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What is the cost of grade retention?

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Abstract

This paper offers a new method to estimate the budgetary cost of grade retention that takes into account a) the impact of grade retention on students' school path; b) the dynamic impact of variations in grade retention on the flow of student enrollment across grades. Using administrative data on students in French secondary schools, I instrument retention by students' date of birth and find that the marginal impact of one year of retention is to increase the number of years of schooling by exactly one year. Modeling student enrollment with a discrete Markov chain model, I simulate a counterfactual scenario where grade retention is completely abolished. I find that budgetary savings increase only gradually and reach a steady state only when students who were entering primary school at the time of the abolition have left high school.

JEL Codes: I21, I22, J20
Keywords:

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1 Introduction

Grade retention is regularly criticized in the public debate for being not only inefficient but also extremely expensive. There is a large literature focusing on the impact of grade retention on short and long term student outcomes (Jacob et Lefgren, 2004; 2009; Manacorda, 2012; Gary-Bobo, Gousse et Robin, 2014). There is however very little evidence on the budgetary cost of grade retention. This question is important for public policy: grade retention is still widely practiced in many developed countries (OECD, 2014). In France for example, at 15 years old, 28 % of students have repeated a grade at least once (PISA 2014).

This paper offers a new method to estimate the cost of grade retention, using comprehensive administrative data on students enrolled in French primary and secondary schools. I focus on the budget cost derived from the impact of grade retention on the stock and flow of enrolled students in the primary and secondary school public system.

The contribution of this paper with respect to the existing literature (Paul et Troncin, 2004; OECD, 2013) is twofold. First, it analyzes the impact of grade retention on students' school path. In particular, I study its impact on high school track (vocational or academic) and on the probability of dropping out. Previous studies assume implicitly grade retention increases mechanically the number of years of schooling by one year. But grade retention can also reduce the number of completed years of training (i.e. the grade reached by students): in that case, previous studies would overestimate the budgetary cost of grade retention. This analysis relies on comprehensive administrative data following the entire school path of the cohort of students born in 1992. Using students' date of birth as an instrument for grade retention, I find that the marginal impact of one year of grade repetition is to increase the number of years of schooling by exactly one year. I also find that grade repetition increases the probability of enrolling in the vocational track of high school. The marginal impact of repeating a year is to increase the number of completed years in the vocational track by 0.3 to 0.4 years.

Second, this paper analyzes the dynamic effect of alternative scenarios were grade retention to be abolished (partially or completely). Previous studies assume implicitly that if grade retention were to be abolished, budget savings would be immediate. But, in fact, budget savings would appear gradually and be completed only after several years, once all students entering primary school at the time of abolition would have left high school. The central counterfactual scenario is the general abolition of grade repetition in primary and secondary school from the beginning of the 2015 academic year. This simulation rely on the assumption that student enrollment can be modeled as a discrete Markov chain model. I find that, in its first year, the abolition of grade retention would cost 20 millions euros. This cost is related to the transitory additional inflow of students towards higher, and more expensive, grade levels (i.e. additional inflow of students from primary school to middle school, and from middle school to high school). Second, the first budget savings would appear only two years after the abolition of grade retention. They would increase gradually until reaching a steady state of two billions euros per year 11 years after the reform, once students who were entering primary school at the time of the reform have left high school.

The remainder of the paper is organised as follows. Section 2 describes the data and a few descriptive statistics. Section 3 presents the empirical strategy and the results of the analysis of the impact of grade retention on students' school path. Section 4 presents the static estimation of the cost of grade retention, assuming that budgetary savings/spendings are immediate. Section 5 shows the dynamic analysis of the cost of grade retention. Section 6 concludes.

2 Data and Descriptive Statistics

2.1 Data

This paper uses comprehensive administrative student records (called FAERE) provided by the French Ministry of Education, and covering the 2004-2012 period. This data includes an encrypted student identifier, students' socio-demographic characteristics, and their grades at the end of middle school examination (called the Diplome national du brevet) taken in 9th grade. The end of middle school examination is externally graded examination and is the same for all students. To ensure their comparability, I standardize these test scores by year. I construct a panel dataset following the school path of all students born in 1992 and enrolled in a public school between 2004 and 2012. I chose to focus on the 1992 cohort because it is the most recent cohort that can be followed until it completely leaves high school. This panel includes 780 112 students and 5 039 973 observations.

I construct the following outcome variables:

- Number of years of schooling: this corresponds to the number of years each student is observed in secondary school, to which I add the number of years in primary school. I do not observe students in primary school therefore I impute the number of years spent in primary school using the age of students when they enter middle school

- Number of completed years of training: the grade level reached by students when they leave secondary school. It is computed using the grade level variable. Its values are between 5 years (number of completed years of a student entering 6th grade) to 13 (number of completed years of a student entering the last year of vocational high school)

- Number of repeated years: the difference between the number of years of schooling and the number of completed years, for each year and for each student.

- Standardized test scores at the standardized and externally graded end of the year 9th grade examination: student achievement is measured with their test scores at the 9th grade written examination. The content of this examination is the same for all students in France. It is anonymous and externally graded. Students' take this examination in three topics: French, Math and History. I standardize this grade by year and region [1]. For students who repeat 9th grade and take this examination twice, I only use their first 9th grade test scores

[1] The examination board is determined at the regional level

2.2 Descriptive Statistics

Table 1 – Cumulated number of repeated years of students born in 1992 at different stages of their schooling

Schooling stage	**Cumulated number of repeated years**
Primary school	0.15
	(0.37)
Middle school	0.20
	(0.40)
High school	0.25
	(0.55)
including: academic track	0.19
	(0.35)
including: vocational track	0.06
	(0.35)
Total	0.60
	(0.75)
Number of observations	780,112

Note: On average, the 1992 cohort accumulated 0.20 repeated years in middle school. Standard errors in parenthesis. **Source**: MENESR-DEPP, FAERE 2003 to 2011.

Table 1 shows that the number of cumulated repeated years increases with the grade level. On average, students accumulate 0.15 year of grade repeatition in primary school, 0.20 year in middle school and 0.25 year in high schooL. This adds up to 0.60 year of grade repetition on average by student in total.

A first approach to analyse the relationship between grade retention and students' school path is to compare the number of completed years of training by students who repeated a grade to those who did not. Figure 1a shows that students who have repeated reach a lower level of training than those who have not repeated. The number of completed years of training is equal to 11.36 years for non-repeaters against 10.78 years for repeaters. Figure 1b shows that repeaters are more likely to take the vocational track than non repeaters (15 % against 35 %) and less likely to take the academic track (70 % against 35 %).

Figure 2 shows that repeaters have a higher number of years of schooling than non repeaters. Students who have repeated at least once spend on average 11.7 years at school against 11.4 years for non repeaters. Therefore, a naive comparison seems to imply that a repeated year would increase the number of years of schooling by 0.3 years.

This naive comparison does not enable us however to conclude that grade retention has a negative causal impact on the level of training. Major confounding factors such as student initial level of achievement and ability are not controlled for and should be taken into account.

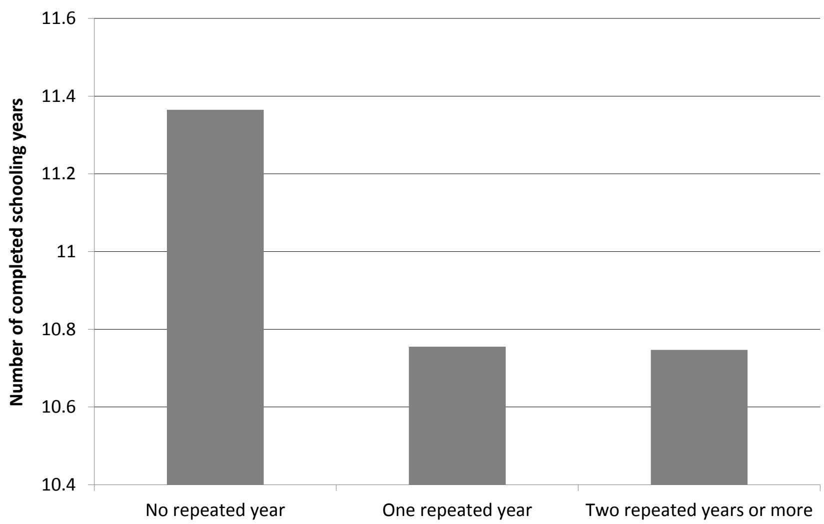
3 The Impact of Grade Retention on Students' Academic Path

3.1 Naive Estimation

Table 2 shows regression estimates of the impact of grade retention on students' school path, by schooling stage with a naive specification controling for students' observable characteristics such as their sex, socioeconomic status. According to this specification, student sex and SES kept equal, an additional year of grade repetition in primary or middle school is associated with a statistically significant reduction (at the 1 % level) of the number of years of schooling, but also of completed training. These results cannot be interpreted causaly due to the omitted variable bias discussed above.

