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Epistemic Quality in the Intended Mathematics Curriculum and Implications for Policy

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Introduction

The discussion in this chapter is in the context of the English school mathematics curriculum and addresses the characterization of epistemic quality in school mathematics: what is it, why does it matter, and what might it look like in a classroom? I develop my arguments by considering which epistemic values are being communicated in official curriculum documents and in the related curriculum and assessment materials used by teachers. I ask how the consequent received curriculum is impinged on by Gericke et al.'s (2018) 'transformation' of knowledge in the classroom, and what impact the related epistemic communications appear to have on the epistemic quality available to students. Within the characterization of epistemic quality, I argue in particular for the valuing of *epistemological*, as well as general *epistemic, ascent* (Winch 2013) in mathematics curriculum planning. I draw on a recent large, longitudinal classroom-close study to explore what that might mean, to what extent each of epistemic and epistemological ascent is being achieved, and how those might be enhanced. Finally, I discuss the implications for curriculum system policy (Schmidt and Prawat 2006), including in relation to curriculum materials, assessments and teacher initial and continuing education.

'Curriculum' itself has been a contested term in the education literature: I use the word to signify the *range* of experiences associated with education institutions, whether *intended, enacted, experienced* or *achieved* (Mullis and Martin 2015). For many jurisdictions in recent years, including England, the *intended curriculum* has been developed as a 'national curriculum' for ages 5 to 16, that in some form embodies centralized intentions for school-associated learning experiences. In England, teachers enjoy a high degree of autonomy in how curriculum is enacted but work with high-stakes student assessments at ages 11, 16 and 18. The English curriculum structure is largely subject-based: arguably, the focus issues are even more central in a thematically or 'problem-based' structure, where the role of disciplines is itself contested.

I draw on work that explored the enactment of the 2014 mathematics curriculum in England, which arguably aligns rather better than its predecessor with the epistemological priorities of the parent discipline. The focus curriculum aims to promote students' mathematical conceptual fluency, reasoning and problem-solving in mathematics. Internationally, such aspirational goals are widely valued but have not been achieved at scale.

I engage with the *epistemic quality* – discussed by Hudson (2018) and understood to be the quality of the syntactical and substantive mathematics – made available to learn in the classroom, and in particular the quality of *epistemology*: the theory of the disciplinary knowledge, especially with regard to its methods, validity and scope, and the approaches to establishing new knowledge as justified belief. I discuss later 'which' mathematics discipline should be the target of school mathematics. I use empirical data to explore how curriculum transformation processes can be constrained by curriculum communication, and by teacher capacity – their knowledge, skills and beliefs (Golding 2017). I discuss the mathematical epistemic quality evidenced in classrooms, and identify valued aspects that appear to be harder to achieve, at least with the curriculum texts being used. Finally, I suggest developments in the analysed curriculum system which might better support widely valued outcomes.

Empirical work drawn on

Between 2016 and 2022, (between 2016 and 2022) I have led a group of ten researchers exploring curriculum enactment: in particular, the ways teachers and students drew on, and were impacted by, related curriculum materials and assessments ('curriculum texts'). We asked 'How is the new curriculum being enacted in classrooms? What curriculum and assessment materials are teachers and students drawing on, and how? What is the impact on teachers and on students?'

The study sampled classes of students aged 5–18, the latter pursuing a calculus-rich pre-university course, and followed classes over at least two years. It drew on the voices of ~400 teachers, ~4,100 students, nearly 200 schools/

colleges, full-lesson observations of ~350 classes, and longitudinal attainment data for those classes. The samples used were representative in terms of several features known to affect teaching and learning in England, such as school size, location and socio-economic intake, assessed inspection quality and typical student attainment. All classes were using curriculum materials and/or high-stakes assessments provided by the market leading mathematics education publisher in England, which also funded the study. Each year, for each class, we observed complete lessons featuring newly emphasized aspects of the intended curriculum, and talked with focus groups of students. We either interviewed or surveyed class teachers and school/college mathematics leads termly and surveyed all students in secondary study classes. We also drew on curriculum, curriculum material and assessment-related documentary analysis, and progression data for students in study classes.

