Engaging Central Banks in Climate Change?

The Mix of Monetary and Climate Policy

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Highlights

The role of central bank in climate change is investigated using an E-DSGE model.

A “climate-augmented” monetary policy rule is pioneeringly proposed and studied.

Different climate policies have different influences on price level and inflation.

Monetary policy can be improved when existing climate policy is considered.

There are risks to care for the climate proactively by using narrow monetary policy.

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Abstract

Given the recent debate on the role of central banks under climate change, this research theoretically investigates the mix of monetary and climate policy and provides insights for central banks who are considering their engagement in the climate change issue. The "climate-augmented" monetary policy is pioneeringly proposed and studied. We build an extended Environmental Dynamic Stochastic General Equilibrium (E-DSGE) model as the method. By this model, we find the following results. First, the making process of monetary policy should consider the existing climate policy since it is a factor that can influence price level and inflation. Second, the reaction coefficients in traditional monetary policy rule can be better set to enhance welfare when climate policy is given. This provides a way to optimise the policy mix. Third, if a typical-form climate target is augmented into the monetary policy rule, a dilemma could be created. This means that it has some risks for central banks to care for the climate proactively by using the narrow monetary policy (interest rate).

Keywords: Central Bank, Climate Change, Monetary Policy, Climate Policy, E-DSGE

1. Introduction

Should central banks engage in climate change issue? In 2015, a report published by the Bank of England[1] proposed that climate change could pose a risk to financial stability and economic development. Since then, and especially after the signing of the Paris Agreement, climate change and broader environmental issue have become a factor that central banks are called on to consider. By forming the Network of Central Banks and Supervisors for Greening the Financial System (NGFS) in 2017, many central banks are starting to investigate ways to manage risks from climate change and to support a green economic transition. However, these arguments and actions do not mean that it is totally justifiable for central banks to engage in climate change issue without condition. Some experts worry that such engagement could not only deviate central banks' market neutrality and original mandate, but also overburden their policy tools (violate the Tinbergen Rule[2]). There are over 50% of the surveyed experts do not support changing the European Central Bank's mandate to incorporate the EU's target of carbon neutrality by 2050.[3] The momentum in policy practice and the debate on the feasibility of the engagement naturally raise the need for research on the monetary policy under climate change considerations.

In academia research, the exacerbated climate change and environmental challenge have brought new waves of research in the "environmental macroeconomics" (Hassler et al., 2016). Since 2010, some theoretical frameworks have been founded and applied to assess how environmental and climate risks and relevant policies could affect the

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[1] Bank of England's Prudential Regulation Authority (2015). The impact of climate change on the UK insurance sector. https://www.bankofengland.co.uk/prudential-regulation/publication/2015/the-impact-of-climate-change-on-the-uk-insurance-sector

[2] This rule argues that one economic policy tool can only be used for one policy target. Accordingly, monetary policy cannot realise both central banks' traditional target and the new climate target.

[3] Ilzetzki, E. & Jia, J. (2021). The ECB's green agenda. https://voxeu.org/article/ecb-s-green-agenda

macroeconomy. The “Environmental Dynamic Stochastic General Equilibrium (E-DSGE)” model has been newly developed as a mainstream method. Angelopoulos (2010), Fischer and Springborn (2011), Heutel (2012), Golosov et al. (2014), Doda (2014), Annicchiarico and Di Dio (2015), and Dissou and Karnizova (2016) investigated relationships between greenhouse gas (GHG)/pollutant emissions and business cycles by setting GHG/pollutant as an externality in the economy and determined how climate/environmental policies influence either fluctuation or economic growth. Other researchers have studied the effect of weather on economic volatility. Chen (2014) built a model with weather shocks embedded and found that it had good explanatory power for China’s business cycle. Gallic and Vermandel (2020) found that weather shocks account for a very significant proportion of economic volatility in the long run. Of those policy-related studies, two types of climate policy, namely cap-and-trade (permitting) and taxing, are the main subjects of focus. For example, Golosov et al. (2014) tried to find the optimal level of taxing fossil fuels. Dissou and Karnizova (2016) compared the different implications of reducing CO2 emissions with carbon permits and carbon taxes in place. Annicchiarico et al. (2021) reviewed existing literature related to business cycles and the design and effects of environmental policies.

At first glance, monetary policy and climate issues are seemingly unrelated. However, such traditional notion starts changing. According to the above research, climate factors and policies are proven to influence either the fluctuation or the growth of the economy, which is exactly what monetary policy cares about. Hence, some researchers have started to investigate the role of central banks and monetary policy under climate change. Pioneering discussions, including Haavio (2010), Campiglio (2016), Ma (2017), Bolton et al. (2020) and Svartzman et al. (2020), have qualitatively explained the linking mechanism between monetary policy and climate change. Particularly, Krogstrup and Oman (2019) point out that the mix of macroeconomic and financial policies for climate change mitigation needs further investigation. McKibbin et al. (2020) argues that central banks should anticipate and respond to inflation increases and output decreases that result from climate policy.

Quantitatively, Annicchiarico and Di Dio (2017) were the first to use an E-DSGE model to study the mix of monetary and climate policy. They compared three specific mixes and showed that the optimal mix allows a slight price fluctuation when GHG emissions are considered. Economides and Xepapadeas (2018) compared monetary policy both with and without considering climate change in the model and found that the reaction of monetary policy to economic shocks will be affected by climate change. Wang et al. (2019) introduced green tilted policies and found that interest subsidies, directional reduction of reserve ratio requirements, and central bank relending could all be effective ways of incentivizing green loans. Punzi (2019) introduced borrowing constraints and heterogeneous production sectors into the model to investigate green financing activity and found that only the differentiated capital requirement policy can sustain green financing. Huang and Punzi (2019) incorporated financial friction, according to Bernanke et al. (1999), and found that environmental regulations can accelerate the risks that the financial system faces. Chan (2020) introduced environmental targeting carbon taxation, fiscal, and monetary policies and compared their different effects in terms of improving the environment and welfare. Dollman et al. (2020) used a specially developed model "G-cubed" and found that carbon tax shock can provoke significant monetary policy action in both the near and the medium to long term. Benmir and Roman (2020) introduced financial frictions and green and dirty production sectors to assess different types of fiscal, monetary, and macroprudential policies aimed at reducing CO2 emissions. Boser and Colesanti Senni (2020) studied "emission-based interest rates" using a dynamic general equilibrium model and showed that it can support the decarbonization of the economy and reduces climate damage. Carattini et al. (2021) introduced pollution market failure and a market failure in the financial sector and found that macroprudential policy alone, without a carbon tax, is not very effective at addressing the pollution externality.

These scholars can be regarded to have started a new discussion on monetary policy and the environment. However, because of the growing global enthusiasm for sustainability, central banks are expected to respond to more concerns about this issue. It includes macroeconomic and financial stability implications of climate change, risks of stranded assets, relationship between monetary policy and both climate change and climate policy, how to encourage green finance, the cost and benefit of "green monetary policy", and many other aspects.[4] Many specific concerns have not been touched upon by previous works.

In this research, we aim to investigate the relationship between and the mix of monetary and climate policy, and so to provide some insights for central banks who are considering their engagement in the climate change issue. We will answer three new and relevant questions: (1) Whether and how monetary policy is influenced by climate policy? (2) Whether and how monetary policy can be improved when the climate policy is considered in the framework of analysis and whether there is an optimal monetary policy? and (3) Should a central bank adopt a "climate-augmented" monetary policy or use monetary policy to care for the climate proactively? Answering these questions can help deepen the understanding of the role of central bank and the design of monetary policy under climate change.

Our research method is an extended E-DSGE model. The basic DSGE setting is in line with the standard New Keynesian framework. The basic "Environmental" features are introduced following Annicchiarico and Di Dio (2017) by incorporating the GHG emissions from production, their negative externality on productivity, and the climate policy that controls emissions[5], i.e., cap-and-trade or carbon tax. To consider the

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[4] Please refer to the NGFS's "Technical Supplement" to the "First Comprehensive Report", "The Macroeconomic and Financial Stability Impacts of Climate Change Research Priorities" a nd NGFS's research priorities listed by The International Network for Sustainable Financial Pol icy Insights, Research, and Exchange (INSPIRE).

[5] We do not consider other climate-related policies, e.g. adaptation policy.

environmental module in a more comprehensive way, we also introduce some novel environmental features into the model: the concealed emissions, the potential penalty for them, and the effectiveness (stringency) of climate policy enforcement. These new features are omitted by most previous E-DSGE models, but actually common in the reality and found to be nontrivial by our analysis.

Based on the E-DSGE model, we first mix monetary policy (of Taylor rule type (Taylor, 1993), which is a close approximation of the real-world) with different climate policies and compare these mixes to see what differences climate policy can bring to monetary policy and the economy. Impulse responses of major economic and environmental variables to shocks and welfare of individuals are calculated. The results show that when monetary policy is mixed with different types and effectiveness (stringency) of climate policy, price level and inflation in the economy are different. The making process of monetary policy should consider the existing climate policy.

We then explore a traditional way to improve the mix of monetary and climate policy. This is to optimise the reaction coefficients in the Taylor rule of monetary policy. The results show that the coefficients can always be better set to enhance welfare when the existing climate policy is considered in the framework of analysis. If the cost-push shock is dominant in the economy, optimal coefficients exist. Both the type and effectiveness of climate policy can affect the value of the optimal coefficients.

Finally, we propose to improve the policy mix by introducing a radical "climate-augmented" monetary policy rule, which can help determine whether it is good for a central bank to use the narrow monetary policy (interest rate rule) to care for the climate proactively. This is to introduce an emission gap target into the Taylor rule of monetary policy. The results show that the welfare of the economy can be enhanced when monetary policy is augmented by the new target and the reaction coefficient of the target is set in a specific interval. However, under some circumstances, such a monetary policy could create a dilemma for central banks. This indicates a risk if we directly use the narrow monetary policy (interest rate) to care for the climate.

The novelty of this research lies in three aspects. First, the research topic. Besides being among the first bunch of discussion and modelling work on monetary policy in the context of climate change, this research pioneeringly investigates the question of "Should central banks engage in climate change issue?" and studies the "'climate-augmented' monetary policy rule" in a formal model. This help answer questions raised by the NGFS community. Second, the research scope. Extending Annicchiarico and Di Dio (2017) who considered either monetary or climate policy as Ramsey type in the policy mix, we work on mixes with both of the two policies non-Ramsey optimised, which can better represent the real-world. Third, the research method. The traditional E-DSGE model is firstly enriched with concealed emission-related features so that its environmental module is more comprehensive and closer to reality, and the analysis of the effectiveness (stringency) of climate policy enforcement is possible.

The paper proceeds as follow. Section 2 describes the extended E-DSGE model. Section 3 compares the mixes of monetary policy with different climate policies. Section 4 investigates the optimisation of policy mixes. Sections 5 concludes.

2. Model

We construct an extended E-DSGE model based on the New Keynesian framework. GHG emissions from production, their negative externality on productivity, and climate policies that control emissions are introduced following Annicchiarico and Di Dio (2017). Innovatively, concealed (illegal) emissions, the potential penalty for them, and the effectiveness (stringency) of climate policy enforcement are introduced into the model. This is to depict the reality that in many countries environmental regulations are not fully effective, and firms have some space to emit more than the legal level. Such new features, although omitted by most previous E-DSGE models, are found to be nontrivial by our analysis.

