Displacement Compensation Units for Integral Abutment Bridges

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Declaration

I, Muhammad Umar Bin Zulkefli confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

Signature
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Date
05/11/2021
Research Impact Statement

This research's primary theme encompasses the strain-induced problem associated with the integral abutment bridge (IAB). In the IAB system, the bridge deck expands and contracts due to thermal variations; these strain cycles cause a densification of the granular fill behind the abutment and the consequent escalation of the stresses in the abutment wall. A number of alternatives to mitigate the problem exist, however, the large stresses and the consequence to the performance of the bridge, over long periods of time have not been addressed until the idea of a Displacement Compensation Unit (DCU), was introduced. Past research, conducted on scaled down versions of an IAB, introduced disc springs between the deck and the wall. This device, a DCU, absorbed the cyclic wall displacement and, at the same time, kept the stresses acting on the abutment wall approximately constant, an ideal situation that practically eliminated the densification of the fill. This solution, however, was anticipated to have issues with corrosion, hence the idea of utilising rubber as the DCU’s material. To understand how the rubber can be used as a DCU, a study on the mechanical and long-term properties of rubber was conducted.

The experimental and modelling analyses conducted in the long-term relaxation of rubber can help rubber component engineers to design more durable products. Besides, the process of developing a new product can be expedited. The research is significant as it paves the way to consider and utilise rubber as the base material for the concept of DCU and creating other possible engineering applications for this natural material. With more research being done in this area, the knowledge of rubber, structural, and geotechnical engineering will be enhanced.

The new method to predict long-term relaxation in rubber offers an insight into a piece of new knowledge. This will excite other researchers to validate and refine the method, perhaps even utilising different materials and geometries. The investigation on the validity of simulations on rubber with changing strains and temperature suggests that the Finite Element Analysis programmes, such as ABAQUS, can serve as a good aid to predict the behaviour of rubber component; despite that, in this study, it was limited to simple shear and buckling deformations.

The introduction of a new rubber component, such as the rubber DCU in a geotechnical problem, will hopefully convince geotechnical engineers to see the potential of
rubber as an engineering component. Therefore, there would be more knowledge integrations between rubber science and civil engineering in the future.

Finally, by demonstrating the benefits of the use of natural rubber in the manufacture of another engineering component, this study has the potential to contribute to an increase in the use of this material, consequently benefitting rubber producing countries such as Malaysia.
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Abstract

Integral abutment bridges (IAB) are increasingly popular in the past few decades due to their design simplicity. The IAB design is currently limited to 60m in the UK and 85m in Malaysia. Past research shows that the IAB’s performance is affected by the cyclic thermal actions that expand and contract the bridge deck, causing an escalation of stress behind the bridge abutment and irreversible settlements near the abutment surface due to the densification of the granular fill.

This research attempts to mitigate the above issues by covering two (2) main parts. On one part, this research investigates the effect of the rubber-based displacement compensation unit (DCU) in the two main performance aspects of the IAB – the stress of the backfill (lateral earth pressure) and the settlement near the wall. The designed rubber DCU mechanism is in its ability to accommodate the change in the bridge’s deck displacement without excessive change of preload. This was achieved by designing the DCU in a hollow rubber cylinder (HRC), which exhibits nonlinear behaviour when axial compressive strains are applied.

Laboratory tests were conducted by deploying the HRC and another type of DCU in the form of conical disc spring (CDS) in the scaled IAB wall with scenarios simulating two magnitudes of thermal displacements; the expected normal displacement for a 60m-long IAB and the displacement for a 120m-long bridge. The results showed that by using both types of DCUs in the wall model with two levels of displacements, the backfill stresses were shown to be approximately unchanged from their initial state, with almost no soil settlement. Whereby the IAB model with the normal thermal displacement showed a rapid backfill stress escalation at the first 50 cycles, with progressive settlements near the surface. The IAB wall with larger displacement showed similar stress escalation and settlement as the normal IAB, except at almost twice in magnitudes.

The effect of the seasons in the IAB wall's initial directional movement was also studied to see how much it would impact the IAB’s performance, with and without the DCU. The results showed that, whether the initial wall position is at ‘winter’ or ‘summer’, there was no apparent difference in peak stress, with and without the DCUs. For the normal IAB walls (without DCUs), regardless of initial wall positions, the backfill stress and settlement values are similar after 20 thermal cycles.
In another part, this research looked to understand the effect of strain and stress magnitudes on creep and stress relaxation of rubber, focusing on the simple shear and buckling deformations. An attempt to evaluate rubber’s long-term properties was conducted, covering two main parts: 1) experimental and 2) numerical methods. The experimental covers the rubber’s stress-strain and the long-term properties, while the numerical method covers the rubber’s constitutive modelling by using the existing viscoelastic model applicable to rubber. Comparisons showed that there is fair agreement in the results between the two methods. Lastly, two accelerated methods – the Time-temperature superposition principle (TTSP) and the Stepped isothermal method (SIM) were used to predict the long-term stress relaxation of unfilled NR, both methods were used with time and temperature dependence and the results compared. The TTSP method was closer in predicting the stress relaxation of unfilled rubber, and the SIM method showed a low degree of confidence in doing so, partly due to the chemical relaxation at high temperature.

The works conducted in both parts, the rubber engineering and the geotechnical engineering, complement each other, as it is important to understand how a rubber-based device behaves under permanent strain when deployed in the geotechnical problem, in this case, the IAB. The amalgamation of the research outcomes will help engineers and rubber technologists to anticipate the issues that present from having such rubber components in the IAB system and also will help to expand the scopes of the research related to these fields.
Table of Content

Declaration .......................................................................................................................... 2
Research Impact Statement ................................................................................................. 3
Acknowledgement ................................................................................................................. 5
Abstract ................................................................................................................................ 6

CHAPTER 1 INTRODUCTION .......................................................................................... 29
  1.1 Background .................................................................................................................. 29
  1.2 Aims and Objectives ................................................................................................... 30
  1.3 Organisation of this Thesis .......................................................................................... 31

CHAPTER 2 LITERATURE REVIEW .............................................................................. 34
  2.1 Integral Abutment Bridge ............................................................................................ 34
    2.1.1 Types of integral abutment bridges ......................................................................... 37
    2.1.2 The issues with integral abutment bridges ............................................................... 39
    2.1.3 Strain ratcheting mechanism ................................................................................ 41
    2.1.4 Thermal strain ...................................................................................................... 47
  2.2 Supported Embedded Walls ......................................................................................... 58
    2.2.1 Temporary props .................................................................................................. 59
    2.2.2 Modular props ..................................................................................................... 59
    2.2.3 Temperature effects on temporary props ............................................................... 60
  2.3 Elements Used to Improve IAB Performance ............................................................... 64
    2.3.1 Compressible inclusion ....................................................................................... 64
    2.3.2 Expanded polystyrene (EPS) .............................................................................. 65
    2.3.3 Tyre-derived aggregates (TDA) ......................................................................... 67
    2.3.4 Displacement Compensation Unit (DCU) ............................................................ 71
  2.4 Rubber as an Engineering Material .............................................................................. 73
    2.4.1 Vulcanisation of rubber and its characteristics ..................................................... 73
    2.4.2 Crosslinking of rubber ...................................................................................... 75
    2.4.3 Filler effect ......................................................................................................... 76
    2.4.4 Elastic Stress-Strain Behaviour of Rubber .......................................................... 78
  2.5 Viscoelasticity .............................................................................................................. 84
    2.5.1 Creep and stress relaxation ............................................................................... 84
    2.5.2 Linear viscoelasticity and the Boltzmann superposition ..................................... 90
    2.5.3 Simple viscoelastic model ............................................................................... 91
    2.5.4 Generalised Maxwell model ............................................................................ 93
    2.5.5 Discretised Prony series .................................................................................. 94
    2.5.6 Time-temperature superposition principle (TTSP) ................................................. 95
    2.5.7 Stepped isothermal method (SIM) ................................................................... 98
  2.6 Rubber in Engineering Applications ........................................................................... 100
    2.6.1 Basic principles for engineering design with rubber ............................................ 101
CHAPTER 3  Viscoelastic Behaviour of Unfilled Natural Rubber ........................................ 112

3.1 Methods of Preparation of Materials and Samples .................................................... 112
  3.1.1 Rubber compound preparation .............................................................................. 112
  3.1.2 Double bonded shear (DBS) sample preparation .................................................. 115
  3.1.3 Hollow rubber cylinder (HRC) preparation ......................................................... 118
  3.1.4 Samples used in this study ...................................................................................... 122

3.2 Experimental Method .................................................................................................. 124
  3.2.1 Stress relaxation test procedure ............................................................................ 124
  3.2.2 Creep test procedure ........................................................................................... 128
  3.2.3 Temperature effect in the measurement of stress relaxation and creep ............. 131

3.3 Stress Relaxation and Creep of Unfilled Natural Rubber ............................................ 134
  3.3.1 Stress relaxation of NR at different strains ........................................................... 134
  3.3.2 Creep of NR at different stresses .......................................................................... 144
  3.3.3 Discussion ............................................................................................................. 151

3.4 Stress Relaxation of Unfilled NR at Arbitrary Strains and Temperatures ..................... 152
  3.4.1 Viscoelastic Model ............................................................................................... 153
  3.4.2 Method ................................................................................................................ 153
  3.4.3 Experimental ...................................................................................................... 153
  3.4.4 Modelling the change in strain (simple shear) .................................................... 155
  3.4.5 Defining stress relaxation by Prony series .......................................................... 156
  3.4.6 Modelling the change in temperature ................................................................. 158
  3.4.7 Complete model for arbitrary strain and temperature ......................................... 160
  3.4.8 Finite element analysis ......................................................................................... 160
  3.4.9 Evaluation on the stress relaxation with the varying strain (constant temperature) .................................................................................................................. 166
  3.4.10 Evaluation on the stress relaxation with varying strain and temperature ......... 170
  3.4.11 Discussion .......................................................................................................... 175

3.5 Accelerated Method for Stress Relaxation of NR ......................................................... 175
  3.5.1 Time-temperature superposition principle (TTSP) and Stepped isothermal method (SIM) .................................................................................................................. 175
  3.5.2 Experimental procedures ..................................................................................... 176
  3.5.3 Effect of temperature on the NR stress relaxation behaviour ............................ 176
  3.5.4 Principles of Stepped isothermal method (SIM) ................................................... 182
  3.5.5 Discussion ............................................................................................................. 193

3.6 Summary ..................................................................................................................... 193

CHAPTER 4  Behaviour of Granular Backfill Under Cyclic Thermal Loadings .................... 195

4.1 The scope of the investigation ..................................................................................... 195

4.2 Methodology .............................................................................................................. 195
  4.2.1 IAB wall experiment and its setup ....................................................................... 196
  4.2.2 Detailing of IAB wall test apparatus .................................................................. 199
CHAPTER 5 Conclusions and Recommendations

5.1 Factors Influencing the Stress Relaxation and Creep of Unfilled NR ................................................................. 275
5.2 Stress Relaxation of Unfilled NR at Arbitrary Strains and Temperatures ............................................................ 276
5.3 Accelerated Method for Stress Relaxation of NR using TTSP and SIM Methods .................................................. 277
5.4 Effectiveness of DCU in Integral Abutment Bridges ............................................................................................... 277
5.5 Recommendations for Future Works .................................................................................................................... 279

References ................................................................................................................................................................. 280

Appendix A ............................................................................................................................................................... 292
List of Figures

Figure 2.1: Schematic diagram of an IAB (b), as opposed to the traditional bridge design (a) (Not to scale). .......................................................... 35

Figure 2.2: Multiple spans continuous IAB with; (a) flexible integral piers (b) self-supporting piers with bearings (Burke Jr, 2009). .......................................................... 35

Figure 2.3: Schematic diagram of an IAB in the United States (Arsoy, Barker and Duncan, 1999) Foundation is mainly made of piles. .......................................................... 37

Figure 2.4: Schematic diagrams of the types of IABs used in the UK (HA, 2003). (a) and (b) frame type abutment; (c) embedded abutment; (d) bank pad abutment; (e) and (f) end screen abutments. .......................................................... 38

Figure 2.5: Semi-integral abutments (BSI PD6694-1, 2011). .......................................................... 38

Figure 2.6: The different patterns of backfill settlement for typical IAB under thermal expansion and contraction. (a) Without the approach slab; (b) With approach slab (Arsoy, Barker and Duncan, 1999). .......................................................... 41

Figure 2.7: Multi-panel filter wall with a tension ring. .......................................................... 42

Figure 2.8: Summary of strain cycling scenarios of wall supporting granular beds (sand). Compression is wall undergoing ‘passive’ loading and extension is wall undergoing ‘active’ loading. ............ 43

Figure 2.9: Rigid wall undergoing constant cyclic strains. .......................................................... 44

Figure 2.10: Flexible wall undergoing repeated strains. .......................................................... 45

Figure 2.11: Flexible wall undergoing strain cycling centred around the isotropic stress state (\(K=1.0\)) .......................................................... 46

Figure 2.12: Settlement variations for 300 thermal cycles. .......................................................... 47

Figure 2.13: Relationship between (a) air temperature and, (b) horizontal bridge displacement for Maple River Bridge, Iowa (Girton, Hawkinson and Greimann, 1991). [Note: 1 inch = 25.4 mm, and \( ^\circ C = (^\circ F - 32) \times \frac{5}{9} \)] .......................................................... 49

Figure 2.14: (a) bridge temperature and, (b) the average backfill pressure over a 33 month period for The Forks Bridge, Maine (Sandford and Elgaaly, 1993) .......................................................... 50

Figure 2.15: Skew effect to the pressure at elevation 177.7m for The Forks Bridge, Maine (Sandford and Elgaaly, 1993) .......................................................... 51

Figure 2.16: (b) Change in average deck temperature and earth pressure on bridge abutments of Haavistonjoki Bridge, Finland. The pressure gauges are labelled as in (a), except gauge ‘V’ and ‘W’ which are in the eastern abutment (Kerokoski and Laaksonen, 2005) .......................................................... 52

Figure 2.17: (a) Abutment movement and (b) pressure variations for the Route 2 New Brunswick bridge (west abutment) (Huntley and Valsangkar, 2013) .......................................................... 53

Figure 2.18: General reproduction of the effective bridge temperatures (EBTs) with the daily and seasonal changes for concrete and composite bridge decks (England, Bush and Tsang, 2000). 54
Figure 2.19: Variables of the IAB used in longitudinal displacement Equation (2.1) (Lock, 2002). This stiff abutment wall with a pinned base is consistent with another model by (England, Bush and Tsang, 2000). ........................................................................................................................................... 55

Figure 2.20: The correlation between the bridge shade temperatures to the effective bridge temperatures (Emerson, 1976b). .................................................................................................................................................. 56

Figure 2.21: The correlation between the bridge shade temperatures to the uniform bridge temperatures (BS EN 1991-1-5:2003). .................................................................................................................................................. 57

Figure 2.22: Types and layout examples of propped excavation (a) Internal support for excavation, elevation view; (b) Internally supported excavation, plan view (William and Waite, 1993)...... 58

Figure 2.23: The relationship between wall movement and soil deformation (Carder, 1995)............ 59

Figure 2.24: The modular unit of a typical hydraulic prop (after Groundforce Shorco 2014). The arrows denote the inflow and outflow inlets of the hydraulic fluid............................................ 60

Figure 2.25: Idealisation of the hydraulic strut modular system (Gould, 2016)............................... 60

Figure 2.26: A brief drop of prop load due to the concrete hydration shown in; (a) Canary Wharf Station (Batten and Powrie, 2000), (b) Crossrail Paddington station (Chambers et al., 2016)........ 63

Figure 2.27: Example of an elastic board behind the abutment. Reinforced backfill comes in the form of geosynthetics mat (Nielsen, 1994). ................................................................................................................................. 64

Figure 2.28: EPS placed behind the abutment. Included are the monitoring instrumentation; steel rod for the displacement and three cells for backfill pressure (Hoppe, 2005)............................... 65

Figure 2.29: Cyclic stress-strain conducted on three EPS specimens, each at different levels of compressive strain. [1 lb/ft2 = 0.05 kPa] (Hoppe, 2005) ........................................................................................................................................... 66

Figure 2.30: Lateral earth pressure for 7 days (Hoppe, 2005). .......................................................... 67

Figure 2.31: (a) Typical full-height abutment and, (b) Isolated full-height abutment with TDA and geogrids (Caristo, Mitoulis and Barnes, 2018).................................................................................................................... 68

Figure 2.32: Backfill pressure escalation on the abutment; solid lines for conventional system and segmented lines for the isolated system (Caristo, Mitoulis and Barnes, 2018) ............... 68

Figure 2.33: Backfill settlement; comparison between the conventional IAB (solid line) and the IAB with TDA (segmented lines) (Caristo, Mitoulis and Barnes, 2018) ......................................................... 69

Figure 2.34: Lateral earth pressure measured for two cases; (a) at rest condition, (b) after support removal (Reddy and Krishna, 2015) ....................................................................................................................... 70

Figure 2.35: The plots showing the wall’s top displacement and % reduction from the effect of having tyre chips for partial sand replacement. ................................................................. 70

Figure 2.36: The possible DCUs placements in IABs in two locations; (a) at the centre of the bridge deck or (b) at the interface between the deck’s ends and the abutments. $d_\text{long}$ is the thermal displacement of the deck in the longitudinal direction................................................................................................. 71

Figure 2.37: (a) The load-deflection characteristics of DCUs used in the model, (b) The influence of the DCU in the soil stresses with the identical thermal deck changes (England et al. 2007)...... 72
Figure 2.38: The different concept of DCU operations (a) linear DCU operation (b) Non-linear DCU operation (England et al., 2010). ............................................................................................................. 72

Figure 2.39: Dependence of network structure on the vulcanisation system. (A) low sulphur/accelerator ratio and high soluble zinc concentration; (B) high sulphur/accelerator ratio or low soluble zinc concentration. S is sulphur and X is an accelerator (Chapman and Porter, 1988). . 74

Figure 2.40: Representation of the network structure of sulphur vulcanisate with different forms of crosslinking. S\[^{\alpha}\], S\[^{\beta}\], S\[^{\gamma}\], and S\[^{\delta}\] indicate the number of S molecules in linkages, with (a, b, x, and y = 1-9) (Chapman and Porter, 1988). ............................................................................................................. 74

Figure 2.41: Schematic rheometer curing curve. 1: minimum viscosity; 2: scorch point (1 torque unit rise above minimum viscosity); T\[^{\text{sc}}\]: scorch time; 3: maximum modulus; 4: 50% crosslinks; 5: optimum cure (90% crosslinks); T\[^{\text{50}}\]: time to 50\% crosslinks; T\[^{\text{90}}\]: time to optimum cure. The cure rate is measured either as T\[^{\text{50}}\]-T\[^{\text{sc}}\] or T\[^{\text{90}}\]-T\[^{\text{sc}}\] (Greensmith and Watson, 1969). .......................................................... 76

Figure 2.42: Tensile stress-strain curve for four NR compounds. A (63 IRHD) contains 45 parts of a reinforcing CB and B (57 IRHD) contains 45 parts of a semi-reinforcing CB. C (44 IRHD) and D (35 IRHD) are unfilled NR (taken from Lindley and Gough (2015)). .......................................................... 77

Figure 2.43: Shear stress-strain curve for four NR compounds. A (65 IRHD) contains 45 parts of a reinforcing CB and B (55 IRHD) contains 30 parts of a semi-reinforcing CB. C (46 IRHD) and D (36 IRHD) are unfilled NR (taken from Lindley and Gough (2015)). .......................................................... 78

Figure 2.44: Pure homogenous strain: (a) unstrained state; (b) strained state ........................................... 79

Figure 2.45: The deformation of chains (after Treloar (1975)). ............................................................... 80

Figure 2.46: Creep plot in (b) following instantaneous stress shown by the dashed line in (a)............... 85

Figure 2.47: Stress relaxation plot in (b) following instantaneous strain shown by the dashed line in (a)........................................................................................................................... 86

Figure 2.48: (a) Stress relaxation and creep rates in extension (Gent 1962a), (b) Stress relaxation rates in extension (Yamaguchi et al. 2015). Both plots are of unfilled NR. .............................................. 87

Figure 2.49: The creep with temperature cycle from 40°C – 60°C – 40°C for; (a) NR, (b) SBR (Derham, 1973). .............................................................................................................................................. 88

Figure 2.50: The stress relaxation rate of NR corresponds to the level of carbon black reinforcement, at 100% extension (Derham, 1973). ........................................................................................................... 89

Figure 2.51: (a) Creep curve at 70°C test temperature, fitted with the relation shown in Equation (2.33) (Derham, Lake and Thomas, 1969). (b) Decomposed form of the creep plot in (a) ........... 90

Figure 2.52: Boltzmann superposition principle ....................................................................................... 91

Figure 2.53: Models of viscoelastic material ............................................................................................. 92

Figure 2.54: Generalised Maxwell model ................................................................................................. 93

Figure 2.55: Stress relaxation master curve for uncrosslinked polyisobutylene and corresponding master curve at 25°C (Tobolsky and Catsiff, 1956). ............................................................................................................. 96
Figure 2.56: The creep test using SIM procedure. (a) correction of the thermal expansion (b) determination of virtual start time, \( t' \) (c) rescaling (d) curve shifting using TTSP shift factor \( a_T \) (Achereiner et al., 2013) ........................................................................................................ 99

Figure 2.57: Different types of load-deflection behaviour for rubber (adapted from Harris & Stevenson 1986). ......................................................................................................................... 101

Figure 2.58: Radial buckling in the outward direction of the axially compressed hollow cylinder in a cross-section view (Willis, 1948). ........................................................................................................ 102

Figure 2.59: Stress-strain curves for cylinders with an outer diameter (OD) of 49.5mm. Figures adjacent to the curves indicate the inner diameter (ID) of the cylinders (Leaver and Lindley, 1976). [Stress = force/rubber cross-section area] ............................................................................. 102

Figure 2.60: Creep of hollow rubber cylinder with carbon black filler, under various load values. The 0.208 ton load reported here is near the hollow rubber cylinder’s critical load (Willis, 1948). [1 ton = 9.81kN] ................................................................................................................................. 103

Figure 2.61: Experimental load-displacement curves for axial compression of square hollow sections (Bakirzis, 1972). ................................................................................................................................. 104

Figure 2.62: Experimental stress-strain curves of compressed hollow rubber cylinders. \( \sigma \) is the stress, and \( \delta L \) is the nominal axial strain. \( D/d \) is the outer-inner diameter ratio (Bakirzis, 1972) .... 105

Figure 2.63: Force–axial compression ratio plot for a short, thick-walled cylindrical rubber column under axial compression. A series of deformed shapes at various levels of axial compression ratio are displayed, showing the formation of creases on the inner wall surface. \( \varepsilon_z \) is the axial strain (Wong et al., 2010). ........................................................................................................ 106

Figure 2.64: (a) Cut section of a 96-year old rubber pad showing a hard surface skin; (b) Drawing of Melbourne Railway Viaduct in 1889 showing the use of natural rubber pads (Ab Malek & Stevenson 1992) ................................................................................................................................. 107

Figure 2.65: Cross-section of Albany Court, the first base-isolated building in the UK (Waller, 1966). ........................................................................................................................................ 108

Figure 2.66: The sectioned steel-laminated rubber bearing for bridges ............................................. 109

Figure 2.67: The compounding mill machine and the finished product. ............................................. 114

Figure 2.68: The rheometer plot for EDS19 compound at 140°C ....................................................... 114

Figure 2.69: DBS samples preparation in sequence ........................................................................... 116

Figure 2.70: Moulded DBS samples (6mm thick). .............................................................................. 117

Figure 2.71: 2mm DBS sample in elevation view. The hatched part on the right is the cross-section of the sample showing the thermocouple inlet. The rubber disks can be 2mm or 6mm in thickness. ........................................................................................................................................ 118

Figure 2.72: The HRC mould; (A) The piston, (B) The main body, with the interchangeable central insert ........................................................................................................................................ 119

Figure 2.73: The 8”x8” Mackey Bowley electric press ........................................................................ 119

Figure 2.74: Moulded HRC sample with inner diameter ID \( \Phi 15 \)mm. All dimensions in mm ........... 120
Figure 3.9: (Left) Cured HRC samples with annular end plates, (Right) HRC cross-section diagram in elevation view. The sample used for the long-term creep and stress relaxation tests.

Figure 3.10: Hollow rubber cylinder sample (HRC); (1) HRC body with HRC ID Ø13mm, (2) Cylindrical test jigs. The sample used for the stress relaxation test with arbitrary strain/temperature and the accelerated relaxation tests.

Figure 3.11: The stress relaxation test rig consisting (A) Temperature-controlled box with the electronic controller, without the lid on; (B) Data Taker DT800 logger with a computer; (C) Fylde Micro Analog logger with a computer; (D) Electronic control unit.

Figure 3.12: DBS sample undergoing permanent shear displacement (δ) or force (P). H is the rubber disk thickness.

Figure 3.13: Applying the strain onto the DBS sample. The tip of the gauge is in contact with an L-shaped bracket attached to the sample.

Figure 3.14: HRC undergoing permanent compressive displacement (δ) or force (P). The sample bottom is fixed to the base.

Figure 3.15: Stress relaxation test setup for an HRC inside the purpose-built temperature-controlled box.

Figure 3.16: Overall setup of the creep test. The temperature-controlled box is shown without the lid on. Fylde datalogger is connected to a computer.

Figure 3.17: The setup and placement of the apparatus for creep test for DBS sample.

Figure 3.18: An HRC sample undergoing creep test. An LVDT is located at the top of the top steel plate (unseen), recording the change in displacement in the vertical direction.

Figure 3.19: Temperature observation inside the stress relaxation test box.

Figure 3.20: Temperature observation inside the creep test box.

Figure 3.21: Stress-strain curve of a 6mm DBS sample, shown here up to 190% shear strain (third cycle). Red circles show the permanent shear strains applied in the stress relaxation tests.

Figure 3.22: Stress relaxation curves of DBS at different shear strains.

Figure 3.23: A stress relaxation curve at 30°C for sample DBS-A4. The possible error due to the temperature fluctuation of ±0.5°C is shown in the red vertical bars.

Figure 3.24: Relaxation modulus at several shear strains.

Figure 3.25: Stress relaxation rate (% per decade) against shear strain.

Figure 3.26: The relaxation moduli at two different times. “Initial” is when the sample was fully strained (t=10s).

Figure 3.27: Stress-strain curve of an HRC under 15mm axial compression.

Figure 3.28: Stress relaxation curves of HRC samples at different strains.

Figure 3.29: Relaxation modulus plots in semi-log for several compressive strains. The linear fit for each plots are represented by black dashed lines, except for strain $\varepsilon=0.16$ (yellow dashed line).

Figure 3.30: Stress relaxation rate (% per decade) against compressive strain.
Figure 3.31: Relaxation modulus at different compressive strains, plotted for two different times. Initial is the time when the sample was fully strained (~10s) .......................................................... 144

Figure 3.32: The shear strain of DBS samples at different stresses. A closer view of the first 20 seconds is on the left .......................................................... 145

Figure 3.33: Creep compliance plots in semi-log for several shear stresses .......................................................... 146
Figure 3.34: Creep rate (% per decade) against shear stress ........................................................................ 147
Figure 3.35: Creep compliance at different shear stress, taken at the initial (~10s) and 1,000,000s. \( r \) in MPa. .................................................................................................................. 147

Figure 3.36: The displacement of the HRC in linear timescale at different permanent axial stresses. \( (\sigma_b \text{ in MPa)} \) .................................................................................................................. 148

Figure 3.37: Creep compliance plots in semi-log for several compressive stresses ........................................ 149
Figure 3.38: Creep rate (% per decade) against compressive stress .......................................................... 150
Figure 3.39: Creep compliance at different buckling stress, shown at the initial (~10s) and 100,000s. \( (\sigma_b \text{ in MPa)} \) .................................................................................................................. 150

Figure 3.40: General experiment setup for the stress relaxation with arbitrary strains and temperatures. Label “A” is the Instron temperature chamber .......................................................... 154

Figure 3.41: Apparatus for the stress relaxation with the arbitrary strain and temperature in the Instron temperature chamber. The thermocouples are shown as; TC-1 (mid box), TC-2 (near sample bottom) and TC-3 (sample top) ........................................................................ 155

Figure 3.42: The strain history with sequence; (1) ramp to \( \gamma_0 \), (2) hold, (3) release strain .................. 155

Figure 3.43: Plane strain DBS model. \( \delta \) is the axial displacement. ENCASTRE constraint is applied at the bottom surface of the model (red segmented line). Dimension is in mm. ........................................ 161

Figure 3.44: (a) The axisymmetric HRC model, with CL as the centreline (b) The mesh part of the HRC, \( \delta \) is the axial displacement, and ENCASTRE constraint is applied at the bottom surface of the model (dimension in mm) ........................................................................ 162

Figure 3.45: Stress relaxation plot at shear strain \( \gamma=1.0 \). The blue is the experimental data with the straining phase ~10s ........................................................................................................ 163

Figure 3.46: Comparison between the experiment and the predicted relaxation behaviour in simple shear at 297K with \( G_c = 0.364 \text{MPa and } H_0 = 0.004 \) ........................................................................ 167

Figure 3.47: Stress-strain behaviour of unfilled rubber (EDS19) in simple shear .................................. 168

Figure 3.48: Comparison between the experiment and the simulation of the relaxation behaviour in compression at 297K ........................................................................................................ 169

Figure 3.49: Force-displacement behaviour of unfilled rubber (EDS19) under compression ............... 170

Figure 3.50: Stress relaxation data (Exp) in shear corresponds to the change in temperature ........... 171

Figure 3.51: (Top) Shear strain and temperature history applied to the DBS sample. (Bottom) The stress relaxation corresponding to both inputs. .................................................................................. 172

Figure 3.52: Stress relaxation data (Experiment) in compressive strain corresponds to the change in temperature ........................................................................................................ 173
Figure 3.53: (Top) Compressive strain and temperature history applied to the HRC sample. (Bottom) The stress relaxation corresponds to both inputs. ................................................................. 174
Figure 3.54: Raw stress relaxation data based on TTSP in shear deformation ($\gamma=1.0$). ............... 179
Figure 3.55: Horizontal shifting according to TTSP (DBS sample). Segmented lines - curves after shifting vertically. Solid lines – curves after shifting vertically and horizontally. Shift factors are given in Table 3.6................................................................. 180
Figure 3.56: Comparison between long-term stress relaxation (SR) experiment at 24°C and the TTSP method in simple shear ($T_0=297K$). ................................................................. 181
Figure 3.57: Comparison between stress relaxation (SR) test at 24°C and the TTSP method for compressed HRC testpieces ($T_0=297K$). ................................................................. 182
Figure 3.58: The shear stress corresponding to the change in temperature (SIM experiment on DBS test piece). ................................................................................................................. 185
Figure 3.59: Overview of the SIM correction due to thermal expansion and determining the virtual starting point, $t'$ for creating individual stress relaxation curve. Virtual time scale = $t-t'$. ...... 186
Figure 3.60: Rescaling the second isothermal curve using the virtual time scale ($t-t'$). ................. 187
Figure 3.61: Vertical and horizontal shifting on the second isothermal ($T=32^\circ C$)........................... 187
Figure 3.62: Stress relaxation master curve (SIM) for DBS at $\gamma=1.0$................................................. 188
Figure 3.63: Comparison plot of stress relaxation between methods; conventional test, TTSP and SIM................................................................................................................. 189
Figure 3.64: SIM raw data for HRC at 37% compressive strain.............................................................. 190
Figure 3.65: Difference in plots between the raw and the thermally-corrected SIM data for HRC... 191
Figure 3.66: Determination of the virtual start time, $t'$ for the individual stress relaxation isothermal curves. ................................................................................................................. 191
Figure 3.67: The overview of the stress relaxation curves comparison between the conventional test and the SIM. ................................................................................................................. 192
Figure 4.1: The sand pluviation involves tracking the hopper back and forth along the rail, ensuring the sand is uniformly filled by layers. (Location: Soil Laboratory, UCL). ............................................. 197
Figure 4.2: The positions of aluminium tins used to retain the pluviated sand....................................... 198
Figure 4.3: The interface of the VEE Pro programme used for data logging and instrumentation monitoring. The top inset graph window shows the pressure, and the bottom shows the wall displacement (Voltage, $V$ represents both). Data were recorded in time function. ............ 199
Figure 4.4: The typical frame type IAB adopted for the scaled model. Note that $d$ is the deck thermal displacement at one end, and $H$ is the abutment wall height........................................... 200
Figure 4.5: The schematic diagram of the scaled IAB wall model apparatus with the instrumentations. ............................................................................................................................... 200
Figure 4.6: The elevation view of the model retaining wall apparatus, shown here filled with the sand................................................................................................................................. 201
Figure 4.7: Pressure sensors embedded in the aluminium plate that acts as the IAB’s retaining wall. Behind the circular protective shims are the pressure sensors. (Location: Soil Laboratory, UCL).

Figure 4.8: The linear potentiometers are located at two locations: (a) LVDT-1: Attached to the rotating retaining wall, (b) LVDT-2: Attached to the moving actuator of the motor. (Location: Soil Laboratory, UCL).

Figure 4.9: The calibration of the pressure sensor by using the hydrostatic pressure at different heights. The water tube is attached to a square steel section.

Figure 4.10: (a) The pressure sensor is sealed within the calibration rod head equipped with a valve, allowing for the water tube connection, (b) The calibration rod being held across the soil tank.

Figure 4.11: Particle size distribution of Leighton Buzzard sand.

Figure 4.12: Sieve set (adhering to ISO 3310-1: 2000)

Figure 4.13: ELE Sieve Shaker used in the particle distribution analysis.

Figure 4.14: The detail of the local instrumentation (hall effect sensor) used in the triaxial test; (left) schematic diagram, (right) actual sample sample fixed on the test pedestal, with two hall effect sensors affixed.

Figure 4.15: The overall triaxial cell setup with Leighton Buzzard sand specimen.

Figure 4.16: Overall setup for the triaxial test done on Leighton Buzzard sand.

Figure 4.17: The Mohr-Coulomb failure envelope shown by three triaxial tests; T1, T2 and T3 with effective cell pressure of 50kPa, 100kPa, and 150kPa respectively.

Figure 4.18: Active and passive movements of a wall.

Figure 4.19: Overview of the HRC static force-deflection test setup (A) Instron 4301 machine, (B) An HRC sample held under the machine actuator, (C) A set of CDS being partially compressed by the actuator, (D) The computer with PicoLog ADC-20 Data Logging System. (Location: Engineering Laboratory, TARRC).

Figure 4.20: A unit of CDS underneath a test machine actuator. A single CDS unit’s dimension is 16mm of outer diameter (OD) and 6mm of inner diameter (ID).

Figure 4.21: The basic configurations of the conical disc spring (CDS).

Figure 4.22: (a) A stack of 10 disc spring in series, placed at the test machine actuator (Instron 4301), (b) The CDS under 57% compression (6.5mm).

Figure 4.23: The CDS force-displacement curves of different configurations. All curves were calculated from the 1-disc curve except “Test 1” and “Test 2” (in red and black markers respectively).

Figure 4.24: Sequence of deformed HRC shapes indicated as (a) the stiff linear state, (b) the soft region, also referred as the ‘plateau’ (c) the stiff jounce state (Refer Figure 4.26)

Figure 4.25: The stress-strain curve of HRCs with a variation of IDs. Solid curves are for the first cycle, and dashed curves are for the third cycle.
Figure 4.26: The 3rd and 10th cycle of the HRC (ID Ø13mm) show almost coinciding force-displacement trace. .................................................................................................................................................. 227

Figure 4.27: The force-displacement behaviour of CDS (10-unit stack) and HRC (ID Ø13mm). ........... 229

Figure 4.28: The locations of two (2) units of DCU (HRC) viewed from the top the wall. ................. 230

Figure 4.29: The installation of the DCUs at the back of the IAB wall model for different tests. (a) IAB0.25 - control test; (b) HRC0.25; (c) CDS0.25. ........................................................................................................................................ 231

Figure 4.30: The locations of every sensor (PrS and LVDTs) and DCU from the bottom hinge. The curvature shows the exaggerated wall displacement at LVDT-1 and LVDT-2 (black dotted line), assumed by the linear representation (blue arrows). .................................................................................. 233

Figure 4.31: Initial stress state for IAB0.25 and IAB0.50. ........................................................................ 234

Figure 4.32: Backfill pressure behind the wall for IAB0.25 test. The segmented lines represent the active and passive pressure profiles at the initial state using the $K_a$ (coefficient of active earth pressure) and $K_p$ (coefficient of passive earth pressure). ............................................................................................................................. 236

Figure 4.33: Backfill pressure behind the wall for IAB0.50 test. The segmented lines represent the active and passive pressure profiles at the initial state using the $K_a$ (coefficient of active earth pressure) and $K_p$ (coefficient of passive earth pressure). ............................................................................................................................. 237

Figure 4.34: Pressure-displacement response of the backfill for IAB0.25 at PrS-2. Point O is the origin loading state, line O-A is the first path during the expansion loading, and A-B is the first contraction unloading. ..................................................................................................................................... 238

Figure 4.35: Pressure-displacement response of the backfill at different cycle for IAB0.25 at PrS-4.239

Figure 4.36: Pressure-displacement response of the backfill at different cycle for IAB0.50 at PrS-4.240

Figure 4.37: The initial stress states, before and after DCU release for CDS0.25 (left), and HRC0.25 and HRC0.50 (right). The ‘0 cycle – after release’ is the new initial stress state. ....................... 241

Figure 4.38: Backfill pressure behind the wall for CDS0.25. The segmented lines represent the active and passive pressure profiles at the initial state using the $K_a$ (coefficient of active earth pressure) and $K_p$ (coefficient of passive earth pressure). ............................................................................................................................. 243

Figure 4.39: Backfill pressure behind the wall for HRC0.25. The segmented lines represent the active and passive pressure profiles at the initial state using the $K_a$ (coefficient of active earth pressure) and $K_p$ (coefficient of passive earth pressure). ............................................................................................................................. 244

Figure 4.40: Backfill pressure behind the wall for HRC0.50. The segmented lines represent the active and passive pressure profiles at the initial state using the $K_a$ (coefficient of active earth pressure) and $K_p$ (coefficient of passive earth pressure). ............................................................................................................................. 245

Figure 4.41: Displacement versus thermal cycle for CDS0.25 test. The red line is the wall movement, the grey is the thermal expansion and contraction movement at the sand surface level, whilst the green is the CDS movement. ............................................................................................................................. 247

Figure 4.42: Displacement versus thermal cycle for HRC0.25 test. The red line is the wall movement, the grey is the thermal expansion and contraction movement at the sand surface level, whilst the blue is the HRC movement. ............................................................................................................................. 248
Figure 4.43: Pressure-displacement response of the wall at different cycle for CDS0.25 at PrS-2 (left) and PrS-4 (right). ................................................................. 250
Figure 4.44: Pressure-displacement response of the wall at different cycles for HRC0.25 at PrS-2 (left) and PrS-4 (right). ................................................................. 250
Figure 4.45: Pressure-displacement response of the wall at different cycle for HRC0.50 at PrS-3. The active and passive stress limits are 1.47kPa and 23.25kPa, respectively. ........................................... 251
Figure 4.46: Force and displacement range of CDS after 5800 cycles (Test: CDS0.25). ................. 252
Figure 4.47: Force and displacement range of HRC after 5800 cycles for HRC0.25, and 740 for HRC0.50 compared to the first cycles. ................................................................. 252
Figure 4.48: (a) - (g) Backfill settlements for three cases; IAB0.25 (top left), HRC0.25 (top right), and CDS0.25 (bottom). Number of cycles shown on the photos. ................................ 257
Figure 4.49: Influence of the number of cycles on settlement profile for amplitude tests with $d/H=0.25\%$. ................................................................................. 257
Figure 4.50: The settlement at the abutment wall for IAB0.25. The inset shows the settlement in log_{10} scale ................................................................. 258
Figure 4.51: Influence of the number of cycles on settlement profile for amplitude tests with $d/H=0.50\%$. ................................................................................. 259
Figure 4.52: The settlement at the abutment wall for IAB0.50. The inset shows the settlement in log_{10} scale ................................................................. 259
Figure 4.53: Progressive settlement behind the IAB wall with the seasonal cycle for two values of rotational displacement ($d/H=0.25\%$ and $0.50\%$) ................................................................. 260
Figure 4.54: Settlement rates for two rotational displacement magnitudes in the log_{10} scale. ....... 260
Figure 4.55: IAB wall initial positions and its movement sequences; (a) middle (b) winter (c) summer. ................................................................................. 261
Figure 4.56: Backfill pressure behind the wall for IAB cases. ................................................................. 262
Figure 4.57: Backfill pressure behind the wall for CDS cases. ................................................................. 263
Figure 4.58: Backfill pressure behind the wall for HRC cases. ................................................................. 264
Figure 4.59: Settlement comparison between three walls at two initial positions; (a) winter, (b) summer. The lines for CDS and HRC are representing 120 cycles. .................................................. 265
Figure 4.60: Influence of initial wall positions and wall rotation amplitudes to the pressure buildup by the number of cycles. ................................................................. 267
Figure 4.61: Influence of initial wall positions on surface settlement after (a) 1, 5 and 10 cycle; (b) 20, 50 and 100 cycle, for $d/H=0.25\%$ wall rotation amplitude. Blue plots represent the IAB_winter and red the IAB_summer. Black lines represent the IAB0.50 test. .................................................. 268
Figure 4.62: The load-temperature relation for the props used in the excavation near the Elephant and Castle Station (Site-1). Prop-1 and Prop-2 refer to the axial load for two different prop units. TC1 and TC2 refer to temperature readings for Prop-1 and Prop-2, respectively. TC3 ambient is the ambient temperature of the site. .................................................. 271
Figure 4.63: The load-temperature relation for five-unit of props used in an excavation on Liverpool Street Station, London (Site-2), assigned as Prop-1 to Prop-5. TC Ambient denotes the ambient temperature on site. ........................................................................................................ 272

Figure 4.64: Simplified illustration for a single HRC unit for one CHS prop installation arrangement (Elevation view). The diagram is not to scale. ................................................................. 273

Figure 0.1: The calibration plots of PCM load transducers Channel 1 to 4 are shown in (a), (b), (c) and (d). On the left of each plots are the load readings in the logger corresponding to the applied dead loads. ........................................................................................................ 294

Figure 0.2: The Type-K thermocouple calibration with; (a) thermocouples at room temperature water beaker (b) thermocouples at elevated temperature from heated beaker (c) real-time temperature logging through Fylde Micro Analog logger with the computer .............. 295

Figure 0.3: Dial gauge used in the test piece straining. The gauge head is in contact with a _-shaped bracket attached to the middle steel part of the Double Bonded Shear (DBS) test piece during the straining stage. ........................................................................................................ 296

Figure 0.4: The simple calibration setup for the dial gauge ....................................................... 297

Figure 0.5: Calibration plot of the dial gauge (based on Table A-2) ........................................ 297

Figure 0.6: Force-displacement plot for a DBS at 100% shear strain (at half cycle). Displacement at 100% strain is 5.65mm and the force is 398.17N ................................................................. 298

Figure 0.7: The calibration plots for five (5) LVDTs used in creep tests. .................................. 299

Figure 0.8: An LVDT being calibrated using a Mitutoyo thickness gauge block, with a digital signal indicator (RDP E525) as a reference. Shown here is the LVDT of RDP ACT500A type. ............ 300

Figure 0.9: Mitutoyo thickness gauge set used in the calibration of LVDTs ............................... 300

Figure 0.10: The electronic system built for the temperature-controlled box for multiple stress relaxation and creep tests .......................................................... 301

Figure 0.11: Shear strains applied on some of the DBS samples; (a) DBS-A3, (b) DBS-A4, (c) DBS-A5, and (d) DBS-A6 .............................................................. 302

Figure 0.12: Compressive strains applied on some of the HRC samples; (a) HRC-A1, (b) HRC-A2, (c) HRC-A4, and (d) HRC-A6 ...................................................... 302

Figure 0.13: Calibration record for Pressure Sensor 1 (PrS-1) .................................................. 303

Figure 0.14: Calibration record for Pressure Sensor 2 (PrS-2) .................................................. 303

Figure 0.15: Calibration record for Pressure Sensor 3 (PrS-3) .................................................. 303

Figure 0.16: Calibration record for Pressure Sensor 4 (PrS-4) .................................................. 303

Figure 0.17: Calibration record for Pressure Sensor 6 (PrS-6) .................................................. 304

Figure 0.18: Calibration record for Pressure Sensor 7 (PrS-7) .................................................. 304

Figure 0.19: Calibration plot for LVDT-1 .............................................................. 304

Figure 0.20: Calibration plot for LVDT-2 .............................................................................. 304

Figure 0.21: Shear strain and temperature history applied to the DBS sample ......................... 308

Figure 0.22: Undeformed DBS model in plane strain ....................................................... 308
Figure 0.23: Shear strain $\gamma=1.00$ and temperature 24°C (297K). ................................................................. 309
Figure 0.24: Shear strain $\gamma=1.00$ and temperature 53°C (326K). ................................................................. 309
Figure 0.25: Shear strain $\gamma=1.00$ and temperature 6.5°C (279.5K) ............................................................... 309
Figure 0.26: Shear strain $\gamma=1.00$ and temperature 6.5°C (279.5K) ............................................................... 310
Figure 0.27: Compressive strain and temperature history applied to the HRC sample. ........................................ 310
Figure 0.28: Underformed HRC model in the ABAQUS. ......................................................................................... 311
Figure 0.29: Compressive strain $\varepsilon_b=37\%$ and temperature 24°C (297K). .................................................... 311
Figure 0.30: Compressive strain $\varepsilon_b=37\%$ and temperature 53°C (326K) ...................................................... 311
Figure 0.31: Calibration plot of the triaxial load cell. ............................................................................................. 312
Figure 0.32: Calibration plot of the external LVDT (triaxial cell). ........................................................................... 313
Figure 0.33: (a) - (h) Backfill settlements for two cases; IAB0.50 (left), and HRC0.50 (right). Number of cycles shown on the photos. ........................................................................................................ 315
List of Tables

Table 3.1: Formulations of the EDS19 compound in part per hundred rubber (pphr) .................. 113
Table 3.2: Experiments and analyses with the DBS and HRC sample identification (ID) .................. 122
Table 3.3: Parameters for the EDS19 in stress relaxation with varying strains (fit to Prony series) 165
Table 3.4: Parameter inputs in ABAQUS for changing temperatures .................................. 165
Table 3.5: The new viscoelastic constants for the reference temperature, \( T_0 \) of 24°C .............. 177
Table 3.6: The horizontal and vertical shift factors for TTSP master curve, according to every sample’s response temperatures (in Kelvin, K). Calculated from Equations (2.49) and (2.53). 180
Table 3.7: The parameters for constructing the SIM master curve (DBS sample) ...................... 188
Table 3.8: The parameters for constructing the SIM master curve (HRC test piece) ............... 192
Table 4.1: The sand density record for the IAB0.25 experiment ........................................ 198
Table 4.2: Calculation for the particle size analysis ......................................................... 206
Table 4.3: The properties of Leighton Buzzard coarse sand ...................................... 212
Table 4.4: The correlations between the shade temperatures to the EBT and UBT (°C) for three main regions of Great Britain ................................................................. 214
Table 4.5: Magnitudes of \( d/H \) (%) selected by researchers for physical modelling. ................. 216
Table 4.6: The details of the model IAB retaining wall tests .............................................. 218
Table 4.7: The stress at buckling for HRC of \( OD = 25\text{mm} \) and varying \( ID s \). ......................... 228
Table 4.8: Difference in peak pressure between the initial and after 100 cycles of wall movement. 246
Table 0.1: Calibration record for load transducer Ch1 ...................................................... 293
Table 0.2: Calibration record for load transducer Ch2 ...................................................... 293
Table 0.3: Calibration record for load transducer Ch3 ...................................................... 294
Table 0.4: Calibration record for load transducer Ch4 ...................................................... 294
Table 0.5: The calibration record for the thermocouples checked against the mercury thermometer .............................................................. 296
Table 0.6: Part 1 - Normal IAB with seasonal rotational displacement \( d/H=0.25\% \) and \( d/H=0.50\% \). 304
Table 0.7: Part 2 - Normal IAB with DCUs inclusions ..................................................... 305
Table 0.8: Part 3 - The effect of initial wall positions according to the seasons ....................... 305
Table 0.9: Part 4 - The HRC under repetitive loadings at different temperature ....................... 306
Table 0.10: Calibration record of the triaxial load cell ................................................... 312
Table 0.11: Calibration record of the external LVDT of triaxial cell ...................................... 313
Nomenclature

