



# 2021 QATAR PROCESS SAFETY SYMPOSIUM

**QPSS 2021**  
**EMBRACING THE RED**  
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# Entropy – stagnation enthalpy interpolation tables for calculation of the critical flow properties of compressible fluids

Sergey Martynov<sup>1</sup>, Haroun Mahgerefteh<sup>1</sup>, Ilias K. Nikolaidis<sup>2</sup>  
and Ioannis G. Economou<sup>3</sup>

<sup>1</sup>*Department of Chemical Engineering, University College London, UK;*

<sup>2</sup>*Institute of Nanoscience and Nanotechnology, NSRF “Demokritos”, Aghia Paraskevi, Greece;*

<sup>3</sup>*Chemical Engineering Program, Texas A&M University at Qatar, Doha, Qatar.*

*E-mail: [s.martynov@ucl.ac.uk](mailto:s.martynov@ucl.ac.uk)*

# Scope

- Motivation and objectives
- Methodology
- Results
- Conclusions and next steps



<https://www.h2-view.com/story/in-focus-transitioning-the-uks-gas-networks-to-hydrogen/>

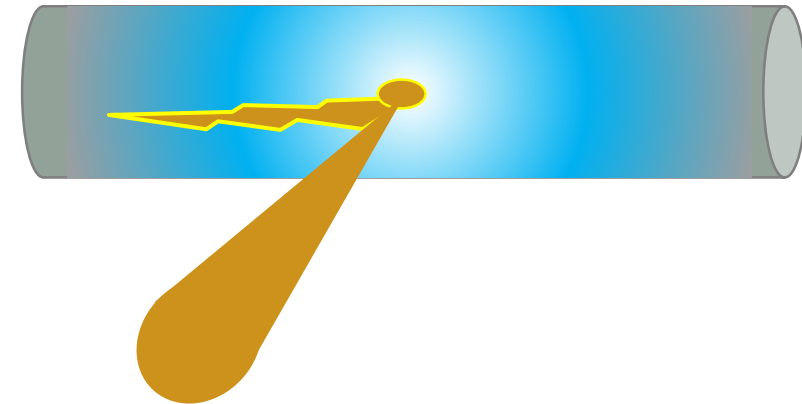
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# Motivation and objectives

- Pipeline transportation of industrial gases is mature technology.
- Due to hazardous nature of transported fluids and high operating pressures, safety of pipeline transportation is of serious concern.
- Mathematical models predicting pipeline decompression and the fluid discharge flow are in the heart of Qualitative Risk Assessment (QRA) predicting consequences of pipeline failure:
  - ✓ Significant cooling and dry ice formation during venting and accidental failure of CO<sub>2</sub> transport pipelines
  - ✓ Low-temperature induced brittle fracture upon accidental failure of ethylene and CO<sub>2</sub> transport pipeline
  - ✓ Safety assessment of hydrogen transport pipelines and facilities – with increasing demand for H<sub>2</sub> transport



# Motivation and objectives

- Models have been developed in the past:
  - Homogeneous Equilibrium Mixture (HEM) and multi-fluid/multiphase models for transient flow of compressible fluids in pipelines
  - Fluid properties calculation methods (EoS, interpolation tables) for single and multi-phase flows
- The accuracy and computational efficiency of the physical properties models is critical for the pipeline decompression flow simulations

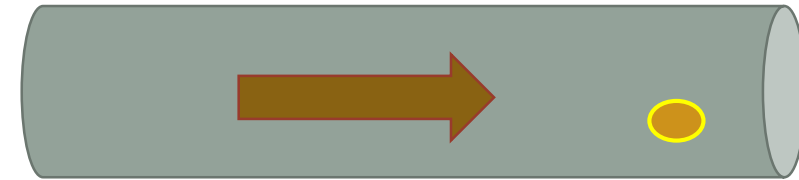
# Physical properties involved

Governing equations for transient flow in a pipeline:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = -S_o$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2}{\partial x} = -\frac{\partial p}{\partial x} - 2f_w \frac{\rho u^2}{D} - u S_o$$

$$\frac{\partial \rho e_{tot}}{\partial t} + \frac{\partial u(\rho e_{tot} + p)}{\partial x} = -2f_w \frac{\rho u^3}{D} + \frac{4q_w}{D} - h_{tot,o} S_o$$



where  $\rho$ ,  $u$ ,  $e_{tot}$  and  $p$  are respectively the mixture density, velocity, total energy and pressure,  $u_o$ ,  $h_{tot,o}$  and  $S_o$  are respectively the velocity, stagnation enthalpy and the mass flux source term associated with the local discharge flow.

# Physical properties involved

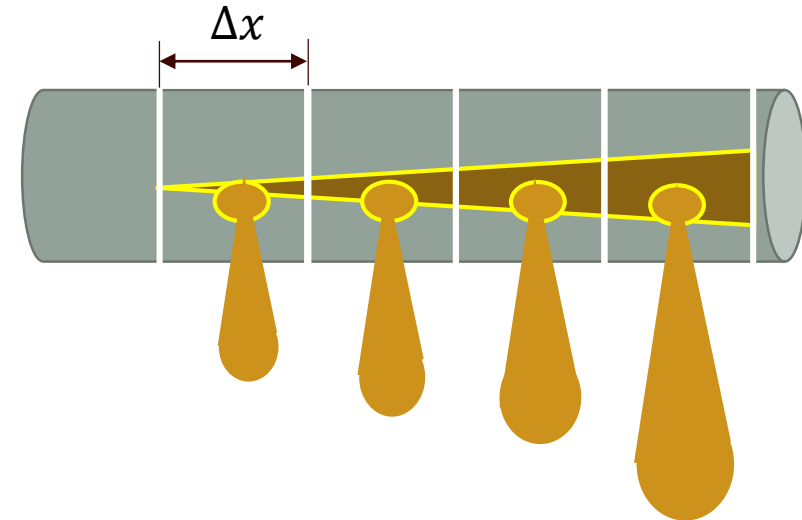
Discharge flow model – choked (critical) flow:

$$S_o = \frac{C_d \rho_o u_o A_o}{A \cdot \Delta x}$$

$$s_o = s_{up}$$

$$h_o + \frac{u_o^2}{2} = h_{tot,up}$$

$$u_o = c_{s,o}$$



where  $C_d$  is the local discharge coefficient for the rupture,  $\Delta x$  is the cell width,  $A$  is the pipe cross-sectional area,  $s$  is the entropy and the index "up" refers to stagnation conditions in the flow.



# Physical properties involved

**Density-energy flash calculations** are performed as part of solution of the conservation equations describing the flow inside the pipe:

$$p, T, x = f(\rho, e)$$

**Stagnation enthalpy – entropy flash calculations** to obtain properties of choked (critical) flow:

$$c_s, \rho = f(s, H)$$

## Physical Properties:

- Density
- Heat Capacity
- Speed of sound
- Joule-Thomson effect
- Phase equilibrium
- Viscosity
- Diffusivity
- Thermal conductivity



