

# Energy Efficiency Optimization for Plane Spiral OAM Mode-Group Based MIMO-NOMA Systems

Jie Tang<sup>1,2(✉)</sup>, Yan Song<sup>1</sup>, Chuting Lin<sup>1</sup>, Wanmei Feng<sup>1</sup>, Zhen Chen<sup>1</sup>,  
Xiuying Zhang<sup>1</sup>, and Kai-kit Wong<sup>3</sup>

<sup>1</sup> School of Electronic and Information Engineering, South China University of Technology, China

eejtang@scut.edu.cn, songyan1222@whut.edu.cn, 13929316612@163.com,  
eewmfeng@mail.scut.edu.cn, chenz@scut.edu.cn, zhangxiuyin@scut.edu.cn

<sup>2</sup> The National Mobile Communications Research Laboratory, Southeast University, Nanjing

<sup>3</sup> Department of Electronic and Electrical Engineering, University College London, UK

kai-kit.wong@ucl.ac.uk

**Abstract.** In this paper, a plane spiral orbital angular momentum (PS-OAM) mode-groups (MGs) based multi-user multiple-input-multiple-output (MIMO) non-orthogonal multiple access (NOMA) system is studied, where a base station (BS) transmits data to multiple users by utilizing the generated PSOAM beams. For such scenario, the interference between users in different PSOAM-mode groups can be avoided, which leads to a significant performance enhancement. We aim to maximize the energy efficiency (EE) of the system subject to the total transmission power constraint and the minimum rate constraint. This design problem is non-convex by optimizing the power allocation, and thus is quite difficult to tackle directly. To solve this issue, we present a bisection-based power allocation algorithm where the bisection method is exploited in the outer layer to obtain the optimal EE and a power distributed iterative algorithm is exploited in the inner layer to optimize the transmit power. Simulation results validate the theoretical findings and demonstrate the proposed system can achieve better performance than the traditional multi-user MIMO system in terms of EE.

**Keywords:** Energy efficiency (EE) · Plane spiral orbital angular momentum (PSOAM) · Non-orthogonal multiple access (NOMA).

## 1 Introduction

The rapid development of Internet-of-Things (IoTs) applications has caused the exponential growth of wireless devices. Consequently, the sixth generation (6G) wireless networks face particular challenges to meet the further requirements in terms of reliable data connectivity and ultra-high data-rate. In addition, the

data rates of devices are severely limited by the insufficient spectrum resources. These trends make spectral efficiency (SE) to become the main indicator of mobile communication networks. On the other hand, a massive number of connected devices also leads to enormous energy consumption, and thus energy efficiency (EE) has become an important and global topic from both environmental and economic reasons.

Orbital angular momentum (OAM) can provide a new degree of freedom for improving the SE due to its orthogonality, thus it can meet the requirements of high data rate [1, 2]. However, the main practice challenge for applying such technology into the electromagnetic (EM) field is the beam divergence and phase singularity caused by the OAM modes and long-distance transmission. To solve this issue, S. Zheng *et al.* proposed a new form of OAM waves called plane spiral orbital angular momentum (PSOAM), which propagates along the transverse plane intelligently, and thereby avoiding the aforementioned issues of phase singularity and the diversity [3]. The authors further analysed the PSOAM beams and put forward the concept of PSOAM mode-groups (MGs), which had the promising prospect in spatial modulation multiple-input-multiple-output (SM-MIMO), smart antenna and MIMO [4]. To demonstrate the performance of PSOAM MGs, the authors in [5] applied PSOAM MGs into a single-user system, where the partial arc sampling receiving (PASR) method was adopted to de-multiplex the PSOAM-MGs-carrying data streams due to its low complexity.

On the other hand, non-orthogonal multiple access (NOMA) is viewed as a key technique to enhance SE in the beyond fifth generation (B5G) communication networks [6]. It can simultaneously serve a large amount of users with the same physical resource via superposition coding (SC), where different users are distinguished with different power levels and the successive interference cancellation (SIC) is used to cancel the multi-user interferences [7]. It has been proved that NOMA can obtain better behaviours from the perspective of SE compared with orthogonal multiple access (OMA).