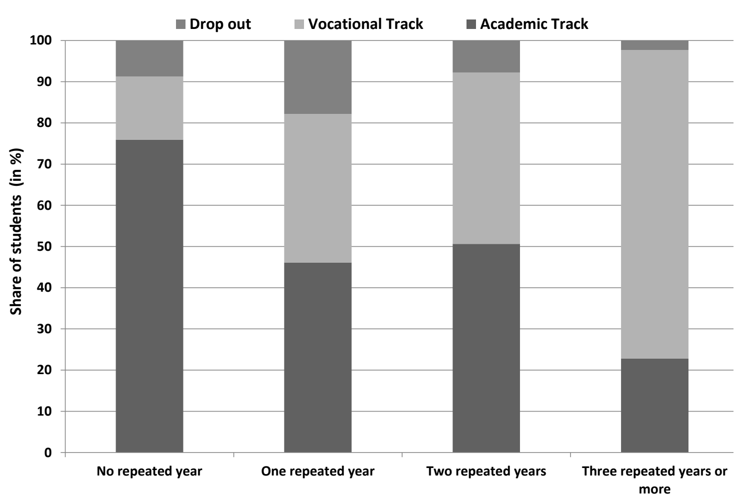
Figure 1 – Number of completed years of training by the number of cumulated repeated years

(a) Number of completed years of training by the number of cumulated repeated years (1992 cohort)



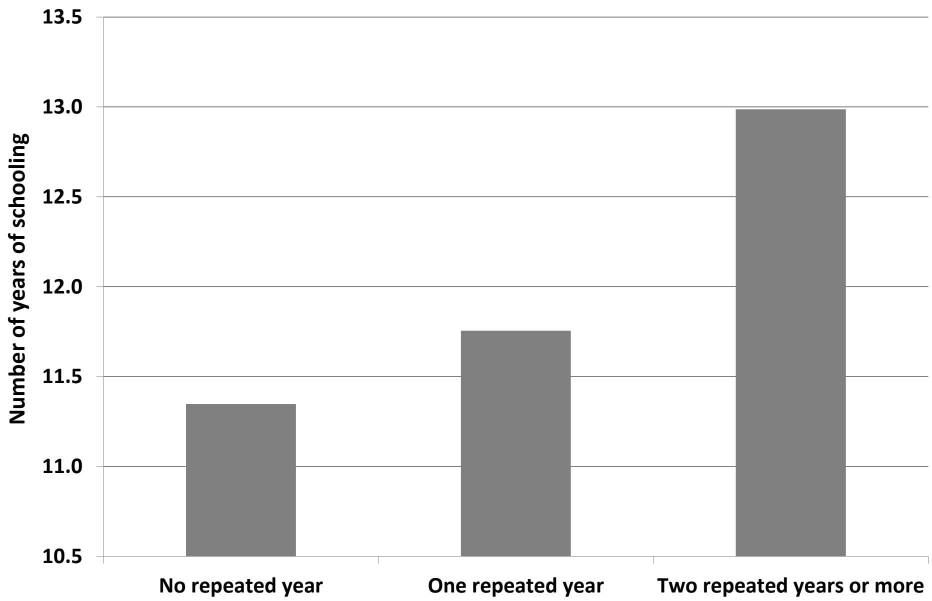
Notes: Students of the 1992 who have accumulated one repeated year complete on average 10.8 years of training. **Source**: MENESR-DEPP, FAERE 2003 to 2011.

(b) High school track by the number of repeated years of the 1992 cohort



Lecture: In high school, close to 70 % of student who have not repeated a grade take the academic track. **Source**: MENESR-DEPP, FAERE 2003 to 2011.

Figure 2 – Average Number of Years of Schooling by Number of Repeated Years for the 1992 cohort



Notes: Students who repeated once have an average number of years of schooling equal to 10.8 years. **Source**: MENESR-DEPP, FAERE 2003 to 2011.

Table 2 – Regression estimates of the impact of grade retention on students' schooling path, by schooling stage

Schooling stage:	Primary		Middle		High School		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Dependent variable:							
Nb of years of schooling	- 0,35***	- 0,29***	0,30***	0,34***	1,31***	1,30***	0,78***
	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)
Nb of years of training	-1,14***	- 1,02***	- 0,52***	-0,44***	0,44***	0,44***	- 0,10***
	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)	(0,00)
Probability to take the vocational track	0,22***	0,16***	0,19***	0,15***	–	–	–
	(0,00)	(0,00)	(0,00)	(0,00)			
Probability to take the academic track	- 0,47***	-0,38***	- 0,31***	- 0,26***	–	–	–
	(0,00)	(0,00)	(0,00)	(0,00)			
Control variables:							
Sex and SES	No	Yes	No	Yes	No	Yes	Yes
Percentile rank at the 9th grade exam (over 100)	No	No	No	Non	No	No	Yes
Missing test score dummy	No	No	No	No	No	No	Yes
Nombre d'observations	780 112	780 112	780 112	780 112	780 112	780 112	616 314

Notes: Each line corresponds to a different regression. Controling for previous student test scores and restricting the sample to students attending high school, we observe that an additional repeated year in high school is associated with a decrease in the number of completed years of 0.10 year (column 7) **Source**: MENESR-DEPP, FAERE 2003 to 2011. *: $p < 0.1$; **: $p < 0.05$; ***: $p < 0.01$. Standard errors in parenthesis.

3.2 Empirical Strategy

Identifying the impact of grade raises two empirical challenges. The first challenge is the omitted variable bias. As established by the literature (see Grenet, 2010 for a review), grade retention is negatively and strongly correlated with initial student performance and ability. The second challenge is the simultaneity bias: each additional school year is an additional "opportunity" to repeat a grade. This challenge is particularly acute for grade repetition which occurs after students have reached the end of compulsory schooling. To overcome this issue, a possibility would be to exclude repetition which occurs after the end of compulsory schooling (sixteen years old in France). However, this would introduce a selection bias because students' who pursue an education after the end of compulsory schooling are higher achievers than those who do not. For example, I observe that the percentile rank at the 9th grade exam of students in high school (after compulsory schooling) is 53.1 (over 100) against 43.1 (over 100) for the other students.

To deal with these empirical challenges, I follow the literature and instrument grade retention by students' date of birth. This is a valid instrument under two conditions. First, it must be strongly correlated with the number of repeated years. Second, students' date of birth must impact students' school path only through its impact on grade repetition. Grenet (2010) shows that month of birth is strongly correlated with students' socioeconomic background, which can bias the results. He also shows that focusing on December and January neutralises this potential bias. In the following analysis, I report results for estimations on i) the whole sample; ii) only students born in January or December.

