POWER ALLOCATION FOR JOINT MMWAVE RADAR SENSING AND COMMUNICATION IN VEHICULAR NETWORK

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ABSTRACT

This paper has proposed a power allocation (PA) design for the vehicle-to-infrastructure (V2I) network, where a road side unit (RSU) provides both localization and communication services to multiple vehicles, via the dual-functional radar-communication (DFRC) transmission. To be specific, we formulate an optimization problem to minimize the summation of the Cramér-Rao bound (CRB) for multiple vehicles, subject to downlink sum-rate constraint. We prove that the problem can be solved in closed-form under certain conditions. Numerical results demonstrate that our approach achieves significantly lower estimation errors while improving the communication rate, as compared to the conventional method.

***Index Terms*—** Vehicular network, mmWave, joint radar-communication

1. INTRODUCTION

The future autonomous vehicular network is expected to have both high-accuracy localization and high-throughput communication capabilities. Conventionally, the vehicle localization and networking are supported by the Global Navigation Satellite System (GNSS) and default protocols, e.g., Dedicated Short Range Communications (DSRC) and LTE-A [1], which are unable to achieve the performance standards demanded by the autonomous vehicles. To address this need, the 5G technology, powered by both millimeter wave (mmWave) and massive MIMO (mMIMO), is envisioned as a game changer, as these techniques provide not only higher wireless capacity but also better resolution for angle and range estimation [2].

In order to improve the spectral-, energy-, and hardware-efficiencies, sensing and communication are often required to be jointly designed, which motivates the research on DFRC transmission [3]. While DFRC has been widely studied over the past few years, its application to the vehicular network remains to be explored. The state-of-the-art approaches on this topic are mostly built upon the IEEE 802.11ad protocol [4, 5, 6], a WLAN standard operating at the mmWave band, which are easy to be implemented in practise. Nevertheless, these works are motivated from the general purposes of reducing the costs and increasing the efficiency for vehicular platforms, which do not reveal the unique performance gain brought by the co-design of sensing and communication. To this end, the work of [7] has proposed a predictive beamforming design for V2I links with the aid of radar sensing, which relies on a state transition model of the vehicles. It has been shown that by using DFRC, the vehicle localization and communication performance can both be improved compared to communication-only methods. As a step further, the vehicular DFRC beamforming scheme proposed in [8] removes the necessity of an explicit state model, which can be applied to more complicated traffic environments.

Following the spirit of [7, 8], in this paper we propose a novel PA design for DFRC transmission in the V2I downlink, where an RSU serves multiple vehicles on the road. The proposed PA aims at reducing the localization errors while guaranteeing the downlink sum-rate, which can be formulated into a convex optimization problem. We prove that under some certain conditions, the formulated problem can be solved in closed-form. Numerical results demonstrate that the proposed PA outperforms the conventional technique in terms of both sensing and communication performance.

2. SYSTEM MODEL

Let us consider a mmWave mMIMO RSU with N_t transmit and N_r receive antennas, forming uniform linear arrays (ULAs) with half-wavelength antenna spacing. The RSU is serving K single-antenna moving vehicles, which locates and communicates with the vehicles via a line-of-sight (LoS) channel. At each transmission block n, the RSU transmits a DFRC signal $\mathbf{s}_n(t) = [s_{1,n}(t), \ldots, s_{K,n}(t)]^T \in \mathbb{C}^{K\times 1}$ at time instant t, with the kth stream $s_{k,n}(t)$ containing the intended information for the kth vehicle.