In fact, the combination of PSOAM MGs and NOMA can greatly improve the SE while considering the interference among all users. Previous works on OAM systems mainly focused on maximizing the spectral efficiency in a PSOAM MGs system [5, 8], and NOMA-based wireless networks [9, 10]. However, seldom works have been studied in EE optimization for the PSOAM MGs based multi-user MIMO-NOMA system. In this paper, a downlink PSOAM-MGs based multi-user MIMO-NOMA system is investigated, where the transmit power is optimized to achieve the maximum EE of the system. The resultant optimization problem considering the constraints of the transmit power and minimum required data rate of users, is non-convex and NP-hard, which cannot be solved directly. To tackle this problem, by applying the fractional programming and the first order Taylor approximation, the original problem is equivalently reformulated as a convex maximization problem, which can be solved by the Lagrange dual method. Particularly, we propose a bisection-based power allocation algorithm, where the bisection method is exploited in the outer layer to obtain the optimal EE and a power distributed algorithm is adopted to optimize the transmit power in the

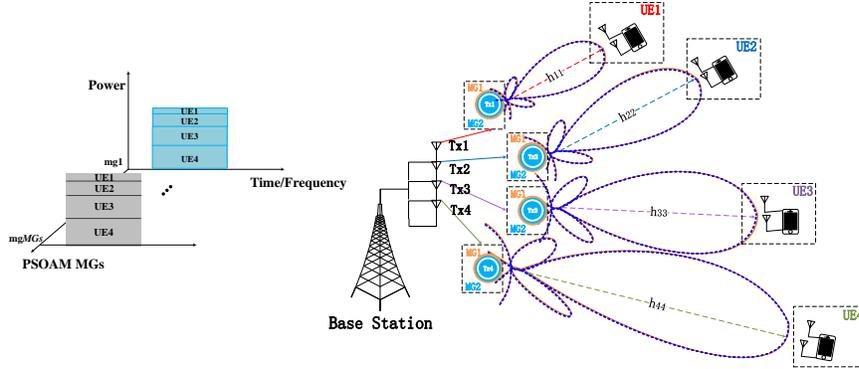


Fig. 1: System model of PSOAM-MGs based multi-user MIMO-NOMA.

inner layer. Simulation results illustrate that the proposed algorithm can achieve the optimal EE in the proposed PSOAM MGs system. In addition, numerical results also demonstrate that the EE achieved in the proposed PSOAM MGs system is superior compared with the conventional multi-user MIMO system.

## 2 System Model and Problem Formulation

### 2.1 System Model

In Fig. 1, the PSOAM MGs based multi-user MIMO-NOMA system includes one base station (BS) with  $N_t$  antennas is deployed to serve  $K$  users. The  $K$  users are randomly distributed in a fan-shaped area and the antenna spacing is  $\zeta$ . At the transmitting side, each antenna only sends data streams to its corresponding user, in which two superposed PSOAM MGs waves are radiated into the free space. Supposing there are  $G$  PSOAM waves in one MG and the equivalent PSOAM MGs phase slope can be calculated by the smallest and the biggest modes [5]. At the receiver, each user is equipped with  $N_r$  receiving antennas, which are placed within the main lobe of the superposed PSOAM MGs waves.

The total transmit power is restricted to  $P_{max}$  and in the  $mg^{th}$  mode group, the signal transmitted to user  $k$  can be written as

$$X_{k,mg} = p_{k,mg} \cdot x_{k,mg}, \quad (1)$$

where  $p_{k,mg}$  denotes the power allocation of the  $mg^{th}$  PSOAM mode group at the  $k^{th}$  user. Let  $\varphi_0^k$  as the initial phase of user  $k$  and for the  $mg^{th}$  mode group, the link of channel gain  $h_{k,nr,nt,mg}$  between the  $nt^{th}$  transmitting antenna and

the  $nr^{th}$  receiving antenna can be written as

$$\begin{aligned} h_{k,nr,nt,mg} &= \beta_{k,nt} \frac{\lambda}{4\pi d_{k,nr,nt}} e^{-j\frac{\lambda}{2\pi} d_{k,nr,nt}} \frac{1}{\sqrt{G^{mg}}} \sum_{g^{mg}=1}^{G^{mg}} e^{-j l_{g^{mg}}^{mg} \varphi_{k,nr,nt}} \\ &= \beta_{k,nt} \frac{\lambda}{4\pi d_{k,nr,nt}} e^{-j\frac{\lambda}{2\pi} d_{k,nr,nt}} \frac{1}{\sqrt{G^{mg}}} e^{-j l_{eq}^{mg} \varphi_{k,nr,nt}}, \end{aligned} \quad (2)$$