$A$ Rate of decrease of stress (%)  
$A_{H}$ Helmholtz free energy  
$A_{DBS}$ Area (mm$^2$)  
$ID$ Inner diameter (mm)  
$A_{hyd}$ Bore area of the of hydraulic strut (mm$^2$)  
$A_{CHS}$ Cross-sectional area of the CHS section (mm$^2$)  
$a_g$ Gravitational acceleration (m/s$^2$)  
$a_T$ Horizontal shift factor (for TTSP)  
$B(t)$ Relaxation modulus (in buckling compression, MPa)  
$B_0$ Buckling modulus at $t_0$ (MPa)  
$C1^o$ Viscoelastic constants for WLF (1$^{st}$)  
$C2^o$ Viscoelastic constants for WLF (2$^{nd}$)  
$C$ Creep rate  
$D(t)$ Creep compliance (in buckling compression)  
$D_0$ Creep compliance at $t_0$ (buckling)  
$d_{long}$ Longitudinal displacement of deck (mm)  
$d$ Wall displacement (mm)  
$E_{hyd}$ Young’s modulus of hydraulic cylinder (MPa)  
$E$ Elastic modulus (MPa)  
$E_0$ Young’s modulus of rubber (MPa)  
$E_{LB}$ Young’s modulus of Leighton Buzzard sand (MPa)  
$E_{CHS}$ Young’s modulus of steel CHS (MPa)  
$F$ Axial force (N)  
$G$ Shear modulus (MPa)  
$G(t)$ Relaxation modulus (in shear, MPa)  
$G_0$ Shear modulus at $t_0$ (MPa)  
$G_{EDS19}$ Shear modulus of EDS19 compound (MPa)  
$G_{EDS31}$ Shear modulus of EDS31 compound (MPa)  
$H$ Abutment wall height (mm)  
$H_r$ Thickness of rubber (mm)  
$H_0$ Relaxation slope  
$H_x$ HRC height (mm)  
$h$ HRC wall thickness (mm)  
$h_w$ Water level height (mm)  
$ID$ Inner diameter of rubber cylinder (mm)  
$J(t)$ Creep compliance (in simple shear)  
$J_0$ Creep compliance at $t_0$ (shear)  
$k$ Boltzmann constant  
$k_s$ Spring stiffness (N/m)
\( K_{EFS} \) Effective length factor
\( K_{test} \) DBS shear stiffness (N/mm)
\( K_{gauge} \) Dial gauge’s stiffness (N/mm)
\( K_{res} \) Resultant stiffness of a modular prop (kN/m)
\( L_{hyd} \) length of hydraulic cylinder
\( L_{CHS} \) is the length of the CHS section (mm)
\( L \) Bridge length (mm)
\( N \) Rubber chain per unit volume
\( n \) Prony series term number
\( OD \) Outer diameter of rubber cylinder (mm)
\( P \) Load (N)
\( P_{cr} \) Critical load at buckling (N)
\( p \) Arbitrary hydrostatic pressure
\( q \) Deviatoric stress (kPa)
\( R \) Average radius of the rubber unit shell (mm)
\( r \) End-to-end rubber chain length (vector)
\( r_g \) Radius of gyration of the cross section
\( s_0 \) Entropy of a single rubber chain at unstrained state
\( S_0 \) Total entropy of a single rubber chain at unstrained state
\( s \) Entropy of a single rubber chain at strained state
\( S \) Stress relaxation rate
\( SD \) Standard deviation
\( T \) Absolute temperature (°C/K)
\( T_{sc} \) Scorch safety time (minute)
\( T_{50} \) Time for 50% crosslinks (minute)
\( T_{90} \) Time for 90% crosslinks (minute)
\( T_g \) Glass-transition temperature (°C/K)
\( T_{ref} \) Reference temperature of material (°C/K)
\( T_s \) Shift temperature (°C/K)
\( T_0 \) Initial reference temperature of test (°C/K)
\( v_5 \) Void ratio of sand
\( \nu \) Poisson’s ratio
\( W \) Strain energy density function
\( t_1 \) Principal stress (direction-1)
\( t_2 \) Principal stress (direction-2)
\( t_3 \) Principal stress (direction-3)
\( t_0 \) Reference time for creep and stress relaxation (s)
\( U \) Internal energy in a single rubber unit (J)
\( \sigma \) Axial stress (MPa)
\( \sigma_s \) Spring stress (MPa)
\( \sigma_d \) Dashpot stress (MPa)
\( \sigma_1 \) Major principal stress (axial) (kPa)
\( \sigma_3 \) Minor principal stress (radial) (kPa)

\( \sigma_b \) Buckling stress (MPa)

\( \sigma_i \) Principal stress (MPa)

\( \varepsilon \) Axial strain (%)

\( \varepsilon_s \) Spring strain (%)

\( \varepsilon_d \) Dashpot strain (%)

\( \varepsilon_f \) Strain at failure (triaxial test) (%)

\( \varepsilon_b \) Buckling strain (%)

\( \varepsilon_0 \) Creep compliance at the reference time, \( t_0 \)

\( \alpha_{\text{conc}} \) Thermal expansion coefficient of concrete (1/°C)

\( \alpha_{\text{steel}} \) Thermal expansion coefficient of steel prop (1/°C)

\( \tau \) Shear stress (MPa)

\( \tau_{\text{min}} \) Lowest point of relaxation time

\( \tau_{\text{max}} \) Highest point of relaxation time

\( \gamma \) Shear strain (%)

\( \bar{\gamma}_3 \) Average dry density of sand (kg/cm³)

\( \lambda_1 \) Principal extension ratio (direction-1) (dimensionless)

\( \lambda_2 \) Principal extension ratio (direction-2) (dimensionless)

\( \lambda_3 \) Principal extension ratio (direction-3) (dimensionless)

\( \mu \) Friction coefficient

\( \xi \) Vertical shift factor (for TTSP)

\( \delta T_{\text{EB}} \) Effective Bridge Temperature (°C/K)

\( \delta \) Rubber displacement (mm)

\( \phi \) Internal friction angle (°)

\( \rho_w \) Density of water (kg/m³)

\( \rho \) Density (kg/cm³)
Abbreviations

BS  British Standards
BSP  Boltzmann superposition principle
CAE  Computer-aided engineering
CBS  N-Cyclohexylbenzothiazole-2-sulphenamide
CHS  Circular hollow section (steel prop)
CV  Constant viscosity
CDS  Conical disc spring
DBS  Double-bonded shear
DCU  Displacement compensation unit
EBT  Effective bridge temperature
EDS  Engineering Datasheet
EPS  Expanded polystyrene
EV  Efficient vulcanisation
FEA  Finite Element Analysis
FEM  Finite Element Modeling
HRC  Hollow rubber cylinder
OWT  Offshore wind turbine
IAB  Integral abutment bridge
IR  Synthetic polyisoprene
IRHD  International Rubber Hardness Degree
ISO  International Organization for Standardization
MDF  Medium-density fibreboard
MRPRA  Malaysian Rubber Producer’s Research Association
NR  Natural Rubber (polyisoprene)
PID  Proportional-integral-derivative
pphr  Part-per-hundred Rubber
SR  Synthetic rubber
SBR  Styrene-butadiene rubber
SIM  Stepped isothermal method
SMR  Standard Malaysian Rubber
TARRC  Tun Abdul Razak Research Centre
TBTD  Tetra-n-butylthiuram disulphide
TBBS  N-tert-Butyl-2-benzothiazolesulfenamide
TC  Thermocouple
TMTD  Tetramethyl thiuram disulphide
TTSP  Time-temperature superposition principle
WLF  William-Landel-Ferry
ZEH  Zinc 2-ethyl-hexanoate
6PPD  N-(1,3-Dimethylbutyl)-N’-phenyl-p-phenylenediamine (Santoflex 13)
CHAPTER 1 INTRODUCTION

1.1 Background

An integral abutment bridge (IAB) is constructed with integrated joints, connecting the deck and the abutments, to form a single rigid frame. Some deck movement is caused by creep and shrinkage but mainly by thermal expansion and contraction. Due to its design and construction method, an IAB is partially or fully restrained by its substructures (Arockiasamy and Sivakumar, 2005; Kim and Laman, 2012; Huntley and Valsangkar, 2013). During service, the thermal actions expand and contract the deck and subsequently move the abutment walls. These cyclic actions cause densification and progressive flow of the backfill material, consequently generating excessive stresses in the concrete - a phenomenon known as strain ratcheting (England, 1994; England, Bush, and Tsang, 2000). The strain ratcheting, however, is not unique to IAB, and is also observed in other engineering structures such as during temporary propping of retaining walls.

In the case of retaining walls, the thermal actions on the temporary props change its length, and subsequently, change the axial load acting in the props. This could induce undesirable deformations to the walls being supported (Batten and Powrie, 2000; Powrie and Batten, 2000; Loveridge, 2001). If the loads acting on the props are not kept sufficiently constant, the embedded walls tend to move inwards (away from the wall). This will cause densification of the soil behind the wall, resulting in a progressive soil settlement (Carder, 1995; Chambers et al., 2016).

In another area, when a constant strain is applied to rubber, the counterforce exerted by the specimen decreases in time. This is called stress relaxation, a phenomenon first reported by Tobolsky, Prettyman, and Dillon (1944) on rubber and rubber-like materials. More contributions include (Gent, 1962a, 1962b; Derham, 1973; Derham and Thomas, 1977; Chai and Thomas, 1981), and later by (Ronan et al., 2007; Fernandes and De Focatiis, 2014; Yamaguchi, Thomas and Busfield, 2015) with studies on predicting the viscoelastic behaviour of rubber as a function of time and temperature. The current information mostly covers the stress relaxation of rubber for the specimens in compression, tension and shear. The industrial applications related to stress relaxation are sealing rings, weather strips and hoses, with the testing standards covering the tests in compression and tension (BS ISO 3384-1, 2013; BS ISO 6914, 2013).
In overcoming the densification problems, a method of changing the cyclic displacement applied by the structure in the soil fill to an approximately constant compressive stress will be described and attempted. This was done by using a Displacement Compensation Unit (DCU) - an elastic device between deck and abutment that accommodates the displacement without excessive change of preload. Scaled DCUs used in the initial proof-of-concept were based on conical disc springs and gave encouraging results (England et al., 2007). As with steel bearings, using conical disc springs to accommodate thermal expansion, corrosion would be anticipated to be a significant issue for steel DCUs, so use of rubber DCUs much more attractive. This research will focus on the attempt to evaluate the efficacy of rubber DCU in the IAB.

Such devices would need to operate over very long periods of time and thus it is important to understand how the stress would reduce over this timescale due to stress relaxation in the rubber devices. Therefore, this research also addresses the need for a better understanding of the mechanical and long-term stress relaxation behaviour of rubber. By understanding the long-term stress relaxation behaviour, components such as the DCU can be designed with better durability and longevity. Furthermore, the process of developing new rubber compounds will be accelerated. To achieve this, a suitable viscoelastic model and a method to predict the long-term stress relaxation in rubber will be described and evaluated.

1.2 Aims and Objectives

This research aims to:

1. Investigate the key mechanical factors that affect the stress relaxation and creep behaviour of unfilled natural rubber (NR), including the strain and stress levels.
2. Introduce a new method to predict the long-term behaviour of unfilled NR under constant deformation over a range of operating temperatures.
3. Introduce a novel rubber-based DCU component as a solution to the densification issue associated with the IAB.
4. Investigate the effect of the seasons in the initial directional movement of the IAB wall to its performance.
Consequently, the main objectives of this research are to:

1. Conduct a systematic experimental campaign to investigate the effect of strain magnitude on the stress relaxation of unfilled NR in simple shear and buckling deformations, and reciprocate the method by evaluating the effect of different stress magnitudes on creep.

2. Assess the validity of the linear viscoelastic theory on the stress relaxation of an unfilled NR in simple shear and buckling deformations. This will be done by implementing the Boltzmann superposition principle to simulate strain changes and the Time-temperature superposition principle or WLF (William-Landel-Ferry) transformation to simulate the changes in temperature. Comparisons are to be made between the experimental results, calculations, and Finite Element Analysis (ABAQUS).

3. Apply and compare the Time-temperature superposition and the Stepped isothermal method (SIM) in predicting the long-term stress relaxation behaviour of unfilled NR in constant simple shear and buckling deformations.

4. Design and evaluate the mechanical and long-term performance of rubber-based DCUs at a range of wall displacement magnitudes, including the comparison with the steel-based DCU.

5. Investigate the factor of the initial position of the IAB wall representing two main seasons; summer and winter, and its effect to the pressure and settlement of the backfill.

1.3 Organisation of this Thesis

This thesis is organised in five (5) chapters and four (4) appendices. Chapter 1 quickly describes the background of the current research work and the aims of this research, the main definitions and concepts that led to this study are also discussed in this chapter, together with the work plans involving the long-term evaluation of rubber and ways to predict its behaviour with strain, time and temperature variables.

Chapter 2 discusses the current trends in bridge construction design in the form of IAB, and its drawbacks, triggering more studies looking at its long-term performance. Similar issues found in other structures – the propped embedded wall and monopile foundation of wind turbines are also discussed.

To elaborate on the process of stress relaxation and creep in rubber, this chapter reprises the fundamentals of linear viscoelasticity in creep and the stress relaxation
phenomena. The theoretical principles of Boltzmann superposition and WLF transform are discussed and linked together. Some existing and new methods to characterise rubber in relaxation are reviewed and a possible improved method is discussed. Finally, the concept of utilising a rubber unit to alter the behaviour of the IAB wall is presented. Other engineering uses of natural rubber are mentioned, citing the importance of long-term performance of NR engineering sectors.

Chapter 3 is dedicated to the evaluation of rubber as the main material in this study, in the creep, and mainly, the stress relaxation behaviour. This part focuses on the long-term properties of NR covering two main parts; 1) Experimental, and 2) Numerical methods. The experimental covers the stress-strain and the long-term properties of the rubber, while the numerical method covers the constitutive modelling of the rubber by using the existing viscoelastic model applicable to the material. A new method to approximate the long-term relaxation modulus of NR with the strain and temperature dependence is attempted and evaluated. Lastly, the use of the accelerated methods in predicting the long-term stress relaxation of unfilled NR with time and temperature dependence is attempted and compared.

Chapter 4 describes the experimental works on the use of the rubber-based DCU and its role in addressing the densification issue affecting the structures with granular backfill materials. The rubber DCU, known as hollow rubber cylinder (HRC), was designed and its efficacy was evaluated by deploying it into the scaled IAB model and monitoring the stress buildup and surface settlement against cyclic wall motion. Performance comparison was also made against the steel-based DCU (conical disc spring (CDS)), and the conventional IAB without the DCU. Several scenarios were simulated in the experiments with the expected thermal displacements for a regular and a longer deck span of the IAB in the UK. The effect of the IAB initial wall positions corresponding to the seasons was also studied.

Chapter 5 brings together the main findings, observations, and suggestions for future work.

In Appendix A, the detail of the stress relaxation and creep test setup, including the calibration of the force transducers, displacement and temperature gauges used in the experiments are briefly described. The circuit diagram for the electronic control unit for the stress relaxation and creep’s temperature-controlled box is also shown.
Appendix B presents the records related to the commissioning works of the scaled IAB wall model in the Soil Mechanics Laboratory at UCL. The calibration record of the pressure transducers and LVDTs for the model retaining wall are presented. The measurements of the sand’s dry density inside the sand tank are tabulated for all experimental cases.

Appendix C shows the double bonded shear sample (DBS) and hollow rubber cylinder (HRC) modelled in ABAQUS. The stress contours of the models are presented here.

Appendix D explains the characterisation test conducted on the backfill material used in this study. The backfill material, Leighton Buzzard sand underwent sieve analysis to check for its particle size distribution, and triaxial tests to determine the material’s internal friction angle, $\phi$. 
This chapter discusses the conceptual design of integral abutment bridges (IAB), the factors governing their emergence in the bridge construction industry, and the drawbacks that are associated with them. The issues found in IAB which also exist in other structures are also highlighted. Hypothetically, the issues with the IABs may possibly be contained with the use of an elastic device – Displacement Compensation Unit (DCU). The idea of using DCU made of rubber is attractive, due to its high displacement capacity and its ability to offer a corrosion-less system. In order to design the rubber DCU system, an understanding of its mechanical and long-term properties are crucial. This prompted the study into reviewing the stress and strain properties of rubber, as well as its viscoelastic behaviour.

2.1 Integral Abutment Bridge

Integral abutment bridge (IAB) can be referred to as a single, of multiple span bridge with a continuous deck integral to the abutment and supported by foundations. Longitudinal forces are accommodated by the movement of the IAB abutments instead of through expansion joints, as is the case for the traditional bridges. A typical design layout of a single span IAB with full height abutment is shown in Figure 2.1, in comparison to the traditional bridge layout. For multiple spans IAB, the pier can be constructed with or independent of the deck structures, as shown in Figure 2.2.
The principal design of an IAB offers a monolithic continuous deck and abutment system, which results in reduced construction elements. The principal advantages of the IAB are lower construction cost, easier maintenance and the longevity derived from the elimination of expansion joints (Soltani and Kukreti, 1992; R. F. Maruri and Petro, 2005; Miletic et al., 2016). Other advantages are the better distribution of internal structural forces and better seismic performance as opposed to identical bridges with joints (Erhan and Dicleli, 2014; Ni Choine, O’Connor and Padgett, 2015).

Over the years around the world, expansion joints were found not to perform well and regarded as a major maintenance issue (Arsoy, Barker and Duncan, 1999; Dicleli and Albhaisi, 2004; Ahn et al., 2011). In the UK, a survey of approximately 200 concrete highway bridges was commissioned by the Department for Transport and revealed that expansion
joints were the primary source of costly and disruptive maintenance work. The joints also leaked and contaminated the piers, soffit and abutments with chloride from the road salt (Wallbank, 1989; Miletic et al., 2016). In response, in 1996, the Highways Agency published the Advice Note BA 42/96, promoting the design of integral abutment bridges and suggesting any bridge with a length of up to 60m and with skewness not exceeding 30° should be constructed with integral supports (Springman, Norrish and Ng, 1996). Skewness affects the IAB’s soil pressure variably, with the obtuse corner moves into the backfill soil more than the acute corner (Sandford and Elgaaly, 1993).

Although IABs have been constructed for almost 60 years (Arsoy, Barker and Duncan, 1999), there are still no specific theories for the design until recently. The behaviour of IABs is not adequately understood by bridge engineers despite the numerous applications, and the design approach was merely empirical rather than a systematic investigation (Arockiasamy, Butrieng and Sivakumar, 2004). The design and construction of IABs was therefore dependent on past experience due to non-existing codes of practice for IABs (Huang, Shield and French, 2008).

The design guidelines of IAB in the UK are still based on document BA 42/96. In Malaysia, there are no official guidelines for IAB design, although it is common for a bridge up to 85m in span to be built integrally, given the lower thermal range (Forouzani and Taib, 2006). IAB construction in other parts of Europe is showing an increasing trend following the introduction of design guidelines - (MDF (Ministerio de Fomento), 2000; ASTRA (Bundesamt für Strassen), 2010; BSI PD6694-1, 2011; BMVBS, 2013). Currently the design guidelines, however, differ within European countries and no unified design guidelines within Eurocodes exists. Similarly, each state within the US has different Bridge Design Manuals, with different sets of design parameters and limitations. The situation is made more complicated because the design approach is specific to the local site conditions and loads and its structural arrangement (Barr et al., 2013).

While more IABs are being constructed nowadays, the traditional bridge with joints is still being built. The joints’ issues, however, became less distressing due to the improvement seen in the expansion joints’ quality and performance. Nevertheless, regular maintenance is still required (Lima and de Brito, 2009).
2.1.1 Types of integral abutment bridges

In general, the IAB design in the US refers to the short stub-type abutment supported by a row of piles acting as its foundation (Burke, 1990; Arsoy, Barker and Duncan, 1999; Hassiotis and Roman, 2005; Dicleli and Erhan, 2010). This system allows the pile to be flexible in undertaking the lateral strain. The European approach, in the UK for example, is to rely on the flexibility of the abutment wall, therefore, the preference in using the frame-type abutment which portrays the structural form of a portal frame (HA, 2003; Caristo, Mitoulis and Barnes, 2018). The common IAB types in the US and the UK are shown in Figure 2.3 and Figure 2.4, respectively.

Figure 2.3: Schematic diagram of an IAB in the United States (Arsoy, Barker and Duncan, 1999) Foundation is mainly made of piles.
Two types of abutment for integral constructions are; 1) the full IAB which has neither expansion joints nor bearings and, 2) the semi-IAB where the structure has vertical support at the deck end in the form of moveable bearings. The abutments do not move into the backfill when the deck expands. Instead, it acts in a similar manner to flexible support abutment with bearings (Burke Jr, 2009). The semi-IABs are depicted in Figure 2.5 below;

As of 2005, there were 9000 fully-IABs and 4000 semi-IAB in the United States based on a survey conducted across 50 states’ department of transportations (Rodolfo F Maruri and Petro, 2005). Another later study suggested that by 2010, 41 out of 50 states have adopted the IAB design whenever possible (Paraschos and Amde, 2011).

This research will focus on the IAB described as the full-height frame abutment with granular fill at the back. The term ‘IAB’ henceforth will be used and automatically referred to as the full IAB structure.
2.1.2 The issues with integral abutment bridges

The IABs are subjected to several types of force (Card and Carder, 1993);

- Dead and live loads
- Temperature gradients in the superstructures
- Creep and shrinkage of the concrete deck
- Differential settlement of the foundation
- Backfill pressures
- Embankment translation and consolidation

With several types of forces acting on the IABs, some design uncertainties and complications arise, particularly with the long-term behaviour. The movement of the abutment over long time periods have been investigated through long-term field monitoring (Arsoy, Barker and Duncan, 1999; Civjan et al., 2007; Kim and Laman, 2012). Measured soil pressures from experimental studies (England, Bush and Tsang, 2000) and field monitoring (Kim and Laman, 2010) indicate that the soil-structure interaction of IAB is cycle-dependent and irreversible.

With the integral joint of the deck to the abutment forming a single rigid frame, the deck movement is partially or fully restrained from the thermal, creep and shrinkage actions (Arockiasamy and Sivakumar, 2005; Kim and Laman, 2012; Huntley and Valsangkar, 2013). The cyclic thermal expansion and contraction acting on the deck will also act on the abutment, causing two major problems; densification and progressive flow of the backfill material. These repetitive actions are gradually generating excessive stress in the concrete, causing a phenomenon known as strain ratcheting (England, 1994; England, Bush and Tsang, 2000). This action densifies the abutment causing settlement in adjacent backfill. In the longer cyclic period, a heaving was observed on the backfill surface. Ratcheting and generation of excessive loads in concrete, driven by thermal cycles, has been highlighted previously in reinforced concrete rings constraining granular material, used in biological filters for wastewater treatment (England, 1994). The mechanics of strain ratcheting is discussed in Section 2.1.3.

The cyclic displacements induced by the thermal expansion and contraction is the main stress contributor to the IABs (Kerokoski 2006). Several states in the US discontinued new construction of IAB due to the serious structural problems resulting from the thermal

The most prominent issue related to IABs in service is the abutment stress escalation, caused by the backfill soil settlement. A backfill settlement was occurred within several months of construction (Lawver, French and Shield, 2000). An experimental work by Springman, Norrish and Ng (1996) suggested that settlement of more than 500mm for a 6m high spread-base wall can be anticipated behind integral abutment walls.

Other researches listed the two major factors that cause backfill settlement: 1) The temperature-induced movements of the abutment causing densification of the abutment soil and the appearance of a void behind the abutment (Wolde-Tinsea and Klinger, 1987; Hoppe and Gomez, 1996; Ng, Springman and Norrish, 1998; Arsoy, Barker and Duncan, 1999; Miletic et al., 2016), and 2) The traffic loads (Wolde-Tinsea and Klinger, 1987; Briaud, James and Hoffman, 1997). The backfill settlement for IABs with and without the approach slabs are depicted in Figure 2.6. The intended functions of the approach slabs are to provide a ramp for the expected settlement between the abutment and the backfill and to span the void that may occur below the slab (Briaud, James and Hoffman, 1997).
Figure 2.6: The different patterns of backfill settlement for typical IAB under thermal expansion and contraction. (a) Without the approach slab; (b) With approach slab (Arsoy, Barker and Duncan, 1999)

Figure 2.6(a) shows the occurrence of a bump when the abutment moves towards the backfill as the result of expanding the IAB deck. Inversely, when the abutment moves away from the backfill, voids will occur, adjacent to the abutment, caused by settlement, subsequently causing a crack on the pavement surface.

Figure 2.6(b) shows the interaction mechanism between the approach system and the abutment. When the abutment moves towards the backfill, the lateral stress is pushed to the end of the slab and therefore, no bump occurs near the abutment. But when the abutment moves away from the fill, the void still develops between the abutment and the backfill. This shows that such an approach system may only mitigate the backfill settlement temporarily, and hence long-term solutions are still needed.

2.1.3 Strain ratcheting mechanism

Strain ratcheting happens when a slender structure supporting granular soil is distorted geometrically due to cyclic expansion and contraction arising from cyclic thermal changes. This causes high stress to the soil and progressively changes the stresses to an extent which causes differential settlements. The resulting incremental settlement densifies the soil causing a progressive increase in the cyclic stresses termed ‘ratcheting’. This ratcheting process may gradually tilt the structure, causing instability and the risk of collapse.

Ratcheting in IAB structures has been widely reported (Springman, Norrish and Ng, 1996; England et al., 1997; Darley, Carder and Baker, 1998; Wood and Nash, 2000; Cosgrove and Lehane, 2003; Hassiotis and Roman, 2005; Clayton, Xu and Bloodworth, 2006; Brena et al., 2007) The mechanism of the strain ratcheting was discussed in detail by England, (1994; England et al., 1997) in a study on the strain simulation of a biological filter wall under cyclic thermal changes. The typical filter wall is described as a cylindrical multi-panel wall with a tension ring holding the panels together, with granular material as the filter beds (see Figure 2.7).
Figure 2.7: Multi-panel filter wall with a tension ring.

The cyclic stress-strain reference data of the filter wall is given in Figure 2.8. It is shown with the stress-strain monotonic loading for approximately 2% compressive and 4% tensile strains.
Figure 2.8: Summary of strain cycling scenarios of wall supporting granular beds (sand). Compression is wall undergoing ‘passive’ loading and extension is wall undergoing ‘active’ loading.

Four main cases of wall’s stress-strain cyclic loadings are illustrated in Figure 2.8;

A : Constant strain cycles; rigid wall
B & C : Variable cyclic stress and strains; flexible walls
D : Variable cyclic strains, almost unchanged stress; flexible wall
E : Constant stress cycles; infinitely flexible wall

Figure 2.8 describes the behaviour of filter wall containing granular beds, with the variation of wall stiffnesses, with “s” as the starting point of the cycles. The isotropic stress state ($\frac{\sigma_h}{\sigma_v} = K = 1.0$) is marked as “0”. $\sigma_h$ is the soil element horizontal stress, and $\sigma_v$ is the vertical stress.
Figure 2.9: Rigid wall undergoing constant cyclic strains.

For case A, the repeated cyclic strains on the rigid wall caused stress escalation. A similar trend is shown for three different strain levels (Figure 2.9).

For case B, the repeated strain caused stress reduction. This is due to its initial state, which is above the isotropic value, which means a huge surcharge will need to be applied to
the granular bed to overcome the vertical stress. This observation, however, is unlikely to be observed in the case of filter wall.

For case C, the repeated strain caused the stress escalation, and the wall position is gradually shifted to ‘active’ position (see Figure 2.10 below). These results mimic the behaviour observed in the containment wall. This case is possible with the initial state below the isotropic stress value. The vertical stress is higher than the horizontal hence explains the differential settlement, which later causes the ratcheting.

Figure 2.10: Flexible wall undergoing repeated strains.
For case D, with repeated strains applied, neither stress escalation nor reduction was observed. This was possible due to the stress state that was centred around its isotropic stress state. For case E, the ratcheting is towards the ‘active’ direction with the same fluctuating stress magnitude. This is made possible when the element state is below isotropic stress value. The behaviour is, however typical for a structure that is very flexible or undergoing cyclic yielding.

From the cases above, the strain ratcheting associated with the filter wall and IAB can be best described by case A (rigid wall) or C (flexible or semi-rigid wall), and obviously the structures are undergoing stress escalation over time. In an ideal condition, a granular backfill with the stress state as in case D will benefit the structures. This can be achieved
through an alteration of the soil-structure system, for example by allowing the wall to be flexible under cyclic strain, and at the same time keeping the stress unchanged, or, if not, with very minimum stress change.

The strain ratcheting occurs from the mobilisation of the soil particles behind a wall due to the inversion of the soil principal stress, $\sigma_h/\sigma_v$. The mobilisation, which is referred to as the settlement, is shown by the settlement profile in Figure 2.12, as studied by England, Bush and Tsang (2000) using a 1/12th scaled model wall representing a 60m long IAB with the cyclic rotational wall displacement of $d/H=0.25\%$. $d$ is the seasonal wall displacement and $H$ is the wall height.

![Figure 2.12: Settlement variations for 300 thermal cycles.](image)

The surface settlement is concentrated close to the wall in a progressive manner, which confirms the occurrence of soil densification, which then leads to the strain ratcheting. The plot shows that the heave of soil surface occurred at the two highest cycles, and located at a far distance from the wall.

### 2.1.4 Thermal strain

This section reviews some examples showing the relationship between the IAB temperature to the deck displacement and the soil pressure behind the abutments. A few studies were conducted on some real-life bridges with variations in dimensions, construction materials,
and locations, in determining their behaviour in backfill pressure due to the effects of deck movements arising from the thermal changes.

A study on Maple Bridge in Iowa was conducted by Girton, Hawkinson and Greimann (1991), where the structure was an IAB with a composite concrete deck and steel girders, 98m long, 10m wide and 30° skew angle. The bridge was a continuous 3-span bridge with two piers located about 30m from each abutment. Two LVDTs were installed at a nickel-iron wire stretching across the bridge length to monitor its longitudinal displacement versus time. To monitor the air and bridge’s temperature, thermocouple wires were soldered to the steel girders and enclosed in electrical boxes. Some other wires were placed inside pre-drilled holes in the bridge deck, which were then sealed with grout. The monitoring and data collection (for temperature and displacement) took place over approximately two years (early 1987 to early 1989), where the data is shown in Figure 2.13.
As seen in Figure 2.13, the maximum expansion and the contraction of the bridge coincided with the maximum and minimum air temperatures, shown by the two seasonal cycles – 1987 to 1989.

Another case is shown by a study conducted by Sandford and Elgaaly (1993) on the Forks Bridge in Maine USA. The subject was a 50m single span IAB with 20° skew angle bridge, and instrumented with pressure cells and temperature gauges for monitoring. The materials for the deck and the abutment were composite reinforced concrete. The bridge’s temperature and pressure behind the abutments were monitored at four elevations since October 1989 for 33 months. The collected data is shown in Figure 2.14 below;
Figure 2.14: (a) bridge temperature and, (b) the average backfill pressure over a 33 month period for The Forks Bridge, Maine (Sandford and Elgaaly, 1993)
The data suggests that the backfill pressure corresponds to the thermal expansion of the deck due to temperature; the rising during the summer pushes the abutments into the backfills, while in winter contraction of the deck will then relieve the pressure. The study also found that the IAB skewness affects the pressure distribution differently, with the obtuse side exhibiting a higher pressure than the acute side at the same distance. The effect of the skew however was observed only at early service life and diminished over a period of two years. This is shown in Figure 2.15.

![Figure 2.15: Skew effect to the pressure at elevation 177.7m for The Forks Bridge, Maine (Sandford and Elgaaly, 1993)](image)

Another study was conducted by Kerokoski and Laaksonen (2005) on the Haavistonjoki Bridge in Finland, where the relations between temperature, earth pressure and structural strain were studied. The superstructure was 50m long continuous 3-span integrally-jointed concrete bridge with no skew. The bridge was constructed and instrumented in 2004. Several thermocouples were used to record the deck, embankment and air temperatures. Twelve pressure gauges were installed in between the abutments-backfill interface, with ten in western abutment and two in the eastern abutment. The correlation between the IAB’s temperature and the IAB’s earth pressure was demonstrated in Figure 2.16.
Another study was conducted on the Route 2 high-speed connector bridge in New Brunswick, Canada, with 76m long, two-span, pile-supported IAB with no skew angle. The work was done by Huntley and Valsangkar (2013). The IAB was constructed in 2004 with instrumentations for structural health monitoring. Three pressure gauges were installed in the middle of each abutment at several heights.
Figure 2.17: (a) Abutment movement and (b) pressure variations for the Route 2 New Brunswick bridge (west abutment) (Huntley and Valsangkar, 2013)

Figure 2.17 shows that the displacement and pressure acting on the abutments simulate changes in ambient temperature. The variations in pressures were increasing in the summer as the abutment wall moves towards the backfill and declining in the winter as the abutment wall contracts.
Based on the field test and monitoring on the IABs above, it was demonstrated that the IAB’s expansion-contraction displacement and the earth pressure changes are governed primarily by the bridge temperature. However, a definite conclusion regarding the presence of earth pressure ratcheting over the study period of approximately 2 to 3 years cannot be made, contradicting some of the research findings mentioned in Section 2.1.3. Some slight increases in pressure variation were observed, but the presence of pressure ratcheting was not consistent among the instrumented bridges. There are differences between one IAB structure to another, as shown in the variability of pressure magnitudes and behaviour over time. Strain ratcheting may also be a longer-term problem rather than a short term one.

The temperature of the bridge changes continuously, shown as effective bridge temperature (EBT) in Figure 2.18. The seasonal temperature variations have the largest amplitude, with a small number of cycles.

![Diagram of effective bridge temperatures (EBTs)](image)

Figure 2.18: General reproduction of the effective bridge temperatures (EBTs) with the daily and seasonal changes for concrete and composite bridge decks (England, Bush and Tsang, 2000)

The compilation of the EBTs of three types of IAB (concrete, composite and steel) are reported at different locations in the UK by Emerson (1976). Although the thermal expansion coefficient for the composite and steel decks are assumed to be identical to concrete, they show higher changes in EBT. The seasonal movements of a 60m composite and steel decks
are 21% and 45% higher than that of a concrete deck respectively (England, Bush and Tsang, 2000).

Several variables have been considered in order to understand what affects the displacement of the deck. Among them are the initial position and the first direction of movement of the abutment, which is determined by whether the IAB starts operating in summer or winter. Both the IABs with the initial movement towards winter and summer positions arrived at their steady-state earth pressure after 15-30 thermal cycles (Caristo et al. 2018).

Another variable is the magnitude of the cyclic movement of the deck. The magnitude of the IAB’s longitudinal displacement, \( d_{\text{long}} \), can be predicted provided the temperature, the deck length and thermal coefficient of expansion are known (Emerson, 1973, 1976b, 1977). Equation (2.1) refers to the variables shown in Figure 2.19.

\[
d_{\text{long}} = \alpha_{\text{conc}} \cdot \Delta T_{EB} \cdot L
\]

where \( L \) is the deck span, \( \Delta T_{EB} \) is the effective bridge temperature (EBT), and \( \alpha_{\text{conc}} \) as the coefficient of thermal expansion, assumed to be \( 12 \times 10^{-6}/^{\circ}\text{C} \) for the concrete deck (BA42/96 2003). This estimation, however, is limited to the following conditions: 1) the creep and shrinkage of the concrete are completed and 2) lateral resistance of backfill soil against axial deck load is almost unchanged (Lock 2002).

Earlier study outlined that the resistance provided by the approach fill is so small that it does not have any significant effect on the expansion of the integral bridge (Oesterle et al.,
1999). Therefore, the amount of expansion and contraction of integral bridges can be calculated using the method recommended by AASHTO (2002), which does not consider the resistance provided by the approach fills.

It is uncommon to measure the actual bridge temperature, and therefore an estimation will greatly help. However, because the bridge temperature estimation is mostly based on the shade air temperature, a guideline was presented by Emerson (1976) to estimate the ‘effective bridge temperature’ (EBT) by using a curved correlation between the shade and the EBT. A standard document was published later in BS EN 1991-1-5:2003 to estimate the maximum and minimum ‘uniform bridge temperature’ (UBT) by using a linear correlation between the shade and the UBT.

The temperature conversion diagram for EBT and UBT are shown in Figure 2.20 and Figure 2.21 respectively. It is worth noting that the term ‘shade temperature’ refers to the air temperature measured in the shaded area under the bridge deck.

![Temperature Conversion Diagram](image)

Figure 2.20: The correlation between the bridge shade temperatures to the effective bridge temperatures (Emerson, 1976b).
The correlation between the bridge shade temperatures to the uniform bridge temperatures (BS EN 1991-1-5:2003).

The code (EN 1991-1-5:2003) categorises the bridge deck into three classes; Type 1: Steel, Type 2: Composite and Type 3: Concrete. This is due to the different thermal properties of the deck materials and consequent different thermal response times. The overall range of the uniform bridge temperature is given by the difference in the $T_{e,max}$ and $T_{e,min}$ values, $\Delta T_N$. The outcome $\Delta T_N$ values may conveniently be used to determine the bridge deck movement, provided that the deck length and its thermal expansion coefficient are known. Only then the magnitude of the cyclic displacement imposed on the abutment and backfill can be determined. The temperature correlation shown in Figure 2.21 is based on the calculation by Emerson (1976), where the linear correlation is considered conservative because the objective of the study was to determine the most extreme possible effective bridge temperature.

It can be suggested that a single value of temperature-induced displacement can be chosen for the IAB simulation study. The seasonal temperature variation is far larger than the daily temperature variation; thus the latter poses a less significant effect on the model displacement. This assumption is supported by past research (England, Bush and Tsang, 2000; Caristo, Mitoulis and Barnes, 2018). The temperature data for UK conditions, as studied by
Emerson (1976), provides the basis of the deck displacement induced by the seasonal thermal changes used in this study.

2.2 Supported Embedded Walls

Deep excavation works are carried out to construct underground infrastructures, including basement and underground railway facilities. Excessive ground movement induced during excavation could damage neighbouring structures. Thus a retaining structure is required to act as a support system. (Twine & Roscoe 1999).

There are three main types of retaining structure as underlined in BS EN 1997:2004 Section 9.1.2 – gravity walls, embedded walls and composite retaining walls. This study focuses on embedded walls with passive resistance of soil which ensure the stability of the structure. The structures adjacent to the excavation are susceptible to movement. In most cases, it may involve cosmetic repairable damage, but in a more serious case, it could lead to collapse. CIRIA 580 outlined that embedded walls are to be reinforced temporarily either with concrete or steel props, anchorage or berms (Gaba et al., 2003).

The typical supports and layouts of propped excavations are illustrated in Figure 2.22.

![Figure 2.22: Types and layout examples of propped excavation](attachment:image.png)

(a) Internal support for excavation, elevation view; (b) Internally supported excavation, plan view (William and Waite, 1993).
The types of embedded wall support may be cantilever, single prop or multi prop. The type used also depends on the construction method used for the excavation, which includes diaphragm walls, sheet pile walls, secant pile walls and contiguous pile walls (Fernie, Puller and Courts, 2012). The wall propping discussed in this study is for temporary propping. During initial excavation, the wall deforms as a cantilever member and the adjacent soil will settle. As the excavation progresses, a prop system or stiffening braces support the upper part of the wall. This will result in the cumulative wall-ground displacement profile shown in Figure 2.23 (c).

![Figure 2.23: The relationship between wall movement and soil deformation (Carder, 1995).](image)

**2.2.1 Temporary props**

Temporary propping systems are typically made of hollow steel sections. The prop is normally orientated perpendicular to the face of a retaining wall and provides lateral support in compression. On an excavation, they transmit the load from one retaining wall to the opposite retaining wall or another source of resistance. Following excavation, the permanent support system, in the form of rigid slabs is constructed within the excavation. The temporary props will then be removed once the permanent works are finished and ready to provide sufficient support. Some temporary props come in modular arrangements, as described in Section 2.2.2.

**2.2.2 Modular props**

A temporary propping system ensures the stability of the excavation and provides a control mechanism against wall deflection and consequent soil deformation. As in other temporary propping systems, the elongation of its section member is translated to the change in stress; therefore it is important to control or limit the elongation in order to avoid damage to adjacent structures.
The modular propping systems use a combination of hollow steel elements or box steel props, connected to hydraulic actuators. The steel elements are normally circular hollow sections (CHS) called struts. These are bolted onto a hydraulic unit, connected to a load cell. The advantages of this system are their high capacity and reusability, which is appealing for construction’s sustainability as compared to the traditional on-site fabricated steel props. It is also favoured because the elongation of the CHS section can be controlled by means of prestressing, so the deformation of the wall can be reduced. The prestressing is done by controlling the hydraulic system. This mechanism also enables the operator to adjust the wall, by pushing it back to its original position in case it is necessary. The hydraulic column is encased in tubular steel with sliding inner and outer sleeves. A typical hydraulic strut construction can be seen in Figure 2.24.

![Figure 2.24: The modular unit of a typical hydraulic prop (after Groundforce Shorco 2014). The arrows denote the inflow and outflow inlets of the hydraulic fluid.](image)

Theoretically, the unit consists of two types of springs in series. The steel component of the prop \( (K_2) \) acts as a very stiff spring in axial compression, and the hydraulic unit \( (K_1) \) is less stiff. The hydraulic unit accommodates the fitting of the prop to the excavation space and allows the length adjustment and pre-stressing.

![Figure 2.25: Idealisation of the hydraulic strut modular system (Gould, 2016).](image)

### 2.2.3 Temperature effects on temporary props

The known drawback of the temporary props, not unique to modular props, is related to the expansion and contraction of the prop due to temperature changes; these length changes
will change the axial load in the props and could induce undesirable deformations in the structure (Batten and Powrie, 2000; Powrie and Batten, 2000; Loveridge, 2001; Gould, 2016). Current practice using the modular props will require the adjustment of the hydraulic unit in order to maintain the prop load approximately at its prestressed level. This is especially required when the load gets too high or too low. This force control method could escalate the operation cost, and if the execution of the load adjustment is badly planned, it may disrupt the excavation works.

If the load is not kept sufficiently constant, the embedded walls tend to move inwards (Carder, 1995). This is likely caused by the densification of the soil behind the wall, resulting in a progressive settlement of the surrounding soil. The settlement may affect and compromise the nearby sub-structures, especially masonry foundations of buildings located in areas near the excavation site.

A single day’s temperature variation may result in a prop load to change tremendously, as shown by Chambers et al. (2016). This thermal effect will be more critical if the position of the prop is closer to the ground top surface, where there is more thermal radiation from the sun. The time and season of prop installation can affect the load changing behaviour of the props. If the props are installed in the winter, it is likely that the thermal loads would increase through the summer and vice versa (Batten and Powrie, 2000).

The thermal variation will pose some additional load onto the prop. To calculate this extra load, the factor of the percentage degree of restraint of the prop, $\beta$ was introduced by Gaba et al. (2003) who recommended the $\beta$ to be 70% for stiff walls in the stiff ground (see Equation (2.2)).

$$\Delta P_{\text{temp}} = \alpha_{\text{steel}} \cdot \Delta T \cdot E \cdot A \left( \frac{\beta}{100} \right)$$

with the $\Delta P_{\text{temp}}$ as the additional load from the temperature effect, $E$ as the Young’s Modulus of the prop material (steel), $A$ as the cross-section of the prop and $\alpha_{\text{steel}}$ as the thermal coefficient of expansion for the prop material, which normally taken as $12 \times 10^{-6}/\degree \text{C}$.

However, because the prop had lower temperature variation than expected due to the shade and position in the excavation box, the observation by Chambers et al. (2016) showed that the actual prop load was only 35% of the design system stiffness (ie $\beta = 35$). Previous studies, however, had supported this by indicating that the degree of restraint shall
fall within 34% to 50% (Powrie and Batten, 2000; Richards et al., 2007; Ivanova, 2012). CIRIA 760 later recommends a degree of 50% for stiff walls in a stiff ground and 30% for flexible walls in a stiff ground (Gaba et al. 2017). Care must be taken as for the case of the modular prop with a hydraulic system, a lower overall stiffness is expected due to the use of the hydraulic units. This is consistent with the modular prop spring diagram shown in Figure 2.25.

The casting of a concrete slab can also cause the prop load to change. Generally, the prop load reduce soon after the concrete is poured. This is caused by the cement hydration reaction, which heats and expands the slab against the retaining wall. Afterwards, the prop load gradually increase as the concrete cools down and shrinks (Chambers et al., 2016). This was observed by Batten and Powrie (2000), where the newly casted concrete base slab of Canary Wharf Station expanded, reducing the prop load as shown in Figure 2.26. The prop regained 60% of its load back after a month. The effect of concrete slab casting is however temporary and shall not affect the prop load over the long term.
The prop change of length can be predicted by using the same approach as the IAB’s shown in Equation (2.1), except by the different value of steel thermal coefficient, $\alpha_{steel}$. For the case of modular props consisting of a steel extension and a hydraulic unit, the initial length is taken as the total length of the prop from one pin to the other end.

The temperature effect is the main factor that leads to the change in prop load. Conventional steel props may experience even higher axial load as the construction stage progresses, although there will be no hassle to adjust the prestressing hydraulic unit as it is in the case of modular prop system. This is made worse with the use of a conservative stiffness factor, $\beta$, before the publication of CIRIA 760.

Modular props have the advantage to limit the deflection of the prop and wall due to the initial prestressing. However, the hydraulic equipment is valuable and vulnerable to damage, so it is not usually desirable to keep it in position for the duration of the construction project. This study considers a new method to control the changing load, by use of a material capable of accommodating high deformation while maintaining little stress change.
2.3 Elements Used to Improve IAB Performance

The strain ratcheting problem has been observed in several engineering practices, as discussed in previous sections. This research will focus on the issues affecting the IAB including work to come up with a possible solution to mitigate the strain ratcheting problem.

IAB structure is thermally active due to the solar heating and cooling but in contact with the thermally inactive backfill material. Over recent years, a few studies have been conducted to improve the IAB performance especially after it is known to be associated with the strain ratcheting problem by modifying the properties of the backfill. In general, the aim was to alter the soil stress ratio behind the abutment so that it can be kept stable with little change in stress. Several mitigation methods for the IAB issues from previous researches are briefly discussed.

2.3.1 Compressible inclusion

Several studies have explored the theoretical applications to solve the excessive settlement of approach embankments, resulting from the repetitive movements of the superstructure. One idea was the inclusion of elastic compressible board or panel right behind the abutment walls. Ideally, the inclusion panel acts as elastic horizontal support which allows abutment movement without excessive deformation within the backfill soil (Nielsen, 1994; Carder and Card, 1997; Horvath, 2000). In particular, Carder and Card (1997) identified several materials for use as compressible layers to absorb lateral stresses behind IAB abutment. Selected candidates include polystyrene, polyethylene foam, geocomposites, and rubbers. Some of the research outcomes were reported by Carder, Barker and Darley (2002).

![Figure 2.27: Example of an elastic board behind the abutment. Reinforced backfill comes in the form of geosynthetics mat (Nielsen, 1994).](image-url)
2.3.2 Expanded polystyrene (EPS)

A detailed study was conducted by Hoppe (2005) by testing an elasticized expanded polystyrene (EPS) as an interface between a rigid abutment wall and a stiff backfill material. The actual bridge was the Route 60 Bridge in Virginia, with 100m three-span continuous steel girder IAB with no skew. It was constructed in 1999 and was monitored for 5 years. The placement of EPS with the instrumentations is shown in Figure 2.28).

