldots, \gamma^1_{(N)})$, where $\gamma^1_{(1)} < 1$; $N^{*,\overrightarrow{\gamma_1}}$ and $N^{*,WTA}$ be the optimal number of participants under $\overrightarrow{\gamma_1}$ and the WTA award scheme, respectively. When $\delta > \overline{\delta}_0^*$, $\delta \geq \overline{\delta}_0|_{N=N^{*,\overrightarrow{\gamma_1}}}$, so Π under the WTA award scheme is larger than Π under $\overrightarrow{\gamma_1}$ by Theorem 2. Since Π under the WTA award scheme and $N^{*,WTA}$ is larger than Π under the WTA award scheme and $N^{*,\overrightarrow{\gamma_1}}$, Π under the WTA award scheme and $N^{*,WTA}$ is larger than Π under $\overrightarrow{\gamma_1}$ and $N^{*,\overrightarrow{\gamma_1}}$. Thus, for any $\delta > \overline{\delta}_0^*$, the WTA award scheme is optimal when the organizer sets N^* and T^*. Next, let $\underline{\delta}_0^* = \min_{N\in\{2,\ldots,\overline{N}\}} \underline{\delta}_0[N]$, where $\underline{\delta}_0[N]$ is the threshold defined in Theorem 2(a) under N participants. When $N = N^{*,WTA}$, for any $\delta < \underline{\delta}_0^* \leq \underline{\delta}_0[N^{*,WTA}]$, there exists $\overrightarrow{\gamma_1}$ with $\gamma^1_{(1)} < 1$ such that Π under the WTA award scheme is smaller than Π under $\overrightarrow{\gamma_1}$. Since Π under $\overrightarrow{\gamma_1}$ and $N^{*,\overrightarrow{\gamma_1}}$ is larger than Π under $\overrightarrow{\gamma_1}$ and $N^{*,WTA}$, it is also larger than Π under the WTA award scheme and $N^{*,WTA}$. Thus, for any $\delta < \underline{\delta}_0^*$, giving multiple awards is optimal under N^* and T^*.

(b) Suppose $\theta(t) = \exp(\rho t)$. By Theorem 2(b), $\underline{\delta}_0[N]$ is increasing in ρ, so is $\underline{\delta}_0^*$.

(c) When $\delta < \underline{\delta}_0[N^*]$, $T^* = \overline{T}$ under N^* for any x, and hence Π is as in (A.14). Thus, we have

$$\begin{aligned}
\left.\frac{\partial \Pi}{\partial x}\right|_{N=N^*} &= \left.\frac{\partial \Pi}{\partial x}\right|_{N=N^*} + \left.\frac{\partial \Pi}{\partial N}\right|_{N=N^*} \left.\frac{\partial N}{\partial x}\right|_{N=N^*} \\
&= \exp\left(-\delta\tau^{-1}\left(\frac{A-N^*F}{cN^*}\left(\frac{xA}{cb}\right)^{\frac{-b}{b-1}}\right)\right)\frac{b(A-N^*F)}{AN^*x^2}\times \\
&\left[\delta(\tau^{-1})'\left(\frac{A-N^*F}{cN^*}\left(\frac{xA}{cb}\right)^{\frac{-b}{b-1}}\right)\frac{b}{(b-1)}\left(\frac{xA}{cb}\right)^{\frac{-1}{b-1}}\left(\frac{b(A-N^*F)}{AN^*x}+E\left[\tilde{\xi}^{N^*}_{(1)}\right]\right)-1\right].
\end{aligned}$$

Let $\Delta^*(x)$ be $\Delta(x)$ in (A.16) under N^*. Then, $\left.\frac{\partial \Pi}{\partial x}\right|_{N=N^*} > 0$ if $\delta > \Delta^*(x)$. We have $\min_N \Delta(x) \leq \Delta^*(x) \leq \max_N \Delta(x)$, $\lim_{x\to 0}\Delta(x) = \lim_{x\to 0} \frac{b-1}{b}\frac{\theta'(\overline{T})}{\theta(\overline{T})}$, and $\lim_{x\to 0}\overline{T} = \infty$ for any N. Thus, $\lim_{x\to 0}\min_N \Delta(x) = \lim_{x\to 0}\max_N \Delta(x) = \Delta(0)$, which is defined in Proposition 2. Thus, by the Squeeze Theorem, $\lim_{x\to 0}\Delta^*(x) = \Delta(0)$. Noting that $\delta_{0,N} = \max\{0, \Delta(0)\}$, when $\delta > \delta_{0,N}$, $\left.\frac{\partial \Pi}{\partial x}\right|_{N=N^*,x=0} > 0$, and hence giving multiple unequal awards is optimal. ■

Appendix B

Additional Analysis of Chapter 2

B.1 Asymmetric Pure-Strategy Nash Equilibrium

In this section, we discuss the robustness of our results when considering asymmetric pure-strategy Nash equilibria. In the following lemma, we show that when $N = 2$, any pure-strategy Nash equilibrium is symmetric, and hence all our results under symmetric Nash equilibria follow.

Lemma 1 *Let e_i^* be agent $i \in \{1,2\}$'s equilibrium effort. Then, $e_1^* = e_2^*$.*

Proof. We first suppose that $e_1^* > 0$ and $e_2^* > 0$. Then, given that agent 2 exerts her equilibrium effort e_2^*, agent 1's utility when exerting effort e_1 is

$$U(e_1,T) = \quad A(1-\gamma_{(1)}) + A(2\gamma_{(1)}-1)\int_{s\in\Xi} H(e_1 - e_2^* + s)h(s)ds - c(e_1)^b \tau(T)^{1-b} - F.$$

Evaluating the first derivative of $U_1(e_1,T)$ with respect to e_1 at $e_1 = e_1^*$ yields

$$\left.\frac{\partial U(e_1,T)}{\partial e_1}\right|_{e_1=e_1^*} = A(2\gamma_{(1)}-1)\int_{s\in\Xi} h(e_1^* - e_2^* + s)h(s)ds - cb(e_1^*)^{b-1}\tau(T)^{1-b} = 0. \tag{B.1}$$

Similarly, noting that $P(e_2 + \widetilde{\xi}_2 > e_1 + \widetilde{\xi}_1) = 1 - P(e_1 + \widetilde{\xi}_1 > e_2 + \widetilde{\xi}_2)$, given that agent 1 exerts her equilibrium effort e_1^*, agent 2's utility when exerting e_2 can be written as

$$\begin{aligned} U(e_2,T) = &A(1-\gamma_{(1)}) + A(2\gamma_{(1)}-1)\times \\ &\left[1 - \int_{s\in\Xi} H(e_1^* - e_2 + s)h(s)ds\right] - c(e_2)^b\tau(T)^{1-b} - F. \end{aligned}$$

Evaluating the first derivative of $U_2(e_2,T)$ with respect to e_2 at $e_2 = e_2^*$ gives

$$\left.\frac{\partial U_2(e_2,T)}{\partial e_2}\right|_{e_2=e_2^*} = -A(2\gamma_{(1)}-1)\int_{s\in\Xi} h(e_1^*-e_2^*+s)h(s)ds - cb(e_2^*)^{b-1}\tau(T)^{1-b} = 0. \tag{B.2}$$

From (B.1) and (B.2), agent 1's and agent 2's equilibrium efforts are

$$e_1^* = e_2^* = \left(\frac{A(2\gamma_{(1)}-1)\int_{s\in\Xi} h(e_1^*-e_2^*+s)h(s)ds}{cb}\right)^{\frac{1}{b-1}} \tau(T).$$

Thus, there does not exist an asymmetric pure-strategy Nash equilibrium where $e_1^* > 0$ and $e_2^* > 0$.

We next suppose that $e_1^* > 0$ and $e_2^* = 0$. Then, $\frac{\partial U_2(e_2,T)}{\partial e_2}\big|_{e_2=0} = -A(2\gamma_{(1)} - 1)\int_{s\in\Xi} h(e_1^*+s)h(s)ds \leq 0$. Thus, $\gamma_{(1)} = 0.5$ because $\int_{s\in\Xi} h(e_1^*+s)h(s)ds > 0$. However, when $e_2^* = 0$ and $\gamma_{(1)} = 0.5$, $\left.\frac{\partial U_1(e_1,T)}{\partial e_1}\right|_{e_1=e_1^*} = -cb(e_1^*)^{b-1}\tau(T)^{1-b} \neq 0$ for $e_1^* > 0$. Thus, by symmetry, there does not exist an asymmetric pure-strategy Nash equilibrium such that $e_1^* > 0$ and $e_2^* = 0$ or $e_2^* > 0$ and $e_1^* = 0$. ■

We next discuss the case where $N > 2$. Specifically, we are interested in whether an asymmetric equilibrium emerges when there is no symmetric one (i.e., $T > \overline{T}$) and how this asymmetric pure-strategy Nash equilibrium changes with T. For ease of illustration, we focus on the WTA award scheme. Let e_i^* be agent i's equilibrium effort. Given that all other agents $j \in \{1,2,...,N\}\setminus i$ exert their equilibrium effort e_j^*, agent i determines her effort e_i to maximize her expected utility

$$U(e_i,T) = A\int_{s\in\Xi} \prod_{j\in\{1,2,...,N\}\setminus i} H(e_i - e_j^* + s)h(s)ds - ce_i^b\tau(T)^{1-b} - F.$$

Let $I(e_i^*|e_{j\neq i}^*) \equiv \int_{s\in\Xi}\sum_{j\in\{1,2,...,N\}\setminus i} h(e_i^*-e_j^*+s)h(s)\prod_{k\in\{1,2,...,N\}\setminus\{i,j\}} H(e_i^*-e_k^*+s)ds$. Evaluating the first-derivative of $U_i(e_i,T)$ with respect to e_i at $e_i = e_i^*$ yields

$$AI(e_i^*|e_{j\neq i}^*) - cb(e_i^*)^{b-1}\tau(T)^{1-b} = 0 \text{ for all } i \in \{1,2,...,N\}. \tag{B.3}$$

In the following lemma, we show that for a sufficiently large T, the agent's participation condition is violated. Thus, consistent with our finding in §2.4, T^* is bounded even when $\delta = 0$.

Lemma 2 *There exists $\overline{T}_a$ such that when the contest duration $T > \overline{T}_a$, an agent's participation condition is violated under a solution to (B.3).*

Proof. Let $P(e_i^*|e_{j\neq i}^*) \equiv \int_{s\in\Xi} \prod_{j\in\{1,2,...,N\}\setminus i} H(e_i^* - e_j^* + s)h(s)ds$. In equilibrium, all agents choose to participate in the contest if and only if the following participation condition is satisfied:

$$AP(e_i^*|e_{j\neq i}^*) - c\left(\frac{AI(e_i^*|e_{j\neq i}^*)}{cb}\right)^{\frac{b}{b-1}} \tau(T) - F \geq 0 \text{ for all } i \in \{1,2,...,N\}. \quad \text{(B.4)}$$

Since $AP(e_i^*|e_{j\neq i}^*) \leq A$, as T approaches ∞, agent i's participation condition is violated unless $I(e_i^*|e_{j\neq i}^*)$ approaches 0 because $\tau(T)$ approaches ∞. Suppose that $\lim_{T\to\infty} I(e_i^*|e_{j\neq i}^*) = 0$. Then, there should exist e_k such that $k \neq i$ and e_k approaches ∞ as T approaches ∞. As e_k approaches ∞ and agent k's expected award is bounded by A, her participation condition is violated. So, for any solution to (B.3), the agent's participation condition is violated for a sufficiently large T. ■

An important implication of Lemma 2 is that even if an asymmetric pure-strategy Nash equilibrium emerges when $T > \overline{T}$, the agent's participation still becomes an issue as T increases. Thus, we next study whether a patient organizer has an incentive to increase the contest duration T up to $\overline{T}_a$ where the agent's participation condition binds, consistent with the effort-participation tradeoff in §2.4. As it is analytically intractable to analyze the impact of T on the organizer's profit Π under an asymmetric pure-strategy Nash equilibrium, we conduct an extensive numerical analysis. We take $\theta(t) = \theta$, and randomly generate 10,000 instances. In each instance, we randomly select parameters according to our numerical analysis setting in footnote 14 (in addition, we select N from Uniform(2,10) and θ from Uniform(0,5)). To focus on the case where there is no symmetric pure-strategy Nash equilibrium, we randomly generate T from Uniform($\overline{T}, 1.05\overline{T}$); and to focus on the case of a patient organizer, we assume that the discount factor $\delta = 0$. In each random instance, we solve (B.3) numerically. Because the symmetric equilibrium effort in (2.5) is a solution to (B.3), to prevent the numerical solver from getting stuck in this symmetric solution, we randomize the initial solutions that we feed to the solver. In 672 instances, we obtain a "valid" asymmetric solution where the sum of squared deviations of agents' first-order conditions from zero $\left(\text{i.e., } \sum_{i=1}^N \left(AI(e_i^*|e_{j\neq i}^*) - cb(e_i^*)^{b-1}\tau(T)^{1-b}\right)^2\right)$ is less than 10^{-15}. (In 250 of these instances, all agents' utilities are non-negative, so there exists an asymmetric solution that satisfies (B.3) and the participation condition (B.4).) To check if Π increases with T at each of these 672 instances, we incrementally increase T to $1.0001T$, $1.001T$, $1.01T$, and $1.1T$, and check if Π increases. We observe that in *all* of these 672 instances, Π increases with T. Thus, we conclude that under an asymmetric pure-strategy Nash equilibrium, the organizer's profit Π increases with the contest duration T when the organizer is patient, and hence by Lemma 2, the agent's participation condition drives the optimal contest duration.

B.2 Mixed-Strategy Nash Equilibrium

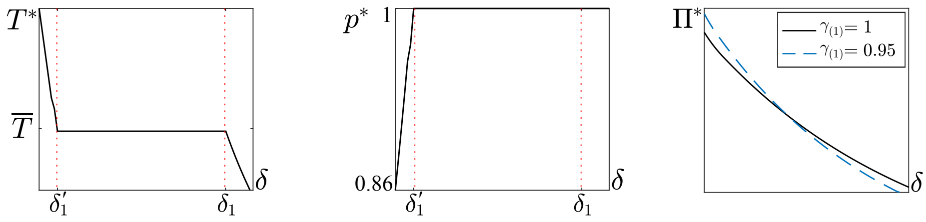
In this section, we consider the case where agents play mixed strategies. For ease of illustration and following the contest literature (e.g., Hu and Wang 2020, Mihm 2010, Seel 2018, Bimpikis et al. 2019), we assume that $N=2$. Each agent $i \in \{1,2\}$ participates in the contest with probability $p_i \in [0,1]$, and exerts effort e_i if both agents participate, and exerts zero effort otherwise. We derive the equilibrium using the best-response argument in a two-stage game. In the second stage, if both agents participate, each agent exerts e^* as in (2.5). In the first stage, given that the other agent participates in the contest with the equilibrium probability of participation p^* and both agents exert e^*, agent i decides on p_i ($\in [0,1]$) to maximize her expected utility

$$U_i(p_i) = p_i p^* \left[\frac{A}{2} - c(e^*)^b \tau(T)^{1-b} - F\right] + p_i(1-p^*)[A_{(1)} - F]. \tag{B.5}$$

The second component in (B.5) (i.e., $p_i(1-p^*)[A_{(1)}-F]$) is always non-negative, so whenever the first component (i.e., $p_i p^* \left[\frac{A}{2} - c(e^*)^b \tau(T)^{1-b} - F\right]$) is also non-negative, the best-response of the agent is to set $p^*=1$. Thus, the agent plays a non-pure strategy (i.e., $p^*<1$) only if she gets negative utility when both agents participate. The following lemma formally shows this result.

Lemma 3 *For any $A_{(1)}, A_{(2)}$, and T such that $\frac{A_{(1)}+A_{(2)}}{2} - c(e^*)^b \tau(T)^{1-b} - F \geq 0$, $p^*=1$.*

Proof. Suppose that $\frac{A_{(1)}+A_{(2)}}{2} - c(e^*)^b \tau(T)^{1-b} - F \geq 0$. When agent i chooses to



(a) T^*. (b) p^*. (c) Π^*.

Figure B.1: (a) T^* under $\gamma_{(1)} = 1$, (b) p^* under $\gamma_{(1)} = 1$, (c) Π^* under $\gamma_{(1)} = 1$ and $(\gamma_{(1)}, \gamma_{(2)}) = (0.95, 0.05)$ when agents play mixed strategies. The setting is the same as Figure 2.1.

participate in the contest, she gets a nonnegative utility. However, when the agent does not participate, she gets zero utility. Therefore, she cannot improve her utility by reducing p_i, and hence $p^* = 1$. ■

Given e^* and p^*, the organizer's profit $\Pi = \exp(-\delta T)(p^*)^2 \left(e^* + E\left[\tilde{\xi}^2_{(1)}\right]\right) - (p^*)^2(A_{(1)} + A_{(2)}) - 2p^*(1-p^*)A_{(1)}$. The following corollary extends Lemma 2.

Corollary 1 *(a) When Π is non-monotonic, $T^* = \widehat{T}$ for any $\delta \geq \delta_1$, and each agent's equilibrium probability of participation $p^* = 1$. (b) There exists $\delta_1'(\leq \delta_1)$ such that for any $\delta < \delta_1'$, $T^* > \overline{T}$ and $p^* < 1$.*

Proof. **(a)** When Π is non-monotonic, by Lemma 2, $T^* = \widehat{T}$ for any $\delta \geq \delta_1$, and hence $T^* = \widehat{T}$ as in (A.7) and $\frac{A}{2} - c(e^*)^b \tau(T^*)^{1-b} - F \geq 0$. Thus, by Lemma 3, $p^* = 1$.

(b) Suppose that $\delta < \delta_1$ or Π is monotonic. For agent $i \in \{1,2\}$, given $p_j = p_j^*$ for $j \neq i$, taking the first derivative of $U_i(p_i)$ with respect to p_i and evaluating it at $p_i = p_i^*$ yields $\frac{\partial U_i(p_i)}{\partial p_i}\Big|_{p_i = p_i^*} = p_j^* \left[\frac{A}{2} - c\left(\frac{xA}{cb}\right)^{\frac{b}{b-1}} \tau(T) - F\right] + (1-p_j^*)[A_{(1)} - F] = 0$. Thus, agent 1's and agent 2's equilibrium probabilities of participation are

$$p_1^* = p_2^* = p^* \equiv \frac{A_{(1)} - F}{A_{(1)} - A/2 + c\left(\frac{xA}{cb}\right)^{\frac{b}{b-1}} \tau(T)}. \tag{B.6}$$

When $T^* = \overline{T}$ as in (A.6), $p^* = 1$ from (B.6), and since p^* decreases with T, $p^* < 1$ if only if $T^* > \overline{T}$.

