

# Energy Efficient Resource Allocation for UCA-based OAM-MIMO System

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**Abstract.** The combination of orbital angular momentum (OAM) and multi-input multi-output (MIMO) is identified as an effective solution to improve energy efficiency (EE) in the next-generation wireless communication. According to the orthogonality of OAM, we adopt uniform circular array (UCA) to establish the transmitter and receiver of the OAM-MIMO system in this paper. Our goal is to maximize the EE of the system whilst satisfying the maximum total transmit power and the minimum capacity requirement of each mode. Due to the inter-interference of different UCA at the same mode, the optimization problem involving the power allocation of modes is non-convex, thus is difficult to solve directly. To tackle this problem, the optimization problem is transformed into two sub-problems by using the fractional programming. Then we develop a dual-layer iteration algorithm where the nonconvex power allocation problem is transformed into a convex problem by exploiting the the first-order Taylor approximation in the inner layer, and the dichotomy is used to update EE in the outer layer. Simulation results confirm the effectiveness of the proposed solution, and demonstrate the superiority of the OAM-MIMO system over the conventional MIMO system from the perspective of EE.

**Keywords:** Energy efficiency (EE) · Orbital angular momentum (OAM) · Multi-input multi-output (MIMO) · Power allocation.

## 1 Introduction

With the phenomenal increase of connected devices, there is the massive growth rate in data traffic and energy consumption in wireless communication. However,

the available spectrum resources are far from enough to support the communication systems with the increasing demand for high data rate. To alleviate this problem, orbital angular momentum (OAM) technology is proposed to improve spectrum efficiency of the wireless communication system [1]. Due to the orthogonality of different OAM modes, OAM can ensure the independence of each channel and increase the degree of freedom of the channel, which is also known as mode division multiplexing [2]. Thus it can greatly improve the transmission rate of point-to-point wireless communication [3]. Recently, OAM has been developed continuously on the way of multiplexing, the transmission distance and the antenna structures of generating OAM [4–6]. Since the antenna structure based on uniform circular array (UCA) is more flexible in the multiplexing of OAM, multiple UCAs are applied to enhance the freedom of the antenna radius and alleviate the divergence of OAM beam [7]. It is proved that the OAM-MIMO system based on UCAs is a potential solution for the future communication system, which has aroused great interest gradually. In [8], the authors studied the transmission characteristics of multiplexing three-OAM-mode based on Butler phase shift and UCA in microwave frequency band. The work in [9] achieved 100 Gbit/s data transmission in the OAM-MIMO multiplexing system based on UCAs in 28 GHz band for the first time, with 11 multiplexed signals and a transmission distance of 10 m. The experience achieved gigabit-class wireless transmission, which enhanced the communication rate.

However, previous research works mostly focused on the optimization of system capacity, which is rarely associated with EE [10–12]. With the requirement of green wireless communication, EE is regarded as an important role in the wireless communication system to balance the total power consumption and the achievable capacity. Motivated by this, the power allocation problem of the OAM-MIMO system based on UCAs is proposed to achieve the maximum EE in this paper. The considered EE optimization problem is nonconvex, due to involving the power allocation of multiple modes on multiple UCAs. To tackle nonconvex optimization problem, we reformulate the objective function into an equivalent subtraction form based on fractional programming accordingly. Based on this, a dual-layer iterative algorithm is proposed, where the power allocation is optimized based on first order Taylor approximation and Lagrange duality method in inner layer, and the optimal EE is obtained by the dichotomy in the outer layer. The simulation results confirm the convergence of the proposed EE maximization algorithm. Besides, we also demonstrate that the proposed EE maximization algorithm can achieve significant performance gain, compared with the traditional point-to-point MIMO algorithm.