Figure 2.28: EPS placed behind the abutment. Included are the monitoring instrumentation; steel rod for the displacement and three cells for backfill pressure (Hoppe, 2005).
EPS specimens with a thickness of 25.4mm were tested cyclically up to several compressive strains as shown in Figure 2.29. In general, up to 10% compression, the EPS exhibits elastic behaviour, but does not recover to its first stress-strain cycle. The change in backfill pressure in response to the air temperature is shown in Figure 2.30.

The backfill pressure in Figure 2.30 shown some distinct variations with changing temperature, with the pressure cell nearest to the pavement showed the highest magnitude. There was no available data for comparison with the conventional abutment. The results, however, suggests that Sensor #2 and #3 exhibited much lower pressure readings than Sensor #1. This showed that the EPS material which covered Sensor #2 and #3 remained elastic during the 7-day observation, and Sensor #1 which is not covered by any EPS exhibited high pressure variations, therefore more or less showed the behaviour of a normal bridge.

Furthermore, long-term creep or stress relaxation of the material is still of primary concern, as bridges have to perform well for many years. The 7-day observation may not sufficient to conclude the efficacy of the EPS material, because the strain problem of soil could affect a bridge in its entire lifetime, which is normally 100 years. It is also interesting to know, whether a slow development of soil plastic strain may occur over long-term because Sensor #1 had no EPS behind it. A slow development of soil strain may mobilise the soil, thus settlement will occur and eventually densifying the abutment.
2.3.3 Tyre-derived aggregates (TDA)

A study on IAB using a panel of tyre-derived aggregates (TDA) behind the abutment was conducted by Caristo, Mitoulis and Barnes (2018). They looked into the soil–structure interaction between the abutment wall and the backfill soil from the effect of seasonal thermal loadings by using the plane-strain finite-element code Plaxis 2D. The inclusion of the TDA was in a similar approach to the EPS geofoam described previously, with the TDA acting as the interface between the abutment and the backfill. The TDA was modelled as the compressible inclusion with geogrid layers forming the mechanically stable soil. The IAB wall with the TDA was compared with the conventionally built IAB of full height abutment, with the details are shown in Figure 2.31. The models were subjected to loadings of the maximum seasonal displacement for UK condition based on 100m long bridge; in accordance with BSI
PD6694-1 (2011). This worked out to be ±27mm, and 120 displacement cycles were imposed. The results are shown in Figure 2.32 and Figure 2.33.

Figure 2.31: (a) Typical full-height abutment and, (b) Isolated full-height abutment with TDA and geogrids (Caristo, Mitoulis and Barnes, 2018).

Figure 2.32: Backfill pressure escalation on the abutment; solid lines for conventional system and segmented lines for the isolated system (Caristo, Mitoulis and Barnes, 2018).

The wall height in Figure 2.32 is shown in ratio as the pattern of pressure distribution behind a wall is expected to be the same regardless of its height. The shape is a horizontal parabola with the peak around the centre height of the wall. The comparison showed a substantial reduction in backfill pressure when the TDA and geogrids were included in the analysis. The settlement behind the abutment for both models by thermal cycles is shown in Figure 2.33.
Figure 2.33: Backfill settlement; comparison between the conventional IAB (solid line) and the IAB with TDA (segmented lines) (Caristo, Mitoulis and Barnes, 2018).

The settlement comparison shows that the IAB has a constant increase in heaving, rather than settlement, over an increasing number of thermal cycles. A small amount of settlement, however, occurred near the wall but it was not quantified to its starting point. The change from settlement to heaving is also sudden.

For IAB with TDA, the settlement is further away from the abutment, around the distance where the geogrids stopped (at 8m in distance from the wall), which could be due to the change in soil stiffness. A small amount of heaving, however, occurred near the wall.

The settlement pattern of the IAB shown in the model was not as in actual IAB where the settlement near the wall is expected to be progressive. The heaving is normally seen away from the wall. The settlement pattern in this model is in contrast to the settlement observed in a model IAB studied by England, Bush and Tsang (2000) as shown in Figure 2.12. The question may lie in the method of the finite-element modelling used in the study especially the interaction between the soil and tyre-derived TDA, which was not elaborated.

Another experimental study on the use of rubber as the backfill material was done by Reddy and Krishna (2015), where tyre chips were used as partial replacement of the backfill sand. Two parameters were used to quantify the effects of the tyre chips; the lateral earth pressure and the displacement of the wall after the release of the support that held the wall vertical. The results indicated that the earth pressure and horizontal displacement were reduced to about 50-60%, as the result of having 30% of tyre chips in the sand as compared to the sand with no tyre chips. These are shown in Figure 2.34 and Figure 2.35.
Figure 2.34: Lateral earth pressure measured for two cases; (a) at rest condition, (b) after support removal (Reddy and Krishna, 2015)

![Figure 2.34: Lateral earth pressure measured for two cases; (a) at rest condition, (b) after support removal (Reddy and Krishna, 2015)](image1)

Figure 2.35: The plots showing the wall’s top displacement and % reduction from the effect of having tyre chips for partial sand replacement.

![Figure 2.35: The plots showing the wall’s top displacement and % reduction from the effect of having tyre chips for partial sand replacement.](image2)

Although the TDA mentioned in the studies mentioned above were in the form of re-used tyre products, no further details were highlighted, including the component detail of TDA or its mechanical stress-strain properties. The possible actual deployment was not presented in the study as it was a conceptual analysis. Another primary concern would be the time-dependent properties of the TDA, which was not considered. This may be important as the TDA was to be derived from rubber tyre, which exhibits time-dependent behaviour under constant stress or strain.
2.3.4 Displacement Compensation Unit (DCU)

The DCU was first studied as a proof-of-concept to offer solutions to the issues found in IABs – the increase of the contact pressure between abutment and backfill, known as strain ratcheting, caused by the irreversible soil strain and the limitation of the IAB deck length due to its sensitivity to the thermal changes (England et al., 2007, 2010). The locations where the DCU could be deployed in the IABs are shown in Figure 2.36 below.

A DCU operates by maintaining a constant, or nearly constant, predetermined longitudinal force while compensating the changes in displacement of its supporting structure (England, 2006). This could be either an IAB’s deck or a temporary excavation prop. A study on a 1/12th scaled model of 60m long IAB showed that the use of a stack of conical disc springs (CDS) could yield a nonlinear force-displacement curve (Figure 2.37a). Two sets of CDS-DCUs were installed in parallel, in between the deck and the abutment in a pre-compressed form. The
pre-compression was targeted to be at the onset of the less stiff region of the DCU load-deflection curve. With the DCU, the IAB model had a peak pressure reduction of almost 35%, with a very small change between loading- and unloading (Figure 2.37b). Also, no backfill settlement was reported as opposed to the progressive settlement shown by the IAB without the CDS-DCU (England et al. 2007).

Figure 2.37: (a) The load-deflection characteristics of DCUs used in the model, (b) The influence of the DCU in the soil stresses with the identical thermal deck changes (England et al. 2007)

The concept of reducing IAB wall pressures and backfill settlements was proven at model scale by using the CDS-DCU. The displacement compensating concept may use a form of linear or nonlinear force-displacement as advocated in Figure 2.38. It is yet to be studied if any form of rubber-based spring can offer the same degree of effectiveness in mitigating the strain ratcheting issues affecting the IAB.

Figure 2.38: The different concept of DCU operations (a) linear DCU operation (b) Non-linear DCU operation (England et al., 2010).
2.4 Rubber as an Engineering Material

Rubber can be divided into two categories; natural rubber (NR), which is mainly extracted from the rubber tree *Hevea brasiliensis* through latex tapping, and synthetic rubber (SR), which is derived from the polymerization process of hydrocarbon oil. NR latex is an emulsion of NR particles in water, which when coagulated forms a solid elastomer. Rubber becomes useful as an engineering material when it is crosslinked; a process called vulcanisation.

Rubber (NR) did not have widespread use until the discovery of vulcanisation by Goodyear in 1839 (Loadman, 2005). The process was altered and commercially exploited afterwards by Thomas Hancock (1786-1865), and many new industrial applications for rubber were developed. The invention of the rubber masticator by Thomas Hancock enabled more products to be made in a shorter time and earned him the nickname, “the father of the rubber product industry” (Duerden, 1986).

2.4.1 Vulcanisation of rubber and its characteristics

Vulcanisation converts rubber from a linear polymer to a three-dimensional network by a reaction called crosslinking. The presence of crosslinks inhibits flow so that the resulting network retains the shape of the mould. Vulcanisation can be achieved by incorporating vulcanising agents, either sulphur, peroxides or urethane reagents in the rubber mixing process. In most applications, accelerators are used to expedite the crosslinking process. Sulphur is the most used vulcanisation method for producing rubber products. Vulcanisation with sulphur gives strength and elastic stability through the insertion of sulphur atoms linking the rubber carbon molecules. In practice, this is done by applying heat and pressure onto the rubber mix, normally by using a moulding press machine.

Typical sulphur vulcanisation in rubber consists of short chains of sulphur atoms linking the long chain of rubber molecules. There are two common sulphur vulcanisation systems; efficient vulcanisation (EV) and conventional vulcanisation as represented in Figure 2.39.
Figure 2.39: Dependence of network structure on the vulcanisation system. (A) low sulphur/accelerator ratio and high soluble zinc concentration; (B) high sulphur/accelerator ratio or low soluble zinc concentration. $S$ is sulphur and $X$ is an accelerator (Chapman and Porter, 1988).

EV systems (A) are simple networks in which the crosslinks are exclusively monosulphidic and in which a high concentration of monosulphidic groups remains. Conventional systems (B) have more complex networks, containing mixtures of mono-, di-, and polysulphide crosslinks, and the rubber chains become more modified with the presence of sulphur donors (accelerators). These changes in structure are accompanied by changes in physical properties (Chapman and Porter, 1988). The monosulphidic structures (EV system) are also more stable to heat, have non-reverting cure behaviour and are stable in service (Russell, Skinner and Watson, 1969).

Figure 2.40: Representation of the network structure of sulphur vulcanisate with different forms of crosslinking. $S_a$, $S_b$, $S_x$ and $S_y$ indicate the number of $S$ molecules in linkages, with $(a, b, x, y = 1-9)$ (Chapman and Porter, 1988).

The first peroxide cure for NR exhibited lower physical strength and heat resistance compared to modern sulphur cure NR (Ostromislensky, 1915). Peroxide vulcanisation occurs by the free radical mechanism which forms direct carbon-to-carbon crosslinks which are
quite stable and result in vulcanisates with good ageing and compression set resistance. The commonly used peroxide is dicumyl peroxide. Peroxide vulcanisates have better heat-ageing stability as compared to sulphur-cured vulcanisates, have low compression set at an elevated temperature, good shelf-life stability, no discolouration of the finished products, and can be used to produce transparent rubbers (Kruzelak, Sykora and Hudec, 2015; Kruželák, Sýkora and Hudec, 2016). The drawbacks of peroxides are slow cure rate, very low scorch safety and poor mechanical properties as compared to sulphur-cured vulcanisates. Scorch safety is the induction period during the early phase of rubber vulcanisation. It allows the rubber mix to flow into the mould cavity or shape before the vulcanisation occurs. The scorch safety time is represented as $T_{sc}$ in the rheometer curing curve shown in Figure 2.41.

### 2.4.2 Crosslinking of rubber

A common instrument used to determine the degree and rate of rubber crosslinking (curing) is the oscillating disc rheometer. An oscillating rotor surrounded by the compounded but uncured rubber specimen is enclosed in a heated chamber. The torque required to oscillate the rotor is monitored as a function of time. The torque rises as the rubber stiffens, and if the degradation of the main carbon chains dominates (chain scission), the torque will decrease. This is known as reversion. However, an increasing torque indicates that crosslinking is dominant. The typical rheometer curve found in practice is shown in Figure 2.41.
Figure 2.41: Schematic rheometer curing curve. 1: minimum viscosity; 2: scorch point (1 torque unit rise above minimum viscosity); $T_{sc}$: scorch time; 3: maximum modulus; 4: 50% crosslinks; 5: optimum cure (90% crosslinks); $T_{50}$: time to 50% crosslinks; $T_{90}$: time to optimum cure. The cure rate is measured either as $T_{50} - T_{sc}$ or $T_{90} - T_{sc}$ (Greensmith and Watson, 1969).

A plateau torque indicates completion of curing and good network stability. Some NR compounds, particularly at high curing temperatures, show reversion. Some compounds show a slowly increasing torque with long curing times. This often occurs in compounds that form many polysulphidic links. With the extended curing times, these linkages will break down and reform new links with lower sulphur rank, thus increasing the total number of crosslinks (Hamed, 2012).

2.4.3 Filler effect

The engineering rubber compounds usually contain antidegradants and fillers. Ingredients in rubber formulations are expressed in parts per hundred of raw rubber by mass (pphr). Carbon black, which is an inorganic filler material, is often incorporated in the rubber and has many grades, categorised by particle size and amount of aggregation of the particles. The presence of such material in rubber products will affect their physical properties. For
example, the inclusion of carbon black will increase the stress relaxation of rubber. One reason for this is strain amplification (Mullins and Tobin, 1965), where due to the filler being inextensible, the strain in the polymer is greater than the overall strain.

Typical stress-strain curves for carbon black-filled and unfilled rubbers in tension and simple shear are shown in Figure 2.42 and Figure 2.43, respectively.

Figure 2.42: Tensile stress-strain curve for four NR compounds. A (63 IRHD) contains 45 parts of a reinforcing CB and B (57 IRHD) contains 45 parts of a semi-reinforcing CB. C (44 IRHD) and D (35 IRHD) are unfilled NR (taken from Lindley and Gough (2015)).

[IRHD = International Rubber Hardness Degree; CB = carbon black]

In Figure 2.43, the shear stress-strain curves for unfilled NR are substantially linear up to strains of about 200%, because there is no geometric sources for nonlinearity. The stiffness, however, increases after 200% strain. The stiffness increases at higher strains is caused by two factors; the molecular chain in the network limiting the extension and the strain-induced crystallisation.
Figure 2.43: Shear stress-strain curve for four NR compounds. A (65 IRHD) contains 45 parts of a reinforcing CB and B (55 IRHD) contains 30 parts of a semi-reinforcing CB. C (46 IRHD) and D (36 IRHD) are unfilled NR (taken from Lindley and Gough (2015)).

The presence of carbon black fillers increases the modulus and contributes an additional source of nonlinearity in the stress-strain behaviour. The increasing stiffness at higher strain for carbon-black filled NR has the same causes as in unfilled NR but begin to occur at smaller strains because the filler amplifies the strain within the rubber matrix (Mullins and Tobin, 1954). The modulus increase is particularly large at low strains, due to the progressive breakdown of the filler as the strain increases (Payne, 1962). Fillers also increase rubber hysteresis and heat build-up. Carbon black fillers protect against ultraviolet degradation and they can provide resistance to mechanical fatigue cracking (Lake and Lindley, 1964).

2.4.4 Elastic Stress-Strain Behaviour of Rubber

Elastomers, including natural rubber (NR) can deform elastically to very large extensions under small stresses, which is known as hyperelasticity. Two approaches can be used to describe the hyperelastic behaviour of rubber – the statistical theory and the phenomenological theory. Both methods can be used to derive the stored or strain energy function. It measures the amount of recoverable elastic energy stored per unit volume of material that has been subjected to a specific state of strain (Flory and Rehner, 1943).
Statistical theory

The statistical theory is based on the molecular approach, considering the random fluctuation of the molecules in the 3-dimensional network. It involves the calculation of the entropy (randomness) of the rubber chain molecules as a function of the strain of the sample, to derive the energy of deformation. Statistical physics is used to predict the elastic behaviour of this structure.

The deformation of rubber can be represented by assuming that a unit cube would be deformed into a cuboid of edge lengths $\lambda_1$, $\lambda_2$ and $\lambda_3$. $\lambda_1$, $\lambda_2$ and $\lambda_3$ are the three principal extension ratios (the ratios of stretched to unstretched length) along three mutually perpendicular axes, which correspond to the three principal stresses $t_1$, $t_2$, $t_3$, (henceforth identified as $\sigma_1$, $\sigma_2$, $\sigma_3$ respectively). This is shown in Figure 2.44.

![Diagram of deformation](image)

Figure 2.44: Pure homogenous strain: (a) unstrained state; (b) strained state

According to the statistical theory of rubber elasticity (Treloar, 1949, 1974), the entropy of a single rubber chain is given by (Kuhn, 1936);

$$ s = c - b^2 k r^2 $$

where $c$ is an arbitrary constant, $b^2$ is the constant in the Gaussian distribution function which is equal to $\frac{3}{2} n l^2$, $k$ is the Boltzmann’s constant and $r$ is the end-to-end rubber chain lengths.

The network contains $N$ chains per unit volume each containing $n$ identical, freely jointed links of length $l$. It is assumed that $r$ has a Gaussian distribution and $r << nl$; that is the end-to-end length of the chains is much less than their fully extended length. The entropy of the network is the sum of the entropies of the individual chains.
Figure 2.45: The deformation of chains (after Treloar (1975)).

Figure 2.45 shows the individual chain represented by vector length $r'$, with components $x'$, $y'$, and $z'$, where:

\[ x' = \lambda_1 x; \quad y' = \lambda_2 y; \quad z' = \lambda_3 z \] (2.4)

the axes of coordinates is chosen to coincide with the principal axes of strain. The number of chains in the unstrained state having length components in the range $dx$, $dy$, $dz$ is:

\[ dN = N \left( \frac{b^3}{\pi^2} \right) e^{-b^2(x^2+y^2+z^2)} dxdydz \] (2.5)

The entropy of the chain in the original state, as given by Equation (2.3), will be;

\[ S_0 = c - kb^2r^2 = c - kb^2(x^2 + y^2 + z^2) \] (2.6)

The total entropy $S_0$ in the unstrained state is, therefore;

\[ S_0 = \int S_0 dN \]

\[ = \frac{Nb^3}{\pi^{3/2}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} [c - kb^2(x^2 + y^2 + z^2)] e^{-b^2(x^2+y^2+z^2)} dxdydz \] (2.7)

This integral can be reduced to a simple form;

\[ S_0 = N(c - 3k/2) \] (2.8)
The entropy of the same chain in the strained state is obtained by substituting the values of \((x, y, z)\), thus;

\[ s = c - kb^2 \left( \lambda_1^2 x^2 + \lambda_2^2 y^2 + \lambda_3^2 z^2 \right) \]  

(2.9)

with a similar approach to Equation (2.7), the total entropy in the strained state is thus;

\[ S = \int s \, dN \]

\[ = \frac{N b^3}{\pi^{3/2}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left[ c - kb^2(\lambda_1^2 x^2 + \lambda_2^2 y^2 + \lambda_3^2 z^2) \right] e^{-b^2(x^2+y^2+z^2)} \, dx \, dy \, dz \]  

(2.10)

and reduced to;

\[ S = N \left[ c - \frac{1}{2} k(\lambda_1^2 + \lambda_2^2 + \lambda_3^2) \right] \]

The entropy difference between the unstrained and strained states is, therefore;

\[ \Delta S = S - S_0 = -\frac{1}{2} N k \left( \lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3 \right) \]  

(2.11)

For reversible deformation, the work is equal to the change in the Helmholtz free energy (Treloar, 1975). The Helmholtz free energy, \(A_H\), is defined as;

\[ A_H = U - TS \]  

(2.12)

where \(U\) is the internal energy and \(T\) is the absolute temperature. Hence, for an isothermal deformation, the work of deformation, \(W\), for a unit volume is given by;

\[ W = \Delta A = \Delta U - T \Delta S \]  

(2.13)

where \(T\) is the absolute temperature. Assuming there no change in internal energy \((\Delta U=0)\);

\[ W = -T \Delta S \]  

(2.14)

Hence from Equation (2.11),

\[ W = \frac{1}{2} N k T \left( \lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3 \right) \]  

(2.15)
The strain energy per unit unstrained volume, \( W \), is also called the strain energy density function. Equation (2.39) may conveniently be written as;

\[
W = \frac{1}{2} G \left( \lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3 \right) \tag{2.16}
\]

with,

\[
G = NkT \tag{2.17}
\]

Rivlin (1948b) shows that \( G \) is the shear modulus, and there is proportionality between stress and strain in simple shear, even for large strains.

Based on the equations above, the stored energy \( W \) is proportional to the absolute temperature \( T \), the number of chain per unit volume \( N \), and is related to a single physical constant – the shear modulus \( G \).

It is reasonable to assume that a rubber body retains a constant volume so;

\[
\lambda_1 \lambda_2 \lambda_3 = 1 \tag{2.18}
\]

So Equation (2.16) can be written as;

\[
W = \frac{1}{2} G \left( \lambda_1^2 + \lambda_2^2 + \left( \frac{1}{\lambda_1 \lambda_2} \right)^2 - 3 \right) \tag{2.19}
\]

So, for an incompressible material, \( W \) is a function of two independent variables; \( \lambda_1 \) and \( \lambda_2 \). For an incompressible rubber, the true stress is given by the equation;

\[
\sigma_i = \frac{\lambda_i}{\lambda_1 \lambda_2 \lambda_3} \frac{\partial W}{\partial \lambda_i} + p \tag{2.20}
\]

With \( p \) represents an arbitrary hydrostatic pressure. \( i \) represents 1, 2, or 3. Since incompressibility implies \( \lambda_1 \lambda_2 \lambda_3 = 1 \), the denominator in (2.20) is omitted.

The differences between any two principal stresses may be determined by;

\[
\begin{align*}
\sigma_1 - \sigma_2 &= G \left( \lambda_1^2 - \lambda_2^2 \right) \\
\sigma_2 - \sigma_3 &= G \left( \lambda_2^2 - \lambda_3^2 \right) \\
\sigma_3 - \sigma_1 &= G \left( \lambda_3^2 - \lambda_1^2 \right)
\end{align*} \tag{2.21}
\]

The stress-strain behaviour for particular types of deformation may be determined from Equation (2.21), by using the suitable extension ratios in relation to the stresses applied.
Some particular types of strain are described here. Simple extension (or uniaxial compression) defined by the extension ratio $\lambda$, then $\lambda_2 = \lambda_3 = 1/\lambda$; $\sigma_2 = \sigma_3 = 0$ (see Figure 2.44b). Therefore the true stress is given by (Treloar, 1974);

$$\sigma_{\text{true}} = G\left(\lambda^2 - \frac{1}{\lambda}\right) \quad (2.22)$$

For the simple shear, produced by tangential stress $\sigma_{12}$ (Figure 2.44b), the relation between shear stress and shear strain $\gamma$ is given;

$$\sigma_{12} \text{ or } \tau = G\gamma \quad \text{with} \quad \gamma = \lambda_1 - \frac{1}{\lambda_1} \quad (2.23)$$

Phenomenological theory

The phenomenological theory is based upon a mathematical description of the stress-strain behaviour from the viewpoint of continuum mechanics. It provides a mathematical description of the rubbery behaviour, so that stress and strain problems may be solved without reference to molecular concepts.

Rivlin (1948a, 1948b) developed the most comprehensive analyses using this approach for large strain elasticity for rubber material, with the assumptions that; 1) the material is isotropic at the unstrained state, and 2) no change of volume occurs during the deformation.

Rivlin suggested the strain energy function, $W$ can be expressed by three ‘strain invariants’ – $I_1$, $I_2$, and $I_3$, which are even powered functions of the extension ratios;

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$$

$$I_2 = \lambda_1^2 \lambda_2^2 + \lambda_2^2 \lambda_3^2 + \lambda_3^2 \lambda_1^2 \quad (2.24)$$

$$I_3 = \lambda_1^2 \lambda_2^2 \lambda_3^2$$

For incompressible rubber, $I_3 \equiv 1$, therefore;

$$I_2 = \lambda_1^{-2} + \lambda_2^{-2} + \lambda_3^{-2} \quad (2.25)$$
This enables for a general form of strain energy function for incompressible rubber, which can be written as a series (Rivlin and Saunders, 1951);

\[
W = \sum_{i=0, j=0}^{\infty} C_{ij} (I_1 - 3)^i (I_2 - 3)^j
\]  

(2.26)

With \( C_{ij} \) are independent elastic constants. The simplest strain energy function using \( I_2 \) is shown as;

\[
W = C_{10} (I_1 - 3)
\]  

(2.27)

where comparison with Equation 2.17, shows that this is the same result as the statistical theory with \( C_{10} = G/2 \). This is similar to Equation (2.16). This model is suitable for general use in Finite Element Analysis (FEA). Equation (2.27) is commonly known as the neo-Hookean material model and is the simplest way to extend Hooke’s Law of proportionality between stress and strain for elastic materials to hyperelastic materials (Muhr, 2005). The model, however, is approximate (Treloar, 1975).

The statistical and phenomenological approaches assume rubber elasticity follows perfectly reversible relations between stress and strain. However, in practice, a departure from such ideal elastic behaviour occurs.

2.5 Viscoelasticity

Viscoelasticity is the property of a material that demonstrates both elastic and viscous behaviour. Also known as time-dependent elasticity, its behaviour is used to describe the long-duration time-dependent behaviours, such as stress relaxation and creep of polymers including rubber.

2.5.1 Creep and stress relaxation

Creep and stress relaxation are inherent mechanical behaviour of viscoelastic materials. Such properties can be associated with the long-term behaviour of vulcanised rubber for engineering applications. Creep is described as the continuous deformation of rubber at constant stress. Similarly, when rubber is held at a constant strain, the initial stress will gradually decrease with time – this is stress relaxation.
Creep and stress relaxation may be related as follow (Gent, 1962a). In general, the creep rate, \( C \) at constant stress, \( \sigma \) is defined as the relative slope of the strain versus time curve;

\[
C = \left( \frac{1}{\varepsilon} \right) \left( \frac{\partial \varepsilon}{\partial t} \right)_t 
\]

where \( \varepsilon \) is the strain at time \( t \) after the stress \( \sigma \) is imposed. The rate of stress relaxation, \( S \) at a constant strain, \( \varepsilon \) is defined similarly:

\[
S = -\left( \frac{1}{\sigma} \right) \left( \frac{\partial \sigma}{\partial t} \right)_\gamma 
\]

If the deformation and stress are equal, the relationship between the two will be;

\[
C = \alpha S \\
\text{with } \alpha = \left( \frac{\sigma}{\varepsilon} \right) \left( \frac{\partial \varepsilon}{\partial \sigma} \right)_t 
\]

Both are often found to be approximately proportional to the logarithm of time (Gent, 1962a). Therefore it is convenient for the time to be quantified as \( \log(t) \) instead of \( t \). Equation (2.30) shows that creep and stress relaxation are related. A typical creep curve for vulcanised rubber is shown in Figure 2.46 whilst the stress relaxation curve is shown in Figure 2.47;

![Figure 2.46: Creep plot in (b) following instantaneous stress shown by the dashed line in (a).](image-url)
Figure 2.47: Stress relaxation plot in (b) following instantaneous strain shown by the dashed line in (a).

Behaviour of the type shown in Figure 2.46, where linearity with log time holds, is usually attributed to physical relaxation, which reflects the rearrangement of molecular networks under strain (Gent, 1962a).
Figure 2.48: (a) Stress relaxation and creep rates in extension (Gent 1962a), (b) Stress relaxation rates in extension (Yamaguchi et al. 2015). Both plots are of unfilled NR.

Figure 2.48(a) shows the stress relaxation rates (●) are independent of extension for unfilled rubber. As the applied stress and, hence, the extension after 1 minute increases, the creep rate (○) increases by a factor of about 2, while the recovery rate (+) decreases by a similar factor. The full curves represent the calculated creep between two extreme values of $T = \frac{C_{01}}{C_{10}}$, 0.5 and 1.0. $C_{10}$ and $C_{01}$ are elastic constants for Mooney-Rivlin elasticity appropriate for nonlinear stress-strain (uniaxial tension deformation). The creep rates, $C$, were calculated when the stress relaxation rates, $S$, are known as shown in (Gent 1962a);

$$C = \alpha S$$

with

$$\alpha = \frac{\lambda(\lambda^2 + \lambda + 1)(\lambda + T)}{\lambda^4 + 2\lambda + 3T}$$

(2.31)

where $\lambda = 1 + \varepsilon$, with $\lambda$ being the extension ratio and $\varepsilon$ the nominal strain. The shear modules ($G$) calculated with both elastic constants by $G = 2(C_{10} + C_{01})$. Thus Gent demonstrated creep and stress relaxation are two manifestations of the same physical processes, and the apparent difference in behaviour is due to the non-linear stress-strain behaviour of rubber in uniaxial tension. Figure 2.48(b) shows that the similar observation on stress relaxation by Yamaguchi, Thomas and Busfield (2015).

Another study by Gent (1962b) on a thin rubber sample found that the creep behaviour changed after about 1,000 to 10,000 minutes, depending on the materials and methods of crosslinking. The tests indicated the presence of an additional and secondary
creep process. This was attributed to an oxidation process which breaks down the rubber molecular network and it is usually referred to as chemical creep. It is characterised by a linear dependence of relaxation with time (rather than logarithmic of time), becoming the dominating effect after very long times.

A study on temperature increase during a creep test on NR suggested that a temporary rise in temperature during the creep experiment gave a marked increase in creep rate, and when the temperature is returned to its original level, significant additional creep had occurred (Derham, 1973). This is true for unfilled rubber (NR and SBR), as shown in Figure 2.49.

![Figure 2.49: The creep with temperature cycle from 40°C – 60°C – 40°C for; (a) NR, (b) SBR (Derham, 1973).](image)

Derham, Lake and Thomas (1969) showed that the relaxation rate of NR increases as the test humidity increase, and to give the linear relaxation in log$_{10}$ time, the test humidity must be maintained between 30-60% (RH%). Derham (1972) then suggested that there is no effect of humidity on NR for RH<40%, which means the uncertainties affected by the humidity can be avoided as long as the air humidity is kept below the threshold throughout the test.

The effect of filler on the relaxation of NR is an increase in the stress relaxation rate, which is approximately proportional to the carbon black loading, as shown in Figure 2.50;
In contrast to unfilled rubber, stress relaxation rates in general increase with increasing strain for filled NR vulcanisates. Filler-filler or filler-polymer breakdown also affect relaxation rates and show different rates with different types of filler (Andrews, 1963; Derham, 1973).

As the creep compliance of rubber-like materials is often found to approximate to a linear dependence on the logarithm of the time, Chai and Thomas (1981) suggested;

\[ J(t) = J_0 \left(1 + A \log \left(\frac{t}{t_0}\right)\right) \]  

(2.32)

where \( J_0 \) is the compliance at reference time \( t_0 \) and \( A \) is a parameter indicative of the rate of increase of \( J(t) \). An equation analogous to this can be written for the relaxation modulus, \( G(t) \). This equation accounts for the physical creep and stress relaxation.

Derham, Lake and Thomas (1969) assert that the total fractional creep is defined by the Equation (2.33);

\[ \frac{\Delta e}{e_0} = A \log \left(\frac{t}{t_0}\right) + B (t - t_0) \]  

(2.33)

with \( A \) as the physical component and \( B \) as the chemical component. \( e_0 \) is strain and \( \Delta e \) is the strain increase at time \( t \). Equation (2.33) was found to be in agreement with experimental work on NR specimens, as shown in Figure 2.51.
2.5.2 Linear viscoelasticity and the Boltzmann superposition

Linear viscoelastic theory can describe the stress relaxation and creep and is generally applicable for unfilled rubbers over a moderate strain, time and temperature range. Linear viscoelastic behaviour means that the Boltzmann superposition principle applies; stress due
to the action of several strains \( \gamma = \gamma_1 + \gamma_2 + \gamma_3 + \ldots \), is equal to the sum of the stresses \( \tau_1, \tau_2, \tau_3, \ldots \) which would be developed as a result of \( \gamma_1, \gamma_2, \gamma_3 \) acting independently (Gent 1992).

Boltzmann superposition is only valid for materials with linear viscoelastic behaviour. For a lot of filled rubbers, linear viscoelasticity cannot predict the observed response, and non-linear viscoelasticity is needed. For the case of simple shear, linear viscoelasticity can be expressed by (Ferry, 1980);

\[
\tau(t) = \sum G(t-t_i)\gamma(t) + G(t-t_i)(\gamma(t) - \gamma(t_i)) + \ldots
\]

\[
\tau(t) = \int_{-\infty}^{t} G(t-t_i) \dot{\gamma}(t_i) dt_i
\]

(2.34)

where \( \tau \) is the shear stress at time \( t \), \( G(t-t_i) \) is the relaxation modulus and \( \dot{\gamma}(t_i) \) is the shear strain rate (written as \( d\gamma(t_i)/dt_i \)). An example of the Boltzmann superposition principle approach is shown in Figure 2.52;

![Figure 2.52: Boltzmann superposition principle](image)

### 2.5.3 Simple viscoelastic model

Various models that represent the linear viscoelastic behaviour, for example, stress relaxation of rubber and rubber-like materials, are cited in the literature; (Gent, 1992; McCrum, Buckley and Bucknell, 1997). A simple representation of linear viscoelastic behaviour can be shown by combining elements and characteristics of the springs and dashpot, which produced two models described in Figure 2.53.
Figure 2.53: Models of viscoelastic material

The linear viscoelastic solid model may be constructed with a Hookean spring with stiffness, \(k\) (N/m) and a Newtonian dashpot containing viscous fluid with viscosity, \(\eta\) (N.s/m\(^2\)). The simple forms to describe the system is to combine a spring and a dashpot either in series or parallel - Maxwell or Voigt models (Figure 2.53). The spring accounts for the elastic component and the dashpot for the viscosity. Each can be represented by;

\[
\sigma_s = E\varepsilon_s
\]  
\[\text{(2.35)}\]

\[
\sigma_d = \eta \left(\frac{d\varepsilon_d}{dt}\right)
\]  
\[\text{(2.36)}\]

where \(\sigma_s\) and \(\sigma_d\) are the stress of spring and dashpot respectively, \(\varepsilon_s\) and \(\varepsilon_d\) are the strain of spring and dashpot respectively, and \(E\) is the elastic modulus. The algebraic derivation of the Maxwell model assumes that the strain is imposed instantaneously and held constant. The expression of the stress at the function of time can be shown as;

\[
\sigma(t) = \sigma_0 e^{-\frac{t}{\tau}}
\]  
\[\text{(2.37)}\]

where \(\tau\) is the relaxation time (\(\tau=\eta/E\)), and \(t\) is the time. Most materials exhibit behaviour that is more complex than these two simple models. For this reason, it is necessary to use generalised models, for instance, the generalised Maxwell or generalised Voigt models, to describe the viscoelastic behaviour of a material quantitatively.
A generalised Maxwell model consists of a number of Maxwell elements in parallel and is characterized by the distribution of elastic moduli $E(\tau)$ as a function of the relaxation time, $\tau$. The generalised Voigt model consists of a number of simple Voigt elements in series and is described by the distribution of compliances $D(\tau)$ as a function of the retardation time, $\tau$. In practice, it is more convenient to describe stress relaxation by a generalised Maxwell model and creep experiments by a generalised Voigt model. For the case of shear deformation which will be discussed from now onwards, the $E(\tau)$ and $D(\tau)$ will be replaced by the shear modulus $G(\tau)$ and the shear compliance $J(\tau)$ respectively.

### 2.5.4 Generalised Maxwell model

The generalised Maxwell model (Figure 2.59) is based on parallel combinations of Maxwell elements, with the addition of $G_\infty$ also in parallel as the ‘long-term’ modulus (Ferry, 1980). As it is based on the linear viscoelastic theory, it can describe stress relaxation and creep and is generally applicable for unfilled NR over a moderate strain range. It is implemented in Finite Element Analysis packages, such as ABAQUS, by a discrete series (also known as Prony series) shown in Equation (2.38);

$$G(t) = G_\infty + \sum G_i e^{-t/\tau_i} \tag{2.38}$$

where $\tau_i$ is the relaxation time for each contribution, $G_i$ to the shear modulus.

According to Tobolsky (1960), a reasonable fit for a viscoelastic material over not too wide a range of log($t$) can be represented by;
\[ G(t) = G_\infty + \int_{t}^{t_{\max}} H_0 e^{-t/\tau} d\ln\tau \]  

(2.39)

with \( \tau_{\min} \ll t \ll \tau_{\max} \). \( G(t) \) is the shear modulus at time \( t \), \( G_\infty \) is the long-term modulus, \( \tau_i \) is the relaxation time for the modulus contribution \( G_i \). \( H_0 \) is the slope of the relaxation plot. In the equation above, there are two parameters; \( G_\infty \) and \( H_0 \). A straight line can be produced with \( G(t = t_{\text{ref}}) \) and \( H_0 = dG(t)/d\ln(t) \) over a \( \log(t) \) span, as proposed by Ahmadi, Kingston and Muhr (2008) and described below.

To allow for an algebraic approach, Equation (2.39) can be discretised as the equation below;

\[ G(t) = G_\infty + \sum_{i=1}^{n} H_0 e^{-t/\tau_i} \delta \ln \tau_i \]  

(2.40)

### 2.5.5 Discretised Prony series

In ABAQUS (Dassault Systèmes, 2014), the material behaviour is described by a Prony series such as;

\[ G(t) = G_0 \left[ 1 - \sum g_i (1 - e^{-t/\tau_i}) \right] \]  

(2.41)

where \( G_0 \) is the instantaneous modulus and \( g_i \) are the dimensionless relaxation modulus. By combining this with Tobolky’s (1960) relaxation function (Equation 2.40), Ahmadi, Kingston and Muhr (2008) proposed a two-parameter implementation of the Prony series which is applicable to rubbers which show stress relaxation behaviour that in which the force decays linearly with \( \log \) time, as follows. By making use of the relationships;

\[
G_\infty = G_0 - \sum G_i \\
g_i = \frac{G_i}{G_0} \\
h_0 = \frac{H_0}{G_0}
\]  

(2.42)

equation (2.40) can be rewritten as;

\[ G(t) = G_0 \left[ 1 - \sum g_i + h_0 \sum e^{-t/\tau_i} \delta \ln \tau_i \right] \]  

(2.43)

and comparison with Equation (2.41) suggests;
\[ g_l = h_0 \delta \ln \tau_i \]  
so that;

\[ G(t) = G_0 \left[ 1 - \sum_{i=1}^{n} h_0 (1 - e^{-t/\tau_i}) \delta \ln \tau_i \right] \]  

This discretisation can be divided into intervals of equal relaxation time (\( \ln \tau_i \)) hence;

\[ \delta \ln \tau_i = \frac{\ln \tau_{\text{max}} - \ln \tau_{\text{min}}}{n} \equiv \ln k \]

with \( k = \left( \frac{\tau_{\text{max}}}{\tau_{\text{min}}} \right)^{\frac{1}{n}} \)

and taking \( \ln \tau \) as the mean of the interval,

\[ \ln \tau_i = \ln \tau_{\text{min}} + \left( i - \frac{1}{2} \right) \cdot \ln k \]  

\[ \ln \tau_i = \ln \tau_{\text{min}} + \ln \left( \frac{\tau_{\text{max}}}{\tau_{\text{min}}} \right)^{\frac{2i-1}{2n}} \]

\[ \tau_i = \tau_{\text{min}} \left( \frac{\tau_{\text{max}}}{\tau_{\text{min}}} \right)^{\frac{2i-1}{2n}} \]

### 2.5.6 Time-temperature superposition principle (TTSP)

Time-temperature superposition principle or WLF (William-Landel-Ferry) transformation is an empirical approach to describe the equivalence of time and temperature for rubber materials (Williams, Landel and Ferry, 1955; Ferry, 1980). Two methods can be used to determine rubber behaviour at longer times. First, by measuring the response at a longer time, or secondly, by conducting a series of short-time measurements at various temperatures and then using these short-time test data from high temperature and shift it to a longer time at a lower temperature (see Figure 2.55).

The superposed curve; called master curve, is constructed by using a horizontal shift factor, \( a_T \) to compensate for the change in timescale by the changing temperature, and a vertical shift factor, \( \xi \) to account for the change in modulus and density.
All the relaxation time $\tau_i$ are shifted to the shift temperature $T_s$ by the temperature-dependent shift factor $a_T$ estimated empirically by the WLF equation;

$$\log a_T(T_s) = -\frac{C_1(T - T_s)}{C_2 + T - T_s}$$

(2.47)

with $T_s = T_g + 50 K$

where $C_1$ and $C_2$ are constants and $T_s$ is the glass transition temperature of the polymer.

Williams, Landel and Ferry (1955) used the viscoelastic constants of $C_1 = 8.86$ and $C_2 = 101.6K$. These values were selected based on the chosen reference temperature, $T_r$ of −30°C for polyisobutylene but found to work well for a wide range of polymers, provided that $T_s$ was set to 50°C above $T_g$. If a different reference temperature, $T_0$ is chosen instead of $T_s$, then the new viscoelastic constants, $C_1^0$ and $C_2^0$ are used, calculated as follows;

From Equation 2.49, the shift factor to shift from a temperature $T$ to $T_s$ is given by;

$$\log a_T(T_s) = -\frac{C_1(T - T_s)}{C_2 + T - T_s}$$

(2.48)

and the shift factor to shift from a reference temperature $T_0$ to $T_s$ is given by;

$$\log a_{T_0}(T_s) = -\frac{C_1(T_0 - T_s)}{C_2 + T_0 - T_s}$$
thus the shift factor for a shift from $T_s$ to $T_0$ is given by;

$$\log a_{T_s(T_0)} = \log a_{T_0(T_s)} = \frac{C_1(T_0 - T_s)}{C_2 + T_0 - T_s}$$

and the shift factor for shifting from $T$ to the required reference temperature, $T_0$ is written as;

$$\log a_{T(T_0)} = \log a_{T(T_0)} + \log a_{T_s(T_0)}$$

$$= \frac{-C_1(T - T_s)}{C_2 + T - T_s} + \frac{C_1(T_0 - T_s)}{C_2 + T_0 - T_s}$$

$$= \frac{-C_1 C_2 (T - T_0)}{(C_2 + T - T_s + T_0 - T_0)(C_2 + T_0 - T_s)}$$

$$\equiv \frac{-C_1^0 (T - T_0)}{C_2^0 + T - T_0} \quad (2.49)$$

where

$$C_1^0 = \frac{C_1 C_2}{C_2 + T_0 - T_s} \quad (2.50)$$

and,

$$C_2^0 = C_2 + T_0 - T_s \quad (2.51)$$

The empirical description of the shift factor, $a_T$, is true for amorphous polymers well above the glass transition temperature $T_g$. The WLF equation, however, is not valid for semi-crystalline polymers below its melting points. For this, the Arrhenius equation is better suited for the shift factor $a_T$;

$$\log a_T = \frac{\Delta H}{2.303 R} \left( \frac{1}{T} - \frac{1}{T_{ref}} \right) \quad (2.52)$$

with $\Delta H$ as the activation energy and $R$ as the universal gas constant (8.3145 JK$^{-1}$mol$^{-1}$) (Seitz and Balazs, 1968).

Apart from the horizontal shift factor, the vertical shift, $\xi$ is included to take into account changes in rubber modulus and density at different temperature. This is represented by Equation (2.53) below;

$$G(t, T) = G(t_{a_T}, T_0) \frac{T_0 \rho_0}{T \rho} \quad (2.53)$$
with \( \xi = \frac{T_0 \rho_0}{T \rho} \)

with \( \rho \) and \( \rho_0 \) are the density at temperature \( T \) and \( T_0 \) respectively, and \( t_{\text{at}} \) is the reduced time. The change in density, is usually small and often neglected (Brinson and Brinson, 2008).

The TTSP test method

For a stress relaxation test using TTSP, the experiment starts by straining several individual samples permanently, each at its individual temperature and letting it relax for a dwell time. To construct a stress relaxation master curve, each individual isothermal curves will then be treated with a horizontal shift factor, \( a_T \), to the reference temperature, and a vertical shift factor, \( \xi \), to consider the density and modulus changes at different temperatures. For validation purpose of the test method, a comparison can be made with the conventional long-term stress relaxation.

2.5.7 Stepped isothermal method (SIM)

The Stepped isothermal method (SIM) is an experimental method to predict the long-term viscoelastic behaviour of a material through a short-term test. It was first utilised for product testing of geosynthetics (Thornton et al., 1998; Thornton, Paulson and Sandri, 1998). The SIM can be explained as a short-term creep or stress relaxation test where the experiment temperature is elevated stepwise using only a single specimen in the experiment. The temperature steps can be rescaled and shifted to generate a master curve, with the aim to match the prediction of long-term creep/stress relaxation resulting from the Time-temperature superposition principle (TTSP) approach. In contrast, the conventional test procedure for TTSP require many experiments to produce a master curve at the reference temperature, implying long test duration, thus incurring higher costs. The SIM is an improvement of the widely used TTSP method.

In engineering testing, care is taken to produce identical samples, although there is still a possibility for sample variations that could exist which may affect the test outcomes. It may come from the inhomogeneity within the rubber mix or from different compound batches, and slight variations in cure conditions used for each samples. The error associated with the sample-to-sample variability can be diminished, as the SIM utilises only a single specimen (Vanhoose et al., 2014). Among the standards and guidelines available for SIM
The SIM test method

SIM requires a single test specimen with a series of step temperature increments. It has been claimed that SIM was successfully applied in characterising the long-term creep behaviour of a tensile strip of polypropylene up to approximately 100 years with a reasonable agreement with the prediction of long-term creep resulting from the conventional approach of TTSP (Achereiner et al. 2013). Due to good agreement with the conventional long-term creep test, the SIM method has been used widely in accelerated ageing tests of many types of geosynthetics and geomembranes (Lechat and Greenwood, 2010; Yeo and Hsuan, 2010; Zhang, Zuo and Liu, 2013). SIM has also been applied in the testing of other polymers like aramid yarn (Alwis and Burgoyne, 2008) (Thornton et al., 1998) and HDPE (Thomas, Nelson and Cuttino, 2010; Bozorg-Haddad and Iskander, 2011). The master curve production technique used by Achereiner et al., (2013) is shown in Figure 2.56 and briefly discussed afterwards.

Figure 2.56: The creep test using SIM procedure. (a) correction of the thermal expansion (b) determination of virtual start time, $t'$ (c) rescaling (d) curve shifting using TTSP shift factor $a_T$ (Achereiner et al., 2013)
For a creep test using SIM (refer Figure 2.56), the experiment starts by stressing a sample permanently at the start temperature and letting it creep for a predetermined dwell time. The temperature then increases stepwise and is held for a similar dwell time. The steps can be repeated accordingly at several temperature levels. After that, a thermal correction experiment is to be conducted in order to account for the additional strain caused by the thermal expansion of the test system. The thermal expansion factor may come from the test jig and the specimen itself. After the thermal correction, the virtual start time, $t'$ for each isothermal curves will be determined. Using the $t'$, the curves will be rescaled as if each isothermal curves starts independently at time zero “0s”. Using the horizontal shift factor, $a_n$, each isothermal curve will be shifted to the reference temperature, constructing the creep master curve. The use of the vertical shift, $\xi$, is considered to account for the density and modulus changes at different temperature levels.

The challenge in using SIM lies in the temperature stepping, where rapid heating can give an advantage to the test setup. In the study by Achereiner et al., (2013), every 10°C of temperature change took 5 minutes. This is true for the specimen of high surface-to-thickness ratio. With a quick temperature change, the transition phase with undefined creep can be considered negligible. However, if the heating is not instantaneous, the undefined creep in the transition region can be excluded from the analysis, but the time scale shall be maintained.

Although such discussions are on the advantage of the SIM in predicting the long-term creep, it must be underlined that the drawback of the method lies in its discrepancy to converge with the actual creep data measured directly in long-term creep experiment (Achereiner et al. 2013). Stiffening of the material due to the chemical ageing may be one reason for the discrepancy (Brinson and Brinson, 2008).

