Waveform Design for Joint Radar-Communications with Low Complexity Analog Components

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Abstract—In this paper, we aim to design an efficient and low hardware complexity based dual-function multiple-input multiple-output (MIMO) joint radar-communication (JRC) system. It is implemented via a low complexity analog architecture, constituted by a phase shifting network and variable gain amplifier. The proposed system exploits the multiple antenna transmitter for the simultaneous communication with multiple downlink users and radar target detection. The transmit waveform of the proposed JRC system is designed to minimize the downlink multi-user interference such that the desired radar beampattern is achieved and the architecture specific constraints are satisfied. The resulting optimization problem is non-convex and in general difficult to solve. We propose an efficient algorithmic solution based on the primal-dual framework. The numerical results show enhanced performance of the proposed approach when compared to existing state-of-the-art fully-digital method.

Index Terms—Joint radar-communications, MIMO, low hardware complexity, waveform design, phase shifter, primal-dual.

I. INTRODUCTION

The shared use of spectrum for sensing and signalling operations provides an efficient approach in decongesting the limited radio frequency (RF) spectrum [1]. An optimization based beamforming design has been addressed for the spectral co-existence of multiple-input multiple-output (MIMO) radar and cellular systems [2]. Going beyond spectral co-existence, joint radar-communication (JRC) systems are designed in order to efficiently share the hardware platform and the frequency band [3]–[6]. The integrated communication and sensing based transmission has been considered for single antenna systems in existing literature [7], [8]. For such systems, there is a considerable performance trade-off between the radar and communication operations, e.g., [7] exhibits high peak average power ratio with a limited dynamic range.

Alternatively, a small number of antennas are employed by 802.11ad wireless local area network protocol [9], [10] operating at 60 GHz band, which can support short-range sensing only. The MIMO antenna setup with multiple radar users and multiple communication users, such as in [2], [11], [12], can provide more degrees of freedom for enhanced performance. The MIMO JRC systems can further benefit from the information embedding approaches [13]. For MIMO JRC systems, the designed transmit waveform enables the transmission at high information rates to the intended users while maintaining a reliable operation of the radar system. In [11], the transmit waveform is designed by minimizing multi-user interference (MUI) such that a desired radar beampattern is achieved. In [2], precoding solutions are proposed such that the designed transmit signals match the desired radar beampattern, and satisfy communication related constraints. In [12], precoding matrix is decomposed into two parts, one for communication and other for the radar system. These techniques are developed for fully-digital transceivers with one RF chain per antenna leading to high power consumption and implementation cost for large systems.

For reduced number of RF chains, hybrid analog-digital transceivers are designed via a two-stage beamformer with a digital part, applied in the base-band followed by an analog beamformer applied in the RF domain [14]–[17], and also with reduced hardware for converting units [18]. For systems with small number of RF chains, multi-user communication may be poorly supported. Reference [19] addresses a symbol level precoding solution based on a transmitter of mainly analog components such as phase shifters (PSs), variable gain amplifier (VGA) etc. Despite its simplicity, the nonlinear design of the transmit signals enables the transmission of multiple parallel streams and thus, it is able to support multi-user communication efficiently while exhibiting significant power consumption and hardware complexity gains.

In this paper, we propose a novel analog architecture for MIMO JRC transmission which is based on a single phase-shifter per antenna and a common VGA that drives the phase shifting network. Then, the optimal transmit waveform is designed by formulating an optimization problem that minimizes the communication MUI while imposing a specific radar beampattern and also satisfying constraints related to the proposed architecture, i.e., per-antenna constant-modulus constraints. The resulting optimization problem is non-convex, difficult to solve and have yet to be addressed in literature. We solve the problem using an efficient algorithm based on the primal-dual framework [20]. The numerical results show high spectral efficiency gains in a MIMO JRC system compared to the state-of-the-art fully-digital architecture while maintaining a desirable radar beampattern performance.

