Ray Optics Model for Trapping of Biconcave Red Blood Cells in Single- and Dual-Beam Optical Tweezers

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Abstract: We present a numerical investigation of the trapping forces and equilibrium configuration for a healthy biconcave red blood cell optically trapped in both single and dual beam optical tweezers using the ray optics approximation. © The Authors 2021.

1. Introduction and motivation

Red blood cells (RBCs), or erythrocytes, are the most abundant type of cell present in blood of most vertebrates. In humans, mature and healthy RBCs are un-nucleated, lack the organelles present in most other cells, and are mainly filled with the protein haemaglobin that binds to oxygen for transport around the body to the tissues. A typical mature RBC assumes the shape of a biconcave disk with an average diameter of ~8 μm and a maximum thickness of ~2-2.5 μm [1]. The series of events that lead to the oxygenation of the tissues starts in the small capillaries of the lungs where the oxygen diffuses within RBCs and binds to hemoglobin, and terminates in the small capillaries of the peripheral tissues where the oxygen diffuses out of the RBC. During this cycle, RBCs deform and elongate to pass through the capillaries the sizes of which are comparable to the diameter of the cell itself [2]. The biomechanical properties of the RBC are, therefore, of primary importance for an efficient oxygen delivery and several pathologies are correlated to the alteration of RBC elasticity, for example diabetes, malaria, sickle cell disease and others [3].

Optical tweezers (OT) have been largely used to sense and screen the mechanical properties of healthy and diseased RBC in a single or double-beam configuration [4]. For both OT configurations, it is observed that when the cell is optically trapped it finds an equilibrium position where its axis of symmetry is perpendicular to the beam propagation direction, as illustrated in Fig. 1 [5, 6].

![Fig 1. Equilibrium position for a RBC trapped by a single (a) and double-beam optical tweezers (b) showing the co-ordinate system used for the calculations.](image)

Previous experimental and theoretical investigations have showed that when a RBC is trapped by a counter-propagating beam optical trap, significant torques arise which align the cell with its axis perpendicular to the direction of the beam propagation [7]. Here we investigate numerically the equilibrium configuration of RBC trapped by a single or dual-beam OT similar to the geometry employed in [6] for the controlled stretching of RBCs to determine their deformability. An extensive investigation of the forces and torques involved in the trapping of the RBC is of paramount importance to allow the experimentalists to optimize the parameters prior to performing experiments. For this reason, we present a fully ray optics investigation of the equilibrium configuration of a healthy RBC trapped by both single and dual-beam OT.

2. Methods

In common with the experiments of [6], the wavelength of the light beam is chosen to be 1.064 μm. In this condition, the geometrical optics approximation is fulfilled since the RBC is much bigger than the wavelength used for trapping [8]. The trapping beam is considered to have a gaussian intensity profile and propagates along the +z direction. Each beam has a total power of 5 mW. We make the simplifying approximation that the RBC is not
deformed by the trapping beam, and is thus considered to be a rigid body, with the cellular membrane approximated as a single layer of negligible optical thickness. We model the cell as being filled by a liquid with isotropic physical properties and optical properties, with a refractive index $n_{\text{cell}} = 1.38$ [9], and suspended in water. In this approximation the reflection and refraction of the light rays happens only at the cell-medium interface. The shape of the RBC is modelled according to the four-parameter equation proposed in [1]. The optical forces and torques are calculated by implementing a new object to describe the cell in the OTGO software [10], and the entire set of numerical simulations are performed with Matlab R2017b.

3. Results

Fig 1(a) illustrates the co-ordinate system for the trapped RBC in single-beam OT. To analyze the confinement in three dimension, we start by analyzing the optical forces for cell displacements along the three Cartesian coordinates as schematically depicted in Fig 1(a). We calculate the force-displacement curves for each of the $x$, $y$ and $z$ directions to locate the equilibrium position and the corresponding trap stiffness. It is important to note that for the axial co-ordinate the effective weight of the cell ($\sim 0.1$ pN), must be included to correctly locate the equilibrium position. To determine the torques that control the orientation of the RBC we rotate the cell around the axes shown that pass through the center of the trap and plot the torque-angle curves to obtain the rotational trap stiffness.

Along $x$ (i.e. the axis that passes along the diameter of the cell), there are two symmetrically located equilibrium points, corresponding to the trap acting near the thickest part of the cell where the overlap between the beam and the cell volume is largest. This result is in good agreement with experiments which show that the cell is trapped on the thickest portion [5]. For $y$ and $z$, a single position of equilibrium is found. The restoring forces for small displacements from equilibrium in each direction are characterized by the trap stiffnesses. For angular displacements a restoring torque is found only for rotations about the $x$ direction, which produces the alignment of cells observed in experiments. In single beam OT there is no confining torque about the $z$ (beam propagation) axis.

We perform a similar investigation for a dual-beam OT with the foci positioned 5.06 $\mu$m apart, and the co-ordinate system shown in Fig 1(b). Note that here for determining the angular stiffness, the rotation axes pass through the trap center which is located half way between the two beam foci. With this dual-beam configuration, we find that the cell is confined along each of the axes $x$, $y$, and $z$, but in this case the presence of a strong torque around the $z$-axis acts to control the cell orientation.

4. Conclusion

In conclusion, we have used the ray optics approximation to describe the optical trapping of a RBC by single and double-beam OT and quantify the forces and torques that control the orientation of the cell. We note that since a rigid body approximation is used, quantitative agreement with experimental values may require the induced deformation of the cell to be accounted for.

References