

# MULTI-OBJECTIVE OPTIMISATION FOR SAFE MULTI-FLOOR PROCESS PLANT LAYOUT USING THE DOW'S FIRE & EXPLOSION INDEX

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## Abstract

In this work, we propose a multi-objective mixed-integer linear programming (MILP) model for evaluation of safe multi-floor process plant layout using the Dow's Fire & Explosion Index (F&EI). Two objectives are considered, namely the total monetary cost and the financial risk, and solved via the  $\epsilon$ -constraint method. The total monetary cost consisted of the piping costs, horizontal and vertical pumping costs, land purchase cost, fixed and area-dependent floor construction cost, and the cost of protection devices. The financial risk was evaluated as the maximum probable property damage cost obtained using the F&EI evaluation procedure. The proposed model was applied to an ethylene oxide plant consisting of 7 equipment items with Pareto-optimal solutions showing that the financial risk can be greatly reduced by layout reconfiguration without the need for protection devices and their associated monetary costs. Further increase in the safety levels of the plant can then be achieved through protection device installation at a cost. These, and more information obtained, which are non-existent in previous single objective considerations, are helpful for a more informed decision making process in the planning stages of the design of chemical process plants.

*Keywords:* Multi-objective optimisation, Dow's Fire and Explosion Index, Mixed-integer linear programming, Multi-floor process plant layout

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## 1. Introduction

It is common practice that safety analysis, and the layout configuration, of chemical process plants are carried out after the detailed design of such plants are completed (Ortiz-Espinoza et al., 2021). This leaves little or no room for design

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modifications to improve on the overall safety levels of the plant which can lead to fatal consequences, disruption of production activities, and irreparable losses to the plant and neighbouring environment (Ejeh et al., 2021). From a safety point of view, this has led to the incorporation of inherent safety principles at the design stages of chemical process plants (Khan and Amyotte, 2004). However, studies have also shown that the layout configuration of such process plants directly and indirectly contribute to about 79% of accidents (Kidam and Hurme, 2012). It then becomes important not just to develop suitable tools for safer process designs (Ortiz-Espinoza et al., 2021), but further extend such tools to obtain safer layout configurations as well.

Roy et al. (2016) presented a review of safety indices applicable to process design where brief descriptions of 25 representative indices were given with their level of application and inputs required. Two key indices were identified as having a direct relation to the layout of a process plant - the Dow's Fire and Explosion index (F&EI) (American Institute of Chemical Engineers, 1994) and the Domino Hazard Index (DHI) (Tugnoli et al., 2008a).

The F&EI is a widely applied method for hazard evaluation of chemical and industrial processes. It was developed by American Dow Chemical Company (American Institute of Chemical Engineers, 1994) and estimates the hazards of a single unit based on the chemical properties of the material(s) within it, and the potential economic risk such equipment poses to itself and neighbouring structures, with or without the installation of protection devices. Figure 1, adapted from Gupta et al. (2003), shows a flow diagram for the calculation procedure of the Dow's F&EI. For an accurate estimation of each metric, the process plant's design and operating data are important e.g. the chemical potential of each unit, quantity of hazardous material it processes, etc. From such data, the F&EI is calculated as a product of the Material Factor (MF) and the process unit hazards factor ( $F_3$ ) and is helpful in determining the degree of hazard (Table 1) a process unit poses in the plant. The chemical properties of the material(s) within a process equipment is quantified by the MF, and the process unit hazards factor is the result of the product of the general ( $F_1$ ) and special ( $F_2$ ) process hazards factors (American Institute of Chemical Engineers, 1994, Gupta et al., 2003, Ortiz-Espinoza et al., 2021). Patsiatzis et al. (2004) gives a detailed procedure for the calculation of each of these factors. All these factors are required in order to determine the radius of exposure, and thus the economic impact of fire and explosion events, for each pertinent equipment item identified. Pertinent equipment items here refer to process units "that could have an impact from a loss prevention standpoint" (American Institute of Chemical Engineers, 1994).

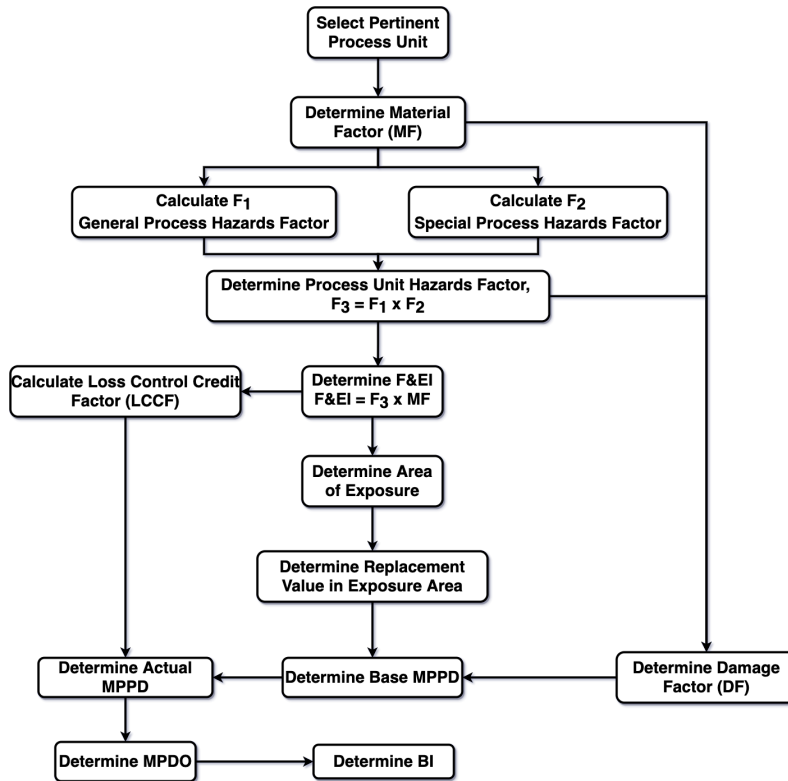


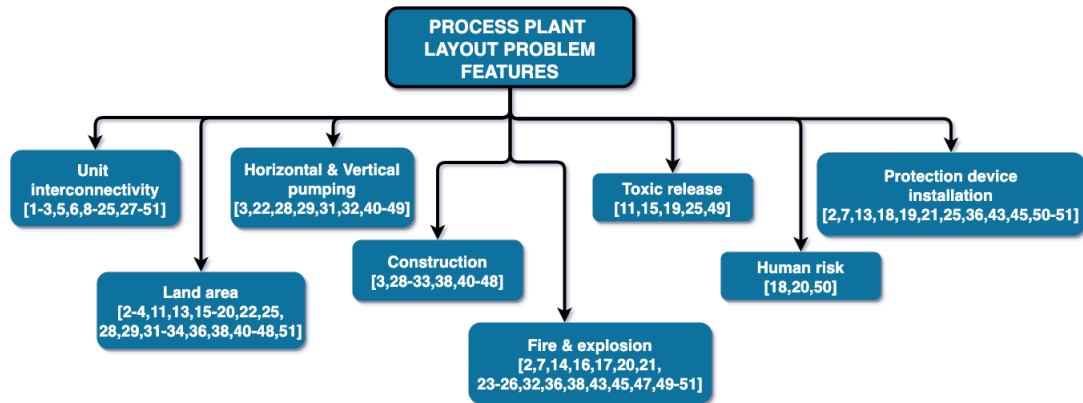
Figure 1: Dow's F&EI calculation procedure

Protection devices are factored into the calculation procedure using the Loss Control Credit Factor (LCCF). For each device, or set of device configurations, known to be beneficial in preventing hazard events and in reducing the probability and magnitude of such events, a numeric value is calculated representing the reduction in the base maximum probable property damage (MPPD) resulting from a hazardous event. Traditionally, the LCCF is not factored into the F&EI calculation procedure, however [Gupta et al. \(2003\)](#) proposed methodology to calculate the Offset F&EI which considers this.

Table 1: Unit classification by F&EI values

F&EI	Degree of hazard
1 - 60	Light
61 - 96	Moderate
97 - 127	Intermediate
128 - 158	Heavy
159+	Severe

As noted in the literature ([Moran, 2017](#)), good process plant layout designs ought to have a healthy balance of efficient, economic, ergonomic and safety factors, and this has been demonstrated in the growing research on the optimal layout of chemical plants with safety considerations. Figure 2 gives a summary of features considered in the literature related to optimal process layout designs, with a substantial number looking to safety considerations - fires, explosion, toxic release, protection device installation and human risk. [Penteado and Ciric \(1996\)](#) addressed safety from the view point of the financial risk associated with accidents. Such risk was quantified as the net present financial loss associated with an accident based on its probability and severity as a function of the proximity of the accident source to neighbouring units. Protection devices were considered to reduce the risk level and their costs were included in the proposed model in a single objective. [Patsiatzis et al. \(2004\)](#) quantified financial risk using the F&EI, with protection devices also being made available to reduce the impact an accident had on neighbouring equipment items. The proposed model was formulated as an MILP problem and considered single-floor layout scenarios. [Park et al. \(2018\)](#) extended the work of [Patsiatzis et al. \(2004\)](#) to multi-floor scenarios using a mixed integer non-linear programming (MINLP) model, while [Wang et al. \(2017\)](#) also employed the F&EI system but over a discrete domain for single floor layouts.