2.1 Household

A representative household maximises its expected lifetime utility, which is determined by consumption C_t and labour L_t and has the form of

$$\mathbb{E}_0\left\{\sum_{t=0}^{\infty}\beta^t S_t\left(\ln C_t-\mu_L\frac{L_t^{1+\eta}}{1+\eta}\right)\right\} \tag{1}$$

where $0<\beta<1$ is the discount factor, $\eta\geq 0$ is the inverse of the elasticity of labour supply, and $\mu_L>0$ is the coefficient of the disutility of labour. S_t represents the stochastic shocks of time-preference, which follows $\ln S_t=\rho_S\ln S_{t-1}+(1-\rho_S)\ln S+e_{S,t}$ to evolve, where $0<\rho_S<1$ and $e_{S,t}\sim i.i.d.N(0,\sigma_S^2)$.

The budget constraint of the household is

$$P_tC_t+R_t^{-1}B_{t+1}=B_t+W_tL_t+D_t+P_tT_t \tag{2}$$

where P_t is the price of final good, B_t and B_{t+1} are the nominal quantity of riskless bonds at period t and $t+1$, R_t is the riskless interest rate of the bonds which is determined by the central bank, W_t is the nominal wage of labour, D_t denotes the nominal dividend derived from enterprises, and T_t is the lump-sum transfer from government.

At the optimum, we have the following first-order conditions

$$\beta R_t\mathbb{E}_t\left[\frac{S_{t+1}}{S_t}\frac{C_t}{C_{t+1}}\frac{1}{\Pi_{t+1}}\right]=1 \tag{3}$$

$$L_t^{\eta}=\frac{W_t}{\mu_L P_t C_t} \tag{4}$$

where $\Pi_{t+1}=P_{t+1}/P_t$ is the inflation of period $t+1$. Equation (3) is the Euler equation, and equation (4) is the labour supply equation.

2.2 Enterprise and the Environment

Consistent with the standard New Keynesian framework, the enterprise sector is formed by final good and intermediate good producers. The final good Y_t is produced by competitive firms using the Constant Elasticity of Substitution (CES) technology

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{\theta_{t-1}}{\theta_t}} dj\right]^{\frac{\theta_t}{\theta_{t-1}}} \tag{5}$$

where $Y_{j,t}$ denotes the intermediate goods produced by monopolistically competitive firms, and the subscript $j \in [0,1]$ denotes the intermediate good firms of a continuum. $\theta_t > 1$ is the elasticity of substitution and is also a stochastic process that describes the cost-push shock (Smets and Wouters, 2003). It follows $\ln\ \theta_t = \rho_\theta \ln\ \theta_{t-1} + (1-\rho_\theta)\ln\theta + e_{\theta,t}$ with $0 < \rho_\theta < 1$ and $e_{\theta,t} \sim i.i.d.N(0,\sigma_\theta^2)$.

Final good producers maximise their profit, which is determined by

$$P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj \tag{6}$$

The first-order condition yields the demand function for intermediate goods

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\ \theta_t} Y_t \tag{7}$$

and

$$P_t = \left[\int_0^1 P_{j,t}^{1-\ \theta_t} dj\right]^{\frac{1}{1-\ \theta_t}} \tag{8}$$

which implies that the price of final good P_t is also the price level.

A typical intermediate good firm has a production function

$$Y_{j,t} = \Lambda_t A_t L_{j,t} \tag{9}$$

where A_t is the total factor productivity (TFP) factor or technology that follows a stochastic process $\ln A_t = \rho_A \ln A_{t-1} + (1-\rho_A)\ln A + e_{A,t}$, in which $0 < \rho_A < 1$ and $e_{A,t} \sim i.i.d.N(0,\sigma_A^2)$. Following Golosov et al. (2014), Λ_t is a damage coefficient that describes the negative externality of GHG emissions on productivity (TFP damage coefficient). It is the pivot linking the economy and the environment. Λ_t is determined by the stock of emissions following

$$\Lambda_t = \mathrm{e}^{-\chi(M_t - \widetilde{M})} \tag{10}$$

where M_t is the stock of emissions of period t, $\widetilde{M}$ is the level before the industrial revolution, and χ>0 measures the intensity of negative externality.

According to Heutel (2012), GHG is a by-product of the production process. The original emissions from production are $Z_{j,t}^{ori}$ which is proportional (measured by φ) to the volume of output of intermediate firms

$$Z_{j,t}^{ori} = \varphi Y_{j,t} \tag{11}$$

To dispose of the original emissions, a firm has three channels to use and trade-off: emission abatement (by, e.g., carbon capture, utilisation and storage, CCUS), legally emitting after paying for tax/permit, and secretly emitting (concealed emission). A firm can choose to abate a percentage of $U_{t,j}$ $(0 \leq U_{t,j} \leq 1)$ of the original emissions which will bring a marginal increasing cost of $\phi_1 U_{j,t}{}^{\phi_2} Y_{j,t}$, where $\phi_1 = \phi_1' \varphi > 0$ and $\phi_2 > 1$ are cost coefficients. A firm can also choose to legally emit some original emissions. This requires the firm to pay for a carbon tax or buy an emission permit in the cap-and-trade system (depending on the existing type of climate policy) at a price $p_{Z,t}$ for every unit of GHG emissions.

The novelty of our model is the introduction of the concealed emission channel and related penalty for such emissions. Normally, a government or environmental authority cannot detect every source of pollution. So, firms have some space to emit secretly, making their real emission level higher than the legal level for which they have either paid tax or bought a permit. The secret or concealed emissions could save some costs for emission abating and tax/permit. Meanwhile, the concealed emissions are subject to a potential penalty. Although a government may not be able to know every concealed emission, they usually have some degree of inspection and regulation on such emissions and will penalise the emitters spotted (most commonly by fine or prosecution). A recent example of concealed emission and related penalty is the Volkswagen emissions scandal in 2015. The Volkswagen company concealed its cars' excessive emissions by technical manipulation for years. It was detected by chance and then the company has faced a huge amount of fine by governments.

To abstract the above facts, we assume that firms can choose to conceal a percentage of $V_{t,j}$ $(0 \leq V_{t,j} \leq 1)$ of the original emissions; the government spots the concealed emissions with a certain probability (the lower, the weaker the effectiveness of climate policy enforcement). If spotted, the government penalises the firm with a certain amount of fine (the fewer, the weaker the effectiveness). To model this, we assume that a firm faces an expected fine that equals to $\frac{\psi}{2}V_{t,j}^2\varphi Y_{j,t}$, where $\varphi Y_{j,t} = Z_{j,t}^{ori}$ is the original emissions; $\psi > 0$ reflects the "effectiveness of climate policy enforcement", which is proportional to the probability of the government spotting concealed emissions and the amount of the fine for every unit of concealed emissions. Using the $\frac{\psi}{2}V_{t,j}^2\varphi Y_{j,t}$ term as the amount of fine is derived from a simple intuition: the more concealed emissions that are emitted and spotted or the more effective the climate policy enforcement is, then the greater the fine. In the term, $V_{t,j}$ is quadratic, which means that the total amount of fine is marginally increasing with regard to $V_{t,j}$. This is derived from another simple intuition: the more a firm emits concealedly, the easier are the emissions to be spotted.

The introduction of concealed emissions and effectiveness of climate policy enforcement relaxes the hidden assumption of the perfect effectiveness of policy enforcement in most previous E-DSGE models and makes our model closer to reality. Such introduction is nontrivial for this particular research since we will show that the differences in effectiveness of policy enforcement will make different policy mixes either more similar or more different and further influence the dynamics of financial and economic variables (see Subsection 3.3).

The three channels by which firms can dispose of their original emissions, namely emission abatement, legally emitting, and concealedly emitting, have now all been explained. This helps illuminate the following variables. The real emissions $Z_{j,t}^{real}$ is the amount of GHG finally emitted to the atmosphere, via both the legal and the concealed channels. It equals to the original emissions $Z_{j,t}^{ori}$ minus the abated emissions $Z_{j,t}^{abate} = U_{t,j}\varphi Y_{j,t}$. The legal emissions $Z_{j,t}^{legal}$ is the amount of GHG emissions that a firm reports to the government. It is the amount that a firm needs to pay for tax or buy permit for. It equals to the real emissions minus the concealed emissions $Z_{j,t}^{concealed}$. Accordingly, we have

$$Z_{j,t}^{real} = Z_{j,t}^{ori} - Z_{j,t}^{abate} = \left(1 - U_{t,j}\right) Z_{j,t}^{ori} = Z_{j,t}^{legal} + Z_{j,t}^{concealed} \tag{12}$$

$$Z_{j,t}^{legal} = Z_{j,t}^{real} - Z_{j,t}^{illegal} = \left(1 - U_{t,j} - V_{t,j}\right) Z_{j,t}^{ori} \tag{13}$$

The above relationship is illustrated in Figure 1.

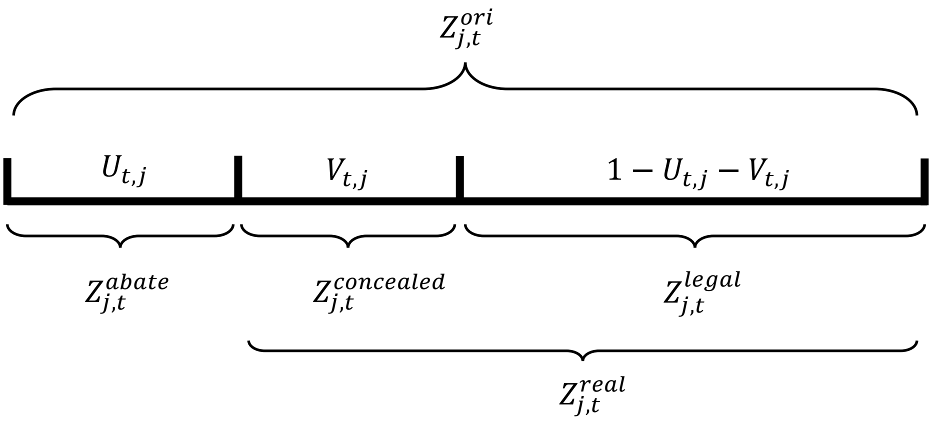

Figure 1: The relationship among emission variables

Considering the cost of disposing of emissions via the three channels and the sticky pricing assumption in the standard New Keynesian framework (Rotemberg, 1982), the objective of an intermediate firm is to maximise

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \Omega_{0,t} \left[\frac{P_{j,t}}{P_t} Y_{j,t} - TC_{j,t} - \frac{\gamma}{2} \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2 Y_t \right] \right\} \tag{14}$$

which is subject to

$$TC_{j,t} = \frac{W_t}{P_t} L_{j,t} + \phi_1 U_{j,t}^{\phi_2} Y_{j,t} + p_{Z,t} \left(1 - U_{j,t} - V_{j,t}\right) \varphi Y_{j,t} + \frac{\psi}{2} V_{j,t}^2 \varphi Y_{j,t} \tag{15}$$

where $\Omega_{0,t} = \beta^t \frac{C_0}{C_t}$ is the stochastic discount factor.