Notation: $\mathbf{A}$, $\mathbf{a}$, a: matrix, vector and scalar variable, respectively; tr(.), re(.), $|.|$, $(.)^T$, $(.)^H$, $\|(.)\|_p$ and $\|(.)\|_F$ denote the trace, real-term, determinant, transpose, complex conjugate transpose, p norm and Frobenius norm, respectively; $\mathbf{A}_{i,j}$ and $\mathbf{a}_i$ are the (i,j)-th and i-th elements in $\mathbf{A}$ and $\mathbf{a}$, respectively; $\mathbf{I}_N$ is N-size identity matrix; $\mathbb{C}$ and $\mathbb{E}(.)$ denote the set of complex numbers and ensemble average, respectively; $\mathcal{CN}(a,b)$ is a complex Gaussian vector with mean a and variance b.

II. SYSTEM MODEL

Let us assume, a MIMO JRC system equipped with a uniform linear array (ULA) of N_T antenna elements which simultaneously serves N_R single-antenna downlink user terminals and transmits radar probing waveforms towards the targets. To reduce the hardware complexity, we propose an analog architecture for the MIMO JRC system as shown in Fig. 1. Such an architecture is implemented by one phase shifter (PS) module connected to each transmit antenna before the power amplifiers (PAs). As shown in Fig. 1, this phase shifting network is driven by a common VGA, with the local oscillator (LO) circuitry, that tunes the common modulus of the output transmission signals.

A. Communication Model

The received signals of the downlink communication users $\mathbf{Y}$ can be expressed as

$$\mathbf{Y} = \mathbf{HXQ} + \mathbf{Z}, \tag{1}$$

where M is the length of the communication frame/radar pulse, $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_M] \in \Omega^{N_T \times M}$ is the transmit signal matrix employed for both radar and communication operations with $\Omega^{N_T \times M}$ being the set of complex unit-modulus numbers, i.e., $\Omega = \{\omega \in \mathbb{C} \mid |\omega| = 1\}$. The channel matrix $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_{N_R}]^T \in \mathbb{C}^{N_R \times N_T}$ represents the frequency flat Rayleigh fading channel entries between the JRC system and the communication users. The channel state information is assumed to be perfectly known at the transceiver. The matrix $\mathbf{Q} = [\mathbf{q}_1, \ldots, \mathbf{q}_M]$ is the $M \times M$ diagonal matrix of complex entries that correspond to the common modulus set by the VGA per frame by assuming that the phase components of the diagonal entries of $\mathbf{Q}$ may be absorbed by the PSs, without generality loss. Note that unlike existing literature where the transmit signal matrix is considered for a fully-digital system [11], in this paper the transmit signal $\mathbf{XQ}$ takes into account the impact of low complexity analog architecture through $\mathbf{Q}$ matrix entries. The matrix $\mathbf{Z} = [\mathbf{z}_1, \ldots, \mathbf{z}_M] \in \mathbb{C}^{N_R \times M}$ represents the independent and identically distributed complex additive white Gaussian noise where $\mathbf{z}_m \sim \mathcal{CN}(0, \sigma_z^2 \mathbf{I}), \forall m$.

The communication part of the system is optimized with the view to maximize its sum-rate, given by,

$$R = \sum_{n=1}^{N_R} \log_2(1 + \psi_n), \tag{2}$$

where, for the n-th user, ψ_n denotes the signal-to-interference-plus-noise ratio per communication frame which is expressed
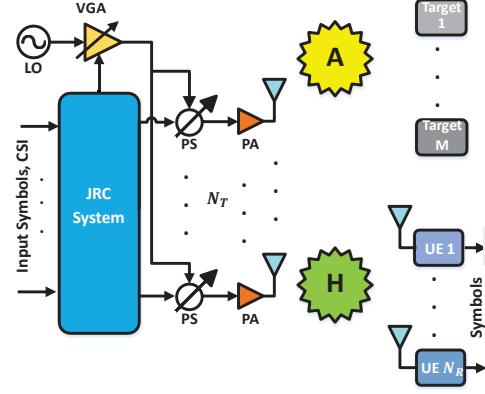

Fig. 1: A MIMO JRC system with low complexity analog architecture.

as the ratio of $\mathbf{S}$ matrix's (n,m)-th element entries, with $\mathbf{S} \in \mathbb{C}^{N_R \times M}$ being the desired transmit symbol matrix that belongs to a given constellation set, and the sum of MUI entity and noise variance as follows [11], [21]:

$$\psi_n = \frac{\mathbb{E}(|\mathbf{S}_{n,m}|^2)}{\mathbb{E}(|\mathbf{h}_n^T \mathbf{x}_m \mathbf{q}_m - \mathbf{S}_{n,m}|^2) + \sigma_z^2}. \tag{3}$$