I estimate the following two stage least square specification:

$$r_i = \pi_0 + m_i \pi_1 + u_i \tag{1}$$

where r_i is the total number of repeated years by student i, m_i her month of birth and u_i the error term.

$$y_i = \alpha + \hat{r}_i \beta + \epsilon_i \tag{2}$$

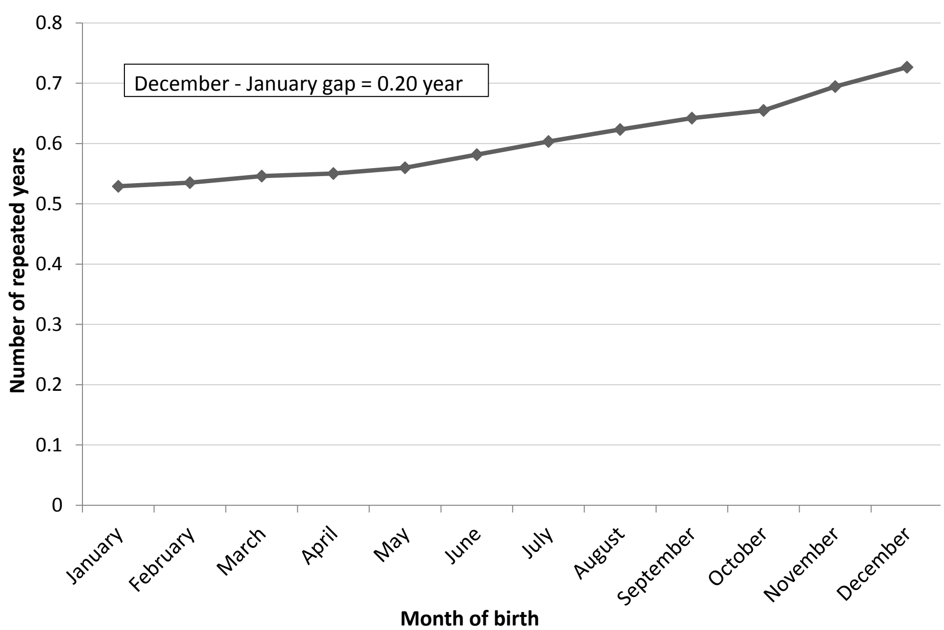
where y_i is either the number of years of schooling or the number of completed years; $\hat{\pi}_1$ is the estimated value of the coefficient π_1 and ϵ_i is the error term. The coefficient of interest is β. It can be interpreted as the causal marginal impact of one additional repeated year. Formally, the coefficient β is equal to the ratio of the covariance between month of birth and academic path, and the covariance between month of birth and the number of repeated years:

$$\beta = \frac{cov(m, y)}{cov(m, r)} \tag{3}$$

3.3 The Impact of Month of Birth on Grade Retention

Figure 3 shows that the number of repeated years increases linearly with the month of birth: students born in January have accumulated at the end of their schooling years on averge 0.53 repeated year against 0.73 year for students born in December. According to the existing literature (see Grenet (2010) for a review), the impact of the month of birth on grade retention appears in the begining of primary school but progressively fades out in middle school. The December-January gap in the number of cumulated repeated years does not fade out and persists in middle school.

Figure 3 – Number of cumulated repeated years by month of birth



Note: The number of cumulated repeated years for students born in January 1992 is equal to 0.53 year against 0.73 year for students born in December 1992.

Table 3 shows estimates of the impact of the month of birth on the number of cumulated repeated years at each stage of schooling (primary school, middle school etc.). Each additional month increases the cumulated number of repeated years by 0.0176 year, with a very high F-stat of 4 424.5. It corresponds to a December - January gap of 0,0176 $\times$ 11 = 0,19 year. The impact of the month of birth on grade repetition

Table 3 – Regression estimates of the impact of month of birth on the number of repeated years at each school stage

	Marginal effect (1)	**December - January gap** (2)
Dependant variable:		
Nb of repeated years	0,0176***	0,19
	(0,000)	
[F-statistic]	[4 424,5]	
Nb of repeated years in primary school	0,0124***	0,13
	(0,000)	
[F-statistic]	[8 032]	
Nb of repeated years in middle school	0,005***	0,05
	(0,002)	
[F-statistic]	[1 279,3]	
Nb of repeated years in high school (all tracks)	0,0004**	0,004
	(0,000)	
[F-statistic]	[4 ,12]	
Nb of repeated years in high school (academic)	0,0002	–
	(0,000)	
[F-statistic]	[1,47]	
Nb of repeated years in high school (vocational)	0,0002	–
	(0,000)	
[F-statistic]	[2,65]	
Nb of observations	780 112	

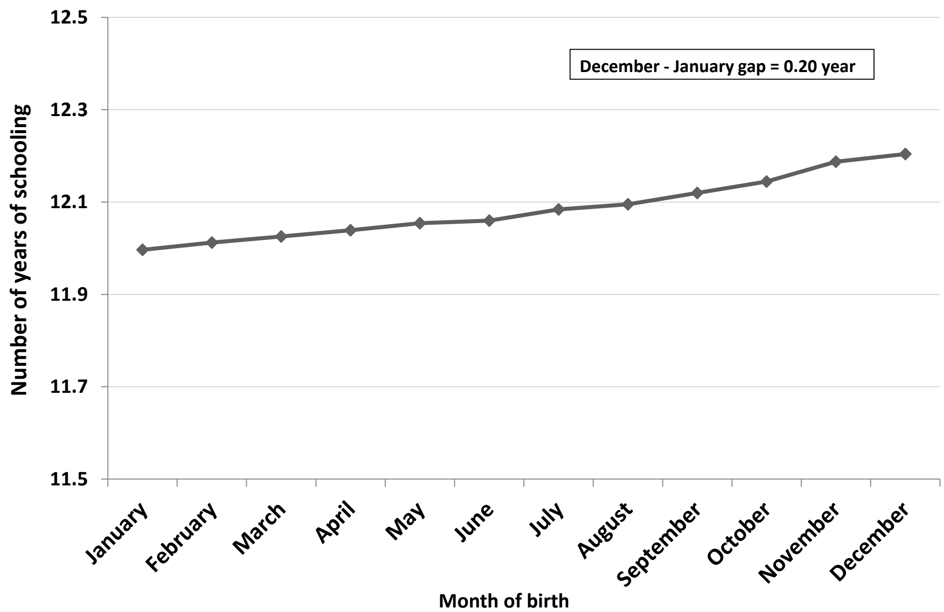
Notes: Each line corresponds to a different regression. Each additional month increases the cumulated number of repeated years by 0.0176 year. This corresponds to a December - January gap of 0,0176 $\times$. 11 = 0,19 year.

is the strongest in primary school. In middle school, this impact is smaller but still statistically significant: each additional month increases the number of repeated years in middle school by 0.005 year, which translates into a December - January gap of 0.05 year. This impact remains stastically significant in high school even though the magnitude becomes much smaller.

Thus, these results show that the impact of the month of birth on the number of repeated years persists throughout students' school path, even if most of the effect is concentrated in primary school. This is important for the interpretation of the instrumental variable estimates using month of birth as an instrument. It implies that this method relies mainly in the exogenous variation in the number of repeated years in primary school and middle school.

3.4 The Impact of Month of Birth on Students' School Path

Figure 4 – Number of Years of Schooling by Month of Birth



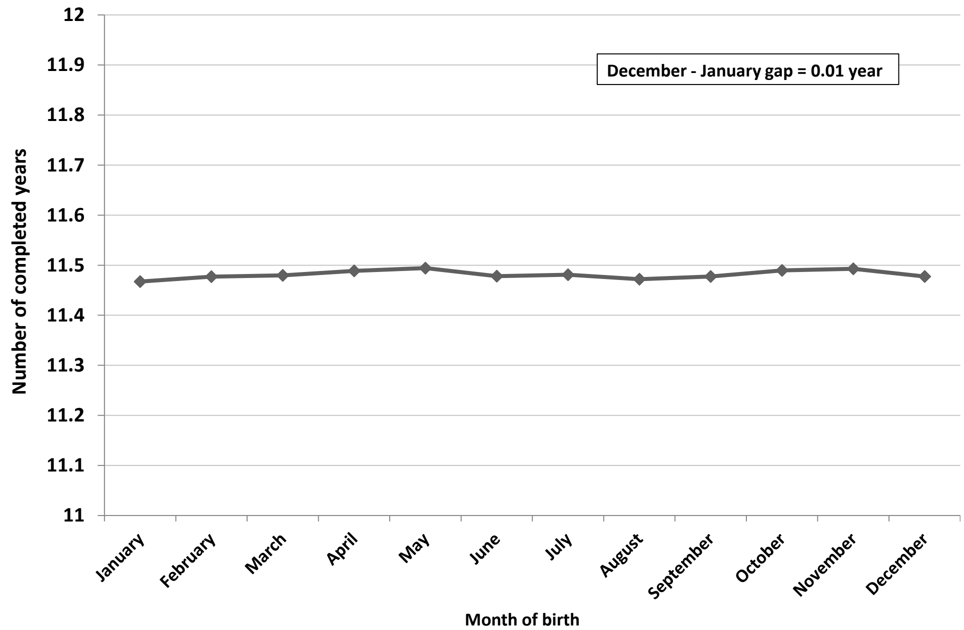
Notes: At the end of their schooling years, students born in January spent on average 11.63 years in school against 11.83 years for students born in December.