The above settings and assumptions yield the following first-order conditions (more details in Appendix)

$$(1-\theta_t)-\gamma(\Pi_t-1)\Pi_t+\beta\gamma\mathbb{E}_t\left[\frac{C_t}{C_{t+1}}(\Pi_{t+1}-1)\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\right]+\theta_t MC_t=0 \quad (16)$$

$$MC_t=\frac{W_t}{\Lambda_t A_t P_t}+\phi_1 U_t^{\phi_2}+p_{Z,t}(1-U_t-V_t)\varphi+\frac{\psi}{2}V_t^2\varphi \quad (17)$$

$$p_{Z,t}=\frac{1}{\varphi}\phi_1\phi_2 U_t^{\phi_2-1} \quad (18)$$

$$V_t=\frac{1}{\psi\varphi}\phi_1\phi_2 U_t^{\phi_2-1}=\frac{p_{Z,t}}{\psi} \quad (19)$$

where MC_t is the marginal cost of production, $\gamma>0$ is the coefficient of price adjusting cost, and $\Pi_t=\frac{P_t}{P_{t-1}}$ denotes inflation. Equation (16) is the New Keynesian Phillips Curve.

2.3 Monetary and Environmental Authorities

The monetary policy authority (central bank) decides the nominal interest rate following a traditional Taylor rule[6]

$$\frac{R_t}{R}=\left(\frac{\Pi_t}{\Pi}\right)^{\rho_\Pi}\left(\frac{Y_t}{Y_t^{na}}\right)^{\rho_Y} \quad (20)$$

where Y_t^{na} is the natural output without price stickiness, R and Π are the steady state of nominal interest rate and inflation, and ρ_Π and ρ_Y are the reaction coefficients for inflation and output gap, respectively. The Taylor rule type monetary policy is a closer approximation of the real-world than the Ramsey monetary policy. We do not consider the latter in this research.

The environmental authority implements climate policy and penalise concealed emitters spotted. In this research, we analyse two major types of climate policy: cap-and-trade (CA) and carbon tax (TX). Under the CA policy, the environmental authority sets an emission cap Z_t^{cap} and sells emission permits to the market at a price decided by the market competition. In equilibrium, the total legal emissions Z_t^{legal} equates to

[6] Different from Annicchiarico and Di Dio (2017), the Taylor rule in this paper targets on output gap rather than output itself. According to Gali (2015), the output gap target is better for improving welfare and is the choice of the standard Taylor rule.

Z_t^{cap}. Under the TX policy, the authority sets a fixed carbon tax level for every unit of legal emissions. The authority does not set a ceiling for total legal emissions. We also include two other types of climate policy, no control on emissions (NO) and Ramsey optimal emission control (RM), for benchmarking purpose in the following analysis. The earnings of environmental authority, including the income from selling emission permits or levying a carbon tax and fines for concealed emissions, are transferred to households.

2.4 Market Clearing and Aggregation

In equilibrium, we have the market-clearing condition

$$Y_t = C_t + \phi_1 U_t^{\phi_2} Y_t + \frac{\gamma}{2}(\Pi_t - 1)^2 Y_t \tag{21}$$

Following Rotemberg (1982), we assume that all the firms are symmetrical. So, the gross variables share the same form of expressions with individual variables. The total production function is

$$Y_t = \Lambda_t A_t L_t \tag{22}$$

The totalities of emissions are

$$Z_t^{legal} = \int_0^1 Z_{j,t}^{legal} dj = (1 - U_t - V_t)\varphi Y_t \tag{23}$$

$$Z_t^{real} = \int_0^1 Z_{j,t}^{real} dj = (1 - U_t)\varphi Y_t \tag{24}$$

The total transfer is

$$B_t + P_t T_t = R_t^{-1} B_{t+1} + \left(p_{Z,t} Z_t^{legal} + \frac{\psi}{2} v_t^2 \varphi Y_t\right) P_t \tag{25}$$

The total stock of emissions is

$$M_t = (1 - \delta_M) M_{t-1} + Z_t^{real} + \tilde{Z} \tag{26}$$

where $\tilde{Z}$ is the emissions from nature without human influence, and $0 < \delta_M < 1$ is the natural rate of decay of GHG stock.

2.5 Calibration

We calibrate the parameters as follows and list them in Table 1. Following Gali

(2015), the discount factor β is set as 0.99, the elasticity of substitution in steady state θ is set as 6, and the inverse of the Frisch elasticity η is set as 1. The adjusting cost coefficient γ, which measures price stickiness, is set as 58.25 so that the stickiness has a duration of three quarters when it is converted into Calvo pricing. The disutility coefficient of labour μ_L is set as 24.9983 so that the steady state of labour is 0.2 without monopoly. Following tradition, the persistent coefficients of shocks (including TFP shock, preference shock, and cost-push shock) are set as 0.9, and the reaction coefficients in Taylor-rule of monetary policy ρ_Π and ρ_Y are set as 1.5 and 0.5, respectively, in Section 3. Following Annicchiarico and Di Dio (2015)'s way of calibration, the scale coefficient of abatement cost ϕ_1 is set as 0.185, and the elasticity ϕ_2 is set as 2.8. The parameter determining the damage caused by emissions on output χ is set as 0.000457. Following Heutel (2012), the decay rate of emission stock δ_M is set as 0.0021. Following Xu et al. (2016), the coefficient measuring the original emissions per unit of output φ is set as 0.601. As for the effectiveness of climate policy enforcement ψ, according to the proportion of government "environmental penalties" in total GDP in China, which is approximately 0.01%,[7] the ψ should be approximately 0.45. This is within the magnitude of 0.1 to 1. For comparison purposes, we need to set a large ψ and a small ψ. Considering the magnitude, the benchmark ψ (in Subsection 3.1 and 3.2) is set as 1, which is the upper bound of the magnitude; the value describing a relative ineffective enforcement is set as 0.1 (in Subsection 3.3), which is the lower bound.

Table 1: Calibrated values of the parameters

Parameter		Value	Target
β	Discount factor	0.99	$\beta = \frac{1}{1+\rho}$, where risk-free (pure time preference) discount rate $\rho \approx 1\%$
η	Inverse of the Frisch elasticity,	1	Literature

[7] Source: The State Council of China http://www.gov.cn/xinwen/2019-02/26/content_5368758.htm

μ_L	Disutility coefficient of labour	24.9983	Steady labour time is 0.2 under fully competition market
θ	Elasticity of substitution in steady state	6	Literature
γ	Adjusting cost coefficient of sticky price	58.25	Literature
ρ_A	Persistent coefficient of TFP shocks.	0.9	Commonly used value
ρ_S	Persistent coefficient of preference shocks.	0.9	Commonly used value
ρ_θ	Persistent coefficient of cost-push shocks.	0.9	Commonly used value
ϕ_1	Scale coefficient of abatement cost	0.185	Literature
ϕ_2	Elasticity of abatement cost	2.8	Literature
χ	Intensity of negative externality	0.000457	Literature
φ	Emissions per unit of output in the absence of abatement	0.601	Literature
ψ	Effectiveness of climate policy enforcement	0.1, 1	Proportion of environmental punishment cost in GDP
δ_M	Decay rate of GHG stock	0.0021	Literature
A	TFP in steady state	5.1151	Steady output is 1 under fully competition market
S	Preference in steady state	1	No influence at steady state
ρ_Π	Policy Response to Inflation	0.5	Literature
ρ_Y	Policy Response to Output Gap	1.5	Literature

To test the robustness, we have tried to vary all the parameter in a reasonable interval and re-run the model. All results do not change qualitatively. Hence, the findings and conclusions in the following sections are robust in terms of parameter choice.

3. The Mixes of Monetary Policy with Different Climate Policies

In this section, we study whether and how monetary policy is influenced by climate policy. To do this, we analyse the mixes of monetary policy with four different types of climate policies: cap-and-trade, carbon tax, no control (with climate policy absent), and

Ramsey optimal, and compare the mixes in terms of differences in fluctuation and welfare. We also consider the differences brought by the (in)effectiveness of climate policy enforcement. This is an extension of Annicchiarico and Di Dio (2017), and also a pre-requisite for optimising the policy mixes in Section 4.

3.1 Fluctuation Comparison

Annicchiarico and Di Dio (2017) initiated investigating the mixes of monetary policy and climate policy by considering one policy as the Ramsey type and the other as varying types. They showed that key macroeconomic variables, including labour, emissions, interest rate, and inflation, respond differently to a productivity shock when the policy type differs. Their work is an inspiring start on such issue, meanwhile, can be extended or improved in some respects. First, at least one policy was assumed as the Ramsey type in any mix they studied. This type of policy is the ideal optimisation but difficult to carry out directly in reality. The mix that purely consists of practically realisable policies is not studied. So, such real-world practical policy mixes can be further investigated. Second, the potential ineffectiveness of policy enforcement that could change the dynamics of the economy can be considered additionally. This relaxes the hidden assumption of the perfect effectiveness of climate policy enforcement. Third, the regimes with "no climate policy" and "Ramsey climate policy" can be introduced into the comparison to serve as benchmarks.

We still compare the response of key macroeconomic variables to the productivity (TFP) shock, but extend the work of Annicchiarico and Di Dio (2017) by including the mixes of Taylor rule type monetary policy with more different (totally four) types of climate policy (constituting four regimes [8]) and by additionally considering the effectiveness of climate policy enforcement. The four types of climate policy include cap-and-trade, carbon tax, no control and Ramsey optimal (see Appendix for equations).

[8] We also call a mix of monetary and climate policy as a "regime".

The first three and the Taylor rule monetary policy are all commonly implemented in the real-world. In this subsection, we compare the fluctuation of the economy in different regimes via impulse response analysis. To be specific, we give a 1% positive TFP shock and then show the dynamics of economic variables. Here, the effectiveness of policy enforcement ψ is set as 1 as a benchmark. The values of tax level and emission target are set so that all regimes (except for the NO regime[9])[10] share the same steady state with the case of Ramsey.

The results of impulse response analysis (absolute deviation from steady states) are shown in Figure 2. It can be found that the responses of endogenous variables to the shock have different paths under the four different regimes. For economic and monetary variables, output under the CA regime increases by less than under the RM regime, whereas output under the TX regime increases by more than under the RM regime. The TFP damage coefficient (Λ_t), inflation, and the resulting interest rate under the CA regime drop less than under the RM regime, whereas under the TX regime the negative changes are larger than is the case under the RM regime. For environmental-related variables, abatement, concealed emissions, and emission price under the CA regime rise by more than under the RM regime, whereas, under the TX regime they either change less than under the RM regime or do not change. Legal emissions and real emissions under the TX regime increase by more than under the RM regime, whereas, under the CA regime, real emissions rise by less than under the RM regime, and legal emissions do not change.

The differences between regimes (note the scales of the y-axes) are not large, because the environmental-related disruption and costs (for abatement, emissions, and

[9] "NO (or TX/CA/RM) regime" is short for "regime with the NO (or TX/CA/RM) type climate policy".

