# Towards a Traffic-Optimal Large-Scale Optical Network Topology Design


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***Abstract*—Designing optical networks for maximum throughput, under diverse traffic demands, is an NP-hard problem. We parameterise the relationship between demand and topology through a polynomial-time objective function, and show it is highly correlated to network throughput, enabling topology design, optimally tailored to the traffic demand.**



***Keywords*—topology design, maximum throughput, demand weighted cost**


## I. Introduction

Wavelength-division-multiplexed (WDM) optical networks provide high-capacity, low-latency and cost-effective solutions for communication services at all time- and distance-scales. One of the important constraints in optical networks is the physical topology, which determines the available routes between nodes, and impacts the performance (including throughput, latency, resilience) and capital/operating expenditure (CAPEX/OPEX) of the network.

Physical topology design has been a long-standing NP-hard optimisation problem for the most commonly used objectives since optical networking emerged [1]. Previous works have mainly focused on minimising network cost [2]–[4], minimising wavelength requirements [5]–[7], power consumption and resilience [8] or the combination of these. However, it is also important to explore the physical topology design problem with the goal to maximise network throughput (the total achievable bitrate of all lightpaths serving a given traffic demand), especially for static or slow-changing traffic scenarios such as backbone networks. Including throughput in topology design will ensure that growing and changing bandwidth requirements can be accommodated, a key requirement of adaptive networks.

The challenge of designing networks with maximum throughput is two-fold, with NP-hard complexity at each stage. First, evaluating the network throughput for a given topology and demand, where conventional methods such as integer linear programming (ILP) are too computationally complex. The second is the search for a near-optimal topology, in a huge solution space of possible topologies, given a number of nodes, total edge length and traffic demand. The combination of the two challenges leads to infeasible computation times especially for large-scale (e.g. 100 node) topology design.


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In this paper, we propose a new, computationally-efficient optimisation objective for maximising network throughput. By parameterising the relationship between the topology and demand, a new, polynomial-time objective function, highly correlated to throughput, is proposed to simplify the analysis. Implementing this objective, within a topology search algorithm, enables the design of large-scale topologies with significant throughput enhancement, for a given demand. The proposed objective function is used as part of a (i) topology selection method with a generative graph model: Prufer sequence (PS) [7] (ii) genetic algorithm optimisation.

## II. Throughput Analysis and Estimation

To reduce the computational complexity when calculating network throughput, the relationship between the topology, traffic demand and network throughput is analysed as shown in Fig. 1.

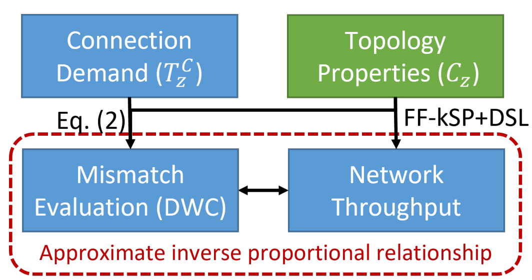


Fig. 1. Relationship between topology properties, demand and throughput

A normalised traffic demand matrix in terms of requested connections ($T^C$) is used to describe the network traffic demand, where $T_z^C$ represents the normalised demand value between node pair $z$ and $\sum_z T_z^C = 1$.

Due to the fact that the ILP is not applicable in calculating throughput for large-scale networks (e.g. 60-100 nodes), we used the First-Fit k-Shortest-Path (FF-kSP) algorithm [9] combined with a demand sequential loading (DSL) scheme to estimate the throughput in this work. More specifically, after obtaining the routing and wavelength assignment (RWA) solution of the current traffic by FF-kSP, we continue to add the load (according to $T^C$) of the network until blocking occurs. Then the signal-to-noise ratio (SNR) is calculated for each lightpath via a closed form Gaussian noise physical layer impairments model [10]. Following this, the Shannon equation [11] is applied to calculate the network throughput. For the physical layer, we assumed a full C-band (1530-1570 nm) transmission with 156 wavelengths (32 GHz Nyquist-spaced) on all fibre links. Colourless, directionless and contentionless, reconfigurable optical add-drop multiplexers (CDC-ROADM) are deployed at all switching nodes.

To reduce the computational complexity needed to estimate throughput, we parameterised the relationship between topology and demand, with the goal of finding an approximate-linear relationship to network throughput. By weighting the communication costs of the node pairs with the traffic demand between them, we propose a new parameter, termed demand weighted cost (DWC).