## 2 System Model and Problem Formulation

### 2.1 UCA-based OAM-MIMO Model

The OAM-MIMO system with multiple UCAs is considered, as shown in Fig. 1. The azimuthal angle of the  $t$ th transmit antenna and the  $r$ th receive antenna are

denoted by  $\phi_t = \frac{2\pi(t-1)}{T}$  and  $\vartheta_r = \frac{2\pi(r-1)}{R}$  respectively, where  $T$  and  $R$  are the number of antenna. To transmit and receive of the multiplexing OAM beams, the transmitter and receiver consist of the Butler matrix and UCAs. We set the number of multiplexed modes on each UCA as  $L$ , thus the Butler matrix of the  $n$ th UCA at the transmitter is  $\mathbf{o}_n = [e^{jl\phi_t}]^H \in C^{T \times L}$  and the Butler matrix of the  $m$ th UCA at the receiver is  $\mathbf{o}_m^H = [e^{jl\vartheta_r}]^H \in C^{L \times R}$ . Therefore, the Butler matrix of the transmitter and receiver are denoted as  $\mathbf{O} = \text{diag}\{\mathbf{o}_1, \dots, \mathbf{o}_N\}$  and  $\mathbf{O}^H = \text{diag}\{\mathbf{o}_1^H, \dots, \mathbf{o}_M^H\}$ , where  $N$  and  $M$  are the number of UCAs at the transmitter and receiver, respectively. The channel matrix between the  $n$ th UCA of transmitter and the  $m$ th UCA of receiver can be expressed as  $\mathbf{H}_{m,n} = [h_{mr,nt}] \in C^{R \times T}$ , where  $h_{mr,nt}$  is the channel gain from the  $t$ th antenna on the  $n$ th UCA of transmitter to the  $r$ th antenna on the  $m$ th UCA of receiver under free space propagation. It can be expressed as

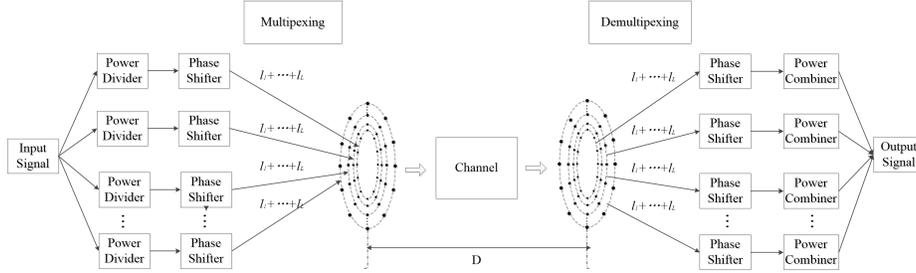


Fig. 1: The System model of the considered OAM-MIMO system based on UCAs.

$$h_{mr,nt} = \beta \frac{\lambda}{4\pi d_{mr,nt}} e^{-j2\pi \frac{d_{mr,nt}}{\lambda}}, \quad (1)$$

where  $\beta$  denotes the antenna gain and  $\lambda$  is wavelength. The distance between the  $t$ th transmit antenna on the  $n$ th UCA and the  $r$ th receive antenna on the  $m$ th UCA is

$$d_{mr,nt} = \sqrt{D^2 + S_n^2 + S_m^2 - 2S_n S_m \cos(\vartheta_r - \phi_t)}, \quad (2)$$

where  $D$  is the vertical distance between the transmit UCA and the receive UCA center.  $S_n$  and  $S_m$  denote the radius of the  $n$ th transmit UCA and the  $m$ th receive UCA. According to  $\sqrt{1-x} \approx 1 - \frac{x}{2}$ ,  $d_{mr,nt}$  is given by

$$d_{mr,nt} \approx \sqrt{D^2 + S_n^2 + S_m^2} - \frac{S_n S_m \cos(\vartheta_r - \phi_t)}{\sqrt{D^2 + S_n^2 + S_m^2}}. \quad (3)$$

Combining (1) with (3), we have

$$h_{mr,n,l} = \sum_{t=1}^T \frac{\beta \lambda e^{-j \frac{2\pi \sqrt{D^2 + S_n^2 + S_m^2}}{\lambda}}}{4\pi D \sqrt{T}} e^{jl_n \phi_t} \cdot \exp\left\{ \frac{j2\pi S_n S_m \cos(\vartheta_r - \phi_t)}{\lambda \sqrt{D^2 + S_n^2 + S_m^2}} \right\}. \quad (4)$$