Since $\lim_{T\to\infty} p^* = 0$, $\lim_{T\to\infty} \Pi = 0$. Thus, $T^*(> \overline{T})$ is interior, and hence $\frac{\partial \Pi}{\partial T}\Big|_{T=T^*} = 0$ is necessary for optimality. The first derivative of the organizer's profit Π with respect to T is

$$\frac{\partial \Pi}{\partial T} = \exp(-\delta T)\left[\left(2p^*\frac{\partial p^*}{\partial T} - \delta(p^*)^2\right)\left(e^* + E\left[\widetilde{\xi}^N_{(1)}\right]\right) + (p^*)^2\frac{\partial e^*}{\partial T}\right] + \frac{\partial p^*}{\partial T}\left[A_{(1)}(2p^* - 2) - 2p^*A_{(2)}\right].$$

Under $\overline{T}$ in (A.6), $p^* = 1$. Thus, a sufficient condition for $p^* < 1$ is that $\frac{\partial \Pi}{\partial T}\big|_{T=\overline{T}} > 0$. We have

$$\left.\frac{\partial \Pi}{\partial T}\right|_{T=\overline{T}} = \exp(-\delta T)\left[\left(2\left.\frac{\partial p^*}{\partial T}\right|_{T=\overline{T}} - \delta\right)\left(e^* + E\left[\widetilde{\xi}^N_{(1)}\right]\right) + \left.\frac{\partial e^*}{\partial T}\right|_{T=\overline{T}}\right] - 2\left.\frac{\partial p^*}{\partial T}\right|_{T=\overline{T}} A_{(2)} > 0$$

if $\left(2\left.\frac{\partial p^*}{\partial T}\right|_{T=\overline{T}} - \delta\right)\left(e^* + E\left[\widetilde{\xi}^N_{(1)}\right]\right) + \left.\frac{\partial e^*}{\partial T}\right|_{T=\overline{T}} \geq 0$, i.e., $\delta \leq \frac{2\left.\frac{\partial p^*}{\partial T}\right|_{T=\overline{T}}\left(e^*+E\left[\widetilde{\xi}^N_{(1)}\right]\right) + \left.\frac{\partial e^*}{\partial T}\right|_{T=\overline{T}}}{e^*+E\left[\widetilde{\xi}^N_{(1)}\right]}$ since $\left.\frac{\partial p^*}{\partial T}\right|_{T=\overline{T}} < 0$. Thus, for any $\delta < \delta_1'$, $\frac{\partial \Pi}{\partial T}\big|_{T^*=\overline{T}} > 0$, and hence $T^* > \overline{T}$ and $p^* < 1$. ■

T being larger than $\overline{T}$ has the following opposing effects on Π. It improves Π by increasing e^*, but it reduces Π by decreasing p^* and discounting the organizer's payoff more. When $\delta = 0$, the organizer still limits T to balance the positive effect of a larger e^* and the negative effect of a smaller p^*. Thus, the effort-participation tradeoff we identify in §2.4 persists when agents play mixed strategies, and this tradeoff drives T^* for the patient organizer. Supplementary to Corollary 1, Figures B.1(a) and B.1(b) illustrate that for $\delta \in [\delta_1', \delta_1]$, $T^* = \overline{T}$ and $p^* = 1$ because negative effects of a smaller p^* and more discounting outweigh the positive effect of a larger e^*.

We next discuss the robustness of Theorem 1. When $T^* = \overline{T}$ and $p^* = 1$, Theorem 1 directly applies. The following corollary extends Theorem 1(a) to the case where $T^* > \overline{T}$ and $p^* < 1$. To analyze a patient organizer while retaining analytical tractability, we assume that $\delta = 0$ and $\theta(t) = \theta$ as in the innovation-contest literature (e.g., Hu and Wang 2020, Mihm 2010).

Corollary 2 *There exists $\underline{\alpha}$ such that T^* is increasing in $\alpha > \underline{\alpha}$.*

Proof. Suppose that $\delta = 0$ and $\theta(t) = \theta$. By Corollary 1, $p^* < 1$, and under the scale parameter α, the optimal contest duration $T^*[\alpha]$ that solves $\frac{\partial \Pi}{\partial T}\big|_{T=T^*[\alpha]} = 0$ is

$$T^*[\alpha] = \frac{\left(\frac{\alpha(A_{(1)}-F)\theta b}{xA} + 2A_{(1)}\right)\left(A_{(1)} - A/2\right) - 4\left(A_{(1)} - F\right)A_{(1)} - 2\left(A_{(1)} - F\right)\left(\alpha E\left[\widetilde{\xi}^N_{(1)}\right] - A\right)}{\alpha^{\frac{-1}{b-1}} 2\left(A_{(1)} - F\right)\theta\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}} - \left(\alpha^{\frac{-1}{b-1}}\frac{(A_{(1)}-F)\theta b}{xA} + 2A_{(1)}\alpha^{\frac{-b}{b-1}}\right)c\left(\frac{xA}{cb}\right)^{\frac{b}{b-1}}}.$$

The first derivative of $T^*[\alpha]$ with respect to α is

$$\begin{aligned}\frac{\partial T^*[\alpha]}{\partial \alpha} &= \frac{\left(\frac{(A_{(1)}-F)\theta b}{xA}\right)\left(A_{(1)} - A/2\right) - 2\left(A_{(1)} - F\right)E\left[\widetilde{\xi}^N_{(1)}\right]}{\alpha^{\frac{-1}{b-1}} 2\left(A_{(1)} - F\right)\theta\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}} - \left(\frac{\alpha(A_{(1)}-F)\theta b}{xA} + 2A_{(1)}\right)c\alpha^{\frac{-b}{b-1}}\left(\frac{xA}{cb}\right)^{\frac{b}{b-1}}} \\ &+ \frac{1}{(b-1)}\frac{\left(\frac{(A_{(1)}-F)\theta b}{xA}\right)\left(A_{(1)} - A/2\right) - 2\left(A_{(1)} - F\right)E\left[\widetilde{\xi}^N_{(1)}\right]}{\alpha^{\frac{-1}{b-1}} 2\left(A_{(1)} - F\right)\theta\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}} - \left(\frac{\alpha(A_{(1)}-F)\theta b}{xA} + 2A_{(1)}\right)c\alpha^{\frac{-b}{b-1}}\left(\frac{xA}{cb}\right)^{\frac{b}{b-1}}} \\ &+ \frac{\left(\frac{(A_{(1)}-F)\theta b}{xA}\right)\left(A_{(1)} - A/2\right) - 2\left(A_{(1)} - F\right)E\left[\widetilde{\xi}^N_{(1)}\right]}{\left(\alpha^{\frac{-1}{b-1}} 2\left(A_{(1)} - F\right)\theta\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}} - \left(\frac{\alpha(A_{(1)}-F)\theta b}{xA} + 2A_{(1)}\right)c\alpha^{\frac{-b}{b-1}}\left(\frac{xA}{cb}\right)^{\frac{b}{b-1}}\right)^2} \times \\ &\left[-\left(2A_{(1)}\alpha^{\frac{-b}{b-1}}\right)c\left(\frac{xA}{cb}\right)^{\frac{b}{b-1}}\right].\end{aligned}$$

$\lim_{\alpha\to\infty}\frac{\partial T^*[\alpha]}{\partial \alpha} > 0$, so by continuity, there exists $\underline{\alpha}$ such that T^* is increasing in any $\alpha > \underline{\alpha}$. ∎

We next discuss how our results about the award scheme extend to the case where agents play mixed strategies. From Corollary 1(a) and Figure B.1(b), we can also deduce that when $\delta > \delta_1'$, $p^* = 1$, and hence Theorem 2(a) directly apply. To analyze the case where $p^* < 1$, we conduct an extensive numerical analysis. We show that for a sufficiently small δ, the WTA award scheme is not optimal.[1] For instance, Figure B.1(c) illustrates that up to some threshold on δ, Π^* is larger under the award scheme $(0.95A, 0.05A)$ than Π^* under the WTA award scheme. The intuition is similar to Theorem 2(a). Specifically, offering multiple awards increases p^*, and hence allows the organizer to set a longer T to elicit a larger expected effort from agents. Opposed to this positive effect, a longer T also leads to more discounting.

[1] We take $\theta(t) = \exp(\rho t)$, and randomly generate 10,000 instances where $p^* < 1$. In each instance, we select parameters according to our numerical analysis setting in footnote 14 (and we select δ from Uniform(0,0.0001)). We observe that in all instances, Π is larger under the award scheme $(0.95A, 0.05A)$ than Π under the WTA award scheme.

When δ is small, the former positive effect dominates the latter negative effect, so offering multiple awards is optimal, as in Theorem 2(a). Also, from Figures B.1(b) and B.1(c), we can deduce that as δ decreases, it first becomes optimal to give multiple awards, and then as δ keeps decreasing, it becomes optimal to set $T^* > \overline{T}$ such that agents play non-pure strategies. Thus, $\underline{\delta}_0$, below which giving multiple awards is optimal, does not change when agents can play mixed strategies, and hence even giving multiple awards is more likely to be optimal as ρ increases, as in Theorem 2(b). Finally, in our numerical analysis, we also observe that when giving multiple awards is optimal, giving unequal awards is almost always better than giving equal awards, as in Proposition 2.

B.3 Existence of Pure-Strategy Nash Equilibrium

In this section, we provide sufficient conditions for e^* in (2.5) to be a pure-strategy Nash equilibrium under $\overline{T}$. We first show sufficient conditions for an interim property in the following lemma, and then use this property in the main result of this section.

Lemma 4 *Suppose that* $\left.\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2}\right|_{e_i=\underline{e}} < 0$ *for some* $\underline{e}$ *and* $b > 2$*. For any* $e_i > \underline{e}$*, when* F *is sufficiently large or when* α *is sufficiently small, we have* $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2} < 0$.

Proof. Under the scale parameter α, given that all other agents exert equilibrium efforts e^*, from (2.8) and $\overline{T}$ in (A.6), the second derivative of agent i's utility $U_i(e_i,\overline{T})$ with respect to e_i is

$$\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2} = \sum_{j=1}^{N} \frac{\partial^2 P_{(j)}^N[e_i,e^*]}{\partial e_i^2}\left(\frac{A_{(j)}}{\alpha^2}\right) - cb(b-1)e_i^{b-2}\left(\frac{A-NF}{cN}\right)^{1-b}\left(\frac{xA}{\alpha cb}\right)^b.$$

Suppose that $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2} < 0$ for some $e_i = \underline{e}$, but there exists $\widehat{e} > \underline{e}$ such that $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2} \geq 0$ for $e_i = \widehat{e}$. Then, $\left.\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2}\right|_{e_i=\widehat{e}} > \left.\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2}\right|_{e_i=\underline{e}}$, i.e.,

$$\sum_{j=1}^{N}\left[\left.\frac{\partial^2 P_{(j)}^N[e_i,e^*]}{\partial(e_i)^2}\right|_{e_i=\widehat{e}} - \left.\frac{\partial^2 P_{(j)}^N[e_i,e^*]}{\partial(e_i)^2}\right|_{e_i=\underline{e}}\right]\frac{A_{(j)}}{\alpha^2}$$
$$> b(b-1)\left[(\widehat{e})^{b-2} - (\underline{e})^{b-2}\right]\left(\frac{A-NF}{N}\right)^{1-b}\left(\frac{xA}{\alpha b}\right)^b. \tag{B.7}$$

Suppose that $b > 2$. Since $\widehat{e} > \underline{e}$, the right-hand side of (B.7) approaches ∞ as F approaches A/N. Thus, when F is sufficiently large, (B.7) cannot be satisfied. Also, as α approaches 0, the right-hand side of (B.7) approaches ∞ faster than the left-hand side of (B.7) (when the left-hand side of (B.7) is positive). Thus, regardless of the sign of the left-hand side of (B.7), when α is sufficiently small, (B.7) cannot be satisfied. Therefore, for any $e_i > \underline{e}$, when F is sufficiently large or when α is sufficiently small, we have $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2} < 0$. ■

The following lemma shows that when the property in Lemma 4 holds for any $\underline{e}$, e^* in (2.5) is a pure-strategy Nash equilibrium under $\overline{T}$.

Lemma 5 *Suppose that for all $\underline{e}$ such that $\left.\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2}\right|_{e_i=\underline{e}} < 0$, we have $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2} < 0$ when $e_i > \underline{e}$. Then, $U_i(e_i,\overline{T})$ is pseudo concave. Thus, e^* in (2.5) is a pure-strategy Nash equilibrium under $\overline{T}$.*

Proof. Suppose that for all $\underline{e}$ such that $\left.\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2}\right|_{e_i=\underline{e}} < 0$, we have $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2} < 0$ when $e_i > \underline{e}$. First, we have $\left.\frac{\partial U_i(e_i,\overline{T})}{\partial e_i}\right|_{e_i=0} = \sum_{j=1}^N \left.\frac{\partial P_{(j)}^N[e_i,e^*]}{\partial e_i} A_{(j)}\right|_{e_i=0} > 0$ and $\lim_{e_i\to\infty} U_i(e_i,\overline{T}) = -\infty$, so there should exist some e_i such that $\frac{\partial U_i(e_i,\overline{T})}{\partial e_i} < 0$ and $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2} < 0$. So, there exists a threshold $e_0 (\geq 0)$ such that for any $e_i < e_0$, $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2} \geq 0$; and for any $e_i > e_0$, $\frac{\partial^2 U_i(e_i,\overline{T})}{\partial e_i^2} < 0$. So, we should have $\frac{\partial U_i(e_i,\overline{T})}{\partial e_i} > 0$ for any $e_i < e_0$, and there should exist another threshold $e_{00} (> e_0)$ such that for any $e_i < e_{00}$, $\frac{\partial U_i(e_i,\overline{T})}{\partial e_i} > 0$; and for any $e_i > e_{00}$, $\frac{\partial U_i(e_i,\overline{T})}{\partial e_i} < 0$. Thus, $U_i(e_i,\overline{T})$ is unimodal with mode e_{00}, and has a unique critical (maximum) point, so it is pseudo concave. Therefore, the first-order condition of the agent's utility-maximization problem in (2.7) is sufficient for optimality. Since e^* in (2.5) satisfies this first-order condition, e^* is the solution to the agent's utility-maximization problem in (2.7). As e^* under $\overline{T}$ also satisfies (2.8), e^* is a pure-strategy Nash equilibrium under $\overline{T}$. ■

B.4 Additional Results

Lemma 6 *$I_{(j)}^N \geq I_{(j+1)}^N$ for any $j \in \{1,2,...,N-1\}$. Furthermore, $\sum_{j=1}^N I_{(j)}^N \gamma_{(j)} \geq 0$ under any distribution of awards $(\gamma_{(1)},\gamma_{(2)},...,\gamma_{(N)})$ such that $\gamma_{(1)} \geq \gamma_{(2)} \geq ... \geq \gamma_{(N)}$.*

Proof. Let $W_{(j)}^N(s) = \frac{(N-1)!}{(N-j)!(j-1)!} H(s)^{N-j}(1-H(s))^{j-1}$. From (2.4), integration by parts yields

$$I_{(j)}^N = \theta \int_{s\in\Xi} \left(W_{(j)}^N\right)'(s)h(s)ds = \theta \lim_{s\to\bar{s}} W_{(j)}^N(s)h(s) - \int_{s\in\Xi} W_{(j)}^N(s)h'(s)ds.$$

$h_{(j)}^N(s) = \frac{N!}{(N-j)!(j-1)!}(1-H(s))^{j-1}H(s)^{N-j}h(s)$, so $W_{(j)}^N(s) = \frac{h_{(j)}^N(s)}{Nh(s)}$. Letting $w_j \equiv \lim_{s\to\bar{s}} \frac{h_{(j)}^N(s)}{N}$, we have $I_{(j)}^N - I_{(j+1)}^N = (w_j - w_{j+1}) + \frac{1}{N}\int_{s\in\Xi}\left[h_{(j+1)}^N(s) - h_{(j)}^N(s)\right]\frac{h'(s)}{h(s)}ds$, $\forall j \in \{1,2,...,N-1\}$.