There are currently no studies on the application and reproducibility of SIM for measuring the relaxation of rubber. This work investigates if the SIM could provide a consistent long-term stress relaxation measurement of rubber.

2.6 Rubber in Engineering Applications

Rubber, especially NR, has been used for more than a hundred years and is very versatile in engineering applications. Major engineering areas that use rubber are automotive, construction, railways and offshore industry. Rubber works mainly as a spring or cushion
against vibration. The literature survey will focus on the application of rubber and elastomers in civil engineering fields. The historical background and current trend of rubber engineering will be discussed.

2.6.1 Basic principles for engineering design with rubber

The working principle of rubber in engineering revolves around its load-deflection behaviour, dynamic mechanical properties and its longevity. The elastic modulus of rubber is of the order of one-thousandth of its bulk modulus, and rubber can deform by several hundred per cent strain without failure (Fuller et al., 1988).

Rubber can be used as a spring. Axially compressed steel spring exhibits a linear load-deflection behaviour, while rubber generally exhibits nonlinear load-deflection behaviour. Harris and Stevenson (1986) explain typical differences between simple shear and axial compression of a solid rubber disc and axial compression of a hollow rubber cylinder. The mode of deformation influences the degree of nonlinearity, as shown in Figure 2.57.

![Figure 2.57: Different types of load-deflection behaviour for rubber (adapted from Harris & Stevenson 1986).](image)

2.6.2 Rubber hollow sections as impact absorbers

The Euler’s buckling of a slender column stresses that the critical stress can be quantified as;

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(K_{ELF} L_g)^2}$$

(2.54)
where $P_{cr}$ is the critical load at buckling, $A$ is the cross-sectional area, $K_{ELF}$ is the effective length factor, $L$ is the unsupported column length, $E$ is Young’s modulus of the material, and $r_g$ is the radius of gyration of the cross-section.

A study on the stability of circular rubber columns under uniaxial compression indicated the existence of a transition slenderness ratio that distinguished the Euler-type buckling behaviour of the long column from the axisymmetric bulging behaviour of the short column. Also, an axially compressed short thick-walled rubber column exhibited axisymmetric bulging and deformation (Beatty and Hook, 1968; Beatty and Dadras, 1976).

Willis (1948) discussed the buckling deformation of rubbery materials with different dimensions in axial compression. More studies on hollow-shaped rubber include Bakirzis (1972) and Leaver & Lindley (1976), who considered various shapes, including NR cylindrical columns and hollow box sections. The radial outward buckling form of a hollow rubber cylinder is shown in Figure 2.58, and the stress-strain curves for the series of unfilled rubber cylinders with different inner diameters as tested by Leaver and Lindley (1976) are shown in Figure 2.59.

![Figure 2.58: Radial buckling in the outward direction of the axially compressed hollow cylinder in a cross-section view (Willis, 1948).](image)

![Figure 2.59: Stress-strain curves for cylinders with an outer diameter (OD) of 49.5mm. Figures adjacent to the curves indicate the inner diameter (ID) of the cylinders (Leaver and Lindley, 1976). [Stress = force/rubber cross-section area](image)
Willis (1948) also demonstrated that when a hollow rubber cylinder is strained near to its critical stress, $\sigma_{cr}$ (also known as the stress at buckling), the creep rate is likely to be higher. Although the study was done only on a series of carbon black filled rubber, a similar observation is expected in unfilled rubber because the deformation sequence of a hollow rubber cylinder shall always be the same, albeit with lower critical load.

A series of creep test with three different initial displacements is shown in Figure 2.60.

![Creep of hollow rubber cylinder with carbon black filler, under various load values. The 0.208 ton load reported here is near the hollow rubber cylinder's critical load (Willis, 1948). [1 ton = 9.81kN].](image)

The plot above shows three creep tests with different loads (with three different initial displacements). The highest load is near the critical load, which means it is in the buckling form. The creep rate begins to increase as the critical deformation is approached, and after a short time a complete collapse of the cylinder occurs. Meanwhile, the other two tests which are far from their critical load, showed slower creep.

Bakirzis (1972) derived the force-deformation characteristics of several hollow rubber sections by using elasticity theory of deformation. Using various parameters,
theoretical predictions were compared with experimental stress-strain curves which showed a satisfactory agreement. The study covered several geometrically simple rubber structure and ratios of cross-sectional dimensions; namely hollow circular sections, half-cylindrical sections, and “D” sections, which were radially compressed, and hollow squares and cylinders that were axially compressed.

For this study, the hollow square sections and hollow cylinders under axial compression are of particular interest due to the buckling deformations which exhibit the non-linear stress-strain curves. The non-linear elastic body can be used as the interface between the IAB wall structure and the backfill soil, as one of the DCU operational concepts shown in Figure 2.38 (England et al., 2010). The experimental results for these are shown in Figure 2.61 and Figure 2.62.

![Graph showing load-displacement curves for axial compression of square hollow sections](image)

Figure 2.61: Experimental load-displacement curves for axial compression of square hollow sections (Bakirzis, 1972).

\( b/d \) is the height-inner diameter ratio, \( P/E_0Ld \) is the unitless load and \( \delta/d \) is the axial strain.

The three thinnest units of hollow square sections buckled at lower axial strain (around \( \delta/d = 1.0 \)), and thicker units showed a linear characteristic with a rising slope for larger deflections. These forms of load-displacement behaviour are less likely beneficial for the IAB system due to the steep stiffnesses increase, which is an adverse approach for the IAB application as it should be a strain control system.
Figure 2.62: Experimental stress-strain curves of compressed hollow rubber cylinders. \( \sigma \) is the stress, and \( \delta/L \) is the nominal axial strain. \( D/d \) is the outer-inner diameter ratio (Bakirzis, 1972).

\[ 1 \text{ kg/cm}^2 = 0.098 \text{ MPa} \]

Similarly, Figure 2.62 shows that the thinner hollow cylinders show a decreasing stress beyond the buckling point until it increased again as the cylinder became solid again. The curves appeared to have three stress regions in series: stiff-soft-stiff, which is likely to be beneficial for a system requiring that large changes of strain are met with small changes of stress. This can be made possible when the cylinder is restrained within the soft region.

The critical stress, \( \sigma_{cr} \) of a hollow rubber cylinder during buckling subjected to compressive stress was given by Bakirzis (1972) as:

\[
\sigma_{cr} = \frac{E_0 h}{R(3(1 - v^2))^{1/2}} \tag{2.55}
\]

where \( E_0 \) is Young’s modulus of rubber, \( h \) is the wall thickness, \( R \) is the average radius of the unit shell and \( v \) is the Poisson’s ratio.

The buckling behaviour can be altered by the change in \( D/d \) ratio. This was similarly observed later by Leaver and Lindley (1976) in an energy storage capacity study. Although similar buckling behaviour of hollow cylinders was first observed and described by Willis (1948).
A study looking at the axial compression of hollow rubber was conducted by Wong et al., (2010) by using the numerical method (ABAQUS) to trigger the instabilities of rubber columns. The focus was towards the forming of the wrinkle and crease on the inner walls. The results demonstrated that the wrinkle and crease could be simulated, and the experimental and numerical analyses were in good conformance. This is shown in Figure 2.63.

Figure 2.63: Force–axial compression ratio plot for a short, thick-walled cylindrical rubber column under axial compression. A series of deformed shapes at various levels of axial compression ratio are displayed, showing the formation of creases on the inner wall surface. $\varepsilon_z$ is the axial strain (Wong et al., 2010).

Carder, Barker and Darley (2002) envisaged the possible use of voided rubber components as an alternative to EPS for IAB, due to rubber’s elasticity. The use of solid rubber, however, may pose a challenge given that it deforms at constant volume, due to its near incompressibility. As a solid rubber does not have space to deform, the stress will be transferred to the surroundings. To overcome this, there was an idea of introducing holes or voids to allow deformation without transferring huge stress.
With that rubber is widely used as an impact-absorbing device which is able to produce large deflections at relatively low material stresses; one example is the rubber dock fender for berthing ships. Scrap tyres are used for small quays and boats, where the energy absorption demand is low but the berthing force of huge vessels will require high energy absorption capacity. This demand can be met by having a large deformable rubber unit (Lau, Leaver and Lindley, 1974).

2.6.3 The ageing aspects of natural rubber

NR or other elastomers are used in many applications where long-term performance is essential, such as engine mountings, sealing rings, and structural and earthquake isolation bearings (Gent, 1992; Coveney, 2006). During service, elastomeric components may be exposed to the environmental conditions causing degradation, e.g. due to temperature, UV and ozone. The service life of some products is long, thus accelerated ageing tests are used to estimate resistance to ageing (Brown and Butler, 2000).

An example is shown by an aged sample of a 96-year old NR pad from the Melbourne Railway Viaduct built in 1890. The degradation was limited to the surface layer of 1-2mm, and the inner rubber was found to be in good condition (Ab Malek and Stevenson, 1992). Such longevity was postulated to be due to the formation of a skin crust which delays the permeation of oxygen (Stenberg, Shur and Jansson, 1979), and oxygen diffusion through the oxidised rubber is much slower than through a new rubber (Gent, 1992). The rubber formulation used was very basic, as antidegradants and antiozonants were only used widely after the Second World War (Huntink, 2003).

Figure 2.64: (a) Cut section of a 96-year old rubber pad showing a hard surface skin; (b) Drawing of Melbourne Railway Viaduct in 1889 showing the use of natural rubber pads (Ab Malek & Stevenson 1992)
The first steel-laminated NR bearings were used in Pelham Bridge in Lincolnshire England, constructed in 1957. After 40 years in service, its ageing showed little changes in its physical properties (Watanabe et al., 1996). Similar conclusions were reached for other long-serving rubber components on the record, such as a 102-year-old sewer joint ring in London (Dunkley, 1964), a 42 years aircraft tyre and inner tube immersed in seawater (Ab Malek & Stevenson 1986) and a submarine sealing gasket with 69 years of seawater exposure (Rubber Development, 1986).

**Natural rubber for structural bearings**

The first building constructed on steel-laminated rubber bearings, in order to isolate the structure from ground-borne vibration, was Albany Court, a block of flats on St. James Park underground station in London; see Figure 2.65 (Waller, 1966). Since then, numerous projects have been undertaken on sites previously deemed to be unacceptable for their high level of noise and vibration. Sharif (2000) listed fifty-five major vibration-isolated structures in the UK. Typical construction of steel-laminated rubber bearings used for bridges is shown in Figure 2.66.

![Figure 2.65: Cross-section of Albany Court, the first base-isolated building in the UK (Waller, 1966).](image-url)
Figure 2.66: The sectioned steel-laminated rubber bearing for bridges.

More recent structures in the UK that use rubber bearings for anti-vibration are Eland House, London (Sharif, 2000) and Park House, London (TARRC, 2012). The creep of rubber bearings used in Albany Court was estimated to be 5.5mm in 100 years at the time of the study (Derham and Waller, 1975).

The steel-laminated rubber bearing was later extended to provide earthquake protection to structures. Three research bodies pioneered seismic isolation bearings based on NR; Delfosse, at Marseille, leading to the isolation of a school in Lambesc in 1977; DSIR New Zealand, leading to William Clayton Building in Wellington in 1983; and TARRC UK, leading to the Foothills Community Law and Justice Centre in California in 1985 (Zulkefli et al. 2014). Rubber bearings demonstrated excellent longevity against environmental factors and are able to maintain their physical properties with very little change (Watanabe et al. 1996 and Ab Malek & Stevenson 1992).

2.7 Summary

The literature discussed here recognised that the IABs are now a common construction and design of choice for many engineers and transport departments as bridges are replaced or newly constructed. However, in spite of its popularity, it is facing an unsolved problem where the escalating stress in the backfill is consequently overstressing the abutment structure. This action is caused by the irreversible settlements near the abutment surface, which is caused by the cyclic thermal displacements that cause the bridge deck to expand and contract (Wolde-Tinsea and Klinger, 1987; Hoppe and Gomez, 1996; Ng, Springman and Norrish, 1998; Arsoy, Barker and Duncan, 1999; England, Bush and Tsang, 2000; Kerokoski, 2006; Kim and
Laman, 2010; Miletic et al., 2016). A similar issue is also found in the propped embedded walls (Carder, 1995; Powrie and Batten, 2000; Chambers et al., 2016).

Several studies were attempted to overcome this issue, where the use of a compressible element behind the wall was tried, and the recycled tyre shreds were used as a partial replacement of the IAB’s granular backfill. One study that showed some enhancement in the performance of the IAB was the use of stackable conical disc springs (CDS), known as DCU.

Following the studies done by Willis, (1948); Bakirzis, (1972); Leaver and Lindley, (1976); and Wong et al., (2010) on hollow rubber columns, the instability from the exertion of axial loading will exhibit bulging deformation. This behaviour provides a large plateau region, where the force changes very little over a large range of displacements. This instability can be exploited, for example, when a specific application is required to maintain its initial prestress while allowing for the changes in strain. This characteristic can be the key feature of the DCU when the initial prestress during the DCU deployment can be set around the middle of the plateau region.

The DCU works by maintaining the load of the structure at approximately constant while absorbing the changes in displacement. It utilises the nonlinear force-deflection behaviour of the DCU, which serves as the ‘working region’ to keep the stress changes low. In real applications, the steel-made CDS may oxidise, and to prevent oxidation and environmental deterioration, the use of rubber can be considered either by encapsulating the DCU or as the base material for the DCU unit. To make it work as a single rubber unit, the idea of using a simple form of nonlinear rubber spring that accommodates the change of displacements while keeping the force constant is attractive and therefore will be pursued.

On another part, methods for predicting long-term properties of rubber have been reviewed, and the Stepped isothermal method was discussed by other researchers for long-term prediction of polymers in creep (Thornton et al., 1998; Thornton, Paulson and Sandri, 1998; Achereiner et al., 2013; Vanhoose et al., 2014). This method can be refined for stress relaxation and has not yet been tried on rubber. Existing theories on the Boltzmann superposition principle (BSP) can be used to model the stress relaxation following changes in strain and the Time-temperature superposition principle (TTSP) or WLF transformation, to allow for changes in temperature. Previous studies covered the stress relaxation of rubber in extension, compression and shear and have not dealt with the long-term performance of
rubber in other deformation modes, such as the compressive strain of hollow rubber body where the buckling state of deformation is possible.

Most applications of rubber in civil engineering are concentrated on its use as anti-vibration bearings for structures, impact absorber for berthing vessels (dock fender), and sealing mechanisms, to name a few. In real practices, rubber bearings are mounted as structural supports and subjected to permanent compressive stresses. The long-term performance can be characterised as creep. For the case of DCU, it is mostly imposed by the permanent compressive strain, therefore characterised as stress relaxation. Such rubber-based DCU is required to withstand long-term strain without developing excessive stress relaxation.

Overall, this study links two distinguished parts; the need to demonstrate the design approach of rubber-based DCU and its capability in changing the problematic thermally induced strain of the IAB to the stress-controlled system, and also, the need to investigate the rubber behaviour that suits the deformation expected for a DCU when deployed in the IAB system. This will involve the works to understand the stress-strain and the long-term viscoelastic behaviour of rubber, especially in stress relaxation. It is worth studying the validity of the existing viscoelastic theory considering the unusual strain shape in buckling. Moreover, to investigate its role in designing a simple rubber component to address issues posed by the thermally induced strain highlighted in the IAB and propped embedded walls.
CHAPTER 3 VISCOELASTIC BEHAVIOUR OF UNFILLED NATURAL RUBBER

This chapter discusses investigations of the effect of strain magnitude on the stress relaxation of unfilled natural rubber (NR) in simple shear and buckling deformations, and reciprocate the method by evaluating the effect of stress magnitudes on creep.

The work to validate the linear viscoelastic theory on the stress relaxation of an unfilled NR in simple shear and buckling deformations will be discussed, which is by implementing the Boltzmann superposition principle to simulate strain changes and the Time-temperature superposition (WLF) transform to simulate the changes in temperature. Comparisons will be made between experiments, calculations, and Finite Element Analysis (ABAQUS).

Lastly, the Time-temperature superposition principle (TTSP) and the Stepped isothermal method (SIM) in predicting the long-term stress relaxation behaviour of unfilled NR in constant simple shear and buckling deformations will be applied and compared.

3.1 Methods of Preparation of Materials and Samples

The preparation of the raw material and the samples used in this study are described in this section.

3.1.1 Rubber compound preparation

A single NR based compound was used in this study, which identified as EDS19. The base material for EDS19 is SMR CV60, a commercially available dry rubber of a high quality grade, obtained by deliberate coagulation of latex by acidification. The ‘CV’ acronym means ‘constant viscosity’, as it is a viscosity-stabilised rubber at 60±5 Mooney unit. This controlled viscosity helps to achieve consistent properties of the end rubber product.

EDS19 is a conventional sulphur-accelerated vulcanisate with no filler, which is suitable for most engineering applications. An unfilled rubber is deliberately chosen in this study because the addition of reinforcing filler greatly complicates the stress-strain behaviour of rubber and increases stress relaxation and creep rates. The focus of this study
is on the long-term stress relaxation behaviour, which stemmed from the earlier work by Muhr (2008) which showed the shear modulus of EDS19’s strong dependence on temperature. Hence its use in investigation of the effect of temperature and strain on unfilled rubber. The other compound ingredients were commercially available and prepared at the mill room in Tun Abdul Razak Research Centre (TARRC), United Kingdom.

The material was mixed on a 2-roll-mill Banbury machine with the milling temperature maintained below 50°C. The NR chunks (SMR-CV60) were loaded and masticated at the beginning of the process. Then the curing agents and other ingredients were added gradually. A compound with a sheen finish was then sheeted out, indicating a well-dispersed rubber mix.

The compound was left for 24 hours before any sample was moulded. A small amount of the compound (5g of mass) was sent for a rheometer analysis, where the cure characteristic of the compound was determined. The rubber was placed in a closed oscillating chamber at the set temperature of 140°C. The cure rate was analysed using a Monsanto Rheometer MDR2000E. The optimum curing condition was determined by the time it requires for the rubber to achieve the maximum torque.

The EDS19 formulation is shown in Table 3.1. The 2-roll-mill Banbury machine and the mixed compound are shown in Figure 3.1. The rheometer curve for the compounded EDS19 mix is shown in Figure 3.2.

Table 3.1: Formulations of the EDS19 compound in part per hundred rubber (pphr)

<table>
<thead>
<tr>
<th>Compound</th>
<th>1 (EDS19)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>pphr</td>
</tr>
<tr>
<td>Natural rubber, SMR CV60(^1)</td>
<td>100</td>
</tr>
<tr>
<td>Zinc oxide</td>
<td>5</td>
</tr>
<tr>
<td>Stearic Acid</td>
<td>2</td>
</tr>
<tr>
<td>Antioxidant 6PPD(^2)</td>
<td>3</td>
</tr>
<tr>
<td>Antiozonant Wax</td>
<td>2</td>
</tr>
<tr>
<td>CBS(^3)</td>
<td>0.6</td>
</tr>
<tr>
<td>Sulphur</td>
<td>2.5</td>
</tr>
<tr>
<td>Cure condition (Time, Temperature)</td>
<td>38 minutes, 140°C</td>
</tr>
</tbody>
</table>

\(^1\) Standard Malaysian Rubber (Constant viscosity of 60 Mooney unit)
\(^2\) N-(1,3-Dimethylbutyl)-N’-phenyl-p-phenylenediamine (Santoflex 13)
\(^3\) N-Cyclohexylbenzothiazole-2-sulphenamide
From the rheometer plot in Figure 3.2, the optimum cure time at 140°C is 38 minutes, indicating the time required for the torque to reach the maximum. Two types of test specimens were moulded: a double-bonded shear (DBS), and a hollow rubber cylinder (HRC). For the DBS sample preparation, a transfer moulding was used, while the HRC was prepared
using the compression moulding method. In preparing both types of samples, a hydraulically-powered electric press with a constant pressure between 70-75bar was used.

The cure temperature of 140°C was used for curing the samples used in this study as it served medium thermal stability rather than 150°C, which is high in thermal delivery, although it means samples can be cured faster. This decision was also due to the DBS and HRC mould designs, where it is important to ensure the cavity of each mould is perfectly filled during the early stage of the moulding. Therefore, using 140°C was thought to be appropriate for the purpose of preparing the DBS and HRC in this work, where the sample production rate was not an issue.

### 3.1.2 Double bonded shear (DBS) sample preparation

In preparing a DBS sample, the rubber disks were bonded to cylindrical steel surfaces during the curing process. The first step for bonding is to clean the steel by dry sandblasting. Afterwards, the steel is immersed in acetone for 5 minutes to remove any remaining impurities. Finally, a rubber-to-metal bonding compound, Chemosil 211 (primer agent) and Chemosil 220 (bonding agent) were applied. The application of the bonding agents was as recommended by the supplier, LORD Corporation UK, for bonding rubber to metal substrates. The combination of Chemosil 211 and Chemosil 220 adhesives are the most used and researched bonding system for natural rubber in engineering applications (Gent, 1992).

A period of two hours, at room temperature, for the primer coat to dry is required before the final bonding agent is applied and left to cure for another 2 hours. The test pieces were moulded within 24 hours of applying the bonding agent to the steel pieces to avoid contamination of the treated surfaces. The treated steel blocks are shown in Figure 3.3(a).

In this work, the DBS samples were transfer moulded, where the rubber is initially placed in the pot and is extruded into the mould cavity through small holes by a piston. Two DBS samples can be prepared from a single moulding, and the mould was preheated at the cure temperature of 140°C. The prepared DBS steel pieces were then placed in the mould (Figure 3.3(b)). The mould was then assembled with the rubber compound placed in the transfer pot with the piston on top. The mould assembly was then placed between the press platens at 140°C and pressure was applied for 38 minutes (Figure 3.3(c)).
(a) DBS cylindrical steel blocks treated with bonding agents

(b) DBS steel blocks arranged in the mould

(c) The DBS mould in the 15”x15” Bradley & Turton electric press

Figure 3.3: DBS samples preparation in sequence.
Flash can be seen coming out through the outlet at the bottom part of the mould, indicating that the mould cavities are filled. The samples were removed from the mould immediately after curing. The moulded testpieces are shown in Figure 3.4.

Once the sample had cooled down, the rubber thickness of each DBS sample was recorded. This is done by measuring the difference between the cured DBS length against the sum of the length of the 3 cylindrical blocks, measured before assembly and at room temperature.

Two types of DBS samples with different rubber thickness were produced; 6mm and 2mm. The 6mm DBS will be used for the long-term creep and stress relaxation tests while
the 2mm DBS will be used for the stress relaxation test with arbitrary strain/temperature and the accelerated stress relaxation tests.

In the first instance, the creep and stress relaxation tests were conducted first, and the DBS with 6mm rubber thickness were used because they were readily available at TARRC, where the tests were conducted. The 6mm DBS samples also adhered to the ISO 23529:2016, where it specifies one of the sample thicknesses to be 6.3±0.3mm. The DBS sample with 2mm thickness is used for the test, which involves the change in temperatures. The aim is to have a thinner rubber specimen, which will allow the rubber to achieve the target temperature more quickly and more uniformly when the temperature changes are loaded to the samples, as opposed to using a thicker rubber disk (6mm).

For the purpose of monitoring the rubber temperature, a blind hole, enough to accommodate a thermocouple, was drilled axially on one of the outer cylinder blocks, stopping approximately 2mm before reaching the rubber disk (Figure 3.6). The thermocouple was pushed into the hole and filled with thermal grease to ensure a proper heat transfer. Diagrams for both DBS samples are shown in Figure 3.5.

![Diagram of 2mm DBS sample](image)

**Figure 3.5**: 2mm DBS sample in elevation view. The hatched part on the right is the cross-section of the sample showing the thermocouple inlet. The rubber disks can be 2mm or 6mm in thickness.

### 3.1.3 Hollow rubber cylinder (HRC) preparation

The HRC samples were prepared in a similar manner but using a specially built mould (Figure 3.6) to produce a hollow cylinder. The process followed consists of: preheating the mould to 140°C; pour the rubber compound in the mould cavity and compress the mixture on a load frame for 38 minutes at 140°C to form the HRC (Figure 3.7). The final moulded HRC is shown in Figure 3.8, together with dimensions.
Figure 3.6: The HRC mould; (A) The piston, (B) The main body, with the interchangeable central insert.

Figure 3.7: The 8”x8” Mackey Bowley electric press.
HRCs with three different inner diameters (ID) were produced by changing the internal inserts of the mould: Ø12mm, Ø13mm, and Ø15mm. The outer diameter (OD) of Ø25mm and the height of 24.5mm were kept constant.

The decision to have HRC with such dimensions is down to several factors; first is due to the existing area for DCU installation in the IAB wall model, which allow a device with no more than 30mm in diameter at the wall’s arm axles. Second, the HRC design is based on several ID variations in reference to the unfilled rubber in buckling deformation as studied by Leaver and Lindley (1976), which suggest the stress at buckling shall be between 0.66MPa to 0.8MPa. Third, a previous research suggests that the working load of a DCU system at the IAB wall model is around 500N. Assuming a DCU system with two HRC unit, the design shall have an approximately 250N force at buckling. For design iterations, therefore, three variations of IDs were chosen to evaluate the HRC – Ø12mm, Ø13mm, and Ø15mm.

Two annular steel end plates were bonded to the HRC sample after moulding to serve as the rigid base for sample fixing in the test machine and in the IAB wall model tests (Figure 3.9). However, for the accelerated relaxation tests (Section 3.4 and 3.5), the 13mm ID HRCs were bonded directly to the cylindrical blocks with threaded rods (Figure 3.10). In all cases, the mating surfaces of both rubber and steel were cleaned with fine grade sandpaper and an
acetone wash, before being bonded to the rubber using Loctite 480; a rubber-toughened instant adhesive with increased flexibility and resistance to shock loads.

Figure 3.9: (Left) Cured HRC samples with annular end plates, (Right) HRC cross-section diagram in elevation view. The sample used for the long-term creep and stress relaxation tests.

Figure 3.10: Hollow rubber cylinder sample (HRC); (1) HRC body with HRC ID Ø13mm, (2) Cylindrical test jigs. The sample used for the stress relaxation test with arbitrary strain/temperature and the accelerated relaxation tests.
3.1.4 Samples used in this study

The samples used in this study, both as the DBS and HRC types are summarised in Table 3.2 below, matched to their respective experiments.

Table 3.2: Experiments and analyses with the DBS and HRC sample identification (ID).

<table>
<thead>
<tr>
<th>Sample</th>
<th>Variant / Dimensions</th>
<th>Experiment / Analysis</th>
<th>Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>DBS</td>
<td>Rubber thickness, $H_r = 6\text{mm}$</td>
<td>Long-term stress relaxation</td>
<td>Sample ID ; strain, $\gamma$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>DBS-A1 ; $\gamma = 0.03$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>DBS-A2 ; $\gamma = 0.11$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>DBS-A3 ; $\gamma = 0.50$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>DBS-A4 ; $\gamma = 0.99$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>DBS-A5 ; $\gamma = 1.23$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>DBS-A6 ; $\gamma = 1.69$</td>
</tr>
<tr>
<td>DBS</td>
<td>Rubber thickness, $H_r = 2\text{mm}$</td>
<td>Long-term creep</td>
<td>Sample ID ; stress, $\tau$(MPa)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>DBS-B1 ; $\tau = 0.08$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>DBS-B2 ; $\tau = 0.21$</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>DBS-B3 ; $\tau = 0.44$</td>
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<td></td>
<td>DBS-B4 ; $\tau = 0.65$</td>
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<td></td>
<td></td>
<td></td>
<td>DBS-B5 ; $\tau = 0.88$</td>
</tr>
<tr>
<td>DBS</td>
<td></td>
<td>Stress relaxation with varying strain</td>
<td>Sample ID ; strain, $\gamma$ and temperature (K)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>DBS-C1 ; $\gamma = 1.0, 1.5, 0.5, 0.0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stress relaxation with varying strain and temperature</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>DBS-C2 ; $\gamma = 1.0, 0.0$ with $T = 297K, 279.5K, 305K$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Accelerated stress relaxation tests:</td>
<td>Sample ID ; strain, $\gamma$ and temperature (K)</td>
</tr>
<tr>
<td></td>
<td>1. Time-temperature superposition (TTSP)</td>
<td></td>
<td>DBS-D1 ; $\gamma = 1.0$ at 297K (24°C)</td>
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<td>DBS-D2 ; $\gamma = 1.0$ at 305K (32°C)</td>
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<td></td>
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<td>DBS-D3 ; $\gamma = 1.0$ at 316K (43°C)</td>
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<td>DBS-D4 ; $\gamma = 1.0$ at 326K (53°C)</td>
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<td>DBS-D5 ; $\gamma = 1.0$ at 336K (63°C)</td>
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<td>DBS-D6 ; $\gamma = 1.0$ at 346K (73°C)</td>
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<td>DBS-D7 ; $\gamma = 1.0$ at 376K (103°C)</td>
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</tbody>
</table>
### Table: Stress Relaxation Experiments

<table>
<thead>
<tr>
<th>HRC</th>
<th>Inner diameter, ID = Ø15mm</th>
<th>2. Stepped isothermal method (SIM)</th>
<th>Step pedothermal method (SIM) DBS-E1; γ = 1.0 at 297K, 305K, 316K, 326K, and 336K (24°C, 32°C, 43°C, 53°C, and 63°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HRC</td>
<td>Inner diameter, ID = Ø13mm</td>
<td>Long-term stress relaxation</td>
<td>Sample ID; strain, ε_b</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>HRC-A1; ε_b = 0.16</td>
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<td></td>
<td>HRC-A2; ε_b = 0.29</td>
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<td></td>
<td>HRC-A3; ε_b = 0.37</td>
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<tr>
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<td></td>
<td>HRC-A4; ε_b = 0.45</td>
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<tr>
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<td></td>
<td></td>
<td>HRC-A5; ε_b = 0.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>HRC-A6; ε_b = 0.57</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Long-term creep</td>
<td>Sample ID; stress, σ_b (MPa)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>HRC-B1; σ_b = 0.13</td>
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<td></td>
<td></td>
<td></td>
<td>HRC-B2; σ_b = 0.21</td>
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<tr>
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<td></td>
<td>HRC-B3; σ_b = 0.24</td>
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<tr>
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<td>HRC-B4; σ_b = 0.32</td>
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<tr>
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<td></td>
<td>HRC-B5; σ_b = 0.37</td>
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<td></td>
<td>HRC-B6; σ_b = 0.43</td>
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<td></td>
<td>HRC-B7; σ_b = 0.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stress relaxation with varying strain</td>
<td>Sample ID; strain, ε_b and temperature (K)</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>HRC-C1; ε_b = 0.2, 0.4, 0.5, 0.0</td>
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<tr>
<td></td>
<td></td>
<td>Stress relaxation with varying strain and temperature</td>
<td>HRC-C2; ε_b = 0.37, 0.0 with T = 297K, 326K, and 280K</td>
</tr>
<tr>
<td></td>
<td>Accelerated stress relaxation tests:</td>
<td>Sample ID; strain, ε_b and temperature (K)</td>
<td></td>
</tr>
<tr>
<td></td>
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<td>HRC-D6; ε_b = 0.37 at 346K (73°C)</td>
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The experiments and analyses associated with the samples will be explained at the following sections.

3.2 Experimental Method

3.2.1 Stress relaxation test procedure

Four stress-relaxation jigs were placed inside the temperature-controlled box. The jigs were machine vices, placed horizontally on the test bench. Machine vices were used given the simplicity and the ability to maintain a fixed displacement through jaw opening by turning a its spindle. Both types of samples, DBS and HRC, were tested using the vice sets. Different modes of deformation were applied to the samples; simple shear for DBS (Figure 3.12) and axial compression for HRC (Figure 3.14). The overall stress relaxation test setup is shown in Figure 3.11.

![Figure 3.11: The stress relaxation test rig consisting (A) Temperature-controlled box with the electronic controller, without the lid on; (B) Data Taker DT800 logger with a computer; (C) Fylde Micro Analog logger with a computer; (D) Electronic control unit.](image)

The same setup was used for both DBS and HRC samples. Two datalogger systems were used to record the signals from the load transducers and thermocouples: a Fylde Micro Analog
and a DataTaker DT800. All load transducers were calibrated against a series of dead weights to ensure the system was capable of recording a load range between 0N to 750N. Thermocouples were calibrated against a glass spirit thermometer. The calibration of the load transducers and thermocouples used in this experiment is shown in Appendix A.

To ensure the correct displacement was applied, a separate dial gauge was used as a straining guide, during the application of a fixed displacement/strain to each sample. The gauge can be seen in Figure 3.13. Because the dial gauge used here came with a long armature, there was a concern if its spring’s stiffness may impose additional force on the sample attached to it. A simple check on the effect of the dial gauge stiffness to the overall DBS sample stiffness was conducted and discussed in Appendix A. The dial gauge stiffness was found to be very small, which was 0.017N/mm as opposed to the shear stiffness of a DBS sample (at 100% shear strain), which was 70.47N/mm. Therefore, its effect was concluded to be insignificant to be able to affect the stiffness value of a DBS test specimen.

Prior to testing, the temperature box and the test jigs were heated up overnight to the test temperature of 30°C. This was to ensure that the desired temperature level can be achieved as the test setup contains four large steel vices with large heat-retaining capacity. DBS sample was fixed to a double-bonded shear test jig before the whole set up was placed between the vice jaw. The jig is placed perpendicular to the load cell’s contact surface.

The stress relaxation test for a DBS sample consists of straining the center piece steel block by a permanent displacement, $\delta$, and monitor the force, $P$ for a period of time, $t$. An example of how a DBS sample is strained is shown in Figure 3.12. The two endpieces are fixed, and the centrepiece moves relative to the endpieces.

Figure 3.12: DBS sample undergoing permanent shear displacement ($\delta$) or force ($P$). $H_r$ is the rubber disk thickness.
The force and temperature readings were started before the straining began. The samples were strained by tightening the vices, an action that move the centerpiece of the jig against the load cell. Once the load cell was in contact with the central steel block of the DBS sample, the vice tightening generated a displacement of the central block relative to the outer blocks (Figure 3.13). Using the dial gauge, a prescribed displacement was applied and the force measured for as long as required (Figure 3.13).

![Figure 3.13: Applying the strain onto the DBS sample. The tip of the gauge is in contact with an L-shaped bracket attached to the sample.](image)

The stress relaxation test for an HRC sample consists of squashing the HRC body in axial direction with a permanent displacement, $\delta$, and monitor the force, $P$ for a period of time, $t$. An example of this is shown in Figure 3.14. The bottom part of the HRC is fixed, and the top part is displaced axially.
Figure 3.14: HRC undergoing permanent compressive displacement ($\delta$) or force ($P$). The sample bottom is fixed to the base.

Similar to the straining method applied on DBS, by tightening the vices with the HRC samples, an axial compression displacement was applied to the HRCs with the resultant force measured by the load transducer (Figure 3.15). Each sample was strained to the desired displacement in 10 seconds, which was deemed achievable for the manual hand winding of the vice. At the beginning of the test, data was captured at every second, and gradually increased to 10s, 100s, and finally 1,000s until the experiment stopped.

Figure 3.15: Stress relaxation test setup for an HRC inside the purpose-built temperature-controlled box.
The stress relaxation test durations were set to be 1,000,000 seconds for all DBS and HRC samples. This has been seen as a reasonable length of test duration, where close physical monitoring can be possibly done on the test setup.

### 3.2.2 Creep test procedure

An identical temperature control system was constructed to be used for creep testing of DBS and HRC samples. A series of five creep test jigs, interchangeable between shear and axial compression mode of deformation, were installed on a laminated wooden bench. Dead weights were used to apply a constant force and these were located below the bench (Figure 3.16). The dead weights were hanging from a threaded bar passing through a small hole under the test jig.

Before installing the sample, the temperature inside the box was preconditioned to the test temperature for 2 hours. Unlike the stress relaxation test box, the creep test box is smaller and contains a set of smaller steel test jigs; therefore, it required less time to achieve a stable test temperature.

The outer steel cylinders of the DBS sample were fixed to the test jigs. The centrepiece was attached to a ring sleeve connected to the threaded bar with the dead weights (Figure 3.17). The same principle was used for the HRC sample, except that the threaded bar was connected to a steel plate located on top of the sample (Figure 3.18) and the threaded bar was carefully positioned at the centre to avoid an uneven distribution of load on the sample. Linear variable differential transducers (LVDTs) were used to record the vertical displacement during the creep test. Similar to the stress relaxation test setup, the creep test apparatus allows several tests to run simultaneously. The LVDTs were calibrated against a variation of Mitutoyo thickness gauge blocks before the test, by using a digital indicator (RDP E525). The calibration data is shown in Appendix A.
Figure 3.16: Overall setup of the creep test. The temperature-controlled box is shown without the lid on. Fylde datalogger is connected to a computer.
Figure 3.17: The setup and placement of the apparatus for creep test for DBS sample.

Figure 3.18: An HRC sample undergoing creep test. An LVDT is located at the top of the top steel plate (unseen), recording the change in displacement in the vertical direction.
The creep test duration for DBS samples was set to be 1,000,000 seconds each, while it was 100,000 seconds for HRC samples. The test duration for the creep test was seen as reasonable because this enabled for close physical monitoring of the test and its setup. The lesser length of test duration for the HRC in creep was due to the limitation of time in using the test equipment and room facility at TARRC. During that time, TARRC was in the middle of managerial changes, and this affected some of the activities in some areas of the building. This meant that the test needed to be settled at a shorter duration (100,000 seconds).

3.2.3 Temperature effect in the measurement of stress relaxation and creep

Since the modulus of the rubber and hence the forces measured, are dependent on the temperature, a temperature controlled test was conducted, due to concerns of having a fluctuating temperature within the test enclosure. As the stress relaxation and the creep tests measure small changes in force or displacement, changes in the rubber modulus would have a comparatively large effect on the measurements, yielding poor results and making correct interpretation of the results impossible. Therefore, accurate temperature control must be a priority for these tests.

This section describes the method to construct a box that is able to maintain its inside temperature at high accuracies (at ±0.5°C). Two boxes were built, and each was deployed in the stress relaxation and creep tests setup described in Section 3.2.1 and 3.2.2 above. The temperature-controlled box was designed and built in the early stage of this research at TARRC. It was built by using widely available electrical and electronic components. The temperature box can withhold only a medium-high range of temperature (from 20°C to 40°C), although this study presents only the temperature level of 30°C. From the current knowledge, this temperature-controlled box is the first equipment being built to serve the purpose of providing a constant temperature for testing multiple samples simultaneously in the stress relaxation and creep tests.

The construction of the temperature-controlled box

A custom-built temperature-controlled box was constructed in order to achieve a constant temperature inside the stress relaxation test rig. The aim was to have system that could heat the inside of a box with 200x50x25cm, made of medium-density fibreboard (MDF), to the desired temperature, ranging from 25°C to 35°C, with an accuracy of ±0.5°C. To keep the budget low, a series of six 70W light bulbs were used as the heating elements.
An electro-contact thermometer acts as a switch for the lighting circuit when a wire inside the thermometer capillary makes contact with the in-glass mercury. The bulbs light up when the electronic circuit opens, which is when the mercury loses contact with the inner wire, as a result of temperature drop. The heat from the bulbs will increase the temperature, and once the mercury expands and touch the wire, the circuit is closed. The repetition of these cycles ensures that the temperature inside the box is maintained within the specified accuracy.

The advantage of this system is that through the use of light bulbs, a rapid on-off heating is achieved, resulting in smaller temperature fluctuations. Also, a light bulb also does not retain as much heat as other heating elements, therefore very little residual heating is added to the system once the bulbs are off. Fans at each end of the box were used at all times to promote quick heat transfer via air circulation to help maintain a constant temperature. The top and bottom openings were fitted with a PVC foam tape around the perimeter, which functions as a seal strip. This was to prevent air leakage and to provide a more controlled environment inside the box.

This temperature-controlled box was built and assigned to each stress relaxation and creep tests setup. All of the electronical apparatus used in the temperature-controlled box was controlled by a specifically-designed circuit on a printed circuit board (PCB), and housed in a universal circuit box. This complete unit is referred to as an ‘electronic control unit’, which is shown in Figure 3.11 and Figure 3.16. The electronics diagram for the electronic control unit is shown in Appendix A.

Figure 3.19 shows the temperatures recorded by three thermocouples inside the box: TC-1, TC-2 and TC-3. The thermocouples were equally spaced inside the box and the target temperature was set to 30°C. This temperature was chosen because an attempt has been made by monitoring the temperature inside the box which was set at 24°C. The record showed that the temperature inside the box was fluctuating, affected by the night time temperature outside of the box. The choice in using 30°C as the test temperature eliminates the fluctuating factor of temperature from outside the box. It was also chosen because it is not too high that it could deteriorate the rubber network structure. Another reason was that 30°C was a temperature easy to achieve for the system designed.
An identical temperature-controlled system was also built for the creep test. A plot of the temperature observation for the second system is shown in Figure 3.20.

It was established that the heating systems intended for the stress relaxation and creep tests were capable of maintaining a temperature of $30\pm0.5^\circ$C, as shown in Figure 3.19 and Figure 3.20. There are occasional temperature spikes on the measurements that went slightly higher.
than 30.5°C. This may be treated as isolated occurrences which are thought to be caused by electrical noise.

3.3 Stress Relaxation and Creep of Unfilled Natural Rubber

Stress relaxation is the reduction of stress required to maintain a permanent strain applied on a specimen. Conversely, if a constant force is applied, the strain in the rubber would increase over time, this process is known as creep. This phenomenon is well known. However, more information is needed, for example, the effect of strain levels on the creep and stress relaxation of rubber in deformation other than uniaxial tension, in particular simple shear and hollow rubber cylinders undergoing buckling. An initial study by Willis (1948) suggested that the creep rate is higher for a hollow rubber cylinder when compressively strained near to its critical stress. However, no subsequent study was done in this area since this early discovery. This has been discussed in Section 2.6.2.

3.3.1 Stress relaxation of NR at different strains

Simple shear

The simple shear test was carried out on a 6mm DBS sample using the Instron 4301 test machine, preconditioned at the temperature of 30°C. The selected temperature was for consistency as all tests have been performed at the same temperature of 30°C. A plot demonstrating the third cycle of shear stress-strain is shown in Figure 3.21. The third cycle is chosen because the stress-strain of the rubber starts to get stable from the third cycle onwards.
A set of shear stress relaxation tests was performed according to Figure 3.21 to determine the influence of strain on the relaxation modulus and the rate of stress decay. The stress relaxation tests on DBS were conducted at six levels of shear strains:

\[ \gamma = [0.03 \quad 0.11 \quad 0.50 \quad 0.99 \quad 1.23 \quad 1.69] \]

The shear modulus, \( G(t) \), is calculated as;

\[ G(t) = \frac{\tau(t)}{\gamma} \quad (3.1) \]

where \( \tau(t) \) is the shear stress acting on the sample at a time \( t \) and \( \gamma \) is the shear strain, maintain constant during the test. \( \tau(t) \) is given by;

\[ \tau(t) = \frac{P(t)}{2A_{DBS}} \quad (3.2) \]

where \( P \) is the shear force and \( A_{DBS} \) is the area of the rubber disk (981.75mm\(^2\)) where the shear force is acting.

The shear stress decay with time is shown in Figure 3.22 below, for six different levels of shear strain.
Some uncertainties exist during the course of the stress relaxation experiment. This is due to the change in the shear modulus, following the fluctuation in temperature as the tolerance of the temperature control system was only able to maintain the temperature, set at 30 °C, with a tolerance of ±0.5°C. These uncertainties can be represented by the equation $G = G_0 \left( \frac{T_0}{T_0 + 0.5} \right)$, where $G$ is the shear modulus, $T$ is the temperature (in Kelvin) with a variation ±0.5 °C, $G_0$ is the relaxation modulus at the test temperature ($T_0=303K$), $\rho_0$ is the rubber density at test temperature ($\rho_0=0.9628g/cm^3$) and $\rho$ is its density at variation ($T_0+0.5$, $\rho=0.9625g/cm^3$ and $T_0-0.5$, $\rho=0.9631g/cm^3$). The density and the temperature govern the change in shear modulus value of rubber. The way both parameters affecting the relaxation of rubber will be demonstrated in Section 3.5.

A stress relaxation test was conducted on sample DBS-A4. The relaxation modulus curve with the effect of ±0.5°C temperature variation is shown in Figure 3.23 below.

The idea of the plot in Figure 3.23 is to estimate uncertainty coming from the effect of temperature fluctuations which need correction. Ideally, the correction can be done if a reliable measurement of the sample's temperature is recorded individually during each experiment. However, to avoid the complication of doing the correction for the relaxation curves, it is much better to keep the temperature as constant as possible unless a change in temperature is needed as part of the research.
If the temperature oscillates quickly, the temperature of the sample will remain constant, but if it drifts slowly, this will be picked up by the sample. This is what the temperature boxes are intended to prevent.

Figure 3.23: A stress relaxation curve at 30°C for sample DBS-A4. The possible error due to the temperature fluctuation of ±0.5°C is shown in the red vertical bars.

Comparison with Figure 3.19 and Figure 3.20 suggests that the relaxation temperature drift is much less than ±0.5°C, so the uncertainty should be less than shown on Figure 3.23. By having a very low fluctuation in temperature in the relaxation experiment, it is fair to presume the accuracy of the model investigated later in Section 3.4 and 3.5.

The plots for relaxation modulus against the logarithm of time are shown in Figure 3.24. The graphs are shown in two separate figures due to the noise that the collected data from the stress relaxation tests at low strains (\( \gamma = 0.03 \) and 0.11) exhibited (Figure 3.24a). The relaxation modulus \( G(t) \) in Equation (3.1) can be fitted by Chai & Thomas (1981) approach as in Equation (2.33).

It is observed that the stress decreased at a slightly faster rate after approximately 10,000 seconds (\( \log(t/t_0) = 4 \)). The best linear fits to the timescale are shown to be between \( \log(t/t_0) = 0 \) to \( \log(t/t_0) = 4 \), as given in Figure 3.24. The linear fits for \( \gamma = 0.03 \) and \( \gamma = 0.11 \) are represented by yellow and white dashed lines respectively in Figure 3.24(a), while the rest are represented by the black dashed lines in Figure 3.24(b).
The relaxation modulus plots for all six strains are shown in two separate graphs due to higher fluctuations exhibited by samples with shear strains $\gamma = 0.03$ and 0.11. The fluctuations were caused by the noise in the load transducer output signal. The sensitivity tolerance of the load transducers was known to be 10% out of its rated output which was 2500N. The loads for strain $\gamma = 0.03$ and 0.11 were 13N and 53N, respectively, meaning that they sat way under the 10% sensitivity tolerance, hence, the high load fluctuations. The load data for both strains, however, deemed valid following the linearity of the transducers’ calibration plots shown in Table 0.2 (for $\gamma = 0.11$) and Table 0.4 (for $\gamma = 0.03$).
The plot of the relaxation rate (% per decade) for different shear strains is shown in Figure 3.25, which suggests that the stress relaxation rate does not depend systematically on shear strain.
The relaxation modulus at the initial and after $1 \times 10^6$ s of test, for several shear strains are shown in Figure 3.26. It can be seen that the relaxation of shear moduli of EDS19 are similar for the different strains tested (0.03 to 1.69). The differences between the initial and final values for the tested strains were between 7.3% – 13.4%.

![Figure 3.26: The relaxation moduli at two different times. “Initial” is when the sample was fully strained ($t_0=10$s).](image)

**Axial compression on cylinder**

To understand the stress-strain behaviour of an HRC sample, a compression test was performed, using an Instron 4301 load frame, preconditioned to a temperature of 30°C. The strain rate of the test was 10mm/min, and the stress-strain behaviour of the HRC is shown in Figure 3.27.

The onset of the buckling state was identified at 29% strain, where the stiff linear response stops and the HRC started to buckle. During the buckling deformation, an HRC will soften showing a dip in force. When further displaced, the internal cusps of the HRC wall will be in contact, and the stress will increase again.