1 Jayakumar and Reklaitis (1994)	2 Penteado and Ciric (1996)	3 Georgiadis and Macchietto (1997)
4 Castell et al. (1998)	5 Papageorgiou and Rotstein (1998)	6 Barbosa-Póvoa et al. (2001)
7 Patsiatzis et al. (2004)	8 Westerlund et al. (2007)	9 Xu and Papageorgiou (2007)
10 Xu and Papageorgiou (2009)	11 Vázquez-Román et al. (2010)	12 Furuholmen et al. (2010)
13 Jung et al. (2010b)	14 Jung et al. (2010a)	15 Díaz-Ovalle et al. (2010)
16 Jung et al. (2011)	17 López-Molina et al. (2013)	18 Han et al. (2013)
19 Díaz-Ovalle et al. (2013)	20 Medina-Herrera et al. (2014)	21 de Lira-Flores et al. (2014)
22 Caputo et al. (2015)	23 Jung (2016)	24 Wang et al. (2017)
25 Latifi et al. (2017)	26 de Lira-Flores et al. (2018)	27 Jayakumar and Reklaitis (1996)
28 Georgiadis et al. (1999)	29 Patsiatzis and Papageorgiou (2002)	30 Barbosa-Póvoa et al. (2002)
31 Patsiatzis and Papageorgiou (2003)	32 Park et al. (2011)	33 Hwang and Lee (2014)
34 Ku et al. (2014b)	35 Ku et al. (2014a)	36 Dan et al. (2015)
37 Lee and Lee (2017)	38 Park et al. (2018)	39 Wrigley et al. (2019)
40 Ejeh et al. (2018a)	41 Ejeh et al. (2018c)	42 Ejeh et al. (2018b)
43 Ejeh et al. (2019a)	44 Ejeh et al. (2019b)	45 Ejeh et al. (2021)
46 Xu et al. (2020)	47 Díaz-Ovalle et al. (2021)	48 He et al. (2019)
49 Wu et al. (2020)	50 Ahumada et al. (2018)	51 Quiroz-Pérez et al. (2021)

**Figure 2: Summary of past literature on features considered by process plant layout optimisation models**

Researchers have also employed the Domino Hazard Index (DHI) system (Tugnoli et al., 2008a) which assesses the domino effects a set of pre-defined hazard events have on other components of a process plant. Hazards accounted for include fires, explosion, toxic release, human risk, blast wave e.t.c. Tugnoli et al. (2008b) first applied the DHI to assess safety levels of different layout configurations, but de Lira-Flores et al. (2014) developed an MINLP model to handle single floor cases. Ejeh et al. (2021) further extended such model to account for multi-floor cases via single and bi-objective MILP models. These models accounted for pool fires, jet fires, fireballs, flash fires and explosions, and the effect such incidents had on other equipment as a function of separation distances and the presence of protection devices.

Other risk estimating methods have also been developed apart from the F&EI and DHI system of estimation. Using dispersion models, Díaz-Ovalle et al. (2010) accounted for toxic release incidents based on a worst-case scenario, with mitigation systems later on introduced in Díaz-Ovalle et al. (2013). Using the

individual risk, [Han et al. \(2013\)](#) also formulated an MILP model to determine layout configurations with minimal risk to human beings.

The Dow’s F&EI index is still a widely used safety metric even in recent ([Ortiz-Espinoza et al., 2021](#)) analysis. However, its application to multi-floor layout considerations is lacking. [Ejeh et al. \(2019a\)](#) proposed an MILP model for safe multi-floor layout designs of chemical process plants. However, the single objective model does not provide design engineers with the necessary flexibility and understanding for a final layout design. In this work, we propose a multi-objective MILP model for the evaluation of safe multi-floor layout configuration with modifications for a more accurate estimation of the areas of exposures within the plant.

The rest of this paper is structured as follows. The bi-objective problem is described in detail in [Section 2](#), and the mathematical model in [Section 3](#). A case study is investigated in [Section 4](#) which demonstrates model performance and merits of the considerations proposed in this work. A summary of findings is discussed in [Section 5](#).

## 2. Problem Description

The bi-objective multi-floor plant layout problem with safety considerations using the Dow’s F&EI is described as follows:

*Given:*

- a set of equipment items and their dimensions (length, breadth and height);
- a subset of pertinent equipment items;
- distance of exposure and damage factors of pertinent equipment items (obtainable from the procedure outline in [Figure 1](#));
- connectivity network amongst equipment items;
- a set of potential floors for layout with known floor height;
- a set of protection device configurations for each pertinent equipment item and its corresponding LCCF if installed on such item;
- cost data (connection, protection device purchase, equipment purchase, pumping, land, and construction);
- space and equipment item allocation limitations;

*to determine:*

- total number of required floors for the layout;
- protection device configuration to be installed on each pertinent equipment item;
- floor area;

- plot layout;

*so as to*: minimise the total monetary costs and financial risk evaluated using the Dow's Fire & Explosion Index (F&EI).

The total monetary cost comprises the plant layout cost - sum of the cost of connecting equipment items by pipes, pumping process fluids through pipes, purchasing the land for the layout based on its area, constructing each floor, installing selected protection devices - and the financial risk represents the monetary equivalent of the total damage to the plant in the event of a fire and/or explosion event.

In the problem, the following assumptions are made:

- The geometry of each equipment item is approximated as a rectangle. Although a number of chemical process plant equipment have a circular footprint, the inclusion of supporting structures about such item approximates to a square;
- The connection distances between equipment items are taken from the respective geometrical centres in the x-y plane, and from a predefined height along the z-plane, based on design calculations;
- For safety considerations, all rectilinear distances between items of concern are measured from the equipment item boundaries to evaluate the probability, magnitude and impact of an incident on a pertinent item. This represents a realistic assumption as the distance of exposure for each item is measured from the item boundaries;
- An item is allowed to rotate through  $90^\circ$  angles about the x-y plane.
- Floors are numbered from bottom to top with a fixed floor height;
- Tall equipment items are allowed to extend through successive floors.

### 3. Mathematical Formulation

#### Nomenclature

##### *Abbreviations/Acronyms*

BI	Business Interruption
DF	Damage Factor
F&EI	Dow's Fire & Explosion Index
LCCF	Loss Control Credit Factor
MF	Material Factor
MPDO	Maximum Probable Days Outage
MPPD	Maximum Probable Property Damage

### Indices

$i, j, n$	equipment item
$k, k'$	floor number
$p$	protection device configuration
$s$	rectangular area sizes

### Sets

$\zeta$	set of ordered pairs of pertinent equipment items and other items; $\zeta = \{(i, j) : i \in I^{pe}, j \neq i\}$
$I$	set of equipment items
$I^c$	set of ordered pairs of connected equipment items; $I^c = \{(i, j) : f_{ij} = 1\}$
$I^{pe}$	set of pertinent equipment items
$I^T$	set of multi-floor equipment items
$P_i$	set of protection device configurations suitable for installation on item $i$

### Parameters

$\alpha_i, \beta_i, \gamma_i$	dimensions of item $i$
$BM$	large number
$C_{ij}^c$	connection/piping costs between items $i$ and $j$
$C_i^e$	purchase cost of item $i$
$C_{ij}^h$	horizontal pumping costs between items $i$ and $j$
$C_{ip}^p$	purchase and installation cost of protection device configuration $p$ for item $i$
$C_{ij}^v$	vertical pumping costs between items $i$ and $j$
$CF_{ip}$	loss control credit factor (LCCF) of protection device configuration $p$ for item $i$
$D_i^e$	distance of exposure of item $i$
$DF_i$	damage factor of pertinent item $i$
$f_{ij}$	1 if flow direction between $i$ and $j$ is positive; 0, otherwise
$FEI_i$	fire and explosion index value of pertinent item $i$
$FC1$	fixed floor construction cost
$FC2$	area-dependent floor construction cost
$FH$	floor height
$IP_{ij}$	distance between the base and input point on equipment $j$ for piping connection between $i$ and $j$
$LC$	land cost
$M_i$	number of floors required by item $i$
$OP_{ij}$	distance between the base and output point on equipment $i$ for piping connection between $i$ and $j$
$\bar{X}_s, \bar{Y}_s$	x-y dimensions of pre-defined rectangular area sizes $s$