Note that the power of the $\mathbb{E}(|\mathbf{S}_{n,m}|^2)$ signal is fixed for a given constellation with fixed energy. Thus, the maximum sum-rate can be achieved by minimizing the so-called "MUI energy" [11], [21], defined as

$$f_{\text{MUI}} = \|\mathbf{HXQ} - \mathbf{S}\|_F^2. \tag{4}$$

In the proposed design, we aim at the optimization of the communication part of the JRC system through the minimization of the MUI energy subject to the constraints related to the radar waveform design and additionally due to the employed low complexity hardware, i.e., unit-modulus signals due to the PSs and maximum transmission power constraints due to the VGA/PAs.

B. Radar Model

The transmit beampattern for the JRC system that points to the targets of interest can be expressed as

$$P_T(\theta) = \mathbf{a}^H(\theta) \mathbf{R}_T \mathbf{a}_T(\theta), \tag{5}$$

where $\mathbf{a}_T(\theta) = [1, e^{j\frac{2\pi}{\lambda} d \sin(\theta)}, \ldots, e^{j(N_T-1)\frac{2\pi}{\lambda} d \sin(\theta)}]^T$ is the transmit array response vector, with λ being the signal wavelength and d is the inter-element antenna spacing. The variable θ denotes the angle of detection. The covariance matrix associated with the transmit signal matrix $\mathbf{XQ}$ is expressed as

$$\mathbf{R}_T = \frac{1}{M} \mathbf{XQQ}^H \mathbf{X}^H, \tag{6}$$

where using the assumption $M \geq N_T$ without loss of generality, $\mathbf{R}_T$ in (6) is positive definite. Following [22], designing the transmit covariance matrix $\mathbf{R}_T$ is equivalent to designing the transmit radar beampattern in (5) for uncorrelated waveforms where MIMO radar exhibits high degree of freedom (DoF) in comparison to the conventional phased array radar. Having

described the desired cost function and the desired features of the radar waveform, the optimization problem via which the transmission signals are derived for the low complexity analog architecture are defined in the next section.

III. OPTIMAL WAVEFORM DESIGN

This section presents the definition of the optimization problem out of which the optimal transmit waveform will be derived and the corresponding proposed algorithmic solution based on the primal-dual framework.

A. Transmit Beampattern Design

We consider $\mathbf{R}_{\mathrm{D}}$ as the Hermitian positive semidefinite covariance matrix corresponding to a well-designed radar beampattern [11]. We can formulate the MUI minimization problem taking into consideration the constraint of $\mathbf{R}_{\mathrm{D}}$ on the waveform $\mathbf{XQ}$ and maximum power budget P_{max} for the entries of the diagonal VGA matrix $\mathbf{Q}$. Thus, the MUI minimization problem can be expressed as

$$(\mathcal{P}_1): \quad \min_{\mathbf{X},\mathbf{Q}} \frac{1}{2}\|\mathbf{S}-\mathbf{HXQ}\|_F^2$$
$$\text{s. t.} \quad \mathbf{X} \in \Omega^{N_{\mathrm{T}}\times M}$$
$$\frac{1}{M}\mathbf{XQQ}^H\mathbf{X}^H = \mathbf{R}_{\mathrm{D}}$$
$$|\mathrm{diag}(\mathbf{Q})|^2 \preceq P_{\mathrm{max}}\mathbf{1}.$$

The Cholseky decomposition for the covariance matrix $\mathbf{R}_{\mathrm{D}}$ can be written as $\mathbf{R}_{\mathrm{D}} = \mathbf{FF}^H$, where the matrix $\mathbf{F}$ is a $N_{\mathrm{T}} \times N_{\mathrm{T}}$ lower triangular matrix. For the matrix $\mathbf{F}$ to be invertible we assume the covariance matrix $\mathbf{R}_{\mathrm{D}}$ to be positive definite without generality loss. The solution to the optimization problem in $(\mathcal{P}_1)$, which considers radar- and low complexity based system-specific constraints, should provide the optimal dual functional transmit waveform and the optimal VGA matrix for an efficient MIMO JRC system.