## Mathematical model of pipeline flow:

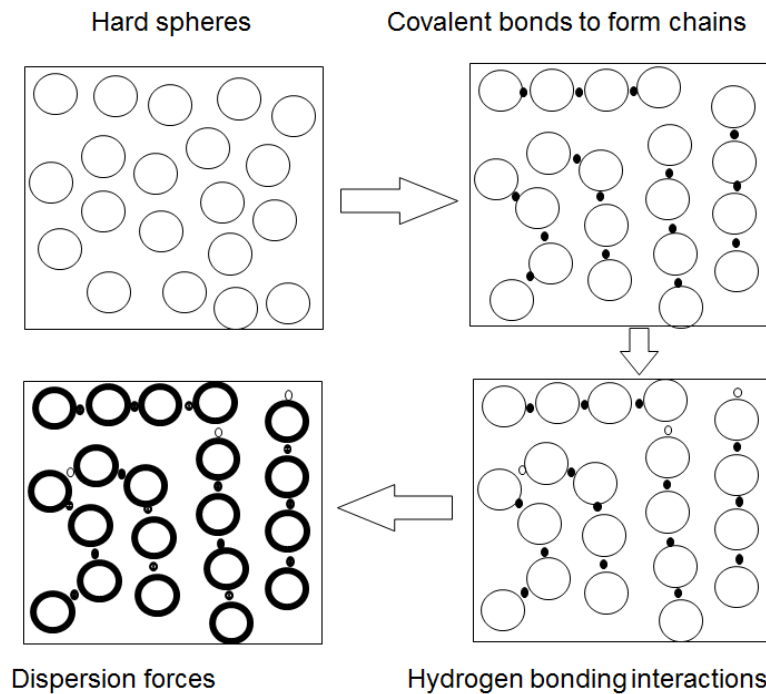
- Mass conservation
- Momentum conservation
- Energy conservation
- Viscous friction
- Heat transfer
- Interphase H&M transfer
- Fluid/structure interaction

# Equation of state models

- Peng – Robinson (PR) EoS: 
$$P = \frac{RT}{v - b} - \frac{a(T)}{v(v + b) + b(v - b)}$$

*Easy to implement,  
computationally efficient*

- Perturbed Chain – Statistical Association Fluid Theory (PC-SAFT) EoS:



$$a(T, \rho) = a^{\text{ideal}}(T, \rho) + a^{\text{hard-sphere}}(T, \rho) + a^{\text{dispersion}}(T, \rho) + a^{\text{chain}}(T, \rho) + a^{\text{association}}(T, \rho)$$

*Highly-accurate, but can be  
computationally demanding*

# Equation of state models

- To speed-up the properties calculations, **interpolation tables** can be used instead of EoS, provided that the interpolation method is:
  - accurate and robust, and
  - suits the *pipe flow* and *discharge flow models*

# Objectives

- To develop the physical properties interpolation method for use in the pipeline decompression flow simulations
- To apply the method for calculation of physical properties of real fluids
- To evaluate the accuracy and computational efficiency of the interpolation method

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# Physical properties inverse interpolation

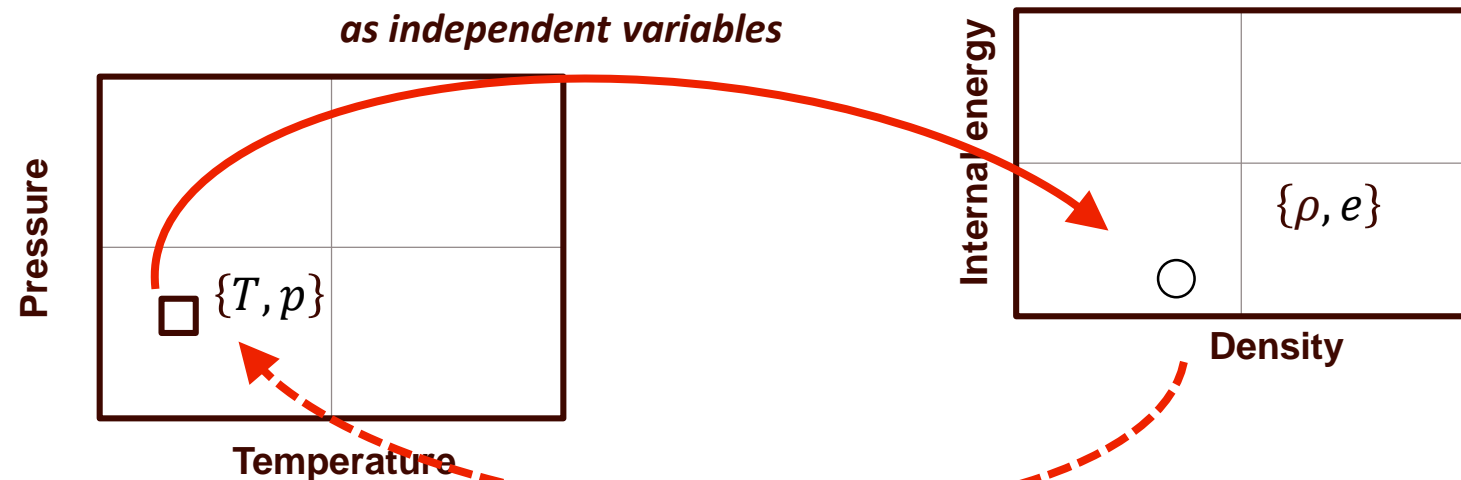
Pipe flow model equations:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = -S_o$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2}{\partial x} = -\frac{\partial p}{\partial x} - 2f_w \frac{\rho u^2}{D} - uS_o$$

$$\frac{\partial \rho e_{tot}}{\partial t} + \frac{\partial u(\rho e_{tot} + p)}{\partial x} = -2f_w \frac{\rho u^3}{D} + \frac{4q_w}{D} - h_{tot,o}S_o$$

Step 1: Construct the interpolant data using EoS (*direct mapping*) with  $\{T, p\}$  as independent variables



Step 2: Interpolation (*inverse mapping*)

$$p, T, x, \dots = f(\rho, e)$$

# Physical properties inverse interpolation

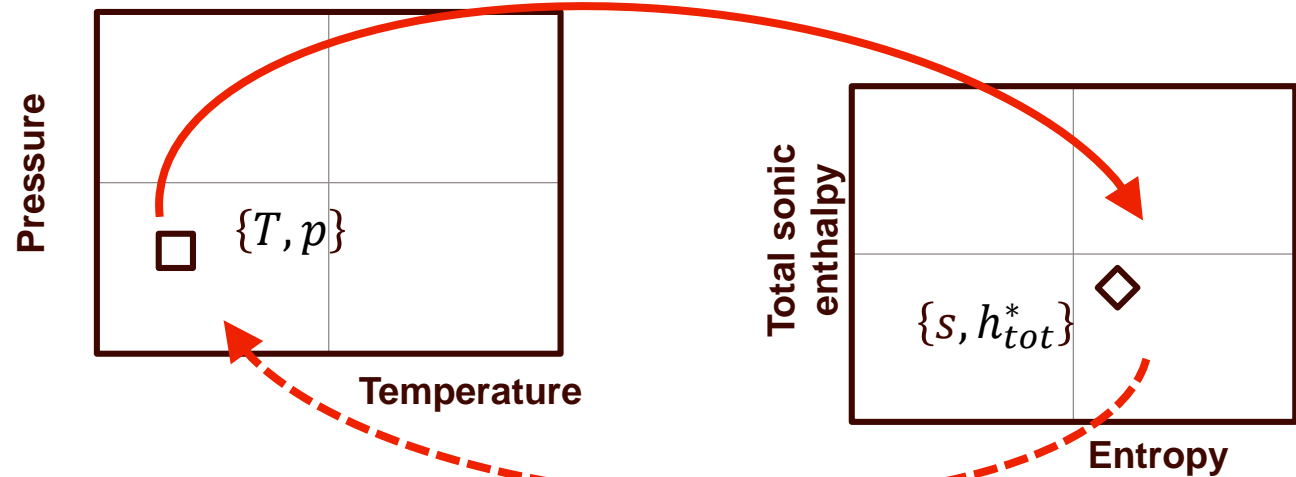
Critical discharge flow model:

$$s_o = s_{up}$$

$$h_o + \frac{c_{s,o}^2}{2} = h_{tot,up}$$

$h_{tot,o}^*$

Step 1: Construct the interpolant data using EoS (*direct mapping*) with  $\{T, p\}$  as independent variables