[10] The No Control regime is equivalent to a TX regime with a tax level at 0. This makes the steady state different and predefined.

fines) are relatively small under current parameters.[11] The differences could be more significant in the future if the climate change problem becomes more serious. Since it could aggravate the external shock (e.g., severer weather extremes) and increase the emission-related costs.

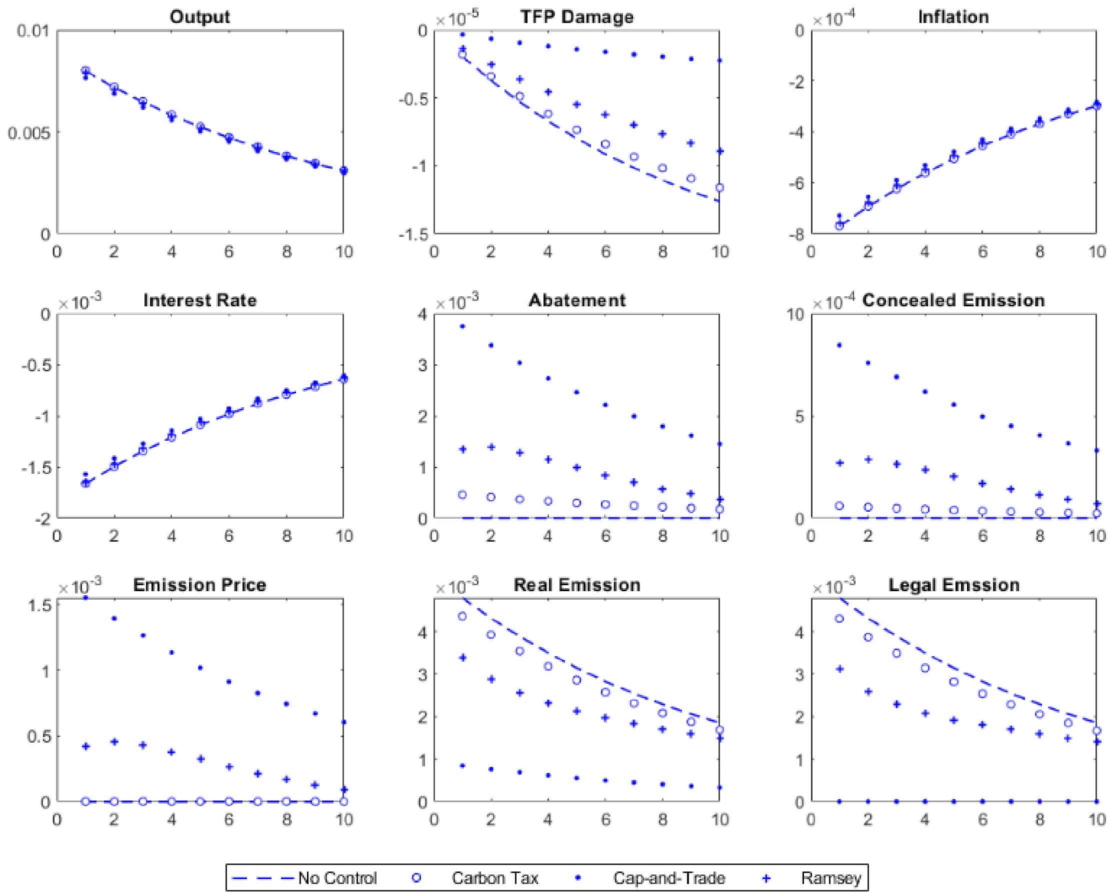

Figure 2: The dynamics of endogenous variables after a 1% positive TFP shock under different regimes ($\psi = 1$)

To understand the mechanism behind the differences in the changes, we first need to understand that after a positive TFP shock, **emission price** and **real emissions** will rise under the RM regime. When the shock happens, the TFP and output will increase linearly (equation (22)), which leads to higher original emissions. The heightened

[11] The standard deviation of Λ_t is less than 0.00027 under the CA and TX regimes. The proportion of environmental-related costs to output (GDP) at steady state is less than 0.7%.

original emissions cause a **higher marginal** damage to TFP (equation (10), non-linear) and output. So, the Ramsey optimisation requires a higher rate of abatement U_t to offset the excessively increased TFP damage. According to equation (18), the **emission price** $\boldsymbol{p_{Z,t}}$ will also be higher simultaneously. Given the raised original emissions and that the social cost of both abatement and emitting (real emissions) are marginal increasing, Ramsey policy maker will enhance both the amount of abatement and **real emission**.

Then, the differences between the CA and TX regimes can be explained. Under the TX regime (and the NO regime), the **emission prices** (for legal emissions) are fixed at the carbon tax level (or 0), irrespective of how much firms emit. After a shock, it will be lower than the Ramsey optimal (increased) emission price. The relative lower emission price has several implications: (1) On **inflation** and **output**. At optimum, all the three channels for emission dispose share the same marginal cost. As the price (marginal cost) for legal emission is fixed and lower than is the case under the RM regime, the marginal costs of abatement and concealed emission are also lower. This brings a lower marginal cost of production, which indicates a lower price (inflation) level and a higher output level (according to the basic New-Keynesian Phillips Curve) than is the RM case. (2) On **real emissions** and the **TFP damage coefficient**. Compare with the RM regime, the relatively lower abatement proportion (caused by lowered emission price, according to equation (18)) and higher output (i.e. higher original emissions) induce higher real emissions. Real emissions accumulate into emission stock and directly decrease the TFP damage coefficient (N.B., it is negative). Therefore, the TFP damage coefficient drops by more than it does under the RM regime. (3) On **legal emissions, abatement, and concealed emissions**. Compared with the RM regime, a relatively lower abatement proportion and concealed proportion (caused by lowered emission price, according to (19)) give a higher legal emission proportion. As both the proportion of legal emission and the amount of original emission are higher, legal emissions become higher than is the case under the RM regime. Although the original emission is higher, the lowered proportion of abatement and concealed emission may play a dominant role. So, the abatement and concealed emissions increase by less than is the case under the RM regime. (4) On **interest rate**. A lowered inflation rate induces a lowered interest rate (according to equation (20)) than is the case under the RM regime.

Under the CA regime, the mechanism of change is the antithesis of that under the TX regime because of an emission price higher than the Ramsey optimal level. The CA regime has a fixed amount of legal emissions, so it is lower than the Ramsey optimised (increased) level when there is a positive TFP shock. A lower legal emission level brings an emission price higher than is the case under the RM regime. The higher than RM regime emission price (which is opposite to the lower than RM regime price under the TX regime) has implications for the endogenous variables that are exactly antithetical to those implications under the TX regime. Therefore, there are differences in the dynamic of variables between the CA and TX regimes after a shock.

In the CA regime, there exists a “fluctuation offsetting” mechanism which can help stabilise the economy. This is because the fixed legal emission volume can bring a higher (lower) price for disposing of emissions when a positive (negative) TFP shock happens. This offsets the lowering (heightening) price level brought by the shock. Price change and any other fluctuation brought by price change is attenuated. Under the TX regime, the fixed emission price does not have such a function.

In general, the above analysis shows that when the type of climate policy is differed, the dynamic of monetary policy (interest rate) and the entire economy (other endogenous variables) will be different in facing an exogenous shock. Under the TX regime, monetary policy fluctuates more than it is under the RM regime; climate policy is looser than is the RM regime, which could make real emissions too high and abatement too low. Conversely, under the CA regime, monetary policy fluctuates less; climate policy is tighter than is the RM regime, which could make real emissions too low and abatement too high.

Two key messages worth emphasis: (1) The cap-and-trade type climate policy could offset the price fluctuation after a shock and become an attenuator of fluctuation. (2) The making process of monetary policy should consider the existing type of climate policy, as price level and inflation (which are the major target of monetary policy) are influenced by the type of climate policy.

3.2 Welfare Comparison

To further investigate the above policy mixes, we compare the welfare of the four regimes in addition to the above fluctuation analysis. This will help us find which of the four mixes are better and which are inferior.

In the comparison, we maintain all parameters, including the reaction coefficients in the Taylor rule and the effectiveness of policy enforcement, fixed. We set the steady states of the CA and TX regimes equal to that of the RM regime. The steady state of the NO regime comes from the $p_{Z,t} = 0$ case of the TX regime. So, the differences in welfare between the CA, TX, and RM regimes are due only to the difference in economic dynamics under different regimes. We follow the welfare criterion of Mendicino and Pescatori (2007) and calculate the conditional welfare of individuals. The expression is

$$W_j = \mathbb{E}_t \sum_{m=0}^{\infty} \beta^m \left(\ln C_{j,t+m} - \mu_L \frac{L_{j,t+m}^{1+\eta}}{1+\eta} \right) \tag{27}$$

where W_j is the conditional welfare, and $j = \{\mathrm{NO}, \mathrm{TX}, \mathrm{CA}, \mathrm{RM}\}$ means the four types of climate policy: no control, carbon tax, cap-and-trade, and Ramsey optimal.

To show results more intuitive, we also calculate the consumption equivalent (CE) of each case. CE is the additional fraction of consumption that households under no policy can obtain if a certain policy is introduced for them. Let

$$W_{j'} = \mathbb{E}_t \sum_{m=0}^{\infty} \beta^m \left[\ln\left(1 + CE_{j'}\right) C_{NO,t+m} - \mu_L \frac{L_{NO,t+m}^{1+\eta}}{1+\eta} \right] \tag{28}$$

we have

$$CE_{j'} = \exp\left\{(1-\beta)\left(W_{j'} - W_{NO}\right)\right\} - 1 \tag{29}$$

where $j' = \{\text{TX, CA, RM}\}$ represents a certain type of climate policy.

The welfares of all four regimes and the corresponding CEs are shown in Table 2.

Table 2: Welfare and Consumption Equivalents of the four regimes

	Welfare	CE
NO	-59.469	0
TX	-58.583	0.0088972
CA	-58.585	0.0088727
RM	-58.566	0.0090715

We can find

$$W_{RM} > W_{TX} > W_{CA} > W_{NO} \tag{30}$$

and equivalently

$$CE_{RM} > CE_{TX} > CE_{CA} > CE_{NO} \tag{31}$$

Specifically: (1) Any regime with a climate policy has better welfare than has the NO regime (increasing CE by 0.89%~0.91%), as any climate policy can somehow reduce emissions, and so does its externality. (2) The RM regime has the highest welfare of all the regimes. This is the nature of the Ramsey policy. (3) The TX regime is a little better than is the CA regime in terms of welfare; however, the differences between them are not big.

In terms of the welfare standard, the TX regime tends to be a better choice among the three real-world implementable regimes (CA, TX, and NO) when a TFP shock happens. However, sensitivity analysis indicates that it is not always the best choice. We find that either when the effectiveness of policy enforcement is small enough or when the shock is changed to demand-type, the result $W_{TX} > W_{CA}$ (equivalently $CE_{TX} > CE_{CA}$) will reverse to $W_{TX} < W_{CA}$ (equivalently $CE_{TX} < CE_{CA}$). Hence, among the three real-world implementable regimes, no one is always dominant over others regardless of parameters and shocks, in terms of the welfare standard.

3.3 The Role of Policy Effectiveness (Stringency)

This section investigates whether the effectiveness (stringency) of climate policy enforcement, in addition to the type of climate policy, will also affect monetary policy and the economy.