The communication cost ($C_z$) between a certain node pair $z$ contains two terms, which are weighted by the network physical connectivity ($\alpha = \frac{2E}{N(N-1)}$) [5], as shown in Eq. (1).

$$C_z = \alpha \cdot L_z + (1 - \alpha) \cdot H_z \quad (1)$$

where $\alpha$ is the network physical connectivity, $L_z$ is the shortest path physical length and $H_z$ is the number of hops in the shortest path.

The reason for using these two terms is that when the topology is sparse ($\alpha$ close to 0), the structural properties represented by the number of hops between each node pair contribute more to network throughput. When the topology is dense ($\alpha$ close to 1), the physical properties represented by the path lengths contribute more. After defining the communication cost, the traffic demand is used to weight this cost and the DWC is defined as in Eq. (2).

$$DWC = \sum_{z \in Z} T_z^C \cdot (\alpha \cdot L_z + (1 - \alpha) \cdot H_z) \quad (2)$$

To explore the relationship between the proposed DWC and the network throughput under different topology and demand scenarios, 10,000 topologies with 60-100 nodes were generated via SNR-BA generative graph model [12], 2,000 at each node scale. To systematically generate different demands, we skewed the demand distribution by randomly selecting half of the node pairs, setting the demand of them as $(1-\gamma)\lambda$ while the other half as $(1+\gamma)\lambda$, where $\lambda$ is the demand value of a node pair under uniform demand distribution $\gamma \in [0, 1]$ weighs how heavily the demand is skewed.

The inverse value of DWC and throughput of the 2,000 generated 100-node topologies are shown in Fig. 2. As $1/DWC$ increases, the networks tend to have higher throughput. The Pearson correlation coefficient ($\rho$) between $1/DWC$ and throughput reaches 0.972. For different node-scale ($N$) topologies, the $\rho$ values stays above 0.969 as shown in Tab. I, which indicates that there is an approximate inverse proportional relationship between DWC and network throughput.

TABLE I
PEARSON CORRELATION COEFFICIENT IN DIFFERENT SCALES

| $N$ | 60 | 70 | 80 | 90 | 100 |
|---|---|---|---|---|---|
| $\rho$ | 0.969 | 0.970 | 0.972 | 0.974 | 0.972 |

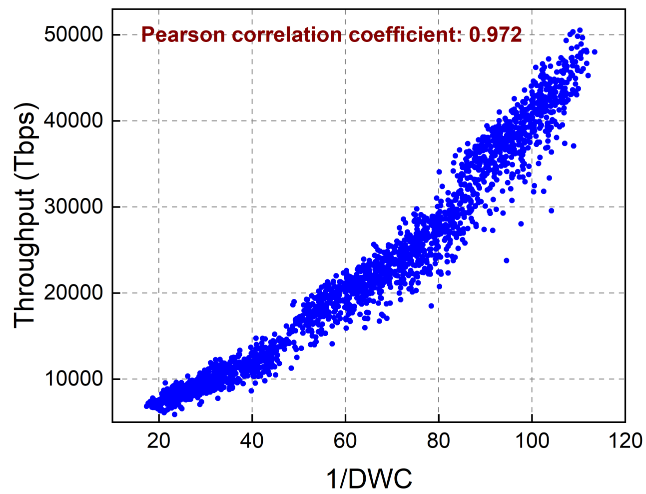


Fig. 2. Relationship between throughput and DWC (100 node)

The computational complexity of evaluating a topology by DWC instead of throughput (estimated by the FF-kSP+DSL method) is now reduced from $O(RKN^3(E + Nlog(N)))$ to $O(N^3)$, where $R, K, N, E$ are the round number of adding network demand, the number of shortest paths considered, node number and edge number, respectively. For the 100-node topologies, the average network evaluation time is reduced from 11649.7s to 0.15s, which is a reduction of 5 orders of magnitude. Therefore, DWC can be used as a computationally efficient objective that approximates the throughput performance of a network in physical topology design.

## III. DWC APPLICATION IN TOPOLOGY DESIGN

In this paper, we formulate the physical topology design problem by starting with a set of nodes ($N$), their positions ($n_i = (x_i, y_i)$), the total edge length limit ($L_{max}$) and the traffic demand matrix ($T^C$), where the goal is to design a bi-connected topology (fulfilling the resilience requirement), maximising the network throughput. To avoid the computational complexity of evaluating throughput in the design process, we set the design target as minimising DWC instead of maximising throughput.