Figure 4 shows that the average number of schooling increases linearly with the month of birth. At the end of their schooling years, students born in January spent on average 11.63 years in school against 11.83 years for students born in December, i.e. a December - January gap equal to 0.20 year. The December - January gap in the number of years of schooling is therefore very similar to the December-January gap in the number of cumulated repeated years.

This first comparison suggests that each additional repeated year increases the number of years of schooling by exactly one year. Therefore, this suggests that grade retention has no statistically significant on students' school path and the level of training they eventually reach. Figure 5 shows that the number of completed years of training is constant across all months of birth. Table 4 shows that there is no statistically significant relationship between the month of birth and the number of completed years of training.

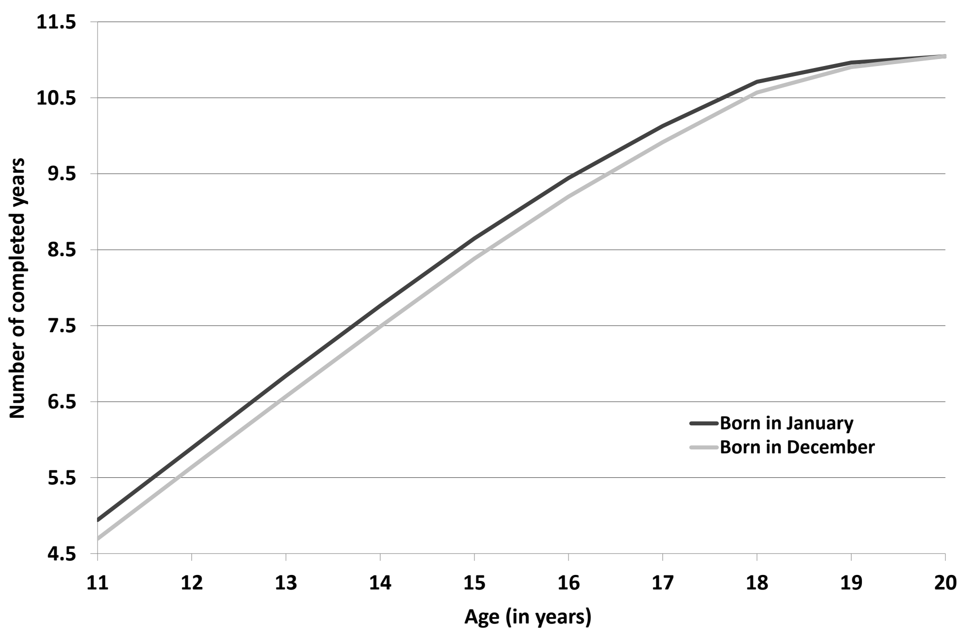
Figure 6 shows the evolution of the number of completed years of training by students' age, for students born in January and those born in December seperately. Its shows that until the age of 19, students born in December have completed, age

Figure 5 – Number of completed years of training by month of birth



Notes: The number of completed years of schooling is constant across all months of birth.

Figure 6 – Number of completed years of training of students born in January and in December by age (in years)



Lecture: Until the age of 19, students born in December have completed, age kept equal, a smaller number of years of training than students born in January.

Table 4 – The Marginal Impact of Month of Birth on Students' School Path

	All		Born in Jan. or in Dec.	
	Marginal impact	Dec.-Jan. gap	Marginal impact	Dec.-Jan. gap
	(1)	(2)	(3)	(4)
Dependent variable:				
Nb of years of schooling	0,0183***	0,201	0,0188***	0,207
	(0,0005)		(0,0008)	
Nb of completed years of training	0,0007	–	0,0009	–
	(0,0004)		(0,0006)	
Probability to take the vocational track	0,0022***	0,024	0,0015***	0,016
	(0,0001)		(0,0002)	
Probability to take the academic track	-0,0025***	-0,027	-0,002***	0,024
	(0,0001)		(0,0002)	
Probability to drop out in 9th grade	0,0002	0,002	0,0003	0,003
	(0,0001)		(0,0002)	
Nombre d'observations	780 112		129 712	

Notes: Each line corresponds to a different regression. The month of birth has a stastically significant positive impact on the probability of taking the vocational track.

kept equal, a smaller number of years of training than students born in January. This December - January gap disapears around the age of 20. Thus, it shows that students born in December progress slower but end up catching up with students born in January at the end of their schooling year. In particular, they end up reaching the same level of training at the end of their schooling year.

Table 4 shows regression estimates of the impact of month of birth on students' school path. These estimates confirm that the December - January gap in the cumulated number of years of repetition is equal to 0.20 years. This gap is equal to the December - January gap in the number of years of schooling. If it does not impact the number of completed years of training, the month of birth impacts the track taken in high school. The month of birth has a statiscally significant positive impact on the probability of taking the vocational track: each additional month increases the probability of taking the vocational track by 0.2 percentage point, which translates into a December - January gap of two percentage points. Symetrically, the month of birth has a negative statiscally significant impact on the probability of taking the academic track. The month of birth has however no statistically significant impact on the probability of dropping out after 9th grade.

3.5 The Impact of Grade Retention on Students' School Path

I use the instrumental variable method to identify the impact of grade retention on students' school path. I use the month of birth as the instrument for the number of years of repetition. The results must be interpretated carefully because they rely on exogenous variations on the subpopulation of compliers. These compliers are students for which the month of birth have an impact on the number of repeated years. As mentioned above, the impact of month of birth on grade retention is concentrated in primary school and the first years of middle school.

Thus, the interpretation of the results relies on the assumption that grade retention has the same impact whenever it occurs: primary, middle or high school.

Table 5 confirms the graphical evidence studied above. The marginal impact of a year of grade retention is to statistically significantly increase the number of years of schooling by exactly one year. Whereas grade retention does not have a stastistically significant impact on the number of completed years of training, it has a significant on the high school track. The marginal impact of one year of grade retention is to reduce the number of completed years of training in the academic track of 0.3 to 0.4 year and to increase by a similar amount in the vocational track.

Table 5 – Regression Estimates of Grade Retention on the Number of Years of Schooling and Students' School Path

	All (1)	**Born in Jan. or Dec.** (2)
Nb of years of schooling	1,04***	1,05***
	(0,02)	(0,03)
Nb of completed years of training	0,04	0,05
	(0,02)	(0,03)
including: general track	-0,38***	-0,26***
	(0,02)	(0,04)
including: vocational track	0,42***	0,32***
	(0,02)	(0,04)
Nb of observations	780 112	129 712

Lecture: Each line corresponds to a single regression. The marginal impact of a year of grade retention is to statistically significantly increase the number of years of schooling by one year.

4 The Cost of Grade Retention: Static Analysis

I estimate the static cost of retention in 2014 by comparing the actual cost of primary and secondary school schooling of students born in 1992 to the counterfactual cost in the case of no grade retention. I scale this cost using the grade retention rates observed in 2015. In the counterfactual scenario with no grade retention, I still keep the actual retention rates in grades leading to a degree (e.g. end of high school) because I still want to give the opportunity to students who fail their degree exam to take it again.

4.1 The Cost of Grade Retention in Primary and Middle School

As discussed above, the December - January gap in the number of cumulated repeated years in primary and middle school represents 95 % of the total number of cumulated repeated years at the end of high school. I estimate the cost of grade retention in primary and middle school by comparing the cost of schooling of students born in December to the cost of schooling of students born in January, and by scaling it by the difference in the observed cumulated number of repeated years between these two groups.