To do this, we set a lower effectiveness parameter ψ equalling to 0.1. This is a much smaller value than the benchmark case in Subsection 3.1, where $\psi = 1$. The small value means that the enforcement of climate policy is less effective. In Figure 3, we show the fluctuation of economy following the same method as in Subsection 3.1. It needs to be noted that the units of some of the vertical axes in Figure 2 and Figure 3 are different. Then, we compare the results in Figure 2 ($\psi = 1$) and in Figure 3 ($\psi = 0.1$) to identify any differences arising from the differed effectiveness of policy enforcement.

It can be found, for variables apart from legal and concealed emissions, that the differences of fluctuation between the CA and TX regimes become smaller when the effectiveness of policy enforcement is lower — mainly because the variables' paths under the CA regime are more approximate to the paths under the TX regime. Under the TX regime, legal emissions change by more than is the case when policy enforcement is more effective. Under the CA regime, concealed emissions change more. This makes the mixes with different types of climate policy become more similar to each other.

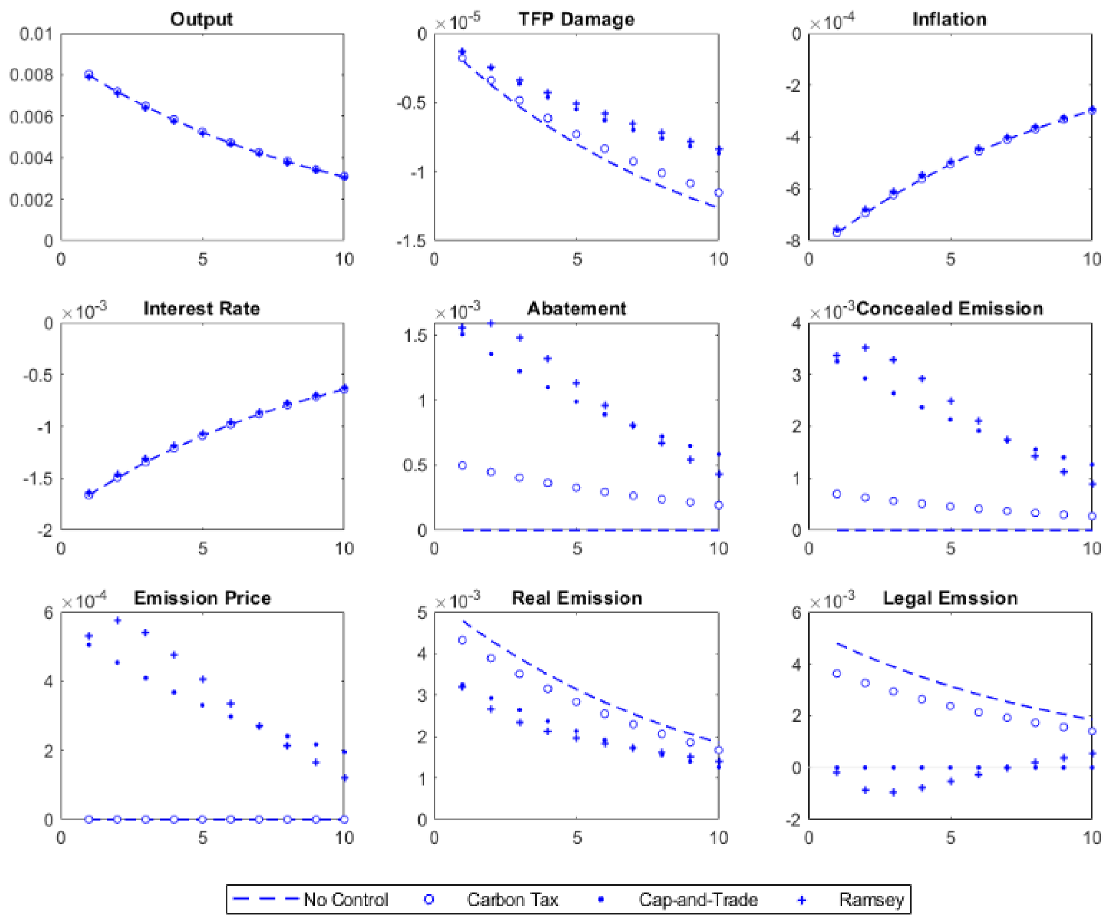

Figure 3: The dynamics of endogenous variables after a 1% positive TFP shock under different regimes ($\psi = 0.1$)

The pivotal reason for the diminishing differences between regimes is that the less effective enforcement of climate policy gives firms more space to dispose of their emissions via the concealed emitting channel and the “fluctuation offsetting” mechanism in the CA regime is weakened. When ψ is lower, the unit cost for concealed emissions and thus the total cost for disposing of every unit of original emissions will decrease. This allows the steady state share of concealed emissions in original emissions (i.e. concealed proportion V_t) and original emissions to increase.

After a positive TFP shock under the TX regime, concealed emissions rise by more than is the case with higher ψ because of the increased steady state of V_t. The path of abatement is almost unchanged because the original emissions after a shock do not change significantly compared to the higher ψ case, and the share of abatement proportion (i.e., U_t) is not changed according to equation (18). Neither does the path of real emissions, whose share is $1 - U_t$, change significantly, for the same reason. The legal emissions rise by less because the legal proportion (i.e., $1 - U_t - V_t$) is reduced due to an increased V_t. The paths of inflation and interest rate are almost unchanged due to a fixed $p_{Z,t}$ under the TX regime.

After a positive TFP shock under the CA regime, $p_{Z,t}$ increases by less than is the case when ψ is higher, as the cost for concealed emissions rises by less.[12] The "fluctuation offsetting" mechanism is weakened. This makes the paths of inflation and the interest rate more similar to regimes without such a mechanism. Concealed emissions rise by more than is the case with a higher ψ for the same reason under the TX regime. Abatement increases by less as more original emissions are disposed of via the concealed emitting channel. Real emissions rise by more because the concealed emissions increase by more and the legal emissions are fixed under the CA regime.

It can be found that, when $\psi = 0.1$, inflation under both the CA and TX regimes are lower than the Ramsey optimised level. This means the reaction coefficient of inflation in monetary policy should be increased to stabilize the price fluctuation when the effectiveness of enforcement of climate policy is low.

In addition to the fluctuation analysis, we also calculate and compare the welfare of each regime after the effectiveness of policy enforcement is changed to 0.1. We find that the order of welfare and the consumption equivalent comparison will change to $W_{ET} > W_{TX}$ and $CE_{CA} > CE_{TX}$. The reason is that consumption, as one of the

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[12] There is a marginal increasing cost for concealed emissions $\frac{\psi}{2} v_{t,j}^2 \varphi Y_{j,t}$. When ψ is lower, the steady state cost for concealed emissions is lower. Hence, the cost for concealed emissions rises less here. Meanwhile, the three channels for disposing of original pollution have the same marginal cost (a natural result of economic optimisation); hence $p_{Z,t}$ equals the cost for concealed emissions.

determinants of welfare, increases by more under the CA regime than under the TX regime. A lower ψ brings a lower cost for concealed emissions. Under the CA regime, this also brings a lower $p_{Z,t}$. Then, the price level decreases, while output and consumption increase. However, under the TX regime, $p_{Z,t}$ is fixed, and, hence, the price level decreases by less than is the case under CA. Then, consumption does not rise by so much.[13] The output under the CA regime rises more than it does under the TX regime, after a shock, which makes the output gap under the CA regime relatively smaller and the welfare larger.

This subsection shows that the existence of concealed emission and ineffectiveness of climate policy enforcement will and make different policy regimes become more similar. The difference between regimes, in terms of fluctuation dynamics of economic variables including price and inflation, will change due to the variation of effectiveness of policy enforcement. Therefore, in addition to the type of climate policy, the effectiveness of enforcement of it also needs to be considered when designing monetary policy. Otherwise, the dynamics of monetary policy and its effect on the economy will be somewhat different (too strong or too weak) from what is envisaged with only considering the type of climate policy. Another implication is that, when making monetary policy, developed countries should consider the existing type of climate policy more carefully than developing countries, as their effectiveness of climate policy enforcement is often higher and the differences between regimes are more significant.

4. The Optimisation of Policy Mixes

From Subsection 3.2, it can be found that, among the three real-world implementable policy mixes (regimes), i.e., CA, TX, and NO, no one is always dominant over others, in terms of the welfare standard. In this section, we try to improve

[13] The fluctuation of price also influences welfare, according to Rotemberg (1982). However, the result here means that the influence of consumption on welfare is stronger.

or "optimise" these regimes by two ways. The first way is to optimise reaction coefficients in the traditional Taylor rule of monetary policy. The second and also a novel way is to introduce a radically "climate-augmented" monetary policy. This is to include the emission gap target into the Taylor rule of monetary policy. We will try to find the best reaction coefficient for the new target and determine whether this inclusion can become a desirable practice. The results will give an answer to central banks' question of "whether it is good for them to proactively care for the climate by using the narrow monetary policy (interest rate rule)".

4.1 Optimisation in the Traditional Monetary Policy

The Ramsey optimal monetary policy, which has been investigated by Annicchiarico and Di Dio (2017), constitutes the ideally optimal policy mix. However, as this kind of policy assumes that all endogenous variables in the economy can be controlled and adjusted by the authority, it is difficult for policy makers to carry out in reality. We do not work more on it here. For real-world implementable regimes (CA, TX, and NO), Subsection 3.2 showed that no one is always dominant.

In this subsection, our way to improve or to "optimise" the policy mix is to first choose a certain regime that is real-world implementable, then optimise the coefficients in them. To do this, we have three potential options. The first is to give a fixed strength of climate policy and optimise the reaction coefficients in the Taylor rule of monetary policy (ρ_Y and ρ_Π). The second is to fix the monetary policy coefficients and optimise the climate policy strength. The third is to optimise the climate strength and the monetary coefficients simultaneously. We choose the first method because this research is on the angle of central banks. The second method is on the angle of climate regulator. The third approach is more comprehensive but is also more complex and difficult for policy makers to coordinate and carry out.

To calculate, we first combine different values of monetary policy coefficients with different types of climate policy (CA or TX[14]) under different effectiveness of policy enforcement and shocks. Shocks include TFP, cost-push, and preference shocks, considering that these three can cover both supply- and demand-side shocks. Then, we derive the welfare and CE of every combination. The reaction coefficients ρ_{π} and ρ_{Y} that maximise the welfare of a certain combination of climate policy type, effectiveness of policy enforcement, and shock, if exist, is the optimised coefficients for it. For simplicity, we only consider the regimes that can solve the model with a unique solution.

We find that under a cost-push shock (a positive θ_t shock), there exist optimal monetary policy coefficients for every type and effectiveness of climate policy, as shown in Table 3. This means that if the cost-push shock is dominant in the economy, the central bank has the best choice of reaction coefficients in the Taylor rule of monetary policy when the type and effectiveness of climate policy are given.