*Integer variables*

$NF$  number of floors required for layout

*Binary variables*

$\psi_{ij}$  1 if item  $j$  is within the distance of exposure of pertinent item  $i$ , 0 otherwise  
 $\mu_{ip}$  1 if protection device configuration  $p$  is installed on pertinent item  $i$ , 0 otherwise  
 $E1_{ij}, E2_{ij}$  non-overlapping binary variables, a set of values which prevents equipment overlap in the  $x, y$  plane  
 $N_{ij}$  1 if items  $i$  and  $j$  are assigned to the same floor; 0, otherwise  
 $O_i$  1 if length of item  $i$  is equal to  $\alpha_i$ ; 0, otherwise  
 $Q_s$  1 if rectangular area  $s$  is selected for the layout; 0, otherwise  
 $S_{ik}^s$  1 if item  $i$  begins at floor  $k$ ; 0, otherwise  
 $Up_{ij}^b$  1 if item  $i$  is located on a floor above item  $j$ , 0 otherwise or if items  $i$  and  $j$  are on the same floor  
 $Dn_{ij}^b$  1 if item  $i$  is located on a floor below item  $j$ , 0 otherwise or if items  $i$  and  $j$  are on the same floor  
 $MB_{ij}^{XY}$  1 if  $XD_{ij} \geq YD_{ij}$ ; 0 otherwise  
 $MB_{ij}^Z$  1 if  $XY_{ij}^{max} \geq VD_{ij}$ ; 0 otherwise  
 $V_{ik}$  1 if item  $i$  is assigned to floor  $k$   
 $W_k$  1 if floor  $k$  is occupied; 0, otherwise  
 $W_{ij}^x$  1 if item  $i$  is to the right of item  $j$  in the  $x$  plane; 0 otherwise  
 $W_{ij}^{\bar{x}}$  1 if the boundary of item  $i$  is strictly to the right or left of item  $j$  in the  $x$  plane; 0 otherwise  
 $W_{ij}^y$  1 if item  $i$  is above item  $j$  in the  $y$  plane; 0 otherwise  
 $W_{ij}^{\bar{y}}$  1 if the boundary of item  $i$  is strictly above or below item  $j$  in the  $y$  plane; 0 otherwise  
 $W_{ij}^z$  1 if item  $i$  is on a higher floor than item  $j$ ; 0 otherwise

*Continuous variables*

$\eta_{ij}^u, \eta_{ij}^d$  positive continuous variables to determine vertical safety distance between items  $i$  and  $j$   
 $\Omega_i^0$  base maximum probable property damage cost for pertinent equipment item  $i$   
 $\Omega_i$  actual maximum probable property damage cost for pertinent equipment item  $i$   
 $\Omega'_{ip}$  linearisation variable denoting the the product of  $\Omega_i$  and  $\mu_{ip}$   
 $A_{ij}$  distance in the  $y$  plane between items  $i$  and  $j$  in the  $x$ - $y$  plane, if  $i$  is above  $j$   
 $AR_s$  predefined rectangular floor area  $s$   
 $B_{ij}$  distance in the  $y$  plane between items  $i$  and  $j$  in the  $x$ - $y$  plane, if  $i$  is below  $j$

$d_i$	breadth of item $i$
$D_{ij}$	connection distance in the z plane between items $i$ and $j$ , if $i$ is lower than $j$
$Dn_{ij}$	vertical separation distance between items $i$ and $j$ , if $i$ is on a lower floor than $j$
$FA$	floor area
$h_i$	height of item $i$
$l_i$	length of item $i$
$L_{ij}$	distance in the x plane between items $i$ and $j$ , if $i$ is to the left of $j$
$NQ_s$	linearised variable expressing the product of $NF$ and $Q_s$
$R_{ij}$	distance in the x plane between items $i$ and $j$ , if $i$ is to the right of $j$
$TD_{ij}^c$	total rectilinear connection distance between items $i$ and $j$
$TD_{ij}^{in}$	total rectilinear safety distance between items $i$ and $j$ , if $j$ is within the distance of exposure of item $i$
$TD_{ij}^{out}$	total rectilinear safety distance between items $i$ and $j$ , if $j$ is outside of the distance of exposure of item $i$
$TD_{ij}^s$	total rectilinear safety distance between items $i$ and $j$
$U_{ij}$	connection distance in the z plane between items $i$ and $j$ , if $i$ is higher than $j$
$Up_{ij}$	vertical separation distance between items $i$ and $j$ , if $i$ is on a higher floor than $j$
$V_i^e$	value of area of exposure of item $i$
$VD_{ij}$	total vertical distance between items $i$ and $j$
$x_i, y_i$	coordinates of the geometrical centre of item $i$
$X^{max}, Y^{max}$	dimensions of floor area
$XY_{ij}^{max}$	maximum value between the horizontal separation distance between items $i$ and $j$ in the x- and y- planes
$XD_{ij}, YD_{ij}$	total horizontal distance between the boundaries of items $i$ and $j$ in the x, y directions respectively
$ZD_{ij}^+, ZD_{ij}^-$	variables to evaluate the difference between $XY_{ij}^{max}$ and $VD_{ij}$ if $XY_{ij}^{max} > VD_{ij}$ and $XY_{ij}^{max} < VD_{ij}$ respectively

The proposed mathematical model presented is a modified form of the MILP model presented by [Ejeh et al. \(2021\)](#) (summarised in [Appendix A](#)) for safe multi-floor process plant layout using the Domino Hazard Index (DHI) and an extension of the considerations of [Patsiatzis et al. \(2004\)](#) to multi-floor process plants. Constraints are modified to evaluate the areas of exposure for pertinent process equipment and the associated MPPD with/without the availability of protection devices.

### 3.1. Floor constraints

First, in addition to the floor constraints outlined in eqs. [\(A.1\)](#) - [\(A.8\)](#) in [Appendix A](#), the equation defining if two items  $i$  and  $j$  are allocated to the same

floor is written to include all possible equipment item pairs ( $j \neq i$ ):

$$N_{ij} \geq V_{ik} + V_{jk} - 1 \quad \forall i, j \neq i, k \quad (1)$$

### 3.2. Distance constraints

Distance constraints for connected equipment items ( $i \in I^c$ ) are as described by [Ejeh et al. \(2021\)](#) with eqs. (A.16) - (A.19) in [Appendix A](#).

#### 3.2.1. Safety distance

For the safety distance between equipment items, the separation distances between equipment item boundaries are adopted.

In addition to the connection distance constraints,  $R_{ij}$ ,  $L_{ij}$ ,  $A_{ij}$  and  $B_{ij}$  are similarly evaluated for all possible combinations between pertinent items and other items ( $(i, j) \in \zeta$ ):

$$R_{ij} - L_{ij} = x_i - x_j \quad \forall i, j : f_{ij} = 1, (i, j) \in \zeta \quad (2)$$

$$A_{ij} - B_{ij} = y_i - y_j \quad \forall i, j : f_{ij} = 1, (i, j) \in \zeta \quad (3)$$

$R_{ij}$  or  $L_{ij}$  determine the distance between the mid-points of items  $i$  and  $j$  if item  $i$  is the right or left of item  $j$  in the x plane respectively. To force one of  $R_{ij}$  or  $L_{ij}$  to zero, eqs. (4) - (5) are introduced. When the binary variable  $W_{ij}^x = 0$ ,  $R_{ij}$  is forced to zero and  $L_{ij}$  takes a positive value equal to  $|x_i - x_j|$ . When  $W_{ij}^x = 1$ , only  $R_{ij}$  takes a positive non-zero value.

$$R_{ij} \leq BM \cdot W_{ij}^x \quad \forall i, j : f_{ij} = 1, (i, j) \in \zeta \quad (4)$$

$$L_{ij} \leq BM \cdot (1 - W_{ij}^x) \quad \forall i, j : f_{ij} = 1, (i, j) \in \zeta \quad (5)$$

The same set of equations are applied to  $A_{ij}$  and  $B_{ij}$  when item  $i$  is above or below item  $j$  in the y plane respectively, using eqs. (6) - (7) and the binary variable  $W_{ij}^y$ :

$$A_{ij} \leq BM \cdot W_{ij}^y \quad \forall i, j : f_{ij} = 1, (i, j) \in \zeta \quad (6)$$

$$B_{ij} \leq BM \cdot (1 - W_{ij}^y) \quad \forall i, j : f_{ij} = 1, (i, j) \in \zeta \quad (7)$$

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as follows. A value of zero is assigned to these distances if items  $i$  and  $j$  overlap at any region on either the x or y plane. For the x-plane, these are evaluated using the following constraints:

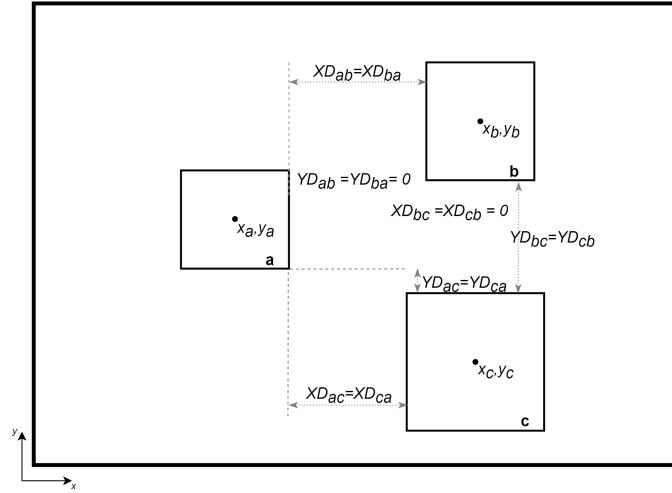
$$x_i - x_j + 2L_{ij} \geq \left( \frac{l_i + l_j}{2} \right) - BM(1 - W_{ij}^{\bar{x}}) \quad \forall (i, j) \in \zeta \quad (8)$$

$$XD_{ij} \leq R_{ij} + L_{ij} - \left( \frac{l_i + l_j}{2} \right) + BM(1 - W_{ij}^{\bar{x}}) \quad \forall (i, j) \in \zeta \quad (9)$$

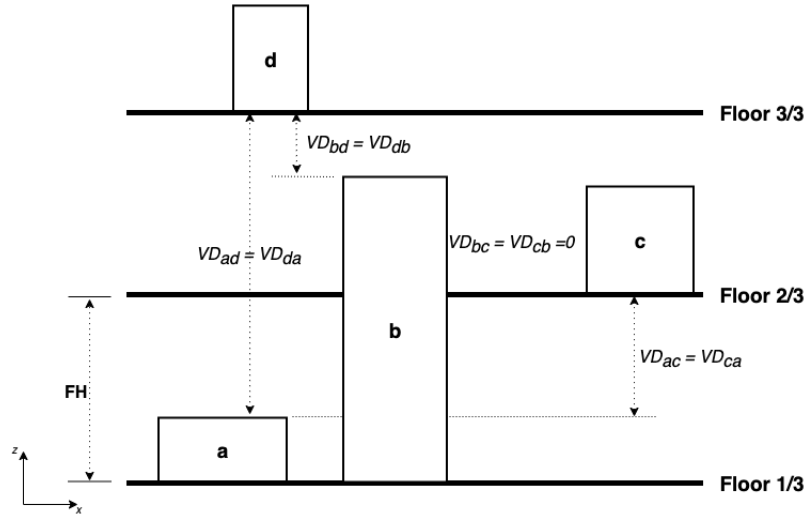
$$XD_{ij} \geq R_{ij} + L_{ij} - \left( \frac{l_i + l_j}{2} \right) - BM(1 - W_{ij}^{\bar{x}}) \quad \forall (i, j) \in \zeta \quad (10)$$

$$XD_{ij} \leq BM \cdot W_{ij}^{\bar{x}} \quad \forall (i, j) \in \zeta \quad (11)$$

Eq. (8) ensures that if the opposing boundaries of item  $i$  is strictly to the right or left of item  $j$ ,  $W_{ij}^x$  takes a value of 1. Eqs. (9) - (11) ensure that the distance between the boundaries of items  $i$  and  $j$  ( $XD_{ij}$ ) in the x plane only takes a positive value when both items do not overlap along any region in the x plane.



(a) Horizontal distances



(b) Vertical distances

**Figure 3: Safety distance between equipment items**

A similar set of constraints (as eqs. (8) - (11)) are written for the y plane

((12) - (15)).

$$y_i - y_j + 2B_{ij} \geq \left( \frac{d_i + d_j}{2} \right) - BM(1 - W_{ij}^{\bar{y}}) \quad \forall (i, j) \in \zeta \quad (12)$$

$$YD_{ij} \leq A_{ij} + B_{ij} - \left( \frac{d_i + d_j}{2} \right) + BM(1 - W_{ij}^{\bar{y}}) \quad \forall (i, j) \in \zeta \quad (13)$$

$$YD_{ij} \geq A_{ij} + B_{ij} - \left( \frac{d_i + d_j}{2} \right) - BM(1 - W_{ij}^{\bar{y}}) \quad \forall (i, j) \in \zeta \quad (14)$$

$$YD_{ij} \leq BM \cdot W_{ij}^{\bar{y}} \quad \forall (i, j) \in \zeta \quad (15)$$

A binary variable  $W_{ij}^{\bar{y}}$  is introduced having a value of 1 if the opposing boundaries of item  $i$  is strictly above or below  $j$ .

The vertical safety distance ( $VD_{ij}$ ) as described in Figure 3b is evaluated such that:

$$VD_{ij} \begin{cases} \geq 0, & N_{ij} = 0 \\ = 0, & N_{ij} = 1 \end{cases} \quad (16)$$

For cases where items  $i$  and  $j$  are assigned to different floors ( $N_{ij} = 0$ ), eqs. (17) or (18) determine the separation distances between items  $i$  and  $j$  if  $i$  is above ( $Up_{ij}$ ) or below ( $Dn_{ij}$ )  $j$  respectively.

$$Up_{ij} = FH \sum_k (k-1)(S_{ik}^s - S_{jk}^s) - \gamma_j \quad \forall (i, j) \in \zeta, N_{ij} = 0 \quad (17)$$

$$Dn_{ij} = FH \sum_k (k-1)(S_{jk}^s - S_{ik}^s) - \gamma_i \quad \forall (i, j) \in \zeta, N_{ij} = 0 \quad (18)$$

The vertical separation distance between the two items  $i$  and  $j$  can then be evaluated as the maximum of  $Up_{ij}$  and  $Dn_{ij}$ . This is determined using the following linear re-formulation:

$$VD_{ij} \leq FH \sum_k (k-1)(S_{jk}^s - S_{ik}^s) - \gamma_i + \eta_{ij}^u + BM \cdot N_{ij} \quad \forall (i, j) \in \zeta \quad (19)$$

$$VD_{ij} \geq FH \sum_k (k-1)(S_{jk}^s - S_{ik}^s) - \gamma_i + \eta_{ij}^d - BM \cdot N_{ij} \quad \forall (i, j) \in \zeta \quad (20)$$

$$VD_{ij} \leq BM \cdot (1 - N_{ij}) \quad \forall (i, j) \in \zeta \quad (21)$$

where  $\eta_{ij}^u$  and  $\eta_{ij}^d$  are positive variables such that:

$$\eta_{ij}^u - \eta_{ij}^d = 2FH \sum_k (k-1)(S_{ik}^s - S_{jk}^s) + \gamma_i - \gamma_j \quad \forall (i, j) \in \zeta \quad (22)$$

$$\eta_{ij}^u \leq BM \cdot W_{ij}^z \quad \forall (i, j) \in \zeta \quad (23)$$

$$\eta_{ij}^d \leq BM \cdot (1 - W_{ij}^z) \quad \forall (i, j) \in \zeta \quad (24)$$

Equations (19) - (21) ensure  $VD_{ij}$  only takes a non-zero value when items  $i$  and  $j$  are on different floors.

Finally, the total safety distance between equipment items  $i$  and  $j$  is then calculated as the Tchebychev distance (eq. (25)) between the equipment boundaries of such items in all x-, y- and z-planes. This distance metric is selected because it is a better estimate to the Euclidean distance when compared to the rectilinear distance between equipment item boundaries without the complexities of a non-linear term (Ejeh et al., 2021).

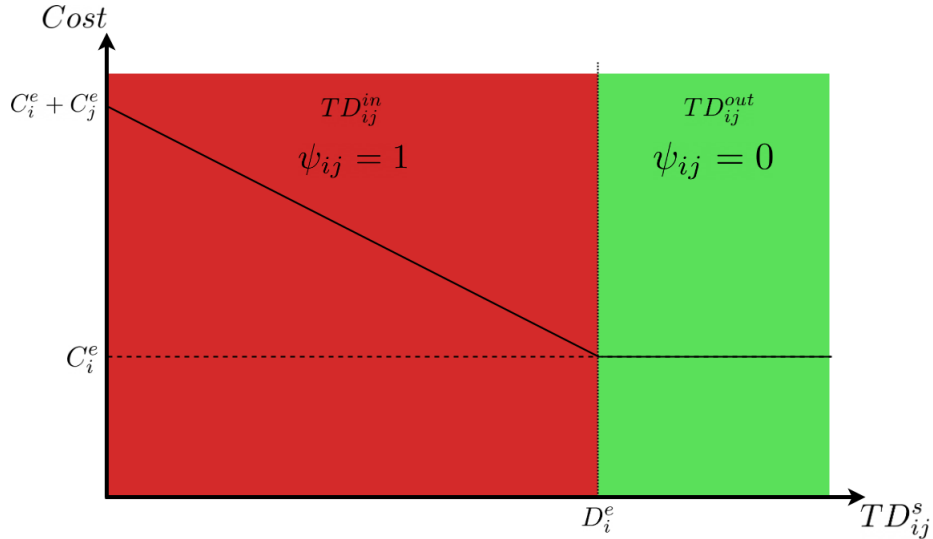
$$TD_{ij}^s = \max(XD_{ij}, YD_{ij}, VD_{ij}) \quad \forall (i, j) \in \zeta \quad (25)$$

A linear re-formulation of eq. (25) is given by eqs. (A.35) - (A.48) in Appendix A.