Noting that $w_1 \geq 0$ and $w_j = 0$ for any $j \in \{2,3,...,N\}$, integration by parts yields

$$I_{(j)}^N - I_{(j+1)}^N \geq \frac{1}{N}\left(\lim_{s\to\bar{s}}\left[H_{(j+1)}^N(s) - H_{(j)}^N(s)\right]\frac{h'(s)}{h(s)} - \int_{s\in\Xi}\left[H_{(j+1)}^N(s) - H_{(j)}^N(s)\right]\left(\frac{h'(s)}{h(s)}\right)' ds\right),$$

for all $j \in \{1,2,...,N-1\}$. Because h is log-concave, $\lim_{s\to\bar{s}}\left[H_{(j+1)}^N(s) - H_{(j)}^N(s)\right]\frac{h'(s)}{h(s)} = 0$ and $\left(\frac{h'(s)}{h(s)}\right)' \leq 0$. Also, $H_{(j+1)}^N(s) - H_{(j)}^N(s) \geq 0$ since $\widetilde{\xi}_{(j)}^N$ first-order stochastically dominates $\widetilde{\xi}_{(j+1)}^N$ for any $j \in \{1,2,...,N-1\}$. Thus, $I_{(j)}^N - I_{(j+1)}^N \geq 0$ for any $j \in \{1,2,...,N-1\}$. Let $k = \max\{j | I_{(j)}^N \geq 0\}$. Because $I_{(j)}^N - I_{(j+1)}^N \geq 0$ for any $j \in \{1,2,...,N-1\}$, we have

$$\sum_{j=1}^N I_{(j)}^N \gamma_{(j)} \geq \sum_{j=1}^k I_{(j)}^N \gamma_{(k)} + \sum_{j=k+1}^N I_{(j)}^N \gamma_{(k)} = \gamma_{(k)} \sum_{j=1}^N I_{(j)}^N = \gamma_{(k)} \sum_{j=1}^N \left.\frac{\partial P_{(j)}^N[e_i, e^*]}{\partial e_i}\right|_{e_i=e^*} = 0. \blacksquare$$

Lemma 7 *When Π is non-monotonic in T and unimodal, $\frac{\theta'(\widehat{T})}{\theta(\widehat{T})} < \frac{\delta(b-1)}{b}$.*

Proof. When Π is non-monotonic in T and unimodal, Π is unimodal with mode $T^* = \widehat{T}$ by Lemma 2, and hence $\frac{\partial \Pi}{\partial T} < 0$ when $T > \widehat{T}$. This is possible only when $\left.\frac{\partial^2 \Pi}{\partial T^2}\right|_{T=\widehat{T}} < 0$ since $\left.\frac{\partial \Pi}{\partial T}\right|_{T=\widehat{T}} = 0$. Thus, we should have $\left.\frac{\partial^2 \Pi}{\partial T^2}\right|_{T=\widehat{T}} < 0$. The second derivative of Π with respect to T is

$$\frac{\partial^2 \Pi}{\partial T^2} = \exp(-\delta T)\times$$
$$\left(-\delta\left[\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}}(-\delta\tau(T) + \tau'(T)) - \delta E\left[\widetilde{\xi}_{(1)}^N\right]\right] + \left[\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}}(-\delta\tau'(T) + \tau''(T))\right]\right),$$

and hence $\left.\frac{\partial^2 \Pi}{\partial T^2}\right|_{T=\widehat{T}} = \exp(-\delta T)\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}}(-\delta\tau'(T) + \tau''(T))$. Thus, $\left.\frac{\partial^2 \Pi}{\partial T^2}\right|_{T=\widehat{T}} < 0$

if and only if $-\delta\tau'(T)+\tau''(T)<0$, i.e., $\frac{\theta'(\widehat{T})}{\theta(\widehat{T})}<\frac{\delta(b-1)}{b}$. When $\theta(t)=\exp(\rho t)$, this condition becomes $\rho<\frac{\delta(b-1)}{b}$. ■

Lemma 8 *Let $\overrightarrow{\gamma_m}=(\gamma^m_{(1)},\gamma^m_{(2)},\dots,\gamma^m_{(N)})$, and $\overrightarrow{\gamma_1}$ and $\overrightarrow{\gamma_2}$ be such that $\sum_{j=1}^N I^N_{(j)}\gamma^1_{(j)}<\sum_{j=1}^N I^N_{(j)}\gamma^2_{(j)}$, $\Phi\leq 0$ under $\overrightarrow{\gamma_1}$ and $\overrightarrow{\gamma_2}$, and Π is non-monotonic in T under $\overrightarrow{\gamma_2}$. Then, Π is also non-monotonic in T under $\overrightarrow{\gamma_1}$, and Π under T^* is smaller under $\overrightarrow{\gamma_1}$ than that under $\overrightarrow{\gamma_2}$.*

Proof. Since Π is non-monotonic in T under $\overrightarrow{\gamma_2}$, there exists some $T=\dot{T}$ such that $\left.\frac{\partial\Pi}{\partial T}\right|_{T=\dot{T}}=\exp(-\delta\dot{T})\left[\left(\frac{xA}{cb}\right)^{\frac{1}{b-1}}\left(-\delta\tau(\dot{T})+\tau'(\dot{T})\right)-\delta E\left[\widetilde{\xi}^N_{(1)}\right]\right]<0$. Because x under $\overrightarrow{\gamma_1}$ is smaller than x under $\overrightarrow{\gamma_2}$, $\left.\frac{\partial\Pi}{\partial T}\right|_{T=\dot{T}}<0$ under $\overrightarrow{\gamma_1}$, and hence, Π is also non-monotonic in T under $\overrightarrow{\gamma_1}$. Thus, given $\overrightarrow{\gamma_1}$ or $\overrightarrow{\gamma_2}$, $T^*=\widehat{T}$. Noting that $\left.\frac{\partial\Pi}{\partial T}\right|_{T=\widehat{T}}=0$, $\left.\frac{\partial\Pi}{\partial x}\right|_{T=\widehat{T}}=\left.\frac{\partial\Pi}{\partial x}\right|_{T=\widehat{T}}+\left.\frac{\partial\Pi}{\partial T}\right|_{T=\widehat{T}}\left.\frac{\partial T}{\partial x}\right|_{T=\widehat{T}}=\exp(-\delta\widehat{T})\left(\left(\frac{A}{cb}\right)^{\frac{1}{b-1}}\tau(\widehat{T})\frac{x^{\frac{2-b}{b-1}}}{b-1}\right)>0$. Then, since x under $\overrightarrow{\gamma_1}$ is smaller than x under $\overrightarrow{\gamma_2}$, Π under $T^*=\widehat{T}$ is smaller given $\overrightarrow{\gamma_1}$ than Π given $\overrightarrow{\gamma_2}$. ■

Appendix C

Proofs of Chapter 3

Proof of Proposition 1. **(a)** We first show that $e^{*,\tau} < e^*$. From (3.4) and (3.11), $e^{*,\tau} < e^*$ if

$$\frac{L_N^\tau}{n} = \frac{N-n}{n(N-1)} \int_{-\infty}^{\infty} (N-1) G(s)^{N-2} g(s)^2 ds < L_N = \int_{-\infty}^{\infty} (N-1) H(s)^{N-2} h(s)^2 ds.$$

Recall that ξ_i follows Normal$(0, \sigma^2)$, where $h(s)$ and $H(s)$ are ξ_i's density function and distribution function, respectively; and ξ_{ti}^τ follows Normal$(\mu_B, \sigma^2 + \sigma_B^2)$, where $g(s)$ and $G(s)$ are ξ_{ti}^τ's density function and distribution function, respectively. Let $z = s/\sigma$. Then, we have

$$\begin{aligned} L_N &= \int_{-\infty}^{\infty} (N-1) H(s)^{N-2} h(s)^2 ds = \int_{-\infty}^{\infty} h(s) dH(s)^{N-1} \\ &= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-s^2/(2\sigma^2)\right) dH(s)^{N-1} = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-z^2/2\right) dH(\sigma z)^{N-1}. \end{aligned}$$

Let $F(z)$ be the distribution function of the standard normal distribution and $I_N \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-z^2/2)\right) dF(z)^{N-1}$ for any N. Noting that $H(\sigma z) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{\sigma z}{\sigma\sqrt{2}}\right)\right] = \frac{1}{2}\left[1 + \text{erf}\left(\frac{z}{\sqrt{2}}\right)\right] = F(z)$, we have $L_N = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-z^2/2)\right) dF(z)^{N-1} = \frac{I_N}{\sigma}$. Similarly, letting $w \equiv \frac{s-\mu_B}{\sqrt{\sigma^2+\sigma_B^2}}$, we have

$$\begin{aligned} \frac{L_N^\tau}{n} &= \left(\frac{N-n}{n(N-1)}\right) \frac{1}{\sqrt{\sigma^2+\sigma_B^2}\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\left((s-\mu_B)^2/(2\left(\sqrt{\sigma^2+\sigma_B^2}\right)^2\right)\right) dG(s)^{N-1} \\ &= \left(\frac{N-n}{n(N-1)}\right) \frac{1}{\sqrt{\sigma^2+\sigma_B^2}\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-w^2/2)\right) dF(w)^{N-1} = \left(\frac{N-n}{n(N-1)}\right) \frac{I_N}{\sqrt{\sigma^2+\sigma_B^2}}. \end{aligned}$$

Since $\frac{N-n}{n(N-1)} < 1$ and $\sqrt{\sigma^2+\sigma_B^2} < \sigma$, $\frac{L_N^\tau}{n} = \left(\frac{N-n}{n(N-1)}\right)\frac{I_N}{\sqrt{\sigma^2+\sigma_B^2}} < L_N = \frac{I_N}{\sigma}$, and hence $e^{*,\tau} < e^*$.

We next show that $(e^{*,\tau} - e^*)$ is decreasing in σ_B and constant in μ_B. From (3.4) and (3.11),

$$e^{*,\tau} - e^* = \left(\frac{AI_N}{cb}\right)^{\frac{1}{b-1}} \left(\left(\frac{N-n}{n(N-1)}\right)^{\frac{1}{b-1}} \frac{1}{(\sqrt{\sigma^2+\sigma_B^2})^{\frac{1}{b-1}}} - \frac{1}{\sigma^{\frac{1}{b-1}}}\right). \quad \text{(C.1)}$$

Thus, $(e^{*,\tau} - e^*)$ is decreasing in σ_B and constant in μ_B.

(b) We first show that $m_N^\tau > m_N$. Letting $z = s/\sigma$ and E_N be the expected value of the maximum of N independent standard normal random variables, we have

$$\begin{aligned} m_N &= \int_{-\infty}^{\infty} sH(s)^{N-1}h(s)ds = \int_{-\infty}^{\infty} \sigma z H(\sigma z)^{N-1}h(\sigma z)d(\sigma z) \\ &= \sigma^2 \int_{-\infty}^{\infty} zH(\sigma z)^{N-1}h(\sigma z)dz = \sigma^2 \int_{-\infty}^{\infty} zF(z)^{N-1}\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-z^2/2\right)dz = \sigma E_N. \end{aligned}$$

Similarly, $m_N^\tau = \int_{-\infty}^{\infty} sG(s)^{N-1}g(s)ds = \mu_B + \sqrt{\sigma^2+\sigma_B^2}E_N$. Since $\sigma_B > 0$ and $\mu_B \geq 0$, $m_N^\tau > m_N$. Furthermore, we have

$$m_N^\tau - m_N = \mu_B + E_N\left(\sqrt{\sigma^2+\sigma_B^2} - \sigma\right), \quad \text{(C.2)}$$

which is increasing in σ_B and μ_B. ■

Proof of Theorem 1. From (3.21), (C.1), and (C.2), we have

$$\begin{aligned} \Pi^{*,\tau} - \Pi^* = &\left(\frac{AI_N}{cb}\right)^{\frac{1}{b-1}} \left(\left(\frac{N-n}{n(N-1)}\right)^{\frac{1}{b-1}} \frac{1}{(\sqrt{\sigma^2+\sigma_B^2})^{\frac{1}{b-1}}} - \frac{1}{\sigma^{\frac{1}{b-1}}}\right) + \mu_B \\ &+ E_N\left(\sqrt{\sigma^2+\sigma_B^2} - \sigma\right). \end{aligned}$$

(a) Suppose $\mu_B = 0$. Then, we have $\lim_{\sigma_B\to\infty}(\Pi^{*,\tau} - \Pi^*) = \infty$, and hence there exists $\overline{\sigma_B}$ (>0) such that when $\sigma_B > \overline{\sigma_B}$, $\Pi^{*,\tau} > \Pi^*$. Also, $\lim_{\sigma_B\to 0}(\Pi^{*,\tau} - \Pi^*) = \left(\frac{AI_N}{cb}\right)^{\frac{1}{b-1}}\left(\left(\frac{N-n}{n(N-1)}\right)^{\frac{1}{b-1}} - 1\right)\sigma^{\frac{-1}{b-1}} < 0$ because $N-n < nN-n$. Thus, there exists $\underline{\sigma_B}$ (>0) such that when $\sigma_B < \underline{\sigma_B}$, $\Pi^{*,\tau} < \Pi^*$.

(b) Suppose $\sigma_B \to 0$. Then, $\Pi^{*,\tau} - \Pi^* = \left(\frac{AI_N}{cb}\right)^{\frac{1}{b-1}}\left(\left(\frac{N-n}{n(N-1)}\right)^{\frac{1}{b-1}} - 1\right)\sigma^{\frac{-1}{b-1}} + \mu_B$. The first term is negative since $N-n < nN-n$. Thus, we have $\lim_{\mu_B\to 0}(\Pi^{*,\tau} - \Pi^*) < 0$, $\lim_{\mu_B\to\infty}(\Pi^{*,\tau} - \Pi^*) = \infty$, and $\frac{\partial(\Pi^{*,\tau}-\Pi^*)}{\partial\mu_B} = 1 > 0$. Therefore, there exists $\overline{\mu_B}$ (>0) such that if $\mu_B \geq \overline{\mu_B}$, $\Pi^{*,\tau} \geq \Pi^*$; otherwise, $\Pi^{*,\tau} < \Pi^*$. Also, we have

$\lim_{\sigma\to 0}(\Pi^{*,\tau}-\Pi^{*}) = -\infty$, $\lim_{\sigma\to\infty}(\Pi^{*,\tau}-\Pi^{*}) = \mu_B \geq 0$, and

$$\frac{\partial(\Pi^{*,\tau}-\Pi^{*})}{\partial\sigma} = \frac{1}{b-1}\sigma^{\frac{-b}{b-1}}\left(\frac{AI_N}{cb}\right)^{\frac{1}{b-1}}\left(1-\left(\frac{N-n}{n(N-1)}\right)^{\frac{1}{b-1}}\right) > 0.$$

Thus, there exists $\overline{\sigma}$ (>0) such that if $\sigma \geq \overline{\sigma}$, $\Pi^{*,\tau} \geq \Pi^{*}$; otherwise, $\Pi^{*,\tau} < \Pi^{*}$. Therefore, if $\sigma \geq \overline{\sigma}$ or $\mu_B \geq \overline{\mu_B}$, $\Pi^{*,\tau} \geq \Pi^{*}$, otherwise, $\Pi^{*,\tau} < \Pi^{*}$. ■

Proof of Corollary 1. This result is directly follows from Proposition 1(a). ■

Proof of Proposition 2. **(a)** From (3.4) and (3.18), we have

$$ne^{*,\tau} - e^{*} = \left(\frac{A}{cb}\right)^{\frac{1}{b-1}}\left[n\left(\frac{1}{n}\left(\frac{N}{n}-1\right)\int_{-\infty}^{\infty} G(s)^{\frac{N}{n}-2}g(s)^2 ds\right)^{\frac{1}{b-1}} - \left(\int_{-\infty}^{\infty}(N-1)H(s)^{N-2}h(s)^2 ds\right)^{\frac{1}{b-1}}\right].$$

Recall that ξ_i follows Normal$(0,\sigma^2)$, where $h(s)$ and $H(s)$ are ξ_i's density function and distribution function, respectively; ξ_t^{τ} follows Normal$(\mu_B, \sigma^2+\sigma_B^2)$, where $g(s)$ and $G(s)$ are ξ_t^{τ}'s density function and distribution function, respectively; and $I_N = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp\left(-z^2/2\right)\right) dF(z)^{N-1}$ for any N. Following the approach in the proof of Proposition 1(a), we have

$$ne^{*,\tau} - e^{*} = \left(\frac{A}{cb}\right)^{\frac{1}{b-1}}\left[n\left(\frac{I_{N/n}}{n\sqrt{\sigma^2+\sigma_B^2}}\right)^{\frac{1}{b-1}} - \left(\frac{I_N}{\sigma}\right)^{\frac{1}{b-1}}\right]. \quad \text{(C.3)}$$

We have $\lim_{\sigma_B\to\infty}(ne^{*,\tau} - e^{*}) = -\left(\frac{A}{cb}\right)^{\frac{1}{b-1}}\left(\frac{I_N}{\sigma}\right)^{\frac{1}{b-1}} < 0$, $\lim_{\sigma_B\to 0}(ne^{*,\tau} - e^{*}) = \left(\frac{A}{cb}\right)^{\frac{1}{b-1}}\left[n\left(\frac{I_{N/n}}{n\sigma}\right)^{\frac{1}{b-1}} - \left(\frac{I_N}{\sigma}\right)^{\frac{1}{b-1}}\right]$, and $\frac{\partial(ne^{*,\tau}-e^{*})}{\partial\sigma_B} < 0$. Noting that $\lim_{b\to\infty}\left(n\left(\frac{I_{N/n}}{n\sigma}\right)^{\frac{1}{b-1}} - \left(\frac{I_N}{\sigma}\right)^{\frac{1}{b-1}}\right) = n-1 > 0$, there exists σ_B' (≥ 0) and $\underline{b}(>1)$ such that when $b > \underline{b}$, $ne^{*,\tau} > e^{*}$ if and only if $\sigma_B < \sigma_B'$. Also, $ne^{*,\tau} - e^{*}$ is decreasing in σ_B and constant in μ_B.

(b) Following the approach in the proof of Proposition 1(b), we have

$$m_{N/n}^{\tau} - m_N = \mu_B + \left(\sqrt{\sigma^2+\sigma_B^2}E_{N/n} - \sigma E_N\right). \quad \text{(C.4)}$$

We have $\lim_{\mu_B\to\infty}(m_{N/n}^{\tau} - m_N) = \infty$, $\lim_{\mu_B\to 0}(m_{N/n}^{\tau} - m_N) < 0$ if and only if $\frac{\sqrt{\sigma^2+\sigma_B^2}}{\sigma} < \frac{E_N}{E_{N/n}}$, and $\frac{\partial(m_{N/n}^{\tau}-m_N)}{\partial\mu_B} > 0$. Thus, there exists μ_B' (≥ 0) such that $m_N^{\tau} > m_N$ if $\mu_B > \mu_B'$; otherwise, $m_N^{\tau} \leq m_N$. Furthermore, we have $\lim_{\sigma_B\to\infty}(m_{N/n}^{\tau} - m_N) = \infty$,

$\lim_{\sigma_B\to 0}(m^{\tau}_{N/n} - m_N) < 0$ if and only if $\mu_B < \sigma\left(E_N - E_{N/n}\right)$, and $\frac{\partial(m^{\tau}_{N/n} - m_N)}{\partial\sigma_B} > 0$. Thus, there exists σ''_B (≥ 0) such that $m^{\tau}_N > m_N$ if $\sigma_B > \sigma''_B$; otherwise, $m^{\tau}_{N/n} \leq m_N$. Also, $(m^{\tau}_{N/n} - m_N)$ is increasing in σ_B and μ_B. ■

Proof of Theorem 2. From (3.23), (C.3), and (C.4), we have

$$\Pi^{*,\tau} - \Pi^* = \left(\frac{A}{cb}\right)^{\frac{1}{b-1}} \left[n\left(\frac{I_{N/n}}{n\sqrt{\sigma^2+\sigma_B^2}}\right)^{\frac{1}{b-1}} - \left(\frac{I_N}{\sigma}\right)^{\frac{1}{b-1}} \right] + \mu_B + \left(\sqrt{\sigma^2+\sigma_B^2}E_{N/n} - \sigma E_N\right).$$

(a) Suppose $\mu_B = 0$. Then, $\lim_{\sigma_B\to\infty}(\Pi^{*,\tau} - \Pi^*) = \infty$, and hence there exists $\overline{\sigma_B}$ (> 0) such that if $\sigma_B > \overline{\sigma_B}$, $\Pi^{*,\tau} > \Pi^*$. Also, $\lim_{\sigma_B\to 0}(\Pi^{*,\tau} - \Pi^*) = \sigma^{\frac{-1}{b-1}}\left(\frac{A}{cb}\right)^{\frac{1}{b-1}}\left[n\left(\frac{I_{N/n}}{n}\right)^{\frac{1}{b-1}} - (I_N)^{\frac{1}{b-1}}\right] + \sigma\left(E_{N/n} - E_N\right)$. Thus, $\lim_{\sigma_B\to 0,\sigma\to 0}(\Pi^{*,\tau} - \Pi^*) = \infty$ if $n\left(\frac{I_{N/n}}{n}\right)^{\frac{1}{b-1}} - (I_N)^{\frac{1}{b-1}} > 0$ (i.e., $b > \underline{b}$ from Proposition 2(a)), and $\lim_{\sigma_B\to 0,\sigma\to\infty}(\Pi^{*,\tau} - \Pi^*) = -\infty$ because $E_{N/n} < E_N$. Therefore, there also exist $\underline{\sigma_B}$ (> 0), $\underline{\sigma}$ (> 0), and $\overline{\sigma}$ (> 0) such that when $\sigma_B < \underline{\sigma_B}$, $\sigma < \underline{\sigma}$, and $b > \underline{b}$, $\Pi^{*,\tau} > \Pi^*$; and when $\sigma_B < \underline{\sigma_B}$ and $\sigma > \overline{\sigma}$, $\Pi^{*,\tau} < \Pi^*$.