Table 6 – Average cost of repeated years of students born in January and in December 1992

	Average number repeated yrs (1)	Average cost yrs of schooling (euros) (2)	Average cost repeated yrs (euros) (3) = (1) × (2)	Average number completed yrs per student (4)	Average cost completed yrs (euros) (5)= (2)× (4)	Average number of schooling (6) = (1)+(4)	Average cost yrs of schooling (euros) (7) = (3)+(5)
A. Students born in January							
Primary school	0.09	6 060	545	5.00	30 300	5,09	30 845
Middle school	0.18	8 410	1 514	3.93	33 414	4.11	34 928
High school: acad. track	0.19	11 310	2 149	1.79	20 245	1.98	22 394
High school: vocational track	0.07	11 960	837	0.68	8 132	0.75	8 969
Total	**0,53**		**5 045**	**11,40**	**92 091**	**11,93**	**97 136**
B. Students born in December							
Primary school	0.23	6 060	1 394	5.00	30 300	5.23	31 694
Middle school	0.24	8 410	2 018	3.93	33 009	4,17	35 027
High school: acad. track	0,19	11 310	2 171	1.74	19 679	1,93	21 850
High school: vocational track	0.06	11 960	717	0.75	8 970	0.81	9 687
Total	**0,73**		**6 300**	**11,41**	**91 958**	**12,14**	**98 258**

Notes: The December - January gap in the number of repeated years cumulated in primary and middle school (0.18 year) corresponds to an additional cost of 1,010 euros per student

The December - January gap in the number of repeated years cumulated in primary and middle school (0.18 year) corresponds to an additional cost of 1,010 euros per student (tableau 6). The cost of one year of grade retention in primary and middle school is therefore equal to 1 010/0.18 = 5 610 euros per student on average. The average number of cumulated repeated years by the 1992 cohort in primary and middle school is 0.35 year. Therefore, the cost of grade retention in primary and middle school is 0.35 x 5,610 x 780,112 = 1.53 billon euros.

The grade retention rate in primary school today is 0.7 time smaller than for the 1992 cohort (DEPP, 2014). Thus, the cost of grade retention in primary and second school today is equal to one billon euros, including 500 millons for grade retention in primary school and 600 millions in middle school.

4.2 The Cost of Grade Retention in High School

The estimation of the cost of grade retention in high school relies on the assumption that the impact of grade retention on the number of years of schooling is the same as primary and secondary school's: the marginal impact of one additional repeated year in high school increases schooling by one additional year.

Table 7 – Average Cost of Repeated Years in High School for Students born in 1992

	Average number of repeated yrs (1)	Average cost of yrs of schooling (euros) (2)	Average cost of repeated yrs (euros) (3) = (1) × (2)
Academic Track	0.13	11 310	1 470
Professional Track	0.00	11 960	44
Total	**0.13**		**1 514**

Notes: The average cost of repeated years in high schools (0.13 years, excluding graduating years) is 1,514 euros per student.

The average cost of repeated years in high schools (0.13 years, excluding graduating years) is 1,514 euros per student. The cost of grade repeatition in high school for students born in 1992 is 1,514 x 780,112 = 1,2 billion euros.

Grade retention rates in high school being 0.8 times smaller for students today than for the 1992 cohort, the cost of grade retention today is equal to 900 millions euros. Almost the totality of this cost can be attributed to grade retention in the academic track.

In total, the total cost of grade of retention in primary, middle school and high school is equal to 2 billions euros.

5 Cost of Grade Retention: Dynamic Approach

The budgetary savings from the reduction or suppression of grade repetition do not occur instantaneously, but only a relatively long transitory period. For example, in the short run, a student repeating first grade will not reduce education spendings because she will cost a year of schooling in second grade rather than a year of schooling in first grade. Therefore, budgetary savings from reducing and suprressing grade retention will reach a steady state only when students who were entering primary school at the time of the change in policy have left high school.

This is why, to accurately measure the budgetary impact of a grade retention reform, I follow a dynamic approach modelling student enrollment flows with a discrete Markov chain model.

5.1 The Conceptual Framework: Discrete Markov Chains

I model student enrollment flows with a discrete Markov chain model. This model have been already used to model hospital patients flows (Kolesar, 1970; Bartolomeo et al., 2008) ou students flows in universities (Bessent et Bessent, 1980; Shah et Burke, 1999; Nicholls, 2007).

Notations. A Markov process (noted X) is a sequence $X_1, X_2, X_3, ...$, of random variables $\{X_t, t \in \mathbb{Z}^+\}$, where X_t take its values in the finite set $S = \{1, ..., N\}$. The values of X_t are called states of the process. I denote $x_t \in S$ the state occupied by the process at time t. The probability of transition between the states x_t and x_{t+1} writes:

$$\begin{aligned} P(x_t, x_{t+1}) &= \Pr[X_{t+1} = x_{t+1} | X_0 = x_0, ..., X_t = x_t] \\ &= \Pr[X_{t+1} = x_{t+1} | X_t = x_t] \end{aligned} \tag{4}$$

The sequence $(x_0, ..., x_t)$ is a discrete Markov chain generated by the process X. This chain is caracteristed by its transition matrix:

$$\mathbf{P} = \begin{bmatrix} P(1,1) & P(1,2) & \cdots & P(1,N) \\ P(2,1) & P(2,2) & \cdots & P(2,N) \\ \vdots & \vdots & \ddots & \vdots \\ P(N,1) & P(N,2) & \cdots & P(N,N) \end{bmatrix} \tag{5}$$

$\mathbf{P}$ is a stochastic matrix. This means that it verifies the two following fundamental properties:

(1) $\forall (i,j) \in S$, $P(i,j) \in [0,1]$.

(2) $\forall i \in S$, $\sum_{j \in S} P(i,j) = 1$

I introduce the vector of the initial distribution of states, called $\mu_\mathbf{0}$. The distribution at time t writes:

$$\mu_\mathbf{t} = \mu_\mathbf{0} \mathbf{P}^t \tag{6}$$

The steady state μ_∞ writes:

$$\mu_\infty = \mu_\infty \mathbf{P} \tag{7}$$

Thus, student enrollment flows can be modeled as follows:

- X_t corresponds to the grade students are in at time t;

- S corresponds to the whole set of grades (1st grade, 2nd grade, etc.) and an absorbing state corresponding to "leaving the schooling system"
- $P(x_t, x_{t+1})$ corresponds to the transition probability between the grade in period t and the grade in $t+1$. Diagonal transition probabilities are grade repeatition probabilities between t et $t+1$.
- The vector $\mu_{\mathbf{0}}$ corresponds to the initial distribution of the number of enrolled students across the different grades

Student enrollment flows, in order to be modelled as Markov chains, must verify the property (4). This property means that students' grade at time $t+1$ depends only of students' grade at time t. This assumption is not plausible at the individual student level. To explain this point further, let us take the example of a fifth grade student at time t, who can either go to 6th grade at time $t+1$ or to repeat fifth grade. If this student has already repeated a grade before, it is unlikely that she will repeat again because multiple grade repetition are rare. In other words, her grade at $t+1$ depends not only on her grade at t, but also on all her previous school path.

The property (4) is more likely to be verified at the aggregate level. At this level of observation, it is more likely to assume that individual schooling paths do not influence macro level of student enrollment flows between grades. This is why our level of observation is the overall student enrollment across grades every year.

I simulate three main counterfactual scenarios: i) grade retention rates remain constant at their 2013 level (see DEPP, 2014 for data); ii) grade repetition is suppressed from 2015 onwards.

5.2 Grade transitions

We consider 18 distinct states: 17 grades [2] and an exit state (exit from schooling). The transition matrix $\mathbf{P}$ has 18 lines and 18 columns. The vector $\mu_{\mathbf{0}}$ gives the distribution of initial student enrollment by grade. This vector corresponds to student enrollment in 2013-2014.