Step 2: Interpolation (*inverse mapping*)

In this work the inverse interpolation method is applied for the entropy - stagnation enthalpy flash calculations

$$p, T, \rho, x, \dots = f(s, h_{tot})$$

# Physical properties of homogeneous vapour-liquid mixture

$$e = e_v x + e_l (1 - x)$$

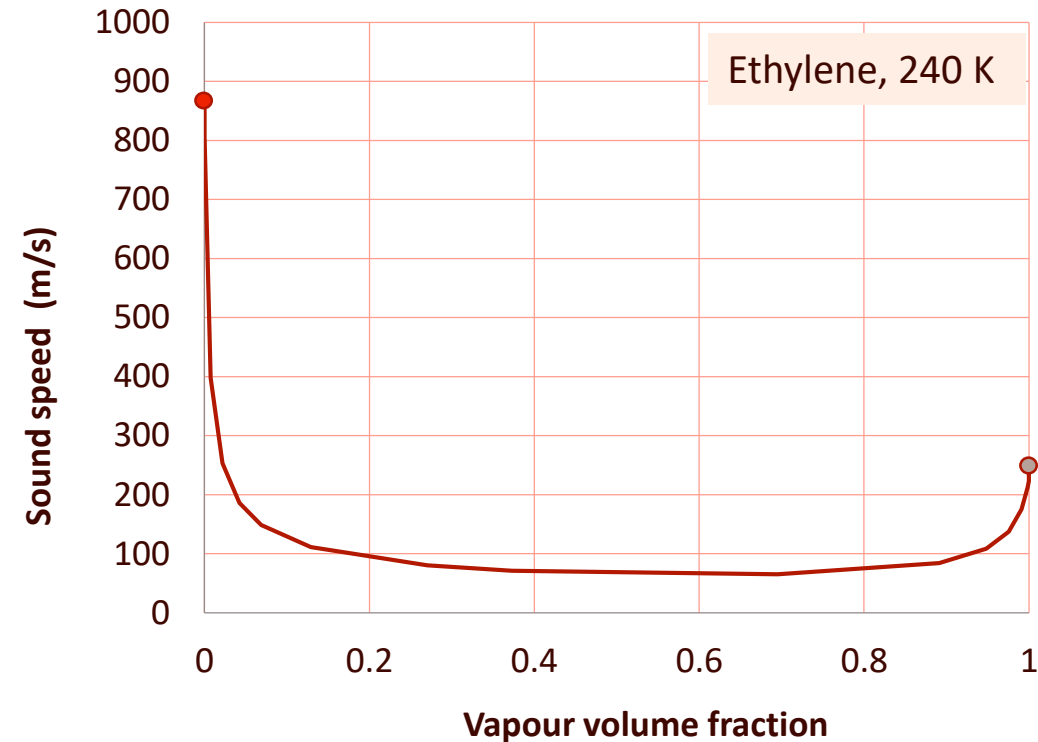
$$s = s_v x + s_l (1 - x)$$

$$\rho = \rho_v \alpha + \rho_l (1 - \alpha)$$

The speed of sound in a two-phase homogeneous frozen mixture (Wood, 1930):

$$c_s = \left[ \frac{\rho \alpha}{\rho_v c_{s,v}^2} + \frac{\rho (1 - \alpha)}{\rho_l c_{s,l}^2} \right]^{-1/2}$$

Where  $x$  is the vapour mass fraction,  
 $\alpha$  is the vapour volume fraction.



The speed of sound in saturated VLE mixture of ethylene, predicted using the homogeneous frozen mixture model.



# Physical properties inverse interpolation method

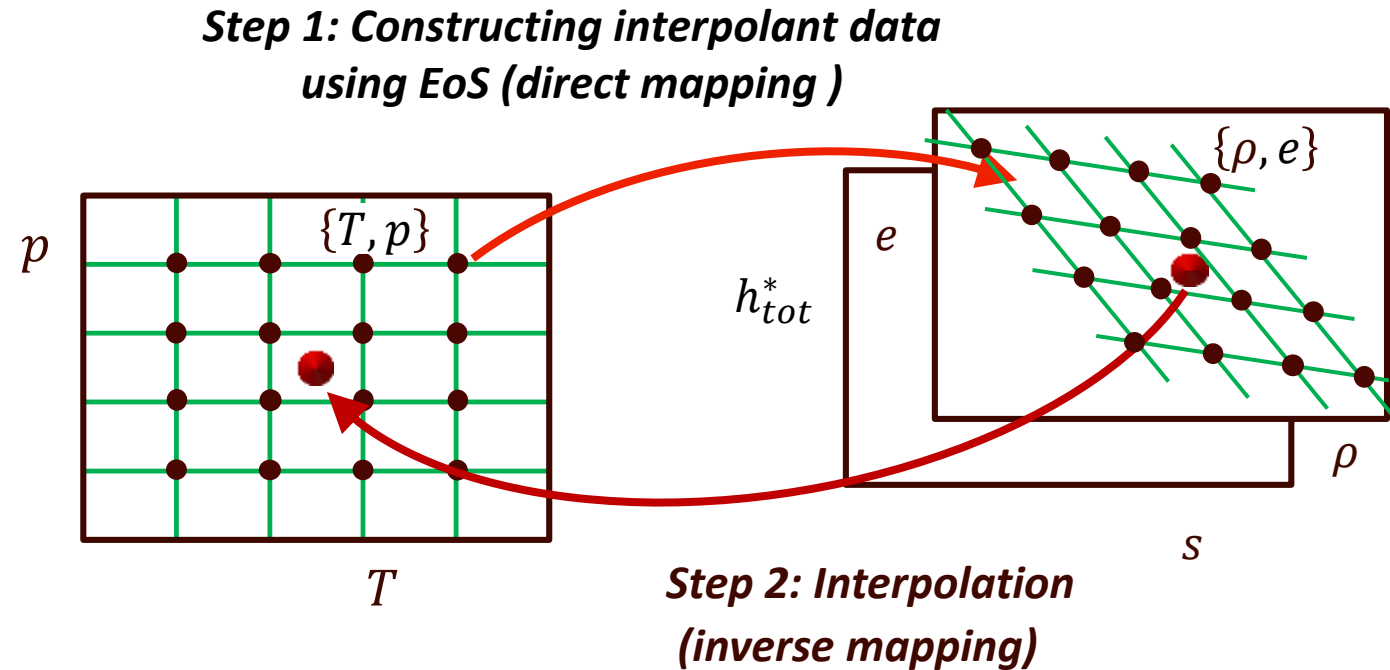
**Step 0:** Construct the  $\{T, p\}$  grid by seeding non-uniformly points along isotherms in the  $T - p$  domain.

**Step 1: Direct mapping:** Use EoS to obtain the interpolant data on  $\{T, p\}$ :

- internal energy ( $e$ ),
- density ( $\rho$ ),
- mass fraction ( $x$ ),
- entropy ( $s$ ), and
- the total sonic enthalpy  $h_{tot}^*$ :

$$h_{tot}^* = h + \frac{c_s^2}{2}$$

**Step 2:** Use the interpolant data to fit Akima splines for the *inverse* interpolation, e.g.:  $p(\rho, e)$  and  $T(\rho, e)$ .