(b) Suppose $\sigma_B \to 0$. Then, $\Pi^{*,\tau} - \Pi^* = \sigma^{\frac{-1}{b-1}}\left(\frac{A}{cb}\right)^{\frac{1}{b-1}}\left[n\left(\frac{I_{N/n}}{n}\right)^{\frac{1}{b-1}} - (I_N)^{\frac{1}{b-1}}\right] + \mu_B + \sigma\left(E_{N/n} - E_N\right)$. Thus, $\lim_{\mu_B\to\infty}(\Pi^{*,\tau} - \Pi^*) = \infty$, and $\frac{\partial(\Pi^{*,\tau} - \Pi^*)}{\partial\mu_B} = 1 > 0$. Thus, there exists $\overline{\mu_B}$ (≥ 0) such that if $\mu_B > \overline{\mu_B}$, $\Pi^{*,\tau} > \Pi^*$; otherwise, $\Pi^{*,\tau} \leq \Pi^*$.

Furthermore, we have $\lim_{\sigma\to\infty}(\Pi^{*,\tau} - \Pi^*) = -\infty$ since $E_{N/n} - E_N < 0$, and $\lim_{\sigma\to 0}(\Pi^{*,\tau} - \Pi^*) = \infty$ if $n\left(\frac{I_{N/n}}{n}\right)^{\frac{1}{b-1}} - (I_N)^{\frac{1}{b-1}} > 0$ (i.e., $b > \underline{b}$ from Proposition 2(a)). Thus, there exist $\underline{\underline{\sigma}}$ (> 0) and $\overline{\overline{\sigma}}$ (> 0) such that when $\sigma < \underline{\underline{\sigma}}$ and $b > \underline{b}$, $\Pi^{*,\tau} > \Pi^*$; and when $\sigma > \overline{\overline{\sigma}}$, $\Pi^{*,\tau} < \Pi^*$. Therefore, if $\mu_B > \overline{\mu_B}$, or $\sigma < \underline{\underline{\sigma}}$ and $b > \underline{b}$, $\Pi^{*,\tau} > \Pi^*$; and if $\mu_B < \overline{\mu_B}$ and $\sigma > \overline{\overline{\sigma}}$, $\Pi^{*,\tau} < \Pi^*$. ■

Proof of Proposition 3. $U^{*,\tau} > U^*$ if and only if

$$e^{*,\tau} - e^* = \left(\frac{A}{cb}\right)^{\frac{1}{b-1}} \left[\left(\frac{I_{N/n}}{n\sqrt{\sigma^2+\sigma_B^2}}\right)^{\frac{1}{b-1}} - \left(\frac{I_N}{\sigma}\right)^{\frac{1}{b-1}} \right] < 0,$$

which is always satisfied because $\frac{I_{N/n}}{n\sqrt{\sigma^2+\sigma_B^2}} < \frac{I_{N/n}}{n\sigma} < \frac{I_N}{\sigma}$, i.e., $\frac{I_{N/n}}{I_N} < n$. ■

Proof of Proposition 4. **(a)** $e^{*,\tau} = n^{\frac{-1}{b-1}} \left(\frac{AL_N^\tau}{cb}\right)^{\frac{1}{b-1}}$ in (3.13) decreases with n.

(b) m_N^τ in (3.13) does not change with team size n.

(c) $U^{*,\tau} = \frac{A}{N} - c(e^{*,\tau})^b$ increases with n because $e^{*,\tau}$ decreases with n. ■

Proof of Proposition 5. **(a)**

We have $\lim_{b\to\infty} ne^{*,\tau} = \lim_{b\to\infty} n^{\frac{b-2}{b-1}} \left(\frac{AL_{N/n}^\tau}{cb}\right)^{\frac{1}{b-1}} = n$, which increases with n. Therefore, $ne^{*,\tau}$ increases with n when b is sufficiently large.

(b) $m_{N/n}^\tau$ in (3.20) decreases with team size n as the expected value of the maximum of N/n random variables decreases with n.

(c) $U^{*,\tau}$ in (3.19) increases with n because $e^{*,\tau} = \left(\frac{AL_{N/n}^\tau}{ncb}\right)^{\frac{1}{b-1}}$ decreases with n. ■

Proof of Proposition 6. From (3.6) and (3.33), we have

$$\begin{aligned}
\Pi^{*,\tau} - \Pi^* &= n^{\frac{b-2\gamma}{(b-1)\gamma}} \left(\frac{AL_{N/n}^\tau}{cb}\right)^{\frac{1}{b-1}} - \left(\frac{AL_N}{cb}\right)^{\frac{1}{b-1}} + m_{N/n}^\tau - m_N \\
&= \left(\frac{A}{cb}\right)^{\frac{1}{b-1}} \left[n^{\frac{b-2\gamma}{(b-1)\gamma}} \left(\frac{I_{N/n}}{\sqrt{\sigma^2+\sigma_B^2}}\right)^{\frac{1}{b-1}} - \left(\frac{I_N}{\sigma}\right)^{\frac{1}{b-1}} \right] + \mu_B \\
&\quad + \left(\sqrt{\sigma^2+\sigma_B^2} E_{N/n} - \sigma E_N\right)
\end{aligned}$$

(a) Suppose $\mu_B = 0$. Then, $\lim_{\sigma_B\to\infty}(\Pi^{*,\tau} - \Pi^*) = \infty$, and hence there exists $\overline{\sigma_B}$ (> 0) such that if $\sigma_B > \overline{\sigma_B}$, $\Pi^{*,\tau} > \Pi^*$. Also, $\lim_{\sigma_B\to 0}(\Pi^{*,\tau} - \Pi^*) = \sigma^{\frac{-1}{b-1}} \left(\frac{A}{cb}\right)^{\frac{1}{b-1}} \left[n^{\frac{b-2\gamma}{(b-1)\gamma}} \left(I_{N/n}\right)^{\frac{1}{b-1}} - (I_N)^{\frac{1}{b-1}} \right] + \sigma\left(E_{N/n} - E_N\right)$. Thus, $\lim_{\sigma_B\to 0,\sigma\to 0}(\Pi^{*,\tau} - \Pi^*) = \infty$ if $n^{\frac{b-2\gamma}{(b-1)\gamma}} \left(I_{N/n}\right)^{\frac{1}{b-1}} - (I_N)^{\frac{1}{b-1}} > 0$ (i.e., $b > \underline{b}$ since $\lim_{b\to\infty}\left(n^{\frac{b-2\gamma}{(b-1)\gamma}} \left(I_{N/n}\right)^{\frac{1}{b-1}} - (I_N)^{\frac{1}{b-1}} \right) = n^{\frac{1}{\gamma}} - 1 > 0$), and $\lim_{\sigma_B\to 0,\sigma\to\infty}(\Pi^{*,\tau} - \Pi^*) = -\infty$ because $E_{N/n} < E_N$. Therefore, there also exist $\underline{\sigma_B}$ (> 0), $\underline{\sigma}$ (> 0), and $\overline{\sigma}$ (> 0) such that when $\sigma_B < \underline{\sigma_B}$, $\sigma < \underline{\sigma}$, and $b > \underline{b}$, $\Pi^{*,\tau} > \Pi^*$; and when $\sigma_B < \underline{\sigma_B}$ and $\sigma > \overline{\sigma}$, $\Pi^{*,\tau} < \Pi^*$.

(b) Suppose $\sigma_B \to 0$. Then, we have

$$\Pi^{*,\tau} - \Pi^* = \sigma^{\frac{-1}{b-1}} \left(\frac{A}{cb}\right)^{\frac{1}{b-1}} \left[n^{\frac{b-2\gamma}{(b-1)\gamma}} \left(I_{N/n}\right)^{\frac{1}{b-1}} - (I_N)^{\frac{1}{b-1}} \right] + \mu_B + \sigma\left(E_{N/n} - E_N\right).$$

$\lim_{\mu_B\to\infty}(\Pi^{*,\tau} - \Pi^*) = \infty$, and $\frac{\partial(\Pi^{*,\tau} - \Pi^*)}{\partial\mu_b} = 1 > 0$. Thus, there exists $\overline{\mu_B}$ (≥ 0) such that if $\mu_B > \overline{\mu_B}$, $\Pi^{*,\tau} > \Pi^*$; otherwise, $\Pi^{*,\tau} \leq \Pi^*$.

Furthermore, we have $\lim_{\sigma\to\infty}(\Pi^{*,\tau}-\Pi^{*})=-\infty$ since $E_{N/n}-E_N<0$, and $\lim_{\sigma\to 0}(\Pi^{*,\tau}-\Pi^{*})=\infty$ if $n^{\frac{b-2\gamma}{(b-1)\gamma}}\left(I_{N/n}\right)^{\frac{1}{b-1}}-(I_N)^{\frac{1}{b-1}}>0$. Thus, there exist $\underline{\underline{\sigma}}$ (>0) and $\overline{\overline{\sigma}}$ (>0) such that when $\sigma<\underline{\underline{\sigma}}$ and $b>\underline{b}$, $\Pi^{*,\tau}>\Pi^{*}$; and when $\sigma>\overline{\overline{\sigma}}$, $\Pi^{*,\tau}<\Pi^{*}$. Therefore, if $\mu_B>\overline{\mu_B}$, or $\sigma<\underline{\underline{\sigma}}$ and $b>\underline{b}$, $\Pi^{*,\tau}>\Pi^{*}$; and if $\mu_B<\overline{\mu_B}$ and $\sigma>\overline{\overline{\sigma}}$, $\Pi^{*,\tau}<\Pi^{*}$.

(c) $\Pi^{*,\tau}=n^{\frac{b-2\gamma}{(b-1)\gamma}}\left(\frac{AL^{\tau}_{N/n}}{cb}\right)^{\frac{1}{b-1}}+m^{\tau}_{N/n}$ decreases with γ because $\frac{\partial\left(\frac{b-2\gamma}{(b-1)\gamma}\right)}{\partial\gamma}=\frac{-b}{(b-1)\gamma^2}<0$. ■

Proof of Proposition 7. $U^{*,\tau}>U^{*}$ if and only if

$$(e^{*,\tau})^b-(e^{*})^b=$$

$$\left(\frac{An^{\frac{1}{\gamma}-2}L^{\tau}_{N/n}}{cb}\right)^{\frac{b}{b-1}}-\left(\frac{AL_N}{cb}\right)^{\frac{b}{b-1}}=\left(\frac{A}{cb}\right)^{\frac{1}{b-1}}\left[\left(n^{\frac{1}{\gamma}-2}L_{N/n}\right)^{\frac{1}{b-1}}-(L_N)^{\frac{1}{b-1}}\right]<0.$$

This is satisfied when $L_N>n^{\frac{1}{\gamma}-2}L^{\tau}_{N/n}$. We have $\lim_{\gamma\to 0^+}n^{\frac{1}{\gamma}-2}=\infty$ and $n^{\frac{1}{\gamma}-2}$ is decreasing in γ. Therefore, there exists $\overline{\gamma}$ such that $U^{*,\tau}>U^{*}$ if and only if $\gamma>\overline{\gamma}$. ■

Appendix D

Additional Analysis of Chapter 3

D.1 Correlated Random Variables

In our main analysis, we consider the case where ξ_{ti}^{B} and ξ_{ti} are independent for each team member i in team t. In the following corollary, we extend Theorem 1 and Corollary 1 to the case where ξ_{ti}^{B} and ξ_{ti} are correlated for each team member i in team t, and hence $\xi_{ti}^{\tau}(=\xi_{ti}^{B}+\xi_{ti})$ follows Normal$(\mu_B, \sigma^2+\sigma_B^2+2\rho\sigma\sigma_B)$, where ρ is the correlation.

Corollary 3 *Theorem 1 continues to hold when ξ_{ti}^{B} and ξ_{ti} are correlated for each team member i in team t.*

Proof. From Π^* in (3.6) and $\Pi^{*,\tau}$ in (3.13), we have

$$\Pi^{*,\tau}-\Pi^* = \left(\frac{AI_N}{cb}\right)^{\frac{1}{b-1}}\left(\left(\frac{N-n}{n(N-1)}\right)\frac{1}{\sqrt{\sigma^2+\sigma_B^2+2\rho\sigma\sigma_B}}-\frac{1}{\sigma}\right)^{\frac{1}{b-1}}+\mu_B$$
$$+E_N\left(\sqrt{\sigma^2+\sigma_B^2+2\rho\sigma\sigma_B}-\sigma\right).$$

Suppose $\mu_B=0$. Then, we have $\lim_{\sigma_B\to\infty}(\Pi^{*,\tau}-\Pi^*)=\infty$, and hence there exists $\overline{\sigma_B}$ (>0) such that when $\sigma_B>\overline{\sigma_B}$, $\Pi^{*,\tau}>\Pi^*$. Also, $\lim_{\sigma_B\to 0}(\Pi^{*,\tau}-\Pi^*) = \left(\frac{AI_N}{cb}\right)^{\frac{1}{b-1}}\left(\left(\frac{N-n}{n(N-1)}-1\right)\frac{1}{\sigma}\right)^{\frac{1}{b-1}}<0$ because $N-n<nN-n$. Thus, there exists $\underline{\sigma_B}$ (>0) such that when $\sigma_B<\underline{\sigma_B}$, $\Pi^{*,\tau}<\Pi^*$.

Suppose $\sigma_B\to 0$ and $\mu_B>0$. Because $\lim_{\sigma_B\to 0}\sqrt{\sigma^2+\sigma_B^2+2\rho\sigma\sigma_B} = \lim_{\sigma_B\to 0}\sqrt{\sigma^2+\sigma_B^2}=\sigma$, the rest of the proof is the same as the proof of Theorem

1(b). ■

In the following corollary, we extend Theorem 2 to the case where ξ_t^B and ξ_t are correlated for each t, and hence $\xi_t^\tau (= \xi_t^B + \xi_t)$ follows Normal$(\mu_B, \sigma^2 + \sigma_B^2 + 2\rho\sigma\sigma_B)$, where ρ is the correlation.

Corollary 4 *Theorem 2 continues to hold when ξ_t^B and ξ_t are correlated for each team t.*

Proof. From Π^* in (3.6) and $\Pi^{*,\tau}$ in (3.20), we have

$$\Pi^{*,\tau} - \Pi^* = \left(\frac{A}{cb}\right)^{\frac{1}{b-1}} \left[n \left(\frac{I_{N/n}}{n\sqrt{\sigma^2 + \sigma_B^2 + 2\rho\sigma\sigma_B}} \right)^{\frac{1}{b-1}} - \left(\frac{I_N}{\sigma}\right)^{\frac{1}{b-1}} \right] + \mu_B$$
$$+ \left(\sqrt{\sigma^2 + \sigma_B^2 + 2\rho\sigma\sigma_B} E_{N/n} - \sigma E_N \right).$$

Suppose $\mu_B = 0$. Then, we have $\lim_{\sigma_B \to \infty}(\Pi^{*,\tau} - \Pi^*) = \infty$, and hence there exists $\overline{\sigma_B}$ (> 0) such that if $\sigma_B > \overline{\sigma_B}$, $\Pi^{*,\tau} > \Pi^*$. Also, $\lim_{\sigma_B \to 0}(\Pi^{*,\tau} - \Pi^*) = \sigma^{\frac{-1}{b-1}} \left(\frac{A}{cb}\right)^{\frac{1}{b-1}} \left[n \left(\frac{I_{N/n}}{n}\right)^{\frac{1}{b-1}} - (I_N)^{\frac{1}{b-1}} \right] + \sigma \left(E_{N/n} - E_N\right)$. Thus, $\lim_{\sigma_B \to 0, \sigma \to 0}(\Pi^{*,\tau} - \Pi^*) = \infty$ if $n\left(\frac{I_{N/n}}{n}\right)^{\frac{1}{b-1}} - (I_N)^{\frac{1}{b-1}} > 0$ (i.e., $b > \underline{b}$), and $\lim_{\sigma_B \to 0, \sigma \to \infty}(\Pi^{*,\tau} - \Pi^*) = -\infty$ because $E_{N/n} < E_N$. Therefore, there also exist $\underline{\sigma_B}$ (> 0), $\underline{\sigma}$ (> 0), and $\overline{\sigma}$ (> 0) such that when $\sigma_B < \underline{\sigma_B}$, $\sigma < \underline{\sigma}$, and $b > \underline{b}$, $\Pi^{*,\tau} > \Pi^*$; and when $\sigma_B < \underline{\sigma_B}$ and $\sigma > \overline{\sigma}$, $\Pi^{*,\tau} < \Pi^*$.