By utilizing the  $l$ -order Bessel function  $J_l(\alpha) = \frac{j^l}{2\pi} \int_0^{2\pi} e^{jl\varphi} e^{j\alpha \cos \varphi} d\varphi$  and defining  $\phi_t = \theta + \vartheta_r$ , the channel gain  $h_{mr,n,l}$  in (4) is converted to

$$h_{mr,n,l} \approx \frac{\beta\lambda\sqrt{T}e^{-j\frac{2\pi\sqrt{D^2+S_n^2+S_m^2}}{\lambda}}e^{jl_n\vartheta_r}}{4\pi D j^l} \cdot J_l\left(\frac{2\pi S_n S_m}{\lambda\sqrt{D^2+S_n^2+S_m^2}}\right). \quad (5)$$

By turning  $e^{jl_n\vartheta_r}$  into continuous  $e^{jl_n\vartheta}$ , we can get the channel gain before spatially sampling. Thus, (5) can be written as follow

$$h_{mn,l} = \frac{\beta\lambda\sqrt{T}e^{-j\frac{2\pi\sqrt{D^2+S_n^2+S_m^2}}{\lambda}}}{4\pi D j^l} \cdot J_l\left(\frac{2\pi S_n S_m}{\lambda\sqrt{D^2+S_n^2+S_m^2}}\right). \quad (6)$$

The multiplexed signal can be recovered based on the zero-forcing successive interference cancellation algorithm [13]. We denote  $\sum_{m,n} = \mathbf{o}_m^H \mathbf{H}_{mn} \mathbf{o}_n$ . The  $l$ th diagonal element of  $\sum_{m,n} \in C^{L \times L}$  is set as  $\delta_{l,m,n}$  and  $\sum_l = [\delta_{l,m,n}] \in C^{M \times N}$ .  $\mathbf{W}_l = (\sum_l^H \sum_l)^{-1} \sum_l^H \in C^{N \times M}$  denotes the filter coefficient of the  $l$ th OAM mode. Since the number of UCA is  $N$ , we calculate  $\mathbf{W}_l$  for  $N$  times to obtain the channel gain of the  $l$ th mode, denoted by  $\mathbf{G}_l$ . Then, we have

$$\mathbf{W}_l^n = (\sum_l^n)^+, \sum_l^n \in C^{N \times N}, \quad (7)$$

where  $\mathbf{W}_l^n$  is the  $n$ th calculations of  $\mathbf{W}_l$ ,  $\sum_l^n$  is the  $n$ th processing of  $\sum_l$  and  $(\cdot)^+$  is pseudo inverse. We denote  $k_n = \arg \min_j ((\mathbf{W}_L)_l^n)_j$  to choose the minimum value except zero of  $(\mathbf{W}_L)_l^n$ , where  $((\mathbf{W}_L)_l^n)_j$  is the  $j$ th value of  $(\mathbf{W}_L)_l^n$  and  $(\mathbf{W}_L)_l^n = [||(\mathbf{W}_l^n)_1||^2, \dots, ||(\mathbf{W}_l^n)_N||^2]$ . After finding the value of  $k_n$ , we set the values of the  $k_n$  columns of the matrix  $\sum_l^n$  to zero, which is expressed as  $\sum_l^{n+1} = (\sum_l^n)_{\bar{k}_n}$ . Based on the pseudo-inverse of  $\sum_l^{n+1}$ ,  $\mathbf{W}_l^{n+1} = (\sum_l^{n+1})^+$  is obtained. Then, we denote  $(\mathbf{W}_{LS})_l^n = [||(\mathbf{W}_l^n)_{k_1}||^2, \dots, ||(\mathbf{W}_l^n)_{k_{N-1}}||^2, ||(\mathbf{W}_l^n)_{k_N}||^2]$ . For the  $l$ th mode, we can get the channel gain  $\mathbf{G}_l$ . It can be shown as

$$\mathbf{G}_l = \begin{pmatrix} \frac{1}{||(\mathbf{W}_l^1)_{k_1}||^2} & \frac{1}{||(\mathbf{W}_l^1)_{k_2}||^2} & \cdots & \frac{1}{||(\mathbf{W}_l^1)_{k_N}||^2} \\ \frac{1}{||(\mathbf{W}_l^2)_{k_1}||^2} & \frac{1}{||(\mathbf{W}_l^2)_{k_2}||^2} & \cdots & \frac{1}{||(\mathbf{W}_l^2)_{k_N}||^2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{||(\mathbf{W}_l^n)_{k_1}||^2} & \frac{1}{||(\mathbf{W}_l^n)_{k_2}||^2} & \cdots & \frac{1}{||(\mathbf{W}_l^n)_{k_N}||^2} \end{pmatrix}. \quad (8)$$