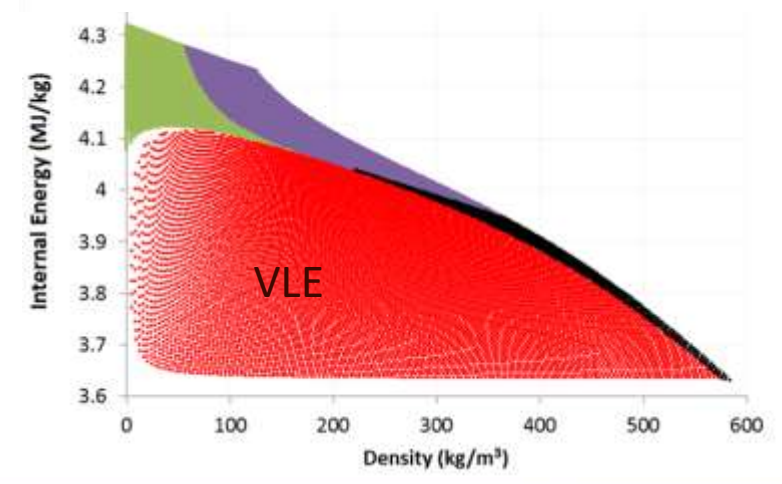
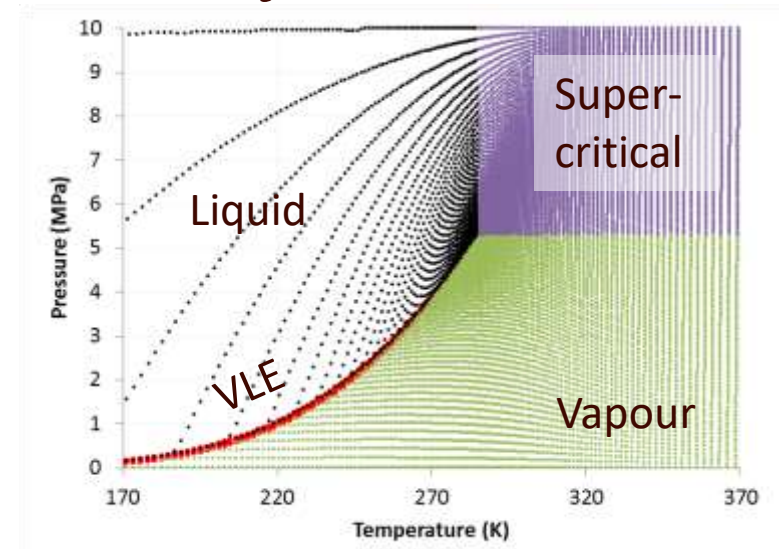
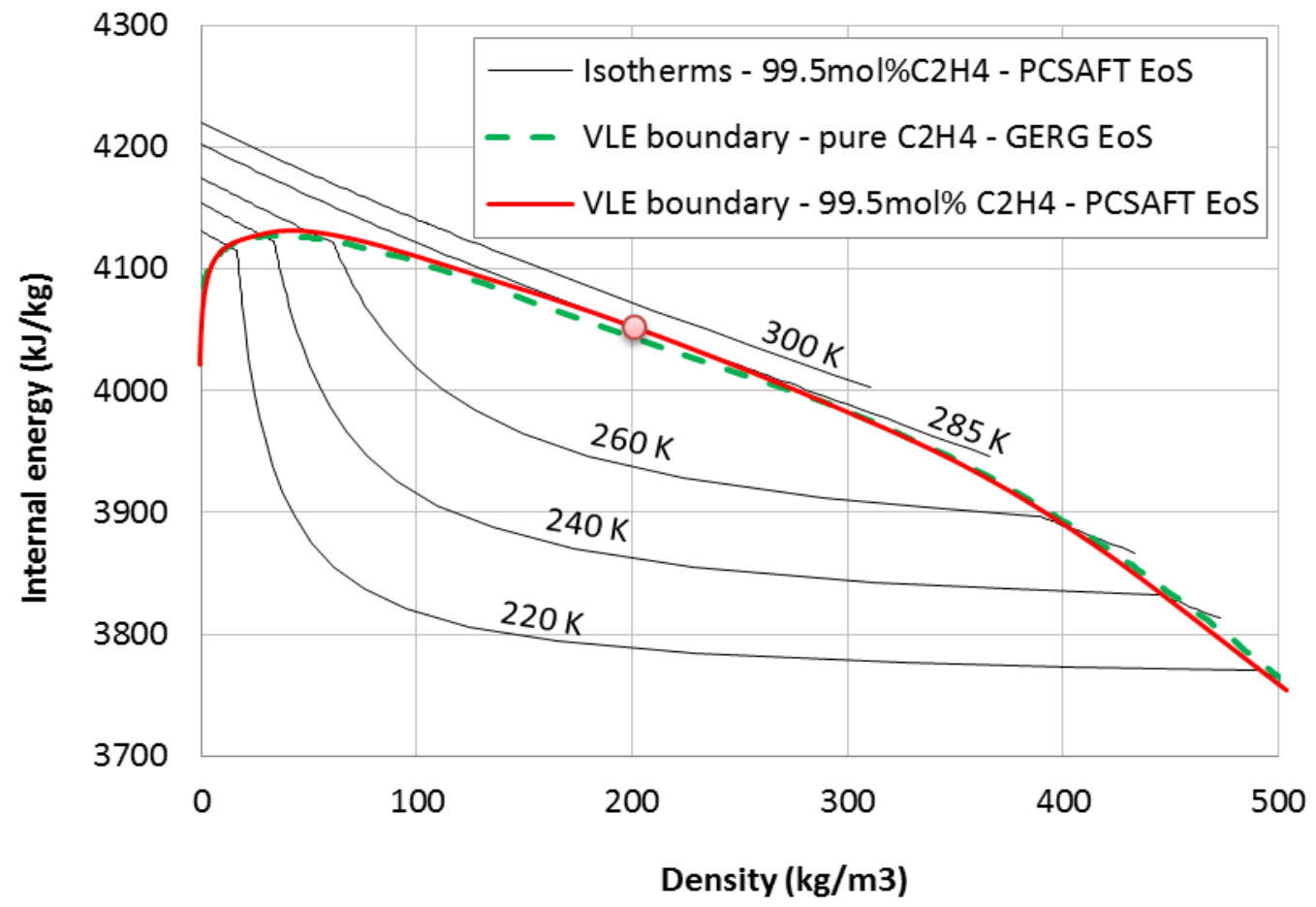