### 3.3. Area of exposure constraints

In order to calculate the value of area of exposure, it needs to be determined if an item  $j$  is within the distance of exposure of a pertinent item  $i$  (Figure 4). The distance of exposure ( $D_i^e$ ) is the region within which a secondary item  $j$  will be affected by a fire and explosion incident originating from a pertinent item  $i$ . This can be calculated according to the procedure described in Figure 1 and eq. (26) below (Patsiatzis et al., 2004):

$$D_i^e = 0.256 \cdot FEI_i \quad \forall (i, j) \in \zeta \quad (26)$$



**Figure 4: Damage cost versus distance between items  $i$  and  $j$**

The total safety distance is then expressed as in eq. (27), where  $\psi_{ij}$  is a binary variable that denotes if a secondary unit  $j$  is within the area of exposure

of pertinent item  $i$ . To ensure only one of  $TD_{ij}^{in}$  and  $TD_{ij}^{out}$  is non-zero, eqs. (28) - (30) are introduced.  $TD_{ij}^{in}$  takes the value of the total distance if item  $j$  is within the distance of exposure ( $D_i^e$ ) of item  $i$ , and  $TD_{ij}^{out}$  if not.

$$TD_{ij}^s = TD_{ij}^{in} + TD_{ij}^{out} \quad \forall i \in I^{pe}, j \neq i \quad (27)$$

$$TD_{ij}^{in} \leq D_i^e \cdot \psi_{ij} \quad \forall i \in I^{pe}, j \neq i \quad (28)$$

$$TD_{ij}^{out} \geq D_i^e \cdot (1 - \psi_{ij}) \quad \forall i \in I^{pe}, j \neq i \quad (29)$$

$$TD_{ij}^{out} \leq BM \cdot (1 - \psi_{ij}) \quad \forall i \in I^{pe}, j \neq i \quad (30)$$

The value of area of exposure (Patsiatzis et al., 2004) is then determined as:

$$V_i^e = C_i^e + \sum_{j \neq i} \left( C_j^e \cdot \psi_{ij} - \frac{C_j^e}{D_i^e} \cdot TD_{ij}^{in} \right) \quad \forall i \in I^{pe} \quad (31)$$

#### 3.4. Maximum probable property damage constraints

The base maximum probable property damage (MPPD) is calculated by eq. (32). It represents the financial losses incurred in a plant as a result of an accident occurring at a pertinent item  $i$  and propagating to all neighbouring items  $j$  within its distance of exposure.

$$\Omega_i^0 = DF_i \cdot V_i^e \quad \forall i \in I^{pe} \quad (32)$$

However, if a protective device is installed on a pertinent item  $i$ , the probability and magnitude of such accident is reduced by a factor referred to as the loss control credit factor (LCCF),  $CF_{ip}$ . Each configuration for a protective device ( $P_i$ ) is thus characterised by a LCCF value representing such reduction. In cost terms, the base MPPD is then re-evaluated as the actual MPPD cost ( $\Omega_i$ ) and represents the financial risk to the plant:

$$\Omega_i = \sum_{p \in P_i} CF_{ip} \cdot \Omega_i^0 \cdot \mu_{ip} \quad \forall i \in I^{pe} \quad (33)$$

Equation (33) is linearised by eqs. (34) - (37) below:

$$\Omega_i = \sum_{p \in P_i} CF_{ip} \cdot \Omega'_{ip} \quad \forall i \in I^{pe} \quad (34)$$

$$\Omega'_{ip} \leq BM \cdot \mu_{ip} \quad \forall i \in I^{pe}; p \in P_i \quad (35)$$

$$\sum_{p \in P_i} \Omega'_{ip} = \Omega_i^0 \quad \forall i \in I^{pe} \quad (36)$$

$$\sum_{p \in P_i} \mu_{ip} = 1; \quad \forall i \in I^{pe} \quad (37)$$

### 3.5. Objective function

The two objective functions are defined below. The total monetary cost ( $f^M$ ) is given by eq. (38);

$$\begin{aligned}
 f^M = & \sum_{i,j:f_{ij}=1} (C_{ij}^c T D_{ij}^c + C_{ij}^v D_{ij} + C_{ij}^h (R_{ij} + L_{ij} + A_{ij} + B_{ij})) \\
 & + FC1 \cdot NF + FC2 \sum_s AR_s \cdot NQ_s + LC \cdot FA \\
 & + \sum_{i,p \in P_i} C_{ip}^p \cdot \mu_{ip}
 \end{aligned} \tag{38}$$

and the financial risk ( $f^R$ ) by eq. (39):

$$f^R = \sum_i \Omega_i \tag{39}$$

The multi-objective model is solved using the  $\epsilon$ -constraint method by converting one of the two objectives into a constraint, setting an upper bound ( $\epsilon^S$ ). In this case, an upper bound is set on the financial risk. The  $\epsilon$ -constraint method (Haimes et al., 1971) is probably the best known technique for multi-criteria optimisation problems (Chiandussi et al., 2012) and has been used in literature for similar problems (Almaraz et al., 2016, Liu and Papageorgiou, 2013, Zhang et al., 2016). Drawbacks do exist for the  $\epsilon$ -constraint method such as the calculation of the nadir values and the guarantee of efficient solution generation, however Bérubé et al. (2009) showed that efficient solutions for bi-objective optimisation problems can be found using this method. The reformulated multi-objective model for the case at hand thus becomes:

$$\begin{aligned}
 \min & \quad f^M \\
 \text{s.t.} & \quad f^R \leq \epsilon^S
 \end{aligned}$$

subject to (1) - (15), (17) - (24), (27) - (32), (34) - (37), and (A.1) - (A.48). This constitutes model PPL\_FEI.

## 4. Case Study

The model proposed was applied to an ethylene oxide (EO) plant (Figure 5) (Patsiatzis et al., 2004). Data on the equipment item geometry, costs, connections and sets of protection device are available in Appendix B. Six protection device configurations are made available for installation, with the first configuration having no protection device purchased or installed. The model was solved until a 0% relative gap was achieved using GAMS modelling system v26.1.0 (GAMS Development Corporation, 2019) with the CPLEX v12.8 MILP solver on an Intel® Xeon® E5-1650 CPU with 32GB RAM.



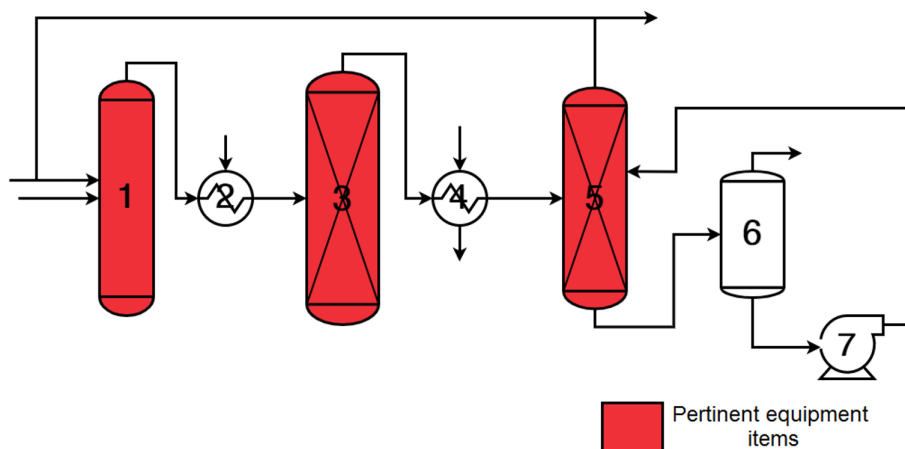


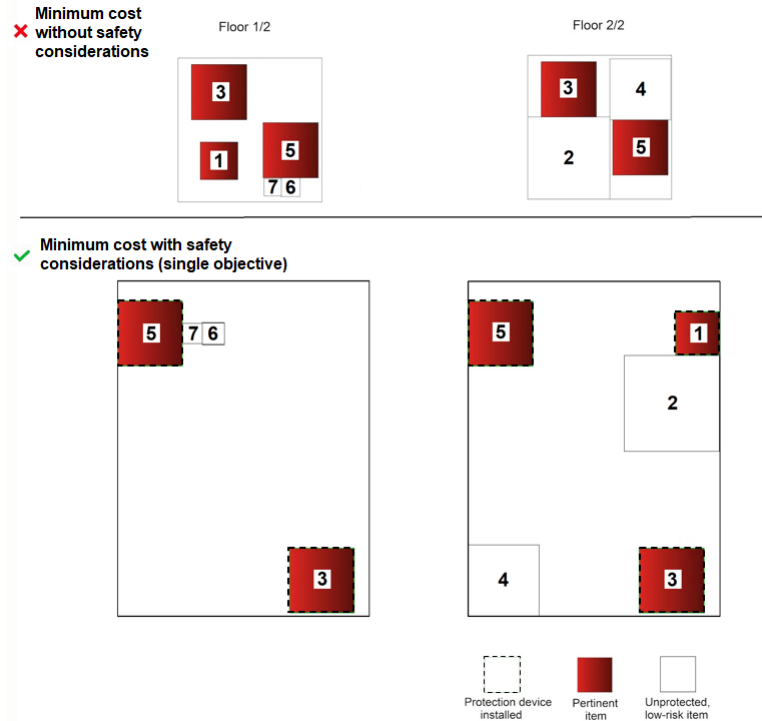
Figure 5: Flow diagram of ethylene oxide plant

