An exploratory study on mathematics teacher educators' beliefs and understandings about computational thinking

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This paper reports an investigation of mathematics teacher educators' views and perceptions on computational thinking (CT) and its impact on mathematical learning. We conducted semi-structured interviews with experienced mathematics teacher educators, all of whom have extensive experience with the use of digital technologies for mathematical teaching and learning and report on data from two of them. Our aim is to offer insights into how CT is perceived and understood by them, to support them in self-assessing their possession of CT practices, and how to support mathematics teachers and students in gaining CT. We offer ideas regarding the promotion of CT and its integration in mathematics teaching and learning.

Keywords: Computational thinking, mathematics teacher educators, digital technologies

Introduction

Over the past decade, there has been upsurge of interest in the teaching and learning of computational thinking (CT). The proponents of CT conceive of CT as a critical skill for all that will enable humankind to harness the power of computers for the common good (e.g., Wing, 2006). As a result, many countries are in the process of introducing CT into their school curricula, either as a new dedicated subject, a cross-curricular theme, or integrated within an existing subject, such as mathematics (see, e.g., Bocconi et al., 2018; see also, Royal Society, 2018). The relationship between CT and mathematics has been of particular interest. Indeed, some see CT as offering the potential to transform school mathematics (eg, Perez, 2018), but realising this potential will be a challenge, for students, mathematics teachers and mathematics teacher educators (MTEs).

Teacher education will be critical in enabling mathematics teachers to realise the potential of CT to transform mathematics. Yet, to date, educational literature on CT, or computational competency or the "new digital age competency" as sometimes is referred to, (e.g., Grover & Pea, 2013, Li et al., 2020) has mainly focused on students' CT. In this paper, we address this gap by investigating MTEs' CT and their computational practices. We present initial findings from an exploratory study, investigating the views of two experienced MTEs, who both have extensive experience with the use of digital technologies for mathematical teaching and learning, including specifically with teachers. We discuss their views on CT from a practitioner and a research perspective, debating about assessing the possession of CT and how to support mathematics teachers and students in gaining CT. We conclude by offering ideas about the promotion of CT and its integration in mathematics teaching and learning.

Computational Thinking and Mathematics Education

CT was first mentioned by Papert (1980) in his seminal *Mindstorms* book, it gained more momentum when Wing re-introduced it in her 2006 pivotal paper making the case for CT as a critical skill for all. Wing (2010) defined CT as "*the thought processes involved in formulating problems and their solutions so that the solutions are represented in a form that can be effectively carried out by an information-processing agent*". (Cuny, Snyder & Wing, 2010). Many researchers (e.g., Brennan & Resnick, 2012; Grover & Pea, 2013) have attempted to clarify this definition. This work emphasises that CT is less about the use of technology and computers and more about the concepts, practices and processes involved (e.g., Lodi, 2020). In Shute et al.'s (2017) terms, CT is "a way of thinking and acting, with or without the assistance of computers" (p.143).

The consensus of research (e.g., Shute et al., 2017) is that, whilst there are practices in common, CT is a distinct and separate discipline to mathematics. However, CT involves practices that are also required in mathematics, such as "decomposition, abstraction, algorithm design, debugging, iteration, and generalization" (Li et al., 2020, p.156). For the purposes of this paper, we will draw on Perez's (2018) summary of the literature on CT practices and dispositions as outlined in Table 1.

Table 1: Perez's (2018) categorization of CT practices and dispositions adapted (see Figure 2, p.428)

Computational Thinking Practices	
Problem Solving	*Abstraction & Generalisation*
Assessing and pursuing different approaches and solutions to a problem	Collecting, organizing, manipulating, and representing data
Generalizing and transferring problem-solving processes to other situations	Abstracting the essential elements of a situation or task
Using incremental and iterative approached, decomposing tasks into smaller pieces	Thinking in levels andunderstanding relationships within a system
Reusing, remixing, and innovating	Choosing effective tools and models for working with data
Efficient and effective combinations of resources, testing and debugging	Developing algorithms and automations
Formulating problems so that they can be analyzed using programs and other tools	Designing and using computational models and simulations
Computational Thinking Dispositions	
• Confidence in dealing with complexity • Persistence in working with difficult problems • Tolerance for ambiguity • The ability to deal with open-ended problems • The ability to communicate and work with others to achieve a common goal or solution	