2.1. Radar Sensing Model

By transmitting DFRC signals towards the vehicles, the received echo signal at the nth block can be expressed as

$$\mathbf{y}_n^R(t) = \sum_{k=1}^{K} \kappa_R \beta_{k,n} \sqrt{p_{k,n}} \mathbf{b}(\theta_{k,n}) \mathbf{a}^H(\theta_{k,n}) \mathbf{F}_n \mathbf{s}_n(t-\tau_{k,n}) + \mathbf{z}_n^R(t), \tag{1}$$

where $\kappa_R = \sqrt{N_t N_r}$ is the array gain, $\beta_{k,n}$ is the reflection coefficient that contains the round-trip pathloss, the radar cross section (RCS) and the Doppler phase shift, $p_{k,n}$ is the transmit power, $\theta_{k,n}$ is the vehicle's azimuth angle, $\mathbf{a}(\theta) \in \mathbb{C}^{N_t \times 1}$ and $\mathbf{b}(\theta) \in \mathbb{C}^{N_r \times 1}$ are transmit and receive steering vectors, which are normalized to have unit norms, $\tau_{k,n} = \frac{2d_{k,n}}{c}$ denotes the round trip delay, with $d_{k,n}$ and c being the distance of the vehicle and the speed of the light, $\mathbf{F}_n = [\mathbf{f}_{1,n}, \mathbf{f}_{2,n}, \ldots, \mathbf{f}_{K,n}] \in \mathbb{C}^{N_t \times K}$ is the transmit beamforming matrix, and finally $\mathbf{z}_n^R(t)$ is the additive white Gaussian noise (AWGN) with variance of σ_R^2. For each vehicle, the corresponding transmit beamformer is given as

$$\mathbf{f}_{k,n} = \mathbf{a}\left(\hat{\theta}_{k,n|n-1}\right), \tag{2}$$

where $\hat{\theta}_{k,n|n-1}$ is the prediction for the real angle θ_n, based on the previous estimates of the vehicle's parameters. By doing so, the RSU formulates K narrow beams pointing to the vehicles for the purpose of communication, as well as of tracking their movements. It is noteworthy that due to the asymptotic orthogonality of the mMIMO array, the inter-user/inter-beam interference vanishes [9]. As a result, the reflected echo from each target vehicle can be processed individually. In other words, the echo signal of the kth vehicle can be expressed as

$$\mathbf{y}_{k,n}^R(t) = \kappa_R \beta_{k,n} \sqrt{p_{k,n}} \mathbf{b}(\theta_{k,n}) \delta_{k,n} s_{k,n}(t - \tau_{k,n}) + \mathbf{z}_{k,n}^R(t), \tag{3}$$

where $\delta_{k,n} = \mathbf{a}^H(\theta_{k,n}) \mathbf{f}_{k,n}$ is defined as the beamforming gain, whose modulus equals to 1 if the angle is perfectly predicted, and is less than 1 otherwise.

2.2. Communication Model

Thanks again to the asymptotic orthogonality of the mMIMO channel, the received signal at each user is modeled as

$$y_{k,n}^C(t) = \alpha_{k,n} \kappa_C \sqrt{p_{k,n}} \mathbf{a}^H(\theta_{k,n}) \mathbf{f}_{k,n} s_{k,n}(t) + z_{k,n}^C(t), \tag{4}$$

where $\kappa_C = \sqrt{N_t}$ is the array gain, $z_{k,n}^C(t)$ is the AWGN with a variance of σ_C^2, and $\alpha_{k,n}$ is the path-loss, giving by

$$\alpha_{k,n} = \alpha_{\text{ref}} d_{k,n}^{-1} e^{j \frac{2\pi f_c}{c} d_{k,n}}, \tag{5}$$

where f_c denotes the carrier frequency, α_{ref} is a known reference path-loss measured at the distance of 1m. It can be seen that $\alpha_{k,n}$ can be readily estimated by estimating $d_{k,n}$. Furthermore, the receive SNR at the kth vehicle is

$$\gamma_{k,n} = p_{k,n} \kappa_C^2 |\alpha_{k,n}|^2 |\delta_{k,n}|^2 \Big/ \sigma_C^2 \triangleq p_{k,n} h_{k,n}, \tag{6}$$

where $\delta_{k,n}$ is the beamforming gain defined above. The achievable sum-rate of the vehicles can be thus obtained as

$$R_n = \sum_{k=1}^{K} \log(1 + \gamma_{k,n}) = \sum_{k=1}^{K} \log(1 + p_{k,n} h_{k,n}). \tag{7}$$

3. PARAMETER ESTIMATION AND PREDICTION

In this section, we briefly introduce the procedure for estimating and predicting the vehicle's parameters. In particular, we are interested in estimating $\mathbf{w}_{k,n} \triangleq [\theta_{k,n}, d_{k,n}, \beta_{k,n}]^T$. For convenience, we denote the estimated and the predicted values of $\mathbf{w}_{k,n}$ as $\hat{\mathbf{w}}_{k,n}$ and $\hat{\mathbf{w}}_{k,n|n-1}$, respectively.