## 2.2 Problem Formulation

According to the UCAs with the same modes, these OAM beam links are sorted, the first mode of the first UCA is set as link  $i = 1$ , and then the second mode of the first UCA represents link  $i = N + 1$ . The total transmit power and the total power consumption from the system hardware can be expressed as  $P = \sum_{i=1}^{N_L} P_i$

and  $PC_{tot} = \sum_{n=1}^N PC_n$  respectively. Hence, the total power consumption  $P_{tot}$  can be written as follows [14]

$$P_{tot}(\mathbf{P}) = \alpha \sum_{i=1}^{N_L} P_i + PC_{tot}, \quad (9)$$

where  $N_L$  is the number of multiplexing OAM channels.

Then,  $i$ -th link of the SINR  $\gamma_i$  is written as

$$\gamma_i(\mathbf{P}) = \frac{P_i(\mathbf{G}_l)_{i' i'}}{\sum_{j=i+1}^Q P_i(\mathbf{G}_l)_{i' j'} + \sigma^2}, \quad (10)$$

where  $i' = i - ((i-1)/N)N$ ,  $j' = j - ((j-1)/N)N$ ,  $l = (i-1)/N + 1$  and  $Q = (1 + (i-1)/N)N$ . In addition,  $(i-1)/N$  represents the integer part of the quotient. From the above conversion, the total capacity  $C_{tot}$  can be rewritten as

$$C_{tot}(\mathbf{P}) = \sum_{i=1}^{N_L} C_i(\mathbf{P}) = \sum_{i=1}^{N_L} \log_2 \left( 1 + \frac{P_i(\mathbf{G}_l)_{i' i'}}{\sum_{j=i+1}^Q P_i(\mathbf{G}_l)_{i' j'} + \sigma^2} \right), \quad (11)$$

where  $C_i$  is the capacity of  $i$ th link. Based on (9) and (11), the EE is written as follows

$$\lambda_{EE} \triangleq \frac{C_{tot}}{P_{tot}} = \frac{\sum_{i=1}^{N_L} \log_2 \left( 1 + \frac{P_i(\mathbf{G}_l)_{i' i'}}{\sum_{j=i+1}^Q P_i(\mathbf{G}_l)_{i' j'} + \sigma^2} \right)}{\sum_{i=1}^{N_L} P_i + PC_{tot}}. \quad (12)$$

According to (12), we can obtain the EE optimization problem

$$\max_{\mathbf{P}} \quad \lambda_{EE}(\mathbf{P}) \quad (13a)$$

$$\text{s.t.} \quad C1 : C_i \geq R_{req}, \forall i, \quad (13b)$$

$$C2 : P_i \geq 0, \forall i, \quad (13c)$$

$$C3 : P^{max} \geq \sum_{i=1}^{N_L} P_i, \forall i. \quad (13d)$$

where  $C1$  is the minimum capacity constraints of each link,  $C2$  guarantees the effectiveness of link  $i$  and  $C3$  is the total transmitting power constraint.

### 3 The EE Maximization Algorithm

Due to the inter-interference from different UCA at the same mode, the objective function of the proposed problem is not concave about power vector  $\mathbf{P}$ . Since it is the ratio of two real valued functions, the proposed optimization problem is a generalized fractional programming problem. In order to tackle the optimization problem, the objective function is converted into a concave function. Then, the maximum EE  $\lambda_{EE}^{opt}$  is expressed as follows

$$\lambda_{EE}^{opt} = \frac{C_{tot}(\mathbf{P}^{opt})}{P_{tot}(\mathbf{P}^{opt})} = \max_{\mathbf{P}} \frac{C_{tot}(\mathbf{P}^{opt})}{P_{tot}(\mathbf{P}^{opt})}. \quad (14)$$