The variations of strain-level were selected to distinguish the relaxation rate at different buckling state. Five strains were chosen within the nonlinearity of the stress-strain curve of the HRC in order to study the effect of several buckling strain levels to the relaxation rates. In contrast, only one strain was chosen within the linear stress-strain curve of the HRC.
Six HRC samples were tested at a different level of axial compressive strains, which are represented by red circles in the stress-strain curve in Figure 3.27. The relaxation moduli will be plotted from the stress relaxation data against time. The relaxation modulus in compression, $B(t)$ is given by compressive relaxation stress, $\sigma_b(t)$ divided by strain, $\varepsilon_b$.

$$\varepsilon_b = [0.16 \ 0.29 \ 0.37 \ 0.45 \ 0.50 \ 0.57]$$

$$B(t) = \frac{\sigma_b(t)}{\varepsilon_b} \quad \sigma_b(t) = \frac{P(t)}{A_{HRC}}$$  \hspace{1cm} (3.3)

$P$ is the force and $A_{HRC}$ is the loaded area of the hollow cylinder in the axial direction (314.16mm$^2$).

The stress decay versus time plots for HRC at several buckling strains is shown in Figure 3.28. Straining the HRC to the desired strain took around 10s for all samples. straining of each HRC sample took between 10 to 12 seconds and the end of the straining was taken as the initial test time, $t_0$. Several samples exhibited humps during the straining phase, those with the target displacements further than the onset of the buckling ($\varepsilon_b > 29\%$). These were due to the cylinders buckling during the process of applying the permanent displacements.
A linear approximation on a semi-logarithmic plot was used here for the stress relaxation in compression; this was similar to Equation (3.3), used for the variation of the shear modulus with temperature. The relaxation modulus $B(t)$, in compression, for HRC can be written as:

$$B(t) = B_0 \left(1 - A \log \frac{t}{t_0}\right)$$

(3.4)

With $B_0$ being the modulus after the reference time $t_0$ and $A$ the rate of decrease of $B(t)$. The plots of the relaxation modulus against the logarithm of time, for the different initial strains, are shown in Figure 3.29, together with the results from the equation above; the equation shows a fairly good fit for the lab data.

Figure 3.30 shows that the rate of relaxation of the modulus in compression $B(t)$ is independent on the strain up to 57%, with the rates taken from the fitted linear equations shown in Figure 3.29. The linear fits (based on Equation (3.4)) are in effect the same as Chai and Thomas (1981) (Equation (2.32)).
Figure 3.29: Relaxation modulus plots in semi-log for several compressive strains. The linear fit for each plots are represented by black dashed lines, except for strain $\varepsilon_b=0.16$ (yellow dashed line).

Figure 3.30: Stress relaxation rate (% per decade) against compressive strain.
The modulus of the cylinder at the linear stiff region ($\varepsilon_b = 0.16$) and the critical buckling state ($\varepsilon_b = 0.29$) are shown to be higher than the modulus within the soft plateau region. This is shown in Figure 3.31. The modulus at compressive strain, $\varepsilon_b = 0.57$ is shown to be low, even though it has a rather high compressive stress, which is close to the critical buckling state. This may be due to the softening from the buckling progression. At the same time the modulus increases slightly again. This is due to the increasing stiffness resulting from the contacting inner walls. In overall, the percentage reduction in modulus for all strains over the period of test time are shown to be 10% on average.

![Relaxation modulus at different compressive strains](image)

Figure 3.31: Relaxation modulus at different compressive strains, plotted for two different times. Initial is the time when the sample was fully strained (~10s).

### 3.3.2 Creep of NR at different stresses

The previous section has shown, that for unfilled rubber in simple shear and compressive deformation, the relaxation rate can be determined even with the time scale up to 100,000 seconds in logarithmic. With that basis, the creep experiment will run within the time scale of up to 100,000 seconds instead of $1\times10^6$ seconds.

The creep tests were done analogously to the stress relaxation tests discussed before, except that the parameters of interest are now the shear and axial compressive
strains for DBS and HRC, respectively. Identical to the setup in stress relaxation, the temperature inside the creep test box was preconditioned to 30°C.

**Simple shear**

The creep for EDS19 in shear has been investigated by plotting the strain elongation data against time. The creep compliance $J(t)$ is given by strain elongation $\gamma(t)$ divided by the stress, $\tau$. The relationship between the applied shear stress and the corresponding strain can be expressed in Equation (3.5). Five DBS samples were tested at different level of shear stresses (in MPa);

$$\tau = [0.08, 0.21, 0.44, 0.65, 0.88]$$

$$J(t) = \frac{\gamma(t)}{\tau} \quad \gamma(t) = \frac{\delta(t)}{H_r} \quad \tau = \frac{P}{2A_{DBS}}$$

(3.5)

With $\delta$ is shear displacement and $H_r$ is the rubber disk thickness (Figure 3.12). $P$ is the permanent force applied by the dead weights, and $A_{DBS}$ is the total area of the rubber disks in contact with both sides of the central steel block of the DBS sample (Figure 3.12). The creep data for several permanent stresses in shear is shown in Figure 3.32. Also with these tests, the straining phase took about 10s to apply the load from the dead weights onto the sample. The release is shown on the left diagram, whilst the waiting period is shown on the right plot.

![Graph showing creep data for different shear stresses](image)

**Figure 3.32:** The shear strain of DBS samples at different stresses. A closer view of the first 20 seconds is on the left.
The creep compliance plots can be linearised in a logarithmic scale, using the same technique previously applied (Equation (2.33)). The plot for DBS at several shear strains are shown in Figure 3.33. The linear fits to the timescale are found to be between \( \log(t/t_0) = 0 \) to \( \log(t/t_0) = 4 \), and the linear functions fit to Equations (2.33) are shown in the plot. The creep rate summary is given in the subsequent Figure 3.34. The linear fits for all plots are represented by black dashed lines, except for \( \tau = 0.08 \) (white dashed line) and \( \tau = 0.88 \) (yellow dashed line).

\[
\begin{align*}
  y &= 0.0188x + 0.989 \\
  y &= 0.0225x + 1.004 \\
  y &= 0.0193x + 1.001 \\
  y &= 0.0188x + 0.995 \\
  y &= 0.0212x + 1.005
\end{align*}
\]

Figure 3.33: Creep compliance plots in semi-log for several shear stresses.

The creep rate, in % per decade against shear stress level is shown in Figure 3.34. It suggests that the creep rate is shown to be independent of the shear stress.
The relationship between the creep compliance, $J(t)$ and shear stress size, $\tau$ is shown in Figure 3.35. Creep compliance measured at two different times, at initial and after 1 million seconds shows that in shear, for the various stress levels, the creep behaviour is approximately linear with the logarithm of the time. The percentage increase in creep compliance for all stress levels are about 12%.
Axial compression on cylinder

Similarly, the approach in testing the creep of HRC in compressive deformation is identical to the DBS in shear. $D(t)$ is the creep compliance of HRC (Equation (3.6)). $P$ is the permanent force acting on the sample, and $A_{HRC}$ is the loaded area of the cylinder in the axial direction. $\delta$ is the axial displacement of the HRC sample and the height of the HRC samples are always 24.5mm. Seven samples were tested at several axial compressive stresses (in MPa);

$$\sigma_b = [ \ 0.13 \ 0.21 \ 0.24 \ 0.32 \ 0.37 \ 0.43 \ 0.50 \ ]$$

$$D(t) = \varepsilon_b(t)/\sigma_b \quad \varepsilon_b(t) = \delta(t)/24.5 \quad \sigma_b = P/A_{HRC} \quad (3.6)$$

The creep plots for HRC at several stresses are shown in Figure 3.36.

![Creep plots for HRC at several stresses](image)

Figure 3.36: The displacement of the HRC in linear timescale at different permanent axial stresses. ($\sigma_b$ in MPa)

In this experiment, the application of an axial stress of 0.50MPa caused the HRC to displace rapidly after the loading phase, collapsing 90s after the beginning of the test (represented by the purple line). This level of stress was just over the stress at the critical buckling state (0.49MPa), where the buckling instability started to form. This demonstrated that any permanent stress equal or higher than 0.50MPa may result in the collapse of the HRC, therefore this creep analysis was excluded from the analysis.
The plots for creep compliance in compressive deformation were linearly estimated in the same manner described by Equation (2.33), except that the compliance is represented by $D(t)$ in Equation (3.7) below:

$$D(t) = D_0 \left(1 + A \log \frac{t}{t_0}\right) \tag{3.7}$$

where $D_0$ is the compliance after reference time $t_0$ and $A$ is the rate of increase of $J(t)$. The compliance plots for the six-level of stresses are shown in Figure 3.37 below.

According to Figure 3.37, linearisation of the curves was found to be reasonable, using Equation (3.7). The slopes of the curves showed no apparent change with applied stress. The creep rate summary is represented in Figure 3.38 below. The linear fits are shown between $\log(t/t_0) = 0$ to $\log(t/t_0) = 4$, with the linear functions fit to Equations (2.33) are shown in the plot. The linear fits for all plots are represented by black dashed lines, except for $\sigma_0=0.13$ (white dashed line).

![Figure 3.37: Creep compliance plots in semi-log for several compressive stresses.](image)

The plot of the creep rate (% per decade) for different shear strains is shown in Figure 3.38, which suggests that the creep rate does not depend systematically on compressive stress.
The graph of creep compliance $D(t)$ versus compressive stresses $\sigma_b$ is shown in Figure 3.39. It shows that the compliance is dependent on the compressive stress level, decreasing as the buckling stresses increases.

**Figure 3.38:** Creep rate (% per decade) against compressive stress.

**Figure 3.39:** Creep compliance at different buckling stress, shown at the initial (~10s) and 100,000s. ($\sigma_b$ in MPa).
3.3.3 Discussion

From the results presented above, it was observed that the stress relaxation and creep for unfilled rubber (EDS19), in shear and compressive buckling of cylinder, can be approximated by a linear function of the logarithm of time, demonstrating that the largest creep or strain relaxation is seen at the beginning of the test.

The linearity in log(time) of the relaxation modulus for shear and compressive buckling indicate that both are dominated by physical relaxation (Figure 3.24 and Figure 3.29). The stress relaxation rates are shown to be independent of the strain levels, and comparable between both deformations, which fell between 1.6 - 2.0% per decade. This is consistent with Gent (1962) findings that the stress relaxation of gum compounds are comparable as only physical processes are involved.

The creep compliance plots for shear and compressive buckling are both linear in log(time) scale, shown in Figure 3.33 and Figure 3.37. The creep rate for shear is shown to be independent of the shear stress (1.9 – 2.3% per decade), and for compressive buckling, it is also independent of the compressive stress (1.1– 1.3% per decade).

The creep for HRC beyond the buckling state was unquantifiable due to the collapse of the sample. But it is also the most interesting test as this has not been observed before. It has been shown that when the HRC is permanently loaded with a load passing its critical buckling state, the creep rate will be very high as the sample gradually collapse.

This section provides evidence to accommodate the subsequent part of this study, where the viscoelastic modelling of rubber in physical stress relaxation will be attempted. The model will not accommodate chemical effects, therefore, for appraising the stress relaxation model, the study will only use data where the chemical effects were insignificant.

The temperature-controlled box used in this study proved to be sufficient to provide a stable but not too high test temperature. The setup allows for multiple tests to be conducted simultaneously, which is convenient for long static stress relaxation or creep tests. The use of an electro-contact thermometer as the switch system, however, may pose a hazard risk due to the in-glass mercury. Alternatively, a switch system using a proportional-integral-derivative (PID) controller with a thermocouple acting as the feedback sensor should be used.
The method of strain application on the samples by clamping the vice for the stress relaxation test, and the manual loading of the weights for the creep test made it difficult to control the loading rate precisely, which made this technique unsuitable for much of the subsequent work. The main advantage of the technique discussed here is that it is low-cost and compact, hence, suitable for carrying out long-term tests and for work which requires a lot of data, for example, to evaluate the relaxation properties of different compounds.

It is important to note that the linearity in log(time) for the relaxation modulus and creep compliance shown in this study were limited to the test times that were not too long (up to 100,000s), and at the temperature that was not too high (30°C). Using the existing experiment setup, with 30°C and permanent stress of strain, it is expected that for longer tests, the relaxation of the sample will be accelerated as the chemical relaxation will be involved, following theories from Gent (1962b) and Derham, Lake and Thomas (1969). The effect of chemical relaxation, however, will not be discussed in this study, because the study in appraising the stress relaxation of unfilled rubber with varying strain and temperature in the following section will be strictly on the physical relaxation.

3.4 Stress Relaxation of Unfilled NR at Arbitrary Strains and Temperatures

NR based components are often required to withstand long-term stress or strain without developing excessive stress relaxation or creep. In the application intended for the IABs, the DCU will be prestressed at a large permanent strain, and along that period it will also be subjected to the additional small changes in strain, and high variations of temperature.

In the previous section, it has been shown that the relaxation modulus and creep for both deformations, tested at 30°C, are dominated by physical processes. And because the stress relaxation and creep presented are linear, they befit the linear viscoelasticity which is known to be suitable to model unfilled NR over moderate strain range. The linear viscoelasticity can be described by a series of Hookean spring with a Newtonian dashpot, which in this part of work will be arranged as the generalised Maxwell model.

This part of the study addresses the attempt to predict the stress relaxation, using a small number of parameters, from a linear viscoelastic model. This includes the attempt to simulate the effect of the varying strains and temperatures on the rubber during a stress relaxation test. The Boltzmann superposition principle is used to predict the stress relaxation
following changes in strain, and the William-Landel-Ferry (WLF) transform is used to model the effect of the temperature changes.

### 3.4.1 Viscoelastic Model

### 3.4.2 Method

The single material used in this study is an unfilled NR, designated EDS19; its composition is tabulated in Table 3.1. For the work described here onwards, the reference temperature $T_0$ will be set at 24°C (297K), and the glass-transition temperature, $T_g$ of EDS19 compound based on NR is -65°C (Lindley and Gough, 2015), so $T_s$ was taken as -15°C (258K).

Experimental works on stress relaxation are considered for two types of deformations; the simple shear (on a DBS sample), and axial compression (on HRC sample). At the first instance, the analytical equations were used to implement the model in simple shear in Microsoft Excel. Comparisons were made to the experimental results and simulation in ABAQUS, using the model inputs described in Section 3.4.8.2. For a non-uniform deformation like axial compression on rubber cylinder (HRC), the mathematical representation is more complex so it was not possible to implement the analytical model, and therefore only the FEA model in ABAQUS was used for comparison with the experimental results.

Two courses of experiments were conducted on both types of deformations; stress relaxation with varying strains, and stress relaxation with varying strains and temperatures.

### 3.4.3 Experimental

The experiments on the double-bonded shear (DBS) and hollow rubber cylinder (HRC) samples were conducted on the Instron 4301 desktop machine. The machine actuator control simulated the strains changing during the long-term stress relaxation, and the changes in temperatures were provided by the Instron insulated temperature chamber. The heating element inside the chamber powered the heating while the pressurised liquid carbon dioxide supplied the cooling. The data was recorded by Picolog Datalogger for the stress and strain, and DTaker 800 for the temperature from three type-K thermocouples. The general experimental setup is shown in Figure 3.40.
Figure 3.40: General experiment setup for the stress relaxation with arbitrary strains and temperatures. Label “A” is the Intron temperature chamber.

The experimental configuration for samples DBS and HRC are shown in Figure 3.41. The left-hand-side of Figure 3.41 is the DBS test setup. Note the TC-2 thermocouple is planted inside the steel block at one end of the DBS. This was to obtain a temperature reading at the nearest possible location to the rubber disk (~2mm). On the right-hand-side is the test setup for HRC.

Tufnol rods were used as the top and bottom connectors. They provide thermal breaks, which minimises the heat transfer between the steel test jigs and the outside of the chamber. Tufnol is a glass fibre laminated plastic material, widely used where electrical and thermal insulations are essential.
Figure 3.41: Apparatus for the stress relaxation with the arbitrary strain and temperature in the Instron temperature chamber. The thermocouples are shown as; TC-1 (mid box), TC-2 (near sample bottom) and TC-3 (sample top).

3.4.4 Modelling the change in strain (simple shear)

This section and onwards discuss the modifications made to accommodate the strain and temperature changes in a linear viscoelastic model in shear, which is represented by the Boltzmann’s superposition. The equations that form the modified model were yielded through a private communication and an unpublished report by Gough (2018).

According to Boltzmann’s superposition principle, the modulus $G$ resulting from the sequential changes in strains $\Delta \gamma_1, \Delta \gamma_2, \ldots, \Delta \gamma_n$ are additive. For a strain that is not applied instantaneously an integral form can be used;

\[
G(t) = \int_{-\infty}^{t} G(t-s) \dot{\gamma} ds ; \quad \frac{dy}{ds} = \frac{\gamma_0}{t} \int_{0}^{t} G(t-s) ds
\]

(3.8)

where $\gamma_0$ is the strain applied to specimen and $t_1$ is the time taken to do so. For an instantaneous strain application at time $t_{\text{app}}$, the modulus is given as;
\[ G(t) = G_G(t - t_{app}) \] (3.9)

Figure 3.42 shown below represents an example of strain history used in Section 3.4.5.

![Figure 3.42: The strain history with sequence; (1) ramp to \( \gamma_0 \), (2) hold, (3) release strain.](image)

### 3.4.5 Defining stress relaxation by Prony series

The relaxation behaviour is given by the Prony series as in Equation (3.10). Prony series requires two material parameters, \( G_0 \) or \( G_\infty \) and \( H_0 \). Another pairs of parameters, \( G_i \) and \( \tau_i \) for each of \( i \) terms in the Prony series are generated using a procedure described previously by Ahmadi, Muhr and Kingston (2008) in Section 2.5.5.

\[ G(t) = G_\infty + \sum_i G_i e^{(-t/\tau_i)} \] (3.10)

Using Equation (3.8) and (3.10), for shear ramp to \( \gamma_0 \), \( t < t_i \):

\[ G(t) = \frac{\gamma_0}{t_1} \left[ \int_0^t G_\infty \, ds + \sum_i G_i e^{\frac{-t}{\tau_i}} \int_0^t e^{\frac{s}{\tau_i}} \, ds \right] \]

\[ = \frac{\gamma_0}{t_1} \left[ G_\infty t + \sum_i G_i \tau_i \left( 1 - e^{\frac{-t}{\tau_i}} \right) \right] \] (3.11)

For holding the strain with \( t > t_i \):

With \( G(t) = \frac{\gamma_0}{t_1} \int_0^{t_1} G(t - s) \, ds \)

\[ G(t) = \frac{\gamma_0}{t_1} \left[ \int_0^{t_1} G_\infty \, ds + \sum_i G_i e^{\frac{t}{\tau_i}} \int_0^{t_1} e^{\frac{s}{\tau_i}} \, ds \right] \] (3.12)
\[
\begin{align*}
&= \frac{\gamma_0}{t_1} \left[ [G_\infty]^{t_1}_{t_0} + \sum G_i e^{\frac{-t}{\tau_i}} \cdot \tau_i \left[ e^{\frac{t_1}{\tau_i}} \right] \right] \\
&= \frac{\gamma_0}{t_1} \left[ G_\infty t_1 + \sum G_i \tau_i \left( e^{\frac{t_1-t}{\tau_i}} - e^{\frac{-t}{\tau_i}} \right) \right]
\end{align*}
\]

For instantaneous strain removal at \( t = t_2 \), there are additional stress terms for \( t \geq t_2 \):

With \( G(t) = -\gamma_0 G(t - t_2) \)

\[
G(t) = -\gamma_0 \left[ G_\infty + \sum G_i e^{-\frac{(t-t_2)}{\tau_i}} \right] \tag{3.13}
\]

The modulus at the strain removal and onwards, \( t \geq t_2 \), is the sum of Equation (3.12) and (3.13). If the strain removal is not applied instantaneously, an integral form may be used as in Equation (3.8).

For more general strain variations, it may not possible to derive the analytical equations. Direct implementation of the equations for many strain events would be cumbersome. Instead, an iterative approach was used by using a small strain step \( \delta y_j \) occurring at the start of each small time increment \( \delta t_j \) (Gough, 2018). The complete equation is obtained by summing many steps as follows;

Using the Boltzmann superposition with the Prony series as in Equation (3.10), the modulus at a time \( t_n = \delta t_1 + \delta t_2 + \cdots + \delta t_n \), is:

\[
G(t)_n = \delta y_0 \left[ G_\infty + \sum G_i e^{-\frac{(\delta t_1 + \delta t_2 + \cdots + \delta t_n)}{\tau_i}} \right] \\
+ \delta y_1 \left[ G_\infty + \sum G_i e^{-\frac{[\delta t_1 + \delta t_2 + \cdots + \delta t_n] - \delta t_1}{\tau_i}} \right] \\
+ \delta y_2 \left[ G_\infty + \sum G_i e^{-\frac{[\delta t_1 + \delta t_2 + \cdots + \delta t_n] - (\delta t_1 + \delta t_2)}{\tau_i}} \right] \tag{3.14} \\
+ \cdots \\
+ \delta y_n \left[ G_\infty + \sum G_i e^{-\frac{(\delta t_n)}{\tau_i}} \right] \\
\equiv G_{\infty,n} + \sum G_{i,n} \tag{3.15}
\]

with
\[ G_{\omega,n} = G_{\omega}(\delta y_1 + \delta y_2 + \cdots + \delta y_n) \]

\[ G_{\omega,n} = G_{\omega,n-1} + \delta y_n G_{\omega} \]  

where \( G_{\omega,1} = G_{\omega} \delta y_1 \)

and

\[ G_{i,n} = \delta y_1 G_i e^{(-\delta \tau_1) \tau_i / \tau_i} e^{(-\delta \tau_2) \tau_i / \tau_i} \cdots e^{(-\delta \tau_n) \tau_i / \tau_i} \]

\[ + \delta y_2 G_i e^{(-\delta \tau_2) \tau_i / \tau_i} \cdots e^{(-\delta \tau_n) \tau_i / \tau_i} \]

\[ + \cdots \]

\[ + \delta y_n G_i e^{(-\delta \tau_n) \tau_i / \tau_i} \]

\[ = (G_{i,n-1} + \delta y_n G_i) e^{(-\delta \tau_n) \tau_i / \tau_i} \]

where \( G_{i,1} = \delta y_1 G_i e^{(-\delta \tau_1) \tau_i / \tau_i} \)

Equation (3.16), (3.19), and (3.20) enable arbitrarystrain to be modelled if the strain jumps are kept small.

### 3.4.6 Modelling the change in temperature

Muhr (2008) proposed a model using the Boltzmann superposition principle to model the effect of changes in strain and predict the temperature dependence by using the WLF superposition (Ferry, 1980). The viscoelastic properties were modelled with a Prony series as described in Section 3.4.5. The model is:

\[ G(t, T) = G_\infty(T_{\text{ref}}) \frac{T}{T_{\text{ref}}} + \sum G_i(T_{\text{ref}}) \frac{T}{T_{\text{ref}}} e^{-t/\alpha_T \tau_i} \]  

(3.18)

where \( \alpha_T \) is the temperature shift factor and \( T_{\text{ref}} \) is the reference temperature. \( G_\infty \) is the modulus at a very long time. \( G_\infty \) and \( G_i \) are proportional to the absolute temperature \( T \). If \( T_{\text{ref}} \) is some value other than \( T \), then \( \alpha_T \) in Equation (3.18), needs to be replaced by \( \alpha_T / \alpha_{T_{\text{ref}}} \).

For an instantaneous strain \( \gamma_0 \) applied at \( t = 0 \), Boltzmann’s superposition equation and Equation (3.18) give:
\[ G(t, T) = \gamma_0 \left[ G_\infty \frac{T}{T_{ref}} + \sum G_i \frac{T}{T_{ref}} e^{\frac{t}{a_T \tau_i}} \right] \quad (3.19) \]

For the strain with the time taken as \( t_n = \sum \Delta t_j \), with the time increment \( \Delta t_j \) at a constant temperature \( T_j \), each Prony term can be written as (Muhr, 2008):

\[ G_i \frac{T}{T_{ref}} \left( e^{-\left( \frac{-\Delta t_1 + \Delta t_2 + \cdots + \Delta t_n}{a_T \tau_i} \right)} \right) = G_i \frac{T}{T_{ref}} e^{\left( \frac{-\Delta t_1}{a_T \tau_i} \right) + \left( \frac{-\Delta t_2}{a_T \tau_i} \right) + \cdots + \left( \frac{-\Delta t_n}{a_T \tau_i} \right)} \]

If the temperature is different, this will change the horizontal shift factor:

\[ G_i \frac{T}{T_{ref}} e^{\left( \frac{-\Delta t_1}{a_T \tau_i} \right) + \left( \frac{-\Delta t_2}{a_T \tau_i} \right) + \cdots + \left( \frac{-\Delta t_n}{a_T \tau_i} \right)} \]

With \( a_{T_2} \ldots a_{T_n} \) are \( a_T/a_{T_{ref}} \) at the temperature during the time interval \( \Delta t_j \).

\[ G_i \frac{T}{T_{ref}} \prod_{j=1}^{m} e^{\left( \frac{-\Delta t_j}{a_T \tau_i} \right)} \quad (3.21) \]

Therefore becomes;

\[ G(t, T) = \gamma_0 \left[ G_\infty \left( \frac{T}{T_{ref}} \right) + G_i \left( \frac{T}{T_{ref}} \right)^n \sum \prod_{i=1}^{m} e^{\left( \frac{-\Delta t_j}{a_T \tau_i} \right)} \right] \quad (3.22) \]

\[ \therefore G(t, T) = \gamma_0 \frac{T_n}{T_{ref}} \left[ G_\infty + G_i \sum \prod_{i=1}^{m} e^{\left( \frac{-\delta t_n}{a_T \tau_i} \right)} \right] \quad (3.23) \]

\( \delta \) replaces \( \Delta \) to indicate that Equation (3.22) is assumed for one time interval per temperature jump, whereas Equation (3.23) can be used for the incrementation of small time intervals. So it is able to model any arbitrary temperature history by choosing sufficiently small \( \delta t \) and hence, sufficiently small \( \delta T \).

The viscoelastic constants \( G_\infty \) and \( G_i \) are not affected by the temperature history (only the exponential terms), and are just multiplied by the \( \frac{T}{T_{ref}} \).
3.4.7 Complete model for arbitrary strain and temperature

The strain Equation (3.18), (3.19), and (3.20) are modified to include the effect of temperature, which become;

\[
G(t, T)_n = \frac{T_n}{T_{ref}} \left( s_{\infty,n-1} + G_{\infty} \delta y_n \right) \\
+ \sum \left( s_{i,n-1} + G_i \delta y_n \right) e^{\left( \frac{-\delta t_i}{\rho \tau_i} \right)}
\]

with

\[
s_{\infty,1} = G_{\infty} \delta y_1
\]

\[
s_{i,1} = G_i \delta y_1 e^{\left( \frac{-\delta t_i}{\rho \tau_i} \right)}
\]

\(s\) are used to represent \(G\) to denote the modulus terms without the temperature correction factor \( \frac{T}{T_{ref}} \).

3.4.8 Finite element analysis

Finite Element Analysis (FEA) and modelling approach for the case of simple shear and buckling compression is discussed in this section. The FEA package, ABAQUS, was used as an alternative evaluation method other than the experimental and calculation methods.

3.4.8.1 The DBS and HRC model in ABAQUS

The DBS and HRC geometry parameters used in ABAQUS are shown in Figure 3.43 and Figure 3.44. Both models replicate the actual sample, which was moulded and tested experimentally.

**DBS model**

Plane strain model was applied for DBS. The plane strain enables a simple homogenous model with a plane strain thickness of 1 unit (displacement per unit length). For the shear model, the unit only moves in its plane (in this case in horizontal), with the assumption of no strain normal to the plane (in the vertical direction). A model which imitates the actual dimension of a DBS sample used in this part of the study is shown in Figure 3.43.
Figure 3.43: Plane strain DBS model. $\delta$ is the axial displacement. ENCASTRE constraint is applied at the bottom surface of the model (red segmented line). Dimension is in mm.

The DBS was modelled as hybrid elements with $25 \times 2 = 50$ 4-node elements (CPE4RH). Reduced integration has been selected for the analysis.

**HRC model**

The symmetry of the HRC and its loading conditions test allows modelling as an axisymmetric deformable model (Figure 3.44). Axisymmetric assumes the model to sweep $360^\circ$. The incompressibility requires the use of hybrid elements. A model, according to Figure 3.44, with $25 \times 5 = 125$ 4-node elements (CAX4RH) with the hybrid formulation and reduced integration were used. The horizontal boundary condition at the top surface of the cylinder has been restricted. This represents the actual condition during the experiment, where the cylinder’s top has been fixed from sliding sideways during the vertical compression. This fixation also prevents the cylinder from collapsing asymmetrically at half-height compression.
3.4.8.2 Fitting the material model

The rubber was modelled as a linear viscoelastic material. The elastic component was modelled with a neo-Hookean strain energy function (Equation (3.26));

\[ W = C_{10}(I_1 - 3) + \frac{1}{D_1}(J_{el} - 1)^2 \]  (3.26)

where \( W \) is the strain energy density, \( C_{10} \) and \( D_1 \) are the material parameters. \( C_{10} = G/2 \) where \( G \) is the shear modulus, and \( D_1 = 2/E_b \) where \( E_b \) is the bulk modulus. \( I_1 \) is the first strain invariant, and \( J_{el} \) is the elastic volume ratio. The shear modulus value (parameter \( C_{10} \)) was derived from the result of double-bonded shear (DBS) test. The bulk modulus value was taken as \( E_b = 2000 \text{MPa} \) after Lindley and Gough (2015). The models’ stress contour for both models is shown in Appendix C. The derivations of Equation (3.26) was discussed in Chapter 2 Section 2.4.4.
The viscoelastic behaviour was modelled with the linear viscoelastic model offered in ABAQUS (Section 4.8 in Dassault Systèmes (2014)). It uses a Prony series (Equation (3.10)) to describe the relaxation behaviour. The parameters were fitted by assuming a relaxation function of the form given by Equation (2.13). In order to determine the two parameters of Equation (3.28), the viscoelastic model was fitted from the stress relaxation data in simple shear as described below. The parameters from the fit will be used to model other cases of stress relaxation, including the buckling deformation. The Prony series may have more than two parameters. Although by using two parameters, it can describe the modulus of unfilled NR at long timescale at ambient temperatures, such as in normal creep and stress relaxation (Muhr, 2008).

A Prony series which serves as a calibration plot has been overlaid on a stress relaxation test conducted on a DBS sample at 100% shear strain. The test ran for 67 hours at a constant 24°C of temperature. To fit the Prony series onto the experimental stress relaxation plot, the $G(t)$ was varied until both plots converge. The $H_0$ was varied to adjust the relaxation slope of the Prony series to match the stress relaxation’s slope. Figure 3.45 shows the Prony calibration plot achieved to the stress relaxation data set at $\gamma=1.0$ for EDS19 using the 2-parameter viscoelastic model.

![Prony calibration plot](image)

**Figure 3.45**: Stress relaxation plot at shear strain $\gamma=1.0$. The blue is the experimental data with the straining phase $\sim10s$.

By iterating the $G(t)$ and $H_0$ values following Ahmadi, Muhr and Kingston (2008) proposed method, a straight line of the plot of $G(t)$ versus log(t) can be produced as shown by the red
line in Figure 3.45, which then was fitted onto an experimental stress relaxation plot. \( G_0 \) and \( G\infty \) values can then be determined by the equations below;

\[
G(\tau_{\text{min}}) = G_0 = G(t) - H_0\ln(\tau_{\text{min}}/t)
\]

\[
G(\tau_{\text{max}}) = G\infty = G(t) - H_0\ln(\tau_{\text{max}}/t)
\]

with \( \tau_{\text{min}} \) and \( \tau_{\text{max}} \) are the limits time range of interest, producing the relaxation terms, \( n \).

Relationship between \( H_0 \) and \( G_0 \) was described in Section 2.5.5. \( n \) and \( g_i \) can now can be expressed by;

\[
g_i = \frac{H_0}{G_0} \ln k \quad ; \quad n = \log \left( \frac{\tau_{\text{max}}}{\tau_{\text{min}}} \right)
\]

with \( n \) as the relaxation terms in logarithmic scale. The decision for the \( n \) value was merely chosen for convenience, with one Prony term for each decade of time. The time range of interest for this fit, \( \tau_{\text{min}} \) and \( \tau_{\text{max}} \) were chosen to be 10s and 10,000,000s respectively. The range was deemed to be reasonable as it was not too short nor too long to produce a smooth fit on an actual stress relaxation test as shown in Figure 3.45.

The viscoelastic model in the subsequent calculation and ABAQUS simulation will be using the parameters resulting from the fit shown above. Values of the parameters used to achieve the fit are shown in Table 3.3, including the neo-Hookean parameters \( C_{10} \) and \( D_2 \). \( C_{10} \) was deduced from the hyperviscoelastic parameter \( (0.5 \times G\infty) \), with \( G\infty \) as in Equation (3.29). \( D_2 \) is relevant to the ABAQUS model but not the Excel calculations.

In the subsequent models, the relaxation time, \( \tau \) of 5-term (\( n=4 \)) was chosen, ranging from 10s to 100,000s. This is due to the maximum range of the experimental tests which end around 100,000s and the relaxation starts after a 10s of strain imposition. The method of calculating \( \tau \) has been explained in Section 2.5.5 and Equation (2.47).

Parameters \( G_0 \) and \( g_i \) will be applied in the model implementation discussed in Section 3.4.9 and 3.4.10.
Table 3.3: Parameters for the EDS19 in stress relaxation with varying strains (fit to Prony series).

<table>
<thead>
<tr>
<th>Hyperviscoelastic Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_0 ) (MPa) (^2)</td>
<td>0.004</td>
</tr>
<tr>
<td>( G(t) ) (MPa); t at 10s</td>
<td>0.410</td>
</tr>
<tr>
<td>( G_\infty ) (MPa)(^1)</td>
<td>0.366</td>
</tr>
<tr>
<td>( G_0 ) (MPa)</td>
<td>0.407</td>
</tr>
</tbody>
</table>

\[
\begin{array}{c|c}
\alpha & \beta \\
0.02298 & 31.62278 \\
0.02298 & 316.2278 \\
0.02298 & 3162.278 \\
0.02298 & 31622.78 \\
0.02298 & 316227.8 \\
\end{array}
\]

\{ For 5-term (\( n = 4 \)) \\

<table>
<thead>
<tr>
<th>neo-Hookean Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{10} ) (MPa) = 0.5( G_\infty )</td>
<td>0.182</td>
</tr>
<tr>
<td>( D_1 ) (MPa(^{-1})) = 2//( E_b )</td>
<td>0.001</td>
</tr>
</tbody>
</table>

\(^1\) Parameter-1  \\
\(^2\) Parameter-2

Table 3.3 shows the hyperviscoelastic parameters of EDS19 at reference temperature (297K). Accordingly, the parameters used at different temperatures have been determined by using Equation (2.48), (2.49), (2.50), (2.51), and (2.52) which results are tabulated in Table 3.4. The description to determine the parameters is explained in Section 2.5.6.

Table 3.4: Parameter inputs in ABAQUS for changing temperatures.

<table>
<thead>
<tr>
<th>Hyperelastic parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{10} )</td>
<td>( D_1 )</td>
</tr>
<tr>
<td>0.1713</td>
<td>0.001</td>
</tr>
<tr>
<td>0.1820</td>
<td>0.001</td>
</tr>
<tr>
<td>0.1869</td>
<td>0.001</td>
</tr>
<tr>
<td>0.1997</td>
<td>0.001</td>
</tr>
</tbody>
</table>

WLF parameters

\[
\begin{align*}
T_0 &= 297 \text{ K} \\
C1^0 &= 6.402 \\
C2^0 &= 140.6
\end{align*}
\]

\( T_0 \) is the reference temperature. The neo-Hookean parameters, \( D_1 \) remained constant across all strain and temperature while \( C_{10} \) and temperature \( T \) change accordingly. TRS is “thermo-
rheologically simple”, which registers the viscoelastic constants $C1^o$ and $C2^o$ for the reduced time concept for temperature dependence (WLF).

3.4.9 Evaluation on the stress relaxation with the varying strain (constant temperature)

This section discusses the comparison of results between the experiment, calculation and ABAQUS simulation on the rubber undergoing stress relaxation with varying strain. In implementing the strain changing in the middle of the relaxation, the Boltzmann superposition principle was applied.

Case 1a: Simple shear

The DBS sample was strained to 100% shear in 10s, then left to relax for 0.03h. It was then released fully for 0.03h before the second strain was applied to 150% shear for 1.1h and rereleased for 0.6h. The third strain was applied to 50% shear for 4.0h and released afterwards for 17.7h. The output of the Excel calculations (in shear) using the 2-parameter model is shown by the green plot. For the ABAQUS simulation, the stress was calculated by force per unit thickness of the model (given as 25, see Figure 3.43). The comparisons between the experiment, Excel calculations, and ABAQUS simulation in stress as a function of time is shown in Figure 3.46.
Figure 3.46: Comparison between the experiment and the predicted relaxation behaviour in simple shear at 297K with $G_\infty = 0.364\text{MPa}$ and $H_0 = 0.004$.

There are some discrepancies between these model and the experimental plot. The experimental stress is generally lower at higher strain ($\gamma = 1.5$), and higher at lower strain ($\gamma = 0.5$). The discrepancies between the experimental and the calculations and simulation can be partly due to the assumption of the model’s neo-Hookean elasticity, and the stress-strain comparison between the experimental and the neo-Hookean model in simple shear is shown in Figure 3.47.
Figure 3.47: Stress-strain behaviour of unfilled rubber (EDS19) in simple shear.

The experimental and simulation plots are shown to be comparable in accordance to the neo-Hookean’s assumption. The result suggests that the proposed model, by making direct use of the Boltzmann superposition principle, gives an adequate prediction of the behaviour of unfilled rubber under an arbitrary strain history. Further refinement of the model, to capture better the change in modulus at different strains, may be possible but has not been attempted in the current work.

Only some details of the linear viscoelastic model implemented in ABAQUS model are documented in the ABAQUS manual, but the similarity of the behaviour suggests that a similar approach, using the Boltzmann superposition principle, may have been used. However, some discrepancies between the calculation and the ABAQUS simulation are observed at every recovery phase when the shear strains were released. The cause of this is yet to be established.

**Case 1b: Axial compression on cylinder**

The strain steps of the stress relaxation; the first compressive strain ($\varepsilon_b = 0.2$) was held for 0.03h and released for 0.03h. It was then strained at ($\varepsilon_b = 0.4$) and held for 2.1h, and released for 0.5h. Lastly, the strain was at ($\varepsilon_b = 0.5$) and held for 3.3h, before being released for 14h. The identical sets of parameters to Case 1a (simple shear) were used in the simulation. The
experiment and simulation plots for compressive deformation with changing strains is given in Figure 3.35.

![Figure 3.48: Comparison between the experiment and the simulation of the relaxation behaviour in compression at 297K. \([\varepsilon_b = 0.2, 0.4, \text{ and } 0.5 \text{ equal to } \delta = 4.9, 9.8, \text{ and } 12.3\text{mm}]\)](image)

The plot shows apparent discrepancies between the experiment and the ABAQUS simulation for the HRC. At low buckling strain, the experimental force is generally higher, and at higher strain, it shows lower force. At higher strains (\(\varepsilon_b = 0.4 \text{ and } 0.5\)), the HRC appears to be within the soft ‘plateau’ region of its force-displacement (blue line in Figure 3.49). The discrepancies in the force-displacement behaviour between the experiment and the ABAQUS simulation for buckling is partly due to the neo-Hookean model. Another source of error is likely due to the mesh limitation of the ABAQUS model. The model parts were simulated in ABAQUS Student Edition 2019, which supports only up to 1000 nodes per analysis. Several iterations of analysis using finer mesh were attempted and resulted in over-distortion leading to aborted analyses. The results shown in this study are the most optimal
part mesh. The force-displacement comparison between the experimental and the ABAQUS model simulation in buckling is shown in

Figure 3.49.

![Force-displacement behaviour of unfilled rubber (EDS19) under compression.](image)

Figure 3.49: Force-displacement behaviour of unfilled rubber (EDS19) under compression.

The effect of coarse mesh can be seen in Figure 3.49. This suggests that the FEA does not capture enough data in the softening of the cylinder during the buckling. The comparison plots show that, until 7mm compressive buckling, the plots for both experiment and simulation are similarly linear, although at different rates. Beyond that point, discrepancies between the two are shown. The discrepancies at three compressive displacement ($\varepsilon_c = 0.2, 0.4, 0.5$ which equal to $\delta = 4.9, 9.8, 12.3$mm) shown in

Figure 3.49 are fairly consistent with Figure 3.48.

### 3.4.10 Evaluation on the stress relaxation with varying strain and temperature

The comparisons between the experiment, calculation and ABAQUS simulation on the rubber undergoing stress relaxation with varying strain and temperature are discussed here.
Case 2a: Simple shear

The DBS sample was tested with an initial shear strain ($\gamma=100\%$). The stress relaxation data is shown in Figure 3.51.

Figure 3.50: Stress relaxation data (Exp) in shear corresponds to the change in temperature.

The strain and temperature inputs for the DBS in the calculation and ABAQUS simulation are summarised in Figure 3.51.
Figure 3.51: (Top) Shear strain and temperature history applied to the DBS sample. (Bottom) The stress relaxation corresponding to both inputs.

The comparison plot above shows some fair agreement between three methods, from the start of the test until the onset of temperature change from 53°C to 6.5°C. The experiment plot shows relatively lower stress getting from 53°C to 6.5°C, and 32°C. Otherwise, there is a good agreement between the calculation and the simulation methods in predicting the relaxation in shear with arbitrary strain and temperature.
Case 2b: Axial compression on cylinder

The HRC sample was tested with an axial compression where the cylinder was in buckling strain ($\varepsilon_b=37\%$). The stress relaxation data with changing strain and temperature is shown in Figure 3.52.

![Figure 3.52: Stress relaxation data (Experiment) in compressive strain corresponds to the change in temperature.](image)

The strain and temperature inputs for the HRC in ABAQUS are summarised in Figure 3.53.
Figure 3.53: (Top) Compressive strain and temperature history applied to the HRC sample. (Bottom) The stress relaxation corresponds to both inputs.

The relaxation plot above suggests that in general, the simulation plot shows a similar pattern to the experiment plot, with some discrepancy from the beginning of the plot.
3.4.11 Discussion

An attempt to predict the stress relaxation behaviour of DBS and HRC, made of unfilled rubber, with changing strain and temperature has been made by applying the Boltzmann superposition principle to model the effect of changes in strain and WLF transform to model the change with temperature, following the model proposed by Muhr (2008). The neo-Hookean model was used to model the hyperelastic behaviour of the unfilled rubber. The neo-Hookean model appeals as it provides a good description of unfilled rubber and is straightforward in the application and worked well alongside the Prony series used in the calculations. The results suggest that fair agreement can be observed between the model and the experimental results. The qualitative effects of changes in both temperature and strain on the subsequent stress were captured well by the model although there were some quantitative discrepancies. These may partly arise from the use of the neo-Hookean model, but some discrepancies were observed which cannot clearly be explained in this way.

There was generally close agreement between the analytical model and the ABAQUS simulation, suggesting that the underlying implementation of the ABAQUS model mirrored that of the Muhr (2008) model implemented here in Excel. However, a few unexplained discrepancies exist.

The modelling of rubber in shear and compression with strain and temperature changes can serve as a guide during the design stage of a rubber product which operates in permanent shear and compression, although the trade-off in discrepancies will need to be considered as shown in the analyses.

3.5 Accelerated Method for Stress Relaxation of NR

3.5.1 Time-temperature superposition principle (TTSP) and Stepped isothermal method (SIM)

The design process of a rubber component subjected to long-term stress relaxation often involves an early-stage product testing which requires experimental measurement of stress-relaxation. An experiment under the product service life conditions would take many years to complete, so a method for accelerating the test is required.
Section 2.5.6 described how the stress relaxation can be predicted by accelerating the stress response following the increase in temperature, by using TTSP method. Another later approach to characterise stress relaxation based on TTSP, known as the Stepped isothermal method (SIM) was also described.

This section aims to evaluate the applicability of using both methods to characterise the long term stress relaxation of unfilled rubber. The study on TTSP and SIM will discuss two types of test samples; double-bonded shear (DBS) and hollow rubber cylinder (HRC); the former represents the simple shear deformation and the latter an axial compressive deformation. The choice of using DBS is because it is common to use simple shear in rubber engineering, and much past research discussed the characterisation of rubber engineering in shear deformation. This study thought that it is worthwhile to evaluate SIM method on compressive deformation of rubber which also relevant to other rubber products and components.

3.5.2 Experimental procedures

For the accelerated stress relaxation experiments, the DBS samples with ~2mm thickness were used. The experiments for the accelerated test methods, covering the TTSP and SIM, used identical experimental setups. The experiments on DBS and HRC test pieces differed slightly due to the fixing of the test jigs and the positions of the thermocouples. A universal test system, Instron 4301, was used together with the insulated temperature chamber. Picolog Datalogger recorded the data for the stress and strain signals, and DTaker 800 logging instrument for the temperature measurements where three thermocouples were used. The experiment setup is almost identical to the one used in the evaluation of stress relaxation of NR at the arbitrary strains and temperatures.

The types of samples and test setup are shown in Figure 3.41 for DBS sample, Figure 3.10 and Figure 3.41 for HRC sample as described in Section 3.4.3.

3.5.3 Effect of temperature on the NR stress relaxation behaviour

An attempt has been made to construct a stress relaxation master curve in shear and buckling mode. A different reference temperature, $T_0$ was chosen instead of $T_s$ ($T_s=T_0+50K$). Similar to
the thermorheological simple (TRS) parameters in Table 3.4, the viscoelastic constants, $C_1^0$ and $C_2^0$ used in this section are shown in Table 3.5.

Table 3.5: The new viscoelastic constants for the reference temperature, $T_0$ of 24°C.

<table>
<thead>
<tr>
<th>Compound</th>
<th>$T_0$ (°C)</th>
<th>$T_0$ (K)</th>
<th>$C_1^0$</th>
<th>$C_2^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EDS19$^a$</td>
<td>-65</td>
<td>297</td>
<td>6.402</td>
<td>140.6</td>
</tr>
</tbody>
</table>

$^a$ Natural rubber SMR-CV60 (Standard Malaysian Rubber constant viscosity grade 60)

**Construction of the TTSP master curve**

The stress relaxation master curve was constructed according to the TTSP procedure, using many different isothermal short-term stress relaxation tests. In this study, seven individual DBS samples were used, each in an individual experiment at different temperatures. The samples were identified as DBS-D1, DBS-D2, DBS-D3, DBS-D4, DBS-D5, DBS-D6, and DBS-D1, with the applied temperatures were 24°C, 32°C, 43°C, 53°C, 63°C, 73°C, and 103°C respectively. Each sample was conditioned at the test temperature before being subjected to 100% shear strain (approximately 2mm), for a period of 11,000s. Similarly, an almost identical set of stress relaxation tests with different isothermals were conducted on six HRC samples; HRC-D1, HRC-D2, HRC-D3, HRC-D4, HRC-D5, and HRC-D6, with the applied temperatures were 24°C, 32°C, 43°C, 53°C, 63°C, and 73°C, respectively. That levels of temperature were chosen with the aim to increase the temperature by approximately 10°C stepwise. Also, these were the temperature steps that can be achieved and retained with good stability by using the Instron temperature chamber. The test durations of 11,000s for each isothermal were chosen as the time (approximately 3 hours) were not too long for the purpose of testing the TTSP model. This enabled close physical monitoring of the test specimens and setup.