Suppose $\sigma_B \to 0$. Because $\lim_{\sigma_B \to 0} \sqrt{\sigma^2 + \sigma_B^2 + 2\rho\sigma\sigma_B} = \lim_{\sigma_B \to 0} \sqrt{\sigma^2 + \sigma_B^2} = \sigma$, the rest of the proof is the same as the proof of Theorem 2(b). ■

D.2 Complementary Tasks

We consider the case where there are two complementary tasks to be performed to develop a solution to the organizer's problem, and in the case of team submissions, the team size is two. As in §3.5.3, we assume a single output shock ξ_i for each solver i (follows Normal$(0, \sigma^2)$, where where $h(s)$ and $H(s)$ are density function and distribution function, respectively); and for each team t, a single output shock ξ_t (follows Normal$(0, \sigma^2)$) related to the problem and a single interaction shock ξ_t^B (follows Normal(μ_B, σ_B^2)) generated through team members' interactions. Recall from §3.3.2 that the team's overall output shock $\xi_t^\tau = \xi_t^B + \xi_t$, where ξ_t^τ follows Normal$(\mu_B, \sigma^2 + \sigma_B^2)$ and $g(s)$ and $G(s)$ are ξ_t's density function and distribution function, respectively.

We first consider the case of individual submissions. As there are two complementary tasks, we derive $P_N[e_i, e^*]$ as follows:

$$P_N[e_i, e^*] = P\{y_i > y_j, \forall\, j \in \mathscr{N}\backslash\{i\}\} = P\{\min\{e_i/2, e_i/2\} + \xi_i$$
$$> \min\{e^*/2, e^*/2\} + \xi_j, \forall\, j \in \mathscr{N}\backslash\{i\}\} = \int_{-\infty}^{\infty} H\left(s + e_i/2 - e^*/2\right)^{N-1} h(s)ds.$$

By considering the first-order condition of (3.3) under $P_N[e_i, e^*]$, we obtain $e^* = \left(\frac{AL_N}{2cb}\right)^{\frac{1}{b-1}}$ and $U^* = \frac{A}{N} - c\left(\frac{AL_N}{2cb}\right)^{\frac{b}{b-1}}$, and the organizer's expected profit in equilibrium as follows:

$$\Pi^* = E\left[\max_{i\in\mathscr{N}}\{y_i\} - A\right] = \min\{e^*/2, e^*/2\} + E\left[\max_{i\in\mathscr{N}}\{\xi_i\}\right] - A = \frac{1}{2}\left(\frac{AL_N}{2cb}\right)^{\frac{1}{b-1}} + m_N - A.$$

We next consider the case of team submissions. We derive team t's probability of winning $P_{N/2}^\tau[e_{ti}^\tau, e^{*,\tau}]$ as follows:

$$\begin{aligned}
P_{N/2}^\tau[e_{ti}^\tau, e^{*,\tau}] &= P\left\{y_t^\tau > y_k^\tau, \forall\, k \in \mathscr{T}\backslash\{t\}\right\} \\
&= P\left\{\min\{e_{ti}^\tau, e^{*,\tau}\} + \xi_t^\tau > \min\{e^{*,\tau}, e^{*,\tau}\} + \xi_k^\tau, \forall\, k \in \mathscr{T}\backslash\{t\}\right\} \\
&= P\left\{\min\{e_{ti}^\tau, e^{*,\tau}\} + \xi_t^\tau > e^{*,\tau} + \xi_k^\tau, \forall\, k \in \mathscr{T}\backslash\{t\}\right\} \\
&= \begin{cases} \int_{-\infty}^{\infty} G\left(s + e_{ti}^\tau - e^{*,\tau}\right)^{\frac{N}{2}-1} g(s)ds & \text{if } e_{ti}^\tau \le e^{*,\tau} \\ \int_{-\infty}^{\infty} G\left(s\right)^{\frac{N}{2}-1} g(s)ds & \text{if } e_{ti}^\tau > e^{*,\tau} \end{cases}
\end{aligned}$$

By taking the first-order condition of (3.17) under $P_{N/2}^\tau[e_{ti}^\tau, e^{*,\tau}]$, we derive $e^{*,\tau} = \left(\frac{AL_{N/2}^\tau}{2cb}\right)^{\frac{1}{b-1}}$ and $U^{*,\tau} = \frac{A}{N} - c\left(\frac{AL_{N/2}^\tau}{2cb}\right)^{\frac{b}{b-1}}$, and the organizer's expected profit in equilibrium as follows:

$$\begin{aligned}
\Pi^{*,\tau} &= E\left[\max_{t\in\mathscr{T}}\{y_t^\tau\} - A\right] = \min\{e^{*,\tau}, e^{*,\tau}\} + E\left[\max_{t\in\mathscr{T}}\{\xi_t^\tau\}\right] - A \\
&= \left(\frac{AL_{N/2}^\tau}{2cb}\right)^{\frac{1}{b-1}} + m_{N/2}^\tau - A.
\end{aligned}$$

In the following corollary, we extend Theorem 2 to the case where there are two complementary tasks to be performed to develop a solution to the organizer's problem.

Corollary 5 *(a) Suppose $\mu_B = 0$. Then, for any σ, there exists $\overline{\sigma_B}$ such that $\Pi^{*,\tau} > \Pi^*$ if $\sigma_B > \overline{\sigma_B}$. Furthermore, there exist thresholds $\underline{\sigma_B}$, $\underline{\sigma}$, and $\overline{\sigma}$ such that when $\sigma_B < \underline{\sigma_B}$ and $\sigma < \underline{\sigma}$, $\Pi^{*,\tau} > \Pi^*$; and when $\sigma_B < \underline{\sigma_B}$ and $\sigma > \overline{\sigma}$, $\Pi^{*,\tau} < \Pi^*$.*
(b) Suppose $\sigma_B \to 0$. Then, there exist thresholds $\overline{\mu_B}$, $\underline{\underline{\sigma}}$, and $\overline{\overline{\sigma}}$ such that $\Pi^{*,\tau} > \Pi^*$ if $\mu_B > \overline{\mu_B}$ or $\sigma < \underline{\underline{\sigma}}$; and $\Pi^{*,\tau} < \Pi^*$ if $\mu_B < \overline{\mu_B}$ and $\sigma > \overline{\overline{\sigma}}$.

Proof. We have

$$\begin{aligned}\Pi^{*,\tau} - \Pi^* = {} & \left(\frac{A}{cb}\right)^{\frac{1}{b-1}} \left[\left(\frac{I_{N/2}}{2\sqrt{\sigma^2 + \sigma_B^2}} \right)^{\frac{1}{b-1}} - \frac{1}{2}\left(\frac{I_N}{2\sigma}\right)^{\frac{1}{b-1}} \right] + \mu_B \\ & + \left(\sqrt{\sigma^2 + \sigma_B^2} E_{N/2} - \sigma E_N \right).\end{aligned}$$

Suppose $\mu_B = 0$. Then, we have $\lim_{\sigma_B \to \infty} (\Pi^{*,\tau} - \Pi^*) = \infty$, and hence there exists $\overline{\sigma_B}$ (> 0) such that if $\sigma_B > \overline{\sigma_B}$, $\Pi^{*,\tau} > \Pi^*$. Also, $\lim_{\sigma_B \to 0} (\Pi^{*,\tau} - \Pi^*) = \sigma^{\frac{-1}{b-1}} \left(\frac{A}{cb}\right)^{\frac{1}{b-1}} \left[\left(\frac{I_{N/2}}{2}\right)^{\frac{1}{b-1}} - \frac{1}{2}\left(\frac{I_N}{2}\right)^{\frac{1}{b-1}} \right] + \sigma \left(E_{N/2} - E_N\right)$. Thus, $\lim_{\sigma_B \to 0, \sigma \to 0} (\Pi^{*,\tau} - \Pi^*) = \infty$ because $I_{N/2} > I_N$, and $\lim_{\sigma_B \to 0, \sigma \to \infty} (\Pi^{*,\tau} - \Pi^*) = -\infty$ because $E_{N/2} < E_N$. Therefore, there also exist $\underline{\sigma_B}$ (> 0), $\underline{\sigma}$ (> 0), and $\overline{\sigma}$ (> 0) such that if $\sigma_B < \underline{\sigma_B}$ and $\sigma < \underline{\sigma}$, $\Pi^{*,\tau} > \Pi^*$, and if $\sigma_B < \underline{\sigma_B}$ and $\sigma > \underline{\sigma}$, $\Pi^{*,\tau} < \Pi^*$.

Suppose $\sigma_B \to 0$. Then, $\Pi^{*,\tau} - \Pi^* = \sigma^{\frac{-1}{b-1}} \left(\frac{A}{cb}\right)^{\frac{1}{b-1}} \left[\left(\frac{I_{N/2}}{2}\right)^{\frac{1}{b-1}} - \frac{1}{2}\left(\frac{I_N}{2}\right)^{\frac{1}{b-1}} \right] + \mu_B + \sigma \left(E_{N/n} - E_N\right)$. Thus, $\lim_{\mu_B \to \infty} (\Pi^{*,\tau} - \Pi^*) = \infty$, and $\frac{\partial(\Pi^{*,\tau} - \Pi^*)}{\partial \mu_b} = 1 > 0$. Thus, there exists $\overline{\mu_B}$ (≥ 0) such that if $\mu_B > \overline{\mu_B}$, $\Pi^{*,\tau} > \Pi^*$; otherwise, $\Pi^{*,\tau} \leq \Pi^*$. Furthermore, we have $\lim_{\sigma \to \infty} (\Pi^{*,\tau} - \Pi^*) = -\infty$ since $E_{N/n} - E_N < 0$, and $\lim_{\sigma \to 0} (\Pi^{*,\tau} - \Pi^*) = \infty$ because $I_{N/2} > I_N$. Thus, there exist $\underline{\underline{\sigma}}$ (> 0) and $\overline{\overline{\sigma}}$ (> 0) such that if $\sigma < \underline{\sigma}$, $\Pi^{*,\tau} > \Pi^*$; and if $\sigma > \overline{\sigma}$, $\Pi^{*,\tau} < \Pi^*$. Therefore, if $\mu_B > \overline{\mu_B}$ or $\sigma < \underline{\underline{\sigma}}$, $\Pi^{*,\tau} > \Pi^*$; and if $\mu_B < \overline{\mu_B}$ or $\sigma > \overline{\overline{\sigma}}$, $\Pi^{*,\tau} < \Pi^*$. We also want to note that in this case, when $\sigma_B \to 0$, the team-effort contribution $e^{*,\tau}$ is always larger than the solver-effort contribution $e^*/2$ because $\left(\frac{I_{N/2}}{2}\right)^{\frac{1}{b-1}} > \frac{1}{2}\left(\frac{I_N}{2}\right)^{\frac{1}{b-1}}$. ■

D.3 Existence of Equilibrium

In this section, we provide sufficient conditions for e^* in (3.4), $e^{*,\tau}$ in (3.11), and $e^{*,\tau}$ in (3.18) to be pure-strategy Nash equilibria. Specifically, in the following lemma, we first show sufficient conditions for interim properties on the individual solver's utility $U = AP_N[e_i, e^*] - ce_i^b$ in (3.3) and the team member's utility $U^\tau = \frac{A}{n}P_N^\tau[e_{ti}^\tau, e^{*,\tau}] - c(e_{ti}^\tau)^b$ in (3.10) and (3.17). Then, we use these properties to show that U in (3.3), U^τ in (3.10), and U^τ in (3.17) are unimodal.

Lemma 9 *(**a**) Suppose that* $\frac{\partial^2 U}{\partial e_i^2}\Big|_{e_i=\underline{e}} < 0$ *for some* $\underline{e}$ *and* $b > 2$. *When* b *or* c *or* σ *is sufficiently high,* $\frac{\partial^2 U_i[e_i,e^*]}{\partial e_i^2} < 0$ *for any* $e_i > \underline{e}$.
*(**b**) Suppose that* U^τ *is as in (3.10) and* $\frac{\partial^2 U_i^\tau}{\partial (e_{ti}^\tau)^2}\Big|_{e_{ti}^\tau=\underline{e}} < 0$ *for some* $\underline{e}$ *and* $b > 2$. *When* b *or* c *or* σ *or* σ_B *is sufficiently high,* $\frac{\partial^2 U_i^\tau}{\partial (e_{ti}^\tau)^2} < 0$ *for any* $e_i^\tau > \underline{e}$.
*(**c**) Suppose that* U^τ *is as in (3.17) and* $\frac{\partial^2 U_i^\tau}{\partial (e_{ti}^\tau)^2}\Big|_{e_{ti}^\tau=\underline{e}} < 0$ *for some* $\underline{e}$ *and* $b > 2$. *When* b *or* c *or* σ *or* σ_B *is sufficiently high,* $\frac{\partial^2 U_i^\tau}{\partial (e_{ti}^\tau)^2} < 0$ *for any* $e_i^\tau > \underline{e}$.

Proof. **(a)** The second derivative of solver i's utility U with respect to e_i is

$$\frac{\partial^2 U}{\partial e_i^2} = A\frac{\partial^2 P_N[e_i,e^*]}{\partial e_i^2} - cb(b-1)(e_i)^{b-2}.$$

Suppose that $\frac{\partial^2 U}{\partial e_i^2} < 0$ for some $e_i = \underline{e}$, but there exists $\widehat{e} > \underline{e}$ such that $\frac{\partial^2 U}{\partial e_i^2} \geq 0$ for $e_i = \widehat{e}$. Then, $\frac{\partial^2 U}{\partial e_i^2}\big|_{e_i=\widehat{e}} > \frac{\partial^2 U}{\partial e_i^2}\big|_{e_i=\underline{e}}$, i.e.,

$$A\left[\frac{\partial^2 P_N[e_i,e^*]}{\partial e_i^2}\bigg|_{e_i=\widehat{e}} - \frac{\partial^2 P_N[e_i,e^*]}{\partial e_i^2}\bigg|_{e_i=\underline{e}}\right] > cb(b-1)\left[(\widehat{e})^{b-2} - (\underline{e})^{b-2}\right]. \quad \text{(D.1)}$$

Suppose that $b > 2$. Since $\widehat{e} > \underline{e}$, the right-hand side of (D.1) approaches ∞ as b or c approaches ∞. Thus, when b or c is sufficiently high, (D.1) cannot be satisfied. Let $x = s + e_i - e^*$. Then, the second derivative of $P_N[e_i,e^*]$ with respect to e_i is

$$\begin{aligned}
&\frac{\partial^2 P_N[e_i,e^*]}{\partial e_i^2} = (N-1)\int_{-\infty}^{\infty}(N-2)H(s+e_i-e^*)^{N-3}h(s+e_i-e^*)^2 h(s)ds \\
&+(N-1)\int_{-\infty}^{\infty}H(s+e_i-e^*)^{N-2}h'(s+e_i-e^*)h(s)ds \\
&=(N-1)\left(\frac{1}{\sigma\sqrt{2\pi}}\right)^3\times \\
&\int_{-\infty}^{\infty}(N-2)\left[\frac{1}{2}\left[1+\operatorname{erf}\left(\frac{x}{\sigma\sqrt{2}}\right)\right]\right]^{N-3}\exp\left(-x^2/(2\sigma^2)\right)^2\exp\left(-s^2/(2\sigma^2)\right)ds \\
&+(N-1)\frac{-2x}{2\sigma^3\sqrt{2\pi}}\times \\
&\int_{-\infty}^{\infty}\left[\frac{1}{2}\left[1+\operatorname{erf}\left(\frac{x}{\sigma\sqrt{2}}\right)\right]\right]^{N-2}\exp\left(-x^2/(2\sigma^2)\right)\exp\left(-s^2/(2\sigma^2)\right)ds.
\end{aligned}$$

The left-hand side of (D.1) approaches 0 as σ approaches ∞, and hence when σ is sufficiently high, (D.1) cannot be satisfied because the right-hand side of (D.1) is positive. Therefore, when b or c or σ is sufficiently high, $\frac{\partial^2 U}{\partial e_i^2} < 0$ for any $e_i > \underline{e}$.

(b) The second derivative of team member i's utility U^τ in (3.10) with respect to e_{ti}^τ is

$$\frac{\partial^2 U^\tau}{\partial (e_{ti}^\tau)^2} = \left(\frac{A}{n}\right) \frac{\partial^2 P_N^\tau [e_{ti}^\tau, e^{*,\tau}]}{\partial (e_{ti}^\tau)^2} - cb(b-1)(e_{ti}^\tau)^{b-2}.$$

Suppose that $\frac{\partial^2 U^\tau}{\partial (e_{ti}^\tau)^2} < 0$ for some $e_{ti}^\tau = \underline{e}$, but there exists $\widehat{e} > \underline{e}$ such that $\frac{\partial^2 U^\tau}{\partial (e_{ti}^\tau)^2} \geq 0$ for $e_{ti}^\tau = \widehat{e}$. Then, $\frac{\partial^2 U^\tau}{\partial (e_{ti}^\tau)^2}\big|_{e_{ti}^\tau = \widehat{e}} > \frac{\partial^2 U^\tau}{\partial (e_{ti}^\tau)^2}\big|_{e_{ti}^\tau = \underline{e}}$, i.e.,

$$\frac{A}{n}\left[\left.\frac{\partial^2 P_N^\tau [e_{ti}^\tau, e^{*,\tau}]}{\partial (e_{ti}^\tau)^2}\right|_{e_{ti}^\tau = \widehat{e}} - \left.\frac{\partial^2 P_N^\tau [e_{ti}^\tau, e^{*,\tau}]}{\partial (e_{ti}^\tau)^2}\right|_{e_{ti}^\tau = \underline{e}}\right] > cb(b-1)\left[(\widehat{e})^{b-2} - (\underline{e})^{b-2}\right]. \tag{D.2}$$

Suppose that $b > 2$. Since $\widehat{e} > \underline{e}$, the right-hand side of (D.2) approaches ∞ as b or c approaches ∞. Thus, when b or c is sufficiently high, (D.2) cannot be satisfied. Let $x = s + e_i - e^* - \mu_B$ and $\sigma_x = \sqrt{\sigma^2 + \sigma_B^2}$. Then, the second derivative of $P_N^\tau [e_{ti}^\tau, e^{*,\tau}]$ with respect to e_{ti}^τ is

$$\frac{\partial^2 P_N^\tau [e_{ti}^\tau, e^{*,\tau}]}{\partial (e_{ti}^\tau)^2} = (N-n)\int_{-\infty}^{\infty} (N-2) H\left(s + e_{ti}^\tau - e^{*,\tau}\right)^{N-3} h\left(s + e_{ti}^\tau - e^{*,\tau}\right)^2 h(s) ds$$
$$+ (N-n)\int_{-\infty}^{\infty} H\left(s + e_{ti}^\tau - e^{*,\tau}\right)^{N-2} h'\left(s + e_{ti}^\tau - e^{*,\tau}\right) h(s) ds$$
$$= (N-n)\left(\frac{1}{\sigma_x\sqrt{2\pi}}\right)^3 \times$$
$$\int_{-\infty}^{\infty} (N-2)\left[\frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x}{\sigma_x\sqrt{2}}\right)\right]\right]^{N-3} \exp\left(-x^2/(2\sigma_x^2)\right)^2 \exp\left(-s^2/(2\sigma_x^2)\right) ds$$
$$+ (N-n)\frac{-2x}{2\sigma_x^3\sqrt{2\pi}} \times$$
$$\int_{-\infty}^{\infty} \left[\frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x}{\sigma_x\sqrt{2}}\right)\right]\right]^{N-2} \exp\left(-x^2/(2\sigma_x^2)\right) \exp\left(-s^2/(2\sigma_x^2)\right) ds.$$

The left-hand side of (D.1) approaches 0 as σ or σ_B approaches ∞, and hence when σ or σ_B is sufficiently high, (D.1) cannot be satisfied because the right-hand side of (D.1) is positive. Therefore, when b or c or σ or σ_B is sufficiently high, $\frac{\partial^2 U^\tau}{\partial (e_{ti}^\tau)^2} < 0$ for any $e_{ti} > \underline{e}$.