According to (14), we define  $F(\lambda_{EE}) = \max_{\mathbf{P}} C_{tot}(\mathbf{P}) - \lambda_{EE} P_{tot}(\mathbf{P})$ . When  $\mathbf{P} = \mathbf{P}^{opt}$ , EE reaches the optimal value. It is clear that  $F(\lambda_{EE}) = 0$  and  $F(\lambda_{EE})$  is a continuous strictly decreasing convex function about  $\lambda_{EE}$ . Hence, we adopt dichotomy to obtain  $\lambda_{EE}^{opt}$  of  $F(\lambda_{EE}^{opt}) = 0$ . The corresponding optimal power can be obtained by the following problem with given  $\lambda_{EE}^z$ , where  $\lambda_{EE}^z$  is the initial EE of the  $z$ th loop.

$$\max_{\mathbf{P}} \quad C_{tot}(\mathbf{P}) - \lambda_{EE}^z P_{tot}(\mathbf{P}) \quad (15a)$$

$$\text{s.t.} \quad C1, C2, C3. \quad (15b)$$

Obviously, (11) can be transformed into the difference between two concave functions about  $\mathbf{P}$ . We turn the objective function of (15) into the following

$$C_{tot}(\mathbf{P}) - \lambda_{EE}^z P_{tot}(\mathbf{P}) = U(\mathbf{P}) - V(\mathbf{P}), \quad (16)$$

$$U(\mathbf{P}) = \sum_{i=1}^{N_L} \log_2 \left( \sum_{j=i}^Q P_j(G_l)_{j'j'} \right) - \lambda_{EE}^z \left( \sum_{i=1}^{N_L} P_i + PC_{tot} \right), \quad (17)$$

$$V(\mathbf{P}) = \sum_{i=1}^{N_L} \log_2 \left( \sum_{j=i+1}^Q P_j(G_l)_{i'j'} + \sigma^2 \right). \quad (18)$$

Therefore, the optimization problem (15) can be rewritten as follows

$$\max_{\mathbf{P}} \quad U(\mathbf{P}) - V(\mathbf{P}) \quad (19a)$$

$$\text{s.t.} \quad C1', C2, C3. \quad (19b)$$

$$C1' : P_i(G_l)_{i'i'} + (1 - 2^{R_{req}}) \left( \sum_{j=i+1}^Q P_j(G_l)_{i'j'} + \sigma^2 \right) \geq 0, \quad (19c)$$

where the feasible set of constraints  $C1'$ ,  $C2$  and  $C3$  is convex. Here, the objective function of the problem (19) is the difference between two concave functions. Hence, it can not be determined to be concave or convex, thus the problem (19) is difficult to be proved as a convex optimization problem. To address it, we turn  $V(\mathbf{P})$  into an affine function by the first order Taylor approximation method. Since the objective function is transformed into a concave function minus an affine function, it is approximated to a concave function. Therefore, the problem (19) is transformed into a convex optimization problem.

Assuming that  $\mathbf{P}^k$  is the transmit power vector of  $N_L$  links in the  $k$ th step, the first order Taylor expansion of  $V(\mathbf{P})$  at  $\mathbf{P}^k$  is given by  $V(\mathbf{P}^k) + \nabla V^T(\mathbf{P}^k)(\mathbf{P} - \mathbf{P}^k)$ , and  $\nabla V(\mathbf{P})$  refers to the gradient of  $V(\mathbf{P})$  as follows

$$\nabla V(\mathbf{P}) = \sum_{i=1}^{N_L} \nabla V_i(\mathbf{P}) = \sum_{i=1}^{N_L} \frac{1}{\sum_{j=i+1}^Q P_j(G_l)_{i'j'} + \sigma^2} \mathbf{e}_i, \quad (20)$$

Table 1: The proposed EE optimization algorithm.