Table 3: Optimal reaction coefficients in the Taylor rule of monetary policy under different types and effectiveness of climate policy (cost-push shock)

φ (effectiveness of climate policy enforcement)	Cap-and-Trade		Carbon Tax	
	ρ_{π}	ρ_{Y}	ρ_{π}	ρ_{Y}
0.1	3.2335	0.4573	3.4792	0.4591
0.5	2.8024	0.4573	3.4948	0.4593
1	2.6819	0.4589	3.4969	0.4593
10	2.5549	0.4619	3.4984	0.4593
100	2.5418	0.4624	3.4985	0.4591

Table 3 shows that ρ_{Y} does not vary significantly across regimes; however, ρ_{Π} is always larger under the TX regime than under the CA regime. This is because the emission price in the CA regime changes when a shock happens. When a cost-push shock (a positive θ_t shock) happens, the price level becomes lower, which increases

[14] We do not incorporate the NO regime as Subsection 3.2 showed that it is always an inferior one.

demand, production output, and emissions. The higher emissions then lead to an increase in the price for disposing of emissions under the CA regime (see Subsection 3.1 for details). Hence, the price level under the TX regime (which is fixed) is relatively lower than is the case under the CA regime. To suppress deflation, a stronger ρ_{Π} is needed. This again shows the "fluctuation offsetting" mechanism in the CA regime and the basic mechanism that differentiates the two regimes. Table 3 also shows that across different effectiveness of climate policy enforcement, only ρ_{π} under the CA regime goes lower significantly when the effectiveness increases. This is because a higher effectiveness pushes up the cost for concealed emissions and increases the demand for legal emissions. Under the CA regime, the emission permit price $p_{Z,t}$ increases more, offsetting the decrease in price level more after the cost-push shock. So, the reaction coefficient for inflation, ρ_{Π}, could be lower.

Under TFP or preference shocks, we find that the welfare become higher when ρ_{π} and ρ_{Y} become larger. This is a common result of the New-Keynesian model. However, this means that there are no optimal values of ρ_{π} and ρ_{Y} if the ranges of the coefficients are not limited and a TFP (or preference) shock is dominant in the economy.

To summarise, we find that when climate policy is considered in the framework, the monetary policy can always be improved by adjusting the reaction coefficients in Taylor rule. If a cost-push shock is dominant in the economy, optimal coefficients exist. Both the type of climate policy and the effectiveness of policy enforcement can affect the value of the optimal coefficients. At this point, we can report that when the existing climate policy is brought into the framework of the central bank's policy making, at least three things can be considered to improve the monetary policy: the type of climate policy, the effectiveness of climate policy enforcement, and the reaction coefficients in the Taylor rule of monetary policy.

4.2 The "Climate-Augmented" Monetary Policy

In this subsection, we propose a radical way to improve the traditional policy mixes. This is to change the form of the Taylor rule of monetary policy by incorporating the emission gap target into it and create a so-called "climate-augmented" monetary policy. We will search for the best coefficient for the new target and determine whether this introduction is good for policy practice. Although no central bank in the real-world has really started this kind of radical practice, our study on it will shed light on central banks' question of "whether it is good for them to proactively care for the climate by using the narrow monetary policy (interest rate rule)".

Our method of constructing the "climate-augmented" monetary policy is to add the emission gap as the third target into the traditional inflation and output gap targeting Taylor rule. The emission gap is the relative deviation of current real emissions to the ideal real emissions (we use the steady state real emissions calculated under Ramsey optimal climate policy[15]). The new form of the Taylor rule is

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi}\right)^{\rho_\Pi} \left(\frac{Y_t}{Y_t^{na}}\right)^{\rho_Y} \left(\frac{Z_{t-1}^{real}}{Z}\right)^{\rho_Z} \tag{32}$$

where Y_t^{na} is the natural output without nominal price stickiness, R, Π, and Z are the steady states of nominal interest rate, inflation rate, and real emissions, respectively. We use Z_{t-1}^{real} to proxy the current real emissions. This is because that real emissions at the period of policy making Z_t^{real} include concealed emissions of the same period, which often cannot be detected simultaneously. The emission gap target is not a replication of the output gap target as we use the real emissions target, not the original emissions who are proportional to output. Real emission incorporates abatement and is the ultimate factor that influences the environment and, thus, can directly reflect the

[15] A more intuitive "ideal real emissions" is the carbon budget measured against the 1.5°C (or lower) target. However, the calculation requires some reliable data in natural science which is currently unavailable.

climate objective. ρ_Z is the reaction coefficient for the emission gap. This new form of Taylor rule makes the monetary policy proactively care for the climate.

Adding such an emission target is not only a radical try, but also theoretically justifiable in terms of macroeconomic fluctuation and welfare when environmental feature is incorporated. According to Gali (2015) (its Appendix of Chapter 4), the theoretical rationality of the traditional inflation and output gap targeting Taylor rule is that, in the baseline DSGE model, the welfare loss function of the household is consists of the variances of inflation and output gap. The purpose of optimised monetary policy is to minimise welfare loss (by offsetting the inflation and output gap). So, it needs to target on the two gaps. In our E-DSGE model, we can find that the loss function $\mathbb{L}_0$ additionally consists of the variance of emission gap, expressed as (see Appendix for derivation)

$$\mathbb{L}_0 = -\frac{\gamma}{2(1-\phi_1 U^{\phi_2})}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{\pi_t^2 + \tilde{a}\left(\hat{Y}_t^{gap}\right)^2 + \tilde{h}\left(\hat{Z}_t^{real}\right)^2 + \tilde{f}\left(\hat{Z}_t^{rp}\right)^2\right\} \quad (33)$$

where U is the steady state of abatement percentage; π_t, $\hat{Y}_t^{gap}$, $\hat{Z}_t^r$ and $\hat{Z}_t^{rp}$ are the logarithm deviation from steady state of "inflation", "output gap", "real emission gap" and "real emission gap minus potential emission" respectively; $\tilde{a} > 0$, $\tilde{h} > 0$ and $\tilde{f} > 0$ are combinations of parameters and steady state variables (see Appendix for full expressions).

So, it is justifiable to add the emission gap into the monetary policy rule. This can help monetary policy better respond to the fluctuation and offset the negative impact on welfare.

In the following, we search for the best coefficient for the newly introduced emission gap target[16] and determine whether such an introduction is good for policy

[16] A more comprehensive analysis is to simultaneously optimise all the coefficients for the three targets. We tried it, however, the coefficients of inflation and output gap (ρ_Π and ρ_Y) turned to be corner solutions which means the optimal values do not exist, given our computing power. Hence, in this paper,

practice. To do this, we set the reaction coefficients for traditional targets of monetary policy as fixed: $\rho_Y = 0.5$ and $\rho_\Pi = 1.5$, and calculate welfare values of the economy with different ρ_Z and different shocks. ρ_Z takes every value in the interval that can produce a unique solution for the equilibrium. Common shocks (TFP, cost-push, and preference) that cover both supply- and demand-side shocks are introduced, respectively. Under a same shock, if the welfare with a ρ_Z is higher than is the welfare with $\rho_Z = 0$, a ρ_Z that can improve the policy mix is found. As ρ_Y and ρ_Π are fixed and allowing ρ_Z to change is introducing a new dimension for optimisation, there must be some ρ_Z that can improve the welfare. It will serve as a supplement of the potentially either over-strong or over-weak ρ_Y and ρ_Π.

Applying the above method, we can find the intervals of ρ_Z that can improve the welfare, as well as the values of ρ_Z that can enhance the welfare at the greatest extent (define as "the best value of ρ_Z") under different regimes and different shocks. Results using parameters calibrated in Subsection 2.5 shown in Table 4. When the TFP or cost-push shock is dominant, the best ρ_Z is negative in both regimes. When the preference shock is dominant, the best ρ_Z lies in the right boundary of possible values, which means that the higher the ρ_Z is, the more the welfare improves.

Table 4: The interval of ρ_Z that can improve welfare and the best ρ_Z under different climate policies and shocks (original price stickiness)

Shock	Cap-and-Trade		Carbon Tax	
	Interval	Best	Interval	Best
TFP shock	(-0.866, 0)	-0.453	(-0.174, 0)	-0.091
Cost-push shock	(-0.509, 0)	-0.261	(-0.12, 0)	-0.062
Preference shock	The higher the better			

we fix the coefficients for the traditional Taylor rule targets and try to optimise the coefficient for the newly introduced emission gap target.

However, sensitivity analysis shows that under TFP or cost-push shock, the best ρ_Z can also be positive under different parameter values. For example, if the price stickiness parameter γ is large enough (e.g., 10 times larger, which is roughly in line with Gertler et al. (2019)) the best ρ_Z becomes positive under both regimes with a cost-push shock, as shown in Table 5.

Table 5: The interval of ρ_Z that can improve welfare and best ρ_Z under different climate policies and shocks (price stickiness 10 times larger)

Shock	Cap-and-Trade		Carbon Tax	
	Interval	Best	Interval	Best
TFP shock	(-0.934, 0)	-0.508	(-0.16, 0)	-0.087
Cost-push shock	(0, 1.342)	0.602	(0, 0.184)	0.085
Preference shock	The higher the better			

We must point out that when the interval of ρ_Z that can improve welfare is negative, there is a dilemma between the traditional welfare objective and the new climate objective. Suppose a positive TFP or cost-push shock happens, then the emission gap is positive due to the lower price level, higher output, and higher emissions. With a negative ρ_Z, an even lower interest rate will be derived, which encourages demand and production, fulfilling the welfare objective. Then the heightened production causes higher emissions, which is, however, adverse to the climate stability objective. On the contrary, if we change the ρ_Z to a positive value to realise the climate objective (emission gap), then it deviates from the interval that can improve welfare. Failing to enhance welfare is incompatible with the fundamental purpose of a central bank. This is the potential dilemma that emerges to a central bank if they add the emission gap target into the traditional monetary policy.

The above analysis gives an answer to the question “whether central banks should adopt ‘climate-augmented’ (emission gap targeting) monetary policy rule” or “whether it is good for the central banks to proactively care for the climate by using the narrow monetary policy (interest rate rule)". If the interval of the new target's coefficient (ρ_Z) that can improve welfare consists of a positive part, it is good to do so by adding the emission gap target into the Taylor rule of monetary policy and setting the reaction coefficient as a value in the positive interval. If the interval consists of only negative values, it is not good to add the emission gap target into the Taylor rule.

Based on the above results and the real-world circumstance, we do not suggest central banks to add the new climate target (the emission gap target) into the Taylor rule of monetary policy without further reviews. Considering that the welfare-improving interval of ρ_Z is not fixed and is determined by many uncertain factors including deep parameters, the type of climate policy, and the type of shock, a central bank cannot assure that the climate augmented Taylor rule monetary policy always does not bring the dilemma between the welfare and the climate objective. Meanwhile, many central banks in the real-world are already overburdened with multiple targets other than price stability and employment.

This subsection shows that, when the strength of the traditional Taylor rule-based of monetary policy is given, incorporating the emission gap target into the rule and setting the coefficient for the new target in a specific interval can improve the policy mix in terms of the welfare standard. The best value of the reaction coefficient for emission target is found under different situations (given the reaction coefficients for inflation and output gap targets fixed). However, under some circumstances, this radically "climate-augmented" monetary policy will create a dilemma between the traditional welfare objective and the new climate objective, making it less valuable of recommendation for central banks to adopt without further reviews.