Teachers' Computational Thinking

Some research has examined the teaching of CT both in general (see Grover & Pea's, 2013, review) and specifically in mathematics and other STEM subjects (see, e.g., Lee et al., 2020). A particularly fruitful avenue of research has investigated the use of Scratch programming in mathematics (e.g., Sung et al., 2018) with some potentially promising results (e.g., Boylan et al., 2018). However, the efficacy of such pedagogic initiatives is dependent on mathematics teachers' knowledge, beliefs and attitudes about CT. Some small-scale research has begun to examine the issues and challenges in this area. Sands et al.'s (2018) survey of 74 US teachers' views of CT suggests that one challenge may be teachers' limited understanding of CT. For example, they found that most respondents viewed CT as synonymous with doing mathematics and using computer or technology. Angeli and Jaipal-Jamani (2018) examined the effects of an intervention on 21 preservice science teachers' CT in a small-scale study. They found that the use of scaffolded programmed scripts resulted in increased CT skills amongst the preservice teachers. However, they found that these increased skills were limited to lower levels of CT and, hence, their study highlights the difficulty of developing higher order CT skills such as generalisation and abstraction. Israel and Lash (2020) carried out a study in one US elementary school examining the integration of CT into mathematics teaching, mostly using the Scratch environment with some lessons using Code.org materials. They found that, despite a very strong commitment from the school and its teachers to CT and integrating CT and mathematics, relatively few of the lessons showed evidence of an integrated approach to teaching CT and mathematics. Chevalier et al.'s (2020) study suggests ways in which these challenges can be addressed. They found out that it is important "not only having a good understanding of CT (e.g., not focusing exclusively on acquiring programming skills), but also thinking and planning carefully in developing and implementing educational activities to develop students' CT" (as cited in Li et al., 2020, p.154).

Teacher Education and MTEs' Computational Thinking

As Lee et al. (2020) observe, teacher education and professional development are critical to the development of effective CT teaching (Weintrop et al., 2016). Yet, we are unaware of any research examining MTEs' knowledge, beliefs and attitudes about CT. To help us reflect on MTEs' CT practices, we decided to have initial discussions with MTE, who have extensive expertise in teacher education and research in the use of digital technologies for mathematical teaching and learning. Such MTEs' beliefs can support our investigation on what CT is, what CT practices are, what the relationship between CT and mathematical thinking is, how CT practices can be promoted among mathematics teachers, why CT practices are useful (or not) and what they offer to mathematics education.

The Exploratory Study

Our aim was to answer the following Research Questions: What are MTEs' CT practices? What are their views on mathematics teachers' CT processes and the link between mathematical thinking and CT? In order to gain insights to these questions, we carried out an exploratory study and interviewed three MTEs with expertise in digital technologies and mathematics. In this paper, due to constraints of space, we present vignettes of only 2 of those MTEs, whom we will refer to as Carole and Naomi. Both Carole and Naomi have school teaching experience, but also lengthy experience as MTEs (Carole over 10 years and Naomi over 7 years). They both have a doctorate in mathematics education and their research interests lie in the use of digital technologies for mathematics teaching and learning, but also in mathematics teacher knowledge.

The interviews consisted of 2 parts. In the 1st part, the interviewees had to solve a task, using the Think-Aloud protocol (Güss, 2018). We asked interviewees to reflect on (a) the programming aspects, (b) mathematical definitions, (c) the structure of the mathematical and the tool's language, and (d) the algorithms. Given that CT is a relatively new area of interest, we wanted a task that would enable the interviewees to articulate various aspects of CT practices. Hence, we chose a task that they were familiar with and involved a digital tool of their choice. This has an advantage of generating a range of ideas in a relatively short space of time, but has some limitations in terms of comparing the MTEs' beliefs. In the 2nd part, we asked them for their own definition of CT and how this relates to the definition by Cuny, Snyder & Wing (2010), which was mentioned earlier and states that CT is "*the thought processes involved in formulating problems and their solutions so that the solutions are represented in a form that can be effectively carried out by an information-processing agent*". Then, we asked them to reflect upon their own approach to solving their task in relation to the CT practices as presented by Perez (2018) in Table 1, identifying which CT practices they used. We asked them to (a) Reflect on mathematics teachers' CT: You have used this task with teachers, what aspects would you highlight to teachers in terms of what is different to pen-and-paper mathematics? How would you design a task to make teachers aware of key elements and features relevant to CT?; (b) Reflect on CT and mathematical thinking, knowledge, teaching, learning, pedagogy. How is CT linked to mathematics? What pedagogical strategies regarding CT would you use with teachers?

Carole's Vignette

A mathematical task

We asked Carole to tackle the 'Rich Aunt' task, a problem that she has used frequently in teacher education at M-level. In this problem, students are asked to decide between one of three gift schemes:

a) £100 now, £90 next year, £80 the year after, and so on;
b) £10 now, £20 next year, £30 the year after, and so on;
c) £1 now, £2 next year, £4 the year after that, and so on.