B. Proposed Method

In order to solve the directional beampattern design problem in $(\mathcal{P}_1)$, we can express it in a form solvable by alternating minimization based approach, called as primal-dual method. There are existing methods which implement alternating minimization based iterative procedures to solve constrained optimization problems efficiently. These methods provide a variant of the standard augmented Lagrangian method that uses partial updates (similar to the Gauss-Seidel method for the solution of linear equations) to solve optimization problems with constraints. This has been successfully applied to non-convex problems as well [23]. For convenience, using the auxiliary matrix $\mathbf{Z}$, the problem $(\mathcal{P}_1)$ is rewritten as

$$(\mathcal{P}_2): \quad \min_{\mathbf{X},\mathbf{Z},\mathbf{Q}} \frac{1}{2}\|\mathbf{S}-\mathbf{HZ}\|_2^2$$
$$\text{subject to} \quad |\mathbf{X}_{i,j}| = 1$$
$$\mathbf{Z} = \mathbf{XQ}$$
$$\frac{1}{M}\mathbf{ZZ}^H = \mathbf{R}_{\mathrm{D}}$$
$$|\mathrm{diag}(\mathbf{Q})|^2 \preceq P_{\mathrm{max}}\mathbf{1}.$$

Algorithm 1 Proposed Method for JRC Waveform Design

1: **Initialize:** $\mathbf{X}$ from the unit-modulus constraints set, $\mathbf{Z}$ to be matrix satisfying $\frac{1}{M}\mathbf{ZZ}^H = \mathbf{R}_{\mathrm{D}}$ and initial $\mathbf{\Lambda}_0$ with random values
2: $\mathbf{R}_{\mathrm{D}} = \mathbf{FF}^H$
3: **while** not converged **do**
4:     $k \leftarrow k+1$
5:     Update auxiliary matrix $\mathbf{Z}_{k+1}$ using solution (12)
6:     Update waveform matrix $\mathbf{X}_{k+1}$ using solution (14)
7:     Update VGA matrix $\mathbf{Q}_{k+1}$ using solution (17)
8:     Update Lagrange multiplier matrix $\mathbf{\Lambda}_{k+1}$ using (19)
9: **end while**
10: **return** $\mathbf{Z},\mathbf{X},\mathbf{Q},\mathbf{\Lambda}$

Following that, the augmented Lagrangian function of the optimization problem can be written as

$$\mathcal{L}(\mathbf{X},\mathbf{Z},\mathbf{Q},\mathbf{\Lambda}) = \frac{1}{2}\|\mathbf{S}-\mathbf{HZ}\|_2^2$$
$$+\mathrm{tr}\big(\mathbf{\Lambda}^H(\mathbf{Z}-\mathbf{XQ})\big)$$
$$+\frac{\rho}{2}\|\mathbf{Z}-\mathbf{XQ}\|_F^2, \quad (7)$$

where ρ is the hyper-parameter which is a scalar quantity and $\mathbf{\Lambda}$ is the Lagrange Multiplier matrix. In terms of the minimization with respect to $\mathbf{Z}$, we have

$$\min_{\mathbf{Z}} -\mathrm{tr}(\mathrm{re}(\mathbf{S}^H\mathbf{HZ})) + \frac{1}{2}\|\mathbf{HZ}\|_F^2] + \mathrm{tr}(\mathrm{re}(\mathbf{\Lambda}^H\mathbf{Z}))$$
$$+ \frac{\rho}{2}[\|\mathbf{Z}\|_F^2 - 2\mathrm{tr}(\mathrm{re}(\mathbf{Z}^H\mathbf{XQ}))]$$
$$\text{subject to} \quad \frac{1}{M}\mathbf{ZZ}^H = \mathbf{R}_{\mathrm{D}}. \quad (8)$$