4.3 Discussion

Although the "climate-augmented" (emission gap targeting) monetary policy is found to be controversial above, it does not mean that this kind of monetary policy is useless from other points of view. The DSGE model is used mainly for fluctuation analysis, so the conclusions are based on short-term standards. Climate change can be attributed as a long-term challenge for mankind. Considering that "climate-augmented" monetary policy of certain forms can limit emission and reduce future climate risks, it could become a preferable choice for policy makers in the long-run. From the modelling perspective, the reasons include: First, the steady state welfare could be higher if emission is limited. This can compensate for the welfare loss shown in the fluctuation analysis. Second, a lower climate risk increases economic stability and decreases welfare loss brought by fluctuation.

The above results neither mean that central banks should not help the climate by broader measures other than the narrow monetary policy (interest rate). Climate change can bring physical and transition risks so that can cause financial and economic instability. Safeguarding financial and economic stability is a major mandate of most central banks. In face of climate change, they could and should use macroprudential and other broader regulatory policy tools, such as environmental stress testing and green asset purchase to fulfil their stability mandate and be helpful with the climate change challenge.

The Tinbergen Rule is not necessarily violated when central banks engage in the climate issue. Within the framework of this paper, targeting on the emission gap is ultimately for controlling fluctuation which could influence welfare. If we do not compromise the welfare target when realising climate objectives (i.e. set ρ_Z in the interval that can improve welfare), the narrow monetary policy (interest rate) will not be overburdened. Such an individual policy tool is still for realising one target. Beyond this paper's framework, central banks normally have multiple policy tools besides the narrow monetary policy, as stated in the previous paragraph. If central banks use other or new policy tools to realise climate objectives, the Tinbergen Rule is still not violated.

5. Conclusion

In this paper, we have studied the relationship between and the mix of monetary and climate policy, providing insights for central banks who are considering their engagement in the climate change issue. By using an Environmental Dynamic Stochastic General Equilibrium (E-DSGE) model augmented with concealed emissions and related penalty, we first mixed Taylor rule-based monetary policy with different climate policies and compared these mixes to see what differences climate policy can bring to monetary policy and the economy. Then, we tried to improve policy mixes by optimising reaction coefficients in the traditional Taylor rule of monetary policy under different climate policies. Lastly, we proposed a "climate-augmented" monetary policy (interest rate rule) and investigated whether it is good for central banks to employ it.

The main findings consist of three parts. First, monetary policy can be influenced by climate policy since when it is mixed with different types and effectiveness (stringency) of climate policy, price level and inflation (which are the major target of monetary policy) in the economy are different. The pivotal reason of the difference is that the cap-and-trade regime can offset the price fluctuation after shocks, whereas the carbon tax regime cannot. The effectiveness of climate policy enforcement also plays a role, as a lower effectiveness can provide more space for concealed emissions. Therefore, the making process of monetary policy should consider the existing climate policy. Developed countries should consider the climate policy more carefully than do developing ones.

Second, the reaction coefficients in the traditional Taylor rule of monetary policy can always be better set to enhance welfare when the existing climate policy is considered in the framework of analysis. If the cost-push shock is dominant in the economy, optimal coefficients exist. Both the type and effectiveness of climate policy can affect the value of the optimal coefficients.

Third, the welfare of the economy can be enhanced by adding the target of emission gap into the rule of monetary policy and setting the reaction coefficient of the new target in a specific interval. The best value of the coefficient can be found under different scenarios. However, under some circumstances, this radically "climate-augmented" (emission gap targeting) monetary policy could create a dilemma between the welfare and the climate objectives. If we do not want central banks to take the risk of such a dilemma, it is better not to introduce the climate target into the monetary policy rule without further reviews. Central banks could and should use measures other than the narrow monetary policy (interest rate) to help the climate.

The above findings give insights to the initial question of this paper "Should central banks engage in climate change issue?" — The making process of monetary policy should consider the existing climate policy; otherwise, the dynamic of monetary policy and its effect on the economy will be different from what is originally envisaged. However, it is not recommended for central banks to add the climate (emission gap) target into the narrow monetary policy rule at the current stage, as this may create a dilemma for them.

The inclusion of concealed emission and related penalty does not change the major results of the research we base on (Annicchiarico and Di Dio, 2017). This is because that most policy mixes studied in this research (those without Ramsey-type policy) are not overlapped or comparable with those in Annicchiarico and Di Dio (2017) (at least one policy is Ramsey-type). The only overlapped and comparable policy mix is the "Taylor rule monetary policy with Ramsey-type climate policy". However, when the climate policy is Ramsey-type, the policy effect (in terms of real emission, output and inflation) is largely determined by such policy. The existence of concealed emission only changes intermediate variables such as the share of legal emission significantly, not those variables that ultimately reflect policy effect.

This research can be extended in several aspects. For example: (1) Set the effectiveness of climate policy enforcement or emission (carbon) price as a shock to study the "transition risk" brought by tightened climate regulation. (2) Set a dynamic rule (e.g., the Taylor type) for climate policy. (3) Improve the form of climate target in the monetary policy rule (e.g., use an ideal real emission that is in line with the 1.5℃ climate target). (4) Introduce more types of shocks (e.g., climate change shock after the tipping point). (5) Introduce more financial fractions and constraints (e.g. zero lower bound of interest rate) to describe the economy more precisely. (6) Find whether the three-target "climate-augmented" monetary policy rule is better than the traditional two-target policy when all the reaction coefficients in Taylor rule are simultaneously optimised.

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Appendix

Derivation of the New Keynesian Phillips Curve

The maximisation problem of firm j is

$$\begin{cases} V_0 = \max \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \Omega_{0,t} \left[\frac{P_{j,t}}{P_t} Y_{j,t} - TC_{j,t} - \frac{\gamma}{2} \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2 Y_t \right] \right\} \\ s.t. \begin{cases} TC_{j,t} = \frac{W_t}{P_t} L_{j,t} + \phi_1 U_{j,t}^{\phi_2} Y_{j,t} + p_{Z,t} \left(1 - U_{j,t} - V_{j,t} \right) \varphi Y_{j,t} + \frac{\psi}{2} V_{j,t}^2 \varphi Y_{j,t} \\ Y_{j,t} = \Lambda_t A_t L_{j,t} \\ Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\theta_t} Y_t \end{cases} \end{cases}$$

We can rewrite the objective function by the Bellman Equation as

$$V_t = \max \left\{ \frac{P_{j,t}}{P_t} Y_{j,t} - TC_{j,t} - \frac{\gamma}{2} \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2 Y_t + \mathbb{E}_t \Omega_{t,t+1} V_{t+1} \right\}$$

which yields the Lagrangian function as

$$\mathcal{L}_t = \frac{P_{j,t}}{P_t} Y_{j,t} - \left[\frac{W_t}{P_t} \frac{Y_{j,t}}{\Lambda_t A_t} + \phi_1 U_{j,t}^{\phi_2} Y_{j,t} + p_{Z,t} \left(1 - U_{j,t} - V_{j,t} \right) \varphi Y_{j,t} + \frac{\psi}{2} V_{j,t}^2 \varphi Y_{j,t} \right]$$
$$- \frac{\gamma}{2} \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2 Y_t + \mathbb{E}_t \left[\Omega_{t,t+1} V_{t+1} \right] + \lambda_{j,t} \left[\left(\frac{P_{j,t}}{P_t} \right)^{-\theta_t} Y_t - Y_{j,t} \right]$$

where $\Omega_{t,t+1} = \beta \frac{C_t}{C_{t+1}}$ is the stochastic discount factor. So, we can obtain the FOC for $U_{j,t}$ and $V_{j,t}$

$$p_{Z,t} = \frac{\phi_1 \phi_2}{\varphi} U_{j,t}^{\phi_2 - 1}$$
$$V_{j,t} = \frac{p_{Z,t}}{\psi}$$

and derive

$$MC_{j,t} = \frac{W_t}{P_t} \frac{1}{\Lambda_t A_t} + \phi_1 U_{j,t}^{\phi_2} + p_{Z,t} \left(1 - U_{j,t} - V_{j,t} \right) \varphi + \frac{\psi}{2} V_{j,t}^2 \varphi$$

The FOCs for $P_{j,t}$ and $Y_{j,t}$ derive

$$1-\ \theta_t-\gamma\left(\frac{P_{j,t}}{P_{j,t-1}}-1\right)\frac{P_{j,t}}{P_{j,t-1}}+\beta\gamma\mathbb{E}_t\left[\left(\frac{P_{j,t+1}}{P_{j,t}}-1\right)\frac{P_{j,t+1}}{P_{j,t}}\frac{C_t}{C_{t+1}}\frac{Y_{t+1}}{Y_t}\right]+\ \theta_t MC_{j,t}$$

$$=0$$

Equation Systems of First Order Conditions

Taylor Rule Monetary Policy Mix Cap-and-Trade Climate Policy

$$\begin{cases}
\beta R_t\mathbb{E}_\mathrm{t}\left[\frac{S_{t+1}}{S_t}\frac{C_t}{C_{t+1}}\frac{1}{\Pi_{\mathrm{t+1}}}\ \right]=1\\
(1-\theta_t)-\gamma(\Pi_t-1)\Pi_t+\beta\gamma\mathbb{E}_\mathrm{t}\left[\frac{C_t}{C_{t+1}}(\Pi_{t+1}-1)\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\right]+\theta_t MC_t=0\\
MC_t=\frac{W_t}{\Lambda_t A_t P_t}+\phi_1\widetilde{U}_t^{\phi_2}+p_{Z,t}(1-U_t-v_t)\varphi+\frac{\psi}{2}v_t^2\varphi\\
L_t^\eta=\frac{W_t}{\mu_L P_t C_t}\\
Y_t=C_t+\phi_1 U_t^{\phi_2}Y_t+\frac{\gamma}{2}(\Pi_t-1)^2 Y_t\\
Z=(1-U_t-v_t)\varphi Y_t+\tilde{Z}\\
M_t=(1-\delta_M)M_{t-1}+(1-U_t)\varphi Y_t+\tilde{Z}\\
Y_t=\Lambda_t A_t L_t\\
p_{Z,t}=\frac{1}{\varphi}\phi_1\phi_2 U_t^{\phi_2-1}\\
v_t=\frac{1}{\psi\varphi}\phi_1\phi_2 U_t^{\phi_2-1}=\frac{p_{Z,t}}{\psi}\\
\frac{R_t}{R}=\left(\frac{\Pi_t}{\Pi}\right)^{\rho_\Pi}\left(\frac{Y_t}{Y_t^{na}}\right)^{\rho_Y}
\end{cases}$$

Taylor Rule Monetary Policy Mix Carbon Tax Climate Policy

$$
\begin{cases}
\beta R_t \mathbb{E}_t\left[\frac{S_{t+1}}{S_t}\frac{C_t}{C_{t+1}}\frac{1}{\Pi_{t+1}}\right] = 1 \\
(1-\theta_t) - \gamma(\Pi_t - 1)\Pi_t + \beta\gamma\mathbb{E}_t\left[\frac{C_t}{C_{t+1}}(\Pi_{t+1}-1)\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\right] + \theta_t MC_t = 0 \\
MC_t = \frac{W_t}{\Lambda_t A_t P_t} + \phi_1 U^{\phi_2} + p_Z(1-U-v)\varphi + \frac{\psi}{2}v^2\varphi \\
L_t^{\eta} = \frac{W_t}{\mu_L P_t C_t} \\
Y_t = C_t + \phi_1 U^{\phi_2} Y_t + \frac{\gamma}{2}(\Pi_t - 1)^2 Y_t \\
M_t = (1-\delta_M)M_{t-1} + (1-U)\varphi Y_t + \tilde{Z} \\
Y_t = \Lambda_t A_t L_t \\
p_Z = \frac{1}{\varphi}\phi_1\phi_2 U^{\phi_2 - 1} \\
v = \frac{1}{\psi\varphi}\phi_1\phi_2 U^{\phi_2-1} = \frac{p_Z}{\psi} \\
\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi}\right)^{\rho_\Pi}\left(\frac{Y_t}{Y_t^{na}}\right)^{\rho_Y}
\end{cases}
$$

Taylor Rule Monetary Policy Mix No Control Climate Policy

No control policy is a special case of the carbon tax policy with $p_Z = 0$. The equation system is all the same as with the "Taylor Rule Monetary Policy Mix Carbon Tax Climate Policy" except that p_Z is set as 0.