where  $d_{k,nr,nt}$  represents the distance between the  $nt^{th}$  transmitting antenna and the  $nr^{th}$  receiving antenna of user  $k$ . Importantly,  $\beta_{k,nt} = \sqrt{G_t G_r}$  is a constant related to the antenna gain of the  $nt^{th}$  transmitting antenna at the  $nr^{th}$  receiving antenna. Specifically,  $G_t$  can be determined by the distribution of the users and the interference of the minor lobe.

In addition,  $\varphi_{k,nr,nt}$  represents the phase between the  $nt^{th}$  transmitting antenna and the  $nr^{th}$  receiving antenna of user  $k$ . For the two PSOAM MGs based system and to calculate the phase  $\varphi_{k,nr,nt}$ , there are two conditions that should be considered. One is the initial phase  $\varphi_0^k > 0$  and the other is  $\varphi_0^k < 0$ . For the condition  $\varphi_0^k > 0$ , three cases are discussed as follows:

**Case 1  $k = nt$ .** We define the vertical distance between the center of the two receiving antennas of user  $k$  and the corresponding transmitting antenna as  $d_k$ . The radius of the arc is marked as  $R_{ad}$ . The distance between the  $nr^{th}$  receiving antenna and the  $nt^{th}$  transmitting antenna can be calculated by  $d_{k,nr,nt,cor} = \sqrt{d_k^2 + (\frac{R_{ad}}{2})^2}$  and the phase  $\varphi_k$  between  $d_k$  and  $d_{k,nr,nt,cor}$  can be regarded as  $\varphi_k = (-1)^{nr} \cdot \arctan(\frac{R_{ad}}{2d_k})$ , where the radius of the receiving antennas  $R_{ad}$  is fixed regardless of the transmission distance, which can be calculated by  $R_{ad} = 2D \tan(\frac{\pi}{2|l_{eq}^{mg1} - l_{eq}^{mg2}|})$  and  $D$  is the relative distance. Thus, the radius of the receiving antennas  $R_{ad}$  is fixed regardless of the transmission distance [5] and the phase  $\varphi_{k,nr,nt,cor}$  is calculated by  $\varphi_k + \varphi_0^k$ .

**Case 2  $k < nt$ .** According to the cosine theorem, we can calculate the distance between the  $nr^{th}$  receiving antenna of the non-intended user  $k$  and the  $nt^{th}$  transmitting antenna as follows

$$d_{k,nr,nt} = \sqrt{((nt - k) \cdot \zeta)^2 + d_{k,nr,nt,cor}^2 - \Theta}, \quad (3)$$

where  $\Theta = 2 \cdot d_{k,nr,nt,cor} \cdot \zeta \cdot (nt - k) \cdot \cos(\frac{\pi}{2} + \varphi_{k,nr,nt,cor})$ . In addition, the azimuthal angle of the  $nr^{th}$  receiving antenna of user  $k$  to the  $nt^{th}$  transmitting antenna can be defined as

$$\varphi_{k,nr,nt} = \frac{\pi}{2} - \omega, \quad (4)$$

where

$$\omega = \arccos\left(\frac{((nt - k) \cdot \zeta)^2 + d_{k,nr,nt}^2 - d_{k,nr,nt,cor}^2}{2 \cdot d_{k,nr,nt} \cdot ((nt - k) \cdot \zeta)}\right). \quad (5)$$