The simplified objective function can be written as, $-\mathrm{tr}\{\mathrm{re}(\mathbf{Z}^H(\mathbf{H}^H\mathbf{S}-\mathbf{\Lambda}+\rho\mathbf{XQ}))\}$. The problem can be further formulated as

$$\min_{\mathbf{Z}} \quad \left\|\mathbf{Z}-\hat{\mathbf{S}}\right\|_F^2$$
$$\text{subject to} \quad \frac{1}{M}\mathbf{ZZ}^H = \mathbf{R}_{\mathrm{D}}, \quad (9)$$

where $\hat{\mathbf{S}} = \mathbf{H}^H\mathbf{S} - \mathbf{\Lambda} + \rho\mathbf{XQ}$. Let $\mathbf{R}_{\mathrm{D}} = \mathbf{FF}^H$ be the Cholesky decomposition of the matrix $\mathbf{R}_{\mathrm{D}}$. The related constraint can be further expressed as, $\frac{1}{M}\mathbf{F}^{-1}\mathbf{ZZ}^H\mathbf{F}^{-1} = \mathbf{I}$. We let $\tilde{\mathbf{Z}} = \sqrt{\frac{1}{M}}\mathbf{F}^{-1}\mathbf{Z}$, thus we have

$$\min_{\mathbf{Z}} \quad \left\|\sqrt{M}\mathbf{F}\tilde{\mathbf{Z}}-\hat{\mathbf{S}}\right\|_F^2$$
$$\text{subject to} \quad \tilde{\mathbf{Z}}\tilde{\mathbf{Z}}^H = \mathbf{I}. \quad (10)$$

The above problem in (10) represents an orthogonal Procrustes problem (OPP) which admits the following closed-form solution:

$$\tilde{\mathbf{Z}} = \tilde{\mathbf{U}}\mathbf{I}_{N_{\mathrm{R}}\times M}\tilde{\mathbf{V}}^H, \quad (11)$$

where $\tilde{\mathbf{U}}\tilde{\mathbf{\Sigma}}\tilde{\mathbf{V}}^H = \mathbf{F}^H\hat{\mathbf{S}}$ represents the singular value decomposition (SVD) of $\mathbf{F}^H\hat{\mathbf{S}}$ where $\tilde{\mathbf{U}}$ and $\tilde{\mathbf{V}}$ are the left and right singular matrices of SVD, respectively, and $\tilde{\mathbf{\Sigma}}$ denotes

the diagonal matrix of singular values. As we execute an iterative procedure in the proposed approach where k denotes the iteration index, the update step for $\mathbf{Z}$ at the $(k+1)$-th iteration is given by

$$\sqrt{\frac{1}{M}}\mathbf{F}^{-1}\mathbf{Z}_{k+1} = \tilde{\mathbf{U}}\mathbf{I}_{N_{\mathrm{R}}\times M}\tilde{\mathbf{V}}^H$$
$$\implies \mathbf{Z}_{k+1} = \sqrt{M}\mathbf{F}\tilde{\mathbf{U}}\mathbf{I}_{N_{\mathrm{R}}\times M}\tilde{\mathbf{V}}^H. \tag{12}$$

In terms of the minimization with respect to $\mathbf{X}$, we have

$$\min_{\mathbf{X}} -\mathrm{tr}(\mathrm{re}(\boldsymbol{\Lambda}^H\mathbf{XQ})) + \frac{\rho}{2}[-2\mathrm{tr}(\mathrm{re}(\mathbf{Z}^H\mathbf{XQ})) + \|\mathbf{XQ}\|_F^2]$$
$$\text{subject to } |\mathbf{X}_{i,j}| = 1, \forall i,j. \tag{13}$$

The gradient with respect to $\mathbf{X}$ is given by, $\mathbf{Q}\boldsymbol{\Lambda}^H - \rho[\mathbf{QZ}^H - \mathbf{QQ}^H\mathbf{X}_k^H]$. With α being the step-size parameter for the proposed algorithm, the update step for $\mathbf{X}$ at the $(k+1)$-th iteration is given by

$$\mathbf{X}_{k+1} = \Pi_{\mathcal{X}}\{\mathbf{X}_k - \alpha\mathbf{Q}_k\boldsymbol{\Lambda}_k^H + \alpha\rho[\mathbf{Q}_k\mathbf{Z}_{k+1}^H] - \mathbf{Q}_k\mathbf{Q}_k^H\mathbf{X}_k^H\}, \tag{14}$$

where the projection operator $\Pi_{\mathcal{X}}$ to project an element x, can be defined as,

$$\Pi_{\mathcal{X}}\{x\} = \begin{cases} \frac{x}{|x|}, & x \neq 0, \\ 1, & x = 0. \end{cases} \tag{15}$$