The research data collection and all non-routine analyses were undertaken by phase- and subject-specific specialist researchers. Further details of the approach taken, including the approach to analysis, and some key outcomes, are given in Golding (2021). Studies of this nature, large-scale and longitudinal but close to the intended objects of curriculum policy, namely teachers and students in classrooms, are unusual, and the institutional ethnographic approach adopted (Smith 2005) allowed distinctive affordances of cross-phase and longitudinal lenses, comparison of teacher with student response, and theorization of student as policy-player. In this chapter, I focus on epistemic characteristics of the data.

Epistemic quality

Hudson (2018) exemplifies high/low epistemic quality in primary school mathematics with brief descriptions of widely contrasting transformations of the intended curriculum; our data suggested a range of quality between those two extremes, varying over time within a single classroom, but also across parallel classrooms and schools, and to some extent across phases. Epistemic quality is judged in relation to the knowledge valued in mathematics learning – which depends on who is doing the valuing, and for what purposes, for whom. I shall show that there is significant overlap between what appears to be communicated as valued in the focus national curriculum, in the target curriculum materials, and in the related assessment criteria, and what is claimed as valued by much of the mathematics education community – and apparently, by policymakers. In school mathematics, I contend that the related range of documentation suggests that 'high epistemic quality' includes access to

- knowledge that is discovered *or* created by the person engaging with it, including
- utilitarian knowledge for everyday purposes
- socially and economically empowering knowledge that enables appreciation and harnessing of the world
- creative know-how that delights and affirms
- knowledge of the syntax and epistemology of school mathematics as a discipline closely related to (but different from) the parent discipline (Golding 2018)
- appreciation of the beauty, power and satisfaction of working with such knowledge as an intellectual endeavour
- over an appropriate range of substantive mathematical content *and processes*.

This is a broader and more detailed characterization of high epistemic quality in school mathematics than found in Hudson (2018). The related knowledge might be explicit, implicit or tacit (Tirosh 1994). By *mathematical epistemology* within school mathematics, I reiterate that I mean the ***mathematical scope, methods including communication, ways of knowing and of coming to know***, that are valued in the parent discipline, suitably transformed for access by the target group of students. I argue that one goal of school mathematics should be to induct school students into such valued knowledge and practices, at a level accessible to current student capacities but increasingly aligned with those of the parent discipline, as well as, when appropriate in application, of related disciplines – because it is only thus that young people have access to the potential of mathematics for their own, and society's, purposes. Morrow (2008: 72) reminds us that access involves not only exposure to opportunity to learn but active agency – commitment and effort – on the part of the student, towards that learning.

For students aiming to transition to higher education courses that are mathematically intense, there is a range of evidence that similar values are also held by those in higher education (Rach and Heinze 2016). For these students, there is a need for support into a university community with a different, and increased, expectation of organization for learning, quite different forms and purposes of curriculum, pedagogy and assessment, and a higher level of rigour, formalization and abstractedness of the espoused epistemology (Gueudet 2008). Although such disjunctures with typical school mathematics provision are widespread globally, they differ in profile across jurisdictions and also between different mathematics-intensive courses and universities within jurisdictions (Gueudet op. cit.). In Bernstein's (2000) terms, students need to acquire the recognition rules in order to recognize the speciality, and hence the potential, of the discourse, yet in terms of epistemic education in schools, I argue the appropriate needs vary.

