Conceptualising and operationalising socio-mathematical agency

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In this paper, I introduce a new theoretical construct of 'socio-mathematical agency' (SMA), which I define as the ability to use mathematics effectively to argue collectively for social change. I present a conceptualisation of SMA which embraces the need to generate powerful mathematical knowledge, and which draws on critical mathematics education in foregrounding the need to attend to learners' individual and collective agency. I propose that developing SMA in learning mathematics can make a significant contribution towards cultivating the collective knowledge and critical understanding needed to address environmental, economic and social challenges facing global society. I present some suggestions for how SMA might be operationalized in the classroom, which I hope will generate further debate about the efficacy and possible future development of SMA.

Keywords: Socio-mathematical agency, powerful knowledge, student agency, social justice.

Introduction: What is 'socio-mathematical agency' and why is it needed?

In this paper, I introduce a new theoretical construct of 'socio-mathematical agency' (SMA), which I define as the ability to use mathematics effectively to argue collectively for social change. The need to develop SMA amongst students is highlighted by recent calls from intergovernmental educational policy-making organisations for a more humanistic school curriculum that cultivates the collective knowledge and critical understanding needed to address the environmental, economic and social challenges facing global society (Organisation for Economic Co-operation and Development [OECD], 2018; United Nations Educational, Scientific and Cultural Organization [UNESCO], 2015). I propose that focusing on the development of SMA in learning mathematics can make a significant contribution towards these aims. The benefits of SMA are exemplified by the impact of a report by the Imperial College COVID-19 Response Team (2020), released soon after the initial outbreak of the pandemic, which used mathematical modelling of coronavirus infections to predict over half a million deaths in the UK if existing precautions were not strengthened. The report led to a significant change in public attitudes towards the virus, prompting the UK Government to introduce additional measures, including social distancing, which prevented an even greater death toll (Skovsmose, 2021).

Mathematical skills are widely recognised as essential for solving real life problems. There is growing consensus among mathematics education researchers that, to develop powerful mathematical knowledge (needed to solve real life problems), students need to be given opportunities to experience processes mathematicians go through in generating new knowledge (Mason et al., 1985; Schoenfeld, 2012). These include working collaboratively (most new knowledge is generated by mathematicians working in teams), posing questions, conjecturing, reasoning, explaining, justifying. Gutstein (2006) argues that engaging in complex mathematical tasks, through a curriculum that emphasises reasoning, communication and problem-solving, is essential for the empowerment of mathematics learners.

The recent Covid-19 pandemic has highlighted how understanding mathematical concepts, such as exponential growth and moving averages, is necessary for individuals to make sense of data presented

in the media and hence arrive at rational decisions about behaviours that take account of the risks to health. Mathematical knowledge is important in helping generate a wider understanding of the many crises currently facing humanity. Coles et al. (2013) highlight how gaining an appreciation of the shape of a normal distribution curve, and how this relates to the mean and standard deviation, can help to explain why global warming is also associated with more extreme cold spells. However, mathematics does not always serve the public good. Skovsmose (2021) highlights how mathematics sometimes (as in the examples presented above) helps to 'picture' a crisis, but at other times can actually 'constitute' a crisis, e.g. in situations where complex mathematical algorithms have led to a collapse in the stock market or to automated reactions of an aeroplane that prevent the pilot from averting a disaster. Mathematics can also 'format' a crisis, e.g. the choice of mathematical models we use to predict climate change can influence how we interact with the climate in future. Careful thought therefore needs to be given to the type of mathematical knowledge, skills, behaviours and dispositions that learners need to develop if they are to make effective use of mathematics to advocate and bring about future social change for the public good.

Theoretical framework: Conceptualising socio-mathematical agency

The content of school curricula has invariably proved contentious, which is hardly surprising given the competing ideological perspectives of those different interest groups involved in their development (Wright, 2012). Even amongst those who champion the empowerment of learners, there is disagreement over how this should be achieved. Social realists, such as Muller and Young (2019), claim that some types of knowledge, particularly those which are formal and specialised, are inherently powerful. They draw on Bernstein's (2000) contention that abstract knowledge is powerful in that it can extend learners' horizons by allowing them to think 'the unthinkable' and the 'not-yet-thought'. It should be noted that Muller and Young's view of 'powerful knowledge' includes an appreciation of 'disciplinary meaning' (how new knowledge is generated within the discipline) as well as understanding abstract concepts, thus endorsing the generally accepted notion of powerful mathematical knowledge described in the Introduction. Muller and Young (2019) blame the involvement of politicians in curriculum-making for an increasing tendency to prioritise the nurturing of skills and competences that are seen as contributing towards economic growth at the expense of powerful knowledge. They claim that knowledge is consequently viewed primarily as an individual asset, rather than for the public good.