(c) Following the same steps in (b), we show that when the team member's utility U^τ is as in (3.17), if b or c or σ or σ_B is sufficiently high, $\frac{\partial^2 U^\tau}{\partial (e^\tau_{ti})^2} < 0$ for any $e_{ti} > \underline{e}$.

■

The following lemma shows that when the properties in Lemma 9 hold for any $\underline{e}$, e^* in (3.4), $e^{*,\tau}$ in (3.11), and $e^{*,\tau}$ in (3.18) are pure-strategy Nash equilibria.

Lemma 10 ***(a)*** *Suppose that for all $\underline{e}$ such that $\left.\frac{\partial^2 U}{\partial e_i^2}\right|_{e_i=\underline{e}} < 0$, $\frac{\partial^2 U}{\partial e_i^2} < 0$ when $e_i > \underline{e}$. Then, U is unimodal. Thus, e^* in (3.4) is a pure-strategy Nash equilibrium.*
(b) *Suppose that U^τ is as in (3.10) and for all $\underline{e}$ such that $\left.\frac{\partial^2 U^\tau}{\partial (e^\tau_{ti})^2}\right|_{e^\tau_{ti}=\underline{e}} < 0$, $\frac{\partial^2 U^\tau}{\partial (e^\tau_{ti})^2} < 0$ when $e^\tau_{ti} > \underline{e}$. Then, U^τ is unimodal. Thus, $e^{*,\tau}$ in (3.11) is a pure-strategy Nash equilibrium.*
(c) *Suppose that U^τ is as in (3.17) and for all $\underline{e}$ such that $\left.\frac{\partial^2 U^\tau}{\partial (e^\tau_{ti})^2}\right|_{e^\tau_{ti}=\underline{e}} < 0$, $\frac{\partial^2 U^\tau}{\partial (e^\tau_{ti})^2} < 0$ when $e^\tau_{ti} > \underline{e}$. Then, U^τ is unimodal. Thus, $e^{*,\tau}$ in (3.18) is a pure-strategy Nash equilibrium.*

Proof. **(a)** Suppose that for all $\underline{e}$ such that $\left.\frac{\partial^2 U}{\partial e_i^2}\right|_{e_i=\underline{e}} < 0$, we have $\frac{\partial^2 U}{\partial e_i^2} < 0$ when $e_i > \underline{e}$. First, we have $\left.\frac{\partial U}{\partial e_i}\right|_{e_i=0} = A\frac{\partial P_N[e_i,e^*]}{\partial e_i} > 0$ and $\lim_{e_i\to\infty} U = -\infty$, and hence there should exist some e_i such that $\frac{\partial U}{\partial e_i} < 0$ and $\frac{\partial^2 U}{\partial e_i^2} < 0$. Thus, there exists a threshold $e_0(\geq 0)$ such that for any $e_i < e_0$, $\frac{\partial^2 U}{\partial e_i^2} \geq 0$; and for any $e_i > e_0$, $\frac{\partial^2 U}{\partial e_i^2} < 0$. Hence, we should have $\frac{\partial U}{\partial e_i} > 0$ for any $e_i < e_0$, and there should exist another threshold $e_{00}(> e_0)$ such that for any $e_i < e_{00}$, $\frac{\partial U}{\partial e_i} > 0$; and for any $e_i > e_{00}$, $\frac{\partial U}{\partial e_i} < 0$. Therefore, the solver's utility U is unimodal with mode e_{00}, and hence the first-order condition of the solver's utility-maximization problem in (3.3) is sufficient for optimality. Thus, e^* is the best-response to the solver's utility-maximization problem, and hence a pure-strategy Nash equilibrium.
(b)-(c) Suppose that for all $\underline{e}$ such that $\left.\frac{\partial^2 U^\tau}{\partial (e^\tau_{ti})^2}\right|_{e^\tau_{ti}=\underline{e}} < 0$, we have $\frac{\partial^2 U^\tau}{\partial (e^\tau_{ti})^2} < 0$ when $e^\tau_{ti} > \underline{e}$. First, we have $\left.\frac{\partial U^\tau}{\partial e^\tau_{ti}}\right|_{e^\tau_{ti}=0} = 0$ and $\lim_{e^\tau_{ti}\to\infty} U^\tau = -\infty$, and hence there should exist some e^τ_{ti} such that $\frac{\partial U^\tau}{\partial e^\tau_{ti}} < 0$ and $\frac{\partial^2 U^\tau}{\partial (e^\tau_{ti})^2} < 0$. Thus, there exists a threshold $e_0(\geq 0)$ such that for any $e^\tau_{ti} < e_0$, $\frac{\partial^2 U^\tau}{\partial (e^\tau_{ti})^2} \geq 0$; and for any $e^\tau_{ti} > e_0$, $\frac{\partial^2 U^\tau}{\partial (e^\tau_{ti})^2} < 0$. Hence, we should have $\frac{\partial^2 U^\tau}{\partial (e^\tau_{ti})^2} \geq 0$ for any $e^\tau_{ti} < e_0$, and there should exist another threshold $e_{00}(> e_0)$ such that for any $e^\tau_{ti} < e_{00}$, $\frac{\partial^2 U^\tau}{\partial (e^\tau_{ti})^2} \geq 0$; and for any $e^\tau_{ti} > e_{00}$,

$\frac{\partial^2 U^\tau}{\partial (e_{ti}^\tau)^2} < 0$. Therefore, U^τ is unimodal with mode e_{00}, and hence the first-order condition of the team member's utility-maximization problem in (3.10) and (3.17) is sufficient for optimality. Thus, $e^{*,\tau}$ is the best-response to the team member's utility-maximization problem, and hence a pure-strategy Nash equilibrium. ■

Appendix E

Proofs of Chapter 4

Proof of Lemma 1. From (4.1), suppose that $\frac{p^2}{q_i(q_i+q_u)} - c(2q_i+q_u-p) \geq 0$. Then, the creator's expected profit is as follows:

$$\Pi = \underbrace{\mathbb{P}\left(v_1 \geq \max\left\{\frac{p}{q_i+q_u}, \frac{p^2}{q_i(q_i+q_u)}\right\}\right)}_{\text{customer 1 pledges and makes a comment}} \times \tag{E.1}$$

$$\underbrace{\mathbb{P}\left(v_2 \geq \frac{p}{q_i+q_u}\right)}_{\text{customer 2 pledges}}(2p-2c(q_i+q_u)^2) - C_i q_i$$

$$= \left(\frac{q_i+q_u-p}{q_i+q_u}\right)^2 (2p-2c(q_i+q_u)^2) - C_i q_i. \tag{E.2}$$

In this case, $\mathbb{P}(improve) = \frac{q_i+q_u-p}{q_i+q_u}$ and $\mathbb{P}(success) = \left(\frac{q_i+q_u-p}{q_i+q_u}\right)^2$. Next, suppose that $\frac{p^2}{q_i(q_i+q_u)} - c(2q_i+q_u-p) < 0$. Then, the creator's expected profit is as follows:

$$\Pi = \left(\frac{q_i-p}{q_i}\right)^2 (2p-2cq_i^2) - C_i q_i. \tag{E.3}$$

In this case, $\mathbb{P}(improve) = 0$ and $\mathbb{P}(success) = \left(\frac{q_i-p}{q_i}\right)^2$. ■

Proof of Proposition 1. The first derivative of the left hand-side of the condition in (4.1) is

$$\frac{\partial\left(\frac{p^2}{q_i(q_i+q_u)} - c(2q_i+q_u-p)\right)}{\partial q_i} = -\frac{p^2(2q_i+q_u)}{(q_i(q_i+q_u))^2} - 2c < 0,$$

and we have $\lim_{q_i \to 0} \frac{p^2}{q_i(q_i+q_u)} - c(2q_i+q_u-p) = \infty$ and $\lim_{q_i \to \infty} \frac{p^2}{q_i(q_i+q_u)} - c(2q_i+q_u-p) = -\infty$. Thus, there exists $\overline{q_i}$ (≥ 0) such that $\frac{p^2}{q_i(q_i+q_u)} - c(2q_i+q_u-p) \geq 0$, and hence $q_f = q_i + q_u$ if and only if $q_i \leq \overline{q_i}$. Also, from Lemma 1, customer

1 pledges and makes a comment with probability $\frac{q_i+q_u-p}{q_i+q_u}$, which is increasing in q_i. Thus, the ex-ante probability that the creator improves the product during the campaign is

$$\mathbb{P}(improve) = \left(\frac{q_i+q_u-p}{q_i+q_u}\right)\cdot 1_{\left\{\frac{p^2}{q_i(q_i+q_u)}-c(2q_i+q_u-p)\geq 0\right\}}. \tag{E.4}$$

Therefore, if $q_i \leq \overline{q_i}$, $\mathbb{P}(improve)$ is increasing in q_i; otherwise, $\mathbb{P}(improve) = 0$. ■

Remark 1 *When* $\lim_{q_i\to p}\frac{p^2}{q_i(q_i+q_u)} - c(2q_i+q_u-p) = \frac{p}{(p+q_u)} - c(p+q_u) > 0$*, i.e.,* $c < \frac{p}{(p+q_u)^2}$*, we have* $\overline{q_i} > 0$*. Also, when* $c \geq \frac{p}{(p+q_u)^2}$*,* $\overline{q_i} = 0$*, and hence* $\mathbb{P}(improve) = 0$ *for any* q_i*.*

Proof of Proposition 2. Recall that $q_f = q_i$ or $q_f = q_i + q_u$ depending on whether there is an improvement or not. In both cases, the first derivative of $\mathbb{P}(success)$ with respect to q_i is $\frac{\partial\mathbb{P}(success)}{\partial q_i} = \frac{2p}{q_f^2}\left(\frac{q_f-p}{q_f}\right)$. Since $q_f \geq q_i > p$, $\frac{\partial\mathbb{P}(success)}{\partial q_i} > 0$. Also, in both cases, the second derivative of $\mathbb{P}(success)$ with respect to q_i is $\frac{\partial^2\mathbb{P}(success)}{\partial q_i^2} = 2\left(-\frac{2p}{q_f^3}+\frac{3p^2}{q_f^4}\right)$, and $\frac{\partial^2\mathbb{P}(success)}{\partial q_i^2} > 0$ if and only if $q_f < \frac{3p}{2}$. Recall from the proof of Proposition 1, $q_f = q_i + q_u$ if and only if $q_i \leq \overline{q_i}$. Therefore, when $q_i > \underline{q_i} = \max\{\overline{q_i}, \frac{3p}{2}\}$, $\mathbb{P}(success)$ is concave in q_i. ■

Proof of Proposition 3. We first identify sub-game perfect equilibrium strategies. When customer k's utility function is as in (4.2), in the fourth stage, if $q_f = q_i + q_u$, customer 2 pledges with probability $\frac{(q_i+q_u)-b(q_i+q_u)^2-p}{(q_i+q_u)-b(q_i+q_u)^2}$; and if $q_f = q_i$, customer 2 pledges with probability $\frac{q_i-bq_i^2-p}{q_i-bq_i^2}$. In the third stage, the creator decides whether to improve the product by comparing

$$\Pi^I = \left(\frac{(q_i+q_u)-b(q_i+q_u)^2-p}{(q_i+q_u)-b(q_i+q_u)^2}\right)\left(2p-2c(q_i+q_u)^2\right) - C_iq_i, \text{ and}$$

$$\Pi^{NI} = \left(\frac{q_i-bq_i^2-p}{q_i-bq_i^2}\right)\left(2p-2cq_i^2\right) - C_iq_i.$$

Thus, the creator improves the product if and only if $\Pi^I \geq \Pi^{NI}$, i.e.,

$$I \equiv \frac{p^2(1-b(2q_i+q_u))}{q_i(q_i+q_u)(1-b(q_i+q_u))(1-bq_i)} - c\left(2q_i+q_u-\frac{p}{(1-b(q_i+q_u))(1-bq_i)}\right) \geq 0. \tag{E.5}$$

In the second stage, when customer 1 pledges, she decides whether to make a comment or not by comparing

$$U_1^C = \left(\frac{(q_i+q_u)-b(q_i+q_u)^2-p}{(q_i+q_u)-b(q_i+q_u)^2}\right)\left(v_1((q_i+q_u)-b(q_i+q_u)^2)-p\right) \text{ and}$$

$$U_1^{NC} = \left(\frac{q_i-bq_i^2-p}{q_i-bq_i^2}\right)\left(v_1(q_i-bq_i^2)-p\right).$$

Thus, customer 1 makes a comment if and only if $U_1^C \geq U_1^{NC}$, i.e., $v_1 \geq \frac{p^2}{(q_i-bq_i^2)((q_i+q_u)-b(q_i+q_u)^2)}$. In the first stage, when customer 1 anticipates an improvement (i.e., $I \geq 0$), customer 1 decides whether to pledge or not by comparing $U_1^P = \left(\frac{(q_i+q_u)-b(q_i+q_u)^2-p}{(q_i+q_u)-b(q_i+q_u)^2}\right)\left(v_1((q_i+q_u)-b(q_i+q_u)^2)-p\right)$ and $U_1^{NP}=0$. So, customer 1 pledges if $v_1 \geq \frac{p}{(q_i+q_u)-b(q_i+q_u)^2}$. When (E.5) holds (i.e., $I \geq 0$), she pledges and makes a comment if

$$\begin{aligned} v_1 &\geq \max\left\{\frac{p^2}{(q_i-bq_i^2)((q_i+q_u)-b(q_i+q_u)^2)}, \frac{p}{(q_i+q_u)-b(q_i+q_u)^2}\right\} \\ &= \frac{p}{(q_i+q_u)-b(q_i+q_u)^2}. \end{aligned} \tag{E.6}$$

Note that the equality in (E.6) holds because $\frac{p}{q_i-bq_i^2} < 1$. So, when $I \geq 0$, customer 1 makes a comment whenever she pledges. In the first stage, when $I < 0$, customer 1 decides whether to pledge or not by comparing $U_1^P = \left(\frac{q_i-bq_i^2-p}{q_i-bq_i^2}\right)\left(v_1(q_i-bq_i^2)-p\right)$ and $U_1^{NP}=0$. Thus, customer 1 pledges if and only if $v_1 \geq \frac{p}{q_i-bq_i^2}$.

(a) From (E.5), as $\lim_{q_i\to 0^+} I = \infty$, there exists $\overline{q_i}$ (≥ 0) such that when $q_i \leq \overline{q_i}$, $\mathbb{P}(improve) = \frac{(q_i+q_u)-b(q_i+q_u)^2-p}{(q_i+q_u)-b(q_i+q_u)^2}$ from (E.6). In this case, $\frac{\partial \mathbb{P}(improve)}{\partial q_i} = \frac{p(1-2b(q_i+q_u))}{((q_i+q_u)-b(q_i+q_u)^2)^2} > 0$ if and only if $b(q_i+q_u) < 0.5$. Also, as $\lim_{q_i\to\infty} I = -\infty$, there exists $\underline{q_i}$ ($\in \mathbb{R}\cup\{+\infty\}$) such that when $q_i > \underline{q_i}$, $\mathbb{P}(improve) = 0$.

(b) When both $q_f = q_i$ and $q_f = q_i+q_u$, the probability of success is $\mathbb{P}(success) = \left(\frac{q_f-bq_f^2-p}{q_f-bq_f^2}\right)^2$. The first derivative of $\mathbb{P}(success)$ with respect to q_i is $\frac{\partial \mathbb{P}(success)}{\partial q_i} = 2\left(\frac{q_i-bq_f^2-p}{q_f-bq_f^2}\right)\left(\frac{p(1-2bq_f)}{\left(q_f-bq_f^2\right)^2}\right)$. Thus, in both cases, when $bq_f < 0.5$, $\mathbb{P}(success)$ is increasing in q_i; and when $bq_f > 0.5$, $\mathbb{P}(success)$ is decreasing in q_i. From the proof of Proposition 3(a), when $q_i < \overline{q_i}$, $q_f = q_i+q_u$. Thus, when $q_i < q_i' = \min\{\overline{q_i}, 0.5/b-q_u\}$, $\mathbb{P}(success)$ is increasing in q_i. Also, since $q_f = q_i$ for $q_i > \underline{q_i}$, when $q_i > q_i'' = \max\{\underline{q_i}, 0.5/b\}$, $\mathbb{P}(success)$ is decreasing in q_i. ■

Proof of Proposition 4. From (4.4), the creator's profit Π with improvement under $q_i = q_f^* - q_u$ is larger than the creator's profit Π without improvement under $q_i = q_f^*$. This is because in the former case, the creator saves $C_i q_u$. When there is an improvement, i.e., $I \geq 0$, the creator's profit

$$\Pi(q_i) = \left(1 - \frac{p}{(q_i+q_u) - b(q_i+q_u)^2}\right)^2 \left(2p - 2c(q_i+q_u)^2\right) - C_i q_i. \qquad \text{(E.7)}$$

Note that by Assumption 1, $\Pi(0) > 0$ and also note that $\Pi(q_i) \leq 0$ under any q_i such that $q_i + q_u - b(q_i+q_u)^2 < p$ because no customer pledges under such q_i. Thus, without loss of optimality, we can restrict attention to $\{q_i \geq 0 | q_i + q_u - b(q_i+q_u)^2 \geq p\}$, which is a compact set. In this region, Π is continuous, so by Weierstrass Theorem, there exists a global maximizer q_i^* of Π in (E.7). Because q_i^* is a maximizer, we have $\Pi(q_i^*) \geq \Pi(0) > 0$. Furthermore, for any $q_i \geq 2p/C_i$, we have $\Pi(q_i) \leq 2p - C_i q_i \leq 0$ since $\mathbb{P}(success) = \left(1 - \frac{p}{(q_i+q_u)-b(q_i+q_u)^2}\right)^2 < 1$. Thus, we should have $q_i^* < 2p/C_i$. Because $\lim_{C_i\to\infty} 2p/C_i = 0$, $\lim_{C_i\to\infty} 0 = 0$, and $0 \leq q_i^* < 2p/C_i$, by Squeeze Theorem, $\lim_{C_i\to\infty} q_i^* = 0$.