For young people whose school mathematics is intended to provide an epistemic basis for less mathematically intense pathways, many of which place a significant demand on mathematical literacy, there is across much of the western world a mismatch between accounts of employer and further education needs and what is perceived to be widely achieved, in terms of confidence and competence in mathematical functioning appropriate to a range of occupational, personal and social needs (e.g. Eurypides 2011). For these different purposes, the epistemic quality of mathematics that is valued would appear to include most of the above characteristics but the target knowledge of syntax and of epistemology, as of substantive knowledge, and the nature of the epistemic *transformation* from parent purpose to classroom, might vary. A key question for curriculum players at all levels is, therefore, what mathematics epistemic provision is appropriate for which young people, at which stage (and who decides)?

I argue in Golding (2018), that school mathematics per se takes place in a constrained context, with novice mathematicians, so that the appropriate epistemic approach should be closely, and increasingly, related to that of the parent discipline – but is necessarily different from it. Within *applications* of school mathematics there are other considerations that serve perhaps to further constrain the alignment of an epistemic approach in order to accommodate the epistemic values of the field of application – a variety of 'rhetorical norms' (Kitcher 1991). I refer above to the challenges associated with then moving to the epistemic values associated with, for example, those of mathematically intensive courses at university, although Golding (2020) shows such challenges are not insuperable.

Yet, even within those, there are significant differences about what is valued epistemically across mathematics-intensive courses. I have vivid recollections of my first term at university, when those following mathematics, physics and engineering courses encountered partial differential equations for the first time. For mathematicians, the epistemic goal was to establish whether there existed at least one solution to a given *class* of PDEs and, if so, whether that was unique, and the nature and asymptotic behaviour of such solution(s); the physicists sought specific (preferably closed) form(s) of solution(s) to a *particular* equation when modelling a particular situation, from which they might explore the physical nature of those solutions, and the engineers wanted a possibly numerical *approximation* to a single solution to a particular equation that modelled a physical situation within a given range of the variables, but also and importantly, to know how stable that approximation was, and within what error bounds. These were all making intensive use of mathematics but for different purposes. One might argue that all experienced a high quality of epistemic access, but the nature of that differed across interest groups. There are therefore choices to be made in relation to the purposes of any intended curriculum and, within that, the characterization of high-quality epistemic access might vary.

Epistemic quality communicated in curriculum texts

The 'Purpose Statement' in the target 2014 curriculum states (DfE p 1),

> Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education … provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject.

This constitutes an aspiration for *expansive* (Engestrom 1987) mathematics learning, well-aligned with the characterization of high epistemic quality suggested above.

The document delineating the intended curriculum continues: 'The national curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately … Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas.

- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalizations, and developing an argument, justification or proof using mathematical language.
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions'.

Similarly, the focus of the pre-university A-Level specification, first taught from September 2017, was developed to provide a continued coherent pathway from the 11–16 curriculum. The key aspirations for deep conceptual fluency, accompanied by mathematical reasoning and problem-solving, represent processes included in previous A-Level specification documents, but the latter two especially enjoy renewed emphases in the current specification. These are well aligned with disciplinary values.

The national curriculum 'programme of study' follows its 'purpose' statement with a list of target content, arranged largely within two-year blocks, within which these process aspirations are intended to be worked out. On analysis, the intentions communicated are to provide foundations for content progression within mathematics, working also towards a grasp of the foundations for the disciplinary epistemology and distinctive mathematics cultural appreciation, together with mathematical literacy for personal, social and occupational purposes. Progression within knowledge of mathematical content is present but not detailed in that document, supported by a strong mathematical hierarchy. There is some indication of intended progression within key processes, and also of epistemological learning.