After receiving the echo signal (3) for the kth vehicle at the epoch n, the RSU will firstly matched-filter the echo with a time-shifted counterpart of $s_{k,n}(t)$, in order to obtain the estimated delay $\hat{\tau}_{k,n}$. Together with the output of the matched filter, the radar measurements for the vehicle k can be defined as

$$\begin{aligned} \mathbf{r}_{k,n} &= \mathbf{g}(\mathbf{w}_{k,n}) + \mathbf{z}_{k,n} \\ &\triangleq \begin{bmatrix} \kappa_R \beta_{k,n} \sqrt{p_{k,n}} \mathbf{b}(\theta_{k,n}) \delta_{k,n} \\ 2d_n/c \end{bmatrix} + \begin{bmatrix} \mathbf{z}_\theta \\ z_\tau \end{bmatrix}, \end{aligned} \tag{8}$$

where $\mathbf{z}_\theta$ is the output noise of the matched filter with a variance of σ_θ^2, and z_τ denotes the estimation noise of the time delay, with a variance of σ_τ^2. Therefore, we have $\mathbf{z}_{k,n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q}_{k,n})$, with the covariance matrix $\mathbf{Q}_{k,n}$ being defined as $\mathbf{Q}_{k,n} = \text{diag}\left(\sigma_\theta^2 \mathbf{1}_{N_r}^T, \sigma_\tau^2\right)$. We note here that the above variances are inversely proportional to the output SNR of the matched filter, which can be therefore modeled as [10]

$$\sigma_\tau^2 = a_1^2 \sigma_R^2 \Big/ \left(G p_{k,n} \kappa_R^2 |\beta_{k,n}|^2 \delta_{k,n}^2\right), \sigma_\theta^2 = a_2^2 \sigma_R^2 / (G p_{k,n}), \tag{9}$$

where G is the matched-filtering gain that equals to the energy of $s_{k,n}(t)$. Finally, a_1 and a_2 are two scaling factors that are determined by the specific signal parameters, such as the spectrum shape and the bandwidth, etc [10]. Note that σ_θ^2 contains no $\beta_{k,n}$, $p_{k,n}$ and $\delta_{k,n}$, since they are already included in the output signal of the matched filter.

Our first goal is to extract the vehicular parameters from the measurement $\mathbf{r}_{k,n}$, which can be done by relying on a well-designed estimator $\mathcal{F}(\cdot)$. This can be expressed in the form as

$$\hat{\mathbf{w}}_{k,n} = \mathcal{F}(\mathbf{r}_{k,n}) = \left[\hat{\theta}_{k,n}, \hat{d}_{k,n}, \hat{\beta}_{k,n}\right]^T. \tag{10}$$

After obtaining $\hat{\mathbf{w}}_{k,n}$, the second goal is to predict the next state of the vehicle via a well-designed predictor $\mathcal{P}(\cdot)$, which is

$$\hat{\mathbf{w}}_{k,n+1|n} = \mathcal{P}(\hat{\mathbf{w}}_{k,n}) = \left[\hat{\theta}_{k,n+1|n}, \hat{d}_{k,n+1|n}, \hat{\beta}_{k,n+1|n}\right]^T. \tag{11}$$

At the $(n+1)$th epoch, the RSU formulates the beamformer for each vehicle based on the predicted value $\hat{\theta}_{k,n+1|n}, \forall k$. By iteratively performing the above estimation and prediction process, the RSU is able to accurately steer multiple beams towards the moving vehicles, and thus to realize high-throughput communication and high-accuracy localization. Due to the strict space limit, we will not elaborate on the design of the estimator and the predictor. We refer readers to [8] for more details on this topic.