**Case 3  $k > nt$ .** Similarly, we can calculate  $\varphi_{k,nr,nt}$  according to case 2.

For the condition  $\varphi_0^k < 0$ , the related angle and distance can be calculated the same as the condition  $\varphi_0^k > 0$ . Considering the mutual interferences among users within one PSOAM MG, NOMA-SIC is applied to the whole system. In the  $mg^{th}$  PSOAM MG, if each transmitting antenna sends  $MGs$  PSOAM mode groups, the corresponding channel gain from the  $l^{th}$  transmitting antenna to the  $k^{th}$  user is indicated as  $h_{kl,mg}$ . Note that if  $k \neq l$ , the channel gain of the  $h_{kl,mg}$  is regarded as the interference signal. The channel model can be denoted as a  $MGs \times MGs$  matrix written as  $\mathbf{H}$  and we use singular value decomposition (SVD) to obtain singular values, denoted as  $\lambda_{kl,mg}$ . Considering the channel gains in one PSOAM mode group of all users satisfy the following condition:  $\lambda_{11,mg} \leq \lambda_{22,mg} \leq \dots \leq \lambda_{KK,mg}$ . To obtain the capacity upper bound, the decoding order of NOMA users is set to  $\{1, 2, \dots, K\}$ . As a result, for the  $mg^{th}$  PSOAM mode group, the data rate of user  $k$ ,  $1 \leq k \leq (K - 1)$ , is given by

$$R_{k,mg} = B \log_2 \left( 1 + \frac{p_{k,mg} \cdot \lambda_{kk,mg}^2}{\sum_{l=k+1}^K p_{l,mg} \lambda_{kl,mg}^2 + \sigma^2} \right). \quad (6)$$

Further, the total rate of all  $K$  users can be formulated as

$$R_{total} = \sum_{mg=1}^{MGs} R_{k,mg} = \sum_{k=1}^K R_k. \quad (7)$$

In general, the power consumption of the PSOAM MGs based multi-user MIMO-NOMA system consists of transmit power and circuit power, which is defined as follows

$$PC_{total} = \alpha \sum_{k=1}^K \sum_{mg=1}^{MGs} p_{k,mg} + N_t \cdot P_{ic}, \quad (8)$$

where  $\alpha$  indicates the power amplifier drain efficiency and  $P_{ic}$  represents the circuit power consumption of system hardware.

## 2.2 Problem Formulation

The work aims to maximize the EE of the PSOAM MGs based multi-user MIMO-NOMA system with the constraint of the minimum required data rate of each user and the total transmit power as well. Therefore, the EE optimization problem is expressed as follows

$$\max_{p_{k,mg}} \frac{\sum_{k=1}^K \sum_{mg=1}^{MGs} B \log_2 \left( 1 + \frac{p_{k,mg} \cdot \lambda_{kk,mg}^2}{\sum_{l=k+1}^K p_{l,mg} \lambda_{kl,mg}^2 + \sigma^2} \right)}{\alpha \sum_{k=1}^K \sum_{mg=1}^{MGs} p_{k,mg} + N_t \cdot P_{ic}} \quad (9)$$

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$$\text{s.t. } C1 : \log_2 \left( 1 + \frac{p_{k,mg} \cdot \lambda_{kk,mg}^2}{\sum_{l=k+1}^K p_{l,mg} \lambda_{kl,mg}^2 + \sigma^2} \right) \geq \frac{R_{req}}{B}, \forall k \in \mathcal{K}, \forall mg \in \mathcal{MG}, \quad (10)$$

$$C2 : \sum_{k=1}^K \sum_{mg=1}^{MGs} p_{k,mg} \leq P_{max}, \quad (11)$$

$$C3 : p_{k,mg} \geq 0, \forall k \in \mathcal{K}, \forall mg \in \mathcal{MG}, \quad (12)$$

where  $\mathcal{K} = \{1, 2, \dots, K\}$  represents the set of users,  $\mathcal{MG} = \{mg1, mg2, \dots, MGs\}$  denotes the set of all PSOAM mode groups.  $C1$  guarantees the constraint of the minimum rate requirement of each user, which is denoted as  $R_{req}$ .  $C2$  guarantees that the total transmit power is limited to  $P_{max}$ . In  $C3$ , the power of each PSOAM MG of user  $k$  is  $p_{k,mg}$ , which ought to be a positive number and  $mg \in \mathcal{MG}$  for any  $k \in \mathcal{K}$  is requested.