For example, in the primary curriculum we read 'explore and make conjectures about … ', 'develop their skills of rounding and estimating as a means of predicting and checking the order of magnitude of their answers', 'checking the reasonableness of their answers' …. (DfE 2013, Primary: 4, non-statutory guidance) and then, in secondary mathematics, 'move freely between different representations', 'make and test conjectures', 'look for proofs and counterexamples', 'explore what can and cannot be inferred … ', 'begin to model situations mathematically and express the results using a range of formal mathematical representations' (DfE 2013: 3, Key Stage 3 programme of study, 'working mathematically'). Then for older students (DfE 2013: 3, Key Stage 4 programme of study), the curriculum suggests

> use mathematical language and properties precisely; make and test conjectures about the generalisations that underlie patterns and relationships; look for proofs or counter-examples; begin to … reason deductively in geometry, number and algebra; … assess the validity of an argument and the accuracy of a given way of presenting information; … use mathematical language and properties precisely; … model situations mathematically and express the results using a range of formal mathematical representations, reflecting on … any modelling assumptions.

Within these excerpts, we see a clear progression in what is expected of students in terms of coming to know, and harness for their own use, valued ways of working mathematically, as they go through their compulsory schooling. There is, then, some clear intention of epistemic, including epistemological, *ascent* (Winch 2013).

There remains, though, a deficit in other aspects of what is valued by those who practise mathematics in a range of fields, which relates to, for example, overt appreciation of the choices mathematicians make in definitions, for example, of a^0, notions of elegance or comparative strengths of different approaches, the search for a fundamental cross-situation or generalized structure that lends power to representations and transformations, the overt valuing of exposure to mathematics as potentially fallible – and I discuss this further below.

The above 'programmes of study' for ages 5–16 (DfE 2013) were developed by a mathematics education 'expert group', moderated by ministers, and initially received a cautious welcome from both the mathematics and mathematics education communities, although aspirations for the years to the end of primary education (typically, age 11 in England) were widely thought to be overly aspirational. I have shown they include knowledge of procedures and processes, of flexible fluency, communication, problem-solving and reasoning, thus mathematical 'know-that' and procedural 'know-how' (Ryle 1946), yet feature little explicit syntactic know-how. The pre-university study focused on a curriculum developed by university mathematics experts in an otherwise parallel process so that some stage-specific 'transformation' of the appropriate epistemic substance was integral to the genesis of the studied curricula.

In the focus studies, similar epistemic analysis was undertaken of the sets of curriculum materials under scrutiny and assessment materials produced to support preparation for high-stakes examinations at ages 16 and 18. We found that the mathematics epistemic quality represented in any one set of the target materials was at least moderately well-aligned with curriculum intentions, representing key processes in discipline-coherent ways largely appropriate to the target students. Support for developing a robust and flexible fluency was well represented, and opportunities for students to build up progression in mathematical reasoning and problem-solving were usually explicitly identified. Explicit exposure to epistemological approaches was well-aligned with that expressed in the programme of study, although sometimes conservative in extent.

In parallel, teacher support materials typically offered lesson plans, identification of likely misconceptions and barriers to student success, tools for probing student thinking around given tasks, ways to build up confidence in approaching the more demanding of those and of supporting resilience in that approach …. Such materials are described as 'teacher educative' by Davis and Krajcik (2005) since they hold the potential to expand and enrich teachers' grasp of subject and subject pedagogic knowledge key to effective teaching. A comparative weakness, in general, though, was the low level of overt communication of mathematical *epistemology* in either teacher or student materials: for example, what was being accepted as validity of approach or argument, and valued ways of communicating that, and of mathematical exploration, remained largely implicit.

We also analysed high-stakes assessments taken by most students at ages 11 and 16 ('GCSE') and by some at 18 ('A-Level'). The curriculum was introduced without significant piloting, meaning exemplar such assessments usually post-dated curriculum materials and first teaching. However, our analysis showed that test materials for eleven-year-olds, nominally focused on 'arithmetic' and 'reasoning', usually featured an interpretation of 'reasoning' less ambitious than that adopted in the related curriculum materials, or arguably, intended in the curriculum, and tended to marginalize measurement, geometry and data handling. At GCSE, early assessment-related materials appeared coherent with both curriculum intentions and with the focus curriculum materials but, over time, targeted levels of reasoning and of problem-solving appeared to decrease. A-Level assessments followed a similar pattern, albeit remaining more aspirational than corresponding curriculum materials.