We designed two types of topology design algorithm with the new objective function of minimising the DWC. The first one is to generate a large number (e.g. 10,000) of topologies using the PS graph generative model [7], selecting one of them with the minimum DWC. We refer to this as the *DWC-selection* method. The reason for using PS model is that it provides a computationally efficient way to search the $N$-node bi-connected graphs, compared to other graph generative models such as Erdos Renyi (ER) [13] and Barabasi-Albert (BA) [14], which do not always generate bi-connected graphs. The second method implements the objective function in a GA framework [15]. The individual in GA that describes a topology is a Prufer sequence, which will generate bi-connected graphs after crossover and mutation, enhancing the searching efficiency compared with using topology vectors extracted from the adjacency matrix. Uniform random crossover and mutation methods are implemented in the GA algorithm.

## IV. Topology Design Results and Discussion

To evaluate the performance of the proposed *DWC-selection* and GA methods in large-scale topology design, we select the best 200 topologies per-demand-matrix with minimum DWC from the 10,000 topologies generated by each method respectively. For each of these matrices, where $\gamma \in \{0.2n | 0 \leq n \leq 5, n \in Z\}$, 200 additional random PS topologies were used as baselines. The node and total edge length limit of the topologies to be designed were set to 100 and 280,000km, respectively according to the node positions, selected uniformly randomly over an area the size of the north-American continent. The minimum distance between two nodes was set to 100 km to mimic the node distances in core networks. The iteration number, population size, parent portion, crossover and mutation rate of the GA method were set to 100, 100, 30%, 80% and 10%, respectively, where these values were determined by grid-search.

The average value of the features for the 1,200 designed topologies are shown in Fig. 3. Compared to the random PS topologies, the GA and *DWC-selection* topologies achieve 33% and 25% higher total throughput ($T$), respectively, with almost the same total fibre length ($L$). The efficiency of fibre deployment, represented by the metric of the throughput per km fibre ($T/L$) is also enhanced by 34% and 25%, which indicates achieving higher throughput with the same or less cost. The average edge length ($L_e$) of GA and DWC-selected topologies are 9.5% and 4.6% shorter, leading to the edge number ($|E|$) increases of 8.9% and 4.7%, respectively. By setting the DWC as the optimisation target, the GA and the *DWC-selection* methods are able to make smarter choices where the edges are actually needed, according to the demand matrix, achieving improved network structure and physical properties compared to the random PS method.

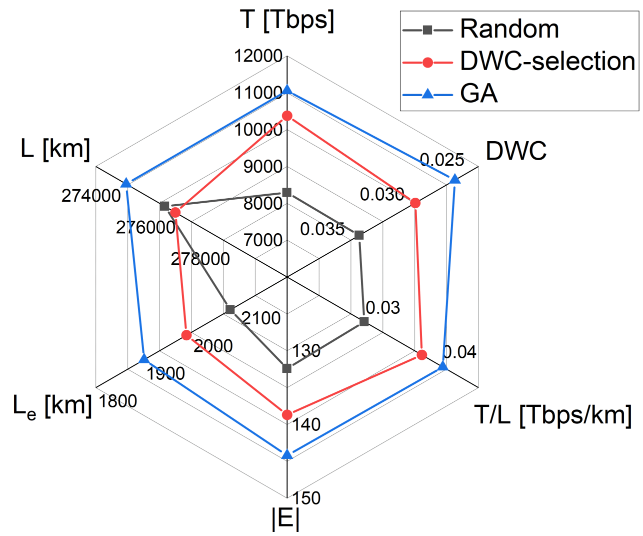


Fig. 3. Features of designed topologies: throughput ($T$), throughput per km fibre ($T/L$), edge number ($|E|$), length per edge ($L_e$) and the total fibre length ($L$)

## V. Conclusions

In this paper, we defined a new parameter, the demand weighted cost, by parameterising the relationship between optical network topology, traffic demand and throughput. The proposed DWC was used as a computationally effective objective (5 orders of magnitude speed-up) to evaluate topologies in the network design process. By implementing DWC alongside two polynomial-time topology optimisation methods, significant increases in throughput were demonstrated for a 100-node topology design. Both the method and the results can be used to develop traffic-tailored network topologies and/or adapt them to deliver bandwidth when and where it is needed. Work is ongoing to extend the current model to quantify the impact of node scales in topology design, and compare them with other topology optimisation methods.

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