Comment on

How particular is the physics of the free energy principle?

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Very particular: Comment on

How particular is the physics of the free energy principle?

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Aguilera *et al* [1] offer a deconstruction of the FEP that rests upon two assertions.

- The FEP relies on "an equivalence between the dynamics of the average states of a system with the average of the dynamics of those states" p25
- The FEP relies on "an absence of perception-action asymmetries that is highly unusual for living systems interacting with an environment" p25

These assertions are wrong and can be traced back to two basic mistakes in their analysis. The first mistake is introduced by their first move:

> *"2.1. First move*: *capturing Bayesian inference with an average flow*
> The FEP starts by describing the average flows of the *external* states of a system as following a gradient minimizing a variational free energy." p30.

The FEP describes the most likely flow of *internal* (not *external*) states as a gradient flow on variational free energy. The ensuing misrepresentation of the FEP is fundamental and goes beyond a simple misreading of the literature for the following reason:

The raison d'être for variational free energy is to account for internal dynamics, where variational free energy is a function of internal states. This means the average flow of external states cannot be a gradient flow on variational free energy, because their flow is not a function of internal states (by condition 1).

This mistake is propagated throughout the paper; for example, the arguments based upon the Laplace assumption are restricted to the dynamics of external states. These arguments may or may not be valid; however, they do not concern the FEP. The second mistake lies in their first assumption:

> *"Assumption 1. Solenoidal couplings between 'blocks' of states (y, s, a, x) are precluded when a Markov blanket emerges under sparse coupling [3].* p32.

This is not an assumption of the FEP. A canonical decomposition of the Jacobian—in terms of flow operators and Hessians that satisfies condition 1 and 3 (i.e., sparse coupling and conditional independence)—can be expressed as follows. Please see [2, 3] for details:

$$\underbrace{\begin{bmatrix} \mathbf{J}_{yy} & \mathbf{J}_{ys} & \mathbf{J}_{ya} & \\ \mathbf{J}_{sy} & \mathbf{J}_{ss} & \mathbf{J}_{sa} & \\ & \mathbf{J}_{as} & \mathbf{J}_{aa} & \mathbf{J}_{ax} \\ & \mathbf{J}_{xs} & \mathbf{J}_{xa} & \mathbf{J}_{xx} \end{bmatrix}}_{\substack{\text{Sparse coupling} \\ \text{(condition 1)}}} = \underbrace{\begin{bmatrix} Q_{yy}-\Gamma_y & Q_{ys} & & \\ -Q_{ys}^T & Q_{ss}-\Gamma_s & & \\ & & Q_{aa}-\Gamma_a & Q_{ax} \\ & & -Q_{ax}^T & Q_{xx}-\Gamma_x \end{bmatrix}}_{\text{Conservative (solenoidal) and dissipative flow operators}} \underbrace{\begin{bmatrix} \mathbf{H}_{yy} & \mathbf{H}_{ys} & & \\ \mathbf{H}_{ys}^T & \mathbf{H}_{ss} & \mathbf{H}_{sa} & \\ & \mathbf{H}_{sa}^T & \mathbf{H}_{aa} & \mathbf{H}_{ax} \\ & & \mathbf{H}_{ax}^T & \mathbf{H}_{xx} \end{bmatrix}}_{\substack{\text{Conditional independencies} \\ \text{(condition 3)}}} \tag{1}$$

$$+\nabla\Omega\cdot\nabla\Im-\nabla\Lambda$$

$$\begin{aligned} \nabla_y\Omega_{\alpha\alpha}&=0,\ \ \nabla_x\Omega_{\beta\beta}=0,\ \ \nabla_y\Lambda_\alpha=0,\ \ \nabla_x\Lambda_\beta=0 \\ \alpha&=(a,x) \\ \beta&=(s,y) \\ \Omega(z)&=Q(z)-\Gamma \end{aligned}$$

This canonical form subsumes the linear cases considered in [1]. The second mistake underwrites several specious conclusions. For example:

> "In this type of [canonical] system, detailed balance is only broken by interactions inside blocks *y, s, a, x,* as all couplings between blocks are symmetric. Therefore, the system is driven out-of-equilibrium only by internal tendencies of these blocks, not by their interactions between them." p36.

This is incorrect. Solenoidal coupling between external and sensory states—and between active and internal states—are (or can be) part of the canonical dynamics that satisfy conditions 1 and 3 (see Equation 1).

Intuitively, it is easy to see why solenoidal coupling between internal and external states does not feature in random dynamical systems under condition 1: solenoidal coupling between internal and external states means that the influence of external states on internal states depends upon the gradients of surprisal with respect to external states (and *vice versa*). However, condition 1 stipulates that external states cannot influence internal states. This means that solenoidal (nonequilibrium, detailed balance breaking) dynamics are mediated by solenoidal interactions within and between external and blanket (sensory) states—and within and between internal and blanket (active) states. A worked (quasi-analytic) example of the ensuing generalised synchronisation can be found in [2].

In addition to these foundational mistakes, the critique rests on intuitions that the authors could be pressed on. For example:

> "In this type of system, detailed balance is only broken by interactions inside blocks …. This precludes for example the existence of asymmetric agent-environment interactions, which may be crucial for … regulating exchanges of matter and energy with the environment." p36.

This intuition implies that solenoidal flow underwrites an exchange of energy. It would be interesting to hear how the authors think solenoidal mixing mediates an exchange of energy. To be more precise, how does solenoidal flow affect energy exchange, when it does not contribute to the dissipation of heat? [4-6]. This speaks to interesting issues about the distinction between the loss of detailed balance—in virtue of solenoidal flows—and entropy production in stochastic thermodynamics—in virtue of gradient flows [7].

Similarly:

> "Intuitively, substituting the true flow by an average flow fixing the blanket state decouples the trajectory of *y* from its previous state, which in most dynamical systems will result in an impoverished description, not capturing its real, history-dependent, behaviour." p41.

Random dynamical systems are Markovian and therefore do not admit "history-dependent behaviour" [8]. Perhaps the authors meant that conditional expectations about external states are not the same as their 'real behaviour'. If this is what the authors meant, it may help to note that the FEP explains how one can read internal states as furnishing an 'impoverished description' of external states—an 'impoverished description' that is Bayes optimal.

One thing that made me smile was:

> "To this end, we explore a class of systems defined by stochastic linear differential equations under a weak-coupling assumption. Such systems are the simplest possible example that can display the dynamics required for the FEP." p25.

This reminded me of the physicist's spherical cow[1]. This joke is particularly prescient in the current setting. This is because the explicanda of the FEP concern that the nature of 'cows' and how they differ from 'spheres'. For example, linear systems preclude the chaotic dynamics [2, 9, 10] that underwrite the itinerancy characteristic of cows and other creatures.

On the difference between 'cows' and 'spheres': one thing that I thought the authors could usefully develop was their ontology of particular kinds implicit in their sensorimotor loop structures in Figure 4. It would be nice if these instances of Markov blankets were supplemented with the accompanying functional form of Equation (1). The behaviour of these distinct kinds can differ in interesting ways, under the FEP—and I am not aware of any previous attempts to systematise particular partitions with progressively sparse coupling.

[1] The physicist to the farmer: "I have the solution, but it works only in the case of spherical cows in a vacuum."

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