The vector $\mu_{\mathbf{e}}$ shows the number of students entering schooling for the first time. We assume this vector to be constant and to take null values except for the first line which is equal to the number of students entering first grade [3].

[2] CP, CE2, CM1, CM2, sixieme, cinquieme, quatrieme, troisieme, seconde GT, premiere GT, terminale GT, seconde pro, premiere pro, terminale pro, premiere annee de CAP et deuxieme annee de CAP

[3] The number of students entering first grade is computed by substracting to the number of students enrolled in first grade in 2013-2014 the number of repeaters.

Student enrollment distributions by grade in period 1 ($\mu_{\mathbf{1}}$) and in period 2 ($\mu_{\mathbf{2}}$) are defined as follows:

$$\mu_{\mathbf{1}} = \mu_{\mathbf{0}}\mathbf{P} + \mu_{\mathbf{e}} \tag{8}$$

$$\mu_{\mathbf{2}} = \mu_{\mathbf{0}}\mathbf{P}^2 + \mu_{\mathbf{e}}\mathbf{P} + \mu_{\mathbf{e}} \tag{9}$$

Thus, the distribution of student enrollment at period t writes:

$$\mu_{\mathbf{t}} = \mu_{\mathbf{0}}\mathbf{P}^t + \mu_{\mathbf{e}} \sum_{k=0}^{t-1} \mathbf{P}^k \tag{10}$$

We write $\mu_{\mathbf{t}}^{(\mathbf{s})}$ the distribution of student enrollment at period t if grade retention were suppressed in 2015 and $\mu_{\mathbf{t}}^{(\mathbf{r})}$ the counterfactual distribution of student enrollment if grade retention rates were kept constant at their 2013 level [4]. The impact of the suppression of grade retention on the distribution of student enrollment in period t is equal to $\mu_{\mathbf{t}}^{(\mathbf{s})} - \mu_{\mathbf{t}}^{(\mathbf{r})}$. The steady state is defined as:

$$\mu_{\mathbf{t+1}}^{(\mathbf{s})} - \mu_{\mathbf{t+1}}^{(\mathbf{r})} = \mu_{\mathbf{t}}^{(\mathbf{s})} - \mu_{\mathbf{t}}^{(\mathbf{r})} = \mu_{\infty}^{(\mathbf{s})} - \mu_{\infty}^{(\mathbf{r})} \tag{11}$$

The yearly budgetary savings generated by the suppression of grade retention are obtained by multiplying the vector $\mu_{\mathbf{t}}^{(\mathbf{s})} - \mu_{\mathbf{t}}^{(\mathbf{r})}$ by the average spending by student for each grade. This computation relies on the assumption that the elasticity of spending with regard to student enrollment is equal to one. In other words, we assume that spendings adjuste instantaneously to variations in student enrollment. This assumption seems plausible in the long run but less in the short run. In the short run, the assumption that the elasticity of spending with regard to student enrollment is smaller than one seems more plausible. An elasticity smaller than one would slow the space of yearly budgetary savings. Thus, our estimation must be interpreted as an upper bound for the short run budgetary savings.

5.3 Simulations of the Suppression of Grade Retention from 2015

Simulation 1: Suppression of Grade Retention in Primary School. Figure 7a shows the dynamic impact on student enrollment of the suppression of grade

[4]The parameters used in this simulation are shown in the appendix

retention in primary school from 2015, keeping grade retention in secondary school at its 2013 level. This shows that the suppression of grade retention in primary school provokes a continuous decrease in enrollment in primary school from 2016 to 2020, but also an increase in enrollment in middle school until 2020, and then in high school from 2020 to 2024. These demographic bulges are due to the fact that grade retention in primary school accelerates students' progression across grades. It "pushes out" students from primary school to secondary school. This increase in enrollment in secondary school is only temporary and disappears progressively from 2024 for middle school, and from 2028 for high school. In total, student enrollment in primary and secondary school remains constant from 2016 to 2019. It starts to decrease only from 2020 and the steady state is reached in 2028. From 2028, the suppression of grade retention in primary school provokes a steady decrease in enrollment (compared to 2015) of 76,500 students per year.

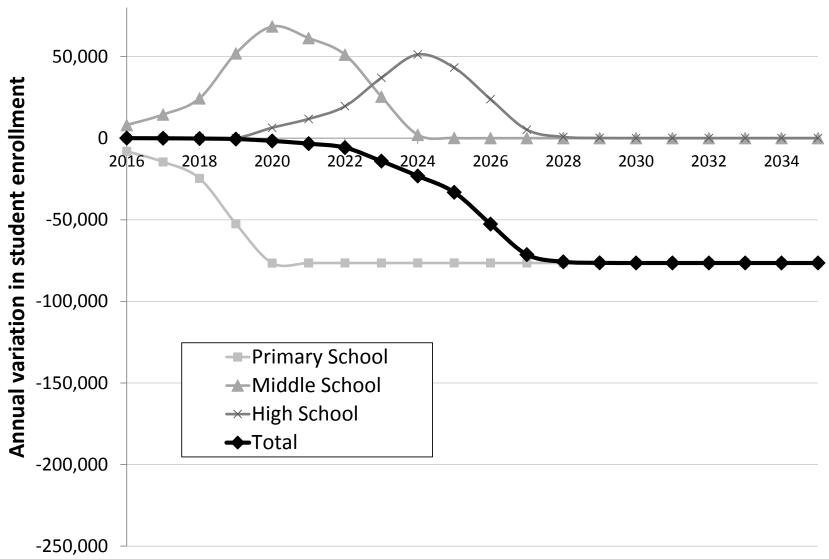
This transitory impact of the suppression of grade retention in primary school on the distribution of students across grades explains the complex dynamic impact of this reform on budgetary spendings(figure 7b). First, this reform provokes an increase in total spendings from 20 millions euros in 2016 to 200 millions euros in 2020. This is explained by the fact that the temporary decrease in enrollment in primary school is offset by the increase in enrollment in secondary school. Spending per student is higher in secondary school than in primary school: this is why total spendings increase temporarily. From 2026, the demographic bulge has disappeared, and the steady state is reached in 2028. From 2028, each year, 465 millions euros are saved thanks to the reform.

Simulation 2: Suppression of Grade Retention in Secondary School. Figure 8 shows the dynamic impact of a reform consisting in maintaining grade retention in primary school at its 2013 level, but to suppress it in secondary school (middle and high school). I still give students who failed their high school degree the opportunity to retake it. By construction, such a reform would have no impact on enrollment in primary school (figure 8a). This reform would however decrease enrollment in secondary school, from 2016 in middle school and 2018 in high school. The transitory enrollment surplus in high school is due to the transitory inflow of students from middle school which offsets temporarily the outflow of students from high school. The steady state is reached in 2024 in middle school, and in 2026 in high school. In total, from 2026, the suppression of grade retention in secondary school provokes an annual decrease in student enrollment of around 150,000 students.

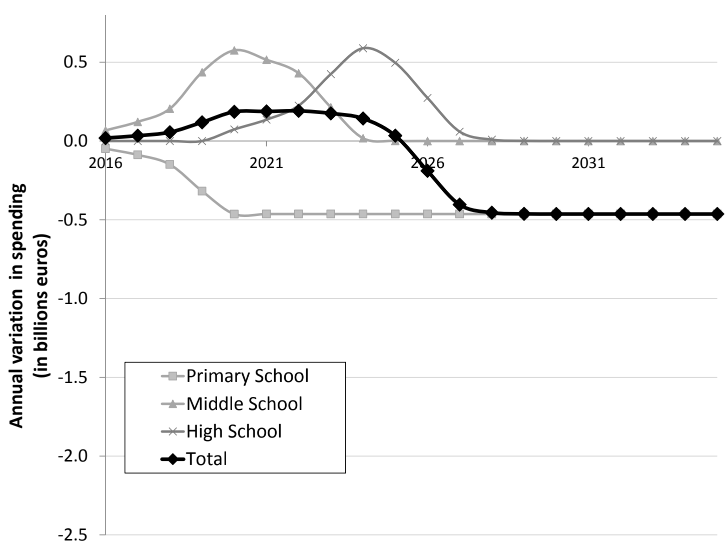
The dynamic impact of the suppression of grade retention in secondary school on