4. POWER ALLOCATION

In this section, we discuss the PA problem among multiple vehicles, which aims to optimize the radar sensing performance while guaranteeing the communication quality-of-service (QoS). In particular, we are interested in minimizing the summation of the Carmér-Rao Bound (CRB) of K vehicles, subject to the communication sum-rate constraints. As the transmit power is re-allocated for every transmission epoch, we will focus on a single epoch with multiple vehicles, and omit the index n in the following discussion for notational convenience, i.e., $\mathbf{w}_k = [\theta_k, d_k, \beta_k]^T$.

Following the similar notations as in Sec. 3, the measurement model for the kth vehicle at the RSU can be formulated as

$$\mathbf{r}_k = \mathbf{g}(\mathbf{w}_k) + \mathbf{z}_k, \tag{12}$$

where the covariance matrix of the measurement noise $\mathbf{z}_k$ is the same as in Sec. 3, and is now denoted as $\mathbf{Q}_k$. The Fisher Information Matrix (FIM) can be computed as

$$\mathbf{J}_k = \mathbf{G}_k^H \mathbf{Q}_k^{-1} \mathbf{G}_k, \tag{13}$$

where $\mathbf{G}_k \triangleq \frac{\partial \mathbf{g}}{\partial \mathbf{w}_k}$ is the Jacobian matrix. It is known that

$$\mathbb{E}\left((\hat{\mathbf{w}}_k - \mathbf{w})(\hat{\mathbf{w}}_k - \mathbf{w})^H \right) \succeq \mathbf{J}_k^{-1} \triangleq \mathbf{C}_k, \tag{14}$$

where $\mathbf{C}_k$ is defined as the CRB matrix. Therefore we have

$$\begin{aligned} &\mathbb{E}\left(\left(\hat{\beta}_k - \beta_k \right)^2 \right) + \mathbb{E}\left(\left(\hat{\theta}_k - \theta_k \right)^2 \right) \\ &+\mathbb{E}\left(\left(\hat{d}_k - d_k \right)^2 \right) \geq \operatorname{tr}(\mathbf{C}_k). \end{aligned} \tag{15}$$

Our aim is to minimize the summation of the CRB of all the target vehicles, subject to communication rate constraints. Let us first denote $\tilde{\mathbf{Q}}_k = p_k \mathbf{Q}_k$, which is a normalized covariance matrix. Then we have

$$\operatorname{tr}(\mathbf{C}_k) = \operatorname{tr}\left(\left(p_k {\mathbf{G}_k}^H \tilde{\mathbf{Q}}_k^{-1} \mathbf{G}_k \right)^{-1} \right) \triangleq p_k^{-1} \operatorname{tr}\left(\tilde{\mathbf{C}}_k \right), \tag{16}$$

By denoting $c_k = \operatorname{tr}\left(\tilde{\mathbf{C}}_k \right)$, the optimization problem can be formulated as

$$\min_{\mathbf{p}} \quad \sum_{k=1}^{K} c_k p_k^{-1} s.t. \;\; R \geq R_0, 1^T \mathbf{p} \leq P_T, p_k \geq 0, \forall k, \tag{17}$$

where $\mathbf{p} = [p_1, p_2, \dots p_K]^T$ is the PA vector to be optimized, R is defined as the sum-rate in (7) by omitting the subscript n, R_0 is the required sum-rate threshold, and P_T is the transmit power budget.