Taylor Rule Monetary Policy Mix Ramsey Optimal Climate Policy

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t S_t \left(\ln C_t - \mu_L \frac{L_t^{1+\eta}}{1+\eta} \right)$$

$$s.t. \begin{cases} \beta R_t \mathbb{E}_t \left[\frac{S_{t+1}}{S_t} \frac{C_t}{C_{t+1}} \frac{1}{\Pi_{t+1}} \right] = 1 \\ (1-\theta_t) - \gamma(\Pi_t - 1)\Pi_t + \beta\gamma \mathbb{E}_t \left[\frac{C_t}{C_{t+1}} (\Pi_{t+1} - 1)\Pi_{t+1} \frac{Y_{t+1}}{Y_t} \right] + \theta_t MC_t = 0 \\ MC_t = \frac{W_t}{\Lambda_t A_t P_t} + \phi_1 U_t^{\phi_2} + p_{Z,t}(1 - U_t - v_t)\varphi + \frac{\psi}{2} v_t^2 \varphi \\ L_t^{\eta} = \frac{W_t}{\mu_L P_t C_t} \\ Y_t = C_t + \phi_1 U_t^{\phi_2} Y_t + \frac{\gamma}{2}(\Pi_t - 1)^2 Y_t \\ M_t = (1-\delta_M) M_{t-1} + (1 - U_t)\varphi Y_t + \tilde{Z} \\ Y_t = \Lambda_t A_t L_t \\ \frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi} \right)^{\rho_\Pi} \left(\frac{Y_t}{Y_t^{na}} \right)^{\rho_Y} \end{cases}$$

Derivation of the Loss Function

We can derive the loss function from households' expected lifetime utility:

$$V_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t S_t U(C_t, L_t)$$

where S_t denotes the preference shock. Generally, the utility function at each period is as follow:

$$U(C_t, L_t) = u(C_t) + v(L_t)$$

And we define:

$$\sigma_t = -C \frac{u''(C_t)}{u'(C_t)}$$

$$\eta_t = L \frac{v''(L_t)}{v'(L_t)}$$

Obviously, the utility function defined in Chapter 2 is a particular case of the one mentioned above:

$$U(C_t, L_t) = \ln C_t - \mu_L \frac{L_t^{1+\eta}}{1+\eta}$$

where

$$\begin{cases}\sigma_t \equiv 1 \\ \eta_t \equiv \eta\end{cases}$$

Therefore, the second-order Taylor expansion of the utility function is:

$$\begin{aligned}\mathbb{U}_t &= S_t U(C_t, L_t) \\ &= U(C, L) + C U_C(C, L)\left(\hat{C}_t + \frac{1-\sigma}{2}\hat{C}_t^2 + \hat{S}_t\hat{C}_t\right) \\ &+ L U_L(C, L)\left(\hat{L}_t + \frac{1+\eta}{2}\hat{L}_t^2 + \hat{S}_t\hat{L}_t\right) + indp. + apprx.\end{aligned}$$

where $indp.$ denotes the independent variables that will not be influenced by any policies, and $apprx.$ denotes the higher-order remainder.

The aggregate conditions including production function, market-clearing condition, and pollution accumulation function are as follows:

$$\begin{cases} Y_t = \tilde{A}_t L_t = \Lambda_t A_t L_t = e^{-\chi(M_t - \widetilde{M})} A_t L_t \\ Y_t = C_t + \left[\frac{\gamma}{2}(\Pi_t - \Pi)^2 + \phi_1 U_t^{\phi_2}\right] Y_t \\ Z_t^r = (1 - U_t)\varphi Y_t \\ M_t = (1 - \delta_M) M_{t-1} + Z_t^r + \tilde{Z} \end{cases}$$

which yield the second-order Taylor expansions as follows:

$$\begin{cases} \hat{L}_t = \hat{Y}_t - \hat{A}_t + \chi M\left(\widehat{M}_t + \frac{1}{2}\widehat{M}_t^2\right) + apprx. \\ \hat{C}_t = (1-\kappa_1)\hat{Y}_t + \kappa_1 \hat{Z}_t^r - \tilde{\gamma}\pi_t^2 - \frac{\kappa_2}{2}\left(\hat{Z}_t^r - \hat{Y}_t\right)^2 + apprx. \\ \widehat{M}_t + \frac{1}{2}\widehat{M}_t^2 = (1-\delta_M)\left(\widehat{M}_{t-1} + \frac{1}{2}\widehat{M}_{t-1}^2\right) + \delta_M\left(1 - \frac{\widetilde{M}}{M}\right)\left(\hat{Z}_t^2 + \frac{1}{2}\left(\hat{Z}_t^r\right)^2\right) + apprx. \end{cases}$$

and yield the Taylor expansion of utility function about major macro-variables:

$$\begin{aligned} V_0 &= C U_C(C, L)(1-\kappa_1) M \chi \frac{(1-\delta_M)\left(\widehat{M}_{-1} + \frac{1}{2}\widehat{M}_{-1}^2\right)}{1 - \beta(1-\delta_M)} + \frac{U(C, L)}{1-\beta} \\ &+ Indp. + apprx. - \frac{\gamma}{2(1 - \phi_1 U^{\phi_2})}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{\pi_t^2 + \tilde{a}\left(\hat{Y}_t^{gap}\right)^2\right. \\ &\left. + \tilde{f}\left(\hat{Z}_t^{gap}\right)^2 + \tilde{h}\left(\hat{Z}_t^r\right)^2\right\} \end{aligned}$$

So, we can define the second-order loss function as follow (according to Gali (2015), Chapter 4):

$$\mathbb{L} = -\frac{\gamma}{2(1-\phi_1 U^{\phi_2})}\left\{Var(\pi_t) + \tilde{a}\cdot Var\left(\hat{Y}_t^{gap}\right) + \tilde{f}\cdot Var\left(\hat{Z}_t^{gap}\right) + \tilde{h}\cdot Var\left(\hat{Z}_t^{r}\right)\right\}$$

where

$$\begin{cases} \hat{Y}_t^{gap} = \hat{Y}_t + \widehat{Abt}_t - \dfrac{1+\eta}{\sigma+\eta}\hat{A}_t^{eff} \\ \hat{Z}_t^{gap} = \hat{Z}_t^{r} - \dfrac{\kappa_2 - (1-\sigma)(1-\kappa_1)^2}{\kappa_2 + (1-\sigma)(1-\kappa_1)\kappa_1}\left(\hat{A}_t^{eff} + \tilde{g}\hat{S}_t\right) \end{cases}$$

in which $\widehat{Abt}_t \propto \widehat{U}_t$ is logarithm deviation of abatement cost, and $\hat{A}_t^{eff} = \hat{A}_t - \chi M\widehat{M}_t$ is the logarithm deviation of effective TFP (see Section 2.2). Other parameters can be proof to equal follows:

$$\tilde{a} = \frac{1-\phi_1 U^{\phi_2}}{\gamma}\left[1 + \frac{\kappa_2 - (1-\sigma)(1-\kappa_1)^2}{(1+\eta)(1-\kappa_1)}\right]$$

$$\tilde{f} = \frac{1-\phi_1 U^{\phi_2}}{\gamma}\left[1 + \frac{\kappa_2 - (1-\sigma)(1-\kappa_1)^2}{(1+\eta)(1-\kappa_1)}\right]$$

$$\tilde{g} = \frac{1-\phi_1 U^{\phi_2}}{\gamma}\frac{\kappa_1}{\kappa_2 + (1-\sigma)(1-\kappa_1)\kappa_1}\left[1 + \frac{\kappa_2 - (1-\sigma)(1-\kappa_1)^2}{(1+\eta)(1-\kappa_1)}\right]$$

$$\tilde{h} = \frac{1-\phi_1 U^{\phi_2}}{\gamma(1+\eta)(1-\kappa_1)}\left[\kappa_1^2 - \frac{\kappa_2(1-\sigma)}{\kappa_2 - (1-\sigma)(1-\kappa_1)^2}\right]$$

$$\kappa_1 = \frac{\phi_1\phi_2 U^{\phi_2}}{1-\phi_1 U^{\phi_2}}\frac{1-U}{U}$$

$$\kappa_2 = \frac{\phi_1\phi_2 U^{\phi_2}}{1-\phi_1 U^{\phi_2}}\frac{1-U}{U^2}\left[\frac{\phi_2(1-U)}{1-\phi_1 U^{\phi_2}} - 1\right] = \frac{\kappa_1}{U}\left[\frac{\phi_2(1-U)}{1-\phi_1 U^{\phi_2}} - 1\right]$$

where U is the efficient steady state of abatement.

Particularly, if the consumption utility is logarithm function (yields $\sigma = 1$), the loss function of social welfare is

$$
\begin{aligned}
\mathbb{L}_0 = -\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Bigg\{ & \frac{\gamma}{1-\phi_1 U^{\phi_2}}\pi_t^2 + \frac{1}{1+\frac{\kappa_2}{(1+\eta)(1-\kappa_1)}}\left(\hat{Y}_t + \widehat{Ab}t_t - \hat{A}_t^{eff}\right)^2 \\
& + \frac{\kappa_2}{\kappa_2 + (1+\eta)(1-\kappa_1)}\left(\hat{Z}_t^r - \hat{A}_t^{eff} - \left(\frac{\kappa_1}{\kappa_2} + \frac{\kappa_1}{(1+\eta)(1-\kappa_1)}\right)\hat{S}_t\right)^2 \\
& + \frac{\kappa_1^2}{(1+\eta)(1-\kappa_1)}\left(\hat{Z}_t^r\right)^2\Bigg\}
\end{aligned}
$$

It can be found that the loss function additionally consists of the variance of emission gap ($\hat{Z}_t^r - \frac{\kappa_2-(1-\sigma)(1-\kappa_1)^2}{\kappa_2+(1-\sigma)(1-\kappa_1)\kappa_1}\left(\hat{A}_t^{eff} + \tilde{g}\hat{S}_t\right)$).