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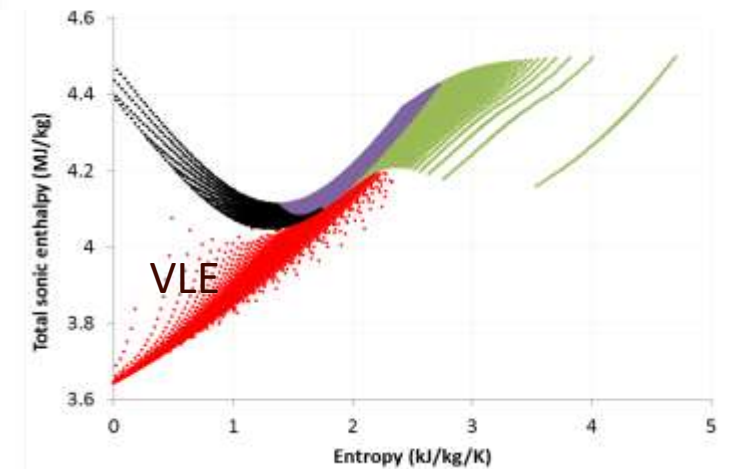
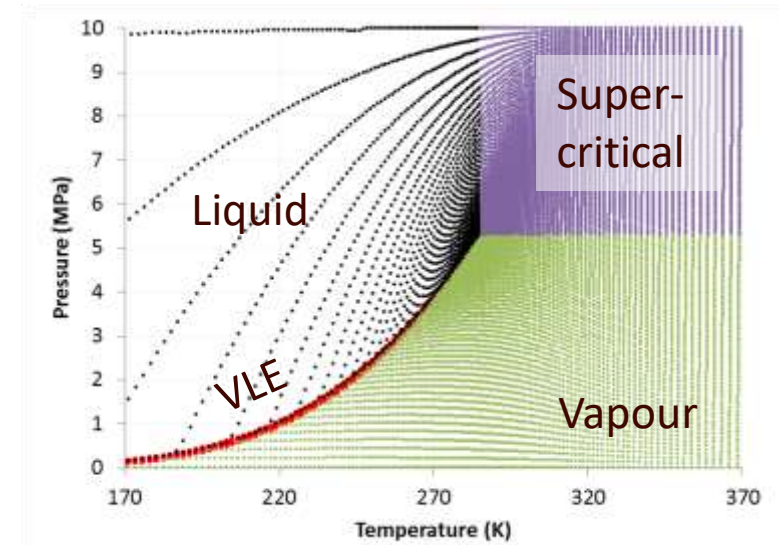
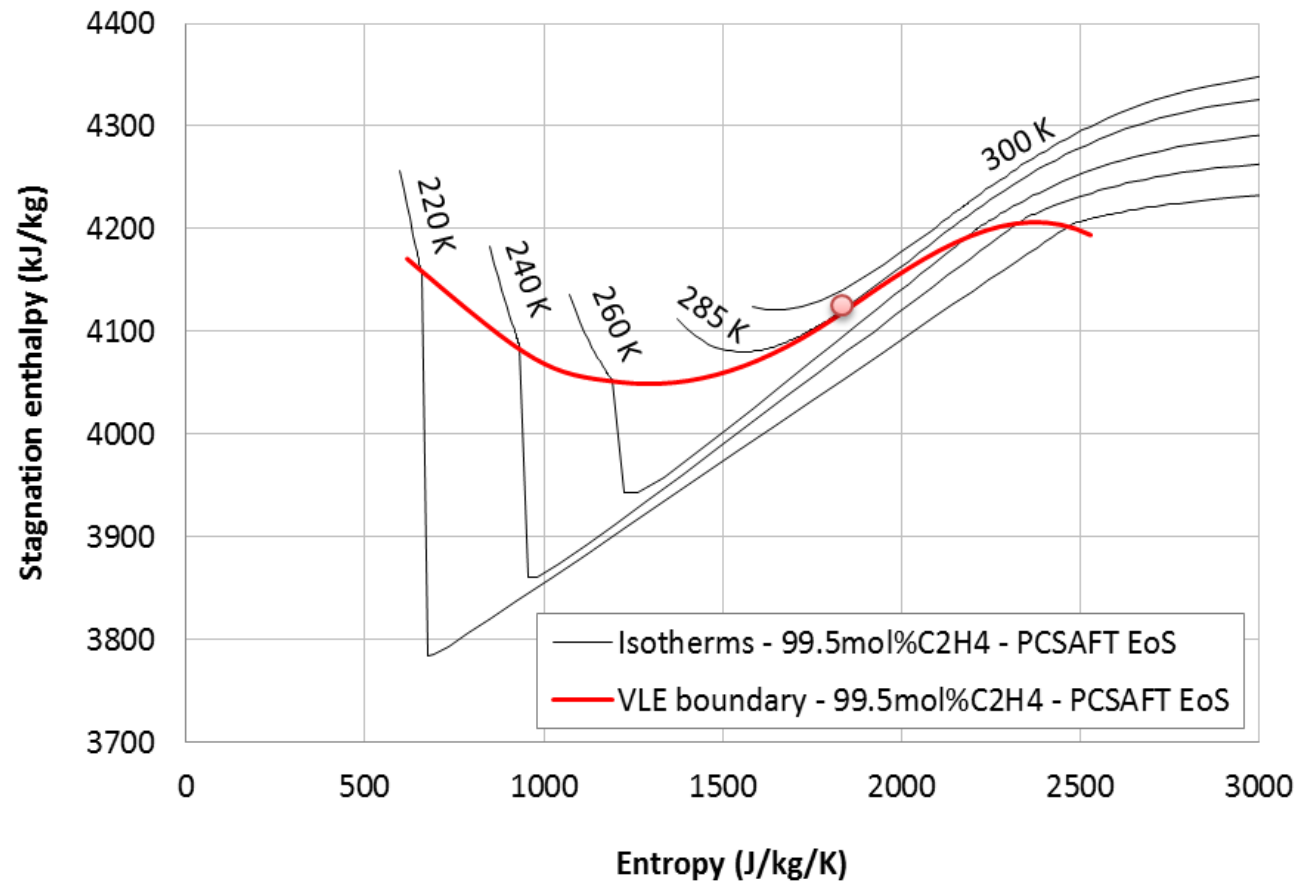