While the PA problem is convex and can be easily solved via numerical tools, we will show that a closed-form solution is obtainable under some certain conditions. Let us firstly formulate the Lagrangian multiplier as follows

$$\begin{aligned} &\mathcal{L}(\mathbf{p}, \lambda, \mu, v_k) \\ &= \sum_{k=1}^{K} c_k p_k^{-1} + \lambda \left(-\sum_{k=1}^{K} \log_2 (1 + h_k p_k) + R_0 \right) \\ &+\mu \left(1^T \mathbf{p} - P_T \right) - \sum_{k=1}^{K} \eta_k p_k, \end{aligned} \tag{18}$$

where λ, μ and η_k are dual variables. As a step further, the Karush-Kuhn-Tucker (KKT) conditions with respect to problem (17) can be formulated as

$$\frac{\partial \mathcal{L}}{\partial p_k} = -\frac{c_k}{p_k^2} - \frac{\lambda h_k}{(1 + h_k p_k) \ln 2} + \mu - \eta_k = 0, \forall k, \tag{19a}$$

$$\lambda \left(-\sum_{k=1}^{K} \log_2 (1 + h_k p_k) + R_0 \right) = 0, \lambda \geq 0, \tag{19b}$$

$$\mu \left(1^T \mathbf{p} - P_T \right) = 0, \mu \geq 0, \tag{19c}$$

$$\eta_k p_k = 0, \eta_k \geq 0, \forall k, \tag{19d}$$

$$1^T \mathbf{p} \leq P_T, \tag{19e}$$

$$p_k \geq 0, \forall k, \tag{19f}$$

One can immediately observe that $p_k > 0, \forall k$. This is because if $p_k = 0$, the objective function becomes unbounded. Based on (19d), we have $\eta_k = 0$. Moreover, it can be noted that the power constraint should be strictly satisfied, i.e., $1^T \mathbf{p} = P_T$. This can be proved by contradiction. Assume that the summation of the optimal p_k is less than P_T. One can always increase any p_k to reach a summation of P_T, which further reduces the objective function while increasing the sum-rate. Therefore, the power budget should be fully exploited. Given the complementary slackness of the KKT conditions, we have $\mu > 0$.

Theorem 1. *Problem (17) has closed-form solutions if*

$$R_0 \in (0, R_{th}] \cup R_{\max}, \tag{20}$$

where

$$R_{th} = \sum_{k=1}^{K} \log_2 \left(1 + \frac{h_k \sqrt{c_k}}{\sum_{k=1}^{K} \sqrt{c_k}} P_T \right), \tag{21}$$

and $R_{\max}$ is the water-filling rate given the power budget P_T and the channel coefficients h_k.

Proof. Since $\eta_k = 0$, we have

$$\frac{\partial \mathcal{L}}{\partial p_k} = -c_k p_k^{-2} - \frac{\lambda h_k}{(1 + h_k p_k) \ln 2} + \mu = 0. \tag{22}$$