The stress decay data for EDS19 under the permanent shear displacement of 2mm are displayed in Figure 3.54. It is known that the TTSP method has an inherent ‘error’ due to the use of many samples and hence a high probability of sample variations. The variations may come from the uncertainty of the rubber thickness measurement (for DBS sample). Other variations could arise from the difference in cure behaviour and inhomogeneity in the rubber mix.
To minimise the effect of sample variations, a strict control in sample preparation was put in place. To ensure each compound batch is well mixed, three rheometer tests were conducted for every mix, sampled from three areas within the rubber mix sheet. A well-dispersed rubber mix is shown by the approximately identical cure times and torque values among the three samples. To ensure the consistency from one compound batch to another, rheometer tests were conducted and compared within batches. The cure conditions of the samples in between batches were kept approximately identical. Otherwise, a new compound will be mixed. For the case of DBS samples, prior to the steel pieces’ treatment with bonding agents, their total thicknesses were measured multiple times at different positions and the average values were used. This was so that the correct rubber thickness is measured when the sample is cured.
As shown in Figure 3.54, each test had a 10-second shear straining phase. The stress decay of each isotherm showed a linear relationship with a logarithmic scale, except for the isothermal curves of 63°C, 73°C and 103°C, where the decline of the modulus is shown after 5,000 to 6,000s for the sample at 63°C, and after 2,000s for the sample at 73°C. A very quick decline in modulus is seen for the sample at 103°C, after approximately 100s. These observations suggest that in these tests, the rubber may have started to degrade chemically; a phenomenon that can be described as chemical stress relaxation (Wood, Bullman and Roth, 1975).

The isothermal curves for 63°C and 73°C were included in the construction of the TTSP master curve, but the 103°C isothermal curve was excluded because the TTSP only describes the physical stress relaxation process. The horizontal and vertical shift factors used for the construction of TTSP master curve for DBS and HRC are summarised in Table 3.6.
Table 3.6: The horizontal and vertical shift factors for TTSP master curve, according to every sample’s response temperatures (in Kelvin, K). Calculated from Equations (2.49) and (2.53).

<table>
<thead>
<tr>
<th>Sample</th>
<th>Temperature, $T$ (K)</th>
<th>Horizontal shift factor, $a_T$</th>
<th>Vertical shift factor, $\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DBS-D1</td>
<td>297$^a$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>DBS-D2</td>
<td>305.4</td>
<td>2.296</td>
<td>0.972</td>
</tr>
<tr>
<td>DBS-D3</td>
<td>315.7</td>
<td>5.644</td>
<td>0.941</td>
</tr>
<tr>
<td>DBS-D4</td>
<td>326</td>
<td>12.438</td>
<td>0.911</td>
</tr>
<tr>
<td>DBS-D5</td>
<td>336</td>
<td>24.563</td>
<td>0.884</td>
</tr>
<tr>
<td>DBS-D6</td>
<td>346</td>
<td>45.147</td>
<td>0.858</td>
</tr>
<tr>
<td>HRC-D1</td>
<td>297$^a$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>HRC-D2</td>
<td>305.5</td>
<td>2.317</td>
<td>0.978</td>
</tr>
<tr>
<td>HRC-D3</td>
<td>316</td>
<td>5.783</td>
<td>0.952</td>
</tr>
<tr>
<td>HRC-D4</td>
<td>326</td>
<td>12.438</td>
<td>0.929</td>
</tr>
<tr>
<td>HRC-D5</td>
<td>336</td>
<td>24.563</td>
<td>0.907</td>
</tr>
<tr>
<td>HRC-D6</td>
<td>346</td>
<td>45.147</td>
<td>0.887</td>
</tr>
</tbody>
</table>

$^a$ Reference temperature, $T_0$ at 24°C (297K)

Figure 3.55: Horizontal shifting according to TTSP (DBS sample). Segmented lines - curves after shifting vertically. Solid lines – curves after shifting vertically and horizontally. Shift factors are given in Table 3.6.
Figure 3.55(a) shows the method for shifting the isothermal curves horizontally. This is for the stress relaxation master curve of DBS at 100% shear strain. Every isothermal curve is shown to have upturns at the beginning, because the initial test data (at t=0) was taken at the early stage of the test where each sample was initially strained. Each curves have straining phase of 10-second which were excluded in this graph as they serve no purpose in creating a stress relaxation master curve.

A benchmark experiment was conducted utilising longer-term direct measurements. A similar stress relaxation experiment was performed on a fresh DBS sample for 67 hours at 24°C. The comparison of the stress relaxation behaviour between the long-term test at 24°C and the TTSP is shown in Figure 3.56.

![Graph showing comparison between long-term stress relaxation (SR) experiment at 24°C and the TTSP method in simple shear (T0=297K).]

As displayed in Figure 3.56, the comparison of the TTSP master curve with the conventional medium-term stress relaxations plots shows a good agreement. A divergence, however, is shown by the TTSP master curve somewhere after 100,000s. The discrepancy is, presumed to be caused by the ageing of the rubber at a high temperature which is shown starting from 63°C of test temperature, indicated in Figure 3.54.
Similarly, the same approach in creating the TTSP master curve was conducted on the isothermal curves of stress relaxation in buckling form (HRC sample). The stress relaxation master curve, with the reference temperature $T_0 = 297K$, is shown in Figure 3.57.

Some divergence can be seen in the TTSP master curve in Figure 3.57 below, developed after 10,000s. It may be caused by the ageing of the rubber at a high temperature which is shown starting from 63°C of test temperature. The similar observation is exhibited by the TTSP curve in simple shear. The conventional stress relaxation test curve (in red) is shown to be stiffer due to the factor of variation in the HRC sample preparation which came from a different rubber compound batch. The noisier curve of the conventional test is due to the use of a load cell which was once installed in the stress relaxation test rig shown in Figure 3.15, and was replaced since then because of its susceptibility to electrical noise.

![Figure 3.57: Comparison between stress relaxation (SR) test at 24°C and the TTSP method for compressed HRC testpieces ($T_0=297K$).](image)

### 3.5.4 Principles of Stepped isothermal method (SIM)

The extended approach of the TTSP, known as the Stepped isothermal method (SIM) is evaluated in this section. The method was introduced to predict the creep behaviour of polymer products (Thornton et al., 1998; Achereiner et al., 2013) and aramid fibre (Alwis and Burgoyne, 2008). Unlike TTSP, which requires multiple samples and experiments to construct a master curve for one strain level, SIM requires only a single sample. For the stress relaxation
of rubber, a constant shear strain ($\gamma = 1.0$) was applied to the DBS sample, with a sequence of increased temperature in steps applied during the test.

The use of a single test sample in SIM is appealing, due to the factor of minimising error and test times. In TTSP, although precautions were taken with sample variations, errors can still take place, for example, during the DBS’ rubber thickness measurement. An error in thickness measurement may lead to the wrong judgement of the permanent displacement which the sample is supposed to hold. When an incorrect displacement is applied during the experiment, the level of the force reading will also be incorrect, therefore causing the isothermal curves to be less likely to converge in order to create the relaxation master curve.

SIM, as in TTSP, involve the shifting of the stress relaxation curves using a horizontal shift factor, $\alpha$, and a vertical shift factor, $\xi$, to construct the master curve. However, an additional step is needed to shift the isothermal curves as the changes in temperature take place within the test time frame. In SIM, each consecutive relaxation curve corresponding to the increase of the temperature will be dealt with as individual isothermal curve. Each curve will have its virtual starting time, known as $t'$, and it is empirically determined.

In a SIM experiment, when the second temperature step $T_2$ starts at time $t_2$, some stress relaxation has already existed within the previous temperature $T_1$. Therefore, a stress relaxation measured between $t_2$ and $t_3$ would have started at a time $t_2' < t_2$. The stress relaxation master curve was plotted on the virtual time scale ($t-t'$).

**SIM on DBS**

A stress relaxation experiment to evaluate the applicability of SIM on a DBS sample was conducted. The experimental setup for SIM is identical to the setup for the TTSP experiment, as explained in Section 3.5.2. A permanent 2mm shear displacement (equals to 100% shear strain) was applied on DBS sample DBS-E1, at reference test temperature, $T_0$ of 24°C for 80 minutes. Subsequently, four more temperature steps (32°C, 43°C, 53°C, and 63°C) were applied during the experiment, while the DBS was maintaining the initial strain. Each temperature step lasted for 80 minutes. The temperature steps chosen for this test were identical to the temperature steps adopted in the TTSP model test in Section 3.5.3, except with the exclusion of 73°C due to the known effect of chemical degradation shown in the TTSP test. The duration of 80 minutes for each temperature step was chosen as this enabled
one set of SIM tests to be completed within one day’s time. This arrangement also made close physical monitoring possible.

The experimental observations are similar to the TTSP experiment, with the magnitude of stress corresponding to the level of temperature. When the temperature is ramped up in the middle of the test, the stress magnitude increased until it reached equilibrium as the DBS temperature reached an equilibrium. The stress then decayed with time, following the stress relaxation behaviour before the next temperature step was applied.

It is worth noting that due to the thermal expansion, which existed in the test system during the experiment, a thermal correction test was conducted to compensate for this error. This was done by repeating the SIM temperature steppings while the sample was held at zero displacement. This was deemed acceptable for the DBS sample as its stress-strain behaviour is linear. For example, a small change in displacement around 0% strain gives the same change in force as the same small change in displacement at 100% strain.

The thermal expansion stress was superimposed with the raw SIM data. To observe this additional thermal stress, the DBS was subjected to a 0% strain with temperature steps, where a pattern of force increase was observed with each step, caused by the sample elongation. This was due to the thermal elongation of the test jigs. Therefore, the stress from the thermal expansion was subtracted from the raw data, leaving only the net stress of the test that corresponds to the change in temperature.

The raw SIM data of DBS at 100% shear strain shows the change in stress corresponding to the changing temperatures, shown by the black plot in Figure 3.58. A complete, corrected SIM data was obtained by subtracting the thermal expansion stress in time-history, shown by the purple plot in Figure 3.58.
Figure 3.58: The shear stress corresponding to the change in temperature (SIM experiment on DBS test piece).

Three temperature probes and records are shown in the SIM plot, with TC-2 was the thermocouple inserted in the DBS test piece, 2-3mm away from the rubber disk (see Figure 3.5). The air temperature inside the chamber achieved the target temperature with stable fluctuations in around 3 minutes, as shown by TC-1 and TC-3 in Figure 3.58. Whereas the rubber disk bulk, as shown by TC-2 plot, achieved the target temperature after 22 minutes. This was due to the significant thermal mass of the testpiece and attached steel test jig. The temperature of the test was controlled to be within ±0.5°C of the nominal temperature. The steps in constructing the master curve are shown from Figure 3.59 to Figure 3.61.
Figure 3.59: Overview of the SIM correction due to thermal expansion and determining the virtual starting point, $t'$ for creating individual stress relaxation curve. Virtual time scale $= t - t'$.

The virtual start time $t'$ for each isothermal was determined by empirical procedure. Starting with an arbitrary value, the virtual starting time, $t'$ was iteratively varied until the slope at the beginning of a temperature step matches the slope at the end of the previous step. This procedure is repeated iteratively for all temperature steps until all the SIM data rescaled in the virtual time scale ($t - t'$). This step is shown visually in Figure 3.60.

The new individual isothermal curves will then be shifted in horizontal and vertical to create a master curve, by using the horizontal shift factor, $a_r$ from Equation (2.49), and the vertical shift factor, $\xi$ from Equation (2.53). The subsequent isothermals were all shifted in the same manner until a complete master curve converged.
Figure 3.60: Rescaling the second isothermal curve using the virtual time scale ($t-t'$).

Figure 3.61: Vertical and horizontal shifting on the second isothermal ($T=32^\circ C$).
The parameters used to construct the stress relaxation master curve in shear are summarised in Table 3.7.

**Table 3.7: The parameters for constructing the SIM master curve (DBS sample)**

<table>
<thead>
<tr>
<th>Temperature, $T$ (°C)</th>
<th>Horizontal shift factor, $a_T$ (Equation (2.49))</th>
<th>Vertical shift factor, $\xi$ (Equation (2.53))</th>
<th>Virtual start time, $t'$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24$^a$</td>
<td>1.000</td>
<td>1.000</td>
<td>$t'_1 = t_1 = 0$</td>
</tr>
<tr>
<td>32</td>
<td>2.296</td>
<td>0.972</td>
<td>$t'_2 = 4300$</td>
</tr>
<tr>
<td>43</td>
<td>5.644</td>
<td>0.941</td>
<td>$t'_3 = 9200$</td>
</tr>
<tr>
<td>53</td>
<td>12.438</td>
<td>0.911</td>
<td>$t'_4 = 14000$</td>
</tr>
<tr>
<td>63</td>
<td>24.563</td>
<td>0.884</td>
<td>$t'_5 = 18500$</td>
</tr>
</tbody>
</table>

$^a$ Reference temperature

The final form of the SIM master curve for shear deformation is shown in Figure 3.62.

![Figure 3.62: Stress relaxation master curve (SIM) for DBS at $\gamma = 1.0$.](image)

Figure 3.62 shows the SIM master curve of DBS with downturns appearing in the early part of each isothermal. These were caused by the increase in the temperature in the chamber, which, using the current setup, took some time to achieve an equilibrium. This transition region has undefined relaxation behaviour because the temperature was yet to reach a stable state, and so did the stress inside the sample. As a result, the data in this transition
region, shown by the downturn curves were excluded from the formation of the main master curve, but the time scale was kept in place.

The comparison plots between the SIM, TTSP and the conventional stress relaxation are shown in Figure 3.63.

![Figure 3.63: Comparison plot of stress relaxation between methods; conventional test, TTSP and SIM.](image)

The SIM master curve in Figure 3.63 shows a divergence from the conventional plot starting from the region after 40,000 seconds. TTSP diverged starting from 100,000 seconds. The divergence may be due to the factor of high temperature and consequent chemical relaxation.

**SIM on HRC**

To evaluate the applicability of SIM with the buckling deformation, a hollow rubber cylinder (HRC) was used. The experimental setup was identical to the ones used in the TTSP and SIM (Figure 3.41), with the temperature steps and duration of tests were similar to the SIM test on DBS. The reference test temperature, \( T_0 \) of 24°C, was used in this test. Subsequently, four more temperature steps (32°C, 43°C, 53°C, and 63°C) were increased stepwise, and each lasted for 80 minutes. In this experiment, the HRC was permanently deformed by 9mm of axial displacement. The sample used in this test was HRC-E1.
The applied 9mm axial displacement, which corresponded to 37% compression, had put HRC-E1 sample in the plateau region of its stress-strain curve (Figure 3.27). During the SIM test on the sample, the small change in length following the increase in temperature steps resulted in a very small addition in force which can be assumed as negligible. The raw SIM data for the HRC which corresponds to the temperature steps is shown in Figure 3.64. The difference between the raw and the thermally-corrected SIM data is shown in Figure 3.65. The final plot of SIM for HRC was constructed by using the corrected SIM data.

![Figure 3.64: SIM raw data for HRC at 37% compressive strain.](image)
Figure 3.65: Difference in plots between the raw and the thermally-corrected SIM data for HRC.

The determination of the virtual start time, $t'$ is identical to the method described for the SIM master curve construction for the DBS sample, and shown in Figure 3.66. The horizontal and vertical shift factor ($a_T$ and $\xi$ respectively), and the $t'$ values are tabulated in Table 3.8.

Figure 3.66: Determination of the virtual start time, $t'$ for the individual stress relaxation isothermal curves.
Table 3.8: The parameters for constructing the SIM master curve (HRC test piece).

<table>
<thead>
<tr>
<th>Temperature, $T$ (°C)</th>
<th>Horizontal shift factor, $a_T$</th>
<th>Vertical shift factor, $\xi$</th>
<th>Virtual start time, $t'$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24$^a$</td>
<td>1.000</td>
<td>1.000</td>
<td>$t'_2 = t'_1 = 0$</td>
</tr>
<tr>
<td>32</td>
<td>2.296</td>
<td>0.972</td>
<td>$t'_3 = 1800$</td>
</tr>
<tr>
<td>43</td>
<td>5.644</td>
<td>0.941</td>
<td>$t'_4 = 5500$</td>
</tr>
<tr>
<td>53</td>
<td>12.438</td>
<td>0.911</td>
<td>$t'_5 = 10500$</td>
</tr>
<tr>
<td>63</td>
<td>24.563</td>
<td>0.884</td>
<td>$t'_6 = 15000$</td>
</tr>
</tbody>
</table>

$^a$ Reference temperature

The temperature loading phases at the beginning of each isothermal curves were excluded to create a smooth SIM master curve. The comparison plots in Figure 3.67 shows the SIM master curve in blue, the TTSP in black and the test result in red.

In general, both curves have a good agreement in axial stress decay from the start of the test, until the SIM master curve started to diverge from the conventional test curve after 6,000 seconds. Meanwhile, the TTSP curve began to diverge after 100,000 seconds. The divergence of the SIM master curve occurs during the test temperature of 43°C (grey line in Figure 3.66) and continues until the test temperature of 63°C. The divergence shows that for the SIM test, the HRC sample is susceptible to chemical relaxation starting from 43°C of temperature, the
point where the chemical reactions started to be acting prominently. For TTSP, chemical relaxation affects the sample from 63°C.

3.5.5 Discussion

The accelerated stress relaxation tests using the TTSP and SIM show some degree of applicability and confidence although strictly at limited period. In the case of shear deformation, the stress decay by TTSP shows an agreement with the conventional test only up to 100,000s, while with SIM, the agreement is only up to 40,000s. After those periods, the curves diverge completely.

In the case of buckling deformation, the TTSP method shows an agreement with the test from the start until around 100,000s. While using the SIM the agreement is only limited for up to 6,000s, after which both curves diverge completely.

Following the results shown in this section, it is clear that TTSP method is more applicable to predict the stress relaxation of unfilled rubber in shear or buckling deformations. In contrast, SIM method shows a low degree of confidence in doing so. This could be due to the chemical relaxation factor at high temperature which is more prevalent in SIM than it is in TTSP.

The continuous use of a single sample throughout the SIM test could impose the sample to a series of temperature increments along the period of the test. As oppose to this, the TTSP test only require each sample to be subjected to a single temperature assignment prior and during the test, which signifies a much less thermal loadings.

In overall, TTSP and SIM methods may appeal as a faster test method as compared to the conventional stress relaxation test. However, the additional chemical relaxation may occur at higher temperature which can limit the validity of the prediction. Thus, it is a factor which need to be taken into consideration if such a method is to be used to predict the long-term stress relaxation or creep behaviour of rubber.

3.6 Summary

The outcomes from this chapter explained how an unfilled rubber in certain geometries (DBS and HRC) and deformations (simple shear and axial compression of cylinder) behave under permanent stress or strain. The study also tested the specimens’ behaviour under the
changes in strain and temperature while they maintained the permanent strains throughout the tests. The tests involved in this part of the work, especially the modelling of HRC with arbitrary strains and temperatures, may help to understand how such components with that geometry may provide a solution to the problem associated with the IAB wall. During its service, the IAB wall could be subjected to changes in the strain as the result of the expansion and contraction of the bridge deck, and at the same time, an HRC installed in the IAB wall system will also be subjected to changes in environmental temperature.

The understanding of rubber properties intended for the application in the strain-induced problem found in IABs will assist rubber technologists and geotechnical engineers to approach the problem. A well-designed rubber component, supported by the research data, will help to solve the problem associated with the IABs, and engineers can anticipate the issue that may arise from the use of such components. The information on rubber properties could bridge the knowledge gap between rubber and geotechnical engineering, therefore, ensuring a rubber component that can be designed and serve as intended in the structure’s service life. This justifies the need for research in rubber characterisation in the framework of the geotechnical problem.

It is also worth noting that the work presented in this Chapter 3 revolves around the characterisation of the unfilled rubber. If any type of filled rubber is used in this study, the approach to the characterisation of rubber will be different, where the filler effect in stress relaxation will need to be considered. This part of the work can be suggested for further researches. Filled rubber is expected to have higher stress relaxation, so low levels of filler are likely to be preferred for the actual full-size DCU. However, the disadvantage is that the devices have to be bigger because the unfilled rubber has a lower modulus. The inclusion of filler in the rubber material may change the way to approach Chapter 4. Therefore, it is imperative to note that the limitation of the work described here is strictly for unfilled rubber.
CHAPTER 4  BEHAVIOUR OF GRANULAR BACKFILL UNDER CYCLIC THERMAL LOADINGS

This chapter discusses the use of the rubber-based load-bearing unit as the Displacement Compensation Unit (DCU), and its role in addressing the densification issue associated with the earth retaining structures such as integral abutment bridge (IAB) and supported embedded walls. The unit is known as Hollow Rubber Cylinder (HRC). The design approach, the possible way of deployment at IAB and the experimental results are discussed here.

4.1 The scope of the investigation

The main issues affecting the IAB’s performance are the escalating stress build up in the backfill and consequently the structure, together with the irreversible settlements near the abutment surface (England, Bush and Tsang, 2000; Kim and Laman, 2010; Miletic et al., 2016). A previous study, using a 1/12 scaled down abutment wall, pinned at the bottom, has shown that by using stacked conical disc springs (CDS) the build up of stresses can be controlled or reduced to match the horizontal design stresses, hence reducing the settlement to values much lower than the values measured in the control test (England et al., 2007).

The evaluation of a rubber-based DCU will be described by analysing two most relevant characteristics: the stress behind the wall (lateral earth pressure) and the backfill settlement near the abutment. A performance comparison with the Conical Disc Spring (CDS), commonly known as the Belleville washer, will also be made. This study considers the use of a rubber-based DCU due to the non-corrosive characteristics of rubber. It is anticipated that with steel based CDS, used to accommodate thermal expansion, corrosion would be a significant issue, in that sense, rubber DCUs would be much more attractive. This is also supported by rubber bearings' long-term performance in services, as highlighted in Section 2.6.3.

4.2 Methodology

The details of the scaled IAB wall apparatus used in this study will be described here. The wall apparatus is located in the Soil Laboratory at UCL. The HRC used in this study must satisfy the scaled IAB requirements in terms of displacement changes, and the principal operating force it needs to retain. The design approach for the HRC used in the tests will be discussed here.
The experimental works were conducted on a scaled IAB wall based on a real structure with a 60m span, known in BA 42/96 design manual as the guideline limit for an IAB designed in the UK. The abutment wall height is 7m, as described in Section 4.2.7, following the guideline in BS EN 1997-1:2004.

A longer bridge (120m in length) is also considered to investigate behavioural trends and make a sensible comparison with the main bridge (60m in length). This is translated as a higher rotational displacement of the IAB wall. The effect of initial position corresponding to the season (winter or summer) where the IAB starts operating was also investigated. The magnitude of the rotational wall displacements used in this study corresponds to the calculations and justifications discussed in Section 4.2.6.

It is also worth noting that similar studies were conducted previously by Movahedifar and Bolouri-Bazaz, (2014) and Zadehmohamad and Bolouri Bazaz, (2017), focusing on the stress buildup, the backfill displacement, and the effect of using different types of geocell as the backfill reinforcement for IAB. The studies were conducted in laboratory settings with downscale walls to model the IAB’s movements.

### 4.2.1 IAB wall experiment and its setup

This section describes the scaled IAB model and its instrumentation. It also describes the factors considered to determine the wall displacement, which simulates the deck seasonal thermal expansion and contraction. The test procedure will also be discussed.

Leighton Buzzard sand was deposited in the tank behind the IAB wall apparatus by the pluviation method to create a consistent sand layers with similar fabric and density. The pluviation was done by funnelling the sand down from a hopper, 1 meter above the sand level (Figure 4.1). Once a layer of around 2.5 inches (63.5mm) is deposited, the hopper’s height is adjusted upwards to maintain a fairly constant pluviation height, resulting with an approximately similar applying energy. To ensure the density of the sand remains constant throughout the tank, densities were monitored at every 80mm of height, during the sand filling. Two cylindrical aluminium tins with known dimensions were filled at certain intervals during the sand filling. The sand-filled tins were weighed on a digital scale, and the densities calculated.
Figure 4.1: The sand pluviation involves tracking the hopper back and forth along the rail, ensuring the sand is uniformly filled by layers. (Location: Soil Laboratory, UCL).

The aluminium tins were positioned in six layers (at every 80mm pluviated sand) referring to position-1 (yellow markers) and position-2 (in blue markers), as shown in Figure 4.2.
Figure 4.2: The positions of aluminium tins used to retain the pluviated sand.

An example of sand density monitoring, for IAB0.25 test, is shown in Table 4.1 below. The densities recorded for the other tests are compiled in Appendix B.

Table 4.1: The sand density record for the IAB0.25 experiment.

<table>
<thead>
<tr>
<th>Height (from the toe of the wall), mm</th>
<th>Density, $\gamma_1$ (kg/cm$^3$)</th>
<th>Density, $\gamma_2$ (kg/cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>480</td>
<td>1744.1</td>
<td>1760.4</td>
</tr>
<tr>
<td>400</td>
<td>1751.3</td>
<td>1748.2</td>
</tr>
<tr>
<td>320</td>
<td>1735.0</td>
<td>1756.4</td>
</tr>
<tr>
<td>240</td>
<td>1744.1</td>
<td>1725.8</td>
</tr>
<tr>
<td>160</td>
<td>1741.1</td>
<td>1738.0</td>
</tr>
<tr>
<td>80</td>
<td>1747.2</td>
<td>1714.6</td>
</tr>
<tr>
<td>Average density, $\bar{\gamma}$</td>
<td>1743.8</td>
<td>1740.6</td>
</tr>
<tr>
<td>Standard deviation, $SD$</td>
<td>5.05</td>
<td>16.34</td>
</tr>
</tbody>
</table>

An Agilent 34970A Data Acquisition Unit, controlled by an in-house software, developed on Agilent VEE Pro, was used for data acquisition. The wall pressure and displacement readings were taken at specific time intervals throughout the test, together with photos, taken using a phone camera with 12-megapixel resolution, used to monitor the sand settlement behind the wall. To easily identify the sand movement, thin layers of green coloured sand were laid on the glass at every 80mm of height. The settlement profiles were analysed and plotted by using the pixel-plotting method available in the WebPlotDigitizer software. Before that, each photo was corrected against skewness and distortion, resulting from the photo-taking angle,
using the photo-editing software GIMP Version 2.10. This ensured that all photos used in settlement profiling were consistent.

![Figure 4.3: The interface of the VEE Pro programme used for data logging and instrumentation monitoring. The top inset graph window shows the pressure, and the bottom shows the wall displacement (Voltage, V represents both). Data were recorded in time function.](image)

Before starting the experiment, a plastic ruler was used to level the surface of the sand. The wall movement was set to 85 seconds per cycle throughout all the tests. The data was captured at every 5 seconds for all experiments. A series of trial runs were conducted using 1, 2 and 5 seconds of data intervals, which suggested that 5 seconds is appropriate for the 1.37mm and 2.63mm peak-to-peak horizontal wall displacements and no peak or trough data points were missed out. The wall displacement cycle resembled a sinusoidal wave shape and the displacement condition will be detailed in Section 4.2.6.

### 4.2.2 Detailing of IAB wall test apparatus

A typical IAB structure shown in Figure 4.4 can be represented by a scaled IAB wall model apparatus described in Figure 4.5.
Figure 4.4: The typical frame type IAB adopted for the scaled model. Note that $d$ is the deck thermal displacement at one end, and $H$ is the abutment wall height.

Figure 4.5: The schematic diagram of the scaled IAB wall model apparatus with the instrumentations.

This study’s experimental apparatus is a model of an IAB wall scaled-down to 1/12 of the real size of an IAB of 60m in length and 7m abutment height. It is composed of a vertical aluminium wall with a total height of 600mm and 300mm in width. The wall is lined with adjustable PTFE seals at both sides, in contact with the 19mm glass panels fixed to a metal frame. The PTFE strips reduce the friction between the wall and the glass panel; hence, the force exerted on the wall is predominantly from the granular backfill soil. The wall is designed to rotate freely about its base hinge and is attached to an actuator via two beams, located at ‘E’ in Figure 4.5 above; one beam is attached to the wall while another is attached to the actuator. The rotational movement of the wall is governed by the movement of the actuator; a rigid connection between the two beams means that the movement of the actuator is the
same as the movement of the wall. Therefore, the system is ideal for simulating any movement associated with earth retaining structures such as IAB wall or any retaining wall.

The model frame is made of aluminium, and the length of the tank is 1130mm. Along the soil tank, there are two stiffener struts affixed in transverse, at the top of the tank, to enhance its rigidity and resist the bending moment expected from the wall’s cyclic loading. The tank is filled up with sand to a height of 540mm. The actuator consists of an electric motor coupled to a gearbox that is connected to an eccentric cam. By adjusting the eccentricity of the cam, the amplitude of the wall movement can be changed.

The scaled IAB wall model with the soil tank is shown in Figure 4.6.

![Figure 4.6: The elevation view of the model retaining wall apparatus, shown here filled with the sand.](image)

4.2.3 Measurement devices

The pressure the backfill applies on the wall is monitored by seven pressure sensors. These are named PrS-1 to PrS-7 and are equally spaced apart at 75mm, with the first unit (PrS-1) placed at the distance of 490mm from the bottom hinge of the wall (Figure 4.5). The pressure sensors are made of pure aluminium ceramic and are of the type P341, sourced from Roxspur Measurement and Control UK. The pressure capacity of each sensor is 50kPa. A view showing the pressure sensors’ location embedded on the wall is shown in Figure 4.7. It is worth noting that the sensors have a plastic shim covering them to avoid damage caused by friction with the sand.
The horizontal displacement of the wall is captured by a linear motion potentiometer installed on the wall at 585mm from the bottom hinge. It was assigned the name LVDT-1 in the wall apparatus diagram in Figure 4.5. Another linear potentiometer unit (assigned as LVDT-2), was placed at the rear of the actuator arm 490mm from the bottom hinge, to cross-check its movement with that of the wall. Both linear potentiometers have a resistance capacity of 1000 ohm (Ω). The calibration data for the linear potentiometers (LVDT-1 and LVDT-2) are shown in Appendix B. The linear potentiometers installed at both locations are shown in Figure 4.8.
Figure 4.8: The linear potentiometers are located at two locations: (a) LVDT-1: Attached to the rotating retaining wall, (b) LVDT-2: Attached to the moving actuator of the motor. (Location: Soil Laboratory, UCL).

The sand movement was monitored by taking pictures at set intervals. The side glass panel was marked with a 40x40mm square grids. Given that the grid-scale is known, the sand movement can be measured. The digital camera used was installed perpendicularly to the tank glass panel to get a consistent set of photos and avoid distortions and other errors. The sand movement was observed by distortion of the different coloured sand layers laid down during the filling process.

To visualise the sand movement, a portion of sand from the same source was set aside and coloured with green acrylic aerosol paint. Every sand grain was fully coated with the paint and left to dry before it was used as a marker. In this experiment, the green sand marker was laid at every two levels of square grids. It was laid and spread evenly at the side surface of the glass wall by using thin cardboard. The green sand marker layer varied around 3 to 5mm thick. The green sand laying was done in a slow and careful manner, as to not cause disruption to the sand layer beneath it, which may affect the density and experiment results.
In total, there are six layers of the green sand marker in each set of experiments. This is shown in Figure 4.6.

### 4.2.4 Calibration of the devices

The calibration of the pressure sensors was performed by applying a range of hydrostatic pressures on the sensors. The pressure was determined by measuring the column of water’s height from the centre of the sensor to the top where the atmospheric pressure was acting. The pressures were varied by raising or lowering the open end of a clear tube.

The hydrostatic water pressures were calculated based on the density of water, $\rho_w$, equal to 1000kgm$^{-3}$ and the gravitational acceleration, $a_g$, equal to 9.81ms$^{-2}$. According to the basic hydrostatic pressure equation, $P = \rho_w a_g h_w$, and for 1-meter of a column of water, the pressure is 9.81 kPa (Nmm$^{-2}$). The hydrostatic pressure of the water at several heights were plotted against the voltage feedback from the sensors. The pressure sensor calibration setup is shown in Figure 4.9 and Figure 4.10.

![Polypropylene tube filled with water](image)

Figure 4.9: The calibration of the pressure sensor by using the hydrostatic pressure at different heights. The water tube is attached to a square steel section.
The linear potentiometers were calibrated against a Mitutoyo calibration gauge, consisting of steel blocks of varied thicknesses, machined with high precision (+/- 0.001mm). The displacements from the potentiometers’ probes were plotted against the voltages. The results of the calibrations of all seven pressure sensors and two potentiometers are shown in Appendix B.

![Image](image1.png)

(a) ![Image](image2.png)

(b)

Figure 4.10: (a) The pressure sensor is sealed within the calibration rod head equipped with a valve, allowing for the water tube connection, (b) The calibration rod being held across the soil tank.

4.2.5 **Backfill material and its characterisation**

Leighton Buzzard coarse sand was used as the backfill material, classified as Class 6P backfill material according to the guideline DMRB Vol.2 Section 1 - BD 30/87 (1995) (Backfilled Retaining Walls and Bridge Abutments). The justification for this study to use Leighton Buzzard sand was for continuation and being consistent with the previous study by England *et al.*, (2007), which used the same backfill material. As this study stemmed from that work, it is important to see if the experimental outcomes would present similar observations as in the previous work.

The description of the works conducted to characterise the backfill material used in this research is explained here. The material, namely Leighton Buzzard, is a medium coarse sand commonly sourced in England.
The particle size distribution test on Leighton Buzzard sand for IAB wall model apparatus.

To understand Leighton Buzzard’s sand classification, the sieve analysis was conducted by using a set of sieves with variation of apertures. The test was conducted in accordance to BS ISO 17892-4: 2016 (Determination of particle size distribution). Three sets of sieve analyses, each with 1000g of sand, were done and the average was taken as the result. The tests were done on ELE Sieve Shaker located in UCL Soil Laboratory, and the results are shown below.

Table 4.2: Calculation for the particle size analysis.

<table>
<thead>
<tr>
<th>Sieve Size (mm)</th>
<th>Mass Retained (g)</th>
<th>Percent retained</th>
<th>Cumulative % retained</th>
<th>Percent % passing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>Average</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>1.18</td>
<td>2</td>
<td>2.5</td>
<td>2</td>
<td>2.2</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2.5</td>
<td>1.5</td>
<td>2.0</td>
</tr>
<tr>
<td>600μ</td>
<td>940</td>
<td>950.5</td>
<td>931</td>
<td>940.5</td>
</tr>
<tr>
<td>425μ</td>
<td>20</td>
<td>16</td>
<td>24</td>
<td>20.0</td>
</tr>
<tr>
<td>300μ</td>
<td>9</td>
<td>7</td>
<td>10</td>
<td>8.7</td>
</tr>
<tr>
<td>150μ</td>
<td>20</td>
<td>17.5</td>
<td>24</td>
<td>20.5</td>
</tr>
<tr>
<td>Pan</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>4.0</td>
</tr>
<tr>
<td>TOTAL</td>
<td>997.5</td>
<td>999.5</td>
<td>998</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.11: Particle size distribution of Leighton Buzzard sand.
The wire cloth sieve set and the shaker machine used in this analysis are shown in Figure 4.12 and Figure 4.13.

**Figure 4.12: Sieve set (adhering to ISO 3310-1: 2000)**

**Figure 4.13: ELE Sieve Shaker used in the particle distribution analysis.**

The triaxial test on Leighton Buzzard sand

The sand friction angle, $\phi$, was determined by conducting a series of triaxial test, in accordance to BS ISO 17892-9: 2018. A triaxial test is also used to determine the elasticity modulus of a backfill material. Alternatively, the friction angle can be determined by a rather simpler test – the direct shear test. However, the triaxial test was chosen due to its higher accuracy in determining the internal friction angle, $\phi$ of the material. The triaxial tests were conducted on Wykeham Farrance Triaxial Testing System. The test data acquisition was done by using Agilent 34970A datalogger, with Agilent VEE Pro software developed previously by Dr. Pedro Ferreira.

**Triaxial sample preparation:**

- A set of brass mould was clamped around the bottom pedestal with two sets of jubilee clips with a latex membrane having a 38mm diameter. On top of the bottom pedestal, a
porous disc was placed to as a barrier for a possible sand entering the back pressure irrigation system. Air suction (for about 30kPa) was constantly applied on the latex membrane to hold it in place. After that, the sand was slowly poured by using a small spoon. The air suction was maintained until the completion of the sample setup.

- The sand pouring was done by batches. At each batch, four (4) spoonful of sands were poured into the mould. A little amount of water was squirted to the inner side wall of the membrane and onto the sand, until the water level was slightly over the sand level. Then the deposited sand was lightly tampered for 1 minute, using a plastic rod. Care was taken during the tampering so that accidental piercing of the latex membrane can be avoided. The steps were repeated until the sand deposit reached the appropriate height close to the mould opening.

- A top cap was placed on top of the sand and an O-ring was secured around the side of the cap’s groove.

- A digital calliper was used to measure each sample’s height in three readings. Diameter of the sample was taken at three positions along the sample’s height. All height and diameter readings were averaged.

- Two units of local instrumentation (hall effect sensor) were affixed on the side of the cylindrical body of the sample, using a cyanoacrylate adhesive. The hall effect sensors were used to ensure a more accurate strain readings can be recorded. This was due to the sand sample which normally failed at a relatively lower shear strain than, say a clay triaxial sample. The diagram and photo showing the sample and hall effect sensors are shown in Figure 4.14.

- After that, the cell was then put in place carefully. The sample’s top cap was ensured to not touching the cell’s piston head. By now the cell was connected to the load cell on the frame’s top. Six threaded rods were secured around the cell, which sealed the cell against water leakage. The setup was then ready for test, and the whole triaxial cell illustration is shown in Figure 4.15.

- The load cell attached to the Wykeham Farrance triaxial test system, together with the external LVDT were calibrated and the records are shown in Appendix D.
Figure 4.14: The detail of the local instrumentation (hall effect sensor) used in the triaxial test; (left) schematic diagram, (right) actual sample sample fixed on the test pedestal, with two hall effect sensors affixed.

Figure 4.15: The overall triaxial cell setup with Leighton Buzzard sand specimen.
**Experiment steps:**

- First, the sample was saturated by applying a back pressure at 20kPa. It gradually went up to 250kPa in about two hours. The increase in back pressure was controlled by the Agilent VEE Pro programme, through an automatic pressure controller and a air/water interface cylinder, with both connected to a an electronic control box and the computer with Agilent 34970A Data Acquisition Unit. Alongside this, the cell pressure was applied to 270kPa.

- After that, the sample was consolidated by increasing the cell pressure to 300kPa. This meant that the sample was having a difference of 50kPa between the back and cell pressures.

- Once the consolidation stabilised, the sample shearing was started by activating the motor which pushed the test pedestal upwards at a slow rate of 0.056 mm/s. Shearing means the sample was compressed axially, where the force and external displacement were recorded by the load cell and the hall effect sensor, respectively.

- The consolidation level was increased, with the cell pressure being varied and increased on more triaxial samples, for example at 350kPa and 400kPa. The variation was needed in order to see the correlation between the major principal stress, $\sigma_1$ (axial) and the minor principal stress, $\sigma_3$ (radial) at failure, which will be plotted in the form of Mohr-Coulomb failure envelopes.

The overall setup of the triaxial test in Soil Mechanics Laboratory at UCL is shown in Figure 4.16.
Figure 4.16: Overall setup for the triaxial test done on Leighton Buzzard sand.

The results of the triaxial test are shown in the Mohr-Coulomb plot in Figure 4.17.

Figure 4.17: The Mohr-Coulomb failure envelope shown by three triaxial tests; T1, T2 and T3 with effective cell pressure of 50kPa, 100kPa, and 150kPa respectively.
Due to the use of cohesionless granular material in Leighton Buzzard sand, the cohesion, $c$ is zero. The internal friction angle is $37^\circ$.

The Young’s modulus of Leighton Buzzard sand can also be determined by using the equation in below:

$$E_{LB} = \frac{\Delta(\sigma_1 - \sigma_3)}{\Delta\varepsilon_f}$$

Where $E_{LB}$ is the Young’s modulus of Leighton Buzzard sand, and $\varepsilon_f$ is the strain at failure. From the stress values plotted in Figure 4.17, and the strain at failure found to be 0.004 following the test, the Young’s modulus of Leighton Buzzard sand used in this study is $12.1\text{MPa}$.

This study assumes the active and passive lateral earth pressure approach by Rankine (1857), which assumes the soil is cohesionless and the glass wall is frictionless. Furthermore, the vertical and horizontal planes are planes on which the minor and major principal stresses act. The frictionless wall assumption is supported by studies which reported that the values of the wall-sand friction from Direct Shear test were very low and close to zero (Movahedifar and Bazaz, 2013; Movahedifar and Bolouri-Bazaz, 2014). However, in the real application, the wall friction factor cannot be totally removed as there will still be an amount of friction from the moving concrete wall and sand interface.

The particle size distribution of Leighton Buzzard sand is described in Figure 4.11 and Table 4.2, and the sand properties are shown in Table 4.3.

<table>
<thead>
<tr>
<th>Table 4.3: The properties of Leighton Buzzard coarse sand.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry density at initial (kN/m$^3$), $\gamma_s$</td>
</tr>
<tr>
<td>Elasticity Modulus (N/mm$^2$), $E$</td>
</tr>
<tr>
<td>Internal friction angle (°), $\phi$</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu_s$</td>
</tr>
<tr>
<td>Void ratio, $e_0$</td>
</tr>
</tbody>
</table>

* After England, Bush and Tsang (2000), which used the same material.

### 4.2.6 Thermal displacement

The displacement verified in the abutment is caused by the deck’s expansion and contraction, caused primarily by the environmental temperature and solar heating. In this experiment,
the change in displacements was simulated by the model IAB wall’s rotational displacements to model the seasonal displacement cycles.

Some theoretical and practical approaches were published to identify the representative bridge temperature (Emerson 1973, 1976 & 1977). A suitable parameter has been defined and termed as Effective Bridge Temperature (EBT), as discussed in Section 2.1.4. The EBT magnitudes correlate to the shade temperature and applicable for the locations in the UK.

For this study, it was assumed that daily temperature variations are smaller than seasonal ones, imposing less significant effects. This assumption is supported by previous research (England, Bush and Tsang, 2000), which suggested that the difference between the seasonal temperature and combinations of seasonal plus daily temperatures are insignificant. Therefore, only a single value of temperature-induced displacement was chosen for the simulations. This study will apply a single IAB deck displacement value, based on the average temperature between the highest in summer and the lowest in winter.

Emerson (1976) outlined a method of estimating the EBT, which depends on the environmental temperature and the type of bridge materials. For a concrete IAB, the EBT correlates to the shade temperature over the previous 48 hours. The UK Meteorological Office published the national temperature map for the minimum and maximum shade temperature at the mean sea level, likely to occur in a 50-year return period (Hopkins and Whyte, 1975). The shade temperature values were later being chosen, with no altitude and wind speed factor considered. In this case, the specific region chosen is London. These values were converted to EBT values using Emerson’s method. The maximum and minimum EBT values for different bridge deck materials (concrete, composite and steel) were summarised in Table 2 and Table 4 of the same document (Emerson, 1976b).

The IAB material considered in this study is reinforced concrete, hence the bridge deck movement was worked out from the knowledge of the EBT values, bridge deck length and thermal expansion coefficient. The magnitude of the cyclic displacement imposed by the wall to the granular backfill was then determined. All temperature values and conversions for the London region are shown in Table 4.4. The seasonal deck displacement, \( d_{\text{long}} \), can be determined from Equation (2.1)(2.1).

For example, the maximum and minimum temperatures are 36°C and -6°C respectively for the London area. The thermal expansion coefficient of concrete, \( \alpha_{\text{conc}} \), was
taken as $12 \times 10^{-6}/^\circ C$ and deck length, $L$ as 60m. Assuming the total deck displacement acts on two abutments (one at each end); therefore, the deck displacement applied to each abutment, $d$, is 15.12mm.

For comparison, the EBT was also determined using the guideline shown in Figure 6.1 of BS EN1991-1-5:2003. The minimum and maximum shade temperatures and the Uniform Bridge Temperatures (UBT, Type 3), are summarised in Table 4.4. The shade temperatures and bridge parameters used were identical to the preceding method.

The same method was used to estimate the deck displacement of a 60m IAB in two other regions, namely Birmingham, representing the British Midlands, and Scottish Highlands, representing the north of the UK. The calculation outcomes are shown in Table 4.4.

Table 4.4: The correlations between the shade temperatures to the EBT and UBT ($^\circ C$) for three main regions of Great Britain.

<table>
<thead>
<tr>
<th>Temperature ($^\circ C$)</th>
<th>a) London</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>b) Birmingham (Midlands)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Shade</td>
<td>-10</td>
<td>-6</td>
<td></td>
<td></td>
<td></td>
<td>-18</td>
<td>-11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum Shade</td>
<td>36</td>
<td>36</td>
<td></td>
<td></td>
<td></td>
<td>34</td>
<td>32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deck displacement, $d$ (mm)</td>
<td>15.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>15.48</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cyclic rotational displacement amplitude, $d/H$ (%)</td>
<td>0.22</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.22</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The seasonal deck displacements resulting from the two approaches; Emerson (1976) for EBT and BS EN1991-1-5 for UBT have different values although close to each other. The difference in their displacement values are because Emerson method is based on a curved correlation between the shade and effective bridge temperature (EBT) (see Figure 2.20). In contrast, BS EN1991-1-5 is based on a linear correlation between the shade and uniform bridge temperature (UBT) (see Figure 2.21). However, Emerson (1976a) commented that it was equally possible to fit a curved envelope to the temperature correlation and the linear envelope because the correlations were only empirical and lacked data at the extremes. Thus, there was no way of deciding which was more accurate. Comparing both curved and linear correlations in Figure 2.20 and 2.21 shows that the linear correlation gives the lowest (and highest) possible effective temperatures, suggesting that BS EN1991-1-5 reports a more conservative UBT value.

Due to the displacement imposed by the deck, the IAB wall rotates about its pinned base. This is termed cyclic rotational displacement, with an amplitude described as \( d/H \) (in percentage); \( d \) is the deck displacement at one end and \( H \) is the abutment wall height. For this study, the highest displacement magnitude was assigned to the IAB with 60m length and 7m wall height, which means the rotational displacement amplitude is \( d/H = 0.25\% \).

As a comparison, Table 4.5 presents the different rotational displacement amplitudes of the wall to its height, \( d/H \) selected by different researchers for their physical wall models.
Table 4.5: Magnitudes of $d/H$ (%) selected by researchers for physical modelling.

<table>
<thead>
<tr>
<th>References</th>
<th>$d/H$ (%) values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ng, Springman and Norrish (1998)</td>
<td>0.1, 0.2, 0.5, 1</td>
</tr>
<tr>
<td>England, Bush and Tsang (2000)</td>
<td>0.13, 0.25, 0.35, 0.9</td>
</tr>
<tr>
<td>Cosgrove and Lehane (2003)</td>
<td>0.2, 0.6</td>
</tr>
<tr>
<td>Tatsuoka et al. (2008)</td>
<td>0.2, 0.6</td>
</tr>
<tr>
<td>Lehane (2011)</td>
<td>0.05, 0.1, 0.2, 0.6</td>
</tr>
<tr>
<td>Movahedifar and Bazaz (2013)</td>
<td>0.1, 0.2, 0.6, 1.0</td>
</tr>
<tr>
<td>This research</td>
<td>0.25, 0.5</td>
</tr>
</tbody>
</table>

When the wall is on the active, it will move to the right, away from the soil. And when the wall is on the passive it will move to the left, against the backfill soil. The active and passive are symbolised by the motion represented by the curved arrows, as shown in Figure 4.18. For the simulation purpose, it is appropriate to assume the difference between the passive and active wall position as the linear deck displacement as described above and in Table 4.4.

![Figure 4.18: Active and passive movements of a wall.](image)

4.2.7 Displacement and dimensions of the IAB wall model

The rotational displacement of the actual IAB was translated to the laboratory model with a downscale factor. The IAB wall model in the UCL Soil Laboratory was readily designed to be of specific dimensions – wall height of 585mm with the sand surface level 540mm above the toe of the wall. This made the downscale factor to be $\frac{1}{12}$th of the actual 7m wall height.