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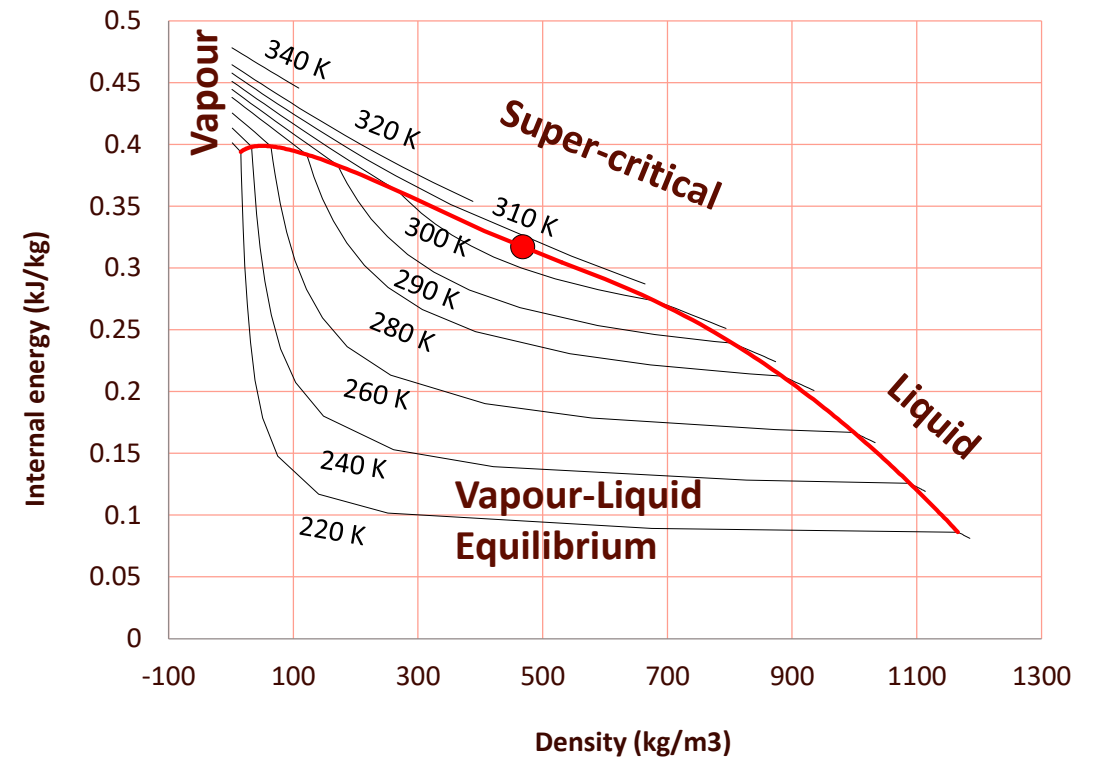
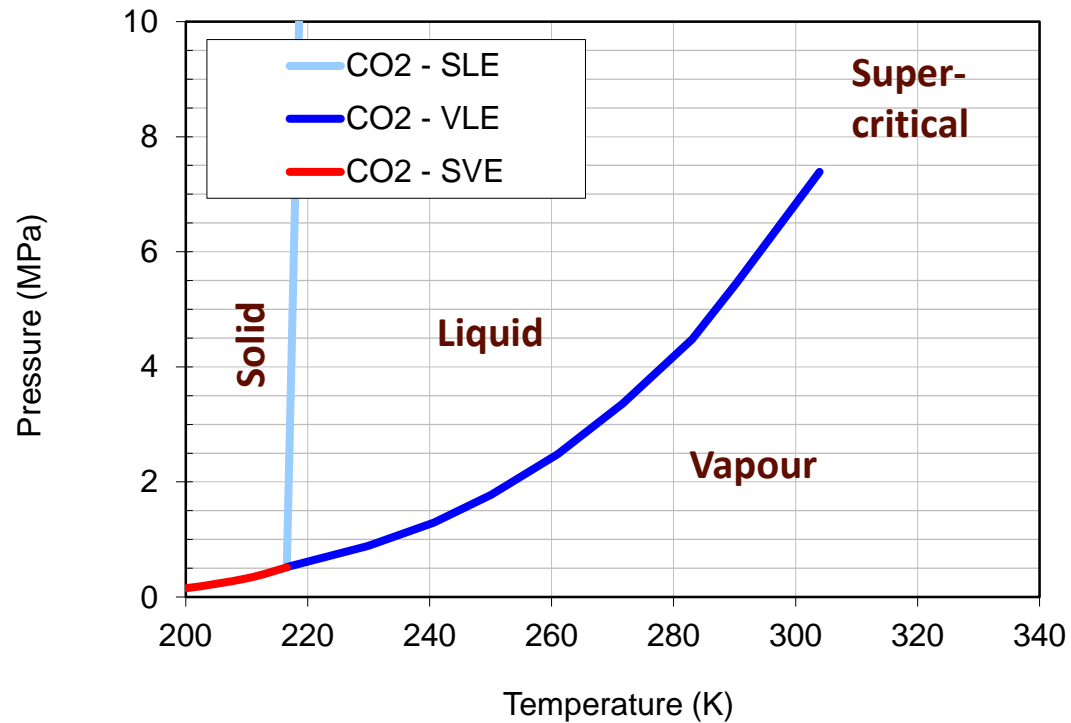
# Density-energy ( $\rho$ - $e$ ) interpolant data – Ethylene



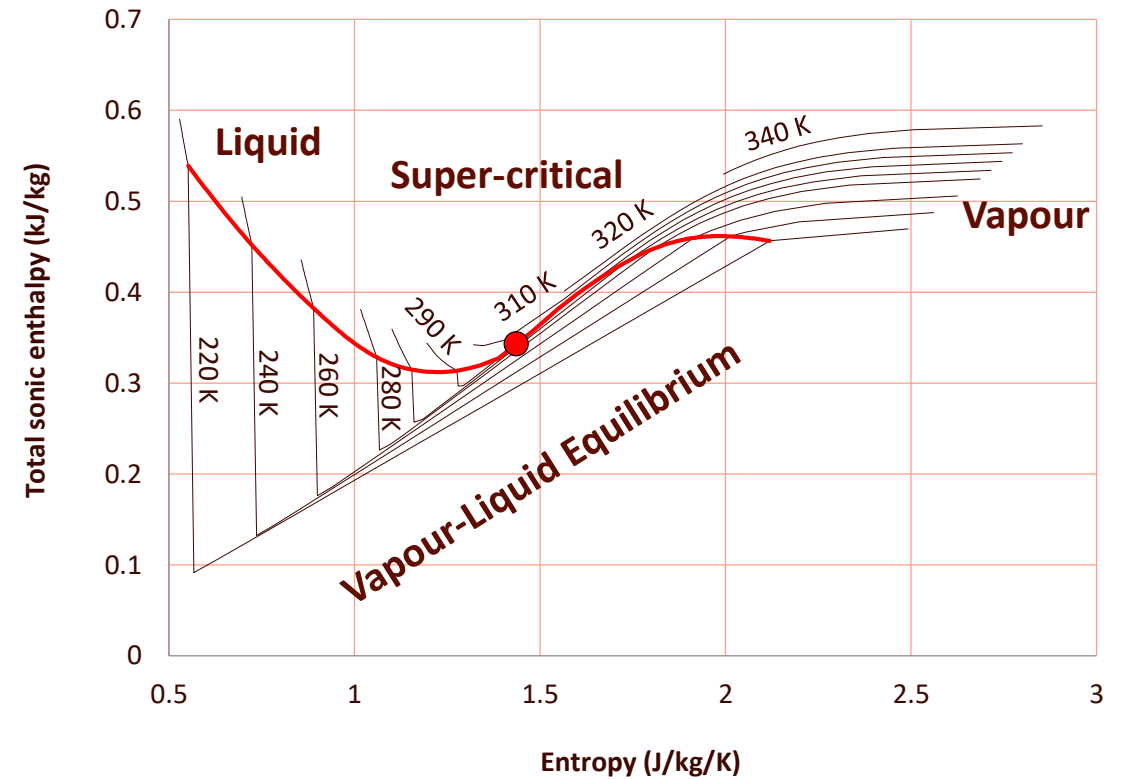
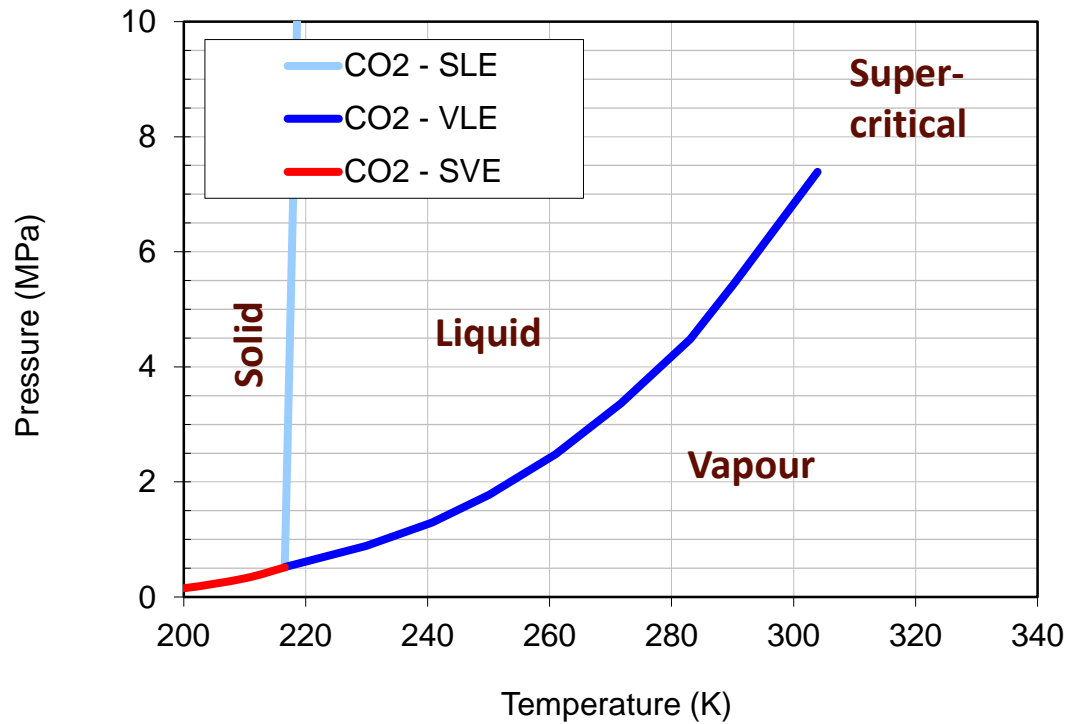
# $(h_{tot} - s)$ interpolant data – Ethylene



