Ergodic Rate Analysis of Multi-IRS Dual-Hop DF Relaying System

Qiang Sun, Panpan Qian, Wei Duan, Jiayi Zhang, Jue Wang, and Kai-Kit Wong

***Abstract*—Intelligent reflecting surface (IRS) has emerged as a promising and low-cost technology for improving wireless communications by collecting dispersed radio waves and redirecting them to the intended receivers. In this letter, we characterize the achievable rate when multiple IRSs are utilized in the manner of decode-and-forward (DF) relaying. Our performance analysis is based on the Nakagami-m fading model with perfect channel state information (CSI). Tight upper bound expressions for the ergodic rate are derived. Moreover, we compare the performance of the multi-IRS DF relaying system with that of the one with a single IRS and confirm the gain. We then optimize the IRS configuration considering the numbers of IRSs and IRS reflecting elements, which provides useful insights for practical design.**

***Index Terms*—Decode-and-forward, Ergodic capacity, Intelligent reflecting surface, Nakagami fading, Relaying.**

I. Introduction

With the fifth-generation (5G) mobile communications being deployed around the world, efforts are beginning to emerge to make another leap towards more capacity, reliability, energy efficiency and lower latency [1]. One particular priority is to focus on low-cost scalable solutions that are upgradable if needed. For this reason, intelligent reflecting surface (IRS) has appeared to fit well with the needs. IRS uses a large tunable surface to collect dispersed radio waves and redirect them to the intended receivers, and finds applications such as coverage extension, capacity gain, secrecy improvement, and etc.

Considerable efforts have been made to study IRS-assisted wireless communication systems. The design of the phase shift matrix and transmit beamformer was analytically investigated in [2], [3] to enhance the symbol error probability performance and for physical-layer secrecy. Later, the authors of [4] studied the robustness of an IRS taking into account the impact of imperfect channel state information (CSI) of the eavesdropping channels. The above results, however, mainly focused on the single IRS scenario and if multiple IRSs are distributed over an area, they are anticipated to provide more robust and flexible links. This was considered in [5] where the outage probability performance was analyzed. The results in [6] demonstrated that the distributed IRS-aided communication system has better outage and rate performance than the one with a single IRS. Although the use of IRS seems promising, the associated pilot overheads for CSI acquisition could outweigh their benefits if the numbers of IRSs and the reflecting elements are large.

While IRS and relaying may be perceived by some people as similar technologies, it has been demonstrated that IRS provides higher rate and energy efficiency than relaying systems [7]. Additionally, decode-and-forward (DF) relaying is known to outperform amplify-and-forward (AF) relaying in terms of rate performance [8]. In [9], communication with mixed DF relaying and IRS was studied to improve the coverage performance. The authors in [10] then discussed the feasibility of the coexistence of DF relaying and IRS. The results in [9] and [10] are encouraging and deserve attention. That said, multi-IRS systems are much less understood. Also, the pilot overhead for CSI acquisition is often overlooked in the design and analysis for IRS-aided wireless communication systems, which motivates the work of this letter.

In this letter, we first investigate a multi-IRS dual-hop DF relaying system in Nakagami-m fading channels, which harnesses the benefits of both relaying and IRSs to compensate for the "double fading" effect[1] and improve the rate performance. Closed-form expressions are derived for the upper bound expressions of the ergodic rate. The analysis allows us to link the pilot overhead to the achievable rate, which indicates that the net ergodic rate can decrease if CSI acquisition takes up too much overhead as the numbers of IRSs and the reflecting elements continue to increase. This leads us to find the optimal numbers of IRSs and the reflecting elements for maximizing the net ergodic rate through our analytical results.