What we see, then, is that to discipline-informed readers, the focus intended curriculum, most curriculum resources, and early assessment materials communicated epistemic values moderately well aligned one with another, although with clear limitations in relation to alignment with disciplinary values, especially in relation to epistemological aspects of those.

Impact of curriculum texts on quality of epistemic access

The early teacher interviews *and* classroom observations across our studies showed that teachers' interpretation of key processes such as mathematical fluency, reasoning and problem-solving showed significant variation: for example, some talked about 'fluency' as meaning rapid rote reproduction, others as the flexible, efficient and reliable use of appropriate facts and procedures. Some identified problem-solving with 'task presented in words' and others as 'successful, sometimes sustained, application to complete an unfamiliar (to the target students), sometimes semi-structured or unstructured, task'. The nature of teachers' *interpretation* of the available curriculum documents, and hence their epistemic aims, therefore varied, and this naturally resulted in students having access to opportunities of variable epistemic quality.

For any one phase in the study, all study classes were using the same set of curriculum materials, designed to be highly teacher-educative (Davis and Krajcik 2005), so this variation is in some ways surprising. Within a single school, the mathematics teacher community sometimes developed a distinctive and apparently influential curriculum discourse which, in common with Smith (2005), we interpreted as curriculum text. However, we also found schools where different teachers interpreted curriculum intentions in very different ways.

Early curriculum enactment, then, appeared very variable in terms of key processes and classroom communication also of mathematics epistemology, even though all study teachers and students were using materials moderately well-aligned with curriculum intentions. As assessment-related materials became available, a clear intertextual hierarchy (Smith 2005) emerged. It was unusual to talk with a teacher who had read the official intended curriculum: almost all relied on curriculum materials or on the school's related 'schemes of work' for their initial interpretation. Within that, there was selection as teachers imposed their own prejudices in relation to the communicated intentions. It was common, for example, across age groups, to find teachers who 'saved' problem-solving or reasoning for their 'quick finishers' or who selected only the most accessible of related questions: '*These students don't do problem-solving*' (Year 11 teacher, Spring 2017). As assessment materials became available, those acquired immediate interpretational authority. In some early cases, this privileging of assessment materials focused teachers' attention on aspirations for problem-solving and reasoning for all students but, in others, as assessment aspirations appeared to dilute over time, teachers analysed that and their practice soon reflected it. Sadly, as teachers became more confident with emergent assessments, there emerged practices which offered students attaining weakly at either 'tier' of GCSE entry an impoverished and sometimes mathematically incoherent experience. Students sometimes talked about teaching directed at strategic approaches that would gain the relatively small number of marks needed for key 'gatekeeper' grades, especially through their examination year: '*We practise spotting where we can get one of the marks in a question, so we're quite good at that*' (Year 11 student, Spring 2018). In each phase of the study, though, there were also teachers who developed an approach that was epistemically of high quality, as characterized above, including in its epistemological communication, and who were able to maintain that.

In classrooms, then, coherence of intended curriculum, curriculum texts and, to some extent, assessments did not always support enactment aligned with those. Over time, incoherence of enacted curriculum with intentions often emerged, supported by high-stakes assessments not fully aligned with epistemic intentions.

How is the enactment of such potential constrained by the quality of the teacher's own knowledge?

In England, most teachers of learners aged 5–13 are not subject specialists, and many beyond that have limited specialist knowledge. For non-subject-specialists, particularly for primary teachers teaching across the curriculum, the typical English one-year initial teacher preparation is likely to be insufficient to inculcate a deeply epistemic, and especially epistemological, grasp of school mathematics from a teacherly perspective. Even as a subject specialist, in a performativity system examination performance pressures mean that as an end, attainment is frequently privileged over depth of subject grasp, arguably, though perhaps fallaciously, consistent with a moral purpose of optimizing the range of pathways subsequently open to young people.