### 3 Proposed iterative resource allocation scheme

The optimization problem (9) is non-convex with respect to the power vector  $\mathbf{P}$ , and it is hard to be solved directly. Fortunately, the optimization problem can be converted into a generalized fractional programming problem. We assume that  $R_{k,mg}(\mathbf{P}) > 0$  and  $PC_{total}(\mathbf{P}) > 0$ . The optimal EE can be denoted as  $\gamma_{EE}^*$  and the optimal power allocation of the considered problem is expressed as  $\mathbf{P}^*$ . We can get the following equation

$$\gamma_{EE}^* = \max_{\mathbf{P} \in \{C1, C2, C3\}} \frac{R_{total}(\mathbf{P})}{PC_{total}(\mathbf{P})} = \frac{R_{total}(\mathbf{P}^*)}{PC_{total}(\mathbf{P}^*)}. \quad (13)$$

Furthermore, according to generalized fractional programming, and let

$$\Upsilon(\gamma_{EE}) = \max_{\mathbf{P} \in \{C1, C2, C3\}} [R_{total}(\mathbf{P}) - \gamma_{EE} PC_{total}(\mathbf{P})], \quad (14)$$

where  $\Upsilon$  is a function and  $\gamma_{EE}$  is the independent variable.

**Theorem 1.** Problem (14) is strictly monotonically decreasing with respect to the  $\gamma_{EE}$ .

Therefore, the optimal EE can be tackled by the bisection method and the detailed information of the method is presented in TABLE 1.

For a given  $\gamma_{EE}^i$ , the optimization problem turns to be

$$\begin{aligned} \max_{\mathbf{P}} \quad & R_{total}(\mathbf{P}) - \gamma_{EE}^i PC_{total}(\mathbf{P}) \\ \text{s.t.} \quad & C1, C2, C3. \end{aligned} \quad (15)$$

To solve the problem (15),  $\Upsilon(\gamma_{EE}^i)$  can be transformed as follows

$$R_{total}(\mathbf{P}) - \gamma_{EE}^i PC_{total}(\mathbf{P}) = F(\mathbf{P}) - H(\mathbf{P}), \quad (16)$$

Table 1: PROPOSED BISECTION-BASED POWER ALLOCATION ALGORITHM.

1: <b>Initialization</b>
Set iteration index $i = 0$ and termination precise $\varepsilon > 0$ .
Set $\gamma_{EE}^{min}$ and $\gamma_{EE}^{max}$ , let $\gamma_{EE}^{min} \leq \gamma_{EE}^* \leq \gamma_{EE}^{max}$ .
2: <b>repeat</b>
3: $\gamma_{EE}^i = (\gamma_{EE}^{max} + \gamma_{EE}^{min})/2$ .
4: Solve (13) with a given $\gamma_{EE}^i$ and get $\mathbf{P}^i$ .
5: <b>if</b> $ \Upsilon(\gamma_{EE}^i)  =  R_{total}(\mathbf{P}^i) - \gamma_{EE}^i P_{total}(\mathbf{P}^i)  \leq \varepsilon$
<b>then</b> $\mathbf{P}^* = \mathbf{P}^i$ and $\gamma_{EE}^* = R_{total}(\mathbf{P}^i)/P_{total}(\mathbf{P}^i)$
6: <b>break.</b>
7: <b>else</b>
8: <b>if</b> $\Upsilon(\gamma_{EE}^i) < 0$ , then
9: $\gamma_{EE}^{max} = \gamma_{EE}^i$ .
10: <b>else</b>
11: $\gamma_{EE}^{min} = \gamma_{EE}^i$ .
12: <b>end if</b>
13: <b>end if</b>
14: $i = i + 1$ .
15: <b>until</b> $ \Upsilon(\gamma_{EE}^i)  =  R_{total}(\mathbf{P}^i) - \gamma_{EE}^i P_{total}(\mathbf{P}^i)  \leq \varepsilon$ .

where different PSOAM MGs can be regarded as the sub-channels paralleling to each other, and

$$F(\mathbf{P}) = f_{mg1}(\mathbf{P}_{mg}) + f_{mg2}(\mathbf{P}_{mg}) + \dots + f_{mgMGs}(\mathbf{P}_{mg}), \quad (17)$$

$$H(\mathbf{P}) = h_{mg1}(\mathbf{P}_{mg}) + h_{mg2}(\mathbf{P}_{mg}) + \dots + h_{mgMGs}(\mathbf{P}_{mg}). \quad (18)$$