II. System Model

Consider a communication system aided by a dual-hop DF relay and L IRSs, as shown in Fig. 1, in which a BS aims to transmit information to a user. In the first hop, the BS transmits the information-bearing signal to the DF relay which also picks up the signals reflecting off from the IRSs. The relay decodes the received signal and then transmits it to the user through the direct link and the reflection paths of the IRSs in the second hop. In our model, we assume that each IRS has N reflecting elements while all other terminals have a single antenna each. In addition, it is assumed that the BS is too far away from the user resulting in a broken direct link between them. Reflection from the IRSs is considered only once and higher-order reflections are ignored.

We denote the (scalar) channels from the BS to the relay, and from the relay to the user, respectively, as $h_{\rm BR}$ and $h_{\rm RU}$.

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[1] The equivalent path loss of the base station (BS)-IRS-user link is the product (instead of the summation) of the path losses of the BS-IRS link and IRS-user link [10].

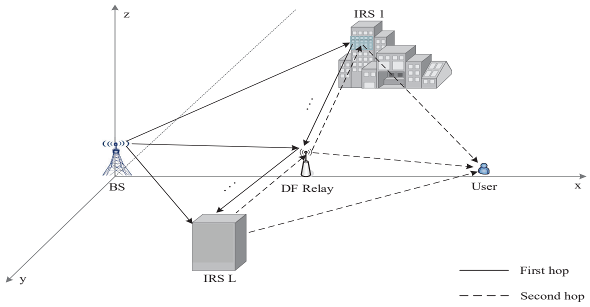

Fig. 1: The multi-IRS dual-hop DF relaying system.

The vector channels between the BS and l-th IRS and between the l-th IRS and relay are denoted by $\mathbf{h}_{\mathrm{BI}_l} \in \mathbb{C}^{N\times 1}$ and $\mathbf{h}_{\mathrm{I}_l\mathrm{R}} \in \mathbb{C}^{N\times 1}$, respectively. Similarly, $\mathbf{h}_{\mathrm{RI}_l} \in \mathbb{C}^{N\times 1}$ and $\mathbf{h}_{\mathrm{I}_l\mathrm{U}} \in \mathbb{C}^{N\times 1}$ are used to denote the channels from the relay to l-th IRS, and that from the l-th IRS to user, respectively.

To account for the path loss and the small-scale fading phenomenon, the channels are modelled in the form:

$$\begin{cases} h_p = \bar{h}_p d_p^{-\frac{\alpha}{2}}, \text{ for } p \in \{\mathrm{BR}, \mathrm{RU}\}, \\ \mathbf{h}_q = \bar{\mathbf{h}}_q d_q^{-\frac{\alpha}{2}}, \text{ for } q \in \{\mathrm{BI}_l, \mathrm{I}_l\mathrm{R}, \mathrm{RI}_l, \mathrm{I}_l\mathrm{U}\}, \end{cases} \tag{1}$$

where α denotes the path loss exponent, and d_p (or d_q) is the respective distance between the terminals. Note that $\bar{h}_p$ and $\bar{\mathbf{h}}_q$ are the small-scale fading channels, which follow independent and identically distributed Nakagami-m distribution, where m denotes the shape parameter and can be chosen appropriately to reflect specific types of fading environments.

In the first hop, the BS transmits the signal to the relay and the L IRSs help reflect the incident signal towards the relay.[2] The received signal at the relay can be written as

$$y_R = \sqrt{P_{\mathrm{B}}}\left(h_{\mathrm{BR}} + \sum_{l=1}^{L} \mathbf{h}_{\mathrm{BI}_l}^T \Theta_l \mathbf{h}_{\mathrm{I}_l\mathrm{R}}\right) x + w_{\mathrm{R}}, \tag{2}$$

