On the Necessity of Importing Neurobiology into Mathematics

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Running Title   Perceptual axioms

Abstract

In Gödel's Incompleteness Theorems, for every mathematical system there are correct statements that cannot be proven to be correct within that system. We here extend this to address the question of axiomatic statements that are *perceived* (or known) to be correct but which mathematics, as presently constituted, cannot prove. We refer to these as *perceptual axioms*.

Keywords: perceptual axioms, "illusions", perceptual reality, logic, incompleteness theorems

Introduction

"The opposite of a correct statement is a false statement. But the opposite of a profound truth can be another profound truth." Niels Bohr

In a famous encounter at a meeting entitled "The Second Conference on the Epistemology of the Exact Sciences (*2. Tagung für Erkenntnislehre der exakten Wissenschaften Wissenschaften)*, held in Königsberg in 1931, Kurt Gödel confronted the famous mathematician David Hilbert with regard to his Incompleteness Theorems; Gödel argued that, for a mathematical system to be consistent, it cannot be complete, in the sense that it cannot prove all correct statements as correct and incorrect ones as incorrect [1]; moreover, that the consistency of the axioms cannot be proven within its own particular axiomatic system. The implication is that, within mathematics, there is an important difference between truth and proof. But Gödel was writing only about the limitations of the mathematical axiomatic theories. Here we address what amounts to the same question in a more neurobiological context. We formulate our question as follows: are there any perceived truths that are universally known but which mathematics cannot prove? We believe that we can address this question within the context of the organization of the perceptual brain and what it allows us to experience.

Truths derived from perception that are unprovable mathematically

The concept of mathematical axioms is well known; it refers to statements which the logical system of the brain accepts without proof; these statements constitute the building blocks which allow us to construct mathematical theories. We use the examples below to propose the concept of another axiomatic system, and introduce the term *perceptual axiomatic* system to do so; this is also universally accepted by the human brain and works in parallel, but independently, of the logical system of the brain underlying mathematics.

The perceptual truths of the three examples given below cannot be doubted but also cannot be proven mathematically. These perceptual truths are so obvious that we refer to them as *perceptual axioms*.

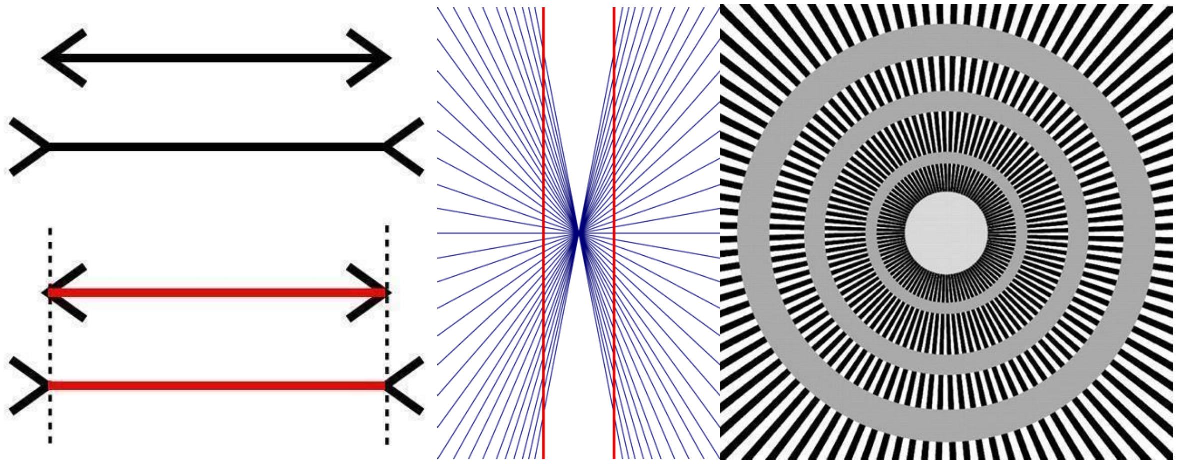

Figure 1. Muller Lyer Illusion (left), Hering Illusion (centre) and a modified (achromatic) version of Isia Leviant's Enigma (right)

Consider first the Muller Lyer "Illusion" (Figure 1, left). In it there are two lines of the same length; both have arrows directed inwards for one line and outwards for the other. That they are of equal length can be proven mathematically. And yet no normal person sees these lines to be of the same length, even when they know them to be mathematically equal; instead the line with the outward arrows is perceived to be shorter than the one with the inward arrows. This is true even if, when trying to convince the viewer of the equality of lengths, the lines are made more conspicuous by being highlighted, for example in red, as in the lower left panel.

A second example is the Hering "illusion" (Figure 1, centre). Here two lines which can be proven to be parallel mathematically are perceived as being bent in a concave way. The *perceptual* fact of them being so bent is one that has universal assent – all humans with a normal visual apparatus (including the visual brain) see these lines as slightly bent and not as parallel; it can therefore be said to be perceptually axiomatic. But mathematical formulae cannot prove these two lines to be so bent, unless a perceptual axiom is taken into consideration, which amounts to incorporating perceptual axioms into mathematics. The only conclusion that mathematics can reach about them is that they are parallel, thus contradicting a profound perceptual axiomatic truth. For normal individuals, it is the latter truth that prevails and which they have to live with while the mathematical truth is the one that is rejected perceptually.

Here again the perceptual truth is contradictory to the mathematical one; this constitutes another example of mathematics not being able to prove a simple perceived reality.

In our third and last example we consider a different type of "illusion" – one in which motion is perceived in a static picture, at least for some individuals (Figure 1, right); the motion is very rapid and reverses direction with prolonged viewing. The reality of this cannot be doubted by individuals who perceive the motion in the rings (not all individuals do). The “illusion” is one prepared by the French artist, Isia Leviant, who has written about the steps in its construction [2]. By sole use of mathematics, which expresses the occurrence of motion in terms of displacement, distance, velocity, acceleration, speed, and time, for example in the form

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}\,, \quad \mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2}$$

it would be impossible to prove the existence of motion in such a picture, and even less so the reversal in its direction. In other words, it will be impossible to prove mathematically to a person who experiences the motion that there is motion. This is another example of perceptual truths which mathematics cannot prove to be true.

Conclusion

We have consistently used the term “illusion” above in quotes – because, by definition, illusion is a departure from “reality”, in the examples given here from “mathematical reality”. However, mathematics depends upon the healthy functioning of brain mechanisms just as perception does; where the two clash, it is the perceptual reality that prevails. It is worth emphasizing that the only perceptual reality that we are capable of experiencing is that which is allowed and made possible by the organization of our brains. Viewed in this context, it is the mathematical reality that is the “illusion” and the perceptual reality that is the truth, something which no argument in mathematics can overcome.

We have given only three examples; there are no doubt many more that fall under the category of perceptual axioms. We use these three examples to make the general point that, without incorporating an understanding of neurobiological mechanisms, the whole field of mathematical logic is limited in its application, as are its results. The only truths that we are sure of, whether mathematical or otherwise, are the ones that our brain organisation allows us to experience. Therefore, mathematics should include in its repertoire an understanding of the neurobiological mechanisms of perceptual reality in order to address not only the problem of incompleteness but the entire problem of knowledge.

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References:
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