The case study above was first solved without safety considerations (using the MILP model proposed by Ejeh et al. (2019b)) and with safety considerations using model PPL\_FEI but minimising the sum of both objective functions. Figure 6 shows the layout results for both cases. Table 3 shows the model statistics and computational results for the EO plant for both cases without safety and with a single objective solve. The single objective solve minimises the total layout, actual MPPD and protection device installation costs. For such case, the optimal solution was obtained in 23.4s having 2 floors with a larger floor area of 30m× 40m and a smaller total cost of 445,660 rmu. Protection devices were selected for each pertinent item costing 135,000 rmu. The green borders over equipment item indicate that protection devices were installed. Each of these devices served to reduce the probability, magnitude and impact of a fire and explosion event on each of these items, resulting in a lower financial risk value of 184,995 rmu.

Table 3: Layout results, model statistics & computational performance

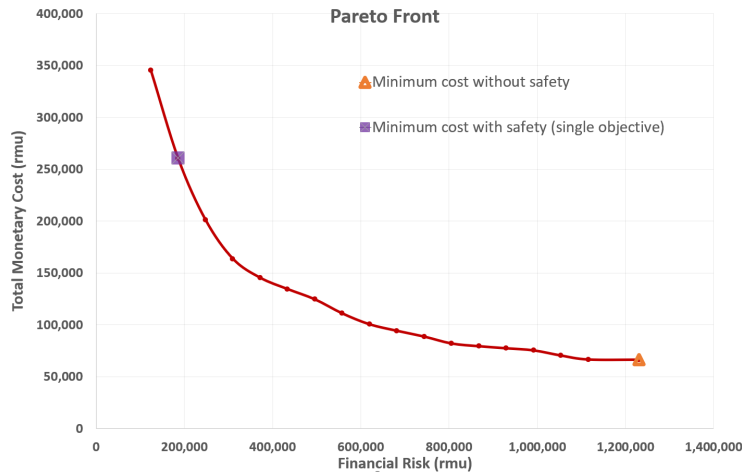
	Minimum cost without safety	Minimum cost with safety (single objective)
Layout cost (rmu)	66,262	125,665
Protection device cost (rmu)	0.0	135,000
Financial risk (rmu)	1,231,128	184,995
<b>Total cost (rmu)</b>	<b>1,297,390</b>	<b>445,660</b>
CPU (s)	0.8	23.4
Number of discrete variables	92	296
Number of continuous variables	157	450
Number of equations	392	1154

Figure 6 shows the optimal layout plot for both cases. The plot, in addition to the cost results in Table 3, establishes the fact that the financial risk, and by implication the total cost, was significantly reduced by the installation of protection devices and increased inter-equipment spacing without the need for additional floors for layout. The financial risk was reduced by about seven times the value to 184,995 rmu.



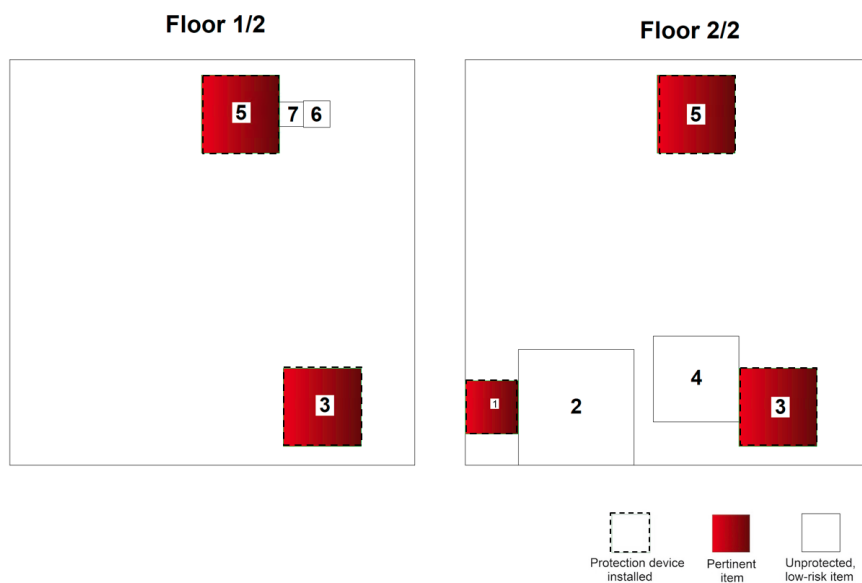
**Figure 6: Layout plant of ethylene oxide plant without and with safety considered as a single objective**

The bi-objective model (PPL\_FEI) was then solved using the  $\epsilon$ -constraint method to better explore the compromises between the total capital/monetary cost ( $f^M$ ) and the financial risk ( $f^R$ ). 20 equi-distance values were used for  $\epsilon$  between 0 and 1,240,000 rmu based on the previously obtained results. Figure 7 shows the Pareto-optimal results for the EO plant.



**Figure 7: Pareto Front: Ethylene oxide plant**

Indications are made on the figure for the two previously discussed solutions. On the far right, where no safety constraints are considered, a configuration with minimum layout costs was achieved of 66,262 rmu with no protection device installed. The purple square indicates the solution of the single objective solve with safety considerations. This solution represents the minimum total cost achievable for the plant considering both the monetary costs and financial risk simultaneously in a single objective model. However, a layout configuration (with protection devices) having a lower financial risk can still be achieved as seen from the Pareto front. Figure 8 shows the layout of the plant with a 40m×40m floor area corresponding to the solution with minimum financial risk. This knowledge is important, as though minimum total costs may be achieved from the solution indicated by the purple square, more risk averse decision makers are given the option of an additionally safer layout configuration and better understand the compromises to be taken. This is as the financial risk, calculated as the maximum probable property damage to equipment items within the plant, may not have equal interpretations by all decision makers. The Pareto-front also shows that the financial risk can be significantly reduced with relatively insignificant increase in monetary commitments.



**Figure 8: Ethylene oxide plant layout with minimum financial risk**

Figure 9 gives a cost breakdown of the total monetary cost for each Pareto-optimal solution. The construction cost dominated in most of the solutions, with the pumping cost following. The protection device cost became much more dominant at lower financial risk values. This also confirms that for the EO plant considered, it was much cheaper to reduce the financial risk by layout re-configuration than by protection device installation. This may not always be the case for all chemical plants as such values will depend on the location of the plant site. However, this knowledge is a useful tool in decision making at the planning stage of the chemical plant, as more resources can be given to purchase a bigger land area or additional protection devices. The combination of both measures also reduces the total financial risk as expected.