where P_{B} denotes the transmit power of the BS, x denotes the normalized information signal with unit energy. Moreover, $\Theta_l = \mathrm{diag}(\eta_{l1}e^{j\theta_{l1}}, \ldots, \eta_{ln}e^{j\theta_{ln}}, \ldots, \eta_{lN}e^{j\theta_{lN}}) \in \mathbb{C}^{N\times N}$ is a diagonal matrix, which captures the reflection coefficients and phase-shifts of the N reflecting elements of the l-th IRS, where $\eta_{ln} \in [0,1]$, $\theta_{ln} \in [0, 2\pi)$ denote the fixed reflection amplitude coefficient and phase shift of the n-th element, respectively. Also, w_{R} is an additive white Gaussian noise (AWGN) at the relay with zero mean and variance of σ_R^2. The received signal-to-noise ratio (SNR) at the relay can be obtained as

$$\gamma_R = \frac{P_{\mathrm{B}}\left|h_{\mathrm{BR}} + \sum\limits_{l=1}^{L} \mathbf{h}_{\mathrm{BI}_l}^T \Theta_l \mathbf{h}_{\mathrm{I}_l\mathrm{R}}\right|^2}{\sigma_R^2}. \tag{3}$$

For the second hop, the relay transmits the decoded information signal, x, to the user via the direct link and the reflection paths from the L IRSs. Therefore, the received signal at the user is given by

$$y_U = \sqrt{P_{\mathrm{R}}}\left(h_{\mathrm{RU}} + \sum_{l=1}^{L} \mathbf{h}_{\mathrm{RI}_l}^T \Phi_l \mathbf{h}_{\mathrm{I}_l\mathrm{U}}\right) x + w_{\mathrm{U}}, \tag{4}$$

with all the variables defined in a similar way as in the first hop. Evidently, P_{R} represents the transmit power of the relay and $\Phi_l = \mathrm{diag}(\tau_{l1}e^{j\phi_{l1}}, \ldots, \tau_{ln}e^{j\phi_{ln}}, \ldots, \tau_{lN}e^{j\phi_{lN}})$ specifies the reflection responses of the elements of the l-th IRS with τ_{ln} and ϕ_{ln} as the magnitude response and phase-shift of the n-th element of the l-th IRS. The AWGN at the user is assumed to have power of σ_U^2. Hence, the SNR at the user is given by

$$\gamma_U = \frac{P_{\mathrm{R}}\left|h_{\mathrm{RU}} + \sum\limits_{l=1}^{L} \mathbf{h}_{\mathrm{RI}_l}^T \Phi_l \mathbf{h}_{\mathrm{I}_l\mathrm{U}}\right|^2}{\sigma_U^2}. \tag{5}$$

As a result, the end-to-end achievable rate is given by

$$R = \frac{1}{2}\log_2(1 + \min\{\gamma_R, \gamma_U\}). \tag{6}$$

III. Ergodic Rate Analysis

By substituting the magnitude and phase responses of the reflecting elements into (3), we can write

$$\gamma_R = \frac{P_{\mathrm{B}}\left|h_{\mathrm{BR}}e^{j\theta_{\mathrm{BR}}} + \sum\limits_{l=1}^{L}\sum\limits_{n=1}^{N} \eta_{ln}[\mathbf{h}_{\mathrm{BI}_l}]_n[\mathbf{h}_{\mathrm{I}_l\mathrm{R}}]_n e^{j(\theta_{ln}+\theta_{\mathrm{BI}_l}+\theta_{\mathrm{I}_l\mathrm{R}})}\right|^2}{\sigma_R^2}. \tag{7}$$

It is known that the optimal phase is given by

$$\theta_{ln} = \arg\max \gamma_R = \theta_{\mathrm{BR}} - (\theta_{\mathrm{BI}_l} + \theta_{\mathrm{I}_l\mathrm{R}}), \tag{8}$$

where $0 \le \theta_{ln} < 2\pi$, and $l \in \{1, \ldots, L\}$, $n \in \{1, \ldots, N\}$. Without loss of generality, we set $\eta_{ln} = \eta$, $\forall n, l$. Hence, the maximum SNR at the relay can be further simplified as

$$\gamma_R = \frac{P_{\mathrm{B}}\left|h_{\mathrm{BR}} + \eta\sum\limits_{l=1}^{L}\sum\limits_{n=1}^{N} [\mathbf{h}_{\mathrm{BI}_l}]_n[\mathbf{h}_{\mathrm{I}_l\mathrm{R}}]_n\right|^2}{\sigma_R^2}. \tag{9}$$