For each PSOAM MG, we can obtain the expression of the function  $f_{mg}(\mathbf{P}_{mg})$  and  $h_{mg}(\mathbf{P}_{mg})$  as follows

$$f_{mg}(\mathbf{P}_{mg}) = \sum_{k=1}^K B \log_2 \left( \sum_{l=k}^K p_{l,mg} \lambda_{kl,mg}^2 + \sigma^2 \right) - \gamma_{EE}^i \cdot \left( \alpha \sum_{l=k}^K p_{k,mg} + \frac{N_t \cdot P_{ic}}{MGs} \right), \quad (19)$$

$$h_{mg}(\mathbf{P}_{mg}) = \sum_{k=1}^K B \log_2 \left( \sum_{l=k+1}^K p_{l,mg} \lambda_{kl,mg}^2 + \sigma^2 \right). \quad (20)$$

Besides, the non-convex constraint  $C1$  in problem (15) is transformed into an equivalent convex linear form mathematically as follows

$$C1' : (1 - 2^{-\frac{R_{req}}{B}}) \left( \sum_{l=k+1}^K p_{l,mg} \lambda_{kl,mg}^2 + \sigma^2 \right) + p_{k,mg} \lambda_{kk,mg}^2 \geq 0, \forall k, \forall mg. \quad (21)$$

Table 2: POWER ALLOCATION ALGORITHM BASED ON PDIA.

1):	<b>Initialization</b> Set the iteration index $q = 0$ . Set the termination precise $\epsilon > 0$ . Set the initial transmit power $\mathbf{P}^{(0)}$ . Calculate $I^0 = F(\mathbf{P}^0) - H(\mathbf{P}^0)$ .
2):	<b>repeat</b>
3):	Solve (13) to get the optimal transmit power $\mathbf{P}^*$
4):	Set $q = q + 1$ , and $\mathbf{P}^q = \mathbf{P}^*$ .
5):	Calculate $I^q = F(\mathbf{P}^q) - H(\mathbf{P}^q)$ .
6):	<b>until</b> $ I^q - I^{q-1}  =  \Xi  \leq \epsilon$ .

Now, the considered problem (15) is equivalent to

$$\begin{aligned} \max_{\mathbf{P}} \quad & F(\mathbf{P}) - H(\mathbf{P}) \\ \text{s.t.} \quad & C1', C2, C3. \end{aligned} \quad (22)$$

Although the constraints of the optimization problem (22) are convex sets, (9), (15) and (22) remain to be the NP-hard problem. We define the expression pairs in (16) as  $f$  minus  $h$ . Each expression pair is regarded as two concave functions. Therefore, the corresponding optimization problem is non-convex. To solve this issue, we can obtain  $\mathbf{P}^q$  through an iterative power allocation algorithm at the  $q^{th}$  iteration. Then, the first-order Taylor expansion at  $\mathbf{P}^q$  is expressed by

$$h_{mg}(\mathbf{P}_{mg}^q) + \nabla h_{mg}^T(\mathbf{P}_{mg}^q)(\mathbf{P}_{mg} - \mathbf{P}_{mg}^q), \quad (23)$$

where  $\nabla h_{mg}(\mathbf{P}_{mg})$  represents the gradient of  $h_{mg}(\mathbf{P}_{mg})$ ,  $\mathbf{P}_{mg} = \mathbf{P}((mg-1)K + 1, mg \cdot k)$  and the optimization problem (22) can be further transformed into

$$\begin{aligned} \max_{\mathbf{P}} \quad & \sum_{mg=1}^{MGs} (f_{mg}(\mathbf{P}_{mg}) - [h_{mg}(\mathbf{P}_{mg}^q) + \nabla h_{mg}^T(\mathbf{P}_{mg}^q)(\mathbf{P}_{mg} - \mathbf{P}_{mg}^q)^T]) \\ \text{s.t.} \quad & C1', C2, C3. \end{aligned} \quad (24)$$

Fortunately, (24) is a standard convex optimization problem, which can be tackled effectively using Lagrange duality algorithm [11]. The detail information of the power distribution iterative algorithm (PDIA) is presented in TABLE 2.