In terms of the minimization with respect to $\mathbf{Q}$, we have

$$\min_{\mathbf{Q}} -\mathrm{tr}(\mathrm{re}(\boldsymbol{\Lambda}^H\mathbf{XQ})) + \frac{\rho}{2}[-2\mathrm{tr}(\mathrm{re}(\mathbf{Z}^H\mathbf{XQ})) + \|\mathbf{XQ}\|_F^2]$$
$$\text{subject to } |\mathrm{diag}(\mathbf{Q})|^2 \preceq P_{\max}\mathbf{1}. \tag{16}$$

The gradient with respect to $\mathbf{Q}$ is given by, $\boldsymbol{\Lambda}^H\mathbf{X} - \rho[\mathbf{Z}^H\mathbf{X} - \mathbf{Q}^H\mathbf{X}^H\mathbf{X}$. The update step for $\mathbf{Q}$ at the $(k+1)$-th iteration is given by

$$\mathbf{Q}_{k+1} = \Pi_{\mathcal{Q}}\{\mathrm{diag}(\mathbf{Q}_k - \alpha\boldsymbol{\Lambda}_k^H\mathbf{X}_{k+1} + \alpha\rho[\mathbf{Z}_{k+1}^H\mathbf{X}_{k+1} - \mathbf{Q}_k^H\mathbf{X}_{k+1}^H\mathbf{X}_{k+1}])\}, \tag{17}$$

where the projection operator $\Pi_{\mathcal{Q}}$ to project an element q, can be defined as,

$$\Pi_{\mathcal{Q}}\{q\} = \begin{cases} q, & |q|^2 \leq P_{\max}, \\ \sqrt{P_{\max}}\frac{q}{|q|}, & |q|^2 > P_{\max}. \end{cases} \tag{18}$$

Furthermore, in relation to the Lagrange Multiplier matrix $\boldsymbol{\Lambda}$, the gradient with respect to $\boldsymbol{\Lambda}$ is given by, $\mathbf{Z} - \mathbf{XQ}$. Following that, the update step with respect to $\boldsymbol{\Lambda}$ for the proposed iterative procedure is expressed as

$$\boldsymbol{\Lambda}_{k+1} = \boldsymbol{\Lambda}_k + \alpha(\mathbf{Z}_{k+1} - \mathbf{X}_{k+1}\mathbf{Q}_{k+1}). \tag{19}$$

Corresponding to the equations (7)-(19), we execute the update steps in an iterative manner in order to obtain the optimal $\mathbf{Z}, \mathbf{X}, \mathbf{Q}$ and $\boldsymbol{\Lambda}$ matrices. Algorithm 1 summarizes the complete algorithmic steps. Next, we evaluate the effectiveness of the proposed method through numerical results.
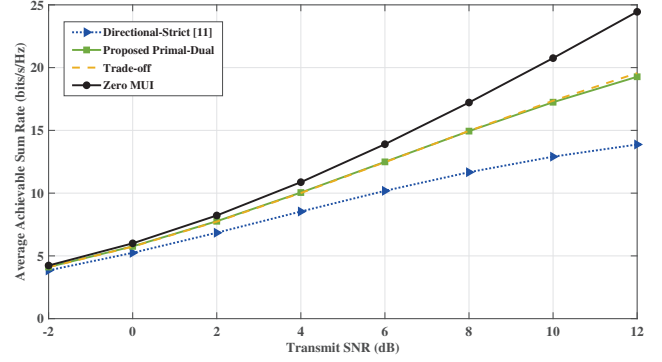

Fig. 2: Sum rate w.r.t. SNR communication performance comparison for different methods with $N_{\mathrm{T}}=16$, $N_{\mathrm{R}} = 6$.
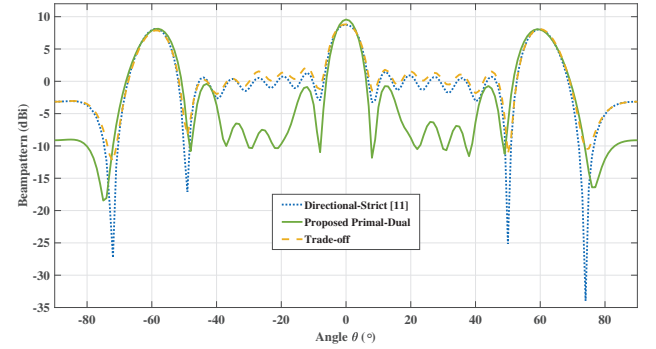

Fig. 3: Radar beampattern performance for different methods with $N_{\mathrm{T}}=16$, $N_{\mathrm{R}} = 6$.