Figure 7 – Simulation 1: Suppression of Grade Retention in Primary School in 2015

(a) Impact on student enrollment (2016-2035)



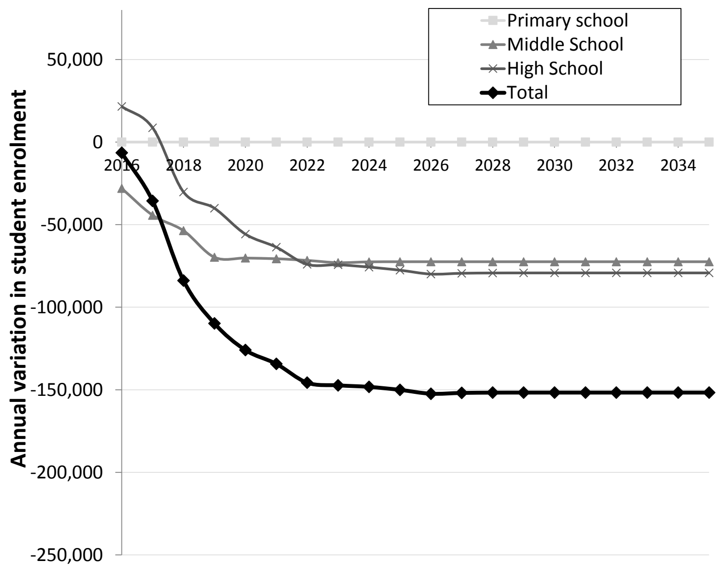
(b) Impact on budgetary spending, in billon euros (2016-2035)



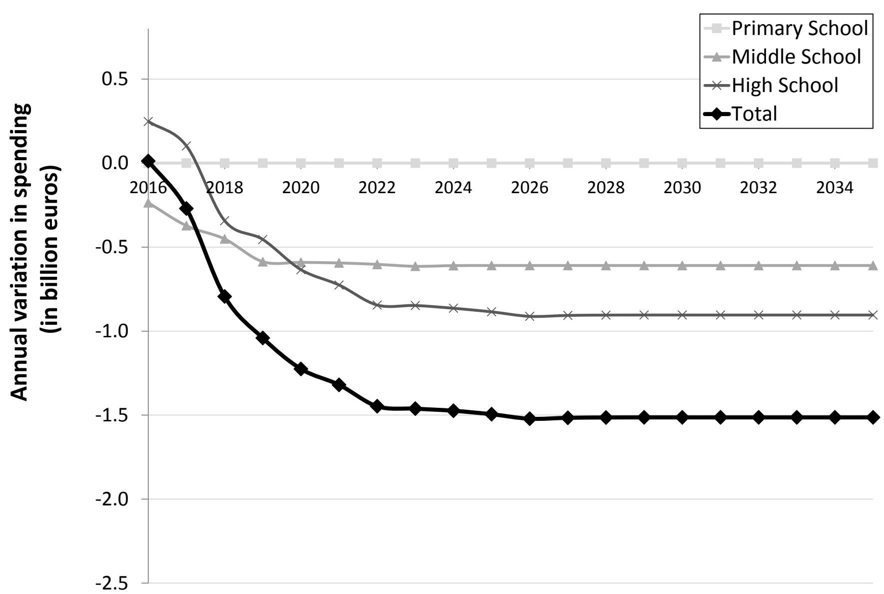
Notes: The suppression of grade retention in primary school provokes a continuous decrease in enrollment in primary school from 2016 to 2020, but also an increase in enrollment in middle school until 2020, and then in high school from 2020 to 2024. These demographic bulges are due to the fact that grade retention in primary school accelerates students' progression across grades. First, this reform provokes an increase in total spendings from 20 millions euros in 2016 to 200 millions euros in 2020.

Figure 8 – Simulation 2: Suppression of Grade Retention in Secondary School from 2015

(a) Impact on student enrollment (2016-2035)



(b) Impact on budgetary spendings, in billon euros (2016-2035)



Notes: see notes of figure 7.

budgetary spending is shown in figure 8b. In 2016, the suppression of grade retention in secondary school provokes an additional cost of 12 millions euros. This comes from the conjonction of two factors: i) the transitory inflow of students from middle to high school; ii) the fact that spending per student is higher in high school than in middle school. The first budgetary savings appear in 2017 and the steady state is reached in 2026. From 2026 onwards, the suppression of grade retention in secondary school creates annual budgetary savings of 1.5 billion euros each year.

Simulation 3: Suppression of Grade Retention in Primary and Secondary School.

The dynamic impact of the total suppression of grade retention (both in primary and secondary school, except for students who repeat their end of high school examination) on student enrollment stems from the combination of the effects studied above. This reform provokes a continuous decrease of enrollment, reaching a steady state in 2027 (figure 9a). From 2027 onwards, the suppression of grade retention generates a decrease of yearly student enrollment of 208 000 students, which translates into a total decrease in annual spending of 2 billions euros (figure 9b).

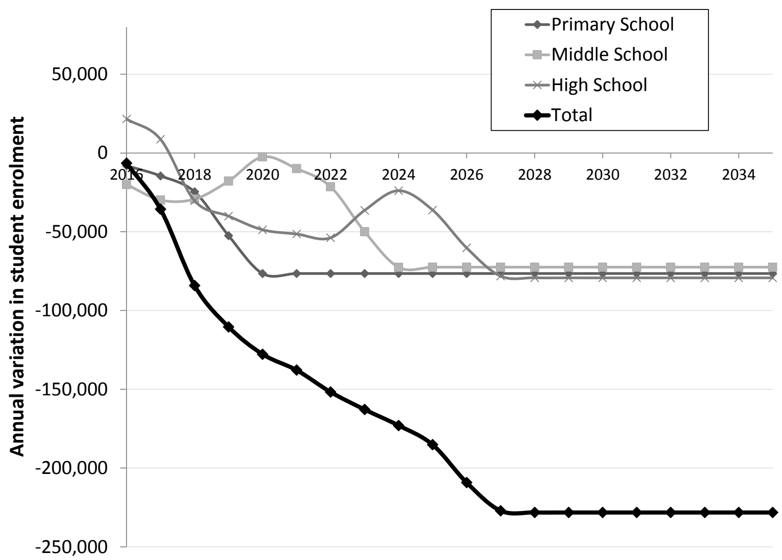
6 Discussion: Reallocation of The Resources from the Suppression of Grade Retention

The resources from the suppression of grade retention could be allocated towards alternative educational interventions which have proven to be more cost-effective according to the economic literature. In particular, two main alternatives can be precisely analysed: the reduction of class size in primary school and summer schools for disadvantaged students. Budgetary savings from the suppression of grade retention would be large enough to allow a class size reduction in primary school of 5.4 students on average. According to Piketty and Valdenaire (2006), the impact of this class size reduction is to increase student test scores by 15 % of a standard deviation. If the new resources from the suppression of grade retention were spent exclusively on 50 % of the most disadvantaged primary schools, class size could be divided by two, and student performance increased by 70 % of a standard deviation in those schools.