# Density-energy ( $\rho$ - $e$ ) phase diagram – CO<sub>2</sub>

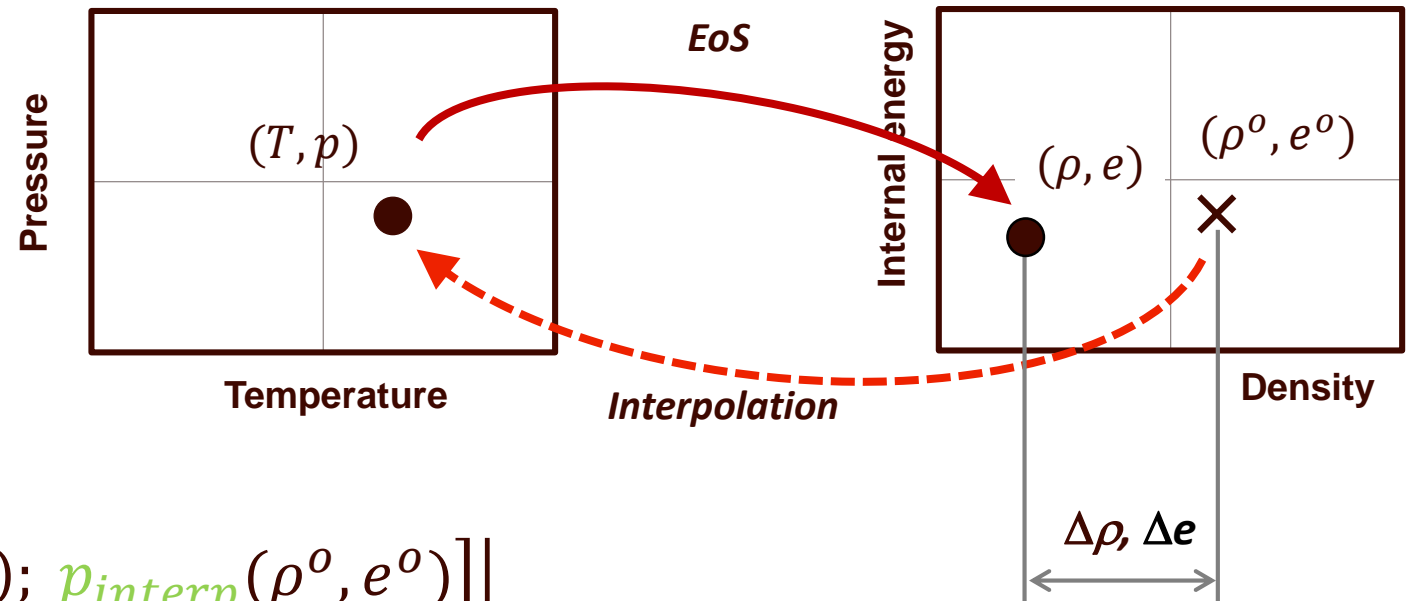


# $(h_{tot} - s)$ phase diagram – CO<sub>2</sub>



# The accuracy of the interpolation method

The accuracy of the inverse interpolation method is assessed by comparing the original input data  $(\rho^0, e^0)$  used for the inverse mapping with the  $(\rho, e)$  data returned by EoS based on the interpolation results  $(T, p)$ :

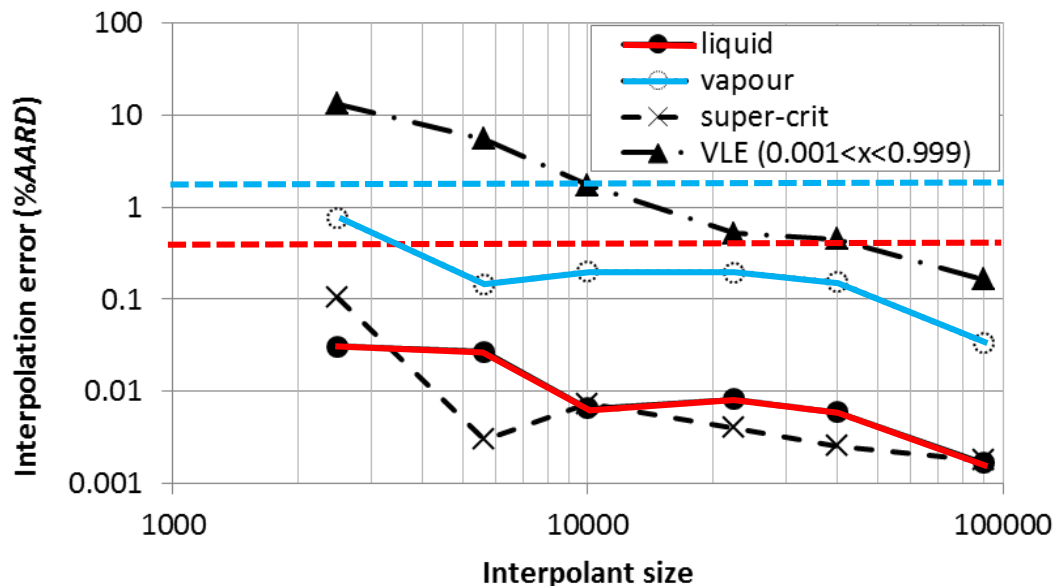


$$\Delta\rho = \left| \rho^0 - \rho_{EOS} \left[ T_{interp}(\rho^0, e^0); p_{interp}(\rho^0, e^0) \right] \right|$$

# The accuracy of the interpolation method (ethylene)

$$\%AARD = \frac{100}{n} \sum_{i=1}^n \left| \frac{\phi_i - \phi_i^o}{\phi_i^o} \right|$$

where  $\phi_i^o$  and  $\phi_i$  are respectively the reference value and the interpolated property at a point  $i$ , and  $n$  is the number of points in the thermodynamic phase region.



The accuracy of the PC-SAFT EoS (Nikolaidis, et al 2018):

- the saturated vapour : 2.37%,
- the saturated liquid: 0.42%,
- the supercritical fluid: 1.24%.

**The interpolation method' errors are the order of magnitude smaller than the accuracy of EoS.**

Relative errors of density predictions by the PC-SAFT based on the interpolated  $p$ - $T$  data, and with the accuracy of PC-SAFT EoS [Nikolaidis et al, 2018).