1:	Initialize the iteration index $z$ , the stopping criterion $\epsilon$ , and the boundary values of $\lambda_{EE}$ .
2:	<b>repeat</b>
3:	$\lambda_{EE}^z = \frac{\lambda_{EE}^{max} + \lambda_{EE}^{min}}{2}$ .
4:	<b>The power allocation scheme</b>
	1): Initialize the iteration index $k$ , the stopping criterion $\epsilon$ for the inner loop, and the initial value of transmit power $\mathbf{P}^{(0)}$ . calculate $I^0 = U(\mathbf{P}^0) - V(\mathbf{P}^0)$ .
	2): <b>repeat</b>
	3): Solve the optimization problem (32) to obtain $\mathbf{P}^*$ , where is the optimal transmit power.
	4): Then $k = k + 1$ , $\mathbf{P}^k = \mathbf{P}^*$ and calculate $I^k = U(\mathbf{P}^k) - V(\mathbf{P}^k)$ .
	6): <b>until</b> $ I^k - I^{k-1}  \leq \epsilon$ .
5:	Let $\mathbf{P}^z = \mathbf{P}^k$
6:	<b>if</b> $ F(\lambda_{EE}^z)  = C_{tot}(\mathbf{P}^z) - \lambda_{EE}^z P_{tot}(\mathbf{P}^z) \leq \epsilon$ then, $\mathbf{P}^{opt} = \mathbf{P}^z$ and $\lambda_{EE}^{opt} = \frac{C_{tot}(\mathbf{P}^z)}{P_{tot}(\mathbf{P}^z)}$ .
7:	break;
8:	<b>else</b>
9:	<b>if</b> $F(\lambda_{EE}^z) < 0$ , then
10:	$\lambda_{EE}^{max} = \lambda_{EE}^z$ .
11:	<b>else</b>
12:	$\lambda_{EE}^{min} = \lambda_{EE}^z$ .
13:	<b>end if</b>
14:	<b>end if</b>
15:	Update iteration index $z = z + 1$ .
16:	<b>until</b> $ F(\lambda_{EE}^z)  = C_{tot}(\mathbf{P}^z) - \lambda_{EE}^z P_{tot}(\mathbf{P}^z) \leq \epsilon$ .

where  $e_i \in C^{N \times 1}$  is a column vector. When  $j' \geq i'$ , we have  $e_i(j') = \frac{(G_i)_{i'j'}}{\ln 2}$ ; otherwise  $e_i(j') = 0$ . Then, the optimization problem can be transformed into the following

$$\max_{\mathbf{P}} \quad U(\mathbf{P}) - [V(\mathbf{P}^k) + \nabla V^T(\mathbf{P}^k)(\mathbf{P} - \mathbf{P}^k)] \quad (21a)$$

$$\text{s.t.} \quad C1', C2, C3, \quad (21b)$$

where the objective function is concave and the constraint set of the optimization problem (21) is convex. Therefore, it is obvious that the problem (21) is a convex optimization problem. However, the considered EE optimization problem is still hard to solve directly. Here, we adopt Lagrange duality method to deal with the problem. The Lagrange function of the optimization problem (21) can be given

by

$$\begin{aligned} \mathcal{L}(\mathbf{P}, \boldsymbol{\mu}, \boldsymbol{\nu}, \psi) = & U(\mathbf{P}) - [V_i(\mathbf{P}^k) + \nabla V_i^T(\mathbf{P}^k)(\mathbf{P} - \mathbf{P}^k)] + \psi \left( P_{max} - \sum_{i=1}^{N_L} P_i \right) \\ & + \sum_{i=1}^{N_L} \nu_i (P_i) + \sum_{i=1}^{N_L} \mu_i \left( P_i(G_l)_{i' i'} + (1 - 2^{R_{req}}) \left( \sum_{j=i+1}^Q P_i(G_l)_{i' j'} + \sigma^2 \right) \right), \end{aligned} \quad (22)$$

where  $\boldsymbol{\mu} \geq \mathbf{0}$ ,  $\boldsymbol{\nu} \geq \mathbf{0}$  and  $\psi \geq \mathbf{0}$  denote the lagrangian multipliers of  $C1'$ ,  $C2$  and  $C3$  respectively. Based on (22), the dual objection function is shown as

$$g(\boldsymbol{\mu}, \boldsymbol{\nu}, \psi) = \max_{\mathbf{P}} \mathcal{L}(\mathbf{P}, \boldsymbol{\mu}, \boldsymbol{\nu}, \psi). \quad (23)$$

Therefore, the dual optimization problem can be expressed as the following

$$\min_{\boldsymbol{\mu}, \boldsymbol{\nu}, \psi} g(\boldsymbol{\mu}, \boldsymbol{\nu}, \psi) \quad (24a)$$

$$\text{s.t. } \boldsymbol{\mu} \geq \mathbf{0}, \boldsymbol{\nu} \geq \mathbf{0} \text{ and } \psi \geq 0. \quad (24b)$$

It is obvious that problem (24) is a convex optimization problem, which can be solved to obtain  $\lambda_{EE}^{opt}$  by CVX. The detailed implementation steps are described in Table 1.