In contrast, critical realists argue that abstract knowledge alone should not be considered powerful, since its power largely depends on the agency of the learner (Manyukhina & Wyse, 2019). Given the possibility that mathematics can constitute or format a crisis (Skovsmose, 2021), particular attention must be given to developing learners' agency, to ensure that mathematical knowledge is used for the public good. Before going further, it is important to clarify what is meant by 'public good' and 'social justice' in this paper. I draw here on Tawney's (1964, cited in Reay, 2012) notion of 'the good society' which strives to eliminate all forms of special privilege within education and within society more widely. A socially-just society is one based around cohesion and solidarity in which individuals share a common interest and treat others in the same way they would like to be treated themselves.

Similarly, a socially-just education system is one which aims to secure for all children what a wise parent would seek for their own child.

Recent events, such as 2020 US Presidential election and the Covid-19 pandemic, have raised awareness of how misleading statistics and media reports can influence the voting habits and behaviour of millions of people (Alderson, 2020). This, in turn, has refocused attention on the school curriculum as a means of fostering the type of critical understanding and collective knowledge needed to promote human rights, equality and social justice (UNESCO, 2015), and to address the social, economic and environmental challenges facing global society (OECD, 2018). Those who misrepresent powerful mathematical knowledge as purely abstract and apolitical and ignore the role that agency plays in empowering learners may be diverting attention away from tackling these challenges. Locating the power required to advance social justice primarily within abstract knowledge is an 'epistemic fallacy', which ignores the reality that such power is dependent on the agency of the 'knower' and rests on the false assumption that school is a level playing field (Alderson, 2020).

Manyukhina and Wyse (2019) describe learners' agency as having two dimensions: 'sense of agency' (a feeling of control over their own learning) and 'agentic behaviour' (exercising control through making decisions and taking actions). Both dimensions need to be present if students are to be empowered as autonomous learners. Instilling learners with a sense of agency is of little use if they are not provided with real opportunities within the curriculum to exercise that agency. Manyukhina and Wyse argue that the structure in which learning takes place and the agency of the learner have mutual causality. Providing a context-sensitive learning environment in which students have space to explore and to be creative helps develop their sense of agency. Conversely, allowing students to exercise their agency and become actively involved in their learning promotes academic achievement and impacts positively on learners' views of themselves and their place in the world. There is a danger that knowledge-based curricula, such as the current National Curriculum in England, which place too much emphasis on acquiring disciplinary knowledge at the expense of shaping learners' identities, neglect the development of the values and attributes students need to contribute towards the public good:

> … it is critical to support young generations in developing the capacity to think critically and independently, engage in autonomous decision-making based on informed choice, and act effectively in a manner that ensures the essential balance between individual and societal interests and priorities. (Manyukhina & Wyse, 2019, p.239)

Skovsmose (2011) highlights how mathematics teaching around the world tends to be dominated by an 'exercise paradigm', in which the teacher presents the solution to a closed mathematical problem on the board before inviting students to complete a series of almost identical problems. Given the status of school mathematics as a gatekeeper qualification, such an approach may be empowering in a pragmatic sense, as it is assumed to help learners acquire the qualifications that they need to access higher-paid employment. However, it is disempowering in a socio-political sense as it stifles opportunities for learners to develop their mathematical agency, i.e. the ability to apply powerful mathematical knowledge in solving real-life problems. Skovsmose proposes an alternative 'critical mathematics education' in which students reflect 'through', 'with' and 'on' mathematics by:

participating in meaningful investigations in which they make their own decisions, pose their own questions, interact and communicate with other learners; carrying out mathematical inquiries which deepen their understanding of their social, cultural, political and economic situations; questioning the nature of mathematics and how it can be used to make decisions affecting them and others.

For mathematical knowledge to be used to advance the public good, consideration needs to be given to collective, as well as individual, mathematical agency. Freire (1974) contends, in his theory of 'education for critical consciousness', that genuine understanding can only be achieved through learners developing a critical awareness of their own situations and how these relate to their studies. From Freire's perspective, the purpose of education should be to meet the collective needs of the community (or society), rather than for individuals to achieve success within the system, through raising awareness of, and challenging, structural inequalities. Emphasis should be placed on mobilising solidarity with those who are marginalised or oppressed and engaging in collective action to challenge exploitation. Freire's (1972) notion of 'praxis', i.e. "reflection and action directed at the structures to be transformed" (p. 96), is used by Gutstein (2006) in proposing a framework for 'reading and writing the world with mathematics', in which students use mathematics to "investigate and critique injustice, and to challenge, in words and actions, oppressive structures and acts" (p. 4). Through generating an understanding of power relations, and how these relate to their own lives and experiences as mathematics learners (both in terms of how they may be exploited themselves as well as being complicit in the exploitation of others), students develop their sense of social agency and self-efficacy, i.e. a belief that they can influence or change society. However, such an approach requires a fundamental shift in students' orientations towards mathematics and in the relationships between mathematics teachers and learners.