Accordingly, (5) can be rewritten as

$$\gamma_U = \frac{P_{\mathrm{R}}\left|h_{\mathrm{RU}} + \tau\sum\limits_{l=1}^{L}\sum\limits_{n=1}^{N} [\mathbf{h}_{\mathrm{RI}_l}]_n[\mathbf{h}_{\mathrm{I}_l\mathrm{U}}]_n\right|^2}{\sigma_U^2}, \tag{10}$$

where $\phi_{ln} = \phi_{\mathrm{RU}} - (\phi_{\mathrm{RI}_l} + \phi_{\mathrm{I}_l\mathrm{U}})$.

A. Tight Upper Bound

The ergodic rate of the proposed system can be found as

$$\mathbb{E}[R] = \frac{1}{2}\mathbb{E}[\log_2(1 + \min\{\gamma_R, \gamma_U\})]. \tag{11}$$

In the presence of the direct link, it is difficult to characterize the exact achievable rate. As such, we resort to an upper bound using the Jensen's inequality, which gives

$$\mathbb{E}[R] \le \frac{1}{2}\log_2(1 + \min\{\mathbb{E}[\gamma_R], \mathbb{E}[\gamma_U]\}). \tag{12}$$

[2] In our narrow-band fading scenario, we assume the maximum delay spread between the direct and the reflected links is less than the symbol period [11].

The remaining task is therefore to obtain the expectation of the SNRs at the relay and user. Now we define the channels through the IRSs as $Y_{\rm BIR} = \eta \sum_{l=1}^{L} \sum_{n=1}^{N} [\mathbf{h}_{{\rm BI}_l}]_n [\mathbf{h}_{{\rm I}_l{\rm R}}]_n$ and $Y_{\rm RIU} = \tau \sum_{l=1}^{L} \sum_{n=1}^{N} [\mathbf{h}_{{\rm RI}_l}]_n [\mathbf{h}_{{\rm I}_l{\rm U}}]_n$. Using (9) and applying the binomial expansion theorem,[3] we get

$$\mathbb{E}[\gamma_R] = \frac{P_{\rm B}}{\sigma_R^2}(\mathbb{E}[h_{\rm BR}^2] + 2\mathbb{E}[h_{\rm BR}]\mathbb{E}[Y_{\rm BIR}] + \mathbb{E}[Y_{\rm BIR}^2]). \quad (13)$$

Based on the property of the Nakagami-m fading model, the expectation of the direct link $h_{\rm BR}$ is given by

$$\mathbb{E}[h_{\rm BR}] = m_{\rm BR}^{-1/2} \frac{\Gamma\left(m_{\rm BR}+\frac{1}{2}\right)}{\Gamma(m_{\rm BR})} d_{\rm BR}^{-\alpha/2}, \quad (14)$$

where $\Gamma(t) = \int_0^\infty x^{t-1}e^{-x}dx$ is the Gamma function. All $\left|[\mathbf{h}_{{\rm BI}_l}]_n [\mathbf{h}_{{\rm I}_l{\rm R}}]_n\right|$, $n \in \{1, 2, \ldots, N\}$ are independent and identically distributed (i.i.d.), based on the central limit theorem (CLT) when each IRS has a sufficiently large number of elements. Following this way, we also have

$$\mathbb{E}[Y_{\rm BIR}] = \eta \sum_{l=1}^{L} \sum_{n=1}^{N} \mathbb{E}[Y_{{\rm BI}_l}]\mathbb{E}[Y_{{\rm I}_l{\rm R}}] = \eta N \sum_{l=1}^{L} \sqrt{\xi_{\rm BIR}}, \quad (15)$$

where

$$\xi_{\rm BIR} = \frac{1}{m_{{\rm BI}_l} m_{{\rm I}_l{\rm R}}} \left[\frac{\Gamma\left(m_{{\rm BI}_l}+\frac{1}{2}\right)\Gamma\left(m_{{\rm I}_l{\rm R}}+\frac{1}{2}\right)}{\Gamma(m_{{\rm BI}_l})\Gamma(m_{{\rm I}_l{\rm R}})} \right]^2. \quad (16)$$