## 4 Simulation Results

In this section, simulation results are presented to validate the behaviour of the proposed bisection-based power allocation scheme. To study the EE performance, we employ a channel with a carrier frequency operating at 10 GHz

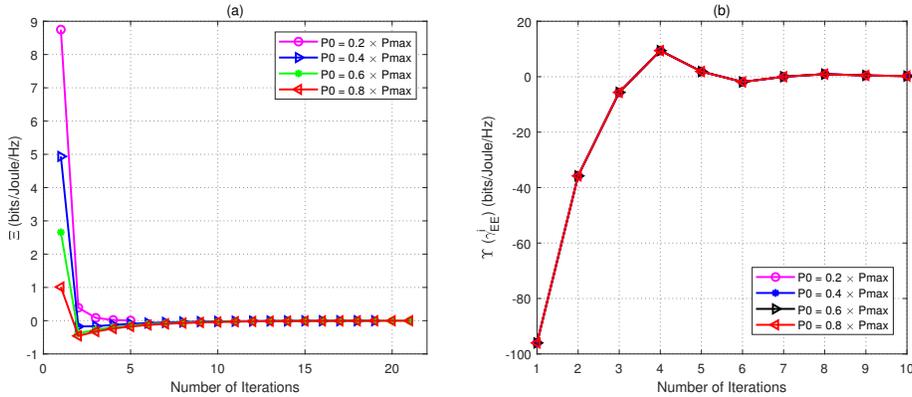


Fig. 2: An example of the convergence behaviour of the proposed bisection-based power allocation algorithm in a PAOAM-MGs based multi-user MIMO-NOMA system. (a) The proposed power resources allocation, (b) The proposed bisection-based EE optimization algorithm.

[5]. The channel noise power  $\sigma^2$  is set to  $1 \times 10^{-5}$ W. The power amplifier drain efficiency is set to  $\alpha = 2$ ;  $P_{ic}$  is set to 6W; In particular, we consider  $K=4$  users randomly distributed in a fan-shaped area, which is 30m away from the BS and the degree of the area ranges from -60 degree to 60 degree. The system we proposed is composed of uniform linear arrays (ULAs) with 4 antennas at the transmitter and the element spacing is  $\zeta = 7\lambda$ . The selected PSOAM MGs of each transmitting antenna are  $mg^1 = \{1, 2, 3, 4, 5, 6, 7, 8\}$  and  $mg^2 = \{9, 10, 11, 12, 13, 14, 15, 16\}$ . When the relative distance is 100m, the  $R_{ad}$  under the PASR method is about 0.7m. The bandwidth of the system is normalized to 1Hz. According to the algorithm we proposed, the termination precises are set to  $\varepsilon = \epsilon = 10^{-3}$  and  $\gamma_{EE} \in [0, 5]$  bit/Joule/Hz. It is worth noting that the parameters in this system are selected to prove the the performance of EE as an example and can be replaced by other reasonable parameters according to the specific scenarios.

First, the convergence behaviour of the proposed bisection-based power allocation algorithm is evaluated by demonstrating how the  $\Xi$  and  $\Upsilon$  behave with the number of iterations. We set  $P_{max} = 2$ W,  $R_{req} = 1$  bit/s/Hz. As seen in Fig. 2(a) that the inner layer of the proposed algorithm on  $\Xi$  can converge to zero, and the  $P_0$  affects the convergence rate of the proposed algorithm. Specifically,  $\Xi$  converges to zero after six iteration when  $P_0 = 0.2P_{max}$ ,  $0.4P_{max}$ ,  $0.6P_{max}$ ,  $0.8P_{max}$ . Moreover, in Fig. 2.(b), the outer layer of the proposed algorithm on  $\Upsilon$  can also converge to zero at approximately eight iterations, which proves that our proposed two-layer algorithm converges to a stable value. This result demonstrates the stability and validity of the proposed algorithm.

Next, we show the  $\gamma_{EE}^*$  of the presented bisection-based power allocation algorithm with different transmit power  $P_{max}$  and minimum required data rate  $R_{req}$ . To demonstrate the effectiveness of our proposed method, we apply the