Furthermore, teaching for the expansive learning envisaged in the intended curriculum requires wide and deep subject-specialist (including pedagogical and here, I argue, epistemological) knowledge (Eurydice 2011), sophisticated skills and positive affect, including beliefs (Golding 2017). Our study showed that teachers of all ages, whatever their mathematical background, usually lacked initial capacity to enact the focus curriculum as intended, though a minority had already developed, or were developing, with the support of either externally provided 'courses' or teacher-educative curriculum materials, curriculum-coherent ways of working. Such development required a significant investment of time and effort and, usually, the support of internal or external colleagues as teachers wrestled with unfamiliar mathematical approaches and the related pedagogy. If epistemic, or especially, epistemological, aspects of curriculum intentions were not explicitly exemplified in such development support, then certainly non-specialist teachers, but often specialists also, remained unaware of them. Examples evidenced included teacher uncertainty about the role of dynamic demonstration in proof, teacher confusion as to whether a square is a rectangle or a cylinder a prism, and teacher unwillingness to engage with alternative arguments presented by learners. Deep, sustained and often collaborative, teacher professional development coherent with curriculum intentions, supported by external expertise perhaps from high-quality teacher-educative resources, was generally needed before teachers could make significant progress towards high-quality epistemic access. Without that, we frequently observed, and teachers reported, lessons where the epistemic quality was apparently limited by teacher capacity: *'I wasn't quite sure I could cope with where they might take that discussion, so I shut it down' (Year 5 teacher, Spring 2018).*

Epistemic quality achieved in the enacted curriculum

High-quality epistemic access then depends on teacher capacity and commitment, curriculum interpretation and on the adopted textual hierarchy (together contributing to Gericke et al.'s (2018) 'transformation'). In our studies, interpretation was usually initially led by teacher-educative resources, sometimes heavily edited with the result of reducing epistemic aspiration. Interpretation later became dominated by high-stakes assessment texts, which for some came to threaten epistemic quality achieved. For others, though, these served to enhance aspirations, particularly for more highly attaining students, since teachers began to acknowledge that, without enactment rather better aligned with emerging assessments, students would under-achieve.

Epistemic and epistemological ascent

Bernstein (2000) theorizes an epistemic quality of *verticality* within a knowledge structure, that is, the hierarchical, cumulative development of knowledge within a discipline, and mathematics represents the archetypal such field. Winch (2013) discusses the relationships needed between (school) subject knowledge, inferential ability within that body of knowledge, and ability to validate and establish new (to student) truths if, as argued, one goal of school curriculum is to support a move from disciplinary novice towards expert. He goes on (128) to argue that 'a (consequent) key feature of good curriculum design is the ability to manage the different types of knowledge in a sequence that matches not just the needs of the (discipline-related) subject but also that of the student, so that the different kinds of disciplinary knowledge are introduced in such a way that the development of expertise is not compromised'. This reminds us that curriculum transformation aimed at achieving high epistemic quality in the classroom is student-, context- and time-dependent.

Within that argument, the clearest instantiations of such *epistemic ascent* might be expected in those school subjects stemming from vertically structured disciplines. The best mathematics curriculum resources, then, support an appropriate enacted epistemic ascent for all learners. Our data suggested that the focus curriculum resources were largely structured to support such ascent, at least in terms of mathematical content and processes. Teachers, though, selected from materials in ways which did not always reflect such structure, such that, for example, they might edit out some aspects of the intended progression. They usually reported this to be because, as in Winch above, they felt that at least some of their students did not at that time have the foundations on which to make such ascent. However, as above and on other occasions, emerging assessments served to restrict access to the epistemic progression reflected in curriculum materials. Overall, though, we observed over time some nascent and widespread classroom growth in the mathematical process progression made available to students.