From the mean square value theorem of Dirichlet L-functions [13], by substituting (14) and (15) into (13), the expected value of the received SNR γ_R at the DF relay can be derived as (17) (see top of next page). Similarly, we can also obtain the expected value of the received SNR γ_U at the user as (18) (see next page). As a result, the upper bound for the ergodic rate is obtained as

$$R_{\rm up} = \frac{1}{2}\log_2\left(1 + \min\left\{\mathbb{E}[\gamma_R], \mathbb{E}[\gamma_U]\right\}\right). \quad (19)$$

B. *Optimal IRS Configuration*

A channel coherence block consists of T symbols covering two phases: the channel training and data transmission phases. Specifically, in the training phase, the user sends a sequence of $LN+1$ pilot symbols to the relay, while each IRS properly sets its reflection over time to facilitate the channel estimation, and then the relay transmits $LN+1$ pilot symbols to the BS. In the data transmission phase, the BS transmits its data to the user using the remaining $T-2LN-2$ symbols. Hence, the net ergodic rate of the user can be expressed as

$$\dot{R}_{\rm net} = \frac{T-2LN-2}{2T}\mathbb{E}[\log_2(1 + \min\{\gamma_R, \gamma_U\})]. \quad (20)$$

Note that it is possible to find the optimal IRS configuration that maximizes the ergodic rate. That is,

$$\max_{L,N} \dot{R}_{\rm net}, \ \text{s.t.}\ N \le LN < T. \quad (21)$$

[3]Other methods for analyzing the performance of IRS-aided communication systems can be found in [7] and [12].

For simplicity, we assume that all the IRSs are in the same circular plane centered at the relay. In what follows, (21) can be rewritten as

$$\max_{L,N} S(L,N) \equiv \frac{T-2LN-2}{2T}\log_2(1 + \bar{\gamma}(aLN^2 + bLN + c)),$$
$$\text{s.t.}\ N \le LN < T, \quad (22)$$

in which L and N are positive integers, $\bar{\gamma} = \frac{P_{\rm B}}{\sigma_R^2}$, $a = d_{{\rm BI}_l}^{-\alpha} d_{{\rm I}_l{\rm R}}^{-\alpha} \eta^2 \xi_{\rm BIR}$, $c = d_{\rm BR}^{-\alpha}$, and

$$b = \frac{2\eta\sqrt{\frac{\xi_{\rm BIR}}{m_{\rm BR}}}\frac{\Gamma\left(m_{\rm BR}+\frac{1}{2}\right)}{\Gamma(m_{\rm BR})}}{d_{\rm BR}^{\frac{\alpha}{2}} d_{{\rm BI}_l}^{\frac{\alpha}{2}} d_{{\rm I}_l{\rm R}}^{\frac{\alpha}{2}}} + \frac{\eta^2(1-\xi_{\rm BIR})}{d_{{\rm BI}_l}^{\alpha} d_{{\rm I}_l{\rm R}}^{\alpha}}. \quad (23)$$

The optimization of (22) is unfortunately non-convex due to the two variables L and N. Although it can be solved by an exhaustive search algorithm, this suffers from huge computational complexity. Thus, we resort to optimizing the IRS configuration in a bi-convex way, i.e., with fixed L or N. For a given number of reflecting elements for each IRS, i.e., with a fixed N, we assume that each IRS activates all the reflecting elements. We can then obtain the optimal number of IRSs to maximize the net ergodic rate by

$$L^* = \arg\max_{1 \le L < \frac{T}{N}} S(L). \quad (24)$$

Since $a, b, c > 0$, it is worth noting that $S(L)$ is strictly concave with L. In the interval $(0, \infty)$, it will first increase, reach a unique maximum, and then decrease. Therefore, the numerical solution of the optimal solution can be obtained by Newton's method [14]. The closed-form solution can be obtained at high SNRs using the following theorem.