By Assumption 1, $\lim_{q_i\to 0} I > 0$, and hence there exists $\underline{q_i}$ such that $I > 0$ for any $q_i < \underline{q_i}$. Also, as $\lim_{C_i\to\infty} q_i^* = 0$, there exists $\underline{C_i}$ such that $q_i^* < \underline{q_i}$ and hence $I > 0$ for any $C_i > \underline{C_i}$. Therefore, it is optimal for the creator to make an improvement and set $q_i^* = q_f^* - q_u$ whenever $C_i > \underline{C_i}$. ■

Appendix F

Additional Analysis of Chapter 4

F.1 Details of LDA Model

In this section, we discuss the details of the LDA method (Blei et al. 2003). The LDA method assumes that each document can be represented as a mixture of topics and each topic can be represented as a mixture of words. So, taking a corpus of documents as an input, the LDA method outputs the distribution of topics in each document and the distribution of words in each topic. The distribution of topics in each document is a vector of weights, where the weight of each topic represents how intensively the topic is used in the document. Similarly, the distribution of words in each topic represents the frequency of words. As product descriptions on campaign pages include explanation of product features, the LDA method is suitable for extracting topics related to these features from product descriptions (e.g., Tirunillai and Tellis 2014, Toubia et al. 2019).

To train the LDA model, we start with 43,536 initial and final descriptions of products in 21,768 campaigns. Following the standard practice (e.g., Tirunillai and Tellis 2014, Toubia et al. 2019), we first pre-process descriptions (e.g., remove stop words, remove descriptions that contain less than ten words and that are not written in English, and stem words). We then fit the LDA model on the corpus of the remaining 42,564 descriptions (from 21,380 campaigns, 196 of which only have a single description after pre-processing) using the standard hyperparameters of $\alpha = 1$ and $\beta = 0.01$ (e.g., Steyvers and Griffiths 2007, Toubia et al. 2019, Ghose et al. 2019), where α and β are parameters of the prior Dirichlet distributions of topics in documents and words in topics, respectively (Blei et al. 2003). Following the rule of $\alpha = 50/T$, where T is the number of topics, (e.g., Steyvers and Griffiths 2007, Tirunillai and Tellis 2014), we set the number of topics to 50. From the trained LDA model, we obtain weights of words in each of 50 topics and weights of topics in each of 42,564 descriptions; all weights are positive. See Figure F.1 for an example.

HAIZE is a new type of navigation system designed for urban cyclists. It works like a magic compass that, instead of pointing north, points to the destination you set in our app. HAIZE leaves you free to choose your own route through the city. It also makes your ride safer by letting you keep your phone in your pocket. HAIZE can easily be attached to any bike and is small and built-to-last so that you can always bring it with you. Our companion app will be available for both Android- and iOS-based smartphones. HAIZE is easy to use. Simply attach it to your bike, set the destination in our companion app, put your phone away and let HAIZE guide you. Its simple LED-based display will point you in the right direction and let you know the distance to your destination. You can select between two different modes of navigation. The compass mode points you in the direction of your final destination and lets you explore along the way. The navigation mode sets a specific route and gives you turn-by-turn directions. Check out the video of HAIZE in action: Check out this video of HAIZE and our app: HAIZE is stripped down to the essence, both the led-based display and the aluminium body combine simplicity with usability. The HAIZE led-display gives you all the information you need at a glance; direction and distance. HAIZE lets you focus on the road and explore the city. If you are interested in additional details about your trip, you can always check the app after your ride. Here you will find stats about your trips and saved routes. The rubber band integrated in the HAIZE body allows you to easily attach it on any bike and keep HAIZE comfortably in your pocket when leaving your bike on the street. The body of HAIZE has been created using machined anodized aluminium, making it both sturdy and stylish. The magnetometer tracks the direction to the destination The accelerometer and gyroscope are used to determine HAIZEs position The light sensor is used to regulate the LED brightness to accommodate different lighting conditions The battery with 300mAh delivers 2 weeks of normal usage and can be easily recharged using a micro usb connection HAIZE is connected to our app via a low power bluetooth 4.0 connection As you can see there is a lot of technology packed into HAIZE...

Existing Topic	0.058*"control" + 0.052*"smart" + 0.037*"sensor" + 0.030*"home" + 0.021*"mode" + 0.021*"button" + 0.019*"connect" + 0.017*"set" + 0.017*"remot" + 0.017*"monitor"

(a) Excerpt from the initial description where words corresponding to an existing topic are highlighted.

HAIZE is minimalist navigation device for urban cyclists. It is designed focusing on high quality materials, style and simplicity. HAIZE works like a magic compass that, instead of pointing north, points to the destination you set in our app. HAIZE leaves you free to choose your own route through the city. It also makes your ride safer by letting you keep your phone in your pocket. If you feel like sticking to the main roads HAIZE also offers turn-by-turn navigation. WHY YOU NEED ONE? 2 navigation modes: "turn-by-turn mode" and "compass mode" Self-regulating LED display for perfect day and night-time visibility Sturdy and high-quality materials Can be used on any bike Wristband to use HAIZE while running, hiking or geo-caching HAIZE is easy to use. Simply attach it to your bike, set the destination in our companion app, put your phone away and let HAIZE guide you. Its simple LED-based display will point you in the right direction and let you know the distance to your destination. You can select between two different modes of navigation. The compass mode points you in the direction of your final destination and lets you explore along the way. The navigation mode sets a specific route and gives you turn-by-turn directions. Check out the video of HAIZE in action: Check out this video of HAIZE and our app: HAIZE is stripped down to the essence, both the led-based display and the aluminium body combine simplicity with usability. HAIZE is made out of aeronautic-grade sandblasted aluminium and shockproof glass. A tested and durable combination that stands out from the first moment. To know a direction you don\'t need to get distracted processing numbers or symbols on a screen. The HAIZE led-display gives you all the information you need at a glance; direction and distance in a simple and intuitive way. HAIZE lets you focus on the road and explore the city. If you are interested in additional details about your trip, you can check them in the app. There you will find stats about your trips and saved routes. HAIZE automatically regulates the brightness of the LEDs to work perfectly under any light condition. It will help you navigate the city no matter what time of the day! The elastic band integrated in the HAIZE body allows you to easily attach it on any bike and keep HAIZE comfortably in your pocket when leaving your bike on the street. And it always stays in place! HAIZE was originally designed for urban cycling. But many of our backers wanted to use it in other situations. That is why we decided to give every backer a wristband to bring HAIZE along to any activity. Be it for hiking, running, or geo-caching. And of course finding your way back to last years perfect mushroom spot. HAIZE will be able to guide you to the best spots while wandering freely. And you can be confident about getting back to the basecamp no matter how many turns you make. The HAIZE wristband is made from high quality silicone and fits perfectly around the aluminum case, allowing you to take HAIZE everywhere. The magnetometer tracks the direction to the destination The accelerometer and gyroscope are used to determine HAIZEs position The light sensor is used to regulate the LED brightness to accommodate different lighting conditions The battery with 300mAh delivers 2 weeks of normal usage and can be easily recharged using a micro usb connection HAIZE is connected to our app via a low power bluetooth 4.0 connection As you can see there is a lot of technology packed into HAIZE...

Existing Topic	0.058*"control" + 0.052*"smart" + 0.037*"sensor" + 0.030*"home" + 0.021*"mode" + 0.021*"button" + 0.019*"connect" + 0.017*"set" + 0.017*"remot" + 0.017*"monitor"
Added Topic	0.059*"materi" + 0.038*"weight" + 0.032*"high" + 0.030*"durabl" + 0.027*"surfac" + 0.023*"made" + 0.022*"blade" + 0.021*"strong" + 0.020*"strength" + 0.020*"resist"

(b) Excerpt from the final description where words corresponding to an existing topic and an added topic are highlighted.

Figure F.1: Initial and final descriptions of the product HAIZE with examples of an "existing" topic that is available in the initial description and an "added" topic that is added to the final description. Tables below excerpts illustrate the most relevant ten words with their weights in these topics.

F.2 Cost of Commenting

In this section, we consider the case where customer 1 incurs cost of d (> 0) when she makes a comment. Suppose that condition (4.1) holds so that customer 1 anticipates an improvement. Then, customer 1 decides whether to make a comment or not by comparing U_1^C when she makes a comment and U_1^{NC} when she does not make a comment, where

$$U_1^C = \left(\frac{q_i+q_u-p}{q_i+q_u}\right)(v_1(q_i+q_u)-p)-d \text{ and } U_1^{NC} = \left(\frac{q_i-p}{q_i}\right)(v_1 q_i - p).$$

Thus, customer 1 makes a comment if and only if $U_1^C \geq U_1^{NC}$, i.e., $v_1 \geq \frac{p^2}{q_i(q_i+q_u)} + \frac{d}{q_u}$. Suppose that $v_1 \geq \frac{p^2}{q_i(q_i+q_u)} + \frac{d}{q_u}$. Then, in the first stage, customer 1 decides whether to pledge or not by comparing U_1^P when she pledges and U_1^{NP} when she does not pledge, where

$$U_1^P = \left(\frac{q_i+q_u-p}{q_i+q_u}\right)(v_1(q_i+q_u)-p)-d \text{ and } U_1^{NP} = 0.$$

Thus, customer 1 pledges if $U_1^P \geq U_1^{NP}$, i.e., $v_1 \geq \frac{p}{q_i+q_u} + \frac{d}{q_i+q_u-p}$, and hence customer 1 pledges and makes a comment if $v_1 \geq \max\left\{\frac{p^2}{q_i(q_i+q_u)} + \frac{d}{q_u}, \frac{p}{q_i+q_u} + \frac{d}{q_i+q_u-p}\right\}$. Next, suppose that $v_1 < \frac{p^2}{q_i(q_i+q_u)} + \frac{d}{q_u}$. Then, customer 1 decides whether to pledge or not by comparing $U_1^P = \left(\frac{q_i-p}{q_i}\right)(v_1 q_i - p)$ and $U_1^{NP} = 0$. Thus, customer 1 pledges if and only if $v_1 \geq \frac{p}{q_i}$. Therefore, customer 1 pledges but does make a comment if $\frac{p}{q_i} \leq v_1 \leq \frac{p^2}{q_i(q_i+q_u)} + \frac{d}{q_u}$. Finally, suppose that condition (4.1) is violated. Then, customer 1 pledges if and only if $v_1 \geq \frac{p}{q_i}$. We characterize all possible outcomes of this model in the following lemma.

Lemma 11 ***(a)*** *Suppose that* $\frac{p^2}{q_i(q_i+q_u)} - c(2q_i+q_u-p) \geq 0$.

(i) *Suppose that* $1 \geq \frac{p}{q_i+q_u} + \frac{d}{q_i+q_u-p} \geq \frac{p^2}{q_i(q_i+q_u)} + \frac{d}{q_u}$.

$$\mathbb{P}(success) = \left(1 - \frac{p}{q_i+q_u} - \frac{d}{q_i+q_u-p}\right)\left(1 - \frac{p}{q_i+q_u}\right). \tag{F.1}$$

(ii) *Suppose that* $\frac{p}{q_i+q_u} + \frac{d}{q_i+q_u-p} < \frac{p^2}{q_i(q_i+q_u)} + \frac{d}{q_u} \leq 1$.

$$\mathbb{P}(success) = \left(1 - \frac{p^2}{q_i(q_i+q_u)} - \frac{d}{q_u}\right)\left(1 - \frac{p}{q_i+q_u}\right) + \left(\frac{p^2}{q_i(q_i+q_u)} + \frac{d}{q_u} - \frac{p}{q_i}\right)\left(1 - \frac{p}{q_i}\right). \tag{F.2}$$

(iii) *Suppose that* $\frac{p}{q_i+q_u} + \frac{d}{q_i+q_u-p} \leq 1 < \frac{p^2}{q_i(q_i+q_u)} + \frac{d}{q_u}$.

$$\mathbb{P}(success) = \left(1 - \frac{p}{q_i}\right)^2. \tag{F.3}$$

(b) *Suppose that* $\frac{p^2}{q_i(q_i+q_u)} - c(2q_i+q_u-p) < 0$.

$$\mathbb{P}(success) = \left(1 - \frac{p}{q_i}\right)^2. \tag{F.4}$$

Proof of Lemma 11. Overall, there are four possible cases:

Case (a). Suppose that $\frac{p^2}{q_i(q_i+q_u)} - c(2q_i + q_u - p) \geq 0$ such that the creator is willing to improve the product further if customer 1 makes a comment.

Case (a-i). Suppose that $1 \geq \frac{p}{q_i+q_u} + \frac{d}{q_i+q_u-p} \geq \frac{p^2}{q_i(q_i+q_u)} + \frac{d}{q_u}$ such that customer 1 makes a comment whenever she pledges. (Note that when $\frac{p}{q_i+q_u} + \frac{d}{q_i+q_u-p} > 1$, customer 1 never pledges.) In this case, the creator's expected profit is as follows:

$$\Pi = \left(1 - \frac{p}{q_i+q_u} - \frac{d}{q_i+q_u-p}\right)\left(1 - \frac{p}{q_i+q_u}\right)(2p - 2c(q_i+q_u)^2) - C_i q_i.$$

So, in this case probability of campaign success is as in (F.1).

Case (a-ii). Suppose that $\frac{p}{q_i+q_u} + \frac{d}{q_i+q_u-p} < \frac{p^2}{q_i(q_i+q_u)} + \frac{d}{q_u} \leq 1$ such that customer 1 *may not* make a comment although she pledges. In this case, the creator's expected profit is as follows:

$$\begin{aligned}\Pi = &\left(1 - \frac{p^2}{q_i(q_i+q_u)} - \frac{d}{q_u}\right)\left(1 - \frac{p}{q_i+q_u}\right)(2p - 2c(q_i+q_u)^2)\\ &+ \left(\frac{p^2}{q_i(q_i+q_u)} + \frac{d}{q_u} - \frac{p}{q_i}\right)\left(1 - \frac{p}{q_i}\right)(2p - 2cq_i^2) - C_i q_i.\end{aligned}$$

So, in this case probability of campaign success is as in (F.2).

Case (a-iii). Suppose that $\frac{p}{q_i+q_u} + \frac{d}{q_i+q_u-p} \leq 1 < \frac{p^2}{q_i(q_i+q_u)} + \frac{d}{q_u}$ such that customer 1 *never* makes a comment although she may pledge. In this case, the creator's expected profit is as follows:

$$\Pi = \mathbb{P}\left(v_1 \geq \frac{p}{q_i}\right) \cdot \mathbb{P}\left(v_2 \geq \frac{p}{q_i}\right)(2p - 2cq_i^2) - C_i q_i = \left(1 - \frac{p}{q_i}\right)^2 (2p - 2cq_i^2) - C_i q_i.$$

So, in this case probability of campaign success is is as in (F.3).

Case (b). Suppose that $\frac{p^2}{q_i(q_i+q_u)} - c(2q_i + q_u - p) < 0$ such that the creator is not willing to improve the product further. In this case, the creator's expected profit is as follows:

$$\Pi = \left(1 - \frac{p}{q_i}\right)^2 (2p - 2cq_i^2) - C_i q_i.$$

So, in this case probability of campaign success is as in (F.4). ■

We numerically analyze these cases according to the setting where we select p from Uniform(0,1), c from Uniform(0,0.01), q_u from Uniform(0,0.5), and cost of commenting d from Uniform(0,0.01). Taking the average of randomly generated

10,000 instances, we show that our theoretical predictions hold.

F.3 Robustness Checks

In this section, we provide results of probit and IV models for robustness checks that we discuss in §4.4.4.

Table F.1: Spline regressions for second-stage estimations in IV models.

	Second Stage of IV Model 1	Second Stage of IV Model 2
	Product improvement	Campaign success
Initial enhancement level (≤ 30)	0.053***	0.073***
	(0.008)	(0.008)
Initial enhancement level (> 30)	-0.115***	-0.056**
	(0.029)	(0.023)
Residuals	-0.036***	-0.046***
	(0.008)	(0.009)
Competition	-0.572	-1.095***
	(0.365)	(0.336)
Technology	0.006	-0.041
	(0.024)	(0.025)
Goal (ln)	-0.022***	-0.344***
	(0.008)	(0.009)
Duration	0.003***	-0.001
	(0.001)	(0.001)
Videos	-0.007	-0.035***
	(0.014)	(0.013)
Pictures	0.003	0.021***
	(0.002)	(0.003)
Risk-section length	-0.000	-0.000**
	(0.000)	(0.000)
Pledge price	0.000	0.000***
	(0.000)	(0.000)
Delivery time	0.001	-0.006**
	(0.002)	(0.003)
Creator experience	0.021*	0.268***
	(0.012)	(0.025)
Individual	-0.148***	-0.360***
	(0.024)	(0.021)
Constant	-0.929***	2.292***
	(0.083)	(0.088)
Observations	18,173	18,173

Nonparametric bootstrap standard errors (100 replications) in parentheses.
**** p<0.01, ** p<0.05, * p<0.1*

Table F.2: Equal time periods before and after IV.