## 4 Simulation Results

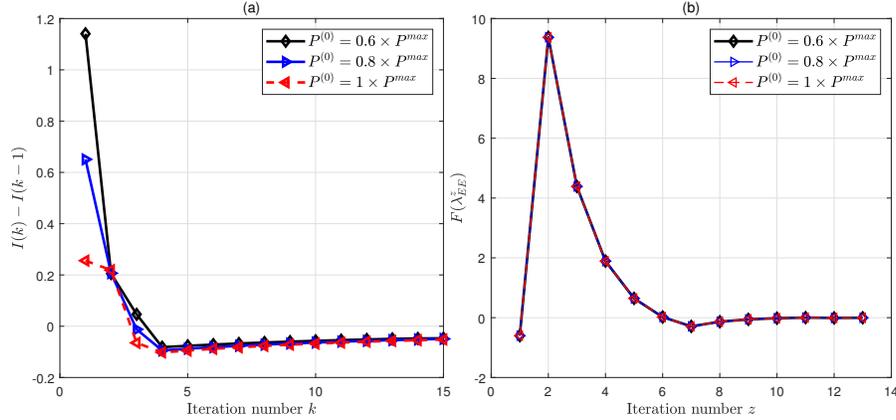


Fig. 2: Convergence performance. (a) the value of  $I^k - I^{k-1}$  vs the number of iteration, (b)  $F(\lambda_{EE}^z)$  vs the number of iteration.

Simulation experiment are carried out to show the performance of the EE maximization algorithm in this section. We assume that the vertical distance  $D$

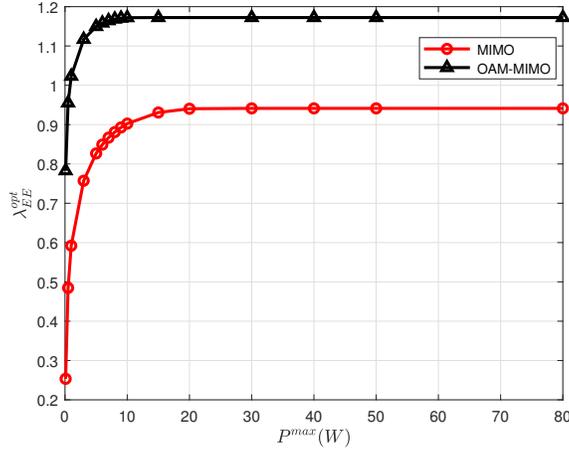


Fig. 3: The EE trend of the OAM-MIMO system and the MIMO system with the variation of  $P^{max}$ .

is 10 m, propagation environment is free space scene and the channel noise is AWGN with power  $\sigma^2 = 1 \times 10^{-5}W$ . The static circuit power of per antenna is set to 3 W and the drain efficiency of the power amplifier is set to 1.5. In addition, termination accuracy  $\varepsilon$  and  $\epsilon$  are set to 0.001. Whereas, the upper and lower limits of EE are considered as  $\lambda_{EE}^{max} = 2$  and  $\lambda_{EE}^{min} = 0$  [15]. The transmitter with 4 UCAs is considered whereas the radius are 0.24 m, 0.36 m, 0.48 m and 0.6 m respectively [7]. Moreover, the number of antenna arrays on each UCA is set to 16.