Theorem 1: As $\bar{\gamma} \to \infty$, (24) has a unique optimal solution

$$L^* = \left\langle \frac{\frac{T}{2}-1}{NW(e(\frac{T}{2}-1)\bar{\gamma}(aN+b))} \right\rangle, \quad (25)$$

where $\langle \cdot \rangle$ denotes the rounding operation, and $W(\cdot)$ is the Lambert's W-function [15].

Proof: See Appendix A. ∎

The number of reflecting elements can also be optimized in a similar way with the fixed number of IRSs.[4]

IV. Numerical Results

In this section, the numerical results are presented to verify the accuracy of the derived expressions, and show the superiority of our proposed system. Under the three-dimensional (3D) Cartesian coordinate system in meter (m), the locations of the BS, relay, and user are set as (0, 0, 10), (40, 0, 10) and (80, 0, 10), respectively[5]. L IRSs are assumed to be located in a circular regime placing in the yz-plane, whose center is 10m away from the relay. In both the IRS-only and DF-only systems, we assume that there is no direct link between the BS

[4]The idea that selecting optimal subsurfaces from a centralized IRS [16] coincides with the one that finding the optimal number of IRSs from multiple distributed IRSs.

[5]The impact of localization of each node will be investigated in our future work.

$$\mathbb{E}[\gamma_R] = \frac{P_{\rm B}}{\sigma_R^2}\left[d_{\rm BR}^{-\alpha} + d_{\rm BR}^{-\alpha/2} d_{{\rm BI}_l}^{-\alpha/2} d_{{\rm I}_l{\rm R}}^{-\alpha/2} 2\eta N \sum_{l=1}^{L} \sqrt{\frac{\xi_{\rm BIR}}{m_{\rm BR}}} \frac{\Gamma\left(m_{\rm BR}+\frac{1}{2}\right)}{\Gamma(m_{\rm BR})} + d_{{\rm BI}_l}^{-\alpha} d_{{\rm I}_l{\rm R}}^{-\alpha} \sum_{l=1}^{L} \eta^2 N((N-1)\xi_{\rm BIR}+1)\right] \tag{17}$$

$$\mathbb{E}[\gamma_U] = \frac{P_{\rm R}}{\sigma_U^2}\left[d_{\rm RU}^{-\alpha} + d_{\rm RU}^{-\alpha/2} d_{{\rm RI}_l}^{-\alpha/2} d_{{\rm I}_l{\rm U}}^{-\alpha/2} 2\tau N \sum_{l=1}^{L} \sqrt{\frac{\xi_{\rm RIU}}{m_{\rm RU}}} \frac{\Gamma\left(m_{\rm RU}+\frac{1}{2}\right)}{\Gamma(m_{\rm RU})} + d_{{\rm RI}_l}^{-\alpha} d_{{\rm I}_l{\rm U}}^{-\alpha} \sum_{l=1}^{L} \tau^2 N((N-1)\xi_{\rm RIU}+1)\right] \tag{18}$$