IV. SIMULATION RESULTS

In order to evaluate the performance of the proposed method we set the following simulation parameters: the number of transmitter antennas $N_{\mathrm{T}} = 16$, the number of users $N_{\mathrm{R}} = 6$, the communication frame/radar pulse length $M = 30$ and maximum power $P_{\max} = 1$ W. We assume an ULA-based setup where antenna elements are spaced by half-wavelength distance, i.e., $d = \lambda/2$ where λ can be based on a standard frequency value [11]. We set the number of targets to be three and the corresponding angular target locations, i.e., θ, at $[-\pi/3, 0, \pi/3]$. The desired $\mathbf{R}_{\mathrm{D}}$ matrix variable is obtained by a least-squares approach such as in [22]. The entries of symbol matrix $\mathbf{S}$ is drawn from a quardature phase shift keying (QPSK) constellation. The signal-to-noise ratio (SNR) is defined as $1/\sigma_n^2$ and channel matrix $\mathbf{H}$ has complex Gaussian elements.

For comparison with the proposed primal-dual method, we consider the following fully-digital baselines:
(a) The state-of-the-art *directional-strict* approach which aims to achieve radar beampattern with strict equality constraints described in [11].
(b) The *directional trade-off* design which allows a tolerable mismatch between the desired radar beampattern and the designed one in order to form a balance with the com-

munication performance. The value of the weighting factor for communication and radar operations in trade-off case is chosen as 0.5;

(c) The *zero MUI* case represents the performance lower bound for MUI and achieves the maximum rate performance.

Fig. 2 shows the average achievable sum-rate performance described in (2) with respect to (w.r.t.) SNR for $N_T = 16$ and $N_R = 6$. It can be observed that the proposed method achieves better rate performance than the existing state-of-the-art directional-strict approach. This is the case, since in the proposed approach a relaxed version of the original problem is treated via the optimization of the augmented Lagrangian. That is, the desired beampattern is imposed in a sense that $\rho\|\mathbf{XQ} - \mathbf{Z}\|_F \to 0$. In general, for small ρ values more weight is given to the MUI term compared to the beampattern imposing term. This explains the observed better performance of the proposed approach compared to the one of the directional-strict solution that imposes the exact beampattern to the desired signals. For example, at 10 dB SNR, the proposed method outperforms the directional-strict case by ≈ 5 bits/s/Hz. Furthermore, the proposed primal-dual approach performs similar to the trade-off method and the performance in comparison with the zero MUI case is also satisfactory given the simplicity of the hardware architecture that implements the proposed approach.

Fig. 3 shows the radar beampattern performance plot for the proposed primal-dual method and the baselines for a 16×6 MIMO radar based JRC system. It can be observed that the proposed primal-dual method exhibits favourable transmit beampattern performance better or similar to the directional-strict baseline at different target locations. In the beampattern plot, the three majorlobes for three targets can be observed at the angle locations of $[-\pi/3, 0, \pi/3]$.

V. Conclusion

This paper proposes a low complexity analog architecture to design an efficient MIMO JRC system. In order to design transmit waveform for the proposed JRC system, a downlink MUI minimization problem is formulated with architecture specific constraints and corresponding to the desired radar beampattern. An efficient solution based on the primal-dual framework is proposed to solve the non-convex optimization problem. The proposed approach achieves high spectral efficiency gains when compared with the state-of-the-art directional-strict baseline, e.g., a gain of about 5 bits/s/Hz rate performance at 10 dB SNR can be observed. A desirable radar beampattern performance is also observed which infers the proposed approach to be an efficient solution for both communication and radar operations.

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