	Probit Model 1 Product improvement	Probit Model 2 Campaign success	First Stage of IV Models 1 and 2 Initial enhancement level	Second Stage of IV Model 1 Product improvement	Second Stage of IV Model 2 Campaign success
Initial enhancement level	.071***	.089***		.086***	.138***
	(.005)	(.005)		(.017)	(.016)
Initial enhancement level × Initial enhancement level	-.002***	-.002***		-.002***	-.003***
	(.000)	(.000)		(.000)	(.000)
Competition	-1.569***	-2.472***	-9.545***	-1.22**	-1.044*
	(.322)	(.314)	(2.015)	(.597)	(.57)
Category: Technology	.075***	.003	1.289***	.06*	-.058
	(.024)	(.026)	(.133)	(.036)	(.037)
Goal (ln)	-.01	-.325***	.691***	-.018	-.357***
	(.009)	(.011)	(.043)	(.014)	(.016)
Duration	.004***	-.002*	-.022***	.004***	-.001
	(.001)	(.001)	(.006)	(.001)	(.001)
Videos	.025	-.028	.705***	.016	-.06***
	(.016)	(.019)	(.143)	(.02)	(.023)
Pictures	.015***	.032***	.328***	.011**	.017***
	(.001)	(.002)	(.009)	(.005)	(.006)
Risk-section length	0***	0**	.013***	0	0*
	(.000)	(.000)	(.001)	(.000)	(.000)
Pledge price	0	0***	0	0	0***
	(.000)	(.000)	(.000)	(.000)	(.000)
Delivery time	.003	-.002	.013	.003	-.002
	(.003)	(.003)	(.012)	(.003)	(.003)
Creator experience	.058**	.329***	.322***	.055**	.319***
	(.026)	(.033)	(.12)	(.026)	(.034)
Individual	-.186***	-.388***	-.093	-.185***	-.387***
	(.032)	(.029)	(.129)	(.032)	(.029)
Before relaxation of rules			2.758***		
			(.209)		
Residuals				-.012	-.046***
				(.016)	(.016)
Residuals×Residuals				0*	.001**
				(.000)	(.000)
Constant	-.959***	2.32***	-1.838***	-1***	2.19***
	(.106)	(.108)	(.49)	(.109)	(.114)
Observations	11764	11764	11764	11764	11764
pseudo R^2	.06	.187	.416	.061	.189

Nonparametric bootstrap standard errors (100 replications) in parentheses.
**** p<.01, ** p<.05, * p<.1*

Table F.3: Campaigns where final enhancement level is greater than or equal to initial enhancement level.

	Probit Model 1	Probit Model 2	First Stage of IV Models 1 and 2	Second Stage of IV Model 1	Second Stage of IV Model 2
	Product improvement	Campaign success	Initial enhancement level	Product improvement	Campaign success
Initial enhancement level	.078***	.09***		.106***	.13***
	(.004)	(.004)		(.009)	(.01)
Initial enhancement level × Initial enhancement level	-.002***	-.002***		-.002***	-.003***
	(.000)	(.000)		(.000)	(.000)
Competition	-1.294***	-2.117***	-1.574	-.676**	-1.284***
	(.24)	(.296)	(1.33)	(.274)	(.373)
Category: Technology	.059**	.035	.965***	.03	-.003
	(.025)	(.026)	(.095)	(.026)	(.029)
Goal (ln)	-.006	-.334***	.571***	-.022**	-.356***
	(.007)	(.008)	(.032)	(.009)	(.009)
Duration	.003***	-.002**	-.016***	.004***	-.002
	(.001)	(.001)	(.004)	(.001)	(.001)
Videos	.028**	-.007	.681***	.009	-.032**
	(.014)	(.015)	(.101)	(.015)	(.015)
Pictures	.015***	.035***	.298***	.006**	.024***
	(.001)	(.001)	(.006)	(.003)	(.003)
Risk-section length	0***	0***	.015***	0	0
	(.000)	(.000)	(.001)	(.000)	(.000)
Pledge price	0	0***	0	0	0***
	(.000)	(.000)	(.000)	(.000)	(.000)
Delivery time	.003	-.005**	.011	.002	-.005**
	(.002)	(.002)	(.009)	(.002)	(.002)
Creator experience	.017	.255***	.221***	.015	.254***
	(.013)	(.026)	(.063)	(.013)	(.026)
Individual	-.145***	-.34***	-.009	-.146***	-.343***
	(.026)	(.029)	(.098)	(.027)	(.028)
Before relaxation of rules			3.606***		
			(.145)		
Residuals				-.029***	-.039***
				(.008)	(.01)
Residuals×Residuals				0	0*
				(.000)	(.000)
Constant	-1.066***	2.257***	-2.679***	-1.107***	2.203***
	(.07)	(.079)	(.369)	(.073)	(.08)
Observations	17005	17005	17005	17005	17005
pseudo R^2	.074	.211	.398	.074	.212

Nonparametric bootstrap standard errors (100 replications) in parentheses.
**** p<.01, ** p<.05, * p<.1*

Table F.4: Treating cancelled campaigns as failed campaigns.

	Probit Model 1	Probit Model 2	First Stage of IV Models 1 and 2	Second Stage of IV Model 1	Second Stage of IV Model 2
	Product improvement	Campaign success	Initial enhancement level	Product improvement	Campaign success
Initial enhancement level	.07***	.086***		.103***	.123***
	(.004)	(.004)		(.007)	(.008)
Initial enhancement level × Initial enhancement level	-.002***	-.002***		-.002***	-.002***
	(.000)	(.000)		(.000)	(.000)
Competition	-.898***	-1.763***	-1.678	-.386	-1.046***
	(.238)	(.253)	(1.118)	(.241)	(.291)
Category: Technology	.054**	.027	1.031***	.026	-.011
	(.021)	(.019)	(.092)	(.022)	(.02)
Goal (ln)	-.005	-.329***	.6***	-.02**	-.351***
	(.007)	(.008)	(.032)	(.008)	(.008)
Duration	.003***	-.003***	-.014***	.003***	-.003***
	(.001)	(.001)	(.005)	(.001)	(.001)
Videos	.018*	-.018	.723***	-.002	-.043***
	(.01)	(.011)	(.084)	(.011)	(.013)
Pictures	.009***	.027***	.287***	.001	.016***
	(.001)	(.001)	(.006)	(.002)	(.003)
Risk-section length	0***	0***	.016***	0**	0**
	(.000)	(.000)	(.001)	(.000)	(.000)
Pledge price	0	0***	0	0	0***
	(.000)	(.000)	(.000)	(.000)	(.000)
Delivery time	.001	-.005**	.007	.001	-.005**
	(.002)	(.002)	(.009)	(.002)	(.002)
Creator experience	.016	.154***	.177***	.014	.153***
	(.01)	(.016)	(.042)	(.01)	(.017)
Individual	-.152***	-.336***	-.072	-.153***	-.337***
	(.023)	(.024)	(.101)	(.023)	(.024)
Before relaxation of rules			3.562***		
			(.134)		
Residuals				-.028***	-.037***
				(.007)	(.008)
Residuals×Residuals				.001***	0*
				(.000)	(.000)
Constant	-1.086***	2.218***	-2.738***	-1.139***	2.167***
	(.078)	(.085)	(.366)	(.075)	(.087)
Observations	21184	21184	21184	21184	21184
pseudo R^2	.049	.177	.399	.051	.178

Nonparametric bootstrap standard errors (100 replications) in parentheses.
**** p<.01, ** p<.05, * p<.1*

Table F.5: When the number of topics is set to 40 in LDA Model.

	Probit Model 1 Product improvement	Probit Model 2 Campaign success	First Stage of IV Models 1 and 2 Initial enhancement level	Second Stage of IV Model 1 Product improvement	Second Stage of IV Model 2 Campaign success
Initial enhancement level	.074***	.103***		.114***	.147***
	(.005)	(.005)		(.011)	(.011)
Initial enhancement level × Initial enhancement level	-.003***	-.003***		-.003***	-.003***
	(.000)	(.000)		(.000)	(.000)
Competition	-1.189***	-2.033***	-2.338*	-.379	-1.166***
	(.257)	(.286)	(1.272)	(.317)	(.331)
Category: Technology	.022	.033	.693***	-.007	.003
	(.022)	(.021)	(.095)	(.024)	(.022)
Goal (ln)	-.007	-.334***	.534***	-.027***	-.356***
	(.008)	(.008)	(.031)	(.009)	(.009)
Duration	.003***	-.002*	-.018***	.004***	-.001
	(.001)	(.001)	(.004)	(.001)	(.001)
Videos	.002	-.006	.609***	-.021	-.03**
	(.012)	(.012)	(.077)	(.014)	(.013)
Pictures	.012***	.033***	.266***	.002	.022***
	(.001)	(.001)	(.005)	(.003)	(.003)
Risk-section length	0***	0***	.013***	0*	0**
	(.000)	(.000)	(.001)	(.000)	(.000)
Pledge price	0*	0***	0	0*	0***
	(.000)	(.000)	(.000)	(.000)	(.000)
Delivery time	0	-.005**	.008	0	-.005*
	(.002)	(.002)	(.009)	(.002)	(.002)
Creator experience	.009	.256***	.163***	.008	.258***
	(.012)	(.023)	(.056)	(.012)	(.023)
Individual	-.155***	-.359***	-.097	-.153***	-.359***
	(.021)	(.02)	(.092)	(.021)	(.019)
Before relaxation of rules			3.245***		
			(.132)		
Residuals				-.04***	-.043***
				(.01)	(.01)
Residuals×Residuals				0	.001**
				(.000)	(.000)
Constant	-1.028***	2.192***	-1.086***	-1.138***	2.078***
	(.08)	(.075)	(.318)	(.085)	(.082)
Observations	18173	18173	18173	18173	18173
pseudo R^2	.044	.206	.411	.045	.207

Nonparametric bootstrap standard errors (100 replications) in parentheses.
**** p<.01, ** p<.05, * p<.1*

Table F.6: When the number of topics is set to 60 in LDA Model.

	Probit Model 1	Probit Model 2	First Stage of IV Models 1 and 2	Second Stage of IV Model 1	Second Stage of IV Model 2
	Product improvement	Campaign success	Initial enhancement level	Product improvement	Campaign success
Initial enhancement level	.066***	.073***		.095***	.116***
	(.004)	(.004)		(.008)	(.009)
Initial enhancement level × Initial enhancement level	-.002***	-.002***		-.002***	-.002***
	(.000)	(.000)		(.000)	(.000)
Competition	-.944***	-2.088***	-2.833*	-.287	-1.128***
	(.257)	(.281)	(1.578)	(.313)	(.328)
Category: Technology	.017	.019	1.239***	-.018	-.031
	(.018)	(.021)	(.114)	(.021)	(.023)
Goal (ln)	-.008	-.328***	.608***	-.025***	-.352***
	(.007)	(.008)	(.037)	(.009)	(.009)
Duration	.003***	-.002*	-.022***	.003***	-.001
	(.001)	(.001)	(.005)	(.001)	(.001)
Videos	.014	-.004	.732***	-.006	-.032**
	(.013)	(.012)	(.1)	(.015)	(.013)
Pictures	.013***	.034***	.319***	.004	.021***
	(.001)	(.001)	(.006)	(.002)	(.003)
Risk-section length	0***	0***	.016***	0	0**
	(.000)	(.000)	(.001)	(.000)	(.000)
Pledge price	0	0***	0*	0	0***
	(.000)	(.000)	(.000)	(.000)	(.000)
Delivery time	.003	-.005**	.016	.002	-.005**
	(.002)	(.002)	(.011)	(.002)	(.002)
Creator experience	.016	.259***	.199***	.015	.26***
	(.012)	(.023)	(.068)	(.012)	(.024)
Individual	-.143***	-.357***	.063	-.146***	-.364***
	(.027)	(.02)	(.115)	(.027)	(.02)
Before relaxation of rules			3.74***		
			(.166)		
Residuals				-.028***	-.041***
				(.007)	(.008)
Residuals×Residuals				0**	0**
				(.000)	(.000)
Constant	-1.035***	2.309***	-3.2***	-1.071***	2.266***
	(.083)	(.074)	(.412)	(.083)	(.077)
Observations	18173	18173	18173	18173	18173
pseudo R^2	.061	.201	.391	.062	.203

Nonparametric bootstrap standard errors (100 replications) in parentheses.
**** p<.01, ** p<.05, * p<.1*

Table F.7: When the threshold is set to 8 while counting the number of topics.

	Probit Model 1	Probit Model 2	First Stage of IV Models 1 and 2	Second Stage of IV Model 1	Second Stage of IV Model 2
	Product improvement	Campaign success	Initial enhancement level	Product improvement	Campaign success
Initial enhancement level	.068***	.081***		.106***	.116***
	(.004)	(.004)		(.009)	(.009)
Initial enhancement level × Initial enhancement level	-.002***	-.002***		-.002***	-.002***
	(.000)	(.000)		(.000)	(.000)
Competition	-.983***	-2.021***	-3.273**	.036	-1.131***
	(.256)	(.287)	(1.599)	(.313)	(.33)
Category: Technology	.031	.029	1.026***	-.012	-.007
	(.02)	(.021)	(.118)	(.023)	(.022)
Goal (ln)	-.001	-.335***	.695***	-.027***	-.358***
	(.007)	(.008)	(.04)	(.008)	(.009)
Duration	.003***	-.002*	-.021***	.003***	-.001
	(.001)	(.001)	(.006)	(.001)	(.001)
Videos	.014	-.007	.777***	-.015	-.031**
	(.011)	(.012)	(.095)	(.014)	(.013)
Pictures	.011***	.033***	.34***	-.002	.022***
	(.001)	(.001)	(.006)	(.003)	(.003)
Risk-section length	0***	0***	.017***	0**	0**
	(.000)	(.000)	(.002)	(.000)	(.000)
Pledge price	0	0***	0	0	0***
	(.000)	(.000)	(.000)	(.000)	(.000)
Delivery time	-.001	-.005**	.01	-.001	-.005**
	(.002)	(.002)	(.011)	(.002)	(.002)
Creator experience	.025**	.257***	.218***	.024*	.258***
	(.012)	(.022)	(.071)	(.013)	(.023)
Individual	-.172***	-.358***	-.109	-.17***	-.358***
	(.022)	(.02)	(.119)	(.023)	(.019)
Before relaxation of rules			4.031***		
			(.163)		
Residuals				-.04***	-.035***
				(.008)	(.008)
Residuals×Residuals				0	0
				(.000)	(.000)
Constant	-1.076***	2.219***	-1.928***	-1.181***	2.126***
	(.071)	(.074)	(.404)	(.077)	(.08)
Observations	18173	18173	18173	18173	18173
pseudo R^2	.051	.206	.411	.053	.207

Nonparametric bootstrap standard errors (100 replications) in parentheses.
**** p<.01, ** p<.05, * p<.1*

Table F.8: When the threshold is set to 12 while counting the number of topics.

	Probit Model 1	Probit Model 2	First Stage of IV Models 1 and 2	Second Stage of IV Model 1	Second Stage of IV Model 2
	Product improvement	Campaign success	Initial enhancement level	Product improvement	Campaign success
Initial enhancement level	.076***	.092***		.123***	.143***
	(.004)	(.005)		(.01)	(.011)
Initial enhancement level × Initial enhancement level	-.002***	-.003***		-.003***	-.003***
	(.000)	(.000)	(.000)	(.000)	(.000)
Competition	-.911***	-2.079***	-2.56**	-.025	-1.129***
	(.254)	(.282)	(1.291)	(.311)	(.329)
Category: Technology	.009	.019	1.017***	-.039*	-.031
	(.019)	(.021)	(.092)	(.022)	(.023)
Goal (ln)	.001	-.328***	.495***	-.02**	-.352***
	(.008)	(.008)	(.031)	(.009)	(.009)
Duration	.003***	-.002*	-.017***	.004***	-.001
	(.001)	(.001)	(.004)	(.001)	(.001)
Videos	.007	-.004	.613***	-.021	-.032**
	(.012)	(.012)	(.08)	(.014)	(.013)
Pictures	.012***	.034***	.262***	.001	.021***
	(.001)	(.001)	(.005)	(.003)	(.003)
Risk-section length	0***	0***	.013***	0***	0**
	(.000)	(.000)	(.001)	(.000)	(.000)
Pledge price	0	0***	0*	0	0***
	(.000)	(.000)	(.000)	(.000)	(.000)
Delivery time	.003	-.005**	.01	.003	-.005**
	(.002)	(.002)	(.009)	(.002)	(.002)
Creator experience	.018	.258***	.187***	.015	.258***
	(.013)	(.023)	(.055)	(.013)	(.023)
Individual	-.14***	-.356***	-.003	-.142***	-.36***
	(.025)	(.02)	(.098)	(.025)	(.02)
Before relaxation of rules			3.043***		
			(.139)		
Residuals				-.046***	-.049***
				(.009)	(.01)
Residuals×Residuals				.001*	.001**
				(.000)	(.000)
Constant	-1.156***	2.295***	-2.364***	-1.213***	2.241***
	(.092)	(.074)	(.347)	(.094)	(.077)
Observations	18173	18173	18173	18173	18173
pseudo R^2	.056	.203	.389	.057	.204

Nonparametric bootstrap standard errors (100 replications) in parentheses.
**** p<.01, ** p<.05, * p<.1*

Table F.9: Control for competition in the first week of each campaign.

	Probit Model 1	First Stage of IV Model 1	Second Stage of IV Model 1
	Product improvement	Initial enhancement level	Product improvement
Initial enhancement level	.072***		.104***
	(.004)		(.009)
Initial enhancement level × Initial enhancement level	-.002***		-.002***
	(.000)		(.000)
Competition	-.996***	-2.614**	-.499*
	(.238)	(1.293)	(.277)
Category: Technology	.053**	1.003***	.023
	(.023)	(.103)	(.025)
Goal (ln)	-.006	.58***	-.024***
	(.008)	(.034)	(.009)
Duration	.003***	-.019***	.003***
	(.001)	(.005)	(.001)
Videos	.014	.697***	-.007
	(.012)	(.089)	(.014)
Pictures	.013***	.296***	.003
	(.001)	(.006)	(.003)
Risk-section length	0***	.015***	0
	(.000)	(.001)	(.000)
Pledge price	0	0	0
	(.000)	(.000)	(.000)
Delivery time	.002	.011	.002
	(.002)	(.01)	(.002)
Creator experience	.02	.203***	.018
	(.015)	(.062)	(.015)
Individual	-.139***	-.037	-.14***
	(.024)	(.107)	(.024)
Before relaxation of rules		3.51***	
		(.15)	
Residuals			-.032***
			(.008)
Residuals×Residuals			0
			(.000)
Constant	-1.082***	-2.277***	-1.107***
	(.085)	(.348)	(.086)
Observations	18173	18173	18173
pseudo R^2	.056	.399	.057

Nonparametric bootstrap standard errors (100 replications) in parentheses.
**** p<.01, ** p<.05, * p<.1*

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