Importantly, we searched in particular for evidence of access to *epistemological ascent*, without which learners cannot fully participate in, or appreciate, the powerful culture of the discipline, and it is to this that I now turn.

Quality of epistemology available in the classroom

Expanding on the initial definition of epistemology adopted, I suggest *high* epistemological quality features opportunity to learn about, for example,

- the nature(s) of mathematical knowledge, contested though that might be: for example, its relationship to sensed – and intrinsically

fallible – knowledge deriving from the world around us; its intrinsic interconnectedness and structure;
- the scope of mathematics study, and aspects of mathematical thinking that are of particular interest to different users;
- justification for new(-to-learner) knowledge, whether created or discovered, explicit, implicit or tacit (Ryle 1946);
- foundations for, and validity of, mathematical belief;
- the authority for new mathematical knowledge as residing within the subject itself, its substance and syntax – rather than with the teacher or the curriculum materials …

and, *for epistemological ascent,* that these should develop over time so that students' ways of mathematical working and being are increasingly aligned with those of mathematics practitioners in different fields.

In the classrooms in our studies, we found access to high-quality appropriate epistemology was unusual, and the typicality for England of our findings is reflected in, for example, Ofsted (2012). We did, though, observe positive examples in all phases from ages 5 to 18, with some primary classes showing a high value for clearly articulated reasoning about classification, about the enumeration of all possibilities, about the comparative strengths of different arguments that the sum of two odd integers must be even …. We observed 11-year-olds wrestling with Goldbach's conjecture that any even number greater than two is the sum of two prime numbers, exploring ways in which this might be proved or disproved – and a delight that the conjecture remains unproved: *'That's what maths is about, really, isn't it?' (Year 7 teacher, Spring 2018, in response to* a *student saying 'So no one knows? Really? That's so cool ….').* 15-year-olds with relatively poor prior attainment were seen using spreadsheets to explore the effect different football scoring systems would have had on last year's teams' league positions, and trying to develop a convincing scoring system that would have resulted in a different champion – but then analysing the effect that such alternative scoring systems would likely have had on teams' tactics. We saw a pre-university class persist with grappling with the nature and location of complex roots of a quartic equation, trying to understand their nature by comparing different representations of the related function and 'playing' with complex approximations and function transformations in order to make better links between those – and then evaluating the relative elegance and power of the different approaches they had explored.

These classroom experiences all brought with them a deep satisfaction for students and, not infrequently, an element of surprise or of a frustration supported and eventually worked through. We consider that in different ways each of the described experiences represented access to high-quality mathematical epistemology, but they were unusual. Each in its own way drew on highly skilled teaching, deeply knowledgeable not only about the mathematics and the epistemology but about the students and their learning of mathematics at that point in time.

Our analysis of the intended mathematics national curriculum is that such opportunities are supported at a high level and are clear to subject-specialist teacher educators but are not presented in detail, applied to individual delineations of target content, so that it is easy for the busy classroom teacher to lose sight of them. Teacher guidance in curriculum documents was generally epistemologically sound, but it often lacked depth and detail, and was sometimes limited in scope: many aspects of widely valued mathematical epistemology are not easy to codify, and so perhaps, to begin to make accessible especially to non-specialist teachers, let alone to students.

As above, though, given that for teachers and even more so for students, curriculum materials and, especially, assessment-related materials are privileged over programmes of study for interpretation of the intended curriculum, the key documents to analyse are assessments. Epistemological grasp at school-appropriate levels is typically not easy to assess in timed written papers and so, in a performativity system, is likely to be marginalized unless teachers have other, compelling, reasons to privilege it. Further, as suggested above, successful development requires in-depth sensitivity to students, their learning and the mathematical opportunity. Identification and harnessing of epistemological potential for particular classroom contexts is therefore highly dependent on teacher capacity, including their awareness and valuing of epistemological ascent in students.

What might be missing?