The IAB dimensions; the deck span and the height, can be checked against BS EN 1997-1:2004 (Geotechnical design, Part 1 - General rules). Following the guideline, a bridge’s earth pressure to abutment wall movement must be considered for active and passive
situations. In this study, both active and passive movement at one bridge end is assumed to be identical; hence, an active or passive displacement is represented by;

\[ d_{0.5} = \frac{d}{2} \]  

(4.1)

with \( d \) being the seasonal deck displacement, as Table 4.4. BS EN 1997-1:2004 (Eurocode 7 – Geotechnical design, Part 1 – General rules) asserts that the value of \( d_{0.5}/H \) for the dense, non-cohesive soil must be within 0.1 – 0.2% for a wall moving about its pinned base.

Using an IAB of 60m span, the seasonal displacement to mobilise an active or passive pressure is, therefore, \( d_{0.5} = 8.64 \text{mm} \) (from \( d = 17.28 \text{mm} \)). Using the \( d_{0.5}/H = 0.1 – 0.2\% \) limit, the IAB wall height can be calculated within 4.32 – 8.64m. Therefore, the 7m wall height used in this study is appropriate as it is within limits. This also confirms that the 7m wall height assumptions in previous studies by England, Bush and Tsang, (2000) and England et al., (2007), were appropriate. The integral bridge with abutment height of 7m is also typical for motorways in Europe (Mitoulis, Argyroudis and Pitilakis, 2014; Caristo, Mitoulis and Barnes, 2018).

4.2.8 Experimental procedure

The IAB wall experiment was divided into three main parts - all were carried out to investigate the influence of seasonal temperature changes to the IAB retaining wall’s strain behaviour. The first part covered the observation of the retaining wall’s backfill pressure and settlement against the seasonal thermal displacement of \( d/H=0.25\% \) and \( d/H=0.50\% \). It consisted of the ‘sand-only’ test condition, and the wall-deck connection of the IABs was made integral. These are referred to as the control tests. The \( d/H=0.50\% \) test signified the test on a longer bridge deck with a larger seasonal displacement. The rotational displacement amplitude of \( d/H=0.50\% \) is equivalent to an IAB deck length of 120m.

The second part consisted of the test on the IAB model with the seasonal displacement of \( d/H=0.25\% \), with the inclusion of Hollow Rubber Cylinder (HRC) and Conical Disc Spring (CDS) systems – both referred as the DCU systems. The reactions of the soil mechanical behaviour from the cyclic strain were observed and compared. Afterwards, another set of a test using the HRC was conducted with the seasonal displacement of \( d/H=0.50\% \). Wall height, \( H \) was assumed to be identical for both displacement levels.
In the first and second parts of the test, the starting position was when the wall was vertical. The third part consisted of observing the backfill behaviour, depending on the season when the IAB is constructed. These initial conditions correspond to; 1) the ‘winter’ where the wall started at the most ‘active’ position, and 2) the ‘summer’ where the wall started at the most ‘passive’ position. These two conditions satisfy the objective of looking at the effects of the wall’s initial position as they represent the two extreme starting positions. This test included the ‘sand only’ condition, plus the inclusion of both types of DCUs of interest (CDS and HRC).

The experimental programme is summarised in Table 4.6.

Table 4.6: The details of the model IAB retaining wall tests

<table>
<thead>
<tr>
<th>Part</th>
<th>Test name</th>
<th>Description</th>
<th>$d/H$ (%)</th>
<th>Seasonal cycles completed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>IAB0.25</td>
<td>IAB (integral deck-abutment)</td>
<td>0.25</td>
<td>2000</td>
</tr>
<tr>
<td></td>
<td>IAB0.50</td>
<td>IAB (integral deck-abutment)</td>
<td>0.50</td>
<td>500</td>
</tr>
<tr>
<td>2</td>
<td>CDS0.25</td>
<td>IAB with CDS</td>
<td>0.25</td>
<td>5800</td>
</tr>
<tr>
<td></td>
<td>HRC0.25</td>
<td>IAB with HRC</td>
<td>0.25</td>
<td>5800</td>
</tr>
<tr>
<td></td>
<td>HRC0.50</td>
<td>IAB with HRC</td>
<td>0.50</td>
<td>740</td>
</tr>
<tr>
<td>3</td>
<td>IAB_winter</td>
<td>Conventional at winter</td>
<td>0.25</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>IAB_summer</td>
<td>Conventional at summer</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CDS_winter</td>
<td>With CDS in winter</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CDS_summer</td>
<td>With CDS in summer</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td></td>
<td>HRC_winter</td>
<td>With HRC in winter</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td></td>
<td>HRC_summer</td>
<td>With HRC in summer</td>
<td>120</td>
<td></td>
</tr>
</tbody>
</table>

* $d/H$ = Seasonal rotational displacement (in percentage), representing one bridge end.

$d/H=0.25\%$ represents the case for the IAB with 60m deck length, while $d/H=0.50\%$ is for 120m deck length. For $d/H=0.25\%$ case, the peak-to-peak horizontal displacement at the wall height of 540mm is 1.37mm, and for $d/H=0.50\%$ case, the wall displacement is 2.63mm at the same height level.

4.2.9 Designing the HRC

An HRC design is governed by its capacity to maintain the scaled retaining wall’s operating load while accommodating the changing displacement. The previous study showed that for the DCU to be effective in preventing the backfill stress buildup and soil settlement, the
compressive load range used for the scaled IAB wall shall be between 400N to 500N (England et al., 2007) (see Figure 2.43(a)). Based on the information, the HRC system must withstand the same load range (400N to 500N). In theory, the total principal load can be achieved by adjusting the number of the HRC units assembled in parallel, provided the individual compressive load capacity of a single HRC unit is known.

The HRC design aims to maintain the backfill stress state fairly constant whilst the bridge deck expands and contracts according to the temperature change. By keeping the stress state close to its initial state, there will be no inversion of the main principal stress direction and the stress ratio ($K^*$) between horizontal and vertical directions ($\sigma_h/\sigma_v$), will be fairly constant. This will prevent the granular fill movement and consequently reduce settlement, helping to maintain a stable structure.

An HRC can achieve the desired load level through the manipulation of its dimensions. Leaver and Lindley (1976) discussed the effect of having a different outer and inner diameter of hollow rubber to its stress and strain relationship. The material used was unfilled rubber with a hardness of 46 IRHD (IRHD – International Rubber Hardness Degree).

To give a preliminary design estimation, reference was made to Figure 2.67. A plot with approximately constant stress under progressing strain was chosen as the benchmark, in this case, cylinders within 23mm to 27mm inner diameter. The selected 45% compressive strain was chosen as the target pre-set form of the cylinder. This is due to its small force change when additional +/- 5% strain is applied. Therefore, the working stress for a 46 IRHD HRC of OD 49.5mm is around 0.60MPa to 0.85MPa. Alternatively, the cylinder’s pre-set form can also be chosen at the onset stress (at around 37% in compressive strain), where any additional +/- 5% strain will return a small change in force.

To make the HRC design manageable and suitable for the existing area of placement in the IAB wall model, the outer diameter (OD) and inner diameter (ID) were factored down to half. This made the OD = 25mm and ID in the range of 13.5mm to 11.5mm. This estimation is valid for the $OD/H_x = 1$ and therefore made the HRC’s height, $H_x$ to be 25mm.

The HRC force, $P_{HRC}$ at 45% strain can be estimated using an empirical method by referring to the stress values in Figure 2.67 as the reference. The estimation is based on the EDS19 as the current base material compared to the material used in Leaver and Lindley (1976) – which was an unfilled rubber with 46 IRHD hardness degree. The material was traced back to Natural Rubber Engineering Data Sheet (MRPRA, 1979) which then identified as
EDS31, with the shear modulus of 0.54MPa. Therefore, the HRC force at 45% strain may be estimated from Equation (4.2).

\[ P_{HRC} = \sigma_b \cdot A \frac{G_{EDS19}}{G_{EDS31}} \]  

(4.2)

With \( \sigma_b \) is the compressive stress, \( A \) is the plan cross-section area of the HRC, \( G_{EDS19} \) is the shear modulus of EDS19 (0.41MPa), and \( G_{EDS31} \) is the shear modulus of EDS31 (0.54MPa). For the HRC with \( ID = 13.5\text{mm} \) and \( 11.5\text{mm} \), the forces were estimated to be \( 158\text{N} \) and \( 250\text{N} \) respectively. This range of HRC forces enables the deployment of two units in parallel to provide sufficient load on the model wall. However, the outcomes from the actual stress-strain test on the HRC with varying \( IDs \) will also be considered and compared with these force estimations.

Three variations of \( IDs \) were chosen to evaluate the HRC – these were 12mm, 13mm and 15mm. The HRC sample preparation is discussed in Section 3.1.3. The test method is explained in Section 4.2.10, including the CDS. The stress-strain evaluations of the CDS and HRC are discussed in Section 4.2.11 and 4.2.12, respectively.

In comparison to the HRC design iterations made by varying the \( IDs \) experimentally, a calculation on the stress at buckling, \( \sigma_{cr} \) for these HRC were attempted and compared in Section 4.2.12.

### 4.2.10 Stress-strain tests on the DCUs

**Experimental setup**

A quasi-static compression test was performed on the CDS and HRC samples. The tests were conducted using a screw-driven universal test machine Instron 4301, located at Engineering Laboratory, TARRC. A load cell with 1kN force capacity was used. A PicoLog ADC-20 data logger recorded the test information. The tests were conducted in displacement control with a 15mm/min loading rate and a room temperature of 23±1°C. The overall test setup is shown in Figure 4.19.
4.2.11 Results: Stress-strain behaviour of the CDS

Conical disc spring (CDS) is also known as the Belleville washer. A single unit of CDS is shown in Figure 4.20.
The CDS were sourced from TFC Europe Ltd and made of chrome vanadium alloy steel (CrV-4), adhering to the DIN 17222 standard (Cold Rolled Steel Strips for Springs- Technical Condition of Delivery). Due to its conical shape, the CDS configuration can be varied to yield different force-displacement plots. Two units of CDS springs can be configured in “series” or “parallel”, as shown in Figure 4.21.

(a) Single – specific force and displacement  
(b) Series – double displacement, no force increase  
(c) Parallel – double force, no displacement increase

Figure 4.21: The basic configurations of the conical disc spring (CDS).

To meet the designated stiffness demand, the CDS design iteration was attempted by manipulating the CDS stiffness by using the knowledge of the force-displacement behaviour of a single unit of the CDS tested beforehand.
To observe the CDS stiffness at the different configuration, two CDS units with ten discs in series were tested in the experimental setup shown in Figure 4.19 earlier. They are labelled as “10-disc – series (Test 1)” and “10disc – series (Test 2)”. The test aims to investigate whether the stack of 10 discs’ stiffness in series is comparable to the prediction. The CDS samples under axial compression are shown in Figure 4.22.

![Figure 4.22: (a) A stack of 10 disc spring in series, placed at the test machine actuator (Instron 4301), (b) The CDS under 57% compression (6.5mm).](image)

The test results are shown in Figure 4.23 below. Several calculated force-displacement curves representing different CDSs' behaviour, either as a single unit, or serial arrangement with the different number of disc are compared.
Figure 4.23: The CDS force-displacement curves of different configurations. All curves were calculated from the 1-disc curve except “Test 1” and “Test 2” (in red and black markers respectively).

From the test, it is shown that the two stacks of CDSs, Test-1 and Test-2, have the stiffness plots close to the iteration plot of a 10-disc in series, although the 14-discs and 16-discs CDS are longer than the 10-disc CDS, these would give slightly lower force fluctuations for a given deck displacement. However, larger and heavier spring units, together with the larger compression displacement, will be needed to achieve the 200N to 250N force range, when compared to the 10-disc CDS. Therefore, the 10-disc CDS unit stacked in series was chosen for inclusion in the wall model experiment. It is worth noting that the identical configuration of a 10-disc CDS in series was also used as a DCU in the previous moving wall study by England et al., (2007).

However, the CDS system’s working range must be limited before the upturn of the stiffness curve is reached. That is because from the start of the upturn, and onwards, the CDS is fully compressed and becomes a solid body which is far too rigid to act as a spring system.

The known unstrained height of the CDS with a stack of 10 units in series is 11.4mm. According to the force-displacement results, to achieve the 500N of working load (as the sums of two units of CDS with 250N of force each), the CDS will be compressed axially for 3.8mm. This will make the CDS’ pre-compressed height 7.6mm when installed at the IAB wall beam. A set of two aluminium spacers with 7.6mm thickness each were manufactured to lock the CDS in place for the test setup (Section 4.2.14).
4.2.12 Results: Stress-strain behaviour of the HRC

The stiffness of the HRC can be controlled by manipulating its dimensions. In the case of this study, the manipulated parameter are the variations of the inner diameter (ID) with no change in outer diameter (OD) and height (H). Based on the design approach discussed in Section 4.2.6, the force-displacement behaviour of the HRC with a variation of IDs is discussed below and shown in Figure 4.25.

Buckling behaviour of HRC

A series of quasi-static axial deformation tests have been conducted on three hollow rubber cylinders (IDs - 12mm, 13mm and 15mm). The physical observations show a visual phenomenon of uniform instability. The fixation at the top and bottom plates constrain the HRC at the ends, so its body bulges radially outwards at the central plane.

During the quasi-static test on the HRCs, a cyanoacrylate bonding agent, suitable for rubber (Loctite 480), was used to fuse the HRC’s to steel end pieces. Several dots of the bonding agents were applied to each of the rubber ends’ surface enough to provide additional friction, consequently avoiding the sideways shearing of the sample expected to happen at the higher compressive stress.

At low strain (10%), the HRC is barrel-shaped, and the curvature is purely convex. As it progresses to the higher strain (20%), a small degree of contra-flexure is observed. When the strain is within 30% to 50%, the end parts of the rubber are creasing due to the bulging of the middle part of the body (Figure 4.24). Beyond this point is where the internal cusp of the HRC come into contact, where a sharp increase in stress can be observed. Further deformation will make the central bulge to come into contact with the test plates, and the whole body behaves as a solid where a much-increased stiffness is expected.
Figure 4.24: Sequence of deformed HRC shapes indicated as (a) the stiff linear state, (b) the soft region, also referred as the ‘plateau’ (c) the stiff jounce state (Refer Figure 4.26)

The stress-strain curves of HRC with three different IDs are shown in Figure 4.25.

Figure 4.25: The stress-strain curve of HRCs with a variation of IDs. Solid curves are for the first cycle, and dashed curves are for the third cycle.

The cylinders' stress-strain curves show higher stiffness in the first cycle, and after a few cycles, a steady state is reached (Figure 4.25 and Figure 4.26). The fall in stiffness when rubber is repeatedly strained is due to the stress-softening attributed to the Mullins effect. Mullins effect is a phenomenon where the rubber's modulus reduced after experiencing deformation (Harwood, Mullins and Payne, 1965; Mullins and Tobin, 1965). The stress-
softening can occur on any rubber vulcanisate, although it is more prevalent in the filler-loaded rubber. The phenomenon is related to the breaking and reforming of the crosslinks during deformation, local rubber network persisting after recovery, and breaking network chains. There may also be a contribution due to changes in rubber-filler interactions.

The HRC’s stiffness begins to stabilise from the third cycle, given an identical force-deflection plot from the third to the tenth cycles. This is shown by the example of the HRC with a 13mm ID, in force (N) and displacement (mm) quantities (see Figure 4.26).

![Diagram of force-displacement curve](image)

**Figure 4.26:** The 3rd and 10th cycle of the HRC (ID Ø13mm) show almost coinciding force-displacement trace.

The force-displacement curve of the HRC with Ø13mm ID shows that the sample has three distinct stiffness regions. Referring to the loading phase, shown “A” in Figure 4.26, initially the sample has a stiff linear behaviour until it reaches 28% strain (7mm). At the onset of the plateau curve, the force is maintained at approximately 260N, which occurs from 8mm to 13mm of axial compressive displacement. Along this plateau region, a slight difference of force (as much as 8N) can be observed between the loading and unloading phases. The force then increases rapidly after reaching 55% strain (13.5mm). The unloading phase (shown as “B”) defines the same three stiffness regions at approximately 10N below the loading phase, along the plateau region. The test was limited to 60% strain because its purpose was to identify the region where it is maintained approximately constant with changing displacement.
In the actual test, for the ID 13mm, the force at 45% (11mm) is 252N (or $\sigma = 0.72\text{MPa}$). While by estimation, for ID 13.5mm and 11.5mm, the forces were estimated to be 158N and 250N respectively. The discrepancy may be contributed by the variations in the modulus of the rubber used in the current test, and the one by Leaver and Lindley (1976) in Figure 2.59. However, the HRC with ID 13mm tested here is shown to have enough force for use as trial DCUs in the scaled wall model.

The critical stress at buckling, $\sigma_{cr}$ of an HRC subjected to compressive stress may be estimated based on Equation (2.55) from Bakirzis (1972). Using the fixed OD = 25mm with three variations of IDs, the $\sigma_{cr}$ are summarised in Table 4.7 below.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Stress at buckling, $\sigma_{cr}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$OD = 25\text{mm} ; ID = 12\text{mm}$</td>
<td>0.576</td>
</tr>
<tr>
<td>$OD = 25\text{mm} ; ID = 13\text{mm}$</td>
<td>0.518</td>
</tr>
<tr>
<td>$OD = 25\text{mm} ; ID = 15\text{mm}$</td>
<td>0.410</td>
</tr>
</tbody>
</table>

The Young’s modulus, $E_0$ for unfilled rubber as in EDS19 was assumed as an isotropic incompressible material which fetches $E_0 = 3G$ (with shear modulus, $G = 0.41\text{MPa}$) and Poisson’s ratio, $\nu = 0.499$ (Lindley, 1974; Gent, 1992). Based on the comparison between the calculated $\sigma_{cr}$ with the results from the stress-strain tests shown in Figure 4.25, the estimated $\sigma_{cr}$ values from the calculation are shown to be underestimated.

### 4.2.13 CDS and HRC force-displacement comparison

The comparison of the force-displacement curves at the third cycle for HRC and CDS under axial compression is shown in Figure 4.27.
Figure 4.27: The force-displacement behaviour of CDS (10-unit stack) and HRC (ID Ø13mm).

The force-deflection characteristic of the HRC showed a nonlinearity under the imposition of a constant displacement rate. The nonlinearity of the HRC can be exploited for a DCU working model by pre-straining within the curve's plateau region (e.g. 11mm). That allows the device to change in displacement while maintaining an approximately constant force. The CDS, alternatively, can be pre-strained at about 4mm to allow for a change in displacement with a small change in force.

4.2.14 The inclusion of DCU in the setup

The IAB wall has an actuator linked to two arms attached to a steel beam, that is fixed to another parallel steel beam of the same dimension, attached to the wall. Between the two beams, a gap was created to accommodate either DCU. This was done by adjusting the axle that passes through each beam. Fitting spacers between the two beams and locking the screws would make the IAB wall system ‘integral’.

The DCU systems, either CDS or HRC, can be installed between these arms, with the locking screws passing through the centre of either DCU units. The existing system uses two CDS units or HRC installed in parallel, perpendicular to the beams given by the configuration.

The installed location of the HRCs can be seen from the top of the wall as shown in Figure 4.28. The HRC arrangements in the IAB wall system are shown in Figure 4.29(a) with
the spacers installed and (b) after spacers were removed, before starting the wall movement. Figure 4.29(c) shows the CDS without spacers.

Figure 4.28: The locations of two (2) units of DCU (HRC) viewed from the top the wall.
Figure 4.29: The installation of the DCUs at the back of the IAB wall model for different tests. (a) IAB0.25 - control test; (b) HRC0.25; (c) CDS0.25.

The HRC units were equipped with annular endplates, as described in Section 3.1.2 and Figure 3.9. Each unit was held in place with the endplates inside 0.3mm deep recesses machined on the beams, preventing the end part of the cylinder from moving sideways. This also prevented it from collapsing under constant buckling deformation and during the cyclic wall movements.

Before the DCU installation, the arms’ axles that run through the beam and the wall were dismantled and pulled away, enough to slot in the CDS or HRC samples on the axles.
Then, both CDS or HRC were positioned so that each axle passed through their inner hollow diameter and fixed to the wall at the designated positions. Then the predetermined precompression was applied. For the HRC it was 10mm, and for CDS it was 3.8mm. The precompression was locked in position by screwing the axles until the specifically designed spacers stop them. At this point, the wall with the DCU is still considered as an integral system, as the DCU is not yet free.

After the precompression of DCUs was completed, and the wall is ensured to be vertical and centred, the tank was filled with sand (Section 4.2.1). Once the sand was levelled, and the backfill pressure was constant, the axles that hold the spacers in place were slowly and carefully unscrewed, allowing the DCUs to load the abutment soil. Once the screws were released, the spacers were removed from the beams, and the wall was no longer directly connected to the cam but to the DCUs.

The wall's behaviour now depended on the mechanical behaviour of the DCUs in accommodating the changes imposed by the bridge deck (actuator). The data collection was started before the unscrewing of DCUs and carried on until the test was finished. The pressure change during the removal of the screws and spacers was observed and discussed further in Section 4.3.4. The spacers removal, or referred to as the DCU release phase, was deemed critical as the DCU and the wall moved together as the DCU expanded and applied some force to the abutment which then loaded the soil.

### 4.2.15 Determination of the wall displacement

Two linear potentiometers were positioned at different heights from the bottom hinge of the scaled wall model; LVDT-1 at 585mm (attached to the moving wall), and LVDT-2 was at 415mm (attached to the rear of the motor).

For $d/H=0.25\%$ case, the passive-to-active rotational wall displacement at 585mm of height was 1.48mm. This made the displacement at the sand surface, at 540mm, to be 1.37mm. Due to the small magnitude, that rotational displacement was assumed to be linear in the lateral direction. Given that the distance of the LVDT-2 and all the pressure sensors are known, the displacement at every level and position can be configured by linear correlation. The wall displacement and the positions of the sensors are shown in Figure 4.30.
4.3 Results: Observation on the backfill pressure

This section discusses the IAB test results under cyclic thermal displacement ($d/H=0.25\%$) and its effect on the backfill pressure and settlement progression. The IAB wall was positioned perfectly centred at the start of the test. The test was conducted to investigate the effect of a longer IAB deck on the backfill pressure and the surface settlement. A longer bridge deck has a larger seasonal displacement, so the rotational displacement amplitude was increased to $d/H=0.50\%$, equivalent to a 120m deck length.
Subsequently, a set of tests with $d/H=0.25\%$ were conducted on the IAB wall with the DCUs (each with CDS and HRC). Later on, the same HRC set was included in a test with larger thermal displacement ($d/H=0.50\%$). The effect of CDS and HRC in these test conditions are summarised.

### 4.3.1 Normal IAB walls

#### The initial state of the wall

The initial stress distribution at the back of the wall, after the sand pluviation, describes the initial condition in a normal, integral IAB system. The initial stress state (at-rest), $K_0$ is the ratio of the soil’s horizontal stress to the vertical stress, $\sigma_h/\sigma_v$, before any cyclic wall movement is applied. The initial stress state of two test models, IAB0.25 and IAB0.50, are shown in Figure 4.31.

![Figure 4.31: Initial stress state for IAB0.25 and IAB0.50.](image)

The initial stress states of IAB0.25 and IAB0.50 are slightly different, as shown in Figure 4.31 above. The differences were not quantified in this work. These might be caused by the variation that may have happened during the laying of the sand in the tank. During the sand
laying by the pluviation method, the hopper that funnels down the sand may not reach the point exactly above the wall, so it became less methodological. This is likely to contribute to the slight difference in the initial stresses found in IAB0.25 and IAB0.50 tests.

At a stable and undisturbed condition, the at-rest earth pressure coefficient, $K_0$ against a nonyielding wall can be calculated using the Jaky (1944) formula, which is simplified in a widely accepted form as;

$$K_0 = 1 - \sin \phi$$  \hspace{1cm} (4.3)

where $\phi$ is the sand internal friction angle (radian). Using $\phi = 37^\circ$, the equation returns the value as $K_0 = 0.6$. During the cyclic wall movement from the deck's strain-imposed loading, the principal horizontal stress, $\sigma_h$ will change while the vertical stress, $\sigma_v$ remains constant. The vertical and horizontal stresses for this experiment are given by;

$$\sigma_v = \gamma_s z$$  \hspace{1cm} (4.4)

$$\sigma_h = K \gamma_s z$$

Where $\gamma_s$ is the dry sand density in kN/m$^3$, $z$ is the depth of soil at from the surface, and $K$ is the lateral earth pressure coefficient.

**The backfill pressure towards the active and passive directions**

The active and passive stress failure lines are drawn in the graph to indicate the limit of each active and passive pressure behind the abutment wall. These active and passive stress limits are represented by;

$$\sigma_{ha} = K_a \gamma_s z$$  \hspace{1cm} (4.5)

$$\sigma_{hp} = K_p \gamma_s z$$

where $\sigma_{ha}$ is the horizontal stress limit for active, and $\sigma_{hp}$ is the horizontal stress limit for passive in kPa, and $z$ is the depth of soil from the surface. For cohesionless soil (sand), $K_a$ is the coefficient of active earth pressure, and $K_p$ is for the passive given by Rankine (1857) in Equation (4.6).

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi}$$  \hspace{1cm} (4.6)

$$K_p = \frac{1 + \sin \phi}{1 - \sin \phi}$$
Using the Leighton Buzzard dry density, $\gamma = 17$ kN/m$^3$ and internal friction angle, $\Theta = 37^\circ$, the passive and active stress failure limit lines can be drawn according to the specific soil depth.

**Backfill pressure – IAB0.25 and IAB0.50**

This part illustrates the influence of different levels of wall rotational displacements on the IAB’s backfill pressure, conducted in two experiments; IAB0.25 and IAB0.50. The backfill pressure plots are shown in Figure 4.32 and Figure 4.33 below.

![Backfill pressure behind the wall](image)

Figure 4.32: Backfill pressure behind the wall for IAB0.25 test. The segmented lines represent the active and passive pressure profiles at the initial state using the $K_a$ (coefficient of active earth pressure) and $K_p$ (coefficient of passive earth pressure).

The behaviour of the IAB0.25 wall under cyclic displacement shows the pressure distribution resembled a horizontal parabola. The passive pressure (when the IAB deck is at its maximum length), suggests that the peak pressure is approximately located at the centre height of the wall (PrS-4), amounting to 12.0kPa after 50 cycles, and 14.8kPa after 2000 cycles. During the unloading phase, or when the deck is contracting, the active pressure at PrS-4 falls to 0.1kPa.
for all cycles. The passive wall pressure (deck expansion) escalates rapidly during the first 50 cycles. The change in pressure afterwards is slower, and from the 50th to the 2000th cycle, the increase in pressure on these two sensors is 5.6%. The increase in passive pressure shows no sign of stopping, and the pressure magnitudes are concentrated in the middle part of the wall.

Another experiment, IAB0.50, was conducted at twice as much displacement as in IAB0.25 but with the wall height remained the same, is shown in Figure 4.33.

![Graph showing backfill pressure behind the wall for IAB0.50 test. The segmented lines represent the active and passive pressure profiles at the initial state using the $K_a$ (coefficient of active earth pressure) and $K_p$ (coefficient of passive earth pressure).](attachment:image.png)

The graph shows that passive pressure for IAB0.50 escalates in the same manner as IAB0.25. The passive pressure increases as the cycle increases, and the active pressure (deck contraction) remained within 2kPa. The maximum passive pressure for IAB0.50 at PrS-4 is 1.4 times higher than it is for IAB0.25 at 100 cycles and 1.5 times more at 1000 cycle. The escalation of passive pressure reached its maximum around 500 cycle and, from then onwards remains the same until the end of the test, at cycle 1000.
Also, for the passive pressure at PrS-1 (the topmost pressure sensor), the values for 500 and 1000 cycles went down close to zero. This was due to the absence of sand in the area, caused by the progressive sand settlement.

**Pressure-displacement response of the backfill (IAB0.25 and IAB0.50)**

An example of the backfill pressure-displacements plot for PrS-2 and PrS-4 are shown in Figure 4.34 and Figure 4.35, respectively.

![Pressure-displacement response of the backfill for IAB0.25 at PrS-2](image)

Figure 4.34: Pressure-displacement response of the backfill for IAB0.25 at PrS-2. Point O is the origin loading state, line O-A is the first path during the expansion loading, and A-B is the first contraction unloading.
In the figures above, the positive sign displacement denotes the deck expansion movement towards the backfill (passive), and the minus sign (-) displacement is the contraction away from the backfill (active). For IAB0.25 at PrS-2 and PrS-4, the graph suggests that the wall displacement towards the passive direction (expansion loading) and active direction (contraction loading) were reduced as the cycle increases. However, the displacement reduction is more apparent in the passive direction due to the sand densification.

The passive pressure is shown to keep increasing up until 2000 cycles and showing no sign of stopping. This increase may be caused by the soil densification as it will increase the soil friction angle. In this study, the increase of friction angle by the soil densification due to progressive wall displacement cycle was not covered as the method to quantify the relationship was not established yet.

The loading-unloading paths shown in Figure 4.34 and Figure 4.35 are similar to the ones observed at sensor PrS-4 in the IAB0.50 test (Figure 4.36 below).
Figure 4.36: Pressure-displacement response of the backfill at different cycle for IAB0.50 at PrS-4.

In the integral system of IAB0.50, a similar wall behaviour to IAB0.25 was observed. The passive pressure increases as the cycle increases. The displacement towards passive direction becomes smaller by cycles, which suggests the characteristics of strain ratcheting. The displacement towards active direction was gradually shifted away from the sand backfill, as shown by the 500 cycle line.

Overall, Figure 4.34 to Figure 4.36 suggest that the IAB0.25 and IAB0.50 systems experience a pressure build up. As the thermal cycle increases, the peak passive pressures on PrS-2 and PrS-4 increase. This can be related to the sand settlement, which after several repeated cycles, densified the backfill. The aspects of the soil densification and settlement of IAB0.25 and IAB0.50 will be discussed more in Section 4.4.
4.3.2 IAB walls with DCUs

The initial state of the IAB wall

After the sand pluviation was completed and before the cyclic wall movement test started, the DCUs’ spacers were removed, so the DCUs would function freely against the imposed wall movements. The release of the pre-compression of the DCUs caused an additional stress to be applied onto the backfill, setting the backfill to an initial stress state, different from the $K_0$ stress state seen in the previous tests. These actions are shown in Figure 4.37.

During the release of the locking spacers, the DCUs pushed against the wall causing the wall to move slightly towards the backfill, as the DCUs extended. This caused an increase in the initial backfill pressure. For example, the increase seen at transducer PrS-2 is 7.5kPa for CDS0.25, 5.7kPa for HRC0.25, and 4.1kPa for HRC0.50 (see Figure 4.37). The stresses resulting from the HRC release for the amplitude of $d/H$ 0.25 and 0.50 are about the same.

For the case of the HRC0.25 test, during the release of the spacers, the wall was shifted from its initial position by 0.59mm towards the backfill. The same shifting was observed during the spacer for the CDS0.25 test, where the wall moved by 0.37mm towards the backfill (passive direction). It is worth noting that in the event of any CDS or HRC deployment in the real situation at the IAB wall, a slight net movement towards the backfill must happen.
In the case of IAB0.50, the increase in the initial pressure at PrS-3 is 4.53kPa. During the HRC release (for HRC0.50 test), the wall was shifted for 0.51mm towards the backfill. When releasing the DCUs, the cyclic wall displacement had not yet started; nevertheless, the backfill pressure was continuously monitored. The actuator which represents the cyclic deck displacement would only be initiated when the pressure had stabilised. A description of the relative movement of the DCUs and the initial wall shifting is shown in the early parts of the displacement versus thermal cycle plots in Section 4.3.4.

**Backfill pressure - CDS0.25 and HRC0.25**

This section investigates how the CDS and HRC use into the IAB affects the stress distribution behind the abutment, by using displacement cycles equivalent to \( d/H = 0.25\% \). The backfill pressure behind the wall for experiments CDS0.25 and HRC0.25 are shown in Figure 4.38 and Figure 4.39, respectively. The highest and lowest pressure for each cycle are plotted, and represented by solid line and dashed line respectively.
Figure 4.38: Backfill pressure behind the wall for CDS0.25. The segmented lines represent the active and passive pressure profiles at the initial state using the $K_a$ (coefficient of active earth pressure) and $K_p$ (coefficient of passive earth pressure).

The graph above shows that the peak passive pressure for CDS0.25 is recorded at PrS-2 and amounts to 8.8kPa at cycle 100 and 9.4kPa at cycle 2000. The wall movements towards the active and passive directions in all cycles are similar. This shows that the cyclic movements applied to the wall were absorbed by the CDS spring. In turn, resulting in a very small wall amplitude change, without change of the principal stress.

The pressure escalation was observed to be small until cycle 1000. Therefore, for curiosity, the test was conducted until 5800 cycles to see if the CDS would continue allowing an escalation of pressures. At 5800 cycles the passive pressure was increased to 10.2kPa from 9.0kPa at 1000 cycles.
Similarly, for the HRC0.25 test, the stress is almost unchanged between the active and passive pressures and between the first and the 2000th. Like CDS0.25, the peak pressure is located at the PrS-2, with 6.6kPa and 6.5kPa of passive and active pressures respectively at the 100th cycle. The 2000th cycle records the stress of 6.7kPa and 6.6kPa each for passive and active pressures.

With the HRC, the backfill pressure was maintained from escalating for up to 2000 cycles. The test was then extended up to 5800 cycles to observe if the pressure behaviour change in the long term. The result shows that the active and passive pressure was kept at 6.7kPa even at 5800 cycles.
Backfill pressure - HRC0.50

The same HRC system was used for the tests with \( d/H = 0.50\% \) since the displacement of 2.63mm of the new configuration is well within the HRC plateau region with 10mm pre-set buckling (see Figure 4.26). According to Figure 4.33, and by calculating the area under the first cycle's passive pressure profile, the backfill force for the first thermal cycle of \( d/H = 0.50\% \) has been worked out to be 690N; higher than the case for \( d/H = 0.25\% \) (399N). The existing HRC design has been used in this experiment to assess the adequacy of using the same HRC configuration for two different deck displacements. The backfill pressure behind the wall for experiment HRC0.50 is shown in Figure 4.40. The highest and lowest pressure for each cycle are distinguished by solid line and dashed line respectively.

![Graph showing backfill pressure behind the wall for HRC0.50. The segmented lines represent the active and passive pressure profiles at the initial state using the \( K_a \) (coefficient of active earth pressure) and \( K_p \) (coefficient of passive earth pressure).](image)

Figure 4.40: Backfill pressure behind the wall for HRC0.50. The segmented lines represent the active and passive pressure profiles at the initial state using the \( K_a \) (coefficient of active earth pressure) and \( K_p \) (coefficient of passive earth pressure).
For HRC0.50 test, a similar pressure pattern to the HRC0.25 was observed, where the difference between the active and passive pressures are almost coincident, except that the maximum pressure occurred at PrS-3. However, the active-passive pressure was shifted and escalated gradually up to 7.3kPa within the first 100 cycles. From 200 to 740 cycle, the pressure remained unchanged at 7.8kPa. The differences between HRC0.25 and HRC0.50 at the top transducers is a consequence of the larger displacement applied at the top of the wall. In the middle of the abutment, when the displacement imposed on the soil for experiment HRC0.50 is equal to the maximum displacement applied in experiment HRC0.25, the pressures measured by the lower transducers have much narrower variations, similar to the ones found in test HRC0.25.

4.3.3 Backfill pressure comparison between normal IAB and IAB with DCUs

To investigate the efficacy of the DCUs in the IAB wall setup, the comparison of backfill pressure between two periods was made. The difference in pressure at the initial state and after 100 cycles of wall movement for five tests are summarised in Table 4.8 below. The reason for selecting the 100 cycles for comparison is due to the bridge's service life which is normally around 100-years.

In this study, one wall cycle is equivalent to a 1 year of wall movement, corresponding to the deck deformation caused by the annual temperature variation. Note that for CDS and HRC tests, the ‘0’ cycle means the wall’s initial state after the DCU release. This is because the wall is fully dependent on the DCU action after the removal of the spacers.

Table 4.8: Difference in peak pressure between the initial and after 100 cycles of wall movement.

<table>
<thead>
<tr>
<th>Test</th>
<th>Cycle</th>
<th>Peak pressure (kPa)</th>
<th>Difference (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IAB0.25</td>
<td>Initial 100</td>
<td>0.12</td>
<td>12.55</td>
</tr>
<tr>
<td>IAB0.50</td>
<td>Initial 100</td>
<td>0.04</td>
<td>17.47</td>
</tr>
<tr>
<td>CDS0.25</td>
<td>0 100</td>
<td>8.391</td>
<td>8.84</td>
</tr>
<tr>
<td>HRC0.25</td>
<td>0 100</td>
<td>6.141</td>
<td>6.59</td>
</tr>
<tr>
<td>HRC0.50</td>
<td>0 100</td>
<td>6.611</td>
<td>7.46</td>
</tr>
</tbody>
</table>

1 Pressure due to the release of pre-stressing in the DCU elements
For the walls with DCUs, the initial state upon release of the pre-stressing pressure, stored on the DCU, will subject the wall to an initial increase in stress. However, this does not pose a problem as the DCU absorbs the subsequent wall movements, hence, keeping the stress changes very low. Also, it is shown in the data that by using DCUs, the peak pressures are much lower than the pressures acting on the normal IAB walls.

4.3.4 The relative displacement of the IAB wall with DCUs

The displacement of the wall and the DCUs during the release of spacers before the cyclic wall displacement tests were discussed in Section 4.3.2. This section discusses the effect of DCU systems in the IAB wall under the thermal cycle movements. The CDS and HRC release phase at the beginning of the tests and its subsequent displacement versus thermal cycle plots are shown in each plot in Figure 4.41 and Figure 4.42, respectively below. The ‘wall’ plots in both figures indicate the wall movement recorded at the sand surface level, and similarly for the ‘thermal movement’ plots. The DCUs movements (CDS and HRC plots) are the displacements at the DCU level, which was calculated linearly to the known displacements recorded by LVDT-1 and LVDT-2, as discussed in Section 4.2.15.

![Figure 4.41: Displacement versus thermal cycle for CDS0.25 test. The red line is the wall movement, the grey is the thermal expansion and contraction movement at the sand surface level, whilst the green is the CDS movement.](image-url)
In the figure above, the CDS movement corresponds to the cam movement in a cycle. Simultaneously, the wall exhibits an almost constant displacement under the thermal expansion and contraction movements. This is made possible as the CDS absorbs the cyclic movement. It is worth noting that the wall is shifted towards the backfill by a tiny amount in the long-term, as shown between cycle 10 to 100, 120 to 500, and cycle 520 to 5770.

Similarly, for HRC0.25 test, the wall’s thermal movement is absorbed by the HRC, leaving an almost constant displacement once the thermal cycle commenced. In the long-term, the wall is also shifted at a small amount towards the backfill, as seen between cycle 10 to 100, 120 to 500, and cycle 520 to 5770.

The DCUs are installed in precompression form. Furthermore, when it is being released to work fully, there will be a tendency that the DCU and the wall will shift slightly. These initial movement will exert additional stress to the IAB wall system’s initial stress state as the DCU will pre-compress the backfill towards the passive direction, consequently increasing the initial backfill pressure. However, the most important is for the wall to have a stable backfill pressure with little change over time, which evidently can be achieved using CDS or HRC (see Section 4.3.2 and Section 4.3.4).

The shifting of the DCU and the wall at the initial part of the test will have to be considered if the DCU will be used in an upscaled test, or if the similar concept of DCU will be implemented in other applications.
4.3.5 Pressure-displacement response of the backfill (CDS0.25, HRC0.25 and HRC0.50)

The pressure-displacement plots for three main cases; CDS0.25, HRC0.25, and HRC0.50 are analysed and compared. The aim is to see how the pressure correlates with the imposed movement on the wall by the DCUs. The plots for CDS0.25 test are shown in Figure 4.43, HRC0.24 test in Figure 4.44, and HRC0.50 in Figure 4.45.

It is worth noting that each pressure sensor has a different horizontal displacement range, given their location on the pinned wall. This is shown in the x-axis of the pressure-displacement plots.

The pressure-displacement plots for CDS0.25 in Figure 4.43 below suggest that as shown by PrS-2 and PrS-4, the pressure changes can be observed, although very small. The pressure bands in the figure show differences between the maximum and minimum; about 1.0kPa and 0.5kPa for PrS-2 and PrS-4, respectively. The pressure remained in the passive direction, and the wall displacement was gradually, and constantly shifted towards the passive direction with no sign of limits, even after 2000 cycles. This shows that under the cyclic displacement of 100 cycles and beyond, the CDS0.25 wall will not return towards the active direction.

For comparison, the pressure-displacement response of HRC0.25 under cyclic wall displacement is shown in Figure 4.44, and the plot for HRC0.50 is in Figure 4.45.

For HRC0.25, the pressure-displacement plots are similar, although smaller than that of CDS0.25; about 0.15kPa and 0.30kPa for PrS-2 and PrS-4, respectively. The wall position was gradually shifted towards the soil (passive direction). The plot for HRC0.50 in Figure 4.45 shows a steeper increase in passive pressure, and the wall displacement is permanently shifted towards the passive direction, at a larger shift magnitude than in CDS0.25 and HRC0.25. The pressure difference shown by the maximum and minimum bands at PrS-3 is about 1.0kPa. The pressure-displacement behaviours are shown to be identical for cycle 500 and 740 of HRC0.50 test, shown by the almost coinciding plots between the two. This suggests that the wall may not move beyond the displacement it reached at the 500 cycles.
Figure 4.43: Pressure-displacement response of the wall at different cycle for CDS0.25 at PrS-2 (left) and PrS-4 (right).

Figure 4.44: Pressure-displacement response of the wall at different cycles for HRC0.25 at PrS-2 (left) and PrS-4 (right).
Figure 4.45: Pressure-displacement response of the wall at different cycle for HRC0.50 at PrS-3. The active and passive stress limits are 1.47kPa and 23.25kPa, respectively.

It is evident that by using DCUs (CDS and HRC) in the wall, as in the CDS0.25, HRC0.25, and HRC0.50, the wall stress changes can be kept low under cyclical displacements, at the value of 1.0kPa and below. This is shown by the difference between the maximum and minimum pressure bands, which were constructed by jointing each cycle's maximum and minimum points.

The lowest difference in pressure among all is shown by HRC0.25 test, at only 0.3kPa. It is also shown in the figures above that overall, the horizontal shift of the wall towards the passive direction is smaller for HRC0.25 than that of CDS0.25.

4.3.6 The force-displacement behaviour of the DCUs under cyclic wall displacement

This part investigates the force-displacement behaviour of the DCUs after a long repetitive cyclic wall displacement, and whether the DCUs are susceptible to losing its elasticity after rigorous tests. The DCUs’ displacements were continuously recorded while being subjected to the cyclic wall displacement. The exact active and passive displacement of the DCU can be calculated as described in Section 4.2.15. After the displacements were calculated, the DCU
force can then be determined by referring it back to the force-displacement curve of the DCUs as in Figure 4.27.

The force-displacement range of the CDS used in the CDS0.25 test is shown in Figure 4.42, while the HRC used in both HRC0.25 and HRC0.50 tests is shown in Figure 4.47.

Figure 4.46: Force and displacement range of CDS after 5800 cycles (Test: CDS0.25).

Figure 4.47: Force and displacement range of HRC after 5800 cycles for HRC0.25, and 740 for HRC0.50 compared to the first cycles.

During the installation between the IAB wall and the actuator beam, the CDS and HRC were pre-compressed for 3.8mm and 10.0mm respectively. As described in Section 4.3.4 and Figure 4.42, both CDS and HRC were shifted towards the soil during the releasing phase.
For CDS, at the first and 5800th cycles, the change in force within a cycle due to expansion and the contraction remained the same at 101N. For HRC, at the first and 5800th cycle, the force difference is shown to be constant at only 10N. This suggests that both types of DCU units’ force and displacement are strictly consistent throughout the test. The active-to-passive displacement range for CDS0.25 and HRC0.25 at the first cycle was 1.44mm and 1.21mm respectively, and at 5800th cycle, the displacement range was 1.47mm for CDS0.25, but remained 1.21mm for HRC0.25. This showed that HRC was able to keep its displacement range constant at long-term. However, the cause to the displacement variation of 1.44mm to 1.47mm observed in CDS is yet to be understood.

In the case of HRC0.50, the identical pre-set displacement used for HRC0.25 was applied. This meant that the HRC was compressed by 10mm in the axial direction before the cyclic wall displacements were applied. The difference in the HRC system’s force was determined using the same methodology as in CDS0.25 and HRC0.25. The results, however, using the linear correlation of the displacement recorded by the LVDT-2, were calculated to be 2.18mm in the first cycle widening to 2.30mm in the 740th cycle, which corresponds to force differences of 8.9N and 9.0N respectively (see Figure 4.47). This suggests that the same HRC system used in HRC0.25 test is adequate to be used at the higher magnitude of IAB deck displacement of HRC0.50.

### 4.4 Results: Observation on the backfill settlement

To investigate the backfill’s settlement behaviour near the abutment wall, tests were conducted on IAB0.25 and IAB0.50. Subsequently, a series of comparative tests were conducted by including the CDS and HRC on the integral IAB wall systems. The settlement patterns for the tests were observed and compared in this section.

#### 4.4.1 Backfill settlement of IAB0.25, CDS0.25, and HRC0.25

**Visual records of the settlement progression**

The green markers in the sand layers reveal the distribution of vertical settlements. Photos were taken at intervals to investigate the sand settlement near the abutment for the case of IAB0.25 against the CDS0.25 and HRC0.25. The settlement observations were made until 2500 cycles for the case of IAB0.25 test, as opposed to 5800 cycles for CDS0.25 and HRC0.25.
tests. The photos are shown in Figure 4.48, and the settlement profiles for the three tests are shown in Figure 4.49.
Figure 4.48: (a) - (g) Backfill settlements for three cases; IAB0.25 (top left), HRC0.25 (top right), and CDS0.25 (bottom). Number of cycles shown on the photos.

Figure 4.49: Influence of the number of cycles on settlement profile for amplitude tests with \(d/H=0.25\%\).

Figure 4.50 shows the settlement record by thermal cycles for IAB0.25. The settlement in logarithmic timescale is shown as an inset in the same figure.
Figure 4.50: The settlement at the abutment wall for IAB0.25. The inset shows the settlement in $\log_{10}$ scale.

The settlement observation photos were taken mainly during the early cycles as the life span of a bridge is around 100 years, therefore a better characterisation of the settlement within that period was needed. The settlement for the IAB0.25 test is more pronounced at the early cycles. From the visuals in Figure 4.48 and the settlement profiles in Figure 4.49 and Figure 4.50, it is evident that there is no settlement observed for the CDS0.25 and HRC0.25 as compared to the IAB0.25 test which records a 20mm settlement after 100 cycles, and 65mm after 2500 cycles. At the early life, up to 20 years, a large settlement is recorded, corresponding to almost 50% of the 100 cycles settlement in volume.

4.4.2 Backfill settlement of IAB0.50 and HRC0.50

The settlement observation was carried out as part of the IAB0.50 test. The settlement profile was plotted and compared with the HRC0.50 test. Later, the settlement profiles of IAB0.25 and IAB0.50 were compared to investigate their correlation. The visual observations for the settlement for both tests are compiled in Appendix D.

The settlement near the abutment at different thermal cycles for IAB0.50 and HRC0.50 are shown in Figure 4.51, and Figure 4.52 shows the settlement recorded for IAB0.50 test. Similar to the IAB0.25, the settlement measured at 20 cycles is around 50% of
the 100 cycles settlement in volume. Figure 4.51 also show that during the first 5 cycles large settlements take place away from the wall. At 20 cycles, the settlements away from the wall seem to stabilise and not much change is seen from 20 to 100 cycles.

Figure 4.51: Influence of the number of cycles on settlement profile for amplitude tests with $d/H=0.50\%$.