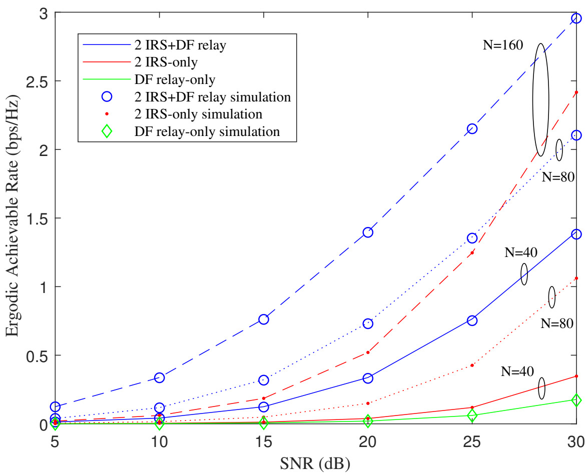

Fig. 2: Ergodic rate versus SNR for various N.

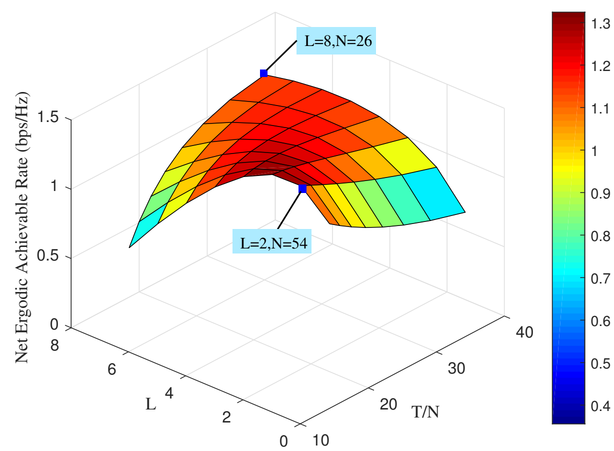

Fig. 4: Net ergodic rate versus L and T/N.

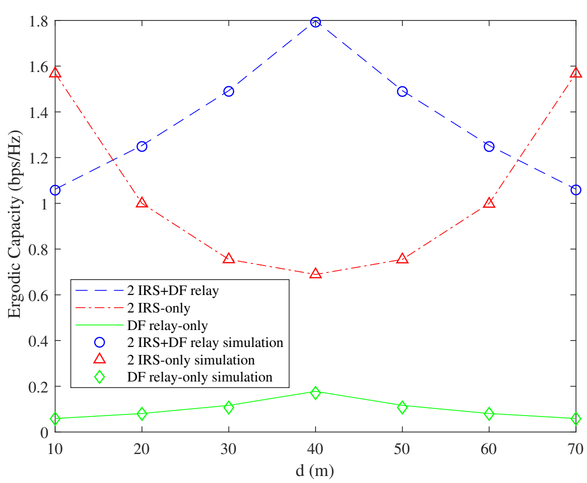

Fig. 3: Ergodic rate versus distance d with $N = 60$.

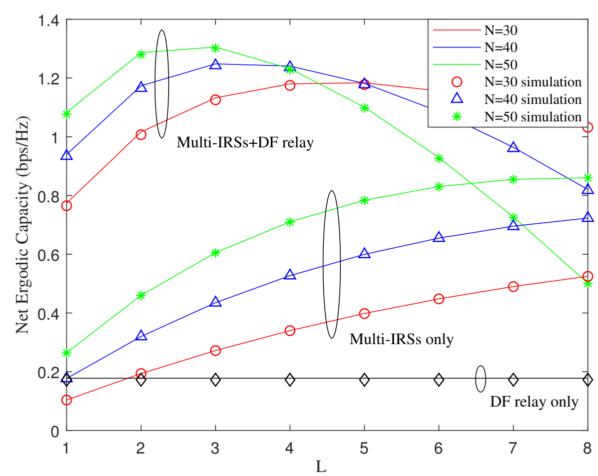

Fig. 5: Net ergodic rate versus L for various N.

and the user, and we constrain the total transmission power at the BS of the IRS-only system by $P = P_R + P_B$. The path loss exponent α is 2.2, and the Nakagami-m fading parameters are all set to 3. Unless stated otherwise, we set $P_B = P_R$, SNR is defined as $\frac{P_B}{\sigma_R^2}$ and the variance of all the noise is 1. For multiple IRSs, the reflection coefficients (η, τ, ζ) are all equal to 0.8. Finally, we give a long channel coherence time $T = 1,000$ to find the optimal L for different values of N.