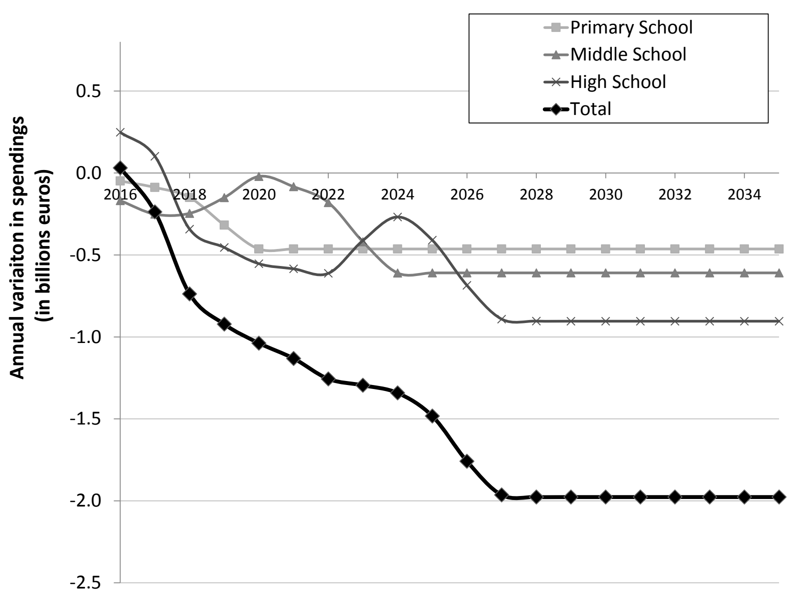
Alternatively, the resources from the suppression of grade retention could be used to fund an intensive program of summer schools. Borman and Dowling (2006) study an intensive summer school program in Baltimore, consisting in seven consecutive weeks

Figure 9 – Simulation 3: Suppression of Grade Retention in Primary and Secondary School from 2015

(a) Impact on student enrollment (2016-2035)



(b) Impact on budgetary spendings (2016-2035)



Notes: see notes of figure 7.

of intensive classes throughout three consecutive years. With a randomized controled trial, they show that the impact of this program is to increase student test scores by 40 % of a standard deviation. A back of envelope computation shows that the suppression of grade retention would fund this program for the 25 % lowest achieving students in primary and middle school.

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Appendix

Table A1 – Transition matrix **P** with 2013 grade retention rates.

	CP	CE1	CE2	CM1	CM2	6e	5e	4e	3e	2GT	1GT	TGT	CAP1	CAP2	2PRO	1PRO	TPRO	Sortie
CP	0,029	0,971	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CE1	0	0,032	0,968	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CE2	0	0	0,012	0,988	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CM1	0	0	0	0,008	0,992	0	0	0	0	0	0	0	0	0	0	0	0	0
CM2	0	0	0	0	0,010	0,99	0	0	0	0	0	0	0	0	0	0	0	0
6e	0	0	0	0	0	0,022	0,969	0	0	0	0	0	0	0	0	0	0	0,009
5e	0	0	0	0	0	0	0,012	0	0,975	0	0	0	0	0	0	0	0	0,013
4e	0	0	0	0	0	0	0	0,019	0,964	0	0	0	0	0	0	0	0	0,017
3e	0	0	0	0	0	0	0	0	0,035	0,626	0	0	0,044	0	0,206	0	0	0,089
2GT	0	0	0	0	0	0	0	0	0	0,079	0,860	0	0	0	0	0,036	0	0,025
1GT	0	0	0	0	0	0	0	0	0	0	0,047	0,932	0	0	0	0	0,003	0,018
TGT	0	0	0	0	0	0	0	0	0	0	0	0,058	0	0	0	0	0	0,942
CAP1	0	0	0	0	0	0	0	0	0	0	0	0	0,039	0,763	0	0	0	0,198
CAP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0,051	0	0,214	0	0,735
2PRO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,042	0,831	0	0,127
1PRO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,018	0,873	0,109
TPRO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,062	0,938
Sortie	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1,000

Table A2 – Transition matrix **P** for simulation 1: suppression of grade retention in primary school

	CP	CE1	CE2	CM1	CM2	6e	5e	4e	3e	2GT	1GT	TGT	CAP1	CAP2	2PRO	1PRO	TPRO	Sortie
CP	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CE1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CE2	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CM1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
CM2	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
6e	0	0	0	0	0	0,022	0,969	0	0	0	0	0	0	0	0	0	0	0,009
5e	0	0	0	0	0	0	0,012	0	0,975	0	0	0	0	0	0	0	0	0,013
4e	0	0	0	0	0	0	0	0,019	0,964	0	0	0	0	0	0	0	0	0,017
3e	0	0	0	0	0	0	0	0	0,035	0,626	0	0	0,044	0	0,206	0	0	0,089
2GT	0	0	0	0	0	0	0	0	0	0,079	0,860	0	0	0	0	0,036	0	0,025
1GT	0	0	0	0	0	0	0	0	0	0	0,047	0,932	0	0	0	0	0,003	0,018
TGT	0	0	0	0	0	0	0	0	0	0	0	0,058	0	0	0	0	0	0,942
CAP1	0	0	0	0	0	0	0	0	0	0	0	0	0,039	0,763	0	0	0	0,198
CAP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0,051	0	0,214	0	0,735
2PRO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,042	0,831	0	0,127
1PRO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,018	0,873	0,109
TPRO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,062	0,938
Sortie	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1,000

Table A3 – Transition matrix **P** for simulation 2: suppression of grade retention in secondary school

	CP	CE1	CE2	CM1	CM2	6e	5e	4e	3e	2GT	1GT	TGT	CAP1	CAP2	2PRO	1PRO	TPRO	Sortie
CP	0,029	0,971	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CE1	0	0,032	0,968	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CE2	0	0	0,012	0,988	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CM1	0	0	0	0,008	0,992	0	0	0	0	0	0	0	0	0	0	0	0	0
CM2	0	0	0	0	0,010	0,99	0	0	0	0	0	0	0	0	0	0	0	0
6e	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
5e	0	0	0	0	0	0	0	0,987	0	0	0	0	0	0	0	0	0	0,013
4e	0	0	0	0	0	0	0	0	0,983	0	0	0	0	0	0	0	0	0,017
3e	0	0	0	0	0	0	0	0	0	0,65	0	0	0,046	0	0,21	0	0	0,092
2GT	0	0	0	0	0	0	0	0	0	0	0,934	0	0	0	0	0,04	0	0,027
1GT	0	0	0	0	0	0	0	0	0	0	0	0,978	0	0	0	0	0,003	0,019
TGT	0	0	0	0	0	0	0	0	0	0	0	0,058	0	0	0	0	0	0,942
CAP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0,794	0	0	0	0,206
CAP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0,051	0	0,214	0	0,735
2PRO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,867	0	0,133
1PRO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,889	0,111
TPRO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,062	0,938
Sortie	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Table A4 – Transition matrix **P** for simulation 3: suppression of grade retention in primary and secondary school

	CP	CE1	CE2	CM1	CM2	6e	5e	4e	3e	2GT	1GT	TGT	CAP1	CAP2	2PRO	1PRO	TPRO	Sortie
CP	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CE1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CE2	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CM1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
CM2	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
6e	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
5e	0	0	0	0	0	0	0	0,987	0	0	0	0	0	0	0	0	0	0,013
4e	0	0	0	0	0	0	0	0	0,983	0	0	0	0	0	0	0	0	0,017
3e	0	0	0	0	0	0	0	0	0	0,65	0	0	0,046	0	0,21	0	0	0,092
2GT	0	0	0	0	0	0	0	0	0	0	0,934	0	0	0	0	0,04	0	0,027
1GT	0	0	0	0	0	0	0	0	0	0	0	0,978	0	0	0	0	0,003	0,019
TGT	0	0	0	0	0	0	0	0	0	0	0	0,058	0	0	0	0	0	0,942
CAP1	0	0	0	0	0	0	0	0	0	0	0	0	0	0,794	0	0	0	0,206
CAP2	0	0	0	0	0	0	0	0	0	0	0	0	0	0,051	0	0,214	0	0,735
2PRO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,867	0	0,133
1PRO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,889	0,111
TPRO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0,062	0,938
Sortie	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Table A5 – Distribution of initial enrollment by grade ($\mu_{\mathbf{0}}$ vector)

	CP	CE1	CE2	CM1	CM2	6e	5e	4e	3e	2GT	1GT	TGT	CAP1	CAP2	2PRO	1PRO	TPRO	Sortie
Enrollment	845 005	855 746	822 034	806 431	803 388	807 069	806 670	808 969	785 408	530 299	473 661	466 601	67 381	53 101	186 093	156 589	173 671	-

Table A6 – Student enrollment by grade ($\mu_{\mathbf{e}}$ vector)

	CP	CE1	CE2	CM1	CM2	6e	5e	4e	3e	2GT	1GT	TGT	CAP1	CAP2	2PRO	1PRO	TPRO	Sortie
Effectifs	820 500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-