In conceptualising SMA, i.e. the ability to use mathematics effectively to argue collectively for social change, I have argued in this section that it is necessary to consider students' development of powerful mathematical knowledge, including an appreciation of disciplinary meaning, as well as their ability to apply this knowledge in solving real-life problems (see elements 1 and 2 below). However, SMA must also involve a readiness of students to use mathematical inquiries to deepen their understanding of exploitative power relationships within society and a disposition towards using mathematical arguments to expose and challenge injustices they encounter (see elements 3 and 4 below). Finally, SMA needs to enable learners to foster a sense of collective agency (see elements 5 and 6 below). Therefore, I propose the following conceptualization of 'socio-mathematical agency' (SMA) which incorporates six elements drawn from the theoretical frameworks presented above (Gutstein, 2006; Manyukhina & Wyse, 2019; Muller and Young, 2019; Skovsmose, 2011):

1) An appreciation of disciplinary meaning (how new knowledge is generated) in mathematics.

2) An ability to apply abstract mathematical concepts in solving meaningful real-life problems.

3) A readiness to use mathematics to explore and develop understanding of social justice issues.

4)A disposition towards using mathematics to expose/challenge exploitation and social injustice.

5) A willingness to work with others in using mathematics to construct an argument for change.

6) Confidence that it is possible to influence society through mathematical argument and action.

Classroom practice: Operationalising socio-mathematical agency

In this section I consider what SMA might look like in the classroom and some strategies/conditions that are likely to promote its development. Firstly, it is worth stressing that I believe that there will always be a place in the mathematics classroom for explaining abstract mathematical concepts and practising routine mathematical procedures. However, there are far more engaging ways of doing this than resorting to the 'exercise paradigm' (described earlier), such as making use of richer tasks that provide opportunities for extensive practice through 'mathematical etudes', which have proved just as effective for attaining procedural fluency (Foster, 2018). Having said that, developing SMA necessitates students engaging regularly with open-ended tasks in which they are given greater control over their own learning through making their own decisions about the direction this will take. They should be encouraged to review their own learning, e.g. by reflecting on any errors they make and non-productive paths they explore along the way. Students should also be provided with regular opportunities to work collaboratively, explain and justify their mathematical reasoning to others, listen to and respect each other's points of view, and appreciate the fallible nature of mathematics in which new knowledge is generated through conjecturing, argumentation and arriving at consensus (Hudson, 2018). Frequent opportunities should also be created for students to generate mathematical models to solve meaningful real-life problems, which involve making simplifying assumptions, choosing which mathematical procedures to apply, and considering the limitations of the solution in relation to the initial assumptions (Schoenfeld, 2012). SMA might then be demonstrated through students discussing these solutions and presenting their findings to others.

Findings from the Teaching Maths for Social Justice (TMSJ) research project (Wright, 2017; 2021) demonstrate students' enthusiasm for exploring social justice issues (such as voting systems, Fairtrade and measures of inequality) in the mathematics classroom. Identifying and building on the strong links that exist between mathematical concepts and social justice issues helps students to develop their understanding of both areas simultaneously and to appreciate the legitimacy of such explorations in the mathematics classroom. These links also provide starting points for teachers to bring social justice issues into the mathematics classroom, whilst navigating the pressures they face in getting through an often-demanding scheme of work. An example of an activity that provides such a starting point is investigating how various methods for counting votes (including Borda Points) can be applied in determining the outcome of an election and then considering which method is 'fairest' (Wright, 2016). Note that Borda Points are based on assigning terms from arithmetic or geometric sequences to different preferences for candidates in an election, which means this activity could easily be attempted as part of a unit of work on sequences (students might go on to explore other types of sequence and consider whether applying these might be 'fairer'). Findings from the TMSJ research project suggest that, as well as developing understanding of social justice issues and related mathematical concepts, such activities also have a positive impact on students' overall engagement with mathematics, as they become more aware of the relevance of the subject to their own lives and society in general (Wright, 2017; 2021).