To solve the task, Carole used Excel to create a table using a formula and dragging across cells to create a table and a graph comparing the annual value of each scheme. Carole uses this task to enable teachers to "experience the power of Excel", because, without Excel, it would be more "time-consuming". However, one doesn't always need to think mathematically: "That's the problem with excel you are not even aware that you actually typed in a formula. … Which is slightly different to having an awareness of a formula behind it, the mathematical formula behind that."

Carole's definition of Computational Thinking

Initially, Carole conceived of CT as synonymous with computer programming such as Scratch and Logo and didn't consider the Rich Aunt task involved CT, saying that she had engaged Excel as a tool using "a simple formula to generate numbers by copying and dragging cells" rather than "thinking the problem through" as would be required for programming. When presented with the list of CT Practices [Table 1], she identified several practices that she had used in the task: pursuing different approaches, using incremental and iterative approaches, innovating, debugging, formulating problems for analysis using tools, organising and representing data and using computational models. She did not consider that she had used algorithms or automation, despite her use of formulae in Excel. She defined CT as "adapting your mathematical thinking to the tool at hand which has got computational power … you think of the mathematics first and then how do I go about using this tool …. You almost have a plan of how you're going to investigate the maths problem and really the tool is just a tool that helps you execute that plan."

Carole said that largely CT is part of mathematics, although she felt that algorithms and automation are "*not necessarily*" to be part of mathematics. She considered CT to be more about working with what is already known in contrast to mathematics which enables one to work with the "*unknown*".

Naomi's Vignette

A mathematical task

We asked Naomi to tackle the task presented in Figure 1 further below, which she has given to mathematics teachers for research purposes. In this task, teachers were asked to explore the 3 diagrams presented in GeoGebra and discuss how to use them when teaching one of the Circle Theorems ("the angle at the centre of the circle is twice the angle at the circumference") to a mathematics class. When discussing this task, Naomi commented on the teachers' keenness to avoid exploring complex diagrams, demonstrating lack of confidence in dealing with complexity. Naomi reflected upon the value of looking at a simpler case to support mathematical thinking and work towards proving a conjecture. Choosing such a special case to make a conjecture simpler to think about, could be considered as decomposing the given task into smaller pieces, which is a CT practice as presented in Table 1. Naomi also reflected upon the importance of task design considering the constraints of the tool in use. GeoGebra may have been chosen due to its dynamicity and the benefit of exploring many different cases, but Naomi argued that you may find some level of dynamicity when using pen and paper. In fact there is some rigidity and inflexibility in GeoGebra as it follows certain mathematical rules, referring to rounding errors as well as the fact that the theorem 'broke' for certain extreme cases. She referred to a tension between Geogebra as a mathematical tool and a tool for mathematics pedagogy and suggested that this tension may prompt productive mathematical thinking.

Naomi's definition of Computational Thinking

Naomi defined CT as "not just about programming, it is about a wider understanding of using computers, but even more broader than that digital technology to solve problems […] kind of using the software tools for an investigative process" to solve a mathematical problem in our case. She later on added that CT is "part of problem-solving or in other words, it's about incorporating another tool into your problem-solving kit". After being presented with the definition as shared in the literature, Naomi reflected: "You need to appreciate that you put things in order… so that could be perhaps tending to be an algorithmic process (putting things in order) so that GeoGebra then becomes a useful tool, as you construct things". It is also worth mentioning Naomi's view on what the "information processing agent" is and that it might be restrictive. This phrase may seem to imply the use of a digital tool, but Naomi offered her own personal definition mentioning any tool that can support the problem-solving processes. "That decision making about what's the right tool to use […] is that fuzzy boundary of computational thinking [and] other kind[s] of tool-based thinking".

When presented with the list of CT Practices [Table 1], Naomi identified several practices that she had used in the task: efficient and effective combinations of resources, testing and debugging, formulating problems so that they can be analyzed using programs and other tools, abstracting the essential elements of a situation of task, thinking in levels and understanding relationships within a system, choosing effective tools and models for working with data. Naomi considered some of the practices to be poorly defined. For example, she argued that "thinking in levels and understanding relationships within a system" is difficult to interpret as she is not sure what is meant by "levels", although she speculated that it might be related to "different levels of abstraction". She commented that “Reusing, remixing and innovating” didn’t make sense to her. She argued that “testing and debugging” should have a more prominent role in CT. To justify this, she gave as an example the GeoGebra diagram and when teachers dragged the points to explore the diagram, ‘testing’ special cases, such as 360 degrees or 0 degrees, and addressing the pedagogical challenges created by rounding errors in GeoGebra.