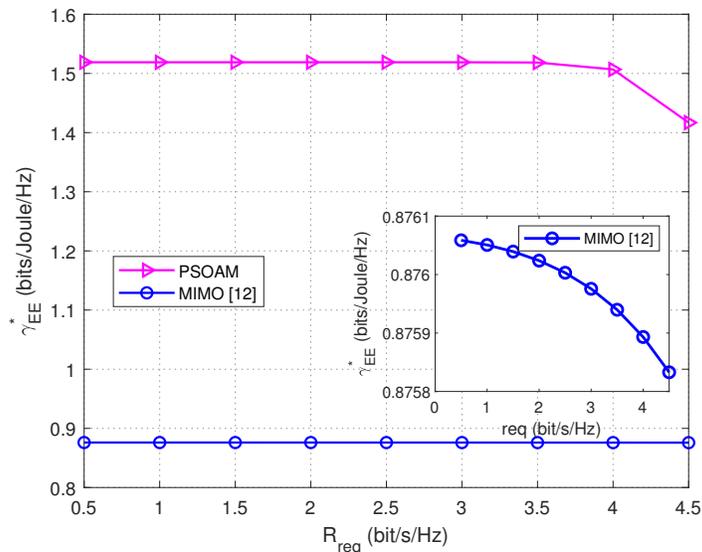


Fig. 3: The  $\gamma_{EE}^*$  vs different minimum rate requirement constraints.

algorithms in the PSOAM MGs based multi-user MIMO-NOMA system and the conventional multi-user MIMO system [12] for comparison. We set  $P_{max} = 2W$ . In Fig. 3, the  $\gamma_{EE}^*$  obtained by all the algorithms are monotonically decreasing with the increase of  $R_{req}$ . For the proposed bisection-based power allocation algorithm, a significant drop occurs when the minimum data rate of users is larger than 4 bit/s/Hz. This is due to the fact that the limitation of transmit power cannot satisfy the QoS requirement of each user.

Finally, we investigate the  $\gamma_{EE}^*$  of the proposed solution with various transmit power  $P_{max}$  as well as different number of users. We set  $R_{req} = 1$  bit/s/Hz. In Fig. 4, the  $\gamma_{EE}^*$  obtained by the two approaches are monotonically non-decreasing under the constraint of  $P_{max}$ . Specifically, the  $\gamma_{EE}^*$  increases swiftly with a lower  $P_{max}$ , and then achieves an asymptotic value when the balance between the available rates and the energy consumption is obtained. Additionally, higher  $P_{max}$  is required to achieve the stable  $\gamma_{EE}^*$  when the number of users increases in the system network. Compared with the traditional multi-user MIMO system, our proposed solution can achieve a significant performance gain in terms of EE due to the degree of freedom provided by PSOAM MGs and NOMA techniques.

## 5 Conclusions

This paper explores the optimization problem of EE for a PSOAM MGs based multi-user MIMO-NOMA system. We aim to maximize the EE while meeting several constraints of total transmit power and minimum required data rate of each user. The corresponding problem of maximizing EE is NP-hard and cannot

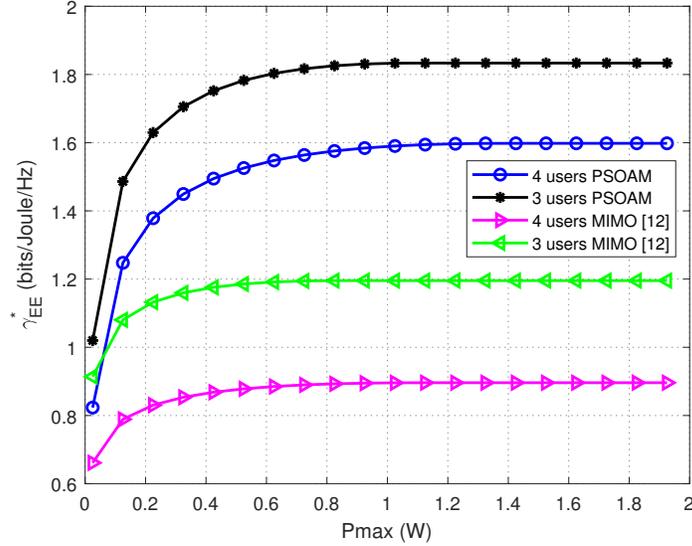


Fig. 4: The  $\gamma_{EE}^*$  vs different entire transmit power constraints.

be tackled directly. Particularly, we obtain the optimal EE in the outer layer via the bisection-based power allocation algorithm and achieve the optimal power allocation in the inner layer through the power distribution iterative algorithm. Numerical results validate the superiority of the PSOAM MGs based multi-user MIMO-NOMA system in EE compared with the conventional multi-user MIMO system. In general, it is worth to further study the joint power resources allocation of EE and SE in the PSOAM MGs based multi-user MIMO-NOMA system in the future.

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