Results in Fig. 2 compare the ergodic rate of the proposed multi-IRS DF relaying system with two benchmark systems: (1) multi-IRS only without the DF relay and (2) DF relay only without IRSs, all for different values of N and $L = 2$. Both the rate upper bound and the exact results from Monte-Carlo simulations are plotted and compared. As we can see, the upper bound is very tight and no noticeable difference is observed. In addition, the ergodic rate monotonically increases with the number of IRS reflecting elements N. The proposed system also shows a significant gain compared to the benchmarks under the same number of reflecting elements N.

TABLE I: Optimal L for different N

Number of reflecting elements N	30	40	50
Optimal L of the proposed system	5	3	3

Fig. 3 studies the impact of the locations of the IRSs relative to the relay when $L = 2$ and $N = 60$. It is noteworthy that the distances between each IRS and the relay are fixed. It is shown that the multi-IRS DF relaying system and the relay-only system have the peak ergodic rates when the IRSs and the relay are in the middle of the BS and the user. Evidently, the performance of the multi-IRS-only system becomes much better when the IRSs are either nearest to the user or the BS, due to stronger line-of-sights from the BS or to the user.

In Fig. 4, we investigate the impact of the number of IRSs L and the ratio of fixed T to different N on the ergodic rate performance. It is noted that a higher number of N has a lower optimal value of L. On the other hand, we can observe that more IRSs can be employed to improve the rate performance if the channel coherence time is longer.

Finally, the results in Fig. 5 are provided for the ergodic rate for a wide range of L, given different values of N. We can observe that the proposed system has the best rate performance when $L < 6$. Nevertheless, the net ergodic rate gain ceases to increase and will decease if L is too large if N remains fixed. There is an optimal value of L for each N. The optimal values of L for different values of N are given in TABLE I.

V. Conclusion

This letter investigated the achievable rate performance for a DF relaying system with the aid of multiple IRSs. We first derived a very tight upper bound in closed form for the ergodic rate of the proposed system under Nakagami-m fading. Utilizing the analytical results, we proposed to obtain the optimal IRSs configuration that maximizes the net ergodic rate. Results indicated that for a given channel coherence time, fewer IRSs with a larger number of reflecting elements tend to have better net rate performance.

Appendix A
Proof of Theorem 1

Based on (22), the expression of L can be written as

$$S(L) = \left(\frac{1}{2} - \frac{LN+1}{T}\right) \log_2(1 + \bar{\gamma}(aN^2 + bN)L + c)). \tag{A.1}$$

At high SNR, (A.1) can be rewritten as

$$S(L) = \left(\frac{1}{2} - \frac{LN+1}{T}\right) \log_2(\bar{\gamma}(aN^2 + bN)L)). \tag{A.2}$$

Setting the first derivative of (A.2) to 0, we have

$$\ln(\bar{\gamma}(aN^2 + bN)L) = \frac{\frac{T}{2} - LN - 1}{LN}. \tag{A.3}$$

Letting $\ln(\bar{\gamma}(aN^2 + bN)L) = z$, L can be obtained by

$$L = \frac{e^z}{\bar{\gamma}(aN^2 + bN)}. \tag{A.4}$$

Thus, (A.3) can be finally obtained from

$$(1 + z)e^{(1+z)} = e\left(\frac{T}{2} - 1\right)\bar{\gamma}(aN + b). \tag{A.5}$$

Using the definition of the Lambert's W-function in [15], substituting it back into (A.3), and rounding the result to its nearest integer, the optimal configuration of L in (25) is obtained. Hence, the proof is completed.

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