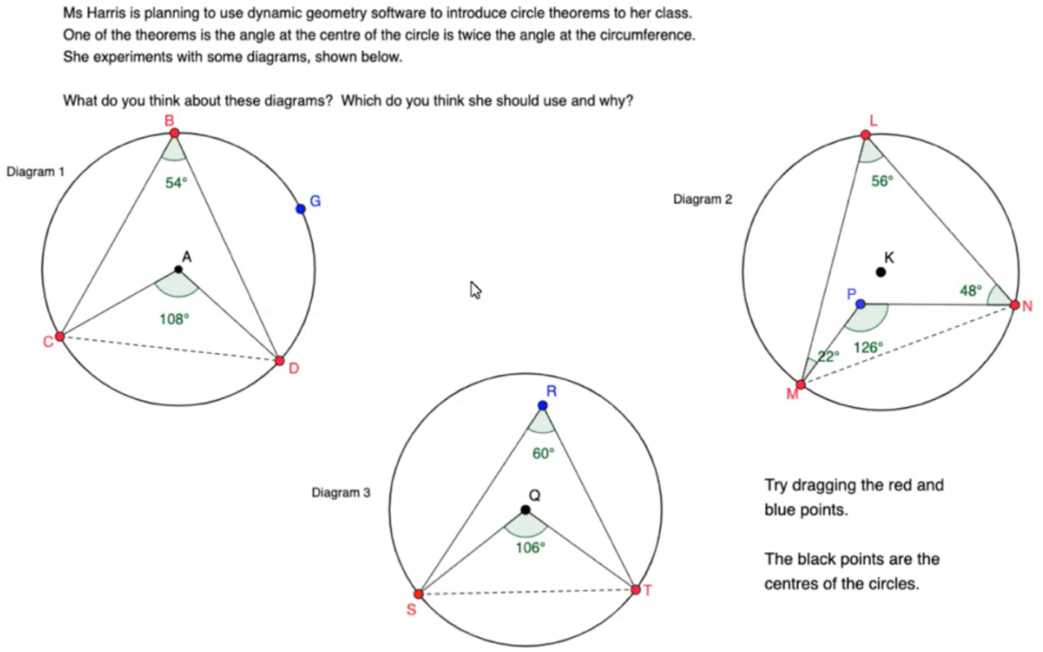

Figure 1: The GeoGebra task Naomi used for research purposes and discussed during her interview

Conclusion

Our study indicated that both Carole and Naomi were skilled in CT and mathematics to solve and discuss a familiar task. Unlike the teachers in Sands et al.’s (2018) neither viewed CT as synonymous with doing mathematics nor simply using digital tools to do mathematics, although both appeared to view CT as closely tied to computers and other digital tools. Nevertheless, their understandings and beliefs about CT were somewhat different. Naomi considered CT to be distinct from mathematics and involving an understanding of how to use digital tools to investigate problems in mathematics and beyond. As such her beliefs were broadly in accordance with the consensus of the academic literature, albeit she appeared to believe CT to be closely related to digital tools. In contrast, Carol appeared to view CT largely as part of mathematics and synonymous with programming. Indeed, she did not consider that using a spreadsheet such as Excel involved any CT. We note here that Carol’s views may have been influenced by the particular task she discussed in that the automation and iteration involved very well-understood mathematics: addition, subtraction and multiplication. Despite Perez’s (2018) claim that the practices identified in his review represent a concensus in mathematics education, the two MTEs found some of the practices identified to be unclear and poorly defined. This is despite both MTEs being highly skilled in the use of digital technologies in mathematics. Whilst Naomi appeared to believe that constructing algorithms provided a focus for thinking mathematically, Carole appeared to believe that using algorithms meant that mathematical thinking was no longer necessary. This highlights a tension between a common purpose in mathematics education of using digital tools to outsource some of the mundane and well-understood mathematical work, whereas in CT, it is crucial to consider, construct and adapt some of the less mundane mathematical processes such as analysing, generalising and abstracting (Pérez, 2018). Crucially, in CT, such processes are designed to be carried out by the information processing agent, which may be, but is not necessarily, a digital tool.

Our study suggests that MTEs would benefit from opportunities to explicitly engage with the nature of CT and its relationship to mathematics and to the application (and non-application) of technology. In doing so, a critical research task is to articulate the nature of Computational Thinking Pedagogical Content Knowledge (CTPCK) as a new term and its relationship to existing work, such as PCK in mathematics and the TPACK (Technological Pedagogical Content Knowledge, Koehler et al., 2013), which we hope to discuss in our future papers. Our future work entails the investigation of mathematics teacher educators' perspectives on what CT is and assess their CT skills, which elements in particular mathematics teachers possess and which ones they should acquire to enrich their mathematics teaching practice.

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