There is clearly, however, *opportunity* to develop epistemological knowledge within the intended curriculum. Further, I have shown there is *epistemological ascent* embedded within the guidelines given, reflected, for example, in expectations of increasing rigour in communication of mathematical argument as students progress through compulsory schooling.

However, the nature of epistemological knowledge is of syntactical know-how which, unlike procedural know-how, is difficult to codify and so to represent effectively in curriculum materials or written teacher support and difficult, especially, then to structure for teaching. It includes, for example, as illustrated above but poorly represented in any of the documents analysed, that:

- some mathematics is contested or ill-defined
- there are easy-to-understand conjectures which are not resolved
- definitions of, for example, a^0 are for mathematicians to agree on – but different decisions bring different implications, including different links with, and potential for coherent working with, existing definitions
- there is frequently mathematical potential in asking 'what if not … ?' ….

It is therefore unsurprising that a well-crafted and detailed approach to a mathematical epistemological ascent is not satisfactorily represented in the materials under study, but there is room for significant development. For example, although 'proof' is expected, the (insufficient) role of multiple examples or of dynamic demonstrations to constitute proof is not, nor their role in inductive thinking. Explicit notions of elegance, of infinity, of invariance or equivalence are missing, the sometimes-competing roles of sense and logic are implicit but not explicit. And the cultural and contextual embedding of mathematical meanings and practices is also hidden: are they global and shared, can they assimilate ethnomathematics, or do they have to change to accommodate that? Teaching for such considerations is highly demanding.

Conclusion

I have argued for particular characterizations of high epistemic quality in school mathematics and, within that, for an explicit characterization of high-quality mathematical epistemology. Our study shows that the quality of access to such epistemic, and especially such epistemological, engagement in the school classroom, and its ascent, varies enormously. For example, observed promoted mathematical authority varied from 'because the textbook says so' to deeply challenging student experiences developing and fully justifying new-to-them knowledge: we observed the range in each age phase. We found that even 'specialist' teachers often marginalized epistemological considerations in the classroom: teaching for high-quality epistemological learning, and its progression, appears to be *highly demanding* of teacher capacity. It is therefore unlikely to happen at scale unless there is sustained, explicit and detailed support for related teacher development, including in teacher-educative curriculum materials, but also in other curriculum texts – and a valuing of that in high-stakes assessments.

There are clear implications for curriculum system innovation. If young people are to learn that mathematics is a meaningful and empowering creative discipline that they can harness for multiple purposes, and communicate to others, requiring shared vocabulary and syntax, then I have argued we need, as a minimum:

- an intended curriculum that is developed by education and mathematics experts to reflect epistemic ascent(s) towards the quality of mathematics practice valued by the range of end-users
- and which overtly values the range of disciplinary epistemology, structured to support high-quality progression within that
- innovative teacher initial and continuing education that prepares teachers for the (demanding) transformation of those qualities for effective classroom use so that they 'know the mathematics' in epistemically and pedagogically powerful ways appropriate to their learners and contexts
- curriculum materials and learning assessments fully and explicitly coherent with those aspirations, developed in detail and depth.

Making progress towards such goals is challenging, but the work reported in this chapter suggests many aspects are at least moderately susceptible to development, and identification of goals is first step to their attainment. It is clear that sustained subject-specific teacher education, both pre- and in-service, is central, and that goal-coherent, teacher-educative curriculum materials and assessments can contribute to that. However, work is needed to develop codification of valued outcomes in detail and in depth, together with identification of those aspects which are necessarily implicit or tacit. In a world where many issues of importance rely on cross-disciplinary approaches, as identified by the OECD's 'Compass 2030' initiative (http://www.oecd.org/education/2030-project/), it is also important to identify which aspects of epistemic development benefit from subject-focused teaching and learning, and which can at least equally well be developed in cross-disciplinary contexts, under what conditions. We have ambitious aspirations for the learning of our young people in the twenty-first century: their flourishing merits investment in innovative teacher development to support those aspirations.

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