In the case $\lambda = 0$, according to the complementary slackness we have

$$\sum_{k=1}^{K} \log_2 (1 + h_k p_k) > R_0, \tag{23}$$

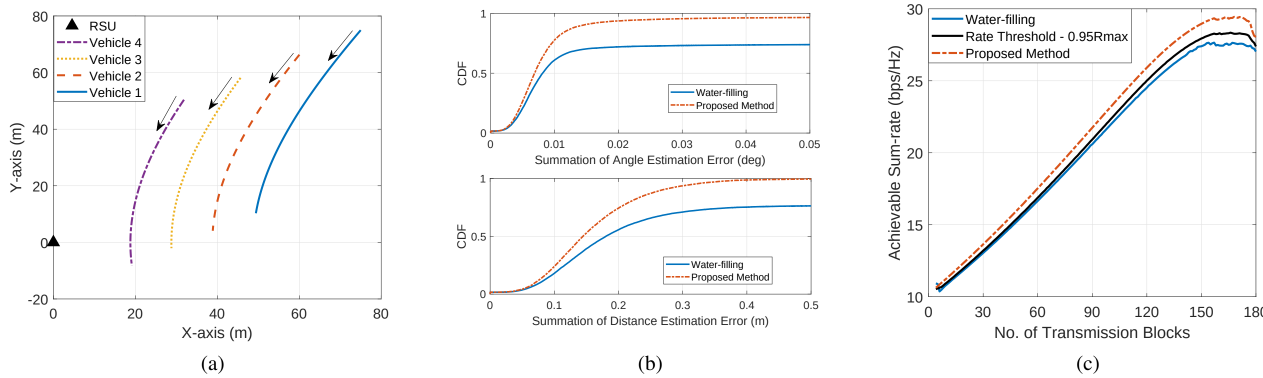

Fig. 1. Numerical results. (a) Trajectories for 4 vehicles; (b) Estimation performance for tracking multiple vehicles, with $N_t = N_r = 64$, SNR = 12dB; (c) Achievable sum-rate for multiple vehicles, with $N_t = N_r = 64$, SNR = 12dB.

and

$$\mu = c_k p_k^{-2}. \tag{24}$$

From the above, it follows that $p_k = \sqrt{\frac{c_k}{\mu}}$. Hence the power constraint can be rewritten as

$$\sum_{k=1}^{K} p_k = \sum_{k=1}^{K} \sqrt{\frac{c_k}{\mu}} = \sqrt{\frac{1}{\mu}} \sum_{k=1}^{K} \sqrt{c_k} = P_T. \tag{25}$$

Therefore

$$\mu = \frac{\left(\sum\limits_{k=1}^{K} \sqrt{c_k}\right)^2}{P_T^2}, \tag{26}$$

which results in the closed-form solution

$$p_k = \frac{\sqrt{c_k} P_T}{\sum\limits_{k=1}^{K} \sqrt{c_k}}. \tag{27}$$

It can be seen that when $R_0 \leq R_{th}$, the sum-rate constraint is inactive, and hence $\lambda = 0$. One can readily obtain the optimal PA expression (27).

When $R_0 = R_{\max}$, it is known that the water-filling PA is unique and optimal, and the feasible region of the problem contains only a single point. Therefore, the solution to the problem is the water-filling solution, which can be again expressed in closed-form. This completes our proof. ■

5. NUMERICAL RESULTS

In this section, we provide simulation results to verify the effectiveness of the proposed PA approaches. For simplicity, we set the noise variances as $\sigma_R^2 = \sigma_C^2 = 1$, and use normalized reference channel coefficients for both radar and communication. Accordingly, we define the transmit SNR as $\frac{P_T}{\sigma_R^2}$. The matched filtering gain is assumed to be $G = 10$. Without loss of generality, we assume that the RSU employs $N_t = N_r = 64$ antennas for both of its Tx and Rx, and that the transmit SNR is $\frac{P_T}{\sigma_R^2} = 12$dB. We study a case of 4 vehicles, with the trajectories shown in Fig. 1(a).

We first show in Fig. 1(b) the simulation results of tracking the 4 vehicles, where the prediction and estimation process follows that of [8]. The proposed PA design is compared with the water-filling PA, with the rate threshold being set as $R_0 = 0.95R_{\max}$. The estimation performance for both angle and distance is shown in terms of the cumulative distribution function (CDF) for estimation errors. It can be observed that by using the proposed technique, the angle and the distance estimation errors can be reduced to the levels of 0.1° and 0.5m, respectively, which is significantly improved compared to the water-filling PA.

In Fig. 1(c), we plot the achievable communication rates at each transmission block. Again, the proposed approach outperforms the water-filling PA. This is because the water-filling scheme is not tailored for angle estimation, which is likely to generate larger estimation errors, and lead to beam misalignment. Consequently, the obtained data rate would be lower than that of the proposed technique.

6. CONCLUSION

In this paper, we have proposed a novel PA design for the V2I network, where an RSU provides simultaneous localization and downlink communication services to multiple vehicles. On top of the predictive beamforming framework proposed in [8], we have formulated a convex optimization problem to further improve the overall radar sensing performance of multiple vehicles, while guaranteeing the downlink communication sum-rate. We have mathematically proved that the problem can be solved in closed-form under certain conditions. Finally, we have numerically verified the effectiveness of our PA approach, showing that the proposed PA design is superior to the conventional water-filling approach, in terms of both vehicle localization and communication performance.

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