Developing SMA necessitates going beyond merely exploring social justice issues and requires students to use the increased awareness they gain by doing so to expose and challenge exploitative power relationships. Another activity generated during the TMSJ research project highlighted the

challenges teachers might face in doing this and how it might require a re-evaluation of the relationships between teachers and students. The activity involved investigating the proportion of the price paid for a chocolate bar that goes to the cocoa producers and other parties (retailers, importers, etc.) and comparing differences in these proportions between Fairtrade and non-Fairtrade chocolate. One teacher researcher was initially frustrated following a heated class discussion in which students began to question the validity of Fairtrade, as they considered the 4% of the price for Fairtrade chocolate that went to the producers to be grossly unfair (despite this being eight times as much as for non-Fairtrade chocolate). On reflection, however, the research team saw this as a positive development, with students beginning to challenge and question commonly held assumptions about Fairtrade (which might perhaps be more accurately described as 'less unfair trade'), and subsequently decided to change the title of the activity to 'How fair is Fairtrade?'

In Freire's (1974) terms, the events described above might be interpreted as students moving away from a position of 'naïve' awareness towards one of 'critical' awareness, as they begin to question unequal power relationships within Fairtrade production, which might be indicative of the development of SMA. The role of the teacher is crucial in such situations. Freire would argue that the teacher should adopt a 'radical' perspective by promoting debate and reflection, working with learners to develop critical awareness, and helping them to arrive at their own solutions (rather than imposing her/his own views). This approach complements the adoption of open-ended and collaborative tasks referred to above that aim to develop mathematical agency. Such a 'radical' stance also requires teachers to reflect critically on their own positions of power, their own views of social justice issues and the extent to which they, themselves, may be privileged. Sticking with the same scenario, a 'radical' teacher might further cultivate students' SMA by prompting debate around whether Fairtrade (despite being only slightly less unfair than conventional trade) might still have strategic benefits, e.g. in putting pressure on chocolate companies to make modest improvements to the conditions of producers, and the potential for the collective power of students (as consumers purchasing Fairtrade products) to impact on the lives of producers in less wealthy countries.

Collective agency, which involves students developing confidence that collaborative action can bring about change and working with others in using mathematics to generate powerful arguments, is an essential element of SMA. This was apparent in the 'Making a Change' activity developed during the TMSJ research project (Wright, 2016; 2017; 2021). In this activity, students worked in small groups, to choose an issue of interest to them, research it and develop a mathematical argument to support a change they would like to see made. Finally, they were asked to present their argument to the rest of the class, which prompted further debate amongst students around the full range of social justice issues explored. The activity prompted exceptionally high levels of engagement amongst students, including those who had previously appeared disinterested in the subject. Students were excited by the opportunity to use mathematics to explore an issue that was of particular interest to them, which was something of a novelty for most students.

The role of the teacher is again of crucial importance in such situations, with the need to maintain a balance between allowing students space to follow their own lines of inquiry and providing them with the support needed to develop a powerful mathematical argument. During the 'Making a Change' activity, the support provided by teachers included discussing issues to be considered in collecting

and analysing statistical data on other students' opinions on the chosen issue, and discussing the difference between mathematical and non-mathematical statements, such as "School absence rates have fallen recently" and "The percentage of students with persistent absence (defined as missing at least 15% of school days) fell from 12.5% in 2006/07 to 8.4% in 2011/12" (Wright, 2016, p.45). Encouraging students to plan in advance, and to evaluate their approach after completing such activities, is important for developing SMA. For the 'Making a Change' activity, students might be asked to reflect on questions such as: 'Is your suggestion for change achievable? How effectively did you use mathematics to strengthen your argument? How well did you work together as a group?' Establishing genuine collaboration within groups of students, based on solidarity, trust and assigning value to communal effort (Radford, 2012), is an essential aspect of SMA. Boaler (2008) offers various strategies for encouraging students to respect each other's views and to take responsibility for everyone's learning (related to the notion of 'relational equity'), including allocating group roles and recognising the achievements of all students.

Concluding remarks

In this paper, I have outlined a new theoretical construct of 'socio-mathematical agency' (SMA) and reflected on some classroom practice that provides a useful starting point for its operationalization. I hope this will stimulate debate amongst researchers and curriculum makers around how mathematics teaching can contribute towards the development of the collective knowledge and critical understanding needed for today's learners to address the environmental, economic and social challenges facing global society. I plan to work collaboratively with teachers in conducting research in schools to further develop and refine the conceptualisation and operationalisation of SMA presented in this paper, and to explore the potential of SMA for cultivating the collective knowledge and critical understanding needed to address the environmental, economic and social challenges facing global society. I aim to report on the findings of this research in the near future.

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