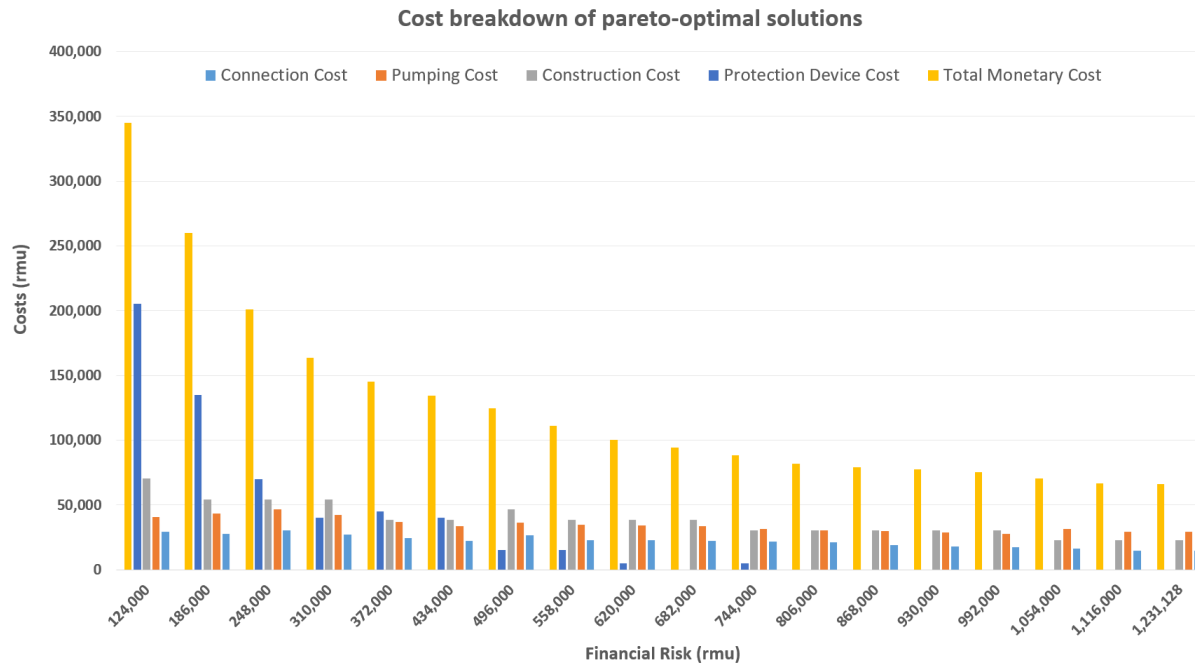


Figure 9: Cost breakdown of Pareto optimal solutions

## 5. Concluding remarks

In this work we presented a multi-objective MILP model for the evaluation of safe multi-floor chemical process plant layout configurations using the widely applied Dow's Fire & Explosion Index (F&EI). Using properties of process fluids within equipment items in the plant, this index determines the potential economic risk a unit poses to itself and neighbouring structures. This information was embedded within an MILP in order to determine the set of Pareto-optimal solutions for an ethylene oxide plant considering two objectives - the total monetary cost and financial risk - using the  $\epsilon$ -constraint method.

Findings showed that solving the problem with a single objective obtained the minimal monetary equivalent for layout costs, protection device costs and financial risk to process plant equipment. However, background information for a better understanding of the comprise between safety and capital costs were lacking. This was achieved using the multi-objective model, which showed that the safety levels can be significantly improved through cost effective layout reconfigurations first. Additional improvements can also be achieved via the installation of protection devices, at a cost. These are all important information at the planning/design stages of a chemical process plant which can lead to safer, yet cheaper layout designs.

Possible areas for future work will include extending existing single or multi-objective optimisation approaches to consider additional hazards of toxic release, and other hazard scenarios not addressed by the F&EI procedure.

## Acknowledgement

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## Appendix A. Additional equations to MILP model

In addition to the constraints outlined in the main sections, the following equations directly obtained from Egeh et al. (2021) are included in the proposed model as described below:

### A.1. Floor constraints

Floor constraints ensure that every non-multi-floor equipment  $i$  is assigned to one floor:

$$\sum_k V_{ik} = 1 \quad \forall i \notin I^T \quad (\text{A.1})$$

Tall/multi-floor equipment items ( $i \in I^T$ ) are assigned to a set of consecutive floors, with the requirement that only the starting floor be available for layout:

$$\sum_k V_{ik} = M_i - \omega_i \quad \forall i \in I^T \quad (\text{A.2})$$

where:

$$\omega_i \geq \sum_k k \cdot S_{ik}^S + M_i - |K| - 1 \quad \forall i \in I^T \quad (\text{A.3})$$

A floor may only be selected, and costed, for layout only if an equipment starts on it:

$$S_{ik}^S \leq W_k \quad \forall i, k \quad (\text{A.4})$$

or the floor above it is also occupied:

$$W_k \leq W_{k-1} \quad \forall k > 1 \quad (\text{A.5})$$

The minimum number of floors required is then given by:

$$NF \geq \sum_k W_k \quad (\text{A.6})$$

### A.2. Multi-floor equipment constraints

Equipment item floor positions are determined with the constraints below:

$$V_{ik} = \sum_{\theta=1}^{M_i} \delta_{i\theta} \cdot S_{i,k-\theta+1}^S \quad \forall i, k \quad (\text{A.7})$$

where  $V_{ik}$  is a binary variable which determines if an equipment  $i$  is assigned to floor  $k$  and  $\delta_{i\theta} = 1$  for all  $\theta \leq M_i$ .  $S_{ik}^S$  is a binary variable that determines if an equipment item  $i$  starts at floor  $k$ . This should only occur on one floor:

$$\sum_k S_{ik}^S = 1 \quad \forall i \quad (\text{A.8})$$

These constraints also ensure tall units occupy consecutive floors.

### A.3. Equipment orientation constraints

A 90° rotation of equipment orientation is allowed in the x-y plane:

$$l_i = \alpha_i O_i + \beta_i (1 - O_i) \quad \forall i \quad (\text{A.9})$$

$$d_i = \alpha_i + \beta_i - l_i \quad \forall i \quad (\text{A.10})$$

### A.4. Non-overlapping constraints

To prevent equipment overlap on the same floor, constraints (A.11) - (A.14) are used:

$$x_i - x_j + BM \cdot (1 - N_{ij} + E1_{ij} + E2_{ij}) \geq \frac{l_i + l_j}{2} + De_{ij}^{min} \quad \forall i, j > i \quad (\text{A.11})$$

$$x_j - x_i + BM \cdot (2 - N_{ij} - E1_{ij} + E2_{ij}) \geq \frac{l_i + l_j}{2} + De_{ij}^{min} \quad \forall i, j > i \quad (\text{A.12})$$

$$y_i - y_j + BM \cdot (2 - N_{ij} + E1_{ij} - E2_{ij}) \geq \frac{d_i + d_j}{2} + De_{ij}^{min} \quad \forall i, j > i \quad (\text{A.13})$$

$$y_j - y_i + BM \cdot (3 - N_{ij} - E1_{ij} - E2_{ij}) \geq \frac{d_i + d_j}{2} + De_{ij}^{min} \quad \forall i, j > i \quad (\text{A.14})$$

To prevent the exclusion of valid optimal solutions:

$$BM \geq \max_s (\bar{X}_s, \bar{Y}_s) + \max_{ij} (De_{ij}^{min}) \quad (\text{A.15})$$

### A.5. Distance constraints

Distance constraints described by eqs. (A.16) - (A.19) determine the distances in the x and y planes respectively for connected equipment items ( $I^c$ ).

$$R_{ij} - L_{ij} = x_i - x_j \quad \forall (i, j) \in I^c \quad (\text{A.16})$$

$$A_{ij} - B_{ij} = y_i - y_j \quad \forall (i, j) \in I^c \quad (\text{A.17})$$

$$U_{ij} - D_{ij} = FH \sum_k (k-1) (S_{ik}^S - S_{jk}^S) + OP_{ij} - IP_{ij} \quad \forall (i, j) \in I^c \quad (\text{A.18})$$

Provision is made for connection between equipment  $i$  and  $j$  at design-specified heights of either equipment items (eq. (A.18)). The total connection distance is then evaluated as follows:

$$TD_{ij}^c = R_{ij} + L_{ij} + A_{ij} + B_{ij} + U_{ij} + D_{ij} \quad \forall (i, j) \in I^c \quad (\text{A.19})$$



### A.6. Layout design constraints

Eqs. (A.20) - (A.23), ensure that equipment are placed within the boundaries of a floor:

$$x_i + \frac{l_i}{2} \leq X^{max} \quad \forall i \quad (\text{A.20})$$

$$y_i + \frac{d_i}{2} \leq Y^{max} \quad \forall i \quad (\text{A.21})$$

with a lower bound:

$$x_i \geq \frac{l_i}{2} \quad \forall i \quad (\text{A.22})$$

$$y_i \geq \frac{d_i}{2} \quad \forall i \quad (\text{A.23})$$

### A.7. Area Constraints

In order to avoid bilinear terms in calculating the floor area,  $FA$ , eqs. (A.24) - (A.28) are introduced. The area of each floor is determined from a set  $S$  of predefined rectangular area sizes,  $AR_s$ , with dimensions  $(\bar{X}_s, \bar{Y}_s)$ .