Figure 4.52: The settlement at the abutment wall for IAB0.50. The inset shows the settlement in log$_{10}$ scale.
The settlement comparison between two integral IAB systems is shown in Figure 4.53 and Figure 4.54.

Figure 4.53: Progressive settlement behind the IAB wall with the seasonal cycle for two values of rotational displacement ($d/H=0.25\%$ and $0.50\%$).

Figure 4.54: Settlement rates for two rotational displacement magnitudes in the log$_{10}$ scale.

The comparison of settlement corresponding to the number of cycle and the settlement rates are shown in Figure 4.54 for both IAB0.25 and IAB0.50 tests. As expected, the larger deck
displacement imposed on IAB0.50 show larger settlements. This is supported by the results shown in Figure 4.51 and Figure 4.54.

4.5 Results: The effect of initial IAB wall positions on backfill pressure and settlement

This part of the study investigates the mechanics of the soil depending on the season when the IAB starts its operation. These initial conditions correspond to the wall's initial positions at the start of the tests: 1) the ‘winter’ position where the wall is at the most ‘active’, and 2) the ‘summer’ position where the wall is most ‘passive’. Both ‘winter’ and ‘summer’ wall positions were applied on three experiment cases; IAB, CDS and HRC, as summarised in Table 4.6. The experiments discussed in this section were run at the rotational wall displacement of $d/H=0.25\%$ unless otherwise stated.

The IAB wall movement sequence for the case of ‘winter’ and ‘summer’ initial positions are described in Figure 4.55(b) and (c) respectively. Figure 4.55(a) is the normal case of the experiment where the wall is set at the centre.

![Figure 4.55: IAB wall initial positions and its movement sequences; (a) middle (b) winter (c) summer.](image)

In the ‘winter’ case, the wall first pushes against the backfill soil from the most active position and subsequently pulls away; that is, an active-passive cycle (A-P). For the case of ‘summer’, it is vice versa or passive-active cycle (P-A). The ‘normal’ case with the wall installed centrally has a middle-passive-active-middle cycle sequence (M-P-A-M).
4.5.1 Backfill pressure

The development of backfill pressure according to the wall initial position for the IAB with and without the DCUs, is shown in Figure 4.56 to Figure 4.58.

![Graph showing backfill pressure for IAB cases](image)

Figure 4.56: Backfill pressure behind the wall for IAB cases.
Figure 4.57: Backfill pressure behind the wall for CDS cases.
The figures above show that an IAB wall that starts operating in summer will have a slightly higher peak passive pressure than the one that starts in winter at about 0.5kPa. In the case of an IAB with CDS, the wall’s peak pressure is higher by 2kPa when it starts in winter than when it starts in the summer. For the IAB with HRC, the peak pressure between the winter and summer cases is almost identical, although the two pressure profiles shows small differences but mainly related to the loads reached after the release of the pre-compression. Overall, no apparent difference in peak pressure is seen between the wall’s initial start positions, with and without the DCUs.

These differences are related to the mode of operation of the systems. The HRC was pre-compressed to its plateau, where changes in deformation will not affect the load applied
in the abutment. With CDS, there are variations in the force displacement curve, therefore if the deformation is against the wall, the forces applied to the soil will be higher when the bridge is built during the winter.

4.5.2 Backfill settlement

The backfill settlement progressions are plotted and compared in Figure 4.59.

![Graph](image-url)

**Figure 4.59:** Settlement comparison between three walls at two initial positions; (a) winter, (b) summer. The lines for CDS and HRC are representing 120 cycles.
Figure 4.59 suggests that the progressive settlement for both ‘winter’ and ‘summer’ cases is similar for the IAB wall. The most significant settlements for the IABs are recorded during the first 20 cycles and becomes slower afterwards. The plots suggest that regardless of the season the wall starts operating, the same settlement progression can be expected. The IAB with CDS or HRC shows no settlement even after 120 cycles.

From the settlement result of all tests, it can be concluded that, in the case of the moving IAB wall, the major principal stress was rotated, from vertical ($\sigma_v$) to horizontal ($\sigma_h$). This rotation of principal stresses had caused the granular soil to move and hence densified. Moreover, with the inclusion of DCUs, they compensate the lateral wall displacements, causing no rotation in the soil principal stresses; $\sigma_h$ (horizontal) and $\sigma_v$ (vertical). This helps to stabilise the backfill soil.

4.5.3 Comparison of pressure and settlement progressions with the ideally vertical IAB wall

The initial wall positions in experiments Part 1 and 2 (see Table 4.6) were set to be perfectly centred and vertical, and the first movements were towards the backfill soil, in passive movement. This section investigates the difference in terms of pressure and settlement progression of an IAB, which starts operating at different initial positions.

Backfill pressure development (3 initial wall positions)

The comparison in passive pressure between three initial wall positions of IAB is discussed here. The initial positions are; 1) centre, which corresponds to the ideally vertical wall, 2) winter, and 3) summer. The comparisons only show the plots for the passive pressure (Figure 4.60). The reason is that passive movement constitutes the greatest change in pressure.
Figure 4.60: Influence of initial wall positions and wall rotation amplitudes to the pressure buildup by the number of cycles.

Figure 4.60 shows that the peak passive pressure for IAB_winter, IAB_summer, and centred wall cases gradually converge and is fairly similar after 20 cycles. This suggests that regardless of initial wall positions, in 20 thermal cycles, the values could be considered similar for all
cases. The $d/H=0.50\%$ case is different, and the higher stresses manifest earlier on and are much larger than the other cases from cycle five onwards. From cycles 20 to 50, the increase in stress is the lowest.

**Settlement progression (3 initial wall positions)**

The comparison of the settlement behind the abutment wall for the IAB_winter and IAB_summer experiments is shown in Figure 4.61. The settlement intervals being compared are: 1, 5, 10, 20, 50, and 100 thermal cycles. The settlement for IAB0.50 case at cycle 5, 10, 50 and 100 are also included for comparison at double of the wall displacement magnitude.

![Figure 4.61](image_url)

*Figure 4.61: Influence of initial wall positions on surface settlement after (a) 1, 5 and 10 cycle; (b) 20, 50 and 100 cycle, for $d/H=0.25\%$ wall rotation amplitude. Blue plots represent the IAB_winter and red the IAB_summer. Black lines represent the IAB0.50 test.*
Figure 4.61 show that the surface settlement for IAB_winter and IAB_summer walls are about the same after 20 cycles. This suggests that after 20 thermal cycles, there is no difference between the two sets of settlement data, regardless of the wall’s positions during the start of the operation. After five cycles, the settlement shows a similarity between winter and summer cases, but the apparent wavy plots suggest that the sand near the surface is still rearranging. The settlement magnitudes for IAB0.50 case are almost double from the IAB_winter and IAB_summer ($d/H=0.25\%$).

### 4.6 Discussion

An attempt to study the efficacy of such a spring system to accommodate the strains in moving IAB wall has been made by designing a new Displacement Compensation Unit (DCU) using unfilled rubber in the form of a hollow cylinder (HRC). The HRC design was attempted by referring to the earlier study by Leaver and Lindley (1976), with several design iterations to satisfy the load range of the scaled IAB wall apparatus. Using the backfill pressure and soil settlement as the main parameters, a series of laboratory tests were conducted, and the results were compared between the normal IAB wall and the IAB equipped with two types of DCUs – CDS and HRC.

For the normal IAB wall without the DCUs, there is also a possibility that friction between the wall and sand contributed to the huge stress buildup. However, due to this study’s limitation, there was no attempt to investigate the friction mechanism of the moving IAB wall and sand.

The results showed that both DCUs; CDS and HRC, were able to suppress the maximum backfill stress to be much lower than the integral IAB wall’s stress. The soil settlement near the wall is non-existent in the IAB case with any DCU type; instead, the normal IAB which showed a progressive settlement with the thermal cycles.

Although both CDS and HRC were shown to be effective, it is evident that HRC exhibits lower stress than CDS’. A test with a larger wall displacement (which used $d/H=0.50\%$), demonstrated that the same set of HRC could still be able to absorb the deck displacement while keeping the backfill pressure at low values.
Using DCUs in the IAB wall will require a pre-stress of the DCUs in order to install the system in the wall. Once these are installed, the removal of the pre-stress will be resisted by the abutment soil, increasing the horizontal stresses.

4.7 Application of DCU in the Supported Embedded Wall

Based on Chapter 2 (Section 2.2.2 and 2.2.3), the embedded wall supported by a temporary prop system faces an almost similar problem as in the IAB wall. This is pertaining to the prop’s expansion and contraction due to temperature changes, which change the prop’s axial load, hence, inducing undesirable deformations on the wall.

This part discusses the load acting on a modular prop, and a simple mathematical calculation will be used to check the capability of a DCU unit, in this case, the HRC, to withstand the stresses imposed by a modular prop.

4.7.1 The Temperature and Load Observation of Props on Actual Sites

To have a clear indication on the behaviour of the actual temporary modular prop at work, two sets of the actual site records of load and temperature of a modular prop have been acquired, courtesy of Groundforce Shorco UK. The props used on the sites are of the 355.6 x 8mm CHS (circular hollow section) type (Gould, 2016), and the data consists of the axial load of the prop and its temperature, either on the prop itself or/and the ambient.

Elephant and Castle Station, London (Site-1)

The first set of data were recorded on an excavation work near Elephant and Castle Underground Station in London, henceforth known as Site-1. The raw data consists of load and temperature records of two steel props, including the ambient site temperature. Each prop has been installed with a load transducer (LC) and thermocouple (TC). The data has been recorded continuously for 102 hours, with 15-minute data acquisition intervals from 6th to 10th Sep 2015. The load-temperature relation of the props on this site is shown in Figure 4.62. Some data affected by the electrical noise was excluded from the plots.
Figure 4.62: The load-temperature relation for the props used in the excavation near the Elephant and Castle Station (Site-1). Prop-1 and Prop-2 refer to the axial load for two different prop units. TC1 and TC2 refer to temperature readings for Prop-1 and Prop-2, respectively. TC3 ambient is the ambient temperature of the site.

The plot in Figure 4.62 confirms an apparent gap of load values between Prop-1 and Prop-2, suggesting the props are supporting different levels of loads. The differences of temperature and load corresponding to the four cycles have been worked out, resulting in the average load per degree Celsius as 17.9kN/°C for Prop-1 and 19.4kN/°C for Prop-2.

**Liverpool Street Station, London (Site-2)**

Another set of load and temperature records is shown in Figure 4.63. The site is an excavation work near Liverpool Street Underground Station in London, henceforth known as Site-2. The site record consists of only one ambient temperature and load for five identical props. The data were recorded for 232 hours from 19th to 29th April 2016. In contrast to Site-1, this is a longer temperature-load observation. The Site-2 plot suggests similar observation to Site-1’s, with the temperature fluctuations governing the props' load change.
It can be seen in the above that each prop is supporting different level of loads. This is due to the different locations where the props were installed. The average load per degree Celsius for 10-day cycles for all five props are; 8.32kN/°C for Prop-1, 7.24kN/°C for Prop-2, 6.57kN/°C for Prop-3, 11.80kN/°C for Prop-4, and 4.24kN/°C for Prop-5.

Based on Figure 4.62 and Figure 4.63, it is shown that the prop’s load fluctuation is larger on Site-1 due to the season it was in which was September, where daily temperature changes are huge. In contrast, the load fluctuation on Site-2 is generally lower, expectedly, due to the relatively colder season in April.

4.7.2 Suitability of HRC for the prop

The acting loads on each prop on the actual sites are known. A simple calculation can be made to check an HRC’s capability to withstand the stresses imposed by the props. A buckling deformation mode of an HRC is chosen due to its plateau load response when being deformed axially. This is shown in the stress-strain curves of HRCs with shear modulus, \( G = 0.41 \text{MPa} \), and the dimension of \( OD \ Ø25\text{mm} \) with various \( ID \); \( Ø12\text{mm} \), \( Ø13\text{mm} \), and \( Ø15\text{mm} \) (refer Figure 4.25 in Section 4.3.4). The plateau range appeals to the prop system’s
application as it maintains the load to be approximately constant under changing displacement. Based on the plateau stress-strain curves, the HRC ideal operating strain, $\varepsilon_b$, shall be within 0.30 to 0.55.

The stress acting on the HRC in axial is calculated using the relationship of stress, $\sigma = F / A$, where $F$ is the axial load from the prop and $A$ is the cross-sectional plan area. For the CHS prop used on the sites, the highest load magnitudes, which are 805.7kN for Site-1 and 579.7kN for Site-2 were used to calculate the stress. Using the stress-strain curve of HRC with $OD$ 25mm and $ID$ 13mm with the $OD/ID$ ratio of 1.92 as references, the highest stress the HRC can withstand was 0.80MPa (at strain $\varepsilon_b$=0.55). This made the cross-sectional area of the HRC to be 1,007,125mm$^2$ for Site-1 and 724,625mm$^2$ for Site-2. Therefore, the possible HRC sizes fit for the calculated areas are of $OD$ 1320mm and $ID$ 686mm for Site-1, and $OD$ 1120mm $ID$ 582mm for Site-2. Following this approach, it is not feasible to deploy the HRC with that sizes at the propping system as their dimensions have become too cumbersome, and could pose logistic and handling issues.

Although the use of a single unit of HRC for a unit of CHS prop may not feasible, a different approach can be considered, for example by using a connecting plate between the two. In this configuration, the HRC will be positioned between the prop’s end and the wall, and a multiple number of HRC units can be arranged in series as such that the desired level of prop’s stress can be met. The feasibility of this method, however, may still depends on how complex the handling can be on site. An illustration of a single HRC unit connected to a single CHS prop is shown in Figure 4.64.

![Figure 4.64: Simplified illustration for a single HRC unit for one CHS prop installation arrangement (Elevation view). The diagram is not to scale.](image)
4.8 Discussion

Data obtained from the actual sites demonstrated that props’ induced load is directly proportional to its temperature change. The relation of load and deflection of prop to temperature fluctuation is similarly observed in the case of integral bridge abutment wall, which saw its effect to the backfill settlement (Wolde-Tinsea and Klinger, 1987; Hoppe and Gomez, 1996; Ng, Springman and Norrish, 1998; Arsoy, Barker and Duncan, 1999; Kim et al., 2014).

Although the temperature changes of the prop shown in Site-1 and Site-2 correspond to its induced load, there is no record which may suggest if wall deflection may contribute to the magnitude of the load. There is also a question of how the modular prop’s hydraulic jack takes this change of length or load. This will be the interest of the next part of this work. Furthermore, this will add another parameter that will affect how a rubber component can be designed for the prop.

Larger load fluctuations seen on Site-1 in September 2015, and lower fluctuations on Site-2 in April 2016 suggest that consideration of the magnitude of load changes according to the season may need to be considered when a propping system is to be installed. The same factor will also need to be considered if such a DCU is to be deployed in a propping system, especially in its capacity to allow for prop length changes due to the temperature changes.

The use of a single HRC unit is shown to be inadequate to withstand the load from a single prop. Further research is needed to explore the possibility of employing a system that can arrange multiple HRC units to cater to the same prop load. For example, one prop can be catered by using two or more HRC units. This can be made possible by using an interface beam placed between the prop and the HRCs.

It is worth noting that the study covered in this chapter is strictly for the 1/12 scaled IAB wall model. The interpretation of the actual behaviour of full-size HRC in the real IAB wall will be different from the laboratory experiments due to the nonlinearity of the HRC element (hollow cylinder). The actual design of the full-size DCU may need the reference to the buckling instability theory, where the critical stress at buckling of the HRC can be estimated from Equation (2.55) by Bakirzis, (1972), or by conducting numerical analysis similar to the one performed by Wong et al., (2010).
CHAPTER 5 CONCLUSIONS AND RECOMMENDATIONS

In this section, the thesis is summarised and main points, research findings and challenges are highlighted. The aim of the research is to investigate the merits of using a rubber-based component, called the DCU, as a solution to the densification of the soil behind the abutment of an integral bridge (IAB). In doing so, the mechanical working concept was studied and some design iterations were made. The rubber DCU, referred to as the hollow rubber cylinder (HRC), was deployed in a scaled IAB model and the system was tested in scenarios simulating several magnitudes of thermal displacements acting on the IAB.

In fulfilling the requirement to have a rubber DCU with suitable properties and performance, experimental works and numerical analyses were planned and executed. The long-term viscoelastic properties of the rubber material used in this study have been evaluated and a predictive model of the behaviour of rubber has been assessed for various geometries and deformation and temperature histories. A new method to predict the relaxation of rubber for long time has also been attempted and discussed.

In essence, the greater part of this research can be divided into four areas of investigation:

1. The key factors influencing stress relaxation and creep.
2. The modelling of the stress relaxation under arbitrary strains and temperature histories and the comparison to the experimental measurements.
3. The accuracy and reliability of the long-term stress relaxation prediction when applying the Stepped isothermal method (SIM).
4. The effect of DCU (HRC) inclusion to the performance of IAB system, using a scaled IAB wall test in the laboratory settings.

5.1 Factors Influencing the Stress Relaxation and Creep of Unfilled NR

The investigations in the first part of Chapter 3 described the first main finding, where it has been found that the stress relaxation and creep of unfilled rubber (EDS19) in simple shear (for DBS) and compressive buckling (for HRC) fit reasonably well with a linear approximation in a logarithmic timescale, for up to 10,000 seconds. The results showed that both were dominated by physical relaxation. From 10,000 seconds onwards, the stress relaxation and creep were shown to be more rapid, suggesting the effect of chemical relaxation (secondary creep or relaxation).
The stress relaxation experiments on the unfilled rubber in shear and compressive buckling at a range of strain sizes showed that their stress relaxation rates were independent of the strain levels. The creep experiments revealed that the creep in shear and buckling deformations were independent of the applied stress levels.

5.2 Stress Relaxation of Unfilled NR at Arbitrary Strains and Temperatures

The second part of Chapter 3 described the NR stress relaxation prediction with the change in strains and temperature. Stress relaxation experiments were carried out for two types of deformations: simple shear and buckling compression.

First, the analytical equations were implemented in simple shear in Microsoft Excel. Comparisons were made to the experimental results and simulation in ABAQUS. For axial compression exhibiting the buckling form, the deformation was non-uniform, thus, requiring a more complex mathematical representation. Therefore, it was not possible to implement the analytical model and only the FEA model in ABAQUS was used for comparison with the experimental results.

For the analytical Excel calculation, the stress relaxation of the unfilled rubber in shear was modelled by Prony series, using material and time parameters. To accommodate the change in strains during the relaxation, modifications were made by using the Boltzmann superposition principle. And to allow for the temperature changes, the William-Landel-Ferry (WLF) transformation was applied, using the temperature shift factor. The reference temperature, $T_{\text{ref}}$ for all the analyses was 24°C (297K). For the identical analysis using ABAQUS, the neo-Hookean model was used to model the behaviour of the unfilled rubber using the inputs deduced from the hyperviscoelastic parameters: shear and bulk moduli.

Two courses of experiments were conducted on both types of deformations: stress relaxation with varying strains (at constant temperature) and stress relaxation with varying strains and temperatures.

The main finding in this part was the fair agreement observed between the experimental results, analytical model (Excel), and the ABAQUS results. The qualitative effects of changes in both temperature and strain on the subsequent stress were captured well by the model although there were some discrepancies. These may partly be caused by
the use of the neo-Hookean model, but some discrepancies were observed which cannot clearly be explained in this way.

In shear deformation, there was a good agreement between the analytical model and the ABAQUS simulation, suggesting that the ABAQUS model’s implementation mirrored that of the analytical model implemented in Excel. However, a few unexplained discrepancies exist.

The experimental and ABAQUS results of the HRC tested under strain and temperature variations show that when the HRC reached the plateau region, changes in strain have insignificant effect on stress. This can be seen by comparing the buckling response between strains $\varepsilon_b = 0.4$ and $\varepsilon_b = 0.5$ (Figure 3.48). This is advantageous for an application like the IAB wall, where changes in strains are expected, but the force applied on the wall is kept approximately constant. In this study, Boltzmann superposition principle together with WLF transformation can model changes in strain and temperature in rubber with simple shear and compressive buckling deformations. The model is also applicable in ABAQUS.

5.3 Accelerated Method for Stress Relaxation of NR using TTSP and SIM Methods

Another main finding in this part was the degree of confidence in using accelerated TTSP and SIM methods to predict the long-term stress relaxation of unfilled rubber (EDS19), in comparison with the conventional long-term tests.

Results showed that the TTSP method was more applicable to predict the stress relaxation of unfilled rubber in shear or buckling deformations. In contrast, the SIM method showed a low degree of confidence in doing so. This was due to the fact that the chemical relaxation at high temperature was more prevalent in SIM than it was in TTSP.

5.4 Effectiveness of DCU in Integral Abutment Bridges

The tests using the IAB model wall demonstrated the ability of the DCUs to significantly improve the IAB wall performance in terms of backfill pressure and settlement near the surface level. Experimental results from the IAB wall test without the DCUs showed that the wall backfill pressure escalated rapidly at the first 50 cycles and increase at a slower rate afterwards. This also caused a settlement near the surface as the granular backfill densified due to the cyclic movements of the wall, mimicking the seasonal temperature variations.
The experiments with DCUs showed small differences between their active and passive pressures, as the number of displacement cycles increased. This has also resulted in no settlement in IABs with CDS or HRC DCUs. This demonstrated the ability of both CDS and HRC to allow cyclic displacement of the bridge deck and, at the same time, holding the principal wall stress approximately constant. This resulted in an abutment wall that moves very little over a long-term period. Overall, the tests showed that the HRC performs slightly better than CDS, with an almost coinciding pressure profile for the case of $d/H=0.25%$.

A test with a larger wall displacement (with $d/H=0.50%$) showed that the same set of HRCs were still able to withhold the wall and accommodate the displacements while suppressing the escalation of the backfill pressure, and no settlement was observed. In comparison, a test of the normal IAB wall showed the pressure escalation pattern similar to the $d/H=0.25%$ test, except for a larger pressure magnitude and settlement.

Another main finding was related to the initial wall positions at the start of the tests: 1) at the ‘winter’ where the wall is at the most ‘active position’, and 2) at the ‘summer’ where the wall is at the most ‘passive position’. Results showed that there was no apparent difference in peak pressure between the wall’s initial starting positions, with and without the DCUs. And for the case of normal IAB walls, regardless of initial wall positions (centred, ‘winter’ or ‘summer’ positions), the backfill pressure and settlement values were similar after 20 thermal cycles, the maximum stresses measured in all experiments are similar after more than 100 cycles, as well as the worse condition.

This study focused on the use of DCU (HRC) with the unfilled rubber made up as its material. The use of filled rubber can be considered for the next research, which can be taken up by any interested parties. In a real application, most engineered rubber products may require the addition of filler for modulus adjustment. If a low stress relaxation is desired, logically, a low level of filler will be preferred for the full-size DCU. But by using low levels of filler, the DCU may have to be bigger to match the required prestress level on the wall. Highly filled rubber may result in higher stress relaxation, and this will pose a challenge for the next research to look at the stress relaxation behaviour and also characterising the viscoelastic behaviour of filled rubber DCU as attempted in this research.

Overall, the results presented in this thesis provide an insight into the issues associated with the IAB wall and possible ways to mitigate it using NR in a hollow cylinder shape as a candidate device. This is also supplemented with some understanding of
predicting the behaviour of unfilled NR to the change in strains and temperatures during stress relaxation. The modelling of NR in shear and buckling with strain and temperature changes can serve as a guide during the design stage of an NR product which operates in permanent shear and buckling.

5.5 Recommendations for Future Works

Based on the experience gathered during this project, the future works may be centred around the following subjects:

1. Research to explore the stress relaxation behaviour and modelling of rubber including the effect of filler. Understanding the behaviour of filled rubber is important as most of engineering rubber products are made with filler materials such as carbon black. With the inclusion of filler, the analytical model may comprise of additive contributions from the reinforcing effect of the filler (elasto-plastic).
2. Extending the predictive model for stress relaxation of rubber to include chemical relaxation effects.
3. A study on the HRC deployment at the IAB wall with a more accurate temperature control, to better replicate the effect of temperature changes on DCU while in service.
4. DCUs could also be used to maintain a constant prestress force in props used in retaining walls. A short analysis demonstrated that the use of a single HRC unit in a prop is inadequate. However different solutions may prove fruitful such as an array of HRCs per prop or the installation of the system between the waler beam and the wall. Therefore, future study would look at the incorporation of these elements into a retaining wall system to avoid the large thermal loads induced by the props.
5. Deployment of a large scale DCU in a pilot study, whether in an IAB or propped wall. The installation of a full scale DCU unit may pose a challenge as there will be a provision of using a spacer to keep the DCU in place. Another challenge will be to convince the engineering authority involved in such structures to consider using a new device in the construction of IAB or propped wall.
6. The work can be extended to other types of spring element such as helical springs, which may serve as a comparison with this study. A work has been done on a rubber column with helical spring reinforcement, resulted with a controlled outward buckling. The spring-reinforced rubber, known as Paton Spring was developed to offer an alternative to conventional MacPherson strut suspension system (Stevenson, 1988).
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Appendix A

A.1 Apparatus setup for stress relaxation test at constant temperature

This section explains the details of the stress relaxation setup, including the calibration of the transducers and gauges used in the experiments.

The load cells used in the stress relaxation tests are a shear beam type model, SB-3410 from PCM (Procter and Chester Measurements UK). The load capacity of the transducer is 2500N with a sensitivity level of 0.08 mV per kilogram (kg) weight.

Four identical load transducers were used in each stress relaxation experiment. Calibrations were conducted prior to the application for data acquisition accuracy. Each load transducer was calibrated by imposing, with several sets of dead weights, a load in ascending magnitude, shown in Figure 0.1. The transducers were connected to Fylde Micro Analog 2 data acquisition system which shows and records the corresponding force values. The transducer input channels were calibrated through the potentiometer wound.

From the calibration exercise, it is shown that satisfying load responses can be produced. This setup has been used in the majority of the stress relaxation test in this study. All four units of PCM SB-4310 load transducers calibration plots are shown below;
Table 0.1: Calibration record for load transducer Ch1

<table>
<thead>
<tr>
<th>Dead load (N)</th>
<th>Readings (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.37</td>
<td>30.7</td>
</tr>
<tr>
<td>44.81</td>
<td>52.5</td>
</tr>
<tr>
<td>89.35</td>
<td>96.5</td>
</tr>
<tr>
<td>137.14</td>
<td>143.0</td>
</tr>
<tr>
<td>202.09</td>
<td>207.4</td>
</tr>
<tr>
<td>291.44</td>
<td>295.0</td>
</tr>
<tr>
<td>454.40</td>
<td>454.5</td>
</tr>
<tr>
<td>738.50</td>
<td>734.0</td>
</tr>
</tbody>
</table>

(a)

Table 0.2: Calibration record for load transducer Ch2

<table>
<thead>
<tr>
<th>Dead load (N)</th>
<th>Readings (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.37</td>
<td>25.0</td>
</tr>
<tr>
<td>44.81</td>
<td>46.0</td>
</tr>
<tr>
<td>89.35</td>
<td>89.0</td>
</tr>
<tr>
<td>137.14</td>
<td>135.0</td>
</tr>
<tr>
<td>202.09</td>
<td>199.0</td>
</tr>
<tr>
<td>291.44</td>
<td>284.0</td>
</tr>
<tr>
<td>454.40</td>
<td>439.0</td>
</tr>
<tr>
<td>738.50</td>
<td>713.0</td>
</tr>
</tbody>
</table>

(b)
Table 0.3: Calibration record for load transducer Ch3

<table>
<thead>
<tr>
<th>Dead load (N)</th>
<th>Readings (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.37</td>
<td>32.5</td>
</tr>
<tr>
<td>44.81</td>
<td>55.4</td>
</tr>
<tr>
<td>89.35</td>
<td>100.2</td>
</tr>
<tr>
<td>137.14</td>
<td>148.0</td>
</tr>
<tr>
<td>202.09</td>
<td>213.0</td>
</tr>
<tr>
<td>291.44</td>
<td>303.6</td>
</tr>
<tr>
<td>454.40</td>
<td>467.2</td>
</tr>
<tr>
<td>738.50</td>
<td>753.0</td>
</tr>
</tbody>
</table>

Table 0.4: Calibration record for load transducer Ch4

<table>
<thead>
<tr>
<th>Dead load (N)</th>
<th>Readings (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.37</td>
<td>22.6</td>
</tr>
<tr>
<td>44.81</td>
<td>44.0</td>
</tr>
<tr>
<td>89.35</td>
<td>88.0</td>
</tr>
<tr>
<td>137.14</td>
<td>134.0</td>
</tr>
<tr>
<td>202.09</td>
<td>198.0</td>
</tr>
<tr>
<td>291.44</td>
<td>285.0</td>
</tr>
<tr>
<td>454.40</td>
<td>443.0</td>
</tr>
<tr>
<td>738.50</td>
<td>720.0</td>
</tr>
</tbody>
</table>

Figure 0.1: The calibration plots of PCM load transducers Channel 1 to 4 are shown in (a), (b), (c) and (d). On the left of each plots are the load readings in the logger corresponding to the applied dead loads.

The plots suggest good agreement between the dead loads and the reading loads shown by the data logger.

A.2 Thermocouple calibration for the stress relaxation test temperature monitoring
Throughout the duration of the stress relaxation test, temperature readings were being recorded for monitoring purposes. Also, to ensure the temperature inside the test box is constant at the desired level; 30±0.5°C.

For the purpose of temperature monitoring, the temperature gauges (thermocouples) were calibrated at different temperatures. Thermocouples used were K-Type thermocouple, with temperature capacity from -50°C to 250°C. Temperature records were taken when the thermocouples were submerged in a beaker with water at room temperature, and later when the beaker was heated on a hot plate; see Figure 0.2. The thermometer readings were then compared to the readings given by Fylde Micro Analog logger. The exercise showed that the thermocouples and the logging system are reliable and can be used in the temperature monitoring of the test.

Figure 0.2: The Type-K thermocouple calibration with; (a) thermocouples at room temperature water beaker (b) thermocouples at elevated temperature from heated beaker (c) real-time temperature logging through Fylde Micro Analog logger with the computer
Table 0.5: The calibration record for the thermocouples checked against the mercury thermometer

<table>
<thead>
<tr>
<th>Thermometer (mercury)</th>
<th>TC-1</th>
<th>TC-2</th>
<th>TC-3</th>
<th>TC-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.5</td>
<td>23.68</td>
<td>23.73</td>
<td>23.52</td>
<td>23.48</td>
</tr>
<tr>
<td>40.8</td>
<td>40.76</td>
<td>40.82</td>
<td>40.77</td>
<td>40.78</td>
</tr>
</tbody>
</table>

Table 0.5 suggests that the thermocouples, labelled as TC-1, TC-2, TC-3 and TC-4, are able to record the actual temperature in agreement to the mercury thermometer’s readings.

A.3 Dial gauge calibration for the use in the strain setup of stress relaxation test

The setup of the stress relaxation tests require the application of different levels of strains to different samples. In order to achieve the accurate strain magnitude, a dial gauge has been used to guide the straining process, as shown in Figure 0.3.

Figure 0.3: Dial gauge used in the test piece straining. The gauge head is in contact with a _-shaped bracket attached to the middle steel part of the Double Bonded Shear (DBS) test piece during the straining stage.
The dial gauge used in this experiment underwent a calibration exercise, as it is important to observe whether the force imposed by the dial gauge during the straining process may contribute to the overall load magnitude of the test. The dial gauge type used was a Mitutoyo 50mm and the force readings were measured by a Sartorius 11kg weight balance; see Figure 0.4. The stiffness of the dial gauge was calculated from its force-displacement readings, shown in Table A-1.

![Figure 0.4: The simple calibration setup for the dial gauge.](image)

<table>
<thead>
<tr>
<th>Displacement (mm)</th>
<th>Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.02</td>
<td>0.941</td>
</tr>
<tr>
<td>10.58</td>
<td>1.011</td>
</tr>
<tr>
<td>20.12</td>
<td>1.249</td>
</tr>
<tr>
<td>27.81</td>
<td>1.299</td>
</tr>
<tr>
<td>35.35</td>
<td>1.502</td>
</tr>
<tr>
<td>45.66</td>
<td>1.589</td>
</tr>
</tbody>
</table>

![Figure 0.5: Calibration plot of the dial gauge (based on Table A-1).](image)

$y = 0.0169x + 0.8594$
Based on the force-displacement plot of the dial gauge, the stiffness has been calculated to be 0.017N/mm. This stiffness value ($K_{\text{gauge}}$) was then compared with the known stiffness of a double bonded shear (DBS) test piece, which was tested beforehand. Here is the force-displacement plot of a typical DBS at 100% shear strain. The shear visualisation and the test setup are shown in Figure 3.12 and Figure 3.41.

![Force-displacement plot for a DBS at 100% shear strain](image)

Based on Figure 0.6, the stiffness of the DBS sample in shear is calculated to be 70.47N/mm ($K_{\text{test}}$). Comparison between $K_{\text{gauge}}$ and $K_{\text{test}}$ suggested that the dial gauge stiffness is 0.024% from the shear stiffness of the DBS test piece. It can be concluded that this amount is insignificant to be able to affect the stiffness value of a DBS test specimen.

**A.4 Calibration of linear variable displacement transformer (LVDT) in creep test**

The calibration record for five (5) units of LVDT transducers used in creep tests are shown below.
The linearity checks, or the calibration of the LVDTs were done by checking the displacement travels at several intervals, by using a variation of Mitutoyo thickness gauge blocks.
Figure 0.8: An LVDT being calibrated using a Mitutoyo thickness gauge block, with a digital signal indicator (RDP E525) as a reference. Shown here is the LVDT of RDP-ACT500A type.

Figure 0.9: Mitutoyo thickness gauge set used in the calibration of LVDTs.

**A.5 Temperature controlled box – diagrams for the box and signal controller.**

The diagram below shows the electronic circuit of the electronic controller used to control the set temperature inside the temperature-controlled box. The electronic controller was designed so that it could maintain the desired temperature inside the box. The temperature
was set in an electro-contact, mercury-in-glass thermometer which acts as a switch. It would close the circuit when the temperature reaches the set temperature, hence, lighting up the incandescent bulbs.

![Diagram of the electronic system](image)

Figure 0.10: The electronic system built for the temperature-controlled box for multiple stress relaxation and creep tests.

### A.6 Photos of DBS and HRC in stress relaxation test imposed with different strains

Some of the photos taken during the multiple stress relaxation and creep tests conducted on double bonded shear (DBS) and hollow rubber cylinder (HRC) samples are shown below.

(a) 50% strain    (b) 99% strain
Figure 0.11: Shear strains applied on some of the DBS samples; (a) DBS-A3, (b) DBS-A4, (c) DBS-A5, and (d) DBS-A6.

Figure 0.12: Compressive strains applied on some of the HRC samples; (a) HRC-A1, (b) HRC-A2, (c) HRC-A4, and (d) HRC-A6.
Appendix B

B.1 Transducers calibration for scaled integral abutment bridge (IAB) wall model

This section presents the records related to the commissioning works of the scaled IAB wall model in UCL’s Soil Laboratory. The calibration record for the pressure sensors (PrS) and linear potentiometers for the IAB wall model are shown below. Seven (7) pressure sensors are represented by PrS-1 to PrS-7, and two linear potentiometers are represented by LVDT-1 and LCDT-2. The positions for all transducers are shown in Figure 4.5.

Figure 0.13: Calibration record for Pressure Sensor 1 (PrS-1)
Figure 0.14: Calibration record for Pressure Sensor 2 (PrS-2)

Figure 0.15: Calibration record for Pressure Sensor 3 (PrS-3)
Figure 0.16: Calibration record for Pressure Sensor 4 (PrS-4)
B.2 Sand dry density, \( \gamma_s \) monitoring

The sand layering in the backfill tank involved the data-taking of the sand densities for every 80mm layer height, at two locations. The sand density records for four (4) main test schemes are presented in Table 0.6 to Table 0.9.

**Abbreviation:**

HRC: Hollow rubber cylinder  
CDS: Conical disc spring

**Table 0.6: Part 1 - Normal IAB with seasonal rotational displacement \( d/H=0.25\% \) and \( d/H=0.50\% \)**

<table>
<thead>
<tr>
<th>Case: IAB0.25</th>
<th>Case: IAB0.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (from the toe of the wall), mm</td>
<td>Density, ( \gamma_s ) (kg/cm(^3))</td>
</tr>
<tr>
<td>480</td>
<td>1744.1</td>
</tr>
<tr>
<td>400</td>
<td>1751.3</td>
</tr>
<tr>
<td>320</td>
<td>1735</td>
</tr>
<tr>
<td>240</td>
<td>1744.1</td>
</tr>
<tr>
<td>160</td>
<td>1741.1</td>
</tr>
<tr>
<td>80</td>
<td>1747.2</td>
</tr>
<tr>
<td>Average density, ( \bar{\gamma}_s )</td>
<td>1743.8</td>
</tr>
<tr>
<td>Standard deviation, SD</td>
<td>5.05</td>
</tr>
</tbody>
</table>
Table 0.7: Part 2 - Normal IAB with DCUs inclusions

<table>
<thead>
<tr>
<th>Height (from the toe of the wall), mm</th>
<th>Density, $\gamma_{S1}$ (kg/cm³)</th>
<th>Density, $\gamma_{S2}$ (kg/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>480</td>
<td>1753.3</td>
<td>1746.2</td>
</tr>
<tr>
<td>400</td>
<td>1766.6</td>
<td>1727.8</td>
</tr>
<tr>
<td>320</td>
<td>1743.1</td>
<td>1720.7</td>
</tr>
<tr>
<td>240</td>
<td>1753.3</td>
<td>1744.1</td>
</tr>
<tr>
<td>160</td>
<td>1772.7</td>
<td>1762.5</td>
</tr>
<tr>
<td>80</td>
<td>1781.9</td>
<td>1753.3</td>
</tr>
</tbody>
</table>

Average density, $\gamma_{S}$ = 1761.8
Standard deviation, $SD$ = 13.17

Case: HRC0.25

<table>
<thead>
<tr>
<th>Height (from the toe of the wall), mm</th>
<th>Density, $\gamma_{S1}$ (kg/cm³)</th>
<th>Density, $\gamma_{S2}$ (kg/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>480</td>
<td>1744.1</td>
<td>1760.4</td>
</tr>
<tr>
<td>400</td>
<td>1730</td>
<td>1725.6</td>
</tr>
<tr>
<td>320</td>
<td>1730.5</td>
<td>1745.1</td>
</tr>
<tr>
<td>240</td>
<td>1726.6</td>
<td>1706.7</td>
</tr>
<tr>
<td>160</td>
<td>1735</td>
<td>1738.8</td>
</tr>
<tr>
<td>80</td>
<td>1759.3</td>
<td>1751.5</td>
</tr>
</tbody>
</table>

Average density, $\gamma_{S}$ = 1737.6
Standard deviation, $SD$ = 11.17

Table 0.8: Part 3 - The effect of initial wall positions according to the seasons

Case: IAB_summer

<table>
<thead>
<tr>
<th>Height (from the toe of the wall), mm</th>
<th>Density, $\gamma_{S1}$ (kg/cm³)</th>
<th>Density, $\gamma_{S2}$ (kg/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>480</td>
<td>1744.1</td>
<td>1760.4</td>
</tr>
<tr>
<td>400</td>
<td>1730</td>
<td>1725.6</td>
</tr>
<tr>
<td>320</td>
<td>1730.5</td>
<td>1745.1</td>
</tr>
<tr>
<td>240</td>
<td>1726.6</td>
<td>1706.7</td>
</tr>
<tr>
<td>160</td>
<td>1735</td>
<td>1738.8</td>
</tr>
<tr>
<td>80</td>
<td>1759.3</td>
<td>1751.5</td>
</tr>
</tbody>
</table>

Average density, $\gamma_{S}$ = 1734.6
Standard deviation, $SD$ = 16.42

Case: IAB_winter

<table>
<thead>
<tr>
<th>Height (from the toe of the wall), mm</th>
<th>Density, $\gamma_{S1}$ (kg/cm³)</th>
<th>Density, $\gamma_{S2}$ (kg/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>480</td>
<td>1766.6</td>
<td>1735</td>
</tr>
<tr>
<td>400</td>
<td>1722</td>
<td>1705.6</td>
</tr>
<tr>
<td>320</td>
<td>1734.3</td>
<td>1713.6</td>
</tr>
<tr>
<td>240</td>
<td>1738.2</td>
<td>1729.5</td>
</tr>
<tr>
<td>160</td>
<td>1741.8</td>
<td>1727.1</td>
</tr>
<tr>
<td>80</td>
<td>1766.6</td>
<td>1735</td>
</tr>
</tbody>
</table>

Average density, $\gamma_{S}$ = 1744.9
Standard deviation, $SD$ = 17.65
## Table 0.9: Part 4 - The HRC under repetitive loadings at different temperature

<table>
<thead>
<tr>
<th>Case: CDS_summer</th>
<th>Case: CDS_winter</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Height (from the toe of the wall), mm</strong></td>
<td><strong>Density, γs₁ (kg/cm³)</strong></td>
</tr>
<tr>
<td>480</td>
<td>1761.5</td>
</tr>
<tr>
<td>400</td>
<td>1760.1</td>
</tr>
<tr>
<td>320</td>
<td>1721.3</td>
</tr>
<tr>
<td>240</td>
<td>1754.6</td>
</tr>
<tr>
<td>160</td>
<td>1764.8</td>
</tr>
<tr>
<td>80</td>
<td>1745.5</td>
</tr>
<tr>
<td><strong>Average density, γ̅</strong></td>
<td><strong>1751.3</strong></td>
</tr>
<tr>
<td><strong>Standard deviation, SD</strong></td>
<td><strong>14.77</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case: HRC_summer</th>
<th>Case: HRC_winter</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Height (from the toe of the wall), mm</strong></td>
<td><strong>Density, γs₁ (kg/cm³)</strong></td>
</tr>
<tr>
<td>480</td>
<td>1744.1</td>
</tr>
<tr>
<td>400</td>
<td>1716.1</td>
</tr>
<tr>
<td>320</td>
<td>1729.4</td>
</tr>
<tr>
<td>240</td>
<td>1738.4</td>
</tr>
<tr>
<td>160</td>
<td>1767</td>
</tr>
<tr>
<td>80</td>
<td>1772.5</td>
</tr>
<tr>
<td><strong>Average density, γ̅</strong></td>
<td><strong>1744.6</strong></td>
</tr>
<tr>
<td><strong>Standard deviation, SD</strong></td>
<td><strong>19.84</strong></td>
</tr>
</tbody>
</table>

Table 0.9: Part 4 - The HRC under repetitive loadings at different temperature

Case: HRC0.50-T19 and HRC0.50-T30

<table>
<thead>
<tr>
<th><strong>Height (from the toe of the wall), mm</strong></th>
<th><strong>Density, γs₁ (kg/cm³)</strong></th>
<th><strong>Density, γs₂ (kg/cm³)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>480</td>
<td>1731.273</td>
<td>1728.51</td>
</tr>
<tr>
<td>400</td>
<td>1726.337</td>
<td>1732.675</td>
</tr>
<tr>
<td>320</td>
<td>1743.359</td>
<td>1739.883</td>
</tr>
<tr>
<td>240</td>
<td>1740.506</td>
<td>1716.201</td>
</tr>
<tr>
<td>160</td>
<td>1725.288</td>
<td>1725.269</td>
</tr>
<tr>
<td>Average density, $\bar{\gamma}$</td>
<td>1739.348</td>
<td>1731.068</td>
</tr>
<tr>
<td>---------------------------------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>Standard deviation, SD</td>
<td>17.83</td>
<td>15.63</td>
</tr>
</tbody>
</table>
Appendix C

This appendix relates to the work described in Section 3.4.10, where the comparisons between the experiment, calculation and ABAQUS simulation on the rubber undergoing stress relaxation with varying strain and temperature were conducted. The figures showing the ABAQUS models for the simple shear (DBS sample) and buckling compression (HRC sample) are shown below.

Case 1: Simple shear

The strain and temperature inputs for the DBS in the ABAQUS simulation are summarised in Figure 0.21.

Figure 0.21: Shear strain and temperature history applied to the DBS sample.

Figure 0.22: Undeformed DBS model in plane strain.
Figure 0.23: Shear strain $\gamma=1.00$ and temperature 24°C (297K).

Figure 0.24: Shear strain $\gamma=1.00$ and temperature 53°C (326K).

Figure 0.25: Shear strain $\gamma=1.00$ and temperature 6.5°C (279.5K).
Case 2: Buckling compression

The strain and temperature inputs for the HRC in the ABAQUS simulation are summarised in Figure 0.27.
Figure 0.28: Underformed HRC model in the ABAQUS.

Figure 0.29: Compressive strain $\varepsilon_b=37\%$ and temperature 24°C (297K).

Figure 0.30: Compressive strain $\varepsilon_b=37\%$ and temperature 53°C (326K).
Appendix D

D.1 Calibration of the load cell and external LVDT used in Wykeham Farrance triaxial test system for Leighton Buzzard characterisation test.

This appendix describes the instrumentations used in the tests involved in the characterising the granular material used in this study – Leighton Buzzard sand.

Instrumentation

Figures and tables below summarised the calibration of instruments used in the triaxial test. The calibration of load cell for the cell axial stress was conducted by observing the voltage response of a series of dead weight stacked gradually onto the load cell. The calibration record is shown in Figure 0.31.

The calibration plot for the external LVDT used for the cell displacement record is shown in Figure 0.32.

<table>
<thead>
<tr>
<th>Load (N)</th>
<th>Voltage reading (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.1117</td>
</tr>
<tr>
<td>98.10</td>
<td>0.1631</td>
</tr>
<tr>
<td>196.20</td>
<td>0.2325</td>
</tr>
<tr>
<td>274.68</td>
<td>0.3054</td>
</tr>
<tr>
<td>353.16</td>
<td>0.3928</td>
</tr>
</tbody>
</table>

Table 0.10: Calibration record of the triaxial load cell.

Figure 0.31: Calibration plot of the triaxial load cell.
Table 0.11: Calibration record of the external LVDT of triaxial cell.

<table>
<thead>
<tr>
<th>Disp (mm)</th>
<th>Volt (V)</th>
</tr>
</thead>
<tbody>
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<td>-5.4991</td>
</tr>
<tr>
<td>1</td>
<td>-4.8499</td>
</tr>
<tr>
<td>2</td>
<td>-4.2042</td>
</tr>
<tr>
<td>3</td>
<td>-3.5629</td>
</tr>
<tr>
<td>4</td>
<td>-2.9228</td>
</tr>
<tr>
<td>5</td>
<td>-2.2856</td>
</tr>
<tr>
<td>6</td>
<td>-1.6417</td>
</tr>
<tr>
<td>7</td>
<td>-0.9988</td>
</tr>
<tr>
<td>8</td>
<td>-0.355</td>
</tr>
<tr>
<td>9</td>
<td>0.2892</td>
</tr>
<tr>
<td>10</td>
<td>0.9313</td>
</tr>
<tr>
<td>11</td>
<td>1.5708</td>
</tr>
<tr>
<td>12</td>
<td>2.2065</td>
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<tr>
<td>13</td>
<td>2.8445</td>
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<tr>
<td>14</td>
<td>3.4894</td>
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<td>15</td>
<td>4.1375</td>
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<td>14</td>
<td>3.4921</td>
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<tr>
<td>12</td>
<td>2.2095</td>
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<tr>
<td>10</td>
<td>0.932</td>
</tr>
<tr>
<td>8</td>
<td>-0.354</td>
</tr>
<tr>
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<td>-4.8471</td>
</tr>
<tr>
<td>0</td>
<td>-5.4932</td>
</tr>
</tbody>
</table>

Figure 0.32: Calibration plot of the external LVDT (triaxial cell).
D.2 Settlement observations for IAB0.50 and HRC0.50 tests.

(a) initial

(b) after 5 cycles

(c) after 10 cycles

(d) after 20 cycles
Figure 0.33: (a) - (h) Backfill settlements for two cases; IAB0.50 (left), and HRC0.50 (right). Number of cycles shown on the photos.