Firstly, the convergence performance of the power allocation scheme is investigated. We assume that one mode of vortex electromagnetic wave is transmitted from each UCA and the value of mode is 1 in Fig. 2. In addition,  $P^{max}$  and  $R_{req}$  are set to 5 W and 1 bits/s/Hz respectively. As shown in Fig. 2, we observe that the value of  $I(k) - I(k-1)$  is approaching to zero gradually with any value of  $P^0$  to obtain the corresponding optimal power allocation vector  $\mathbf{P}$  for given  $\lambda_{EE}$ . Obviously, the result is in consistent with our theoretical analysis, which proves that the power allocation scheme is efficient and convergent. In the next simulation, the convergence of the EE maximization algorithm is demonstrated and the influence of  $P^0$  on the iteration number required to reach the stable value is studied. It can be seen from Fig. 2 that the stable value can achieve converge under different values of  $P^0$ . In other words, the convergence of the proposed EE maximization algorithm is not affected by the initial value of  $P^0$ , which verifies the convergence and effectiveness of the EE maximization algorithm.

We then compare the performance of the proposed EE maximization algorithm on the OAM-MIMO system with the traditional point-to-point MIMO system in Fig. 3 and Fig. 4. In this simulation, the multiplexed modes on each UCA is within the range of  $l \in [-6, 6]$ . For fairness comparison, we set the

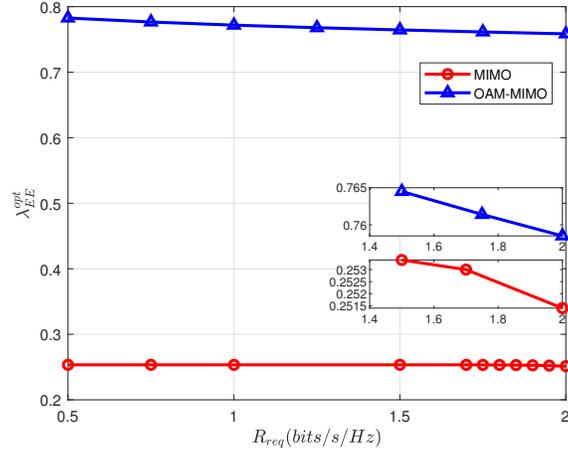


Fig. 4: The EE trend of the OAM-MIMO system and the MIMO system with the variation of  $R_{req}$ .

MIMO system the same antennas as the OAM-MIMO system. In addition, the area covered by all antennas of the MIMO system is the same as the area covered by the UCA with largest radius in the OAM-MIMO system. In order to ensure that the channels can reach the minimum capacity requirement, we select the channel with relatively large eigenvalues after singular value decomposition of the channel matrix in the MIMO system, which means the relatively superior channel [16].

In the following simulation, the performance of the proposed EE optimization algorithm in two systems is studied, with the variation of the total transmit power budget within a range of  $0.1 \text{ W} \leq P^{max} \leq 80 \text{ W}$ . In addition, we set  $R_{req} = 0.5$ . From Fig. 3, when  $P^{max}$  is relatively low,  $\lambda_{EE}^{opt}$  increases sharply with the increase of  $P^{max}$  until large than 20 W. It reveals that the extra power budget no longer constitutes the extra gain of  $\lambda_{EE}^{opt}$ , leading to a balance between the total capacity and the total power consumption. Fig. 4 shows the variation trend of  $\lambda_{EE}^{opt}$  with the minimum capacity constraint  $R_{req}$  varying from 0.2 to 1.  $P^{max}$  is fixed as  $1 \times 10^{-1} \text{ W}$  in this simulation. As  $R_{req}$  increases, it can be observed that  $\lambda_{EE}^{opt}$  decreases continuously. From Fig. 3 and Fig. 4, we can observe that the stability values of the OAM-MIMO system is higher than the MIMO system. This is because different OAM mode of the OAM-MIMO system are mutually orthogonal, resulting in the lower interference of channel compared with the MIMO system.

## 5 Conclusions

In this paper, we investigate the EE optimization problem of the OAM-MIMO system with aligned UCAs. The corresponding optimization problem involves

power allocation of multiple modes, which is non-convex. To obtain a feasible solution, we propose a dual-layer iterative EE optimization algorithm. The optimal power allocation scheme is obtained by convex programming based on first order Taylor approximation in inner layer and the optimal EE is obtained by the dichotomy in the outer layer. Simulation results demonstrate the validity of the EE optimization proposed algorithm.

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