$$FA = \sum_s AR_s Q_s \quad (\text{A.24})$$

$$\sum_s Q_s = 1 \quad (\text{A.25})$$

The floor length and breadth is selected from the chosen rectangular area size dimensions:

$$X^{max} = \sum_s \bar{X}_s Q_s \quad (\text{A.26})$$

$$Y^{max} = \sum_s \bar{Y}_s Q_s \quad (\text{A.27})$$

Also, a new term  $NQ_s$  is introduced in order to linearise the cost term associated with the number of floors:

$$NF = \sum_s NQ_s \quad (\text{A.28})$$

$$NQ_s \leq |K| \cdot Q_s \quad \forall s \quad (\text{A.29})$$

### A.8. Symmetry breaking constraints

To reduce the occurrence of symmetric solutions, the following constraints are introduced:

$$x_i + y_i - x_j - y_j \geq \sigma \cdot N_{ij} \quad \forall (i, j) = \arg \max_{i \in I^T, j \in I^T} C_{ij}^c \quad (\text{A.30})$$

$$E1_{ij} = 0 \quad \forall (i, j) = \arg \max_{i \in I^T, j \in I^T} C_{ij}^c \quad (\text{A.31})$$

### A.9. Integer cut

The following integer cuts were applied to the model to reduce the solution space by eliminating unrealistic overlap considerations:

$$\frac{E1_{in} + E2_{in}}{2} \geq E1_{ij} + E2_{ij} + E1_{jn} + E2_{jn} - 3 \quad \forall i < j < n \quad (\text{A.32})$$

$$N_{ij} \geq E1_{ij} \quad \forall i, j > i \quad (\text{A.33})$$

$$N_{ij} \geq E2_{ij} \quad \forall i, j > i \quad (\text{A.34})$$

### A.10. Tchebychev distance

The Tchebychev distance for the safety distance between two equipment items is given as:

$$TD_{ij}^s = \max(XD_{ij}, YD_{ij}, VD_{ij}) \quad \forall (i, j) \in \zeta$$

The above expression can be re-written as:

$$\max(\max(XD_{ij}, YD_{ij}), VD_{ij})$$

which is linearised as follows.

The first part,  $XY_{ij}^{max} = \max(XD_{ij}, YD_{ij})$ , is linearised using eqs. as follows:

$$XY_{ij}^{max} \leq XD_{ij} + BM \cdot (1 - MB_{ij}^{XY}) \quad \forall (i, j) \in \zeta \quad (\text{A.35})$$

$$XY_{ij}^{max} \geq XD_{ij} - BM \cdot (1 - MB_{ij}^{XY}) \quad \forall (i, j) \in \zeta \quad (\text{A.36})$$

$$XY_{ij}^{max} \leq YD_{ij} + BM \cdot MB_{ij}^{XY} \quad \forall (i, j) \in \zeta \quad (\text{A.37})$$

$$XY_{ij}^{max} \geq YD_{ij} - BM \cdot MB_{ij}^{XY} \quad \forall (i, j) \in \zeta \quad (\text{A.38})$$

$$XD_{ij} \geq YD_{ij} - BM \cdot (1 - MB_{ij}^{XY}) \quad \forall (i, j) \in \zeta \quad (\text{A.39})$$

$$YD_{ij} \geq XD_{ij} - BM \cdot MB_{ij}^{XY} \quad \forall (i, j) \in \zeta \quad (\text{A.40})$$

$$XY_{ij}^{max} \leq BM \cdot (W_{ij}^{XO} + W_{ij}^{YO}) \quad \forall (i, j) \in \zeta \quad (\text{A.41})$$

where  $MB_{ij}^{XY}$  is a binary variable equal to 1 when  $XD_{ij} \geq YD_{ij}$ . Integer cuts are included to select  $YD_{ij}$  if  $XD_{ij}$  is zero (eqs. (A.42)), and  $XD_{ij}$  if both  $XD_{ij}$  and  $YD_{ij}$  are zero (eqs. (A.43)):

$$MB_{ij}^{XY} \leq W_{ij}^{XO} + W_{ij}^{YO} \quad \forall (i, j) \in \zeta \quad (\text{A.42})$$

$$W_{ij}^{XO} \geq MB_{ij}^{XY} \quad \forall (i, j) \in \zeta \quad (\text{A.43})$$

The second part of eq. (25),  $\max(XY_{ij}^{max}, VD_{ij})$  is linearised as:

$$TD_{ij}^s = XY_{ij}^{max} + ZD_{ij}^+ \quad \forall (i, j) \in \zeta \quad (\text{A.44})$$

$$ZD_{ij}^+ - ZD_{ij}^- = VD_{ij} - XY_{ij}^{max} \quad \forall (i, j) \in \zeta \quad (\text{A.45})$$

$$ZD_{ij}^+ \leq BM \cdot MB_{ij}^Z \quad \forall (i, j) \in \zeta \quad (\text{A.46})$$

$$ZD_{ij}^- \leq BM \cdot (1 - MB_{ij}^Z) \quad \forall (i, j) \in \zeta \quad (\text{A.47})$$

$$MB_{ij}^Z \leq 1 - N_{ij} \quad \forall (i, j) \in \zeta \quad (\text{A.48})$$

where  $MB_{ij}^Z$  is a binary variable indicating when  $VD_{ij} \geq XY_{ij}^{max}$ .

## Appendix B. Data for case study

The dimensions of equipment items in the ethylene oxide plant are shown in Table B.1. Tables B.2, B.3, B.4 and B.5 show the key parameters for the pertinent equipment items, and the protection device configurations and data for the Reactor and Absorbers respectively. These values were obtained according to the F&EI procedure shown in Figure 1 and outlined in detail in Patsiatzis et al. (2004) for the case study in question. Table B.6 shows the connection cost, connection heights, horizontal and vertical pumping costs, as well as other required data.

**Table B.1: Equipment dimensions & purchase cost for ethylene oxide plant**

Equipment item	Description	$\alpha_i$ (m)	$\beta_i$ (m)	$\gamma_i$ (m)	Purchase cost (€)
1	Reactor	5.22	5.22	4.50	335,000
2	Heat exchanger 1	11.42	11.42	2.21	11,000
3	Ethylene oxide absorber	7.68	7.68	7.42	107,000
4	Heat exchanger 2	8.48	8.48	2.21	4,000
5	CO <sub>2</sub> absorber	7.68	7.68	6.40	81,300
6	Flash tank	2.6	2.6	3.50	5,000
7	Pump	2.4	2.4	1.20	1,500

**Table B.2: Pertinent item system factors; EO plant**

Equipment item	Description	Material factors	$D_i^e$	$DF_i$
1	Reactor	29	40	0.87
3	Ethylene oxide absorber	29	21.8	0.73
5	CO <sub>2</sub> absorber	24	18.06	0.66

**Table B.3: Protection device configurations (Patsiatzis et al., 2004)**

Device	Configuration type
1	Additional cooling water
2	Additional overpressure relief devices
3	Additional fire relief devices
4	Second skin on reactor
5	Explosion protection system of reactor
6	Duplicate control system with interlocking flow on reactor
7	Duplicate control shutdown system on absorption tower

**Table B.4: Protection device data for the Reactor, item 1**  
(Patsiatzis et al., 2004)

Configuration ( $p$ )	Device	$CF_{ip}$	$C_{ip}^p$
1	–	1	0
2	1	0.900	5,000
3	3	0.750	15,000
4	1,3,6	0.365	40,000
5	1,3,5,6	0.292	60,000
6	1,3,4,5,6	0.117	125,000

$CF_{ip}$  - loss control credit factor of protection device configuration  $p$  for item  $i$

$C_{ip}^p$  - purchase and installation cost of protection device configuration  $p$  for item  $i$

**Table B.5: Protection device data for the ethylene oxide, item 3,**  
**and CO<sub>2</sub>, item 5, absorber** (Patsiatzis et al., 2004)

Configuration ( $p$ )	Device	$CF_{ip}$	$C_{ip}^p$
1	–	1	0
2	1	0.900	5,000
3	2	0.760	20,000
4	1,2	0.684	25,000
5	1,7	0.612	35,000
6	1,2,7	0.465	55,000

$CF_{ip}$  - loss control credit factor of protection device configuration  $p$  for item  $i$

$C_{ip}^p$  - purchase and installation cost of protection device configuration  $p$  for item  $i$

**Table B.6: Parameters for the ethylene oxide plant**

(a) Connection and pumping costs, and connection heights

Connection	$C_{ij}^c$ (rmu/m)	$C_{ij}^h$ (rmu/m)	$C_{ij}^v$ (rmu/m)	$OP_{ij}$ (m)	$IP_{ij}$ (m)
1.2	200	400	4000	4.5	1.11
2.3	200	400	4000	1.11	3.71
3.4	200	300	3000	7.42	1.11
4.5	200	300	3000	1.11	3.2
5.1	200	100	1000	6.40	2.25
5.6	200	200	2000	0.0	1.75
6.7	200	150	1500	0.0	0.60
7.5	200	150	1500	1.20	4.80

**(b) Other Parameters**

Parameters	Value
$K$	3
$FC1$ (rmu)	3,330
$FC2$ (rmu/m <sup>2</sup> )	6.6
$LC$ (rmu/m <sup>2</sup> )	26.6
$FH$ (m)	5

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