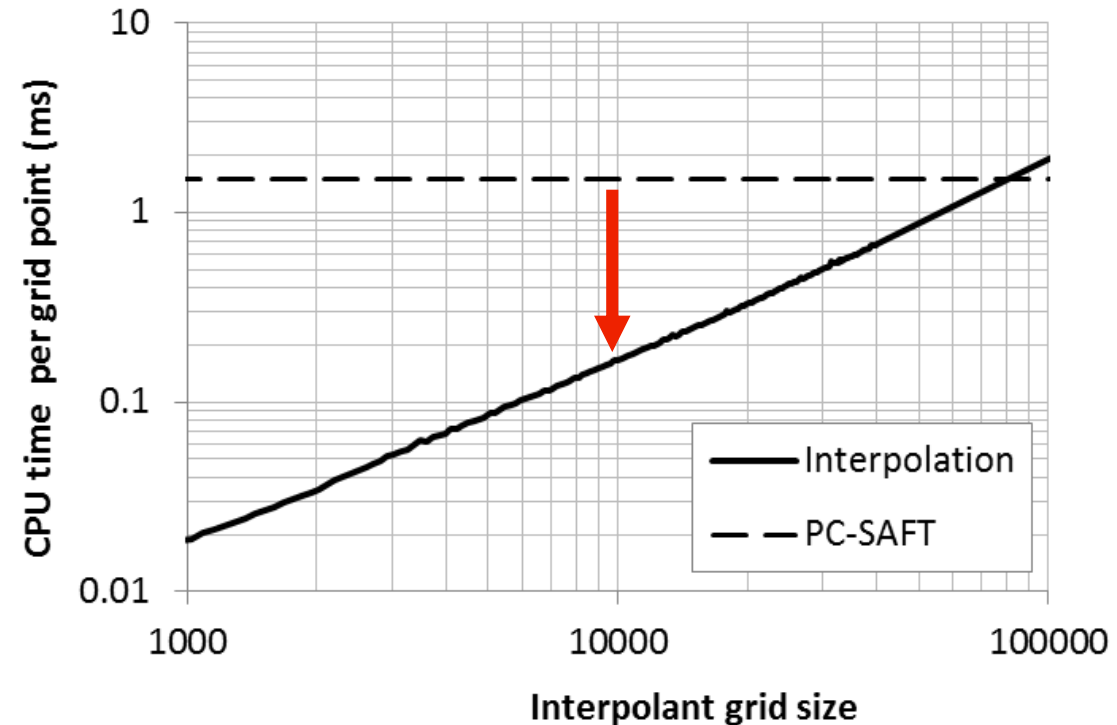


# The computational runtime (ethylene)

$$\tau_{CPU} = \frac{1}{N_{ref}} \sum_{i=1}^{N_{ref}} \tau_i$$

where  $\tau_i$  is the CPU time spent on interpolation at a point  $i$

**The proposed interpolation method can speed up the property calculations, compared to using directly the PC-SAFT EoS, when using less than 80,000 interpolant points**



Average computational runtimes ( $\tau_{CPU}$ ) spent on the interpolation of density and using directly the PC-SAFT EoS.

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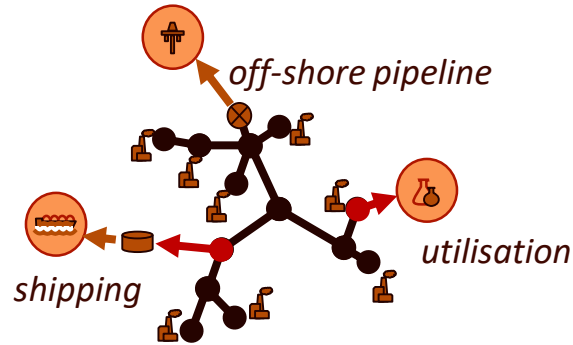


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# Conclusions and next steps

- ✓ Constructed  $h_{tot}^* - s$  phase diagrams for the inverse interpolation flash calculations of critical (choked) flow
- ✓ Developed methodology for assessment of accuracy & computational efficiency of interpolation tables
- ✓ Demonstrated the methods for ethylene
  
- ✓ Ongoing work: validation against pipeline decompression data for ethylene and applying the method to carbon dioxide

# H2020 project: C<sup>4</sup>U - Advanced Carbon Capture for Steel Industries Integrated in CCUS Clusters



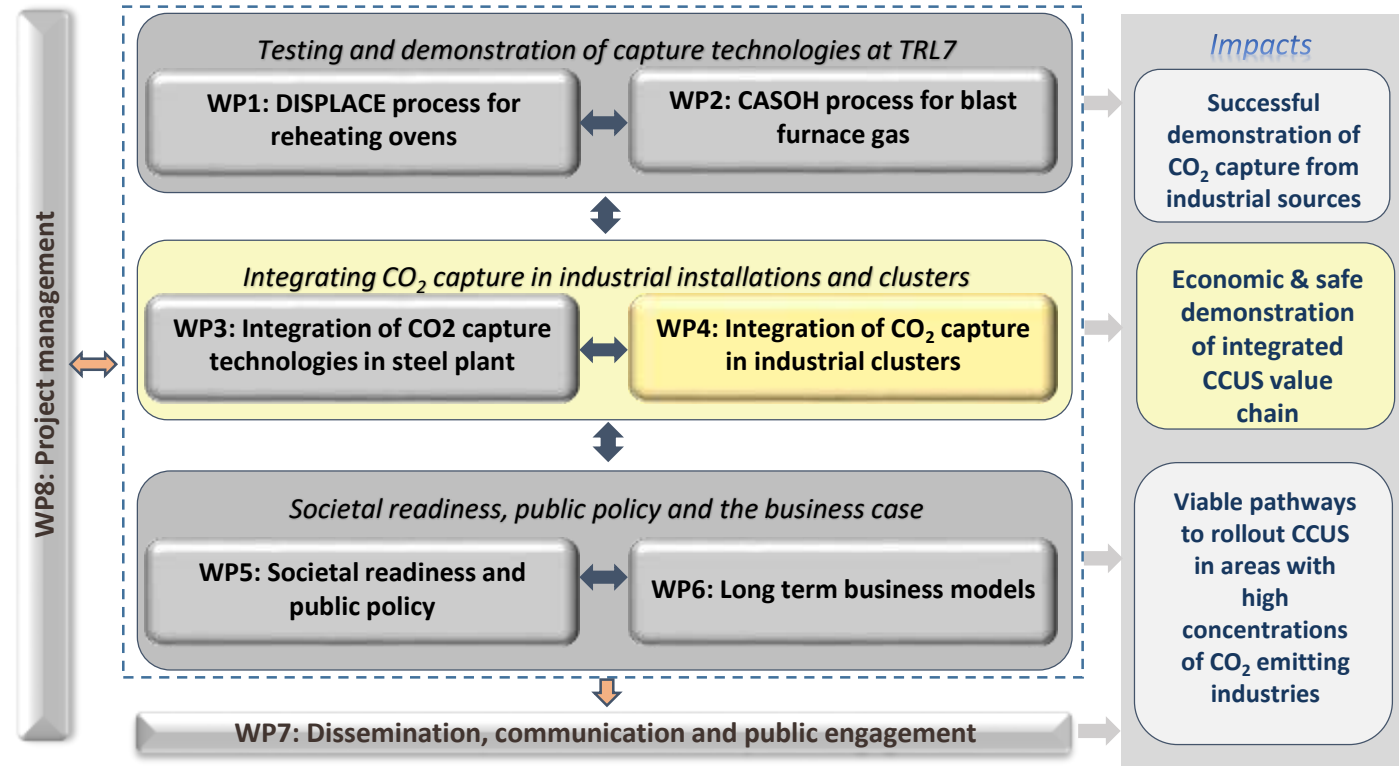
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# References

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- Diamantonis, N. et al. (2013). Evaluation of cubic, SAFT, and PC-SAFT equations of state for the vapor-liquid equilibrium modeling of CO<sub>2</sub> mixtures with other gases. *Industrial and Engineering Chemistry Research*, 52, 3933–3942.
- Nikolaidis, I., e al. (2018) Modeling of Physical Properties and Vapor - Liquid Equilibrium of Ethylene and Ethylene Mixtures with Equations of State. *Fluid Phase Equilib*, 1–15.

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- The statements made